S1: Hapnewidyn Arwahanol

Gaeaf 2005

(8) (a) $\Sigma P(X=x) = 1$

$0.1 + a + b + 0.3 + 0.2 = 1$

$0.6 + a + b = 1$

$a + b = 0.4$ ——(1) ✓

(b) (i) $E(X) = 3.4$

$\Sigma x\, P(X=x) = 3.4$

$1 \times 0.1 + 2 \times a + 3 \times b + 4 \times 0.3 + 5 \times 0.2 = 3.4$

$0.1 + 2a + 3b + 1.2 + 1 = 3.4$

$2a + 3b + 2.3 = 3.4$

$2a + 3b = 1.1$ ——(2)

(ii) (1) $\Rightarrow a = 0.4 - b$. ——(3)

Yn amnewid am a o (3) i (2):

$2(0.4 - b) + 3b = 1.1$

$0.8 - 2b + 3b = 1.1$

$b = 1.1 - 0.8$

$b = 0.3$

Yn amnewid yn ôl yn (3): $a = 0.4 - 0.3$

$a = 0.1$

(c) $E\left(\frac{1}{1+x}\right) = \Sigma \frac{1}{1+x} P(X=x)$

$= \frac{1}{1+1} \times 0.1 + \frac{1}{1+2} \times 0.1 + \frac{1}{1+3} \times 0.3 + \frac{1}{1+4} \times 0.3 + \frac{1}{1+5} \times 0.2$

$= \frac{1}{2} \times 0.1 + \frac{1}{3} \times 0.1 + \frac{1}{4} \times 0.3 + \frac{1}{5} \times 0.3 + \frac{1}{6} \times 0.2$

$= \frac{151}{600}$

Haf 2005

(7) (a) $\sum P(X=x) = 1$

$K(1+1) + K(1+2) + K(1+3) + K(1+4) + K(1+5) = 1$

$2K + 3K + 4K + 5K + 6K = 1$

$20K = 1$

$K = \frac{1}{20}$ ✓

(b) $E(X) = \sum x\, P(X=x)$

$= 1 \times 2K + 2 \times 3K + 3 \times 4K + 4 \times 5K + 5 \times 6K$

$= 2K + 6K + 12K + 20K + 30K$

$= 70K$

$= 70 \times \frac{1}{20}$

$= 3.5$

$E(X^2) = \sum x^2 P(X=x)$

$= 1^2 \times 2K + 2^2 \times 3K + 3^2 \times 4K + 4^2 \times 5K + 5^2 \times 6K$

$= 2K + 12K + 36K + 80K + 150K$

$= 280K$

$= 280 \times \frac{1}{20}$

$= 14$

$Var(X) = E(X^2) - [E(X)]^2$

$= 14 - 3.5^2$

$= 1.75$

(c) $P(X_1 + X_2 = 4) = P(X_1=1, X_2=3 \text{ neu } X_1=2, X_2=2 \text{ neu } X_1=3, X_2=1)$

$= P(X_1=1, X_2=3) + P(X_1=2, X_2=2) + P(X_1=3, X_2=1)$

$= 2K \times 4K + 3K \times 3K + 4K \times 2K$

$= 8K^2 + 9K^2 + 8K^2$

$= 25K^2$

$= 25\left(\frac{1}{20}\right)^2$

$= 0.0625$

(ch) $Y = 2X + 3$

$E(Y) = 2E(X) + 3$
$= 2 \times 3.5 + 3$
$= 10$

$Var(Y) = 2^2 \times Var(X)$
$= 4 \times 1.75$
$= 7$

Gaeaf 2006

(8) (a) Rhaid bod $\Sigma P(X = x) = 1$ a bod pob tebygolrwydd yn y tabl rhwng 0 ac 1.
Dim ond y gwerthoedd $0 \leq \theta \leq 0.7$ sy'n sicrhau hyn.

(b) (i) $E(X) = 3$

$\Sigma x P(X = x) = 3$

$1 \times 0.1 + 2 \times 0.2 + 3 \times \theta + 4 \times (0.7 - \theta) = 3$

$0.1 + 0.4 + 3\theta + 2.8 - 4\theta = 3$

$3.3 - \theta = 3$

$\theta = 0.3$

(ii) $E(X^3) = \Sigma x^3 P(X = x)$

$= 1^3 \times 0.1 + 2^3 \times 0.2 + 3^3 \times 0.3 + 4^3 \times 0.4$

$= 0.1 + 1.6 + 8.1 + 25.6$

$= 35.4$

Haf 2006

⑦ (a) $\sum P(X=x) = 1$

$K + 2K + 3K + 4K + 5K = 1$

$15K = 1$

$K = \frac{1}{15}$ ✓

(b) $E(X) = \sum x\, P(X=x)$

$= 1\times K + 2\times 2K + 3\times 3K + 4\times 4K + 5\times 5K$

$= K + 4K + 9K + 16K + 25K$

$= 55K$

$= 55\left(\frac{1}{15}\right)$

$= \frac{11}{3}$

$E(X^2) = \sum x^2 P(X=x)$

$= 1^2\times K + 2^2\times 2K + 3^2\times 3K + 4^2\times 4K + 5^2\times 5K$

$= K + 8K + 27K + 64K + 125K$

$= 225K$

$= 225\left(\frac{1}{15}\right)$

$= 15$

$Var(X) = E(X^2) - [E(X)]^2$

$= 15 - \left(\frac{11}{3}\right)^2$

$= \frac{14}{9}$

(c) $Y = X_1 + X_2$

$P(Y=6) = P(X_1=1, X_2=5 \text{ ou } X_1=2, X_2=4 \text{ ou } X_1=3, X_2=3$
$\text{ou } X_1=4, X_2=2 \text{ ou } X_1=5, X_2=1)$

$= P(X_1=1, X_2=5) + P(X_1=2, X_2=4) + P(X_1=3, X_2=3)$
$+ P(X_1=4, X_2=2) + P(X_1=5, X_2=1)$

$= K\times 5K + 2K\times 4K + 3K\times 3K + 4K\times 2K + 5K\times K$

$= 5K^2 + 8K^2 + 9K^2 + 8K^2 + 5K^2$

$= 35K^2$

$= \frac{7}{45}$

Iaeaf 2007

⑥ (a) $\sum P(X=x) = 1$

$0.3 + p + 0.1 + q + 0.05 = 1$

$0.45 + p + q = 1$

$p + q = 0.55$ —— ① ✓

(b) $E(X) = 2.75$

$\sum x\,P(X=x) = 2.75$

$1\times0.3 + 2\times p + 3\times0.1 + 4\times q + 5\times0.05 = 2.75$

$0.3 + 2p + 0.3 + 4q + 0.25 = 2.75$

$2p + 4q + 0.85 = 2.75$

$2p + 4q = 1.9$ —— ②

① $\Rightarrow p = 0.55 - q$ —— ③

Yn amnewid am p o ③ i ②:

$2(0.55 - q) + 4q = 1.9$

$1.1 - 2q + 4q = 1.9$

$2q = 1.9 - 1.1$

$2q = 0.8$

$q = 0.4$ → Yn amnewid yn ôl i ③:

$p = 0.55 - 0.4$

$p = 0.15$

(c) $E(X^2) = \sum x^2 P(X=x)$

$= 1^2\times0.3 + 2^2\times0.15 + 3^2\times0.1 + 4^2\times0.4 + 5^2\times0.05$

$= 0.3 + 0.6 + 0.9 + 6.4 + 1.25$

$= 9.45$

$Var(X) = E(X^2) - [E(X)]^2$

$= 9.45 - 2.75^2$

$= 1.8875$

(ch) $Y = 4X + 2$

$E(Y) = 4E(X) + 2$
$= 4 \times 2.75 + 2$
$= 13$

$Var(Y) = 4^2 \times Var(X)$
$= 16 \times 1.8875$
$= 30.2$

(ii) $P(Y < 15) = P(4X + 2 < 15)$
$= P(4X < 13)$
$= P(X < 3.25)$
$= P(X=1) + P(X=2) + P(X=3)$
$= 0.3 + 0.15 + 0.1$
$= 0.55$

Haf 2007

⑥ (a) $\Sigma P(X=x) = 1$

$K(1^2) + K(2^2) + K(3^2) + K(4^2) = 1$

$K + 4K + 9K + 16K = 1$

$30K = 1$

$K = \frac{1}{30}$ ✓

(b) $E(X) = \Sigma x P(X=x)$
$= 1 \times K(1^2) + 2 \times K(2^2) + 3 \times K(3^2) + 4 \times K(4^2)$
$= K + 8K + 27K + 64K$
$= 100K$
$= 100\left(\frac{1}{30}\right)$
$= \frac{10}{3}$

$E(X^2) = \Sigma x^2 P(X=x)$
$= 1^2 \times K(1^2) + 2^2 \times K(2^2) + 3^2 \times K(3^2) + 4^2 \times K(4^2)$
$= K + 16K + 81K + 256K$
$= 354K$
$= 354\left(\frac{1}{30}\right)$
$= 11.8$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$
$$= 11.8 - \left(\frac{12}{5}\right)^2$$
$$= \frac{31}{75}$$

Haf 2008

(6) (a) Rhaid bod $\Sigma P(X=x) = 1$ a rhaid bod bob tebygoliaeth rhwng 0 ac 1. Mae'r gwerthoedd $0 \leq \theta \leq \frac{1}{3}$ yn sicrhau hyn.

(b) (i) $E(X) = 2.2$
$$\Sigma x P(X=x) = 2.2$$
$$1 \times \theta + 2 \times 2\theta + 3 \times (1-3\theta) = 2.2$$
$$\theta + 4\theta + 3 - 9\theta = 2.2$$
$$-4\theta + 3 = 2.2$$
$$-4\theta = -0.8$$
$$\theta = 0.2$$ ✓

(ii) $E(X^2) = \Sigma x^2 P(X=x)$
$$= 1^2 \times \theta + 2^2 \times 2\theta + 3^2 \times (1-3\theta)$$
$$= \theta + 8\theta + 9 - 27\theta$$
$$= 9 - 18\theta$$
$$= 5.4$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$
$$= 5.4 - 2.2^2$$
$$= 0.56$$
Gwyriad Safonol $(X) = \sqrt{0.56}$
$$\approx 0.75 \text{ i } 2 \text{ le degol}$$

(iii) $E\left(\frac{1}{x}\right) = \sum \frac{1}{x} P(X=x)$

$= \frac{1}{1} \times \theta + \frac{1}{2} \times 2\theta + \frac{1}{3} \times (1-3\theta)$

$= \theta + \theta + \frac{1}{3} - \theta$

$= \theta + \frac{1}{3}$

$= 0.2 + \frac{1}{3}$

$= \frac{8}{15}$

Haf 2008

(6) (a) $P(X=x) = 1$

$K(1+1) + K(1+2) + K(1+3) = 1$

$2K + 3K + 4K = 1$

$9K = 1$

$K = \frac{1}{9}$ ✓

(b) $E(x) = \sum x P(X=x)$

$= 1 \times K(1+1) + 2 \times K(1+2) + 3 \times K(1+3)$

$= 2K + 6K + 12K$

$= 20K$

$= 20\left(\frac{1}{9}\right)$

$= \frac{20}{9}$

(c) $E\left(\frac{1}{x}\right) = \sum \frac{1}{x} P(X=x)$

$= \frac{1}{1} \times K(1+1) + \frac{1}{2} \times K(1+2) + \frac{1}{3} \times K(1+3)$

$= 2K + 1.5K + \frac{4}{3}K$

$= \frac{29}{6}K$

$= \frac{29}{6}\left(\frac{1}{9}\right)$

$= \frac{29}{54}$

Gacaf 2009

(8) (a) $E(X) = \sum x P(X=x)$

$= 2\left(\frac{10-2}{20}\right) + 4\left(\frac{10-4}{20}\right) + 6\left(\frac{10-6}{20}\right) + 8\left(\frac{10-8}{20}\right)$

$= 2 \times \frac{2}{5} + 4 \times \frac{3}{10} + 6 \times \frac{1}{5} + 8 \times \frac{1}{10}$

$= 4$

$E(X^2) = \sum x^2 P(X=x)$

$= 2^2 \times \frac{2}{5} + 4^2 \times \frac{3}{10} + 6^2 \times \frac{1}{5} + 8^2 \times \frac{1}{10}$

$= 20$

$Var(X) = E(X^2) - [E(X)]^2$

$= 20 - 4^2$

$= 4$

(b) $P(X_1 + X_2 = 8) = P(X_1 = 2, X_2 = 6$ atau $X_1 = 4, X_2 = 4$ atau $X_1 = 6, X_2 = 2)$

$= P(X_1 = 2, X_2 = 6) + P(X_1 = 4, X_2 = 4) + P(X_1 = 6, X_2 = 2)$

$= \frac{2}{5} \times \frac{1}{5} + \frac{3}{10} \times \frac{3}{10} + \frac{1}{5} \times \frac{2}{5}$

$= 0.25$

Haf 2009

⑥ (a) (i) $E(X) = \sum x P(X=x)$

$= 1\times0.1 + 2\times0.2 + 3\times0.3 + 4\times0.3 + 5\times0.1$

$= 0.1 + 0.4 + 0.9 + 1.2 + 0.5$

$= 3.1$

(ii) $E(X^2) = \sum x^2 P(X=x)$

$= 1^2\times0.1 + 2^2\times0.2 + 3^2\times0.3 + 4^2\times0.3 + 5^2\times0.1$

$= 0.1 + 0.8 + 2.7 + 4.8 + 2.5$

$= 10.9$

$Var(X) = E(X^2) - [E(X)]^2$

$= 10.9 - 3.1^2$

$= 1.29$

(b) $P(X_1 = X_2) = P(X_1=1, X_2=1 \text{ neu } X_1=2, X_2=2 \text{ neu } X_1=3, X_2=3 \text{ neu } X_1=4, X_2=4 \text{ neu } X_1=5, X_2=5)$

$= P(X_1=1, X_2=1) + P(X_1=2, X_2=2) + P(X_1=3, X_2=3) + P(X_1=4, X_2=4) + P(X_1=5, X_2=5)$

$= 0.1\times0.1 + 0.2\times0.2 + 0.3\times0.3 + 0.3\times0.3 + 0.1\times0.1$

$= 0.01 + 0.04 + 0.09 + 0.09 + 0.01$

$= 0.24$

Gorffennaf 2010

(4) (a) Rhaid bod $\sum P(X=x) = 1$ a bod pob tebygolrwydd yn y tabl rhwng 0 ac 1. Dim ond $0 \leq \lambda \leq 0.4$ sy'n sicrhau hyn.

(b) (i) $E(X) = 4.25$

$\sum x P(X=x) = 4.25$

$2 \times 0.1 + 3 \times 0.2 + 4 \times 0.3 + 5 \times \lambda + 6 \times (0.4 - \lambda) = 4.25$

$0.2 + 0.6 + 1.2 + 5\lambda + 2.4 - 6\lambda = 4.25$

$4.4 - \lambda = 4.25$

$\lambda = 0.15$

(ii) $E(X^2) = \sum x^2 P(X=x)$

$= 2^2 \times 0.1 + 3^2 \times 0.2 + 4^2 \times 0.3 + 5^2 \times \lambda + 6^2 \times (0.4 - \lambda)$

$= 0.4 + 1.8 + 4.8 + 25 \times 0.15 + 36 \times 0.25$

$= 7 + 3.75 + 9$

$= 19.75$

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 19.75 - 4.25^2$

$= 1.6875$

Haf 2010

(6) (a) $\sum P(X=x) = 1$

$K + 3K + 5K + 7K = 1$

$16K = 1$

$K = \frac{1}{16}$ ✓

(b) (i) $E(x) = \sum x P(X=x)$

$= K + 3\times 3K + 5\times 5K + 7\times 7K$

$= K + 9K + 25K + 49K$

$= 84K$

$= 84\left(\frac{1}{16}\right)$

$= 5.25$

(ii) $E\left(\frac{1}{x}\right) = \sum \frac{1}{x} P(X=x)$

$= \frac{1}{1}\times K + \frac{1}{3}\times 3K + \frac{1}{5}\times 5K + \frac{1}{7}\times 7K$

$= K + K + K + K$

$= 4 \times \frac{1}{16}$

$= 0.25$

(c) (i) $P(X_1 + X_2 = 6) = P(X_1 = 1, X_2 = 5$ neu $X_1 = 3, X_2 = 3$
neu $X_1 = 5, X_2 = 1)$

$= P(X_1 = 1, X_2 = 5) + P(X_1 = 3, X_2 = 3)$
$+ P(X_1 = 5, X_2 = 1)$

$= K \times 5K + 3K \times 3K + 5K \times K$

$= 5K^2 + 9K^2 + 5K^2$

$= 19K^2$

$= 19 \times \left(\frac{1}{16}\right)^2$

$= \frac{19}{256}$

$(= 0.07421875)$

(ii) $P(X_1 = X_2) = P(X_1=1, X_2=1 \text{ neu } X_1=3, X_2=3 \text{ neu } X_1=5, X_2=5 \text{ neu } X_1=7, X_2=7)$

$= P(X_1=1, X_2=1) + P(X_1=3, X_2=3) + P(X_1=5, X_2=5) + P(X_1=7, X_2=7)$

$= K \times K + 3K \times 3K + 5K \times 5K + 7K \times 7K$

$= K^2 + 9K^2 + 25K^2 + 49K^2$

$= 84K^2$

$= 84 \left(\frac{1}{16}\right)^2$

$= \frac{21}{64}$

$(= 0.328125)$

Gaeaf 2011

(7) (a) (i) Rhaid bod $\Sigma P(X=x) = 1$ a bod bob tebygolrwydd yn y tabl rhwng 0 ac 1. Dim ond $0 \leq \alpha \leq 0.4$ sy'n siarhau hyn.

(ii) $E(X) = \Sigma x P(X=x)$

$= .1 \times (0.4-\alpha) + 2 \times 2\alpha + 3 \times (0.6-\alpha)$

$= 0.4 - \alpha + 4\alpha + 1.8 - 3\alpha$

$= 2.2$ (Adim yn dibynnu ar α).

(iii) $E(X^2) = \Sigma x^2 P(X=x)$

$= 1^2 \times (0.4-\alpha) + 2^2 \times 2\alpha + 3^2 \times (0.6-\alpha)$

$= 0.4 - \alpha + 8\alpha + 5.4 - 9\alpha$

$= 5.8 - 2\alpha$

$Var(X) = 0.66$

$E(X^2) - [E(X)]^2 = 0.66$

$5.8 - 2\alpha - 2.2^2 = 0.66$

$5.8 - 2\alpha - 4.84 = 0.66$

$-2\alpha = -0.3$

$\alpha = 0.15$

(b) $P(X_1 = X_2) = P(X_1=1, X_2=1 \text{ atau } X_1=2, X_2=2 \text{ atau } X_1=3, X_2=3)$

$= P(X_1=1, X_2=1) + P(X_1=2, X_2=2) + P(X_1=3, X_2=3)$

$= (0.4-\alpha)(0.4-\alpha) + (2\alpha)(2\alpha) + (0.6-\alpha)(0.6-\alpha)$

$= 0.15^2 + 0.5^2 + 0.35^2$

$= 0.395$

Ifaf 2011

⑤ (a) $\sum P(X=x) = 1$

$K(1^2) + K(2^2) + K(3^2) + K(4^2) = 1$

$K + 4K + 9K + 16K = 1$

$30K = 1$

$K = \frac{1}{30}$ ✓

(b) $E(X) = \sum x\, P(X=x)$

$= 1 \times K(1^2) + 2 \times K(2^2) + 3 \times K(3^2) + 4 \times K(4^2)$

$= K + 8K + 27K + 64K$

$= 100K$

$= 100\left(\frac{1}{30}\right)$

$= \frac{10}{3}$

$E(X^2) = \sum x^2 P(X=x)$

$= 1^2 \times K(1^2) + 2^2 \times K(2^2) + 3^2 \times K(3^2) + 4^2 \times K(4^2)$

$= K + 16K + 81K + 256K$

$= 354K$

$= 354\left(\frac{1}{30}\right)$

$= 11.8$

$Var(X) = E(X^2) - [E(X)]^2$

$= 11.8 - \left(\frac{10}{3}\right)^2$

$= \frac{31}{45}$

(c) $P(X_1 + X_2 = 4) = P(X_1=1, X_2=3 \text{ atau } X_1=2, X_2=2 \text{ atau } X_1=3, X_2=1)$

$= P(X_1=1, X_2=3) + P(X_1=2, X_2=2) + P(X_1=3, X_2=1)$

$= K(1^2) \times K(3^2) + K(2^2) \times K(2^2) + K(3^2) \times K(1^2)$

$= 9K^2 + 16K^2 + 9K^2$

$= 34K^2$

$= 34\left(\frac{1}{30}\right)^2$

$= \frac{17}{450}$

Jaeaf 2012

(7) (a) $E(X) = \Sigma x P(X=x)$

$= 1 \times 0.1 + 2 \times 0.1 + 3 \times 0.2 + 4 \times 0.2 + 5 \times 0.4$

$= 0.1 + 0.2 + 0.6 + 0.8 + 2$

$= 3.7$

$E(X^2) = \Sigma x^2 P(X=x)$

$= 1^2 \times 0.1 + 2^2 \times 0.1 + 3^2 \times 0.2 + 4^2 \times 0.2 + 5^2 \times 0.4$

$= 0.1 + 0.4 + 1.8 + 3.2 + 10$

$= 15.5$

$Var(X) = E(X^2) - [E(X)]^2$

$= 15.5 - 3.7^2$

$= 1.81$

(b) $E\left(\frac{1}{X^2}\right) = \Sigma \frac{1}{x^2} P(X=x)$

$= \frac{1}{1^2} \times 0.1 + \frac{1}{2^2} \times 0.1 + \frac{1}{3^2} \times 0.2 + \frac{1}{4^2} \times 0.2 + \frac{1}{5^2} \times 0.4$

$= 0.1 + 0.025 + \frac{1}{45} + 0.0125 + 0.016$

$= \frac{3163}{18000}$

(c) (i) $P(X_1 + X_2 = 6) = P(X_1 = 1, X_2 = 5$ ou $X_1 = 2, X_2 = 4$ ou
$X_1 = 3, X_2 = 3$ ou $X_1 = 4, X_2 = 2$ ou
$X_1 = 5, X_2 = 1)$

$= P(X_1 = 1, X_2 = 5) + P(X_1 = 2, X_2 = 4) +$
$P(X_1 = 3, X_2 = 3) + P(X_1 = 4, X_2 = 2) +$
$P(X_1 = 5, X_2 = 1)$

$= 0.1 \times 0.4 + 0.1 \times 0.2 + 0.2 \times 0.2$
$+ 0.2 \times 0.1 + 0.4 \times 0.1$

$= 0.04 + 0.02 + 0.04 + 0.02 + 0.04$

$= 0.16$

(ii) $P(X_1 = X_2) = P(X_1 = 1, X_2 = 1$ ou $X_1 = 2, X_2 = 2$ ou
$X_1 = 3, X_2 = 3$ ou $X_1 = 4, X_2 = 4$ ou
$X_1 = 5, X_2 = 5)$

$= P(X_1 = 1, X_2 = 1) + P(X_1 = 2, X_2 = 2) +$
$P(X_1 = 3, X_2 = 3) + P(X_1 = 4, X_2 = 4) +$
$P(X_1 = 5, X_2 = 5)$

$= 0.1^2 + 0.1^2 + 0.2^2 + 0.2^2 + 0.4^2$

$= 0.01 + 0.01 + 0.04 + 0.04 + 0.16$

$= 0.26$

Haf 2012

(8) (a) Rhaid bod $\Sigma P(X=x) = 1$ a bod bob tebygolrwydd yn y tabl rhwng 0 ac 1. Oim ond $0 \leq \theta \leq 0.3$ sy'n siarhau hyn.

(b) $E(X) = \Sigma x P(X=x)$

$= 2 \times (0.3 - \theta) + 3 \times 2\theta + 4 \times (0.7 - \theta)$

$= 0.6 - 2\theta + 6\theta + 2.8 - 4\theta$

$= 3.4$ (dim yn dibynnu ar θ)

(c) Gwyriad Safonol $(X) = 0.8$ felly $Var(X) = 0.8^2$

$= 0.64$

$E(X^2) = \Sigma x^2 P(X=x)$

$= 2^2 \times (0.3 - \theta) + 3^2 \times 2\theta + 4^2 \times (0.7 - \theta)$

$= 1.2 - 4\theta + 18\theta + 11.2 - 16\theta$

$= 12.4 - 2\theta$

$Var(X) = E(X^2) - [E(X)]^2$

$0.04 = 12.4 - 2\theta - (3.4)^2$

$0.04 = 12.4 - 2\theta - 11.56$

$2\theta = 12.4 - 11.56 - 0.04$

$2\theta = 0.2$

$\theta = 0.1$

(ii) $P(X_1 + X_2 = 6) = P(X_1 = 2, X_2 = 4$ neu $X_1 = 3, X_2 = 3$ neu $X_1 = 4, X_2 = 2)$

$= P(X_1 = 2, X_2 = 4) + P(X_1 = 3, X_2 = 3) + P(X_1 = 4, X_2 = 2)$

$= (0.3 - \theta)(0.7 - \theta) + (2\theta)(2\theta) + (0.7 - \theta)(0.3 - \theta)$

$= 0.2 \times 0.6 + 0.2 \times 0.2 + 0.6 \times 0.2$

$= 0.28$

Graeaf 2013

⑥ (a) $\sum P(X=x) = 1$

$K(1+1) + K(1+2) + K(1+3) + K(1+4) = 1$

$2K + 3K + 4K + 5K = 1$

$14K = 1$

$K = \frac{1}{14}$ ✓

(b) $E(X) = \sum x P(X=x)$

$= 1 \times K(1+1) + 2 \times K(1+2) + 3 \times K(1+3) + 4 \times K(1+4)$

$= 2K + 6K + 12K + 20K$

$= 40K$

$= 40(\frac{1}{14})$

$= \frac{20}{7}$

$E(X^2) = \sum x^2 P(X=x)$

$= 1^2 \times K(1+1) + 2^2 \times K(1+2) + 3^2 \times K(1+3) + 4^2 \times K(1+4)$

$= 2K + 12K + 36K + 80K$

$= 130K$

$= 130(\frac{1}{14})$

$= \frac{65}{7}$

$Var(X) = E(X^2) - [E(X)]^2$

$= \frac{65}{7} - (\frac{20}{7})^2$

$= \frac{55}{49}$

(c) $P(X_2 = 1 + X_1) = P(X_1=1, X_2=2 \text{ new } X_1=2, X_2=3 \text{ new } X_1=3, X_2=4)$

$= P(X_1=1, X_2=2) + P(X_1=2, X_2=3) + P(X_1=3, X_2=4)$

$= K(1+1) \times K(1+2) + K(1+2) \times K(1+3) + K(1+3) \times K(1+4)$

$= 2K \times 3K + 3K \times 4K + 4K \times 5K$

$= 6K^2 + 12K^2 + 20K^2$

$= 38K^2$

$= \frac{19}{98}$

Haf 2013

(7) (a) $P(X=x) = 1$

$$\frac{K}{1} + \frac{K}{2} + \frac{K}{4} + \frac{K}{8} = 1$$

$$\frac{15K}{8} = 1$$

$$15K = 8$$

$$K = \frac{8}{15} \quad \checkmark$$

(b) $E(X) = \sum x P(X=x)$

$$= 1 \times \frac{K}{1} + 2 \times \frac{K}{2} + 4 \times \frac{K}{4} + 8 \times \frac{K}{8}$$

$$= K + K + K + K$$

$$= 4 \times \frac{8}{15}$$

$$= \frac{32}{15}$$

$E(X^2) = \sum x^2 P(X=x)$

$$= 1^2 \times \frac{K}{1} + 2^2 \times \frac{K}{2} + 4^2 \times \frac{K}{4} + 8^2 \times \frac{K}{8}$$

$$= K + 2K + 4K + 8K$$

$$= 15K$$

$$= 15\left(\frac{8}{15}\right)$$

$$= 8$$

$Var(X) = E(X^2) - [E(X)]^2$

$$= 8 - \left(\frac{32}{15}\right)^2$$

$$= \frac{776}{225}$$

(c) (i) $P(X_1 = X_2) = P(X_1 = 1, X_2 = 1$ neu $X_1 = 2, X_2 = 2$ neu $X_1 = 4, X_2 = 4$ neu $X_1 = 8, X_2 = 8)$

$= P(X_1 = 1, X_2 = 1) + P(X_1 = 2, X_2 = 2) + P(X_1 = 4, X_2 = 4) + P(X_1 = 8, X_2 = 8)$

$= \frac{K}{1} \times \frac{K}{1} + \frac{K}{2} \times \frac{K}{2} + \frac{K}{4} \times \frac{K}{4} + \frac{K}{8} \times \frac{K}{8}$

$= K^2 + \frac{1}{4}K^2 + \frac{1}{16}K^2 + \frac{1}{64}K^2$

$= \frac{85}{64}K^2$

$= \frac{85}{64}\left(\frac{8}{15}\right)^2$

$= \frac{17}{45}$

(ii) $P(X_1 > X_2) + P(X_1 = X_2) + P(X_1 < X_2) = 1$

Gan ddefnyddio 'cymesuredd' y rhifau, gallwn ddweud fod $P(X_1 > X_2) = P(X_1 < X_2)$.

Felly $P(X_1 = X_2) + 2P(X_1 > X_2) = 1$

$\frac{17}{45} + 2P(X_1 > X_2) = 1$

$2P(X_1 > X_2) = 1 - \frac{17}{45}$

$2P(X_1 > X_2) = \frac{28}{45}$

$P(X_1 > X_2) = \frac{14}{45}$.

Gwirio: $P(X_1 > X_2)$:

2, 1 neu 4, 1 neu 8, 1 neu 4, 2 neu 8, 2 neu 8, 4

$\frac{K^2}{2} + \frac{K^2}{4} + \frac{K^2}{8} + \frac{K^2}{8} + \frac{K^2}{16} + \frac{K^2}{32}$

$= \frac{35K^2}{32}$

$= \frac{35}{32} \times \left(\frac{8}{15}\right)^2$ $= \frac{14}{45}$ ✓

S1 Gaeaf 2014

(7) (a) $E(X) = \sum x\,P(X=x)$

$= 1\times0.1 + 2\times0.2 + 3\times0.3 + 4\times0.1 + 5\times0.3$

$= 3.3$

$E(X^2) = \sum x^2\,P(X=x)$

$= 1^2\times0.1 + 2^2\times0.2 + 3^2\times0.3 + 4^2\times0.1 + 5^2\times0.3$

$= 12.7$

$Var(X) = E(X^2) - [E(X)]^2$

$= 12.7 - 3.3^2$

$= 1.81$

(b) (i) $P(S=4) = P(X_1 + X_2 + X_3 = 4)$.

Cyfuniadau posib:

X_1	X_2	X_3
1	1	2
1	2	1
2	1	1

Felly $P(S=4) = (0.1\times0.1\times0.2) + (0.1\times0.2\times0.1) + (0.2\times0.1\times0.1)$

$= 3\times0.002$

$P(S=4) = 0.006$

(ii) $P(S\leq4) = P(S=4) + P(S\leq3)$.

Dim ond un ffordd sydd o gael cyfanswm llai na neu'n hafal i 3, sef $X_1=1$, $X_2=1$, $X_3=1$, efo tebygolrwydd $0.1\times0.1\times0.1 = 0.001$.

Felly $P(S\leq4) = 0.006 + 0.001$

$P(S\leq4) = 0.007$

S1 Haf 2014

(7) (a) Rhaid bod $\Sigma P(X=x) = 1$ a rhaid bod bob tebygolrwydd rhwng 0 ac 1. Mae'r gwerthoedd $0 \leq \theta \leq 0.4$ yn sicrhau hyn.

(b) $E(X) = \Sigma x P(X=x)$

$= 1 \times 0.1 + 2 \times 0.3 + 3 \times \theta + 4 \times 0.2 + 5 \times (0.4 - \theta)$

$= 0.1 + 0.6 + 3\theta + 0.8 + 2 - 5\theta$

$= 3.5 - 2\theta$

Gyda θ ar ei leiaf (0) mae $E(X) = 3.5 - 2 \times 0$

$= 3.5$

Gyda θ ar ei fwyaf (0.4) mae $E(X) = 3.5 - 2 \times 0.4$

$= 2.7.$

Felly $2.7 \leq E(X) \leq 3.5$

(c) $\text{Var}(X) = 1.5$

$E(X^2) - [E(X)]^2 = 1.5$

Nawr $E(X^2) = \Sigma x^2 P(X=x)$

$= 1^2 \times 0.1 + 2^2 \times 0.3 + 3^2 \times \theta + 4^2 \times 0.2 + 5^2 \times (0.4 - \theta)$

$= 0.1 + 1.2 + 9\theta + 3.2 + 10 - 25\theta$

$= 14.5 - 16\theta.$

Felly $E(X^2) - [E(X)]^2 = 1.5$

$14.5 - 16\theta - (3.5 - 2\theta)^2 = 1.5$

$14.5 - 16\theta - (12.25 - 14\theta + 4\theta^2) = 1.5$

$2.25 - 2\theta - 4\theta^2 = 1.5$

$0 = 4\theta^2 + 2\theta + 1.5 - 2.25$

$0 = 4\theta^2 + 2\theta - 0.75$

$4\theta^2 + 2\theta - 0.75 = 0$

Fformiwla Gwadratig: $\theta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\theta = \frac{-2 \pm \sqrt{2^2-4\times4\times-0.75}}{2\times4}$

$\theta = \frac{-2 \pm \sqrt{16}}{8}$

$\theta = \frac{-2 \pm 4}{8}$

Unai $\theta = \frac{-2+4}{8}$ neu $\theta = \frac{-2-4}{8}$

$\theta = \frac{2}{8}$ neu $\theta = \frac{-6}{8}$

$\theta = \frac{1}{4}$ $\theta = \frac{-3}{4}$

Rhaid bod $0 \leq \theta \leq 0.4$

felly rhaid cael $\theta = \frac{1}{4}$

S1 Haf 2015

7) (a) $\sum P(X=x) = 1$

$$\frac{K}{2} + \frac{K}{3} + \frac{K}{4} + \frac{K}{6} = 1$$

$$\frac{5K}{4} = 1$$

$$K = \frac{4}{5}$$ ✓

(b) $E(X) = \sum x P(X=x)$

$$= 2\times\left(\frac{K}{2}\right) + 3\times\left(\frac{K}{3}\right) + 4\times\left(\frac{K}{4}\right) + 6\times\left(\frac{K}{6}\right)$$

$$= K + K + K + K$$

$$= 4K$$

$$= \frac{16}{5}$$

(c) $P(X_1X_2 = 12) = P(X_1=2, X_2=6 \text{ neu } X_1=3, X_2=4 \text{ neu } X_1=4, X_2=3 \text{ neu } X_1=6, X_2=2)$

$$= P(X_1=2, X_2=6) + P(X_1=3, X_2=4) + P(X_1=4, X_2=3) + P(X_1=6, X_2=2)$$

$$= \frac{K}{2}\times\frac{K}{6} + \frac{K}{3}\times\frac{K}{4} + \frac{K}{4}\times\frac{K}{3} + \frac{K}{6}\times\frac{K}{2}$$

$$= \frac{K^2}{12} + \frac{K^2}{12} + \frac{K^2}{12} + \frac{K^2}{12}$$

$$= \frac{4K^2}{12}$$

$$= \frac{K^2}{3}$$

$$= \frac{\left(\frac{4}{5}\right)^2}{3}$$

$$= \frac{16}{75}$$

S1 Haf 2016

(7) (a) (i) $\sum P(X=x) = 1$

$0.3 + 0.2 + 0.1 + a + b = 1$

$0.6 + a + b = 1$

$a + b = 1 - 0.6$

$a + b = 0.4$ ✓

(ii) $E(X) = \sum x\, P(X=x)$

$2.85 = 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 + 4a + 5b$

$2.85 = 1 + 4a + 5b$

$1.85 = 4a + 5b$ ——— (1)

$a + b = 0.4 \Rightarrow a = 0.4 - b$

Amnewid am a i mewn i (1):

$1.85 = 4(0.4 - b) + 5b$

$1.85 = 1.6 - 4b + 5b$

$1.85 - 1.6 = b$

$b = 0.25$

Felly $a = 0.4 - 0.25$

$a = 0.15$

(b) $P(X_1 + X_2 \leq 4) = P(X_1 = 1, X_2 = 1) + P(X_1 = 1, X_2 = 2)$
$+ P(X_1 = 1, X_2 = 3) + P(X_1 = 2, X_2 = 1)$
$+ P(X_1 = 2, X_2 = 2) + P(X_1 = 3, X_2 = 1)$

$= 0.3 \times 0.3 + 0.3 \times 0.2$
$+ 0.3 \times 0.1 + 0.2 \times 0.3$
$+ 0.2 \times 0.2 + 0.1 \times 0.3$

$= 0.09 + 0.06 + 0.03$
$+ 0.06 + 0.04 + 0.03$

$= 0.31$

S1 Haf 2017

6) $P(X=x) = \frac{x^2}{54}$ ar gyfer $x = 2, 3, 4, 5$

$P(X=x) = 0$ fel arall

a)

x	2	3	4	5
$P(X=x)$	$\frac{4}{54}$	$\frac{9}{54}$	$\frac{16}{54}$	$\frac{25}{54}$

$$E(X) = \sum x P(X=x)$$
$$= 2 \times \frac{4}{54} + 3 \times \frac{9}{54} + 4 \times \frac{16}{54} + 5 \times \frac{25}{54}$$
$$= \frac{112}{27}$$
$$E(X) = 4\frac{4}{27}$$

$$E(X^2) = \sum x^2 P(X=x)$$
$$= 4 \times \frac{4}{54} + 9 \times \frac{9}{54} + 16 \times \frac{16}{54} + 25 \times \frac{25}{54}$$
$$= \frac{163}{9}$$

$$Var(X) = E(X^2) - [E(X)]^2$$
$$= \frac{163}{9} - \left(\frac{112}{27}\right)^2$$
$$Var(X) = 0.903978052\ldots \quad \left(\text{neu } \frac{659}{729}\right)$$

b) $P(X_1 + X_2 + X_3 = 14) = P(X_1 = 4, X_2 = 5, X_3 = 5$ neu
$X_1 = 5, X_2 = 4, X_3 = 5$ neu
$X_1 = 5, X_2 = 5, X_3 = 4)$

$$= \frac{16}{54} \times \frac{25}{54} \times \frac{25}{54} + \frac{25}{54} \times \frac{16}{54} \times \frac{25}{54} + \frac{25}{54} \times \frac{25}{54} \times \frac{16}{54}$$
$$= \frac{1250}{6561}$$