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## Geometric Series

(Gaeaf 2005)

8. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1 - r^n)}{1 - r} .$$

Given that  $|r| < 1$ , write down the sum to infinity of the series. [4]

- (b) The sum to infinity of a geometric series is equal to 4. The sum of the first two terms of the series is 3. Find the common ratio, given that it is positive. [5]

(Haf 2005)

4. The sum of the first two terms of a geometric series is 6.4, and the sum to infinity of the series is 10.

(a) Given that the common ratio is positive, find its value. [5]

(b) Find, correct to three decimal places, the sum of the first eleven terms of the series. [3]

(Gaeaf 2006)

4. (a) A geometric series has first term  $a$  and common ratio  $r$ . Write down the  $n$ th term of the series and prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1 - r^n)}{1 - r} . \quad [4]$$

(b) The fourth term of a geometric series is 2 and the seventh term is 54.

(i) Find the common ratio of the series.

(ii) Find the sum of the first ten terms of the series, giving your answer correct to one decimal place.

(iii) Find the least value of  $n$  for the  $n$ th term to exceed 125 000. [10]

(Haf 2006)

5. The second term of a geometric series is nine times the fourth term of the series.

(a) Find the possible values of the common ratio. [4]

(b) Given that the common ratio is positive and the sum to infinity of the series is 12, find the third term of the series. [3]

(Gaeaf 2007)

3. (a) A geometric series has first term  $a$  and common ratio  $r$ . Write down the  $n$ th term and prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Given that  $|r| < 1$ , write down the sum to infinity of the series. [5]

- (b) The sum of the first term and the second term of a geometric series is equal to twice the sum of the second term and the third term of the series.
- (i) Given that the common ratio of the series is positive, find the value of the common ratio. [4]
- (ii) The sum to infinity of the series is 12. Find, correct to two decimal places, the sum of the first eight terms of the series. [4]

(Haf 2007)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Given that  $|r| < 1$ , write down the sum to infinity of the series. [4]

- (b) The sum to infinity of a geometric series with first term  $a$  and common ratio  $r$  is 10. The sum to infinity of a second geometric series with first term  $a$  and common ratio  $2r$  is 15.
- (i) Find the value of  $r$ . [4]
- (ii) Find the sum of the first four terms of the **first** series, giving your answer correct to two decimal places. [3]

(Gaeaf 2008)

4. A geometric series has first term  $a$  and common ratio  $r$ . The fifth term of the geometric series is 135 and the eighth term is 5.
- (a) Show that  $r = \frac{1}{3}$  and find the value  $a$ . [5]
- (b) Find the sum to infinity of the series. [2]

(Haf 2008)

5. A geometric series has first term  $a$  and common ratio  $r$ . The sum of the first two terms of the geometric series is 7.2. The sum to infinity of the series is 20. Given that  $r$  is positive, find the values of  $r$  and  $a$ . [6]

(Gaeaf 2009)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r} . \quad [3]$$

- (b) Find the sum of the first eighteen terms of the geometric series

$$10 + 9 + 8 \cdot 1 + \dots$$

Give your answer correct to the nearest whole number. [3]

- (c) The second term of another geometric series is  $-4$ . The sum to infinity of the series is 9.

- (i) Show that  $r$ , the common ratio of the series, satisfies the equation

$$9r^2 - 9r - 4 = 0.$$

- (ii) Find the value of  $r$ , giving a reason for your answer. [6]

(Haf 2009)

5. (a) The ninth and tenth terms of a geometric series are 36 and 108 respectively. Find the seventh term of the geometric series. [3]

- (b) Another geometric series has first term  $a$  and common ratio  $r$ . The second term of this geometric series is 9 and the sum to infinity of the series is 48.

- (i) Show that  $r$  satisfies the equation

$$16r^2 - 16r + 3 = 0.$$

- (ii) Find the two possible values for  $r$  and the corresponding values of  $a$ . [6]

(Gaeaf 2010)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r} . \quad [3]$$

- (b) The common ratio of a geometric series is positive. The sum of the first four terms of the series is  $73 \cdot 8$ . The sum to infinity of the series is 125. Find the common ratio and the first term of the geometric series. [6]

10. The  $n$ th term of a number sequence is denoted by  $t_n$ . The  $(n + 1)$ th term of the sequence satisfies

$$t_{n+1} = 2t_n + 1,$$

for all positive integers  $n$ . Given that  $t_4 = 63$ ,

- (a) evaluate  $t_1$ , [2]
- (b) without carrying out any further calculations, explain why 6043582 cannot be one of the terms of this number sequence. [1]

(Haf 2010)

6. (a) Find the sum to infinity of the geometric series  
 $40 - 24 + 14.4 - \dots$  [3]
- (b) Another geometric series has first term  $a$  and common ratio  $r$ . The fourth term of this geometric series is 8. The sum of the third, fourth and fifth terms of the series is 28.

(i) Show that  $r$  satisfies the equation

$$2r^2 - 5r + 2 = 0.$$

(ii) Given that  $|r| < 1$ , find the value of  $r$  and the corresponding value of  $a$ . [6]

(Gaeaf 2011)

5. (a) The second term of a geometric series is 6 and the fifth term is 384.
- (i) Find the common ratio of the series.
- (ii) Find the sum of the first eight terms of the geometric series. [6]
- (b) The first term of another geometric series is 5 and the common ratio is 1.1.
- (i) The  $n$ th term of this series is 170, correct to the nearest integer. Find the value of  $n$ .
- (ii) Dafydd, who has been using his calculator to investigate various properties of this geometric series, claims that the sum to infinity of the series is 940. Explain why this result cannot possibly be correct. [5]

(Haf 2011)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}. \quad [3]$$

- (b) The sum to infinity of a geometric series is equal to four times the first term of the series.
- (i) Find the value of the common ratio of the series.
- (ii) Given that the sum of the first two terms of the series is 35, find the sum of the first nine terms of the series. Give your answer correct to the nearest whole number. [6]

(Gaeaf 2012)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}. \quad [3]$$

- (b) The sum of the first two terms of a geometric series is  $25.2$ . The sum to infinity of the series is  $30$ . Given that the common ratio is positive, find the common ratio and first term of this geometric series. [6]

(Haf 2012)

5. A geometric series has first term  $a$  and common ratio  $r$ . The sum of the first and second terms of the series is  $72$ . The sum of the first and third terms of the series is  $120$ .

- (a) Show that  $r$  satisfies the equation

$$3r^2 - 5r - 2 = 0. \quad [4]$$

- (b) Given that  $|r| < 1$ , find the value of  $r$  and the sum to infinity of the series. [5]

(Gaeaf 2013)

5. (a) The  $p$ th term of a geometric series is  $16$ . The  $(p + 1)$ th term of this series is  $24$ . Find the  $(p + 4)$ th term of the series. [3]

- (b) The sum of the first three terms of another geometric series is  $22.8$ . The sum to infinity of the series is  $18.75$ . Find the common ratio and the first term of this geometric series. [6]

(Haf 2013)

5. (a) Find the sum of the first eighteen terms of the geometric series

$$100 + 80 + 64 + \dots$$

Give your answer correct to the nearest whole number. [3]

- (b) The second term of a geometric series is  $-20$ . The sum to infinity of the series is  $64$ .

- (i) Show that  $r$ , the common ratio of the series, satisfies the equation

$$16r^2 - 16r - 5 = 0.$$

- (ii) Find the value of  $r$ , giving a reason for your answer. [6]

(Gaeaf 2014)

4. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}. \quad [3]$$

- (b) The fourth term of a geometric series is  $-108$  and the seventh term is  $4$ .

(i) Find the common ratio of the series.

(ii) Find the sum to infinity of the series. [6]

(Haf 2014)

5. A geometric series has first term  $a$  and common ratio  $r$ . The sum of the second and third terms of the series is  $-216$ . The sum of the fifth and sixth terms of the series is  $8$ .

(a) Prove that  $r = -\frac{1}{3}$ . [5]

(b) Find the sum to infinity of the series. [3]

(Haf 2015)

5. (a) The eighth and ninth terms of a geometric series are  $576$  and  $2304$  respectively. Find the fifth term of the geometric series. [3]

- (b) Another geometric series has first term  $a$  and common ratio  $r$ . The third term of this geometric series is  $24$ . The sum of the second, third and fourth terms of the series is  $-56$ .

(i) Show that  $r$  satisfies the equation

$$3r^2 + 10r + 3 = 0.$$

(ii) Given that  $|r| < 1$ , find the value of  $r$  and the sum to infinity of the series. [8]

(Haf 2016)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms of the series is given by

$$S_n = \frac{a(1-r^n)}{1-r}. \quad [3]$$

- (b) The sum of the first five terms of a geometric series is  $275$ . The sum to infinity of the series is  $243$ . Find the common ratio and the first term of the geometric series. [6]

(Haf 2017)

5. A rich businessman makes one donation per year to a certain charity. He starts by donating £100 in the first year. In each subsequent year, the value of the donation is 1.2 times the value of the previous year's donation.
- (a) Find the value of the businessman's donation in the 12<sup>th</sup> year. Give your answer correct to the nearest pound. [2]
- (b) After receiving the  $n$ th donation, the charity's treasurer calculates that over the years, the businessman has donated a **total** of £15 474, correct to the nearest pound. Find the value of  $n$ . [5]

(Haf 2018)

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms of the series is given by

$$S_n = \frac{a(1-r^n)}{1-r} . \quad [3]$$

- (b) The sum of the first and third terms of a geometric series is 340. The sum of the fourth and sixth terms of the series is 73.44.
- (i) Prove that  $r = 0.6$ , where  $r$  denotes the common ratio of the series.
- (ii) Find the sum to infinity of the series. [8]