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Stationary Points

(Gaeaf 2005)

3. A curve C has equation

$$y = 2x^3 - 6x^2 + 12.$$

- (a) Find the coordinates of the stationary points of C and determine the nature of each of those points. [7]
- (b) Sketch C . [3]
- (c) State, giving a reason, the number of real roots of the equation

$$2x^3 - 6x^2 + 12 = 0. \quad [2]$$

(Haf 2005)

7. The curve C has equation $y = x^3 - 3x^2$.

- (a) Find the coordinates of the stationary points of C and determine the nature of each of these points. [7]
- (b) Sketch C . [3]
- (c) Find the range of values of k for which there are three real and distinct solutions of the equation $x^3 - 3x^2 = k$. [2]

(Gaeaf 2006)

10. The curve C has equation

$$y = 2 + 6x^2 - 2x^3.$$

- (a) Find the coordinates and the nature of each of the stationary points of C . [7]
- (b) Sketch C , indicating clearly the nature of each of the stationary points. [3]
- (c) State clearly, giving a reason, the number of real roots of the equation

$$2 + 6x^2 - 2x^3 = 0. \quad [2]$$

(Haf 2006)

10. The curve C has equation

$$y = x^3 - 3x^2 - 9x + 2.$$

Find the coordinates of the stationary points of C and determine the nature of each of these stationary points. [7]

(Gaeaf 2007)

9. The curve C has equation

$$y = 4x^3 - 12x + 3.$$

- (a) Find the coordinates of the stationary points of C and determine the nature of each of these points. [7]
- (b) Sketch C , indicating the coordinates of the stationary points. [3]
- (c) Given that $f(x) = 4x^3 - 12x + 3$, sketch the curve $y = f(x - 1)$, indicating the coordinates of **each** of the stationary points. [3]

(Haf 2007)

10. The curve C has equation

$$y = x^3 - x^2 - x + 2.$$

Find the coordinates of the stationary points of C and determine the nature of each of these stationary points. [7]

(Gaeaf 2008)

10. The curve C has equation

$$y = x^3 - 12x + 11.$$

- (a) Find the coordinates and nature of each of the stationary points of C . [7]
- (b) Sketch C , indicating the coordinates of each of the stationary points. [2]
- (c) Given that the equation

$$x^3 - 12x + 11 = k$$

has only one real root, find the range of possible values for k . [2]

(Haf 2008)

9. The curve C has equation

$$y = -2x^3 + 3x^2 + 12x - 5.$$

Find the coordinates and nature of each of the stationary points of C . [7]

(Gaeaf 2009)

10. The curve C has equation

$$y = x^3 + 3x^2 - 9x - 13.$$

- (a) Find the stationary points of C and determine the nature of each of these points. [6]
- (b) Sketch C , indicating the coordinates of the stationary points. [2]
- (c) State, giving a reason, the number of real roots of the equation

$$x^3 + 3x^2 - 9x - 13 = 0. \quad [2]$$

(Haf 2009)

10. The curve C has equation

$$y = x^3 - 3x^2 + 3x + 5.$$

- (a) Show that C has only one stationary point. Find the coordinates of this point. [4]
- (b) Verify that this stationary point is a point of inflection. [2]

(Gaeaf 2010)

10. The curve C has equation

$$y = x^3 - 6x^2 + 20.$$

- (a) Find the coordinates and the nature of each of the stationary points of C . [6]
- (b) Sketch C , indicating the coordinates of each of the stationary points. [2]
- (c) Given that the equation

$$x^3 - 6x^2 + 20 = k$$

has three **distinct** real roots, find the range of possible values for k . [2]

(Haf 2010)

10. The curve C has equation

$$y = \frac{1}{2}x^3 - 6x + 3.$$

Find the coordinates and the nature of each of the stationary points of C . [6]

(Gaeaf 2011)

10. The curve C has equation

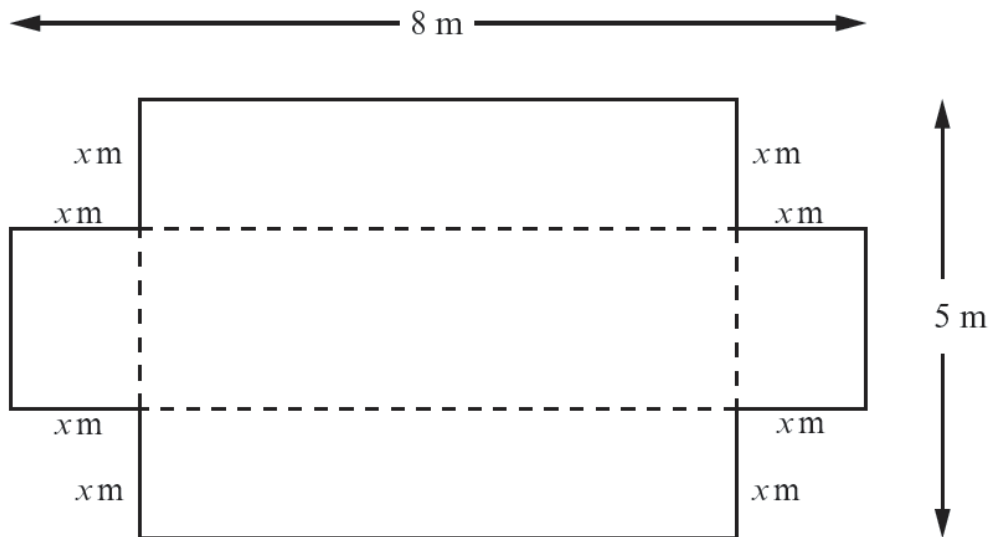
$$y = x^3 + kx^2 - 9x - 10,$$

where k is a constant. The two stationary points on the graph of C are denoted by Q and R . The x -coordinate of Q is -1 .

- (a) Find $\frac{dy}{dx}$ and hence show that $k = -3$. [3]
- (b) Find the x -coordinate of R . [2]
- (c) Determine the nature of each of the stationary points Q and R . [2]

(Haf 2011)

10. A rectangular sheet of metal has length 8 m and width 5 m. Four squares, each of side x m, where $x < 2.5$, have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



- (a) Show that the volume $V\text{m}^3$ of this tank is given by
- $$V = 4x^3 - 26x^2 + 40x. \quad [2]$$
- (b) Find the maximum value of V , showing that the value you have found is a maximum value. [5]

(Gaeaf 2012)

10. The curve C has equation

$$y = x^3 - 6x^2 + 12x - 9.$$

(a) Show that C has only one stationary point. Find the coordinates of this point. [4]

(b) Verify that this stationary point is a point of inflection. [2]

(Haf 2012)

10. The curve C has equation

$$y = x^3 + 3x^2 - 1.$$

(a) Find the coordinates and the nature of each of the stationary points of C . [6]

(b) Sketch C , indicating the coordinates of each of the stationary points. [2]

(c) Write down the number of **positive** real roots of the equation

$$x^3 + 3x^2 - 1 = 0. \quad [1]$$

(Gaeaf 2013)

10. The curve C has equation

$$y = x^3 - 3x^2 - 9x + 14.$$

(a) Find the coordinates and the nature of each of the stationary points of C . [6]

(b) Sketch C , indicating the coordinates of each of the stationary points. [2]

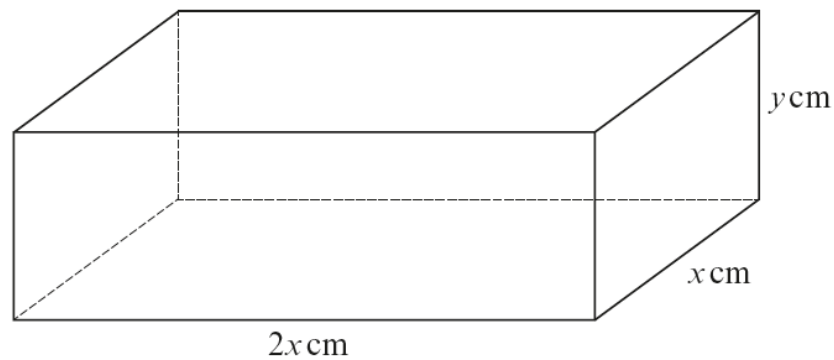
(c) Given that the equation

$$x^3 - 3x^2 - 9x + 14 = k$$

has only one real root, find the range of possible values for k . [2]

(Haf 2013)

10. The diagram shows a **closed** box in the form of a cuboid. The length of the box is $2x$ cm, its width is x cm and its height is y cm.



The total surface area of the box is 108 cm^2 .

- (a) (i) Write down an equation involving x and y and hence show that

$$xy = 18 - \frac{2}{3}x^2.$$

- (ii) Hence show that the volume $V \text{ cm}^3$ of the box is given by

$$V = 36x - \frac{4}{3}x^3. \quad [3]$$

- (b) Find the maximum value of V , showing that the value you have found is a maximum value. [5]

(Gaeaf 2014)

10. The curve C has equation

$$y = -2x^3 + 12x^2 - 18x + 5.$$

- (a) Find the coordinates and the nature of each of the stationary points of C . [6]
(b) Sketch C , indicating the coordinates of each of the stationary points. [2]
(c) Given that the equation

$$-2x^3 + 12x^2 - 18x + 5 = k$$

- has three distinct real roots, find the range of possible values for k . [2]

(Haf 2014)

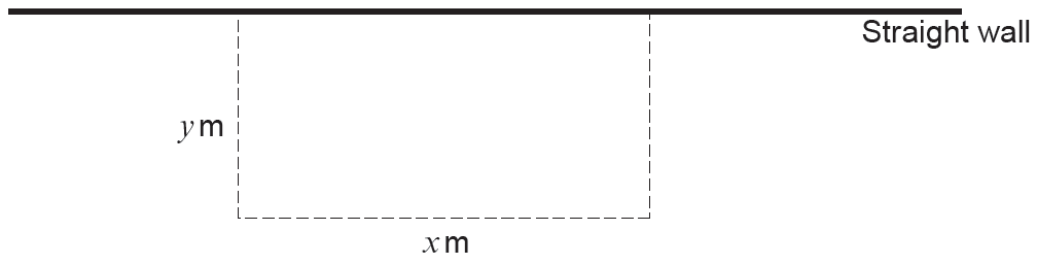
10. The curve C has equation

$$y = x^3 + 9x^2 + 27x + 31.$$

- (a) Show that C has only one stationary point. Find the coordinates of this point. [4]
- (b) Verify that this stationary point is a point of inflection. [2]
- (c) Sketch the graph of C , indicating the coordinates of its stationary point. [1]

(Haf 2015)

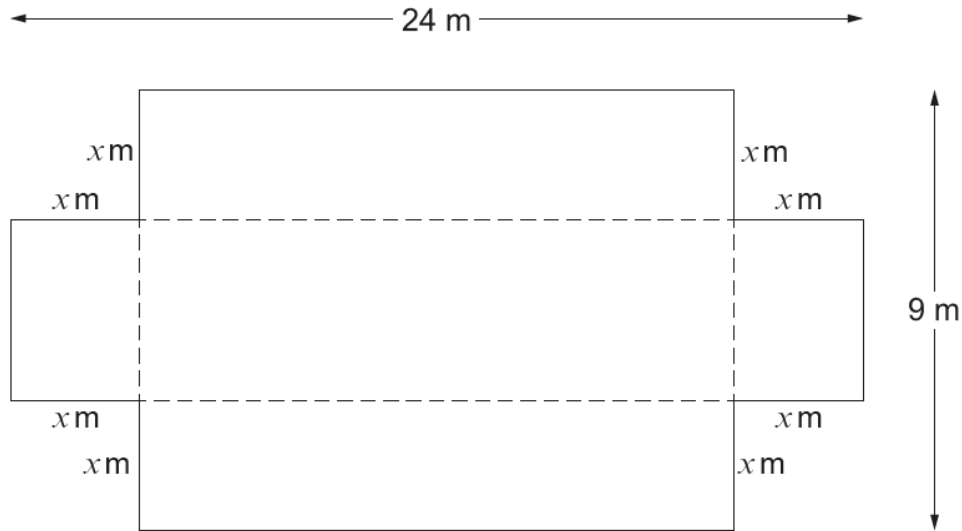
10. A sheep farmer wishes to construct a rectangular enclosure for his animals. He decides to use a straight wall as one side of the enclosure and fencing for the other three sides. The area of the enclosure is to be 800 m^2 . The lengths of the sides of the rectangular enclosure are $x \text{ m}$ and $y \text{ m}$, as shown in the diagram, and the total length of the **fencing** is $L \text{ m}$.



- (a) Show that $L = x + \frac{1600}{x}$. [2]
- (b) Find the minimum value of L , showing that the value you have found is a minimum value. [5]

(Haf 2016)

10. A rectangular sheet of metal has length 24 m and width 9 m. Four squares, each of side x m, where $x < 4.5$, have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



- (a) Show that the volume $V \text{ m}^3$ of this tank is given by

$$V = 4x^3 - 66x^2 + 216x. \quad [2]$$

- (b) Find the maximum value of V , showing that the value you have found is a maximum value. [5]

(Haf 2017)

10. The curve C has equation

$$y = x^3 - 9x^2 + 15x + 10.$$

- (a) (i) Find the coordinates of each of the stationary points of C .
(ii) Determine the nature of each of these stationary points. [6]

- (b) Sketch C , indicating the coordinates of each of the stationary points. [2]

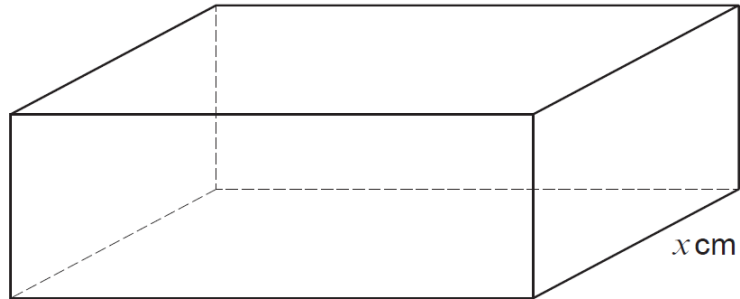
- (c) Given that the equation

$$x^3 - 9x^2 + 15x + 10 = k$$

- has only one real root, find the range of possible values for k . [2]

(Haf 2018)

10. A closed box, in the form of a cuboid, is such that the length of its base is three times the width of its base. The volume of the box is 6000cm^3 . The total length of the twelve edges of the box is denoted by L cm.



- (a) Show that $L = 16x + \frac{8000}{x^2}$, where x cm denotes the width of the base. [3]
- (b) Find the minimum value of L , showing that the value you have found is a minimum value. [5]