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Arithmetic Series

(Gaeaf 2005)

5. In an arithmetic series the sum of the first term and the fifth term is zero. The thirteenth term is 20.
- (a) Find the first term and the common difference of the series. [5]
- (b) Calculate the sum of the first twenty terms of the series. [2]

(Haf 2005)

3. (a) An arithmetic series has first term a and common difference d . Write down the n th term and prove that the sum of the first n terms is given by

$$S_n = \frac{n}{2} [2a + (n-1)d] . \quad [4]$$

- (b) The seventh term of an arithmetic series is twice the third term. The sum of the first ten terms of the series is 195.
- (i) Find the common difference of the series.
- (ii) Find the sum of the first sixty terms. [7]

(Gaeaf 2006)

5. The sum of the first two terms of an arithmetic series is 3. The eighth term of the arithmetic series is 47.

Find

- (a) the first term and the common difference of the series, [4]
- (b) the sum of the first twenty terms of the series. [2]

(Haf 2006)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]. \quad [3]$$

- (b) The sum of the first twenty terms of an arithmetic series is 540 and the sum of the first thirty terms of the series is 1260.
- (i) Find the first term and the common difference of the series.
- (ii) Calculate the **50th term** of the series. [6]

(Gaeaf 2007)

4. In an arithmetic series, the eighth term is twice the third term. The twentieth term of the series is 11. Find the common difference and the first term of the series. [5]

(Haf 2007)

4. The third term of an arithmetic series is four times the sixth term of the series. The sum of the first twenty terms of the series is 350.

(a) Find the first term and the common difference of the series. [6]

(b) Given that the n th term of the series is 125, find the value of n . [2]

(Gaeaf 2008)

3. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]. \quad [3]$$

(b) Find an expression, in terms of n , for the sum of the first n terms of the arithmetic series

$$1 + 3 + 5 + \dots$$

Simplify your answer. [2]

(c) The twentieth term of an arithmetic series is 98 and the sum of the first twenty terms of the series is 1010. Find the first term and the common difference of the series. [4]

(Haf 2008)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] . \quad [3]$$

(b) The sum of the first ten terms of an arithmetic series is 320. The sum of the twelfth and sixteenth terms of the series is 166. Find the first term and the common difference of the series. [5]

(Gaeaf 2009)

4. (a) The thirteenth term of an arithmetic series is 51. The ninth term of the series is five times the second term. Find the first term and common difference of the arithmetic series. [5]

(b) The first term of another arithmetic series is 5 and the twentieth term is 62. Find the sum of the first twenty terms of this arithmetic series. [2]

(Haf 2009)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]. \quad [3]$$

- (b) The eighth term of an arithmetic series is 46. The sum of the first nine terms of the series is 225. Find the first term and the common difference of the series. [4]

- (c) Find an expression, in terms of n , for the sum of the first n terms of the arithmetic series

$$3 + 7 + 11 + 15 + \dots$$

Simplify your answer. [3]

(Gaeaf 2010)

4. The sum of the first eight terms of an arithmetic series is 124 and the sum of the first twenty terms of the series is 910.

- (a) Find the first term and common difference of the series. [5]

- (b) The n th term of the series is 183. Find the value of n . [2]

(Haf 2010)

5. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]. \quad [3]$$

- (b) The first term of an arithmetic series is 4 and the common difference is 2. The sum of the first n terms of the arithmetic series is 460.

Write down an equation satisfied by n . Hence find the value of n . [3]

- (c) The fifth term of another arithmetic series is 9. The sum of the sixth term and the tenth term of this series is 42. Find the first term and the common difference of the arithmetic series. [5]

(Gaeaf 2011)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]. \quad [3]$$

- (b) The eighth term of an arithmetic series is 28. The sum of the first twenty terms of the series is 710. Find the first term and common difference of the arithmetic series. [5]

- (c) The first term of another arithmetic series is -3 and the fifteenth term is 67. Find the sum of the first fifteen terms of this arithmetic series. [2]

(Haf 2011)

4. (a) The sum of the first fifteen terms of an arithmetic series is 780. The sum of the second, fourth and tenth terms of the series is 100. Find the first term and the common difference of the series. [5]
- (b) The p th term of another arithmetic series is 1023. The $(p + 4)$ th term of this series is 1059. Find the $(p + 7)$ th term of the series. [3]

(Gaeaf 2012)

4. The fifteenth term of an arithmetic series is seven times the fifth term. The sum of the first eleven terms of the series is 88.
- (a) Find the first term and common difference of the arithmetic series. [6]
- (b) Given that the n th term of the series is 143, find the value of n . [2]

(Haf 2012)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d]. \quad [3]$$

- (b) The sum of the third, fourth and tenth terms of an arithmetic series is 79. The sum of the sixth and seventh terms of the series is 61. Find the first term and the common difference of the series. [4]
- (c) Find an expression, in terms of n , for the sum of the first n terms of the arithmetic series

$$15 + 13 + 11 + 9 + \dots$$

Simplify your answer. [3]

(Gaeaf 2013)

4. (a) The first term of an arithmetic series is 1 and the common difference is 4.
- (i) Show that the n th term of the arithmetic series is $4n - 3$.
- (ii) The sum of the first n terms of this series is given by

$$S_n = 1 + 5 + \dots + (4n - 7) + (4n - 3).$$

Prove from first principles, without using the formula for the sum of the first n terms, that

$$S_n = n(2n - 1). \quad [4]$$

- (b) The sum of the first ten terms of another arithmetic series is 55. The sum of the fourth, seventh and ninth terms of the series is 27. Find the first term and the common difference of this arithmetic series. [5]

(Haf 2013)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]. \quad [3]$$

- (b) The sum of the first ten terms of an arithmetic series is 115. The sum of the **next** four terms of this series is 130. Find the first term and the common difference of the arithmetic series. [5]

(Gaeaf 2014)

3. (a) The sum of the third and eighth terms of an arithmetic series is zero. The sum of the fifth, seventh and tenth terms of the series is 22. Find the first term and the common difference of the series. [4]

- (b) The first term of another arithmetic series is 9 and the common difference is 2. The sum of the first $2n$ terms of this arithmetic series is 3 times the sum of the first n terms of the series. Find the value of n . [5]

(Haf 2014)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]. \quad [3]$$

- (b) The first term of an arithmetic series is 3 and the common difference is 2. The sum of the first n terms of the series is 360.

Write down an equation satisfied by n . Hence find the value of n . [3]

- (c) The tenth term of another arithmetic series is seven times the third term. The sum of the eighth and ninth terms of the series is 80. Find the first term and common difference of this arithmetic series. [4]

(Haf 2015)

4. (a) The first term of an arithmetic series is 4 and the common difference is 6.

(i) Show that the n th term of the arithmetic series is $6n - 2$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 4 + 10 + \dots + (6n - 8) + (6n - 2).$$

Without using the formula for the sum of the first n terms of an arithmetic series, **prove** that

$$S_n = n(3n + 1). \quad [4]$$

- (b) The tenth term of another arithmetic series is four times the fifth term. The sum of the first fifteen terms of the series is 210.

(i) Find the first term and common difference of this arithmetic series.

(ii) Given that the k th term of the series is 200, find the value of k .

[6]

(Haf 2016)

4. (a) Gwilym has decided to run in a half marathon race. In order to get himself fit, he devises a training programme whereby he runs around his local track each day, gradually increasing the distance he runs from day to day. On the first day, he runs 6 laps of the track and subsequently, on any given day, he runs 2 laps further than he did on the day before.
- (i) How many laps does he run on the 20th day of his programme?
- (ii) After how many days will the total number of laps he has run since the beginning of his training programme be equal to 750? [6]

(b) The n th term of an **arithmetic** series is denoted by t_n . It is known that

$$t_{12} + t_{13} = 50.$$

- (i) **Write down** the value of $t_{11} + t_{14}$. [1]
- (ii) Find the sum of the first twenty-four terms of this arithmetic series. [2]

(Haf 2017)

4. (a) An arithmetic series has first term a and common difference d . Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]. \quad [3]$$

- (b) The sum of the first eight terms of an arithmetic series is 156 and the sum of the first sixteen terms of the series is 760. Find the first term and common difference of this series. [4]
- (c) The p th term of another arithmetic series is 2057. The $(p + 5)$ th term of this series is 2102. Find the $(p + 8)$ th term of the series. [3]

(Haf 2018)

4. Dafydd enters into an agreement with a finance company to provide him with a loan to buy a car. Under the terms of the agreement, he is to repay a total of £3900 over a period of two years by making 24 monthly repayments. The first repayment is £ P , and in each subsequent month, the value of the repayment for that month is £ x **less** than the repayment for the previous month. Given that Dafydd's eighth repayment is £185, find the value of P and the value of x . [5]