

**CYD-BWYLLGOR ADDYSG CYMRU
WELSH JOINT EDUCATION COMMITTEE**

**Tystysgrif Addysg Gyffredinol
Safon Uwch/Uwch Gyfrannol**

**General Certificate of Education
Advanced Level/Advanced Subsidiary**

MATHEMATEG

LLYFRYN FFORMIWLÂU

**CBAC
WJEC**

Cyhoeddwyd 2004

Mathemateg Bur

Mesureg

$$\text{Arwynebedd arwyneb sffêr} = 4\pi r^2$$

$$\text{Arwynebedd arwyneb crwm côn} = \pi r \times \text{uchder oledd}$$

Cyfrës Rhifydddeg

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2} n(a + l) = \frac{1}{2} n [2a + (n - 1)d]$$

Cyfrës Geometrig

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ ar gyfer } |r| < 1$$

Symiannau

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

Cyfrës Fionomial

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{lle mae } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithmau ac esbonyddolion

$$e^{x \ln a} = a^x$$

Rhifau Cymhlyg

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\text{Rhoddir gwreiddiau } z^n = 1 \text{ gan } z = e^{\frac{2\pi k i}{n}}, \text{ ar gyfer } k = 0, 1, 2, \dots, n-1$$

Cyfrs Maclaurin a Chyfrs Taylor

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^r}{r!} f^{(r)}(a) + \dots$$

$$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!} f''(a) + \dots + \frac{x^r}{r!} f^{(r)}(a) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{ar gyfer pob } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{ar gyfer pob } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{ar gyfer pob } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{ar gyfer pob } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{ar gyfer pob } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

Ffwythiannau Hyperbolig

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln \{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln \{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Geometreg Gyfesurynnol

Conigau

| | Elips | Parabola | Hyperbola | Hyperbola Petryal |
|-----------------|---|---------------|--|--------------------------------|
| Ffurff Safonol | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | $y^2 = 4ax$ | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $xy = c^2$ |
| Ffurff Bamedrig | $(a \cos \theta, b \sin \theta)$ | $(at^2, 2at)$ | $(a \sec \theta, b \tan \theta)$ $(\pm a \cosh \theta, b \sinh \theta)$ | $(ct, \frac{c}{t})$ |
| Echreiddiad | $e < 1$ $b^2 = a^2(1 - e^2)$ | $e = 1$ | $e > 1$ $b^2 = a^2(e^2 - 1)$ | $e = \sqrt{2}$ |
| Ffocysau | $(\pm ae, 0)$ | $(a, 0)$ | $(\pm ae, 0)$ | $(\pm\sqrt{2}c, \pm\sqrt{2}c)$ |
| Cyfeirliniau | $x = \pm \frac{a}{e}$ | $x = -a$ | $x = \pm \frac{a}{e}$ | $x + y = \pm\sqrt{2}c$ |
| Asymptotau | dim | dim | $\frac{x}{a} = \pm \frac{y}{b}$ | $x = 0, y = 0$ |

Unfathiannau Trigonometrïg

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\text{Ar gyfer } t = \tan \frac{1}{2}A: \quad \sin A = \frac{2t}{1+t^2}, \quad \cos A = \frac{1-t^2}{1+t^2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Fectorau

$$\text{Cydran } \mathbf{a} \text{ yng nghyfeiriad } \mathbf{b} \text{ yw } \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

$$\text{Y pwynt sy'n rhannu } AB \text{ yn y gymhareb } \lambda : \mu \text{ yw } \frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$$

Trawsffurfiadau Matrics

$$\text{Cylchdro gwrthglocwedd trwy } \theta \text{ o amgylch } O: \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$\text{Adlewyrchiad yn y llinell } y = (\tan \theta)x: \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

Differu

Ffwythiant

$$\frac{f(x)}{g(x)}$$

$$\tan x$$

$$\sec x$$

$$\cot x$$

$$\operatorname{cosec} x$$

$$\sin^{-1} x$$

$$\cos^{-1} x$$

$$\tan^{-1} x$$

$$\sinh x$$

$$\cosh x$$

$$\tanh x$$

$$\sinh^{-1} x$$

$$\cosh^{-1} x$$

$$\tanh^{-1} x$$

Deilliad

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\sec^2 x$$

$$\sec x \tan x$$

$$-\operatorname{cosec}^2 x$$

$$-\operatorname{cosec} x \cot x$$

$$\frac{1}{\sqrt{1-x^2}}$$

$$-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{1+x^2}$$

$$\cosh x$$

$$\sinh x$$

$$\operatorname{sech}^2 x$$

$$\frac{1}{\sqrt{1+x^2}}$$

$$\frac{1}{\sqrt{x^2-1}}$$

$$\frac{1}{1-x^2}$$

Integru (+ cysonyn; a > 0 lle'n berthnasol)

Ffwythiant

Integrol

$\tan x$

$\ln|\sec x|$

$\cot x$

$\ln|\sin x|$

$\operatorname{cosec} x$

$-\ln|\operatorname{cosec} x + \cot x| = \ln\left|\tan\left(\frac{1}{2}x\right)\right|$

$\sec x$

$\ln|\sec x + \tan x| = \ln\left|\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right|$

$\sec^2 x$

$\tan x$

$\sinh x$

$\cosh x$

$\cosh x$

$\sinh x$

$\tanh x$

$\ln \cosh x$

$\frac{1}{\sqrt{a^2 - x^2}}$

$\sin^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$

$\frac{1}{a^2 + x^2}$

$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$

$\frac{1}{\sqrt{x^2 - a^2}}$

$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$

$\frac{1}{\sqrt{a^2 + x^2}}$

$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\{x + \sqrt{x^2 + a^2}\}$

$\frac{1}{a^2 - x^2}$

$\frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$

$\frac{1}{x^2 - a^2}$

$\frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$

$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Arwynebedd sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{cyfesurynnau pegynol})$$

Hyd arc

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cyfesurynnau Cartesaidd})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{ffurf bamedrig})$$

Arwynebedd arwyneb cylchdro

$$S_x = 2\pi \int y ds = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$S_y = 2\pi \int x ds = 2\pi \int x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Mathemateg Rhifiadol

Integru rhifiadol

Rheol y trapesium: $\int_a^b y dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, lle mae $h = \frac{b-a}{n}$

Rheol Simpson: $\int_a^b y dx \approx \frac{1}{3} h \{(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\}$,

lle mae $h = \frac{b-a}{n}$ ac n yn eilrif

Datrys Hafaliadau'n Rhifiadol

Iteriad Newton-Raphson ar gyfer datrys $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Mecaneg

Mudiant mewn cylch

Cyflymder ardraws: $v = r\dot{\theta} = \omega r$

Cyflymiad Rheiddiol: $-r\dot{\theta}^2 = -\frac{v^2}{r} = -\omega^2 r$

Craidd Mâs Gwrthrychau Unffurf

Laminâu Trionglog: $\frac{2}{3}$ ar hyd y llin ganol o'r fertig

Tebygolrwydd ac Ystadegaeth

Tebygolrwydd

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

$$\text{Theorem Bayes: } P(A_j | B) = \frac{P(A_j)P(B | A_j)}{\sum P(A_i)P(B | A_i)}$$

Dosraniadau arwahanol

Ar gyfer hapnewidyn arwahanol X yn cymryd gwerthoedd x_i gyda thebygolrwyddau p_i

$$\text{Disgwyliant (cymedr): } E(X) = \mu = \sum x_i p_i$$

$$\text{Amrywiant: } \text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$$

$$\text{Ar gyfer ffwythiant } g(X): E(g(X)) = \sum g(x_i) p_i$$

Dosraniadau arwahanol safonol:

| Dosraniad X | $P(X = x)$ | Cymedr | Amrywiant |
|-----------------------|-------------------------------------|-----------|-----------|
| Binomial $B(n, p)$ | $\binom{n}{x} p^x (1-p)^{n-x}$ | np | $np(1-p)$ |
| Poisson $Po(\lambda)$ | $e^{-\lambda} \frac{\lambda^x}{x!}$ | λ | λ |

Dosraniadau di-dor

Ar gyfer hapnewidyn di-dor X gyda ffwythiant dwysedd tebygolrwydd f

$$\text{Disgwyliant (cymedr): } E(X) = \mu = \int x f(x) dx$$

$$\text{Amrywiant: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{Ar gyfer ffwythiant } g(X): E(g(X)) = \int g(x) f(x) dx$$

$$\text{Ffwythiant dosraniad cronnus: } F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

Dosraniadau di-dor safonol:

| Dosraniad X | Ffwythiant Dwysedd Tebygolrwydd (P.D.F) | Cymedr | Amrywiant |
|--|---|--------------------|-----------------------|
| Unffurf (Petryal) ar $[a, b]$ $U[a, b]$ | $\frac{1}{b-a}$ | $\frac{1}{2}(a+b)$ | $\frac{1}{12}(b-a)^2$ |
| Normal $N(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | μ | σ^2 |

Algebra Disgwyliant

Ar gyfer hapnewidynnau annibynnol X ac Y

$$E(XY) = E(X)E(Y), \quad \text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Dosraniadau samplu

Ar gyfer hapsampl X_1, X_2, \dots, X_n o n arsylw annibynnol o ddosraniad â chymedr μ ac amrywiant σ^2

$$\text{Mae } \bar{X} \text{ yn amcangyfrifyn diduedd ar gyfer } \mu, \text{ gyda } \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

$$\text{Mae } S^2 \text{ yn amcangyfrifyn diduedd ar gyfer } \sigma^2, \text{ lle mae } S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$$

Ar gyfer hapsampl o n arsylw o $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

$$\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{(n-1)}$$

Os mai X yw'r nifer o lwyddiannau a arsylwir mewn n prawf Bernoulli annibynnol y mae p yn debygolrwydd llwyddiant ym mhob un ohonynt, ac $Y = \frac{X}{n}$, yna mae

$$E(Y) = p \quad \text{a} \quad \text{Var}(Y) = \frac{p(1-p)}{n}$$

Ar gyfer hapsampl o n_x arsylw o $N(\mu_x, \sigma_x^2)$ ac, yn annibynnol, hapsampl o n_y arsylw o $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

Dull Sgwariau Lleiaf

Os yw $y_i = \alpha + \beta x_i + e_i, i = 1, 2, \dots, n$,

yna amcangyfrifon swm sgwariau lleiaf β a α yn ôl eu trefn, yw

$$b = S_{xy} / S_{xx}, \quad a = \bar{y} - b\bar{x},$$

$$\text{lle mae } S_{xy} \equiv \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - (\sum x_i)(\sum y_i) / n$$

$$S_{xx} \equiv \sum (x_i - \bar{x})^2 = \sum x_i^2 - (\sum x_i)^2 / n$$

Os yw e_1, e_2, \dots, e_n yn annibynnol a phob un yn cael ei ddosrannu fel $N(0, \sigma^2)$, yna

(i) mae a yn arsylw o $N\left\{\alpha, \frac{\sigma^2 \sum x_i^2}{nS_{xx}}\right\}$,

(ii) mae b yn arsylw o $N\left\{\beta, \frac{\sigma^2}{S_{xx}}\right\}$

(iii) Mae $\hat{y}_0 = a + bx_0$, amcangyfrif swm sgwariau lleiaf $y_0 = \alpha + \beta x_0$,

$$\text{yn arsylw o } N\left\{y_0, \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]\right\}$$