C1 Haf 2005

① $A = (1, 7)$ $B = (5, -1)$ $C = (8, 3)$ $D = (6, 7)$

(a) Graddiant $AB = \frac{y_2 - y_1}{x_2 - x_1}$ Graddiant $CD = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - 7}{5 - 1}$ $= \frac{7 - 3}{6 - 8}$

$= \frac{-8}{4}$ $= \frac{4}{-2}$

$= -2$ $= -2$

Felly mae AB a CD yn baralel gan bod eu graddiant yn hafal.

(b) Hafaliad AB: $y - y_1 = m(x - x_1)$

$y - 7 = -2(x - 1)$

$y - 7 = -2x + 2$

$y = -2x + 9$ —— ①

(c) Graddiant $L = \frac{1}{2}$ (negatif cilydd -2).

Hafaliad L: $y - y_1 = m(x - x_1)$

$y - 7 = \frac{1}{2}(x - 6)$

$2y - 14 = x - 6$

$0 = x - 2y - 6 + 14$

$0 = x - 2y + 8$

$x - 2y + 8 = 0$ ✓ —— ②

(ch) Yn amnewid am y o ① i ②:

$x - 2(-2x + 9) + 8 = 0$

$x + 4x - 18 + 8 = 0$

$5x - 10 = 0$

$5x = 10$

$x = 2$

Yn amnewid yn ôl i ①:

$y = -2(2) + 9$

$y = -4 + 9$

$y = 5$

Felly $E = (2, 5)$

(d) F= canolbwynt AB $A=(1,7)$ $B=(5,-1)$

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+5}{2}, \frac{7+-1}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{6}{2}\right)$$

$$= (3,3)$$

$E=(2,5)$ $F=(3,3)$

Hyd EF $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(3-2)^2+(3-5)^2}$$

$$= \sqrt{1^2+(-2)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5} \text{ uned}$$

(2) (a) $\sqrt{45}+\sqrt{80}-\sqrt{125} = \sqrt{9\times5}+\sqrt{16\times5}-\sqrt{25\times5}$

$$= 3\sqrt{5}+4\sqrt{5}-5\sqrt{5}$$

$$= 2\sqrt{5}$$

(b) $\frac{6+\sqrt{2}}{2+\sqrt{2}} = \frac{(6+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$

$$= \frac{12-6\sqrt{2}+2\sqrt{2}-2}{4-2\sqrt{2}+2\sqrt{2}-2}$$

$$= \frac{10-4\sqrt{2}}{2}$$

$$= 5-2\sqrt{2}$$

③ (a) $f(x) = 3x^3 + 5x^2 + ax - 4$

$f(1) = 0$

Felly $0 = 3(1)^3 + 5(1)^2 + a(1) - 4$

$0 = 3 + 5 + a - 4$

$0 = 4 + a$

$\underline{a = -4}$ ✓

(b) $f(x) = 3x^3 + 5x^2 - 4x - 4$

Mae $x-1$ yn ffactor o $f(x)$

$$
\begin{array}{r}
3x^2 + 8x + 4 \\
x-1 \;\overline{)\; 3x^3 + 5x^2 - 4x - 4} \\
3x^3 - 3x^2 \\
\hline
8x^2 - 4x - 4 \\
8x^2 - 8x \\
\hline
4x - 4 \\
4x - 4 \\
\hline
\end{array}
$$

Patrys: $3x^3 + 5x^2 - 4x - 4 = 0$

$(x-1)(3x^2 + 8x + 4) = 0$

$(x-1)(3x + 2)(x + 2) = 0$

Unai $x - 1 = 0$ neu $3x + 2 = 0$ neu $x + 2 = 0$

$\underline{x = 1}$ $3x = -2$ $\underline{x = -2}$

$\underline{x = -\frac{2}{3}}$

(c) $f(-1) = 3(-1)^3 + 5(-1)^2 - 4(-1) - 4$

$= 3(-1) + 5(1) + 4 - 4$

$= -3 + 5 + 4 - 4$

$= 2$

Y gweddill pan gaiff $3x^3 + 5x^2 - 4x - 4$ ei rannu â $x + 1$ yw 2.

④

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
```

$$(1+2x)^6 = 1^6 + 6(1^5)(2x) + 15(1^4)(2x)^2 + 20(1^3)(2x)^3 + \cdots$$
$$= 1 + 6(2x) + 15(4x^2) + 20(8x^3) + \cdots$$
$$= 1 + 12x + 60x^2 + 160x^3 + \cdots$$

⑤ $y = x^2 - 7x + 2$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$f(x) = x^2 - 7x + 2$

$f(x+\delta x) = (x+\delta x)^2 - 7(x+\delta x) + 2$

$= x^2 + 2x\delta x + (\delta x)^2 - 7x - 7\delta x + 2$

$f(x+\delta x) - f(x) = x^2 + 2x\delta x + (\delta x)^2 - 7x - 7\delta x + 2 - x^2 + 7x - 2$

$= 2x\delta x + (\delta x)^2 - 7\delta x$

$\frac{f(x+\delta x) - f(x)}{\delta x} = 2x + \delta x - 7.$

$\frac{dy}{dx} = \lim_{\delta x \to 0} 2x + \delta x - 7$

$\frac{dy}{dx} = 2x - 7.$

⑥ $y = 16\sqrt{x} + \frac{32}{x} + 2$

(a) $y = 16x^{\frac{1}{2}} + 32x^{-1} + 2$

$\frac{dy}{dx} = 16\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - 32x^{-2}$

$\frac{dy}{dx} = 8x^{-\frac{1}{2}} - 32x^{-2}$

$\frac{dy}{dx} = \frac{8}{\sqrt{x}} - \frac{32}{x^2}$

Os yw $x = 4$ yna $\frac{dy}{dx} = \frac{8}{\sqrt{4}} - \frac{32}{4^2}$

$= \frac{8}{2} - \frac{32}{16}$

$= 4 - 2$

$= 2.$

(b) Os yw $x = 4$ yna $y = 16(\sqrt{4}) + \frac{32}{4} + 2$

$y = 16 \times 2 + 8 + 2$

$y = 32 + 8 + 2$

$y = 42.$

Graddiant y tangiad yw 2 $\left(\frac{dy}{dx}\right)$

Graddiant y normal yw $-\frac{1}{2}$ (negatif y cilydd)

Hafaliad y normal: $y - y_1 = m(x - x_1)$

$y - 42 = -\frac{1}{2}(x - 4)$

$2y - 84 = -x + 4$

$2y + x - 84 - 4 = 0$

$2y + x - 88 = 0$

⑦ $y = x^3 - 3x^2$

(a) $\frac{dy}{dx} = 3x^2 - 6x$

Pwyntiau arhosol: angen datrys $\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$ (rhannu â 3)

$x(x-2) = 0$

Unai $x = 0$ neu $x - 2 = 0$

$x = 2$

Felly $y = 0^3 - 3(0)^2$

$y = 0 - 0$

$y = 0$

Felly $y = 2^3 - 3(2^2)$

$y = 8 - 12$

$y = -4$

Natur: $\frac{d^2y}{dx^2} = 6x - 6$

Os yw $x = 0$ yna $\frac{d^2y}{dx^2} = 6(0) - 6$

$= -6$

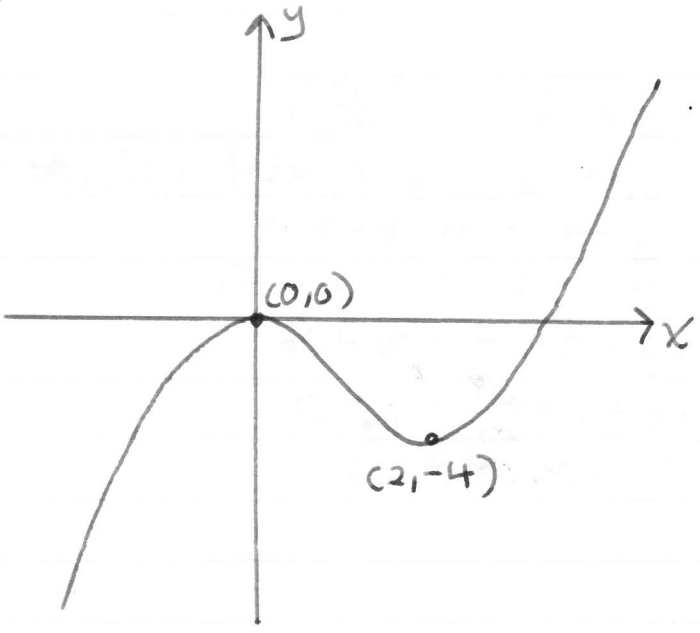
Felly pwynt macsimwm yw $(0, 0)$

Os yw $x = 2$ yna $\frac{d^2y}{dx^2} = 6(2) - 6$

$= 6$

Felly pwynt minimwm yw $(2, -4)$.

(b)



(c) Ble mae'n bosib llunio llinell lorweddol sy'n croestorri gromlin mewn 3 pwynt gwahanol? Mae'r gwerthoedd rhwng -4 a 0 yn gweithio felly'r ateb yw $-4 < k < 0$.

(8) (a) $x^2-6x+16 = (x-3)^2-9+16$

$= (x-3)^2+7$

(Felly $a=3$, $b=7$).

Gwerth lleiaf $x^2-6x+16$ yw 7.

(b) $(x+1)^2 \leq 4x+9$

$(x+1)(x+1) \leq 4x+9$

$x^2+x+x+1 \leq 4x+9$

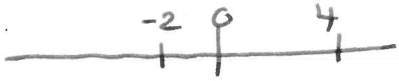
$x^2+2x+1 \leq 4x+9$

$x^2-2x-8 \leq 0$

$(x-4)(x+2) \leq 0$

Pwyntiau critigol: $x-4=0$ $\quad x+2=0$

$x=4$ $\quad x=-2$



Graff siâp U fellŷr aleb yw $-2 \leq x \leq 4$.

(9) Llinell $y=2x+c$ $\quad$ Cromlin $y=x^2+6x+7$

(a) Rydym angen darganfod ble mae'r llinell a'r gromlin yn croestorri. $\quad 2x+c = x^2+6x+7$

$0 = x^2+4x+7-c$

I gyffwrdd mewn un pwynt rhaid bod $b^2-4ac=0$

$4^2-4(1)(7-c)=0$

$16-4(7-c)=0$

$16-28+4c=0$

$4c=28-16$

$4c=12$

$\underline{c=3}$

(b) Graddiant y llinell yw 2.

Ble mae graddiant y tangiad yn 2?

Rydym angen datrys $\frac{dy}{dx}=2$.

$\frac{dy}{dx} = 2x + 6$ felly rydym angen datrys

$2x + 6 = 2$

$2x = -4$

$x = -2$

Yn amnewid yn ôl i hafaliad y llinell:

$y = 2x + 3$

$y = 2(-2) + 3$

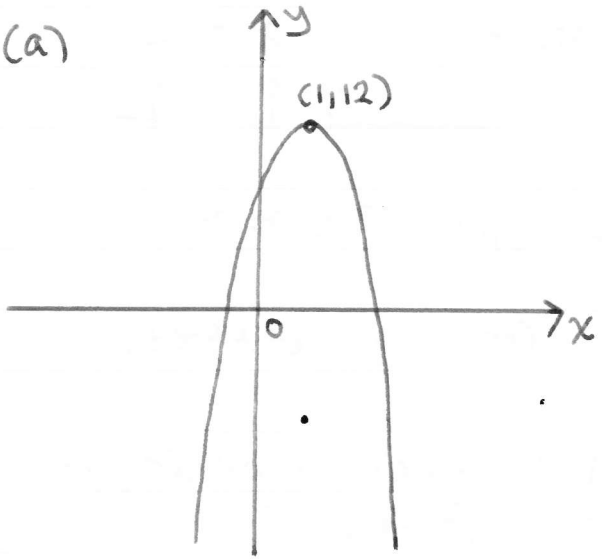
$y = -4 + 3$

$y = -1$

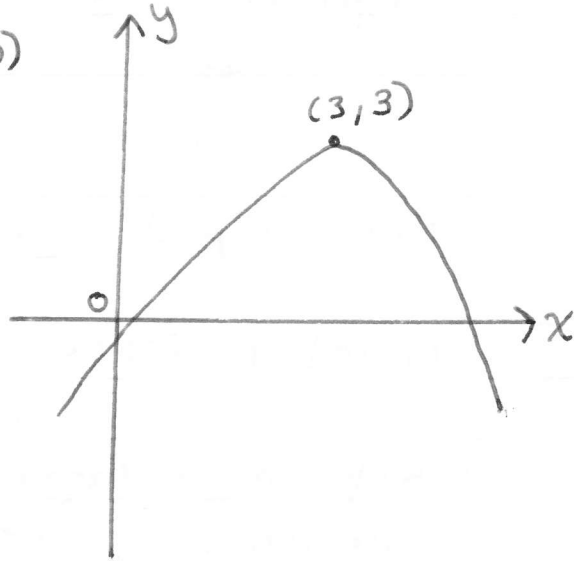
Mae pwynt cyffwrdd y tangiad ar gromlin yn $(-2, -1)$.

(10)

(a)



(b)



(c)