wjec cbac

GCE AS/A level

0985/01

S16-0985-01

MATHEMATICS – S3
Statistics

A.M. MONDAY, 27 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains one 50p coin, two 10p coins and three 2p coins. A random sample of three coins is selected from the bag. Calculate the expected value of the coin of highest value in the sample. [8]

2. A car manufacturer claims that the average mileage per gallon for a new model on a motorway journey is 61. However, a motoring organisation claims that the average mileage per gallon is less than this. A trial is therefore set up in which 10 cars of this new model undertake a long motorway journey and the mileage per gallon for each car is recorded as follows.

60·2 59·9 61·2 62·3 58·5 59·7 61·2 60·7 59·4 60·3

You may assume that this is a random sample from a normal distribution with mean μ and variance σ^2.

(a) State suitable hypotheses to test these claims. [1]

(b) Calculate unbiased estimates of μ and σ^2. [5]

(c) Carry out an appropriate hypothesis test at the 5% significance level. State your conclusion in context, explaining clearly how you reached it. [7]

3. *(a)* Alwyn plays a game on his computer. Each time he plays, he has a constant probability *p* of winning. Successive games are independent. In order to estimate *p*, he plays the game 80 times and he wins 44 of these games. Calculate an approximate 90% confidence interval for *p*. [6]

(b) Beti also plays the game and she has a probability *q* of winning. Again, successive games are independent. She plays the game 100 times and she calculates the following approximate confidence interval for *q* based on these games.

[0·555, 0·705]

Determine

(i) how many of these games she won,

(ii) the confidence level of this interval, giving your answer as a percentage to the nearest integer. [7]

4. A supermarket sells olive oil from two different suppliers A and B. The manager wishes to test whether or not the mean amounts of olive oil in bottles from A and bottles from B are equal. He therefore selects 80 bottles from each of A and B and he measures the amount of olive oil, x ml, in each bottle from A and the amount of olive oil, y ml, in each bottle from B. His results are summarised below.

$$\sum x = 20128 : \sum x^2 = 5064256 : \sum y = 20112 : \sum y^2 = 5056222$$

(a) State suitable hypotheses for the manager's test. [1]

(b) Determine an approximate *p*-value for these results and state your conclusion. [11]

(c) Explain briefly where the Central Limit Theorem is used in your analysis. [1]

5. The amount, y grams, of chemical that dissolves in 1 litre of water at a temperature of x°C satisfies the relationship $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	10	20	30	40	50	60
y	162	183	201	225	248	267

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 1·5.

(i) Determine a 95% confidence interval for β, giving your limits correct to three significant figures.

(ii) A 95% confidence interval is to be determined for the value of y when $x = x_0$. Giving a reason, state the value of x_0 for which the confidence interval has minimum width. [7]

6. A random sample $X_1, X_2, \ldots, X_n$ is taken from a probability distribution with mean μ and variance σ^2. The sample mean is denoted by $\bar{X}$.

(a) (i) Show that $\bar{X}$ is an unbiased estimator for μ.

(ii) Show that the standard error of $\bar{X}$ is $\dfrac{\sigma}{\sqrt{n}}$. [4]

(b) (i) Show that

$$E\left(X_i^{\,2}\right) = \mu^2 + \sigma^2 .$$

(ii) Given that

$$S^2 = \frac{\left(\displaystyle\sum_{i=1}^{n} X_i^{\,2}\right) - n\bar{X}^2}{n-1},$$

show that S^2 is an unbiased estimator for σ^2. [5]

(c) By considering the variance of S, show that S is not an unbiased estimator for σ. [4]

END OF PAPER

