



GCE AS/A level

0978/01



S16-0978-01

MATHEMATICS – FP2
Further Pure Mathematics

A.M. FRIDAY, 24 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{16-x^4}} dx,$$

giving your answer in the form $\frac{\pi}{n}$, where n is a positive integer. [6]

2. (a) (i) Evaluate $(3 - i)^2$, giving your answer in the form $a + ib$.
 (ii) Using your result, show that

$$(3 - i)^4 = 28 - 96i. \quad [3]$$

(b) Hence write down the four 4th roots of $28 - 96i$. [3]

3. (a) Use de Moivre's Theorem to prove that, for $\sin \theta \neq 0$,

$$\frac{\sin 4\theta}{\sin \theta} = 4\cos \theta(1 - 2\sin^2 \theta). \quad [4]$$

(b) Hence evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin 4\theta}{\sin \theta} d\theta.$$

Give your answer correct to three significant figures. [4]

4. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, find the general solution, in radians, to the equation

$$\sin x + \tan x + \tan\left(\frac{x}{2}\right) = 0. \quad [11]$$

5. The function f is defined by

$$f(x) = \frac{3x^2 + x + 6}{(x+2)(x^2+4)}.$$

(a) Determine whether f is even, odd or neither even nor odd. [1]

(b) Express $f(x)$ in partial fractions. [5]

(c) Hence evaluate

$$\int_0^1 f(x) dx,$$

giving your answer correct to three significant figures. [6]

6. (a) Show that the general hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

can be represented parametrically by $x = a \sec \theta$, $y = b \tan \theta$.

[2]

- (b) The equation of the hyperbola H is

$$x^2 - y^2 = 1.$$

- (i) Show that the equation of the normal to H at the point $P(\sec \theta, \tan \theta)$ is

$$x \sin \theta + y = 2 \tan \theta.$$

- (ii) This normal meets the x -axis at the point Q . Show that the locus of the midpoint of PQ as θ varies is a hyperbola. Determine its eccentricity and the coordinates of its foci.

[12]

7. The function f is defined by

$$f(x) = \frac{x^3 - 8}{x^3 - 1}.$$

- (a) Write down the equations of the asymptotes on the graph of f .

[2]

- (b) Find the points of intersection of the graph of f with the coordinate axes.

[2]

- (c) Find the coordinates of the stationary point on the graph of f and identify it as a maximum, a minimum or a point of inflection.

[5]

- (d) Sketch the graph of f , including the asymptotes.

[3]

- (e) The set $S = [-2, 2]$. Determine

(i) $f(S)$.

(ii) $f^{-1}(S)$.

[6]

END OF PAPER