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Completing the Square

(Gaeaf 2005)

5. Express the quadratic expression $x^2 - 14x + 55$ in the form $(x - a)^2 + b$, where the values of the constants a and b are to be determined. Hence show that $x^2 - 14x + 55$ is positive for all values of x . [5]

(Haf 2005)

8. (a) Express the quadratic expression $x^2 - 6x + 16$ in the form $(x - a)^2 + b$, where the values of the constants a and b are to be determined. **Deduce** the least value of $x^2 - 6x + 16$. [3]
- (b) Solve the inequality

$$(x + 1)^2 \leq 4x + 9. \quad [4]$$

(Gaeaf 2006)

9. (a) Express $23 + 6x - x^2$ in the form $b - (x - a)^2$, where the constants a and b are to be determined. Hence find the greatest value of $23 + 6x - x^2$ and the corresponding value of x . [4]
- (b) Use the results found in (a) to deduce the least value of $\frac{1}{30 + 6x - x^2}$. [2]

(Haf 2006)

4. (a) Given that the equation

$$kx^2 - 4x + k - 3 = 0$$

has equal roots, find the values of k . [5]

- (b) Express $x^2 + 8x + 2$ in the form $(x + a)^2 + b$. Hence write down the least value of $x^2 + 8x + 2$. [3]

(Gaeaf 2007)

8. (a) Express $x^2 + 4x + 9$ in the form $(x + a)^2 + b$, where the values of a and b are to be determined.
Deduce the maximum value of

$$\frac{1}{x^2 + 4x + 9}. \quad [4]$$

- (b) Show that the line $y = x + 2$ touches the curve $y = x^2 - 5x + 11$, and find the coordinates of the point of contact. [4]

(Haf 2007)

7. (a) Express $2x^2 + 4x + 5$ in the form $a(x + b)^2 + c$, where a , b and c are to be determined. [3]
- (b) Use the result derived in (a) to find the greatest value of $\frac{1}{2x^2 + 4x + 9}$. [2]

(Gaeaf 2008)

7. Show that $x^2 + 1 \cdot 8x - 3 \cdot 19$ may be expressed in the form $(x + p)^2 - 4$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1 \cdot 8x - 3 \cdot 19 = 0$. [5]

(Haf 2008)

5. (a) Express $x^2 + 6x - 4$ in the form $(x + a)^2 + b$ where the values of a , b are to be determined. [2]
- (b) **Use your results to part (a)** to find the least value of $2x^2 + 12x - 8$ and the corresponding value of x . [2]

(Gaeaf 2009)

4. Express $3x^2 - 12x + 17$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found.
Hence, sketch the graph of $y = 3x^2 - 12x + 17$, indicating the coordinates of its stationary point. [5]

(Haf 2009)

4. (a) (i) Express $x^2 - 5x + 8$ in the form $(x + a)^2 + b$, where the values of the constants a and b are to be found.
(ii) Deduce the greatest value of $-x^2 + 5x - 8$. [3]

(Gaeaf 2010)

4. (a) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$, where a , b and c are constants whose values are to be found. [3]
- (b) Use your answer to part (a) to find the greatest value of

$$\frac{1}{4x^2 - 8x + 7}. \quad [2]$$

(Haf 2010)

5. (a) Express $2x^2 + 12x - 7$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
- (b) **Use your answer to part (a)** to find the least value of $6x^2 + 36x - 17$. [2]

(Gaeaf 2011)

6. Show that $x^2 - 1.4x - 8.51$ may be expressed in the form $(x + p)^2 - 9$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 - 1.4x - 8.51 = 0$. [5]

(Haf 2011)

4. Express $-x^2 + 6x - 7$ in the form $-(x + a)^2 + b$, where the values of the constants a and b are to be found.
Hence sketch the graph of $y = -x^2 + 6x - 7$, indicating the coordinates of its stationary point. [4]

(Gaeaf 2012)

5. (a) Express $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a , b and c are constants whose values are to be found. [3]
(b) **Use your answer to part (a)** to find the greatest value of

$$\frac{1}{3x^2 - 6x + 11} . \quad [2]$$

(Haf 2012)

5. (a) Express $3x^2 - 12x + 29$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
(b) **Using your answer to part (a)**, write down the stationary value of $y = 3x^2 - 12x + 29$. State whether this stationary value is a maximum or a minimum. [2]

(Gaeaf 2013)

4. (a) (i) Express $x^2 + 8x + 5$ in the form $(x + a)^2 + b$, where the values of a , b are to be determined.
(ii) **Use your answers to part (i)** to find the least value of $3x^2 + 24x + 15$ and the corresponding value of x . [4]

(Haf 2013)

4. (a) Express $2x^2 - 16x - 8$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
(b) **Using your answer to part (a)**, find the least value of $x^2 - 8x - 4$ and the corresponding value of x . [2]

(Gaeaf 2014)

4. Show that $x^2 + 1.6x - 24.36$ may be expressed in the form $(x + p)^2 - 25$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1.6x - 24.36 = 0$. [5]

(Haf 2014)

5. (a) Express $4x^2 - 8x + 11$ in the form $a(x + b)^2 + c$, where a , b and c are constants whose values are to be found. [3]
- (b) Use your answer to part (a) to find the greatest value of $\frac{1}{4x^2 - 8x + 29}$. [2]

(Haf 2015)

4. (a) Express $4x^2 - 24x - 189$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
- (b) Using your answer to part (a), solve the equation
- $$4x^2 - 24x - 189 = 0. \quad [3]$$

(Haf 2016)

5. (a) Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$, where a and b are constants whose values are to be found. [2]
- (b) Use an algebraic method to solve the simultaneous equations $y = x^2 + 4x - 8$ and $y = 2x + 7$. [4]
- (c) Draw a sketch illustrating geometrically the results of both part (a) and part (b). [4]

(Haf 2017)

4. (a) Express $-2x^2 - 20x + 35$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
- (b) Without carrying out any further calculation, write down the stationary value of $y = -2x^2 - 20x + 35$ and state whether this stationary value is a maximum or a minimum. [2]

(Haf 2018)

4. (a) Express $4x^2 + 40x - 69$ in the form $a(x + b)^2 + c$, where the values of the constants a , b and c are to be found. [3]
- (b) Using your answer to part (a), solve the equation
- $$4x^2 + 40x - 69 = 0. \quad [3]$$