WJEC CBAC

GCE AS/A level

983/01

MATHEMATICS S1

Statistics

P.M. WEDNESDAY, 21 January 2009

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

JD*(W09-983-01)

1. The two events A, B are such that

$$P(A) = 0{\cdot}65,\ P(A \cup B) = 0{\cdot}93.$$

Evaluate $P(B)$ given that

(a) A and B are mutually exclusive, [2]

(b) A and B are independent. [4]

2. In a class of 30 students, 12 are studying French, 15 are studying Spanish and 8 are studying neither French nor Spanish. A student is selected at random from this class.

(a) Find the probability that the student is studying both French and Spanish. [4]

(b) Find the probability that the student is studying French but not Spanish. [2]

3. *(a)* The number of accidents per week at a certain roundabout has a Poisson distribution with mean 2·75. **Without the use of tables**, find the probability that, during a randomly chosen week, the number of accidents at this roundabout is

(i) exactly 4,

(ii) more than 2. [5]

(b) The number of accidents per week at a certain crossroads has a Poisson distribution with mean 3. **Using tables**, find the probability that, during a randomly chosen week, the number of accidents at the crossroads is

(i) less than 5,

(ii) exactly 3. [5]

4. The random variable X has a Poisson distribution with mean 4. The random variable Y is given by

$$Y = 3X - 7.$$

(a) Find the mean and variance of Y. [5]

(b) Find the probability that Y is positive. [3]

5. A pack of 16 cards contains 4 red cards, 4 blue cards, 4 green cards and 4 yellow cards. Ann selects 3 of these cards at random **without replacement**.
Calculate the probability that the 3 cards selected are

(a) all of the same colour, [3]

(b) all differently coloured. [4]

6. For a certain type of tulip bulb, the probability of producing a red flower is 0·6. A gardener plants 20 of these bulbs.

(a) Find the probability that the number of red flowers produced is

(i) exactly 10,

(ii) at least 12. [6]

(b) The probability that this type of tulip bulb fails to produce a flower of any colour is 0·04. A park-keeper plants 80 of these bulbs. Use a Poisson approximation to find the probability that fewer than 5 bulbs fail to produce a flower of any colour. [3]

7. *(a)* Two fair cubical dice with faces numbered 1, 2, 3, 4, 5, 6 respectively are thrown. Find the probability that the sum of the two numbers on the uppermost faces is 5. [2]

(b) Jo tosses a fair coin. If it falls ‘heads’, she throws one of the dice and her score is equal to the number on the uppermost face. If it falls ‘tails’, she throws the two dice and her score is equal to the sum of the two numbers on the uppermost faces.

(i) Find the probability that her score is equal to 5.

(ii) Given that her score is equal to 5, find the probability that she obtained a ‘head’ when she tossed the coin. [6]

8. The probability distribution of the discrete random variable X is given by

$$P(X = x) = \frac{(10 - x)}{20}, \quad \text{for } x = 2, 4, 6, 8,$$
$$P(X = x) = 0 \qquad \text{otherwise.}$$

(a) Find the mean and variance of X. [6]

(b) Given that X_1, X_2 are independent observations on X, calculate

$$P(X_1 + X_2 = 8)$$ [3]

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0, && \text{for } x < 0, \\ F(x) &= kx^3, && \text{for } 0 \leqslant x \leqslant 2, \\ F(x) &= 1, && \text{for } x > 2, \end{aligned}$$

where k is a constant.

(a) Show that

$$k = \frac{1}{8}.$$ [2]

(b) Find the probability that the value of X lies between 0·5 and 1·5. [2]

(c) Find the median of X. [2]

(d) Evaluate $E(X)$. [6]