



**GCE AS/A level**

983/01

**MATHEMATICS S1**

**Statistics**

A.M. THURSDAY, 11 June 2009

1½ hours

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

### **INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A school committee contains 9 members of which 2 are teachers, 3 are boys and 4 are girls. A sub-committee of 3 members is to be formed and it is decided to choose the 3 members at random. Calculate the probability that the sub-committee contains

(a) no teachers, [2]

(b) 1 teacher, 1 boy and 1 girl. [3]

2. Events  $A$  and  $B$  are such that

$$P(A) = 0.2, P(B) = 0.3.$$

(a) Evaluate  $P(A \cup B)$  when

(i)  $A$  and  $B$  are mutually exclusive,

(ii)  $A$  and  $B$  are independent. [5]

(b) Given that  $P(A \cup B) = 0.4$ , calculate  $P(A|B)$ . [4]

(c) What is the smallest possible value for  $P(A \cup B)$  and when does this occur? [2]

3. The random variable  $X$  has the binomial distribution  $B(25, 0.8)$ .

(a) State the mean and variance of  $X$ . [2]

(b) The random variable  $Y$  is defined by

$$Y = aX - b$$

where  $a, b$  are positive constants.

(i) Given that  $a = 2, b = 3$ , find the mean and variance of  $Y$ .

(ii) Given that  $E(Y) = 0$  and  $\text{Var}(Y) = 1$ , find the values of  $a$  and  $b$ . [8]

4. Dafydd is a fisherman. When he fishes in a certain lake, the number of fish that he catches in  $t$  hours has a Poisson distribution with mean  $0.6t$ .

(a) One morning, he fishes for 4 hours. Find the probability that he catches

(i) exactly 3 fish,

(ii) at least 3 fish. [5]

(b) One day, the probability of Dafydd catching no fish was 0.5. For how long did he fish? [4]

5. It is known that 5% of the population suffer from a certain disease. When a test is applied to a person with the disease, it gives a positive response with probability 0.99. When the test is applied to a person who does not have the disease, it gives a positive response with probability 0.02. The test is applied to a randomly selected member of the population.

- (a) Find the probability that a positive response is obtained. [3]
- (b) Given that a positive response is obtained, find the probability that the person has the disease. [3]

6. The probability distribution of the discrete random variable  $X$  is given in the following table.

$x$	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.3	0.1

- (a) Evaluate

- (i)  $E(X)$ ,
- (ii)  $\text{Var}(X)$ . [6]

- (b) Given that  $X_1, X_2$  are independent observations on  $X$ , calculate

$$P(X_1 = X_2). \quad [3]$$

7. (a) Ann tosses 3 fair coins and Bob tosses 2 fair coins. Find the probability that Ann obtains more heads than Bob. [6]

- (b) Ceri and Mair now toss a fair coin alternately, starting with Ceri. The winner is the one who obtains the first head. Find the probability that Mair

- (i) wins on her first toss,
- (ii) wins on her second toss,
- (iii) is the winner. [7]

8. The continuous random variable  $X$  has probability density function  $f$  given by

$$f(x) = \frac{1}{2} (1 + 2x) \quad \text{for } 0 \leq x \leq 1,$$

$$f(x) = 0 \quad \text{otherwise.}$$

- (a) Calculate  $E(X)$ . [4]

- (b) Obtain an expression for  $F(x)$ , valid for  $0 \leq x \leq 1$ , where  $F$  denotes the cumulative distribution function of  $X$ . [3]

- (c) Calculate

- (i)  $P(0.4 \leq X \leq 0.5)$ ,
- (ii) the median of  $X$ . [5]