GCE AS/A level

0974/01

S16-0974-01

MATHEMATICS – C2
Pure Mathematics

A.M. WEDNESDAY, 25 May 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_3^6 \frac{7-\sqrt{x}}{7+\sqrt{x}}\,\mathrm{d}x\,.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$6\sin^2\theta + 1 = 2(\cos^2\theta - \sin\theta).$$ [6]

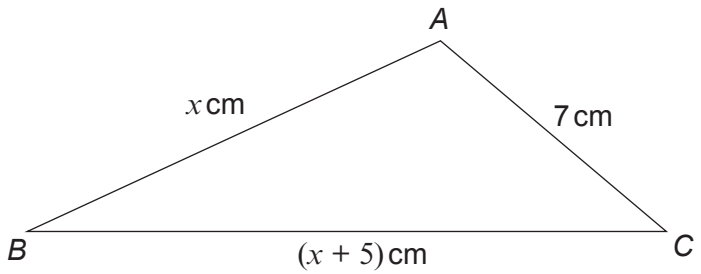
(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan(3x - 57°) = -0{\cdot}81.$$ [4]

(c) Without carrying out any calculations, explain why there are no values of ϕ which satisfy the equation

$$2\sin\phi + 4\cos\phi = -7.$$ [1]

3. The diagram below shows a sketch of the triangle *ABC* with $AB = x$ cm, $BC = (x + 5)$ cm, $AC = 7$ cm and $\cos B\hat{A}C = -\frac{3}{5}$.



(a) Write down an equation satisfied by x. Hence show that $x = 15$. [3]

(b) Find the exact value of the area of triangle *ABC*. [3]

(c) The point *D* lies on *BC* and is such that *AD* is perpendicular to *BC*. Find the length of *AD*. [2]

4. *(a)* Gwilym has decided to run in a half marathon race. In order to get himself fit, he devises a training programme whereby he runs around his local track each day, gradually increasing the distance he runs from day to day. On the first day, he runs 6 laps of the track and subsequently, on any given day, he runs 2 laps further than he did on the day before.

(i) How many laps does he run on the 20th day of his programme?

(ii) After how many days will the total number of laps he has run since the beginning of his training programme be equal to 750? [6]

(b) The nth term of an **arithmetic** series is denoted by t_n. It is known that

$$t_{12} + t_{13} = 50.$$

(i) **Write down** the value of $t_{11} + t_{14}$. [1]

(ii) Find the sum of the first twenty-four terms of this arithmetic series. [2]

5. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{a(1-r^n)}{1-r} \ .$$ [3]

(b) The sum of the first five terms of a geometric series is 275. The sum to infinity of the series is 243. Find the common ratio and the first term of the geometric series. [6]

6. *(a)* Find $\int\left(\frac{3}{\sqrt[4]{x}} - 9x^{\frac{5}{2}}\right)\mathrm{d}x$. [2]

(b) The region *R* is bounded by the curve $y = 2x^2 + \frac{6}{x^2}$, the x-axis and the lines $x = 1$, $x = 4$. Find the area of *R*. [5]

7. *(a)* Given that $x > 0$, show that

$$\log_a x^n = n \log_a x.$$ [3]

(b) Solve the equation

$$4^{3x+1} = 22.$$

Show your working and give your answer correct to two decimal places. [3]

(c) Given that

$$\log_d z = 2\log_d 6 - \log_d 9 - 1,$$

express z in terms of d, giving your answer in a form **not** involving logarithms. [4]

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8. The circle C_1 has centre A and equation

$$x^2 + y^2 + 6x - 20y + 59 = 0.$$

(a) (i) Find the coordinates of A and the radius of C_1.

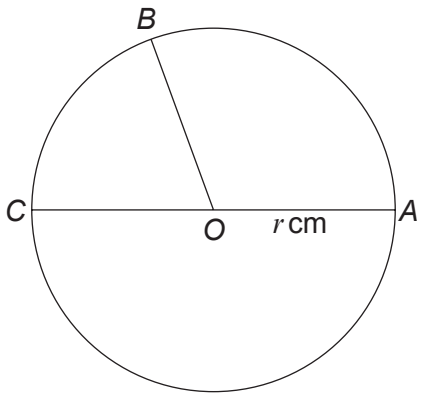
(ii) Find the shortest distance from the origin to the circle C_1. Give your answer correct to two decimal places. [5]

(b) The line L has equation $y = 3x - 1$. The line L and the circle C_1 intersect at the points P and Q.

(i) Find the coordinates of P and Q.

(ii) The circle C_2 has centre $B(6, 7)$ and is such that PQ is the common chord of C_1 and C_2. Find the equation of C_2. [7]

9.



The diagram shows a sketch of a circle with centre O and radius r cm. Three points A, B and C lie on the circle. The line AC is a diameter of the circle and $A\hat{O}B = 2{\cdot}15$ radians.

Given that the area of sector BOC is 26 cm^2 less than the area of sector AOB, find the value of r. Give your answer correct to one decimal place. [5]

END OF PAPER

