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Discrete Random Variables

(Gaeaf 2005)

8. The following table gives the probability distribution of the discrete random variable X , where a and b are positive constants.

x	1	2	3	4	5
$P(X = x)$	0.1	a	b	0.3	0.2

- (a) Show that

$$a + b = 0.4. \quad [2]$$

- (b) Given that $E(X) = 3.4$,

- (i) write down and simplify another equation for a and b ,
(ii) find the values of a and b . [5]

- (c) Evaluate $E\left(\frac{1}{1+X}\right)$. [3]

(Haf 2005)

7. The discrete random variable X has probability distribution given by

$$\begin{aligned} P(X = x) &= k(1 + x) && \text{for } x = 1, 2, 3, 4, 5, \\ P(X = x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Show that $k = \frac{1}{20}$. [2]

- (b) Find the mean and variance of X . [5]

- (c) Given that X_1, X_2 are two independent observations of X , evaluate

$$P(X_1 + X_2 = 4). \quad [4]$$

- (d) The random variable Y is defined by

$$Y = 2X + 3.$$

- Find the mean and variance of Y . [4]

(Gaeaf 2006)

8. The following table gives the probability distribution of the discrete random variable X , where θ is a constant.

x	1	2	3	4
$P(X = x)$	0.1	0.2	θ	$0.7 - \theta$

- (a) State the range of possible values of θ . [2]
- (b) Given that $E(X) = 3$,
- (i) find the value of θ ,
- (ii) evaluate $E(X^3)$. [7]

(Haf 2006)

7. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$

- (a) Show that $k = \frac{1}{15}$. [2]
- (b) Find the mean and variance of X . [6]
- (c) The random variable Y is defined by $Y = X_1 + X_2$, where X_1, X_2 are independent observations on X . Find $P(Y = 6)$. [4]

(Gaeaf 2007)

6. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3	4	5
$P(X = x)$	0.3	p	0.1	q	0.05

- (a) Show that $p + q = 0.55$. [1]
- (b) Given that $E(X) = 2.75$, show that $p = 0.15$ and $q = 0.4$. [4]
- (c) Find the variance of X . [3]
- (d) The random variable Y is defined by $Y = 4X + 2$.
- (i) Find the mean and variance of Y .
- (ii) Find $P(Y < 15)$. [6]

(Haf 2007)

6. The discrete random variable X has a probability distribution given by

$$\begin{aligned} P(X = x) &= kx^2, & x = 1, 2, 3, 4, \\ P(X = x) &= 0, & \text{otherwise.} \end{aligned}$$

(a) Show that $k = \frac{1}{30}$. [2]

(b) Find the mean and variance of X . [7]

(Gaeaf 2008)

6. The discrete random variable X has the following probability distribution.

x	1	2	3
$P(X = x)$	θ	2θ	$1 - 3\theta$

(a) State the range of possible values of the constant θ . [2]

(b) Given that $E(X) = 2.2$,

(i) show that $\theta = 0.2$,

(ii) calculate the standard deviation of X ,

(iii) evaluate $E\left(\frac{1}{X}\right)$. [10]

(Haf 2008)

6. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= k(1 + x) & \text{for } x = 1, 2, 3, \\ P(X = x) &= 0 & \text{otherwise.} \end{aligned}$$

(a) Show that $k = \frac{1}{9}$. [2]

(b) Evaluate $E(X)$. [3]

(c) Evaluate $E\left(\frac{1}{X}\right)$. [3]

(Gaeaf 2009)

8. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= \frac{(10 - x)}{20}, & \text{for } x = 2, 4, 6, 8, \\ P(X = x) &= 0 & \text{otherwise.} \end{aligned}$$

(a) Find the mean and variance of X . [6]

(b) Given that X_1, X_2 are independent observations on X , calculate

$$P(X_1 + X_2 = 8) \quad [3]$$

(Haf 2009)

6. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.3	0.1

(a) Evaluate

(i) $E(X)$,

(ii) $\text{Var}(X)$.

[6]

(b) Given that X_1, X_2 are independent observations on X , calculate

$$P(X_1 = X_2).$$

[3]

(Gaeaf 2010)

4. The probability distribution of the discrete random variable X is given in the following table, where λ is a constant.

x	2	3	4	5	6
$P(X = x)$	0.1	0.2	0.3	λ	$0.4 - \lambda$

(a) Find the range of possible values of λ .

[2]

(b) Given that $E(X) = 4.25$,

(i) find the value of λ ,

(ii) evaluate $\text{Var}(X)$.

[6]

(Haf 2010)

6. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= kx && \text{for } x = 1, 3, 5, 7, \\ P(X = x) &= 0 && \text{otherwise.} \end{aligned}$$

(a) Show that $k = \frac{1}{16}$.

[2]

(b) Determine

(i) $E(X)$,

(ii) $E\left(\frac{1}{X}\right)$.

[5]

(c) Given that X_1, X_2 are two independent values of X , determine

(i) $P(X_1 + X_2 = 6)$,

(ii) $P(X_1 = X_2)$.

[7]

(Gaeaf 2011)

7. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3
$P(X = x)$	$0.4 - \alpha$	2α	$0.6 - \alpha$

- (a) (i) State the range of possible values of the constant α .
(ii) Show that $E(X)$ is independent of α .
(iii) Given that $\text{Var}(X) = 0.66$, find the value of α . [7]
- (b) Assume now that $\alpha = 0.25$. Given that X_1, X_2 are two independent values of X , determine the value of $P(X_1 = X_2)$. [4]

(Haf 2011)

5. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= kx^2 && \text{for } x = 1, 2, 3, 4, \\ P(X = x) &= 0 && \text{otherwise,} \end{aligned}$$

where k is a constant.

- (a) Show that $k = \frac{1}{30}$. [2]
(b) Calculate the mean and variance of X . [5]
(c) Two independent observations X_1, X_2 are taken from the distribution of X . Calculate $P(X_1 + X_2 = 4)$. [4]

(Gaeaf 2012)

7. The probability distribution of the discrete random variable X is given by

x	1	2	3	4	5
$P(X = x)$	0.1	0.1	0.2	0.2	0.4

- (a) Calculate the mean and variance of X . [5]
(b) Calculate $E\left(\frac{1}{X^2}\right)$. [3]
(c) Two independent observations X_1, X_2 are taken from the distribution of X .
(i) Calculate $P(X_1 + X_2 = 6)$. [4]
(ii) Calculate $P(X_1 = X_2)$. [3]

(Haf 2012)

8. The probability distribution of the discrete random variable X is given by

x	2	3	4
$P(X = x)$	$0.3 - \theta$	2θ	$0.7 - \theta$

- (a) State the range of possible values of the constant θ . [2]
- (b) Show that $E(X)$ is independent of θ . [2]
- (c) You are now given that the standard deviation of X is 0.8 .
- (i) Find the value of θ .
- (ii) Two independent observations X_1, X_2 are taken from the distribution of X . Calculate $P(X_1 + X_2 = 6)$. [8]

(Gaeaf 2013)

6. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= k(1 + x) && \text{for } x = 1, 2, 3, 4, \\ P(X = x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Show that $k = \frac{1}{14}$. [2]
- (b) Find the mean and variance of X . [5]
- (c) Given that X_1, X_2 are independent observations on X , determine the value of $P(X_2 = 1 + X_1)$. [4]

(Haf 2013)

7. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= \frac{k}{x} && \text{for } x = 1, 2, 4, 8, \\ P(X = x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Show that $k = \frac{8}{15}$. [2]
- (b) Determine the mean and variance of X . [5]
- (c) Given that X_1, X_2 are independent observations on X ,
- (i) find the value of $P(X_1 = X_2)$,
- (ii) use your answer to (i) to deduce the value of $P(X_1 > X_2)$. [5]

(Gaeaf 2014)

7. The discrete random variable X has the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.1	0.3

(a) Determine the mean and the variance of X . [5]

(b) Three independent observations X_1, X_2, X_3 are taken from the distribution of X and $S = X_1 + X_2 + X_3$. Calculate

(i) $P(S = 4)$,

(ii) $P(S \leq 4)$. [6]

(Haf 2014)

7. The probability distribution of the discrete random variable X is given by

x	1	2	3	4	5
$P(X = x)$	0.1	0.3	θ	0.2	$0.4 - \theta$

(a) State the range of possible values of the constant θ . [1]

(b) State the range of possible values of $E(X)$. [3]

(c) Given that $\text{Var}(X) = 1.5$, determine the value of θ . [8]

(Haf 2015)

7. The discrete random variable X has the following probability distribution.

$$P(X = x) = \frac{k}{x} \quad \text{for } x = 2, 3, 4, 6,$$

$$P(X = x) = 0 \quad \text{otherwise.}$$

(a) Show that $k = \frac{4}{5}$. [2]

(b) Determine $E(X)$. [2]

(c) Given that X_1 and X_2 are independent observations from the distribution of X , determine the probability that the product X_1X_2 is equal to 12. [4]

(Haf 2016)

7. The discrete random variable X has the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	0.3	0.2	0.1	a	b

where a, b are positive constants.

(a) (i) Show that

$$a + b = 0.4.$$

(ii) Given that $E(X) = 2.85$, obtain a second equation involving a and b .
Hence determine the value of a and the value of b . [5]

(b) Given that X_1 and X_2 are independent observations of the random variable X , determine $P(X_1 + X_2 \leq 4)$. [4]

(Haf 2017)

6. The discrete random variable X has the following probability distribution.

$$P(X = x) = \frac{x^2}{54} \quad \text{for } x = 2, 3, 4, 5,$$

$$P(X = x) = 0 \quad \text{otherwise.}$$

(a) Calculate the mean and the variance of X . [5]

(b) Three independent observations X_1, X_2, X_3 are taken from the distribution of X .
Determine the value of $P(X_1 + X_2 + X_3 = 14)$. [4]

(Haf 2018)

7. The discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	0.3	α	β	0.2

(a) Given that $E(X) = 2.2$, determine the value of α and the value of β . [5]

(b) Three independent observations X_1, X_2, X_3 are taken from the distribution of X .
Determine the value of $P(X_1 = X_2 = X_3)$. [3]