

973/01

MATHEMATICS C1

Pure Mathematics

P.M. TUESDAY, 10 January 2006

(1½ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A , B , C have coordinates $(-2, -3)$, $(6, 1)$ and $(k, 3)$ respectively. The line AB is perpendicular to BC .

(a) Find the gradient of AB . [2]

(b) Show that $k = 5$. [3]

(c) The line L is parallel to BC and passes through A . Find the equation of L . [2]

(d) The line L intersects the y -axis at D . Calculate the length of CD . [3]

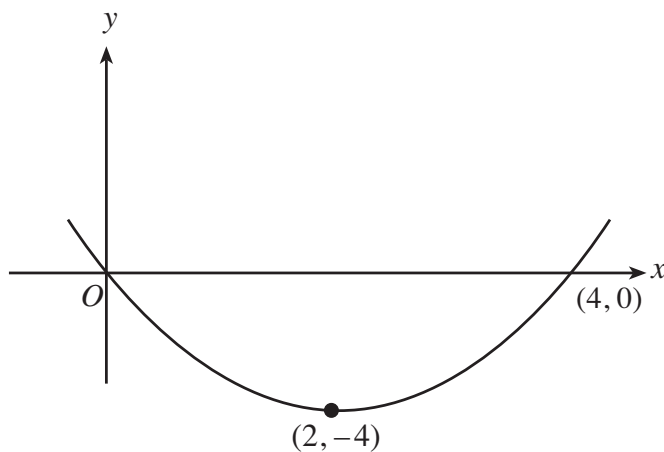
2. (a) Simplify the following.

$$\sqrt{48} + \sqrt{27} - \frac{6}{\sqrt{3}} \quad [4]$$

(b) Simplify $\frac{2 + \sqrt{7}}{3 + \sqrt{7}}$, expressing your answer in surd form. [4]

3. Find the equation of the normal to the curve $y = 4x^2 - 7x + 2$ at the point $(2, 4)$. [4]

4.



The diagram shows the graph of $y = f(x)$. The curve passes through the origin, the point $(4, 0)$ and has a minimum point at $(2, -4)$. Sketch on separate diagrams the graphs of

(a) $y = -f(x)$, [2]

(b) $y = f(x - 2)$, [3]

in each case giving the coordinates of the points of intersection of the graph with the x -axis and the coordinates of the stationary point.

5. Given that the quadratic equation

$$(k + 2)x^2 + 4x + k + 5 = 0$$

has no real roots, show that

$$k^2 + 7k + 6 > 0.$$

Find the range of values of k satisfying this inequality.

[7]

6. (a) Given that when the polynomial $ax^3 - x^2 - 7x + 6$ is divided by $x - 2$ the remainder is 4, show that $a = 2$. [2]

(b) Solve the equation $2x^3 - x^2 - 7x + 6 = 0$. [5]

7. (a) Using the binomial theorem, expand $(3x + 2)^3$, simplifying each term of the expansion. [3]

(b) In the binomial expansion of $(1 + 2x)^n$ the coefficient of x^2 is twice the coefficient of x . Given that $n > 0$, find the value of n . [4]

8. (a) Given that $y = 2x^2 - 5x + 3$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = \frac{a}{x} + 2x^{\frac{3}{2}}$ and $\frac{dy}{dx} = 7$ when $x = 4$, find the value of the constant a . [4]

9. (a) Express $23 + 6x - x^2$ in the form $b - (x - a)^2$, where the constants a and b are to be determined. Hence find the greatest value of $23 + 6x - x^2$ and the corresponding value of x . [4]

(b) Use the results found in (a) to deduce the least value of $\frac{1}{30 + 6x - x^2}$. [2]

10. The curve C has equation

$$y = 2 + 6x^2 - 2x^3.$$

(a) Find the coordinates and the nature of each of the stationary points of C . [7]

(b) Sketch C , indicating clearly the nature of each of the stationary points. [3]

(c) State clearly, giving a reason, the number of real roots of the equation

$$2 + 6x^2 - 2x^3 = 0. [2]$$