

SI: Hapnewidyn Di-dor

Gaeaf 2005

① $f(x) = \frac{1}{21}x^2$ ar gyfer $1 \leq x \leq 4$
 $f(x) = 0$ fel arall

$$\begin{aligned}
 (a) E(X) &= \int x f(x) dx \\
 &= \int_1^4 x \left(\frac{1}{21}x^2 \right) dx \\
 &= \int_1^4 \frac{1}{21}x^3 dx \\
 &= \frac{1}{21} \int_1^4 x^3 dx \\
 &= \frac{1}{21} \left[\frac{x^4}{4} \right]_1^4 \\
 &= \frac{1}{21} \left[\frac{4^4}{4} - \frac{1^4}{4} \right] \\
 &= \frac{1}{21} \left(64 - \frac{1}{4} \right) \\
 &= \frac{85}{28}
 \end{aligned}$$

$$\begin{aligned}
 (b) F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_1^x f(t) dt \\
 &= \int_1^x \frac{1}{21}t^2 dt \\
 &= \frac{1}{21} \int_1^x t^2 dt \\
 &= \frac{1}{21} \left[\frac{t^3}{3} \right]_1^x \\
 &= \frac{1}{21} \left(\frac{x^3}{3} - \frac{1^3}{3} \right)
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow \frac{1}{21} \left(\frac{x^3}{3} - \frac{1}{3} \right) \\
 &= \frac{1}{63} (x^3 - 1)
 \end{aligned}$$

$$\begin{aligned}
 (c) P(2 \leq X \leq 3) &= F(3) - F(2) \\
 &= \frac{1}{63}(3^3 - 1) - \frac{1}{63}(2^3 - 1) \\
 &= \frac{26}{63} - \frac{7}{63} \\
 &= \frac{19}{63}
 \end{aligned}$$

(ch) Canolrif X: $F(x) = 0.5$

$$\frac{1}{63}(x^3 - 1) = 0.5$$

$$x^3 - 1 = 0.5 \times 63$$

$$x^3 = 31.5 + 1$$

$$x = \sqrt[3]{32.5}$$

$$x = 3.19 \text{ i } 2 \text{ ie degol}$$

Itaf 2005

⑧ $F(x) = 0 \quad \text{ar gyfer } x < 0$

$$F(x) = 4x^3 - 3x^4 \quad \text{ar gyfer } 0 \leq x \leq 1$$

$$F(x) = 1 \quad \text{ar gyfer } x > 1$$

(a) $P(0.2 \leq X \leq 0.8) = F(0.8) - F(0.2)$
 $= 4(0.8^3) - 3(0.8)^4 - (4(0.2)^3 - 3(0.2)^4)$
 $= 0.792$

(b) $F(0.45) = 4(0.45^3) - 3(0.45^4)$
 $= 0.24148125.$

$$F(0.46) = 4(0.46^3) - 3(0.46^4)$$

 $= 0.25502032.$

Mae 0.25 rhwng 0.24148125 a 0.25502032

Felly mae'r chwarter isaf rhwng 0.45 a 0.46.

(c) $F(x) = \frac{d}{dx} (F(x))$
 $= \frac{d}{dx} (4x^3 - 3x^4)$
 $= 12x^2 - 12x^3 \quad (\text{ar gyfer } 0 \leq x \leq 1)$

(ch) $E(X) = \int x F(x) dx$
 $= \int_0^1 x(12x^2 - 12x^3) dx$
 $= 12 \int_0^1 x^3 - x^4 dx$
 $= 12 \left[\frac{x^4}{4} - \frac{x^5}{5} \right]_0^1$

$\rightarrow = 12 \left(\left(\frac{\frac{1}{4} - \frac{1}{5}}{4} \right) - \left(\frac{0^4}{4} - \frac{0^5}{5} \right) \right)$ $= 12 \left(\frac{1}{4} - \frac{1}{5} \right)$ $= 0.6$

Graef 2006

⑨ $f(x) = Kx^2$ ar gyfer $1 \leq x \leq 4$
 $f(x) = 0$ fel arall

(a) (i) $\int f(x) dx = 1$
 $\int_1^4 Kx^2 dx = 1$
 $K \int_1^4 x^2 dx = 1$
 $K \left[\frac{x^3}{3} \right]_1^4 = 1$
 $K \left(\frac{4^3}{3} - \frac{1^3}{3} \right) = 1$
 $K \left(\frac{64}{3} - \frac{1}{3} \right) = 1$
 $\frac{63}{3} K = 1$
 $K = \frac{3}{63}$
 $K = \frac{1}{21}$ ✓

(ii) $E(x) = \int x f(x) dx$
 $= \int_1^4 x (Kx^2) dx$
 $= K \int_1^4 x^3 dx$
 $= K \left[\frac{x^4}{4} \right]_1^4$
 $= K \left(\frac{4^4}{4} - \frac{1^4}{4} \right)$
 $= K \left(64 - \frac{1}{4} \right)$
 $= \frac{1}{21} \left(64 - \frac{1}{4} \right)$
 $= \frac{85}{28}$

(b) (i) $F(x) = \int_{-\infty}^x F(t) dt$
 $= \int_1^x F(t) dt$
 $= \int_1^x Kt^2 dt$
 $= K \int_1^x t^2 dt$
 $= K \left[\frac{t^3}{3} \right]_1^x$
 $= K \left(\frac{x^3}{3} - \frac{1^3}{3} \right)$
 $= \frac{1}{21} \left(\frac{x^3}{3} - \frac{1}{3} \right)$
 $= \frac{1}{63} (x^3 - 1)$

(ii) $P(2 \leq X \leq 3)$
 $= F(3) - F(2)$
 $= \frac{1}{63}(3^3 - 1) - \frac{1}{63}(2^3 - 1)$
 $= \frac{26}{63} - \frac{7}{63}$
 $= \frac{19}{63}$

(iii) Canolrif: $F(x) = 0.5$
 $\frac{1}{63}(x^3 - 1) = 0.5$
 $x^3 - 1 = 0.5 \times 63$
 $x^3 = 31.5 + 1$
 $x = \sqrt[3]{32.5}$
 $x = 3.19 \text{ i } 2 \text{ le degol}$

Haf 2006

⑧ $F(x) = 0 \quad \text{ar gyfer } x < 0$
 $F(x) = \frac{1}{2}(x^2 + x) \quad \text{ar gyfer } 0 \leq x \leq 1$
 $F(x) = 1 \quad \text{ar gyfer } x > 1$

(a) (i) $P(0.25 \leq X \leq 0.5) = F(0.5) - F(0.25)$
 $= \frac{1}{2}(0.5^2 + 0.5) - \frac{1}{2}(0.25^2 + 0.25)$
 $= 0.21875$

(ii) canolrif X : $F(x) = 0.5$
 $\frac{1}{2}(x^2 + x) = 0.5$
 $x^2 + x = 1$
 $x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2 \times 1}$$

Unai $x = \frac{-1 + \sqrt{1+4}}{2}$ neu $x = \frac{-1 - \sqrt{1+4}}{2}$

$$x = 0.618 \text{ i 3 ll.d.} \quad x = -1.618 \text{ i 3 ll.d.}$$

Gan fod $0 \leq x \leq 1$ rhaid bod $x = 0.618$ i 3 ll.d.

(b) (i) $f(x) = \frac{d}{dx}(F(x))$
 $= \frac{d}{dx}\left(\frac{1}{2}(x^2 + x)\right)$
 $= \frac{1}{2}(2x) + \frac{1}{2}$
 $= x + \frac{1}{2}$

(ar gyfer $0 \leq x \leq 1$)

(ii) $E(X) = \int x f(x) dx$
 $= \int_0^1 x \left(x + \frac{1}{2}\right) dx$
 $= \int_0^1 x^2 + \frac{1}{2}x dx$
 $= \left[\frac{x^3}{3} + \frac{x^2}{4}\right]_0^1$

$$= \left[\left(\frac{1^3}{3} + \frac{1^2}{4}\right) - \left(\frac{0^3}{3} + \frac{0^2}{4}\right)\right]$$

$$= \frac{1}{3} + \frac{1}{4}$$

$$= \frac{7}{12}$$

Gaeaf 2007

⑦ $f(x) = 20(x^3 - x^4)$ ar gyfer $0 \leq x \leq 1$
 $f(x) = 0$ fel arall

$$\begin{aligned}
 (a) E(x) &= \int x f(x) dx \\
 &= \int_0^1 x (20(x^3 - x^4)) dx \\
 &= \int_0^1 20x^4 - 20x^5 dx \\
 &= \left[\frac{20x^5}{5} - \frac{20x^6}{6} \right]_0^1 \\
 &= \left[\left(\frac{20 \times 1^5}{5} - \frac{20 \times 1^6}{6} \right) - \left(\frac{20 \times 0^5}{5} - \frac{20 \times 0^6}{6} \right) \right] \\
 &= \frac{20}{5} - \frac{20}{6} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 (b) (i) F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_0^x f(t) dt \\
 &= \int_0^x 20(t^3 - t^4) dt \\
 &= \left[\frac{20t^4}{4} - \frac{20t^5}{5} \right]_0^x \\
 &= [5t^4 - 4t^5]_0^x \\
 &= [(5x^4 - 4x^5) - (5(0)^4 - 4(0)^5)] \\
 &= 5x^4 - 4x^5
 \end{aligned}$$

$$\begin{aligned}
 (\text{iii}) \text{ Chwarter uchaf: } \\
 F(x) &= 0.75 \\
 \text{Yn defnyddio } q, \\
 F(q) &= 0.75 \\
 5q^4 - 4q^5 &= 0.75 \\
 5q^4 - 4q^5 &= \frac{3}{4} \\
 20q^4 - 16q^5 &= 3 \\
 0 &= 16q^5 - 20q^4 + 3 \\
 16q^5 - 20q^4 + 3 &= 0 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (\text{ii}) P(0.4 \leq X \leq 0.6) &= F(0.6) - F(0.4) \\
 &= 5(0.6^4) - 4(0.6^5) - (5(0.4^4) - 4(0.4^5)) \\
 &= \frac{781}{3125} \\
 &= 0.24992
 \end{aligned}$$

Haf 2007

⑦ $f(x) = \frac{6}{5}x(x-1)$ ar eftir $1 \leq x \leq 2$
 $f(x) = 0$ fel arall

(a) $E\left(\frac{1}{x}\right) = \int \frac{1}{x} f(x) dx$
= $\int_1^2 \frac{1}{x} \left(\frac{6}{5}x(x-1)\right) dx$
= $\int_1^2 \frac{6}{5}(x-1) dx$
= $\frac{6}{5} \int_1^2 x-1 dx$
= $\frac{6}{5} \left[\frac{x^2}{2} - x \right]_1^2$
= $\frac{6}{5} \left[\left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right) \right]$
= $\frac{6}{5} \left[0 - \frac{1}{2} + 1 \right]$
= 0.6

(b) (i) $F(x) = \int_{-\infty}^x f(t) dt$
= $\int_1^x f(t) dt$
= $\int_1^x \frac{6}{5}t(t-1) dt$
= $\frac{6}{5} \int_1^x t^2 - t dt$
= $\frac{6}{5} \left[\frac{t^3}{3} - \frac{t^2}{2} \right]_1^x$
= $\frac{6}{5} \left[\left(\frac{x^3}{3} - \frac{x^2}{2}\right) - \left(\frac{1^3}{3} - \frac{1^2}{2}\right) \right]$
= $\frac{6}{5} \left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{6} \right)$
= $0.4x^3 - 0.6x^2 + 0.2$

(ii) $P(X \leq 1.75) = F(1.75)$
= $0.4 \times 1.75^3 - 0.6 \times 1.75^2 + 0.2$
= 0.50625

(iii) Mae canolrif X yn 1.1ai;
na 1.75 gan fod
 $F(1.75)$ yn fwy na
0.5.

Göraf 2008

⑧ $f(x) = 4 - 2x$ ar gyfer $1 \leq x \leq 2$
 $f(x) = 0$ fel arall

$$\begin{aligned}
 (a) E(x) &= \int x f(x) dx \\
 &= \int_1^2 x(4-2x) dx \\
 &= \int_1^2 4x - 2x^2 dx \\
 &= \left[\frac{4x^2}{2} - \frac{2x^3}{3} \right]_1^2 \\
 &= \left[\left(2(2^2) - \frac{2(2^3)}{3} \right) - \left(2(1^2) - \frac{2(1^3)}{3} \right) \right] \\
 &= 8 - \frac{16}{3} - 2 + \frac{2}{3} \\
 &= \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 (b) F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_1^x f(t) dt \\
 &= \int_1^x 4 - 2t dt \\
 &= \left[4t - \frac{2t^2}{2} \right]_1^x \\
 &= [(4x - x^2) - (4(1) - 1^2)] \\
 &= 4x - x^2 - 4 + 1 \\
 &= 4x - x^2 - 3
 \end{aligned}$$

$$\begin{aligned}
 (c) P(X > 1.2) &= 1 - P(X \leq 1.2) \\
 &= 1 - F(1.2) \\
 &= 1 - (4(1.2) - (1.2)^2 - 3) \\
 &= 1 - 0.36 \\
 &= 0.64
 \end{aligned}$$

$$\begin{aligned}
 (\text{ch}) \text{ Canolrif } X: \\
 F(x) &= 0.5 \\
 4x - x^2 - 3 &= 0.5 \\
 0 &= x^2 - 4x + 3 + 0.5 \\
 0 &= x^2 - 4x + 3.5 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x &= \frac{4 \pm \sqrt{(4)^2 - 4(1)(3.5)}}{2 \times 1}
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{4 \pm \sqrt{16 - 14}}{2} \\
 \text{unai } x &= \frac{4 + \sqrt{2}}{2} \text{ neu } x = \frac{4 - \sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 x &= 2.71 & x &= 1.29 \\
 ; 2 \text{ le degol} & ; 2 \text{ le degol}
 \end{aligned}$$

Gan fod $1 \leq x \leq 2$, rhaid bod
 $x = 1.29$; 2 le degol.

Haf 2008

⑧ $F(x) = 0$ ar gyfer $x < 0$
 $F(x) = 4x^3 - 3x^4$ ar gyfer $0 \leq x \leq 1$
 $F(x) = 1$ ar gyfer $x > 1$

(a) $P(0.25 \leq X \leq 0.75) = F(0.75) - F(0.25)$
= $4(0.75^3) - 3(0.75^4) - (4(0.25^3) - 3(0.25^4))$
= 0.6875

(b) $F(0.6) = 4(0.6^3) - 3(0.6^4)$
= 0.4752.

Mae canolrif X yn fwy na 0.6 gan fod 0.4752 yn lloain a 0.5.

(c) $f(x) = \frac{d}{dx} F(x)$
= $\frac{d}{dx} (4x^3 - 3x^4)$
= $12x^2 - 12x^3$ (ar gyfer $0 \leq x \leq 1$)

(ch) $E(X) = \int x f(x) dx$
= $\int_0^1 x(12x^2 - 12x^3) dx$
= $\int_0^1 12x^3 - 12x^4 dx$
= $\left[\frac{12x^4}{4} - \frac{12x^5}{5} \right]_0^1$
= $\left[\left(\frac{12(1^4)}{4} - \frac{12(1^5)}{5} \right) - \left(\frac{12(0^4)}{4} - \frac{12(0^5)}{5} \right) \right]$
= $3 - \frac{12}{5}$
= 0.6

Graef 2009

⑨ $F(x) = 0$ ar gyfer $x < 0$
 $F(x) = Kx^3$ ar gyfer $0 \leq x \leq 2$
 $F(x) = 1$ ar gyfer $x > 2$

(a) Rhaid bod $F(2) = 1$
 $K(2^3) = 1$
 $8K = 1$
 $K = \frac{1}{8}$ ✓

(b) $P(0.5 \leq X \leq 1.5) = F(1.5) - F(0.5)$
 $= K(1.5^3) - K(0.5^3)$
 $= \frac{1}{8} \times 3.375 - \frac{1}{8} \times 0.125$
 $= 0.40625$

(c) Canolrif X : $F(x) = 0.5$
 $Kx^3 = 0.5$
 $\frac{1}{8}x^3 = 0.5$
 $x^3 = 4$
 $x = \sqrt[3]{4}$
 $x = 1.59$ i 2 llodegol.

(d) $f(x) = \frac{d}{dx} F(x)$
 $= \frac{d}{dx} (Kx^3)$
 $= 3Kx^2$
 $= \frac{3}{8}x^2$

$E(X) = \int x f(x) dx$
 $= \int_0^2 x \left(\frac{3}{8}x^2\right) dx$
 $= \frac{3}{8} \int_0^2 x^3 dx$
 $= \frac{3}{8} \left[\frac{x^4}{4}\right]_0^2$
 $= \frac{3}{8} \left[\frac{2^4}{4} - \frac{0^4}{4}\right]$
 $= \frac{3}{8} \times \frac{16}{4}$
 $= 1.5$

Haf 2009

⑧ $f(x) = \frac{1}{2}(1+2x)$ ar gyfer $0 \leq x \leq 1$
 $f(x) = 0$ fel arall

$$\begin{aligned} \text{(a)} \quad E(X) &= \int x f(x) dx \\ &= \int_0^1 x \left(\frac{1}{2}(1+2x)\right) dx \\ &= \int_0^1 \frac{1}{2}(x+2x^2) dx \\ &= \int_0^1 \frac{1}{2}x + x^2 dx \\ &= \left[\frac{x^2}{4} + \frac{x^3}{3} \right]_0^1 \\ &= \left[\left(\frac{1^2}{4} + \frac{1^3}{3}\right) - \left(\frac{0^2}{4} + \frac{0^3}{3}\right) \right] \\ &= \frac{1}{4} + \frac{1}{3} \\ &= \frac{7}{12} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad F(x) &= \int_{-\infty}^x F(t) dt \\ &= \int_0^x f(t) dt \\ &= \int_0^x \frac{1}{2}(1+2t) dt \\ &= \int_0^x \frac{1}{2} + t dt \\ &= \left[\frac{1}{2}t + \frac{t^2}{2} \right]_0^x \\ &= \left[\left(\frac{1}{2}x + \frac{x^2}{2}\right) - \left(\frac{1(0) + 0^2}{2}\right) \right] \\ &= \frac{1}{2}(x+x^2) \\ &= \frac{1}{2}x(1+x) \quad (\text{ar gyfer } 0 \leq x \leq 1). \end{aligned}$$

$$\begin{aligned} \text{(c) (i)} \quad P(0.4 \leq X \leq 0.5) &= F(0.5) - F(0.4) \\ &= \frac{1}{2}(0.5)(1+0.5) - \frac{1}{2}(0.4)(1+0.4) \\ &= 0.375 - 0.28 \\ &= 0.095 \end{aligned}$$

$$\begin{aligned} \text{(ii) Canolrif } X: \quad F(x) &= 0.5 \\ \frac{1}{2}x(1+x) &= 0.5 \\ x(1+x) &= 1 \\ x+x^2 &= 1 \\ x^2+x-1 &= 0 \end{aligned}$$

→ Unai $x = \frac{-1+\sqrt{5}}{2}$ neu $x = \frac{-1-\sqrt{5}}{2}$
 $x = 0.62$ $x = -1.62$
 i 2 le degol i 2 le degol
 ond mae $0 \leq x \leq 1$ felly rhaid
 bod $x = 0.62$ i 2 le degol.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2 \cdot 1} \\ x &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

Graef 2010

$$\textcircled{8} \quad F(x) = 0 \quad \text{ar gyfer } x < 1$$

$$F(x) = \frac{1}{10}(x^2 + x - 2) \quad \text{ar gyfer } 1 \leq x \leq 3$$

$$F(x) = 1 \quad \text{ar gyfer } x > 3$$

(a) (i) $P(2 \leq X \leq 2.5) = F(2.5) - F(2)$

$$= \frac{1}{10}(2.5^2 + 2.5 - 2) - \frac{1}{10}(2^2 + 2 - 2)$$

$$= 0.675 - 0.4$$

$$= 0.275$$

(ii) canolrif X : $F(x) = 0.5$

$$\frac{1}{10}(x^2 + x - 2) = 0.5$$

$$x^2 + x - 2 = 5$$

$$x^2 + x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(-7)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{29}}{2}$$

Unai $x = \frac{-1 + \sqrt{29}}{2}$ new $x = \frac{-1 - \sqrt{29}}{2}$

$$x = 2.19 \text{ i } 2 \text{ le degol} \quad x = -3.19 \text{ i } 2 \text{ le degol.}$$

Onn $1 \leq x \leq 3$ felly rhaid bod $x = 2.19$ i 2 le degol.

(b) (i) $f(x) = \frac{d}{dx} F(x)$

$$= \frac{d}{dx} \left(\frac{1}{10}(x^2 + x - 2) \right)$$

$$= \frac{1}{10}(2x + 1)$$

$$= \frac{2}{10}x + \frac{1}{10}$$

Car gyfer $1 \leq x \leq 3$.

(ii) $f(4) = 0$

gan nad yw 4 yn
yramrediad $1 \leq x \leq 3$.

(iii) $E(X) = \int x f(x) dx$

$$= \int_1^3 x \left(\frac{2}{10}x + \frac{1}{10} \right) dx$$

$$= \int_1^3 \frac{2}{10}x^2 + \frac{1}{10}x dx$$

$$= \left[\frac{2x^3}{30} + \frac{x^2}{20} \right]_1^3$$

$$= \left[\left(\frac{2 \times 3^3}{30} + \frac{3^2}{20} \right) - \left(\frac{2 \times 1^3}{30} + \frac{1^2}{20} \right) \right]$$

$$= 2.25 - \frac{7}{60}$$

$$= \frac{32}{15}$$

Haf 2010

⑧ $F(x) = Kx(1-x^2)$ ar gyfer $0 \leq x \leq 1$
 $F(x) = 0$ Fel arall

(a) $\int f(x)dx = 1$
 $\int_0^1 Kx(1-x^2)dx = 1$
 $K \int_0^1 x - x^3 dx = 1$
 $K \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = 1$
 $K \left[\left(\frac{1^2}{2} - \frac{1^4}{4} \right) - \left(\frac{0^2}{2} - \frac{0^4}{4} \right) \right] = 1$
 $K \left(\frac{1}{2} - \frac{1}{4} \right) = 1$
 $\frac{1}{4}K = 1$
 $K = 4 \quad \checkmark$

(b) $E(x) = \int x f(x)dx$
 $= \int_0^1 x Kx(1-x^2)dx$
 $= \int_0^1 4x^2 - 4x^4 dx$
 $= \left[\frac{4x^3}{3} - \frac{4x^5}{5} \right]_0^1$
 $= \left[\left(\frac{4(1^3)}{3} - \frac{4(1^5)}{5} \right) - \left(\frac{4(0^3)}{3} - \frac{4(0^5)}{5} \right) \right]$
 $= \frac{4}{3} - \frac{4}{5}$
 $= \frac{8}{15}$

(c) (i) $F(x) = \int_{-\infty}^x f(t)dt$
 $= \int_0^x f(t)dt$
 $= \int_0^x Kt(1-t^2)dt$
 $= \int_0^x 4t - 4t^3 dt$
 $= \left[\frac{4t^2}{2} - \frac{4t^4}{4} \right]_0^x$
 $= \left[2t^2 - t^4 \right]_0^x$
 $= [(2x^2 - x^4) - (2(0)^2 - (0)^4)]$
 $= 2x^2 - x^4$
 (ar gyfer $0 \leq x \leq 1$).

(ii) $P(0.25 \leq X \leq 0.75)$
 $= F(0.75) - F(0.25)$
 $= (2 \times 0.75^2 - 0.75^4) -$
 $(2 \times 0.25^2 - 0.25^4)$
 $= \frac{20}{256} - \frac{3}{256}$
 $= 0.6875$

(iii) Ganoirif X : $F(x) = 0.5$
 $2x^2 - x^4 = 0.5$
 $0 = x^4 - 2x^2 + 0.5$

Gadewchi: $y = x^2$. Yna

$$0 = y^2 - 2y + 0.5$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(0.5)}}{2 \times 1}$$

$$y = \frac{2 \pm \sqrt{2}}{2}$$

$$\text{Unai } y = \frac{2+\sqrt{2}}{2} \quad \text{neu } y = \frac{2-\sqrt{2}}{2}$$

$$y = 1.707106781\cdots \quad \text{neu } y = 0.2928932188\cdots$$

ond $y = x^2$. Felly $x = \sqrt{y}$

$$\text{unai } x = \sqrt{1.707106781\cdots} \quad \text{neu } x = \sqrt{0.2928932188\cdots}$$

$$x = 1.31 \text{ i } 2 \text{ le degol}. \quad x = 0.54 \text{ i } 2 \text{ le degol}$$

ond $0 \leq x \leq 1$ felly rhaid bod $x = 0.54$ i 2 le degol.

Gaeaf 2011

$$\textcircled{9} \quad f(x) = \frac{1}{6}(x+1) \quad \text{ar gyfer } 1 \leq x \leq 3.$$

$$f(x) = 0 \quad \text{fel arall}$$

$$\begin{aligned} (a) \quad E(X) &= \int x f(x) dx \\ &= \int_1^3 x \left(\frac{1}{6}(x+1) \right) dx \\ &= \int_1^3 \frac{1}{6}x^2 + \frac{1}{6}x dx \\ &= \left[\frac{x^3}{18} + \frac{x^2}{12} \right]_1^3 \end{aligned}$$

$$= \left[\left(\frac{3^3}{18} + \frac{3^2}{12} \right) - \left(\frac{1^3}{18} + \frac{1^2}{12} \right) \right]$$

$$= 2.25 - \frac{5}{36}$$

$$= \frac{19}{9}$$

$$\begin{aligned}
 (b) (i) \quad F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_1^x f(t) dt \\
 &= \int_1^x \frac{t}{6}(t+1) dt \\
 &= \frac{1}{6} \int_1^x t^2 + t dt \\
 &= \frac{1}{6} \left[\frac{t^2}{2} + \frac{t^3}{3} \right]_1^x \\
 &= \frac{1}{6} \left[\left(\frac{x^2}{2} + x \right) - \left(\frac{1^2}{2} + 1 \right) \right] \\
 &= \frac{1}{6} \left(\frac{x^2}{2} + x - \frac{3}{2} \right) \\
 &= \frac{x^2}{12} + \frac{x}{6} - \frac{3}{12} \\
 &= \frac{x^2}{12} + \frac{x}{6} - \frac{1}{4}
 \end{aligned}$$

$$(ii) \quad F(4) = 1 \quad \text{gan fod } 4 > 3.$$

$$\begin{aligned}
 (iii) \quad P(1.5 \leq X \leq 2) &= F(2) - F(1.5) \\
 &= \left(\frac{2^2}{12} + \frac{2}{6} - \frac{1}{4} \right) - \left(\frac{1.5^2}{12} + \frac{1.5}{6} - \frac{1}{4} \right) \\
 &= \frac{5}{12} - \frac{3}{16} \\
 &= \frac{11}{48}
 \end{aligned}$$

$$(iv) \quad \text{Canalrif } X: \quad F(x) = 0.5$$

$$\frac{x^2}{12} + \frac{x}{6} - \frac{1}{4} = 0.5$$

$$x^2 + 2x - 3 = 6 \quad (\text{Muosiefb } 12)$$

$$x^2 + 2x - 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2 \cdot 1}$$

$$x = \frac{-2 \pm \sqrt{40}}{2}$$

Unai
 $x = \frac{-2 + \sqrt{40}}{2}$
 $x = 2.16$ i 2 ledegol
 neu
 $x = \frac{-2 - \sqrt{40}}{2}$
 $x = -4.16$ i 2 ledegol
 and $1 \leq x \leq 3$ felly
 rhaid bod
 $x = 2.16$
 i 2 ledegol.

Itaf 2011

⑧ $f(x) = 12x^2(1-x)$ ar gyfer $0 \leq x \leq 1$
 $f(x) = 0$ Fel arall

(a) (i) $E(x) = \int x f(x) dx$
 $= \int_0^1 x(12x^2(1-x)) dx$
 $= \int_0^1 12x^3 - 12x^4 dx$
 $= \left[\frac{12x^4}{4} - \frac{12x^5}{5} \right]_0^1$
 $= \left[\left(\frac{12(1^4)}{4} - \frac{12(1^5)}{5} \right) - \left(\frac{12(0^4)}{4} - \frac{12(0^5)}{5} \right) \right]$
 $= \frac{12}{4} - \frac{12}{5}$
 $= 0.6$

(ii) $E(\frac{1}{x}) = \int \frac{1}{x} f(x) dx$
 $= \int_0^1 \frac{1}{x}(12x^2(1-x)) dx$
 $= \int_0^1 12x - 12x^2 dx$
 $= \left[\frac{12x^2}{2} - \frac{12x^3}{3} \right]_0^1$
 $= [6x^2 - 4x^3]_0^1$
 $= [(6(1)^2 - 4(1)^3) - (6(0^2) - 4(0^3))]$
 $= 6 - 4$
 $= 2$

(iii) $P(0.2 \leq x \leq 0.5) = \int_{0.2}^{0.5} f(x) dx$
 $= \int_{0.2}^{0.5} 12x^2(1-x) dx$
 $= \int_{0.2}^{0.5} 12x^2 - 12x^3 dx$
 $= \left[\frac{12x^3}{3} - \frac{12x^4}{4} \right]_{0.2}^{0.5}$
 $= [4x^3 - 3x^4]_{0.2}^{0.5}$
 $= [(4 \times 0.5^3 - 3 \times 0.5^4) - (4 \times 0.2^3 - 3 \times 0.2^4)]$
 $= 0.3125 - 0.0272$
 $= 0.2853$

$$(b) F(y) = ay + by^2 \text{ ar gyfer } 1 \leq y \leq 2.$$

$$F(1) = 0$$

$$a(1) + b(1^2) = 0$$

$$a + b = 0$$

$$a = -b \quad \text{--- (1)}$$

$$F(2) = 1$$

$$a(2) + b(2^2) = 1$$

$$2a + 4b = 1 \quad \text{--- (2)}$$

Yn amnewid am a o (1) i (2):

$$2(-b) + 4b = 1$$

$$-2b + 4b = 1$$

$$2b = 1$$

$$\underline{b = \frac{1}{2}}$$

Yn amnewid yn ôi i (1):

$$\underline{a = -\frac{1}{2}}$$

Gaeaf 2012

(9)

$$F(x) = 0 \quad \text{ar gyfer } x < 1$$

$$F(x) = K(x^2 - x) \quad \text{ar gyfer } 1 \leq x \leq 3$$

$$F(x) = 1 \quad \text{ar gyfer } x > 3.$$

$$(a) (i) \quad F(3) = 1$$

$$K(3^2 - 3) = 1$$

$$K(9 - 3) = 1$$

$$6K = 1$$

$$K = \frac{1}{6}$$

$$(ii) P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - F(2)$$

$$= 1 - K(2^2 - 2)$$

$$= 1 - \frac{1}{6}(4 - 2)$$

$$= 1 - \frac{2}{6}$$

$$\approx \frac{2}{3}$$

(iii) canolrif X: $F(x) = 0.5$

$$K(x^2 - x) = 0.5$$

$$\frac{1}{6}(x^2 - x) = 0.5$$

$$x^2 - x = 3$$

$$x^2 - x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2 \times 1}$$

$$x = \frac{1 \pm \sqrt{13}}{2}$$

$$\text{Unai } x = \frac{1 + \sqrt{13}}{2} \text{ neu } x = \frac{1 - \sqrt{13}}{2}$$

$$x = 2.30 \text{ i 2 ldegol} \quad x = -1.30 \text{ i 2 ldegol}$$

ond $1 \leq x \leq 3$ felly rhaid bod $x = 2.30$ i 2 ldegol.

$$\begin{aligned}(\text{b}) (\text{i}) \quad F(x) &= \frac{d}{dx} F(x) \\&= \frac{d}{dx} (K(x^2 - x)) \\&= \frac{d}{dx} \left(\frac{1}{6}(x^2 - x) \right) \\&= \frac{1}{6}(2x - 1) \\&= \frac{1}{3}x - \frac{1}{6} \quad (\text{ar gyfer } 1 \leq x \leq 3)\end{aligned}$$

$$\begin{aligned}(\text{ii}) \quad E(X) &= \int_1^3 x f(x) dx \\&= \int_1^3 x \left(\frac{1}{3}x - \frac{1}{6} \right) dx \\&= \int_1^3 \frac{1}{3}x^2 - \frac{1}{6}x dx \\&= \left[\frac{x^3}{9} - \frac{x^2}{12} \right]_1^3 \\&= \left[\left(\frac{3^3}{9} - \frac{3^2}{12} \right) - \left(\frac{1^3}{9} - \frac{1^2}{12} \right) \right] \\&= \left(3 - \frac{3}{4} \right) - \left(\frac{1}{9} - \frac{1}{12} \right) \\&= \frac{20}{9}\end{aligned}$$

Haf 2012

⑨ $f(x) = \frac{1}{10}(2x+3x^2)$ ar gyfer $1 \leq x \leq 2$
 $f(x) = 0$ fel arall

(a) (i) $E(x) = \int x f(x) dx$
 $= \int_1^2 x \left(\frac{1}{10}(2x+3x^2)\right) dx$
 $= \int_1^2 \frac{1}{5}x^2 + \frac{3}{10}x^3 dx$
 $= \left[\frac{x^3}{15} + \frac{3x^4}{40} \right]_1^2$
 $= \left[\left(\frac{2^3}{15} + \frac{3(2^4)}{40} \right) - \left(\frac{1^3}{15} + \frac{3(1^4)}{40} \right) \right]$
 $= \frac{8}{15} + \frac{48}{40} - \frac{1}{15} - \frac{3}{40}$
 $= \frac{191}{120}$

(ii) $E(x^2) = \int x^2 f(x) dx$
 $= \int_1^2 x^2 \left(\frac{1}{10}(2x+3x^2)\right) dx$
 $= \int_1^2 \frac{1}{5}x^3 + \frac{3}{10}x^4 dx$
 $= \left[\frac{x^4}{20} + \frac{3x^5}{50} \right]_1^2$
 $= \left[\left(\frac{2^4}{20} + \frac{3(2^5)}{50} \right) - \left(\frac{1^4}{20} + \frac{3(1^5)}{50} \right) \right]$
 $= \frac{16}{20} + \frac{96}{50} - \frac{1}{20} - \frac{3}{50}$
 $= 2.61$

$\text{Var}(x) = E(x^2) - [E(x)]^2$
 $= 2.61 - \left(\frac{191}{120}\right)^2$
 ~~$= 0.08$~~ i 2 ie degol

$$\begin{aligned}
 (b) (i) \quad F(x) &= \int_{-\infty}^x f(t) dt \\
 &= \int_1^x f(t) dt \\
 &= \int_1^x \frac{1}{10}(2t+3t^2) dt \\
 &= \int_1^x \frac{1}{5}t + \frac{3}{10}t^2 dt \\
 &= \left[\frac{t^2}{10} + \frac{3t^3}{30} \right]_1^x \\
 &= \left[\left(\frac{x^2}{10} + \frac{3x^3}{30} \right) - \left(\frac{1^2}{10} + \frac{3 \cdot 1^3}{30} \right) \right] \\
 &= \frac{x^2}{10} + \frac{x^3}{10} - \frac{1}{10} - \frac{1}{10} \\
 &= \frac{1}{10}(x^2 + x^3 - 2) \quad (\text{ar gyfer } 1 \leq x \leq 2)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad P(X \leq 1.4) &= F(1.4) \\
 &= \frac{1}{10}(1.4^2 + 1.4^3 - 2) \\
 &= 0.2704
 \end{aligned}$$

(iii) Mae chwarter isaf X yn llai na 1.4 gan fod
 0.2704 yn fwy na 0.25

Gaeaf 2013

⑧

$$\begin{aligned}
 F(x) &= 0 && \text{ar gyfer } x < 0 \\
 F(x) &= 2x^2 - x^4 && \text{ar gyfer } 0 \leq x \leq 1 \\
 F(x) &= 1 && \text{ar gyfer } x > 1
 \end{aligned}$$

$$\begin{aligned}
 (a) (i) \quad P(0.25 \leq X \leq 0.75) &= F(0.75) - F(0.25) \\
 &= (2(0.75^2) - 0.75^4) - (2(0.25^2) - 0.25^4) \\
 &= \frac{207}{256} - \frac{31}{256} \\
 &= 0.6875
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \text{Canolrif } X: \quad F(x) &= 0.5 \\
 2x^2 - x^4 &= 0.5 \\
 4x^2 - 2x^4 &= 1
 \end{aligned}$$

$$0 = 2x^4 - 4x^2 + 1$$

$$2x^4 - 4x^2 + 1 = 0$$

OS yw'r canolrif yn m gna maer canolrif yn bodloni
 $2m^4 - 4m^2 + 1 = 0 \quad \checkmark$

(iii) Gadeuwch i $y = m^2$. Yna

$$2y^2 - 4y + 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{4 \pm \sqrt{(4)^2 - 4(2)(1)}}{2 \times 2}$$

$$y = \frac{4 \pm \sqrt{8}}{4}$$

Unai $y = \frac{4+\sqrt{8}}{4}$ neu $y = \frac{4-\sqrt{8}}{4}$
 $y = 1.707106781\cdots$ $y = 0.2928932188\cdots$

ond $y = m^2$ felly $m = \sqrt{y}$.

Felly unai $m = \sqrt{1.707106781\cdots}$ neu $m = \sqrt{0.2928932188\cdots}$
 $m = 1.31$ i 3 ff. yst. $m = 0.54$ i 3 ff. yst.

ond $0 \leq m \leq 1$ Felly $m = 0.54$ i 3 ffigur ystyrlion.

(b) (i) $f(x) = \frac{d}{dx} F(x)$
 $= \frac{d}{dx} (2x^2 - x^4)$
 $= 4x - 4x^3$ (ar gyfer $0 \leq x \leq 1$)

$$\begin{aligned} \text{(ii)} \quad E(\sqrt{x}) &= \int \sqrt{x} f(x) dx \\ &= \int_0^1 \sqrt{x} (4x - 4x^3) dx \\ &= 4 \int_0^1 x^{\frac{1}{2}} (x - x^3) dx \\ &= 4 \int x^{\frac{3}{2}} - x^{\frac{5}{2}} dx \\ &= 4 \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{9}{2}}}{\frac{9}{2}} \right]_0^1 \\ &= 4 \left[\left(\frac{1^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1^{\frac{9}{2}}}{\frac{9}{2}} \right) - \left(\frac{0^{\frac{5}{2}}}{\frac{5}{2}} - \frac{0^{\frac{9}{2}}}{\frac{9}{2}} \right) \right] \end{aligned}$$

$\Rightarrow = 4 \left(\frac{1}{\frac{5}{2}} - \frac{1}{\frac{9}{2}} \right)$
 $= 4 \left(\frac{2}{5} - \frac{2}{9} \right)$
 $= \frac{32}{45}$

Haf 2013

⑨ $f(x) = K(1 - \frac{x^2}{4})$ ar gyfer $0 \leq x \leq 2$
 $f(x) = 0$ fel arall

(a) $\int f(x) dx = 1$

$$\int_0^2 K(1 - \frac{x^2}{4}) dx = 1$$

$$K \int_0^2 1 - \frac{x^2}{4} dx = 1$$

$$K \left[x - \frac{x^3}{12} \right]_0^2 = 1$$

$$K \left[\left(2 - \frac{2^3}{12} \right) - \left(0 - \frac{0^3}{12} \right) \right] = 1$$

$$K \left(2 - \frac{8}{12} \right) = 1$$

$$K \left(\frac{4}{3} \right) = 1$$

$$K = \frac{3}{4} \quad \checkmark$$

(b) $E(x) = \int x f(x) dx$

$$= \int_0^2 x \left(K(1 - \frac{x^2}{4}) \right) dx$$

$$= \int_0^2 \frac{3}{4} x - \frac{3x^3}{16} dx$$

$$= \left[\frac{3x^2}{8} - \frac{3x^4}{64} \right]_0^2$$

$$= \left[\left(\frac{3(2^2)}{8} - \frac{3(2^4)}{64} \right) - \left(\frac{3(0^2)}{8} - \frac{3(0^4)}{64} \right) \right]$$

$$= \frac{12}{8} - \frac{3}{4}$$

$$= \frac{3}{4}$$

$$\rightarrow = \frac{3}{4} \left[\left(x - \frac{x^3}{12} \right) - \left(0 - \frac{0^3}{12} \right) \right]$$

$$= \frac{3}{4} \left(x - \frac{x^3}{12} \right)$$

(ar gyfer $0 \leq x \leq 2$)

(c) (i) $F(x) = \int_{-\infty}^x f(t) dt$

$$= \int_0^x f(t) dt$$

$$= \int_0^x K(1 - \frac{t^2}{4}) dt$$

$$= K \int_0^x 1 - \frac{t^2}{4} dt$$

$$= K \left[t - \frac{t^3}{12} \right]_0^x$$

(ii) $P(0.5 \leq X \leq 1.5) = F(1.5) - F(0.5)$

$$= \frac{3}{4} \left(1.5 - \frac{1.5^3}{12} \right) - \frac{3}{4} \left(0.5 - \frac{0.5^3}{12} \right)$$

$$= \frac{117}{128} - \frac{47}{128}$$

$$= 0.546875$$

S1 Graef 2014

(9)

$$F(x) = 0 \quad \text{ar gyfer} \quad x < 1$$

$$F(x) = K(x^3 - x) \quad \text{ar gyfer} \quad 1 \leq x \leq 2$$

$$F(x) = 1 \quad \text{ar gyfer} \quad x > 2$$

(a) (i) Rhaid bod $F(2) = 1$

$$K(2^3 - 2) = 1$$

$$K(8 - 2) = 1$$

$$6K = 1$$

$$K = \frac{1}{6}$$



$$(ii) P(1.25 \leq x \leq 1.75) = F(1.75) - F(1.25)$$

$$= \frac{1}{6}(1.75^3 - 1.75) - \frac{1}{6}(1.25^3 - 1.25)$$

$$= \frac{77}{128} - \frac{15}{128}$$

$$= \frac{31}{64}$$

$$(b) (i) f(x) = \frac{d}{dx}[F(x)]$$

$$= \frac{d}{dx}\left[\frac{1}{6}(x^3 - x)\right]$$

$$= \frac{d}{dx}\left[\frac{1}{6}x^3 - \frac{1}{6}x\right]$$

$$= \left(\frac{1}{6}\right)3x^2 - \frac{1}{6}$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{6} \quad \text{ar gyfer } 1 \leq x \leq 2$$

$$(ii) E(x) = \int x f(x) dx$$

$$= \int_1^2 x \left(\frac{1}{2}x^2 - \frac{1}{6}\right) dx$$

$$= \int_1^2 \frac{1}{2}x^3 - \frac{1}{6}x dx$$

$$= \left[\left(\frac{1}{2}\right)\frac{x^4}{4} - \left(\frac{1}{6}\right)\frac{x^2}{2} \right]_1^2$$

$$= \left[\frac{x^4}{8} - \frac{x^2}{12} \right]_1^2$$

$$= \left[\left(\frac{2^4}{8} - \frac{2^2}{12}\right) - \left(\frac{1^4}{8} - \frac{1^2}{12}\right) \right]$$

$$\rightarrow = \left(\frac{16}{8} - \frac{4}{12}\right) - \left(\frac{1}{8} - \frac{1}{12}\right)$$

$$= \frac{5}{3} - \frac{1}{24}$$

$$= \frac{13}{8}$$

$$= 1.625$$

S1 Haf 2014

⑨ $F(x) = 0$ ar gyfer $x < 0$
 $F(x) = 2x^3 - x^6$ ar gyfer $0 \leq x \leq 1$
 $F(x) = 1$ ar gyfer $x > 1$

(a) (i) $P(0.4 \leq x \leq 0.6)$
 $= F(0.6) - F(0.4)$
 $= 2(0.6^3) - 0.6^6 - [2(0.4^3) - 0.4^6]$
 $= 0.385344 - 0.123904$
 $= 0.26144$

(ii) Canolrif: $F(x) = 0.5$
 $2x^3 - x^6 = 0.5$
 $2x^3 - x^6 - 0.5 = 0$
 $x^6 - 2x^3 + 0.5 = 0$

Gadewch i $y = x^3$. $y^2 = x^6$.

Felly angen datrys $y^2 - 2y + 0.5 = 0$
 $2y^2 - 4y + 1 = 0$
 ~~$(2y - 1)(y - 1) = 0$~~

Fformula gwadratig $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2 \times 2}$$

Unai $y = \frac{4 + \sqrt{16 - 8}}{4}$ neu $y = \frac{4 - \sqrt{16 - 8}}{4}$

$$y = \frac{2 + \sqrt{2}}{2}$$

$$y = \frac{2 - \sqrt{2}}{2}$$

Felly $x = \sqrt[3]{\frac{2 + \sqrt{2}}{2}}$

$$x = \sqrt[3]{\frac{2 - \sqrt{2}}{2}}$$

$x = 1.195$ i 3 lle degol
Na (dim rhwng 0 a g 1) $\underline{x = 0.664}$
i 3 lle degol

$$\text{b) i) } f(x) = \frac{d}{dx}(F(x)) \\ = \frac{d}{dx}(2x^3 - x^6) \\ = 6x^2 - 6x^5 \quad (\text{är gyter } 0 \leq x \leq 1)$$

$$\text{ii) } E(x^3) = \int x^3 f(x) dx \\ = \int_0^1 x^3 (6x^2 - 6x^5) dx \\ = \int_0^1 6x^5 - 6x^8 dx \\ = 6 \int_0^1 x^5 - x^8 dx \\ = 6 \left[\frac{x^6}{6} - \frac{x^9}{9} \right]_0^1 \\ = 6 \left[\left(\frac{1^6}{6} - \frac{1^9}{9} \right) - \left(\frac{0^6}{6} - \frac{0^9}{9} \right) \right] \\ = 6 \left[\left(\frac{1}{6} - \frac{1}{9} \right) - (0 - 0) \right] \\ = 6 \times \frac{1}{18}$$
$$E(x^3) = \frac{1}{3}$$

SI Haf 2015

9) $f(x) = \frac{4}{9}(4x - x^3)$ av gyfer $1 \leq x \leq 2$
 $f(x) = 0$ fel avall

a) $E\left(\frac{1}{x}\right) = \int \frac{1}{x} f(x) dx$
= $\int_1^2 \frac{1}{x} \left(\frac{4}{9}(4x - x^3) \right) dx$
= $\frac{4}{9} \int_1^2 4 - x^2 dx$
= $\frac{4}{9} \left[4x - \frac{x^3}{3} \right]_1^2$
= $\frac{4}{9} \left[\left(4 \times 2 - \frac{2^3}{3} \right) - \left(4 \times 1 - \frac{1^3}{3} \right) \right]$
= $\frac{4}{9} \left[\left(8 - \frac{8}{3} \right) - \left(4 - \frac{1}{3} \right) \right]$
= $\frac{4}{9} \left[\frac{16}{3} - \frac{11}{3} \right]$
= $\frac{4}{9} \left[\frac{5}{3} \right]$
= $\frac{20}{27}$

b) (i) $F(x) = \int_{-\infty}^x f(t) dt$
= $\int_1^x f(t) dt$
= $\int_1^x \frac{4}{9}(4t - t^3) dt$
= $\frac{4}{9} \int_1^x 4t - t^3 dt$
= $\frac{4}{9} \left[\frac{4t^2}{2} - \frac{t^4}{4} \right]_1^x$
= $\frac{4}{9} \left[2t^2 - \frac{t^4}{4} \right]_1^x$
= $\frac{4}{9} \left[\left(2x^2 - \frac{x^4}{4} \right) - \left(2 \times 1^2 - \frac{1^4}{4} \right) \right]$
= $\frac{4}{9} \left[\left(2x^2 - \frac{x^4}{4} \right) - \left(2 - \frac{1}{4} \right) \right]$
= $\frac{4}{9} \left[2x^2 - \frac{x^4}{4} - \frac{7}{4} \right]$

$$\begin{aligned}
 &= \frac{8x^2}{9} - \frac{x^4}{9} - \frac{7}{9} \\
 &= \frac{8x^2}{9} - \frac{x^4}{9} - \frac{7}{9} \\
 &= \frac{1}{9}(8x^2 - x^4 - 7)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad P(1.25 \leq x \leq 1.75) &= F(1.75) - F(1.25) \\
 &= \frac{1}{9}(8 \times 1.75^2 - 1.75^4 - 7) - \frac{1}{9}(8 \times 1.25^2 - 1.25^4 - 7) \\
 &= \frac{1}{9}\left(\frac{2079}{256}\right) - \frac{1}{9}\left(\frac{783}{256}\right) \\
 &= \frac{23}{256} - \frac{87}{256} \\
 &= \frac{9}{16} \\
 &= \underline{\underline{0.5625}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \text{Canolrif } X: \quad F(X) = 0.5 \\
 \frac{1}{9}(8x^2 - x^4 - 7) &= 0.5 \\
 8x^2 - x^4 - 7 &= 0.5 \times 9 \\
 8x^2 - x^4 - 7 &= 4.5 \\
 0 &= x^4 - 8x^2 + 7 + 4.5 \\
 0 &= x^4 - 8x^2 + 11.5
 \end{aligned}$$

Gadewch i $y = x^2$. Yna

$$\begin{aligned}
 0 &= y^2 - 8y + 11.5 \\
 y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 y &= \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 11.5}}{2 \times 1}
 \end{aligned}$$

$$y = \frac{8 \pm \sqrt{18}}{2}$$

$$\text{Unai } y = \frac{8+\sqrt{18}}{2} \quad \text{neu } y = \frac{8-\sqrt{18}}{2}$$

ond $y = x^2$. Felly $x = \sqrt{y}$

$$\text{Unai } x = \sqrt{\frac{8+\sqrt{18}}{2}}$$

$x = 2.47$ i 21e degol

$$\text{neu } x = \sqrt{\frac{8-\sqrt{18}}{2}}$$

$x = 1.37$ i 21e degol.

ond $1 \leq x \leq 2$ felly rhaid bod $x = 1.37$ i 21e degol.
Hwn yw canolrif X.

SI Haf 2016

(9) $f(x) = K(2x-1)$ ar gyfer $1 \leq x \leq 2$
 $f(x) = 0$ fel arall

a) (i) $F(x) = \int_{-\infty}^x f(t)dt$

$$= \int_1^x K(2t-1)dt$$

$$= K \int_1^x 2t-1 dt$$

$$= K \left[\frac{2t^2}{2} - t \right]_1^x$$

$$= K [x^2 - x]$$

$$= K [(x^2 - x) - (1^2 - 1)]$$

$$= K [(x^2 - x) - 0]$$

$$= K(x^2 - x)$$

(ii) Yn defnyddio $F(2) = 1$
 $K(2^2 - 2) = 1$
 $K(4 - 2) = 1$
 $K = \frac{1}{2}$ ✓

b) (i) $E(x) = \int x f(x) dx$

$$= \int_1^2 x \left(\frac{1}{2} (2x-1) \right) dx$$

$$= \int_1^2 x^2 - \frac{1}{2} x dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{4} \right]_1^2$$

$$= \left[\left(\frac{2^3}{3} - \frac{2^2}{4} \right) - \left(\frac{1^3}{3} - \frac{1^2}{4} \right) \right]$$

$$= \left[\left(\frac{8}{3} - 1 \right) - \left(\frac{1}{3} - \frac{1}{4} \right) \right]$$

$$= \frac{19}{12}$$

(ii) Canolrif X : $F(x) = 0.5$

$$\frac{1}{2}(x^2 - x) = 0.5$$

2

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2 \times 1}$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$\text{Naillai } x = \frac{1 + \sqrt{5}}{2}$$

$$\text{neu } x = \frac{1 - \sqrt{5}}{2}$$

$$\underline{x = 1.6180} \text{ i 411.d.}$$

$$x = -0.6180 \text{ i 411.d.}$$

(0dim shung 1a 2)

$$(iii) P(X > 1.5) = 1 - P(X \leq 1.5)$$

$$= 1 - F(1.5)$$

$$= 1 - \frac{1}{2}(1.5^2 - 1.5)$$

$$= 1 - 0.375$$

$$P(X > 1.5) = 0.625$$

SI Haf 2017

8) $F(x) = 0$ ar gyfer $x < 1$
 $F(x) = K(x^4 - x^2)$ ar gyfer $1 \leq x \leq 2$
 $F(x) = 1$ ar gyfer $x > 2$

a) (i) Mae angen $F(2) = 1$
 $K(2^4 - 2^2) = 1$
 $K(16 - 4) = 1$
 $12K = 1$
 $K = \frac{1}{12}$ ✓

(ii) 95ed canradd: $F(x) = 0.95$

$$\frac{1}{12}(x^4 - x^2) = 0.95$$

$$x^4 - x^2 = 11.4$$

$$x^4 - x^2 - 11.4 = 0$$

Gadewch i $y = x^2$. Rydym angen datrys

$$y^2 - y - 11.4 = 0$$

Itafaliad cwadratig: $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-11.4)}}{2 \times 1}$$

$$y = \frac{1 \pm \sqrt{46.6}}{2}$$

Naill ai $y = \frac{1 + \sqrt{46.6}}{2}$ neu $y = \frac{1 - \sqrt{46.6}}{2}$

$$y = 3.913209633$$

$$y = -2.913209633$$

Felly

$$x = \sqrt{y}$$

$$x = \sqrt{3.913209633}$$

$$x = 1.98$$

$$x = \sqrt{y}$$

$$x = \sqrt{-2.913209633}$$

Dim datrysiaid

$$\begin{aligned}
 \text{(iii)} \quad & P(X < 1.25 \mid X < 1.75) \\
 & = \frac{P(X < 1.25 \cap X < 1.75)}{P(X < 1.75)} \\
 & = \frac{P(X < 1.25)}{P(X < 1.75)} \\
 & = \frac{F(1.25)}{F(1.75)} \\
 & = \frac{\frac{1}{12}(1.25^4 - 1.25^2)}{\frac{1}{12}(1.75^4 - 1.75^2)} \\
 & = \frac{1.25^4 - 1.25^2}{1.75^4 - 1.75^2} \\
 & = \frac{75}{539}
 \end{aligned}$$

b) (i) $f(x) = \frac{d}{dx} (F(x))$

$$\begin{aligned}
 & = \frac{d}{dx} \left(\frac{1}{12}(x^4 - x^2) \right) \\
 & = \frac{1}{12}(4x^3 - 2x) \\
 F(x) & = \frac{x^3}{3} - \frac{x}{6} \quad (\text{diliys ar gyfer } 1 \leq x \leq 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad E(\sqrt{X}) &= \int \sqrt{x} f(x) dx \\
 &= \int_1^2 \sqrt{x} \left(\frac{x^3}{3} - \frac{x}{6} \right) dx \\
 &= \int_1^2 x^{\frac{1}{2}} \left(\frac{x^3}{3} - \frac{x}{6} \right) dx \\
 &= \int_1^2 \frac{x^{3\frac{1}{2}}}{3} - \frac{x^{1\frac{1}{2}}}{6} dx \\
 &= \left[\frac{x^{4\frac{1}{2}}}{4\frac{1}{2} \times 3} - \frac{x^{2\frac{1}{2}}}{2\frac{1}{2} \times 6} \right]_1^2 \\
 &= \left[\frac{2x^{4\frac{1}{2}}}{27} - \frac{x^{2\frac{1}{2}}}{15} \right]_1^2
 \end{aligned}$$

$$\begin{aligned}&= \left(\frac{2(2^{4\frac{1}{2}})}{27} - \frac{2^{2\frac{1}{2}}}{15} \right) - \left(\frac{2(1^{4\frac{1}{2}})}{27} - \frac{1^{2\frac{1}{2}}}{15} \right) \\&= 1.298981346 - \frac{1}{135} \\&= 1.291573939 \\&= \underline{1.2916} \text{ i 4 11e degol}\end{aligned}$$

S1 Itaf 2018

9) $F(x) = 0$ ar gyfer $x < 1$
 $F(x) = \frac{1}{10} (x^2 + x - 2)$ ar gyfer $1 \leq x \leq 3$
 $F(x) = 1$ ar gyfer $x > 3$

a) i) $P(2 < x < 2.5) = F(2.5) - F(2)$
 $= \frac{1}{10} (2.5^2 + 2.5 - 2) - \frac{1}{10} (2^2 + 2 - 2)$
 $= 0.675 - 0.4$
 $= \underline{\underline{0.275}}$

ii) Chwarter uchaf X: $F(x) = 0.75$

$$\frac{1}{10} (x^2 + x - 2) = 0.75$$

$$x^2 + x - 2 = 7.5$$

$$x^2 + x - 9.5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(-9.5)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{39}}{2}$$

$$\text{Naill a } x = \frac{-1 + \sqrt{39}}{2} \quad \text{neu } x = \frac{-1 - \sqrt{39}}{2}$$

$$x = 2.622498999$$

$$x = -3.622498999$$

$x = 2.6225$

i 4 lle degol: e

(Ddim yn bevhnasol)

$$b) i) f(x) = \frac{d}{dx}(F(x))$$

$$= \frac{d}{dx}\left(\frac{1}{10}(x^2 + x - 2)\right)$$

$$= \frac{d}{dx}\left(\frac{1}{10}x^2 + \frac{1}{10}x - \frac{2}{10}\right)$$

$$f(x) = \frac{2}{10}x + \frac{1}{10} \quad (\text{ar gyfer } 1 \leq x \leq 3)$$

$$ii) E(X) = \int x f(x) dx$$

$$= \int_1^3 x\left(\frac{2}{10}x + \frac{1}{10}\right) dx$$

$$= \int_1^3 \frac{2}{10}x^2 + \frac{1}{10}x dx$$

$$= \left[\frac{2}{10} \frac{x^3}{3} + \frac{1}{10} \frac{x^2}{2} \right]_1^3$$

$$= \left[\frac{x^3}{15} + \frac{x^2}{20} \right]_1^3$$

$$= \left(\frac{3^3}{15} + \frac{3^2}{20} \right) - \left(\frac{1^3}{15} + \frac{1^2}{20} \right)$$

$$= \frac{9}{4} - \frac{7}{60}$$

$$= \underline{\underline{\frac{32}{15}}}$$

51 Haf 2019

8) $f(x) = \frac{3}{4}x^2(2-x)$ ar gyfer $0 \leq x \leq 2$
= 0 fel arall

a) i) $E(X) = \int_{-\infty}^{\infty} xf(x) dx$
= $\int_0^2 x \left(\frac{3}{4}x^2(2-x)\right) dx$
= $\int_0^2 \frac{3}{4}x^3(2-x) dx$
= $\int_0^2 \frac{3}{4}(2x^3) - \frac{3}{4}x^4 dx$
= $\int_0^2 \frac{3}{2}x^3 - \frac{3}{4}x^4 dx$
= $\left[\frac{3x^4}{2 \times 4} - \frac{3x^5}{4 \times 5} \right]_0^2$
= $\left(\frac{3(2^4)}{8} - \frac{3(2^5)}{20} \right) - \left(\frac{3(0^4)}{8} - \frac{3(0^5)}{20} \right)$
= $(6 - 4.8) - (0 - 0)$
= 1.2

ii) Mae angen ffeindio pwynt macsimum $f(x)$.

$$f(x) = \frac{3}{2}x^2 - \frac{3}{4}x^3$$

$$f'(x) = 3x - \frac{9}{4}x^2$$

Pwyntau arhosol $\Rightarrow f'(x) = 0$

$$3x - \frac{9}{4}x^2 = 0$$

$$x(3 - \frac{9}{4}x) = 0$$

Naillai $x=0$ neu $3 - \frac{9}{4}x = 0$

$$3 = \frac{9}{4}x$$

$$\frac{12}{9} = x$$

$$x = \frac{4}{3}$$

$$f''(x) = 3 - \frac{9}{2}x$$

os yw $x=0$ mae $f''(x) = 3 - \frac{9}{2}(0)$
= 3

Felly pwynt minimum yw $x=0$

$$\text{os yw } x = \frac{4}{3} \text{ mae } f''(x) = 3 - \frac{9}{2}\left(\frac{4}{3}\right) \\ = -3$$

Felly mae puntyr macsimum pan fo $x = \frac{4}{3}$

$$\begin{aligned} b) (i) F(x) &= \int_{-\infty}^x f(t) dt \\ &= \int_0^x \frac{3}{2}t^2 - \frac{3}{4}t^3 dt \\ &= \left[\frac{3t^3}{2 \times 3} - \frac{3t^4}{4 \times 4} \right]_0^x \\ &= \left(\frac{3x^3}{6} - \frac{3x^4}{16} \right) - \left(\frac{3(0)^3}{6} - \frac{3(0)^4}{16} \right) \\ &= \frac{x^3}{2} - \frac{3x^4}{16} \end{aligned}$$

(ii) Os yw $x = 1.2$ (y cymedr) gna

$$F(x) = \frac{1.2^3}{2} - \frac{3(1.2^4)}{16} \\ = 0.4752$$

Os yw $x = \frac{4}{3}$ (y modd gna

$$F(x) = \frac{\frac{4}{3}^3}{2} - \frac{3\left(\frac{4}{3}\right)^4}{16} \\ = 0.5926 \text{ i 4 llawdegol}$$

Mae 0.5 yn gorwedd rhwng 0.4752 a 0.5926

Felly mae canolnif X yn gorwedd rhwng y modd ari cymedr.