Name:

Revision Cards for WJEC Foundation Tier GCSE Mathematics

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01 Numbers and Rounding Off

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
								4	2	5	0	7	0	0	.	6		

Four million, two hundred and fifty thousand and seven hundred point six.

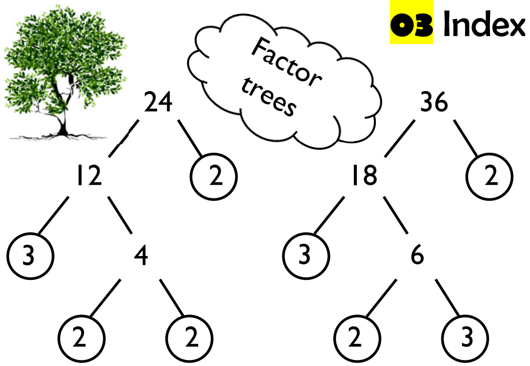
Round off 4753.4852 to two decimal places: 4,753.49
to one decimal place: 4,753.5
to the nearest unit: 4,753
to one significant figure: 5,000
to the nearest thousand: 5,000
to the nearest hundred: 4,800
to the nearest ten: 4,750
to two significant figures: 4,800

Unit 2 only: **estimation**

Negative five is less than negative two, $-5 < -2$.

Odd numbers: 1, 3, 5, 7, 9, 11, 13, ... (end with 1, 3, 5, 7 or 9).
Even numbers: 2, 4, 6, 8, 10, 12, 14, ... (end with 2, 4, 6, 8 or 0).
Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, ... (the 6 times table).
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 (numbers that divide exactly into 24).
Prime numbers: 2, 3, 5, 7, 11, 13, 17, ... (numbers with exactly two factors).
Triangle numbers: 1, 3, 6, 10, 15, 21, ... (add one more each time).

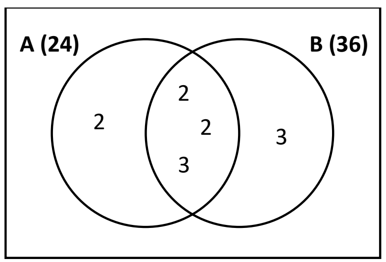
03 Index Form



Product of prime factors:
$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

Index Form: $24 = 2^3 \times 3$
$36 = 2^2 \times 3^2$

36 is a square number because all its indices in index form are even numbers. Halve the indices for the square root: $\sqrt{36} = 2^1 \times 3^1 = 6$.



To change 24 to be a square number, the least number to multiply by (or divide by) is $2 \times 3 = 6$ (multiply by the base of each odd index).

Greatest Common Divisor of 24 and 36 is $2 \times 2 \times 3 = 12$ (middle of the Venn diagram).
Least Common Multiple of 24 and 36 is $2 \times 2 \times 2 \times 3 \times 3 = 72$ (multiply all the numbers in the Venn diagram). [$24 \times 36 = 12 \times 72$]

05a Fractional and Percentage Changes

What is the **percentage change** if an item's price changes from £2.50 to (a) £2.80, (b) £1.80?

(a) The change in the price is £0.30.
As a percentage of the original price:
$0.30 \div 2.50 \times 100 = 12\%$ increase.

(b) The change in the price is £0.70.
As a percentage of the original price:
$0.70 \div 2.50 \times 100 = 28\%$ decrease.

Multiples
Find 4% of £56: the multiple is 4%, or 0.04.
Increase £56 by 4%: the multiple is 104%, or 1.04.
Decrease £56 by 4%: the multiple is 96%, or 0.96.

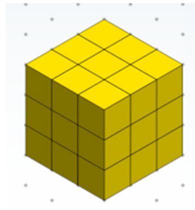
Add or subtract from 100%

05b Using a Calculator Efficiently

Make sure you can obtain the following answers on **your** calculator.
(a) $17^2 = 289$. (b) $\sqrt[3]{729} = 9$. (c) $\sqrt{36} + 4^3 = 70$. (d) $\frac{1}{7} - \frac{1}{8} = \frac{1}{56}$.
(e) $\frac{2}{5} + \frac{3}{4} = 1\frac{3}{20}$. (f) $3\frac{2}{7} \times 2\frac{4}{9} = 8\frac{2}{63}$. (g) 3 minutes 47 seconds + 19 minutes 39 seconds = 23 minutes 26 seconds. (h) 37% of £83 = £30.71.
(i) $\pi \times 7.5 = 23.56$ to 2 decimal places.
(j) $2 \div 5000 = 0.0004$.

SETTINGS: Choose MathI/DecimalO

02a Indices (Units 1, 2 and 3)



Square numbers: $1^2, 2^2, 3^2, 4^2, 5^2, \ldots$ 7^2 means $7 \times 7 = 49$.
Square root reverses squaring. E.g. $\sqrt{49} = 7$.
Cube numbers: $1^3, 2^3, 3^3, 4^3, 5^3, \ldots$ 7^3 means $7 \times 7 \times 7 = 343$.
Cube root reverses cubing. E.g. $\sqrt[3]{125} = 5$.
Higher powers: 2^5 means $2 \times 2 \times 2 \times 2 \times 2 = 32$.

Hint: Turn on the '*digit separator*' on your calculator to better read numbers.

02b Rules of Indices (Unit 2 only)

Rules:	Examples:
$n^a \times n^b = n^{a+b}$	$5^7 \times 5^8 = 5^{15}$
$n^a \div n^b = n^{a-b}$ or $\frac{n^a}{n^b} = n^{a-b}$	$8^6 \div 8^2 = 8^4$ or $\frac{3^{15}}{3^5} = 3^{10}$
$(n^a)^b = n^{a \times b}$	$(6^3)^4 = 6^{12}$

02c Bounds (Unit 1 only)

The length of an item, correct to the nearest centimetre, is 7 cm. What could the true length be? The true length lies between 6.5 cm and 7.5 cm. So, the true length could be 7.2 cm, but it could not be 7.6 cm. The capacity of a cup, correct to the nearest 10 ml, is 280 ml. The true capacity is between 275 ml and 285 ml.

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Version 2.0 (August 2026)

Unit 1
Financial Mathematics and Other Applications of Numeracy
1 hour 30 minutes, 65 marks
30% of the course

Unit 2
Non-calculator
1 hour 30 minutes, 65 marks
30% of the course

The green formulae appear on page 2 of the exam paper

Unit 3
Calculator-allowed
1 hour 45 minutes, 75 marks
40% of the course

Number	Algebra
Shape	Data Handling

Created by a person
100% no AI

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06a Repeated Proportional Changes

A new car costs £15,000. If its value decreases 7% per year, what is the car worth after 4 years?
£15,000 × 93%^{4} = £11,220.78, correct to the nearest penny.

Steffan buys a house for £200,000. If its value increases $\frac{1}{20}$ per year, what will the house be worth after 3 years?
£200,000 × $\left(\frac{21}{20}\right)^3$ = £231,525.

06b Ratio

Share £32 between Alaw and Bob according to the ratio 5 : 3. Bar model

£32

£4	£4	£4	£4	£4	£4	£4	£4
Alaw					Bob		

There are 5 + 3 = 8 parts to the ratio. One part of the ratio: £32 ÷ 8 = £4.
Alaw: £4 × 5 = £20. Bob: £4 × 3 = £12. Check: £20 + £12 = £32 ✓.

Simplify the ratio 30 : 45. We must divide by a common factor until this is not possible. 30 : 45 = 6 : 9 = 2 : 3. (Dividing by 5 and 3, respectively.)
It is possible to compare ratios by writing them all in the form 1 : n (or n : 1).

04 Fractions, Percentages and Decimals

Fraction	Percentage		Decimal
$\frac{1}{4} = \frac{25}{100}$	25%	→ ÷ 100	0.25
$\frac{40}{100} = \frac{2}{5}$	40%		0.4
$\frac{2.5}{100} = \frac{5}{200} = \frac{1}{40}$	2.5%	← × 100	0.025

(Start with the red numbers and finish with the blue numbers.)

Simplifying a fraction: divide by a common factor until this is not possible.
Example: $\frac{24}{60} = \frac{12}{30} = \frac{4}{10} = \frac{2}{5}$. (Dividing by 2, 3 and 3, respectively.)

Express 15 as a percentage of 75:
Without a calculator: $\frac{15}{75} = \frac{5}{25} = \frac{20}{100} = 20\%$.
With a calculator: 15 ÷ 75 × 100 = 20%.

Calculate 34% of £25:
Without a calculator: [10%] £25 ÷ 10 = £2.50
[1%] £2.50 ÷ 10 = £0.25 [30%] £2.50 × 3 = £7.50
[4%] £0.25 × 4 = £1 [34%] £7.50 + £1 = £8.50.
With a calculator: 25 × 34% = £8.50. (Or 25 × 0.34.)

Calculate $\frac{4}{5}$ of £30:

£30

£6	£6	£6	£6	£6

$\frac{4}{5}$

Bar model

£30 ÷ 5 = £6
£6 × 4 = £24

07 Calculation

Order of operations: **BODMAS** [**B**rackets **O**rder **D**ivision **M**utiplication **A**ddition **S**ubtraction]. To start, complete any calculations in brackets ( ). Then, complete any division ÷ and multiplication × sums, working left to right. Finally, complete any addition + and subtraction – sums, again working left to right.

Checklist: can you complete the following **without** and **with** a calculator?

743 + 2839	508 – 46	349 × 57	768 ÷ 12
7.43 + 28.39	508 – 4.6	3.49 × 5.7	76.8 ÷ 0.12
$1\frac{2}{3}+\frac{3}{5}$	$1\frac{2}{3}-\frac{3}{5}$	$1\frac{2}{3}\times\frac{3}{5}$	$1\frac{2}{3}\div\frac{3}{5}$
7 + –3	7 – –3	–7 + –3	–7 – –3

On a calculator, take care to round off an answer reasonably, considering the context of the question. Also, do not round off too much in the middle a question, so that the final answer is accurate.

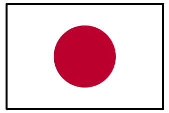
09 Foreign Money

One day, the pounds to euros exchange rate was £1 = €1.16.

(a) How many euros would correspond to £173.58?
£173.58 × 1.16 = €201.35, correct to the nearest cent.

(b) How many pounds would correspond to €260?
€260 ÷ 1.16 = £224.14, correct to the nearest penny.

You are going on holiday to Japan. As part of the preparations, you go to the Post Office to change £150 into yen. The sign at the Post Office shows the following information:
"We sell yen at the rate £1 = ¥143.28".



The smallest number of yen available today at the Post Office is a ¥1,000 note. What is the maximum number of yen you can get today at the Post Office, and how much change (in pounds) would you receive?

£150 × 143.28 = ¥21,492. (Change the money into yen.)
Round down to the nearest thousand: ¥21,000.
(This is the maximum number of yen you can get today at the Post Office.)
Cost of the yen: ¥21,000 ÷ 143.28 = £146.57, correct to the nearest penny.
Change: £150 – £146.57 = £3.43.
(So, you walk out of the Post Office with ¥21,000 and £3.43.)

11 Household Bills

At the end of one quarter, the reading on Mr. Smith's electricity meter was 53,799, whilst the previous reading was 52,471. The electricity company charges 17.4 pence per unit along with a standing charge of £35. On top of this, value added tax is charged at a rate of 5%. Calculate the total of Mr Smith's electricity bill.

Units used	53,799 – 52,471 = 1,328.
Electricity cost	1,328 × 17.4 = 23,107.2 pence.
Change to pounds	23,107.2 ÷ 100 = £231.07, to the nearest penny.
Add the standing charge	£231.07 + £35 = £266.07.
Calculate the VAT	£266.07 × 5% = £13.30, to the nearest penny.
Total bill	£266.07 + £13.30 = £279.37.

You could also face questions on the following bills...

- Gas bill (a charge for each kilowatt hour of gas used);
- Water and sewerage bill (a charge for each m³ of water used);
- Mobile phone bill (a charge for each gigabyte of data used);

...and other types of bills not listed here.

13 Rules of Algebra

$5\times x = 5x$
$a+a+a=3a$
$4\times(a+b)=4(a+b)$

$x\times x = x^2$
$a\times b = ab$
$x\times x\times x = x^3$

$1x = x$
$a\div b=\frac{a}{b}$

Term	Example 1	Example 2
Expression	$2x+3$	$4f-7g$
Formula	$C=\pi d$	$P=2a+2b$
Equation	$x+2=9$	$x^2+6x+8=0$

Forming expressions: Four less than x is $x-4$; three times y is $3y$; five more than double z is $2z+5$.

Collecting like terms: $2a+5b+3a-4b=5a+b$.

Substituting $a=4$, $b=3$ into $5a-2b$: $5\times4-2\times3=20-6=14$.

Expanding one set of brackets: $3(x-4)=3x-12$; $2y(3y+7)=6y^2+14y$.

Factorising (Unit 2 only):
$14x+16=2(7x+8)$
$12x^2+20xy=4x(3x+5y)$
$7x^2-5x=x(7x-5)$
$2x^3+6x^2+10x=2x(x^2+3x+5)$

10a Simple Interest and Compound Interest

Tom wants to borrow £18,000 over a period of four years.

Barclays offer **simple interest** at a rate of 4.2% per year.

HSBC offer **compound interest** at a rate of 4.1% per year.

Which bank should Tom choose?

Barclays
£18,000 × 4.2% = £756
£756 × 4 = £3,024
£18,000 + £3,024 = £21,024

100% + 4.1% = 104.1%

HSBC
£18,000 × 104.1%^{4} = £21,138.56, correct to the nearest penny.

Tom should choose Barclays as the total to pay back is less.

10b Best Bargain

Which of the following is the best bargain?

2 litres of Diet Coke for £2.10 — ÷2 → 1 litre of Diet Coke for £1.05

1.25 litres of Diet Coke for £1.40 — ÷ 1.25 → 1 litre of Diet Coke for £1.12

The 2 litre bottle is the better bargain because the cost per litre is less.

08 Gross Wage and Net Wage

Cara's gross wage, before tax, is £42,500. The tax rates are shown in the table on the right.

Band	Taxable income	Tax rate
Personal allowance	Up to £16,000	0%
Basic rate	£16,000 to £40,000	20%
Higher rate	Over £40,000	35%

(a) Calculate how much tax Cara pays.

Personal allowance: £16,000 × 0% = £0.
(No tax on the first £16,000.)
Basic rate (20%):
£24,000 × 20% = £4,800.
(£40,000 – £16,000 gives the £24,000.)
Higher rate (35%):
£2,500 × 35% = £875. (£42,500 – £40,000 gives the £2,500.)
Total tax: £0 + £4,800 + £875 = £5,675.

(b) If Cara pays £2,394 of National Insurance each year, and yearly Pension Contributions of £3,782.50, what is Cara's net wage?

£42,500 – £5,675 – £2,394 – £3,782.50 = £30,648.50.
(Gross wage – tax – national insurance – pension = net wage.)

14 Solving Equations

$x+3=9$ $x=6$ [subtract 3]	$x-3=9$ $x=12$ [add 3]	$2x=24$ $x=12$ [divide by 2]
$\frac{x}{2}=4$ $x=8$ [multiply by 2]	$2x-3=17$ $2x=20$ [add 3] $x=10$ [divide by 2]	$5x-2=2x+25$ $5x=2x+27$ [add 2] $3x=27$ [subtract $2x$] $x=9$ [divide by 3]
$\frac{1}{2}(3x+5)=10$ $3x+5=20$ [multiply by 2] $3x=15$ [subtract 5] $x=5$ [divide by 3]	$2(x-3)=7$ $2x-6=7$ [expand] $2x=13$ [add 6] $x=6.5$ [divide by 2]	$2(x-3)=7$ $x-3=3.5$ [divide by 2] $x=6.5$ [add 3]

Gruff thinks of a number x. The sum of the number and 3, multiplied by 4, is equal to five less than double his number. Form and solve an equation to find x.

$4(x+3)=2x-5$ [form the equation]
$4x+12=2x-5$ [expand]
$2x+12=-5$ [subtract $2x$]
$2x=-17$ [subtract 12]
$x=-8.5$ [divide by 2]

Check:
$4(-8.5+3)=2\times -8.5-5$
$4\times -5.5=-17-5$
$-22=-22$ ✓

12a Assessing Written Communication

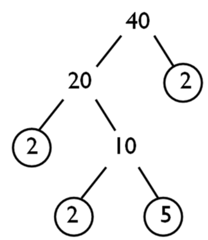
There is one question in each paper where the "quality of your organisation, communication and accuracy in writing" is assessed.

Sum	Explanation
Write down any mathematical sums that are required.	Explain why you have done each sum on the left.

12b Recurring Decimals (Unit 2 only)

Does a fraction (such as $\frac{46}{80}$) correspond to a **terminating** decimal (like 0.4), or a **recurring** decimal (like $0.\dot{4}$)?

To start, simplify the fraction: $\frac{46}{80}=\frac{23}{40}$. Then, write the denominator as a product of its prime factors:
40 = 2 × 2 × 2 × 5. As only 2 and 5 appear (and no other prime numbers), $\frac{46}{80}$ corresponds to a finite decimal (0.575).



Use a division frame to change a fraction to a decimal, looking for the pattern if the decimal is recurring.

15a Algebra and Rules of Indices

$x^4 \times x^7 = x^{11}$

$4a^4b^5 \times 2a^3b^7 = 8a^7b^{12}$

$\frac{y^8}{y^4} = y^4$

$y^8 \div y^2 = y^6$

$(2x^3)^4 = 2^4 \times (x^3)^4 = 16x^{12}$

$16x^{10} \div 2x^5 = 8x^5$

15b Simplifying Algebraic Fractions

$$\frac{3x}{7} + \frac{5x}{4} = \frac{4(3x)}{4\times7} + \frac{7(5x)}{7\times4} = \frac{12x}{28} + \frac{35x}{28} = \frac{47x}{28}$$

$$\frac{6x-10}{4x-8} = \frac{\cancel{2}(3x-5)}{\cancel{2}(2x-4)} = \frac{3x-5}{2x-4}$$

Unit 2 factoring

Remember to simplify, if possible

$$\frac{6n^4}{2n^8} = \frac{3}{n^4} \text{ (or } 3n^{-4}\text{)}$$

$$\frac{3x+2}{3} - \frac{2x-5}{5} = \frac{5(3x+2)}{15} - \frac{3(2x-5)}{15} = \frac{15x+10}{15} - \frac{6x-15}{15} = \frac{15x+10-6x+15}{15} = \frac{9x+25}{15}$$

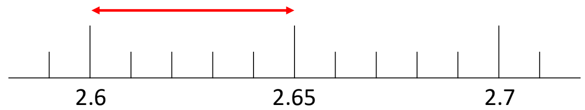
Form a common denominator before adding or subtracting

17 Trial and Improvement

If we cannot solve an equation through algebraic methods, we can attempt to solve it using **trial and improvement**. In this method, we start with a sensible guess for the solution (a *trial*), and then *improve* upon it, until we have found the solution to the required degree of accuracy.

Use the trial and improvement method to find, correct to one decimal place, the solution to $x^3 + 4x - 29 = 0$ lying between $x = 0$ and $x = 10$. With the `TABLE MODE`:

Trial	Value of $x^3 + 4x - 29$	Too high / Too low?
2	−13	Too low
3	10	Too high
2.6	−1.024	Too low
2.7	1.483	Too high
2.65	0.209625	Too high



The solution to the equation lies between $x = 2.6$ and $x = 2.65$ so, to one decimal place, the solution to the equation is $x = 2.6$.

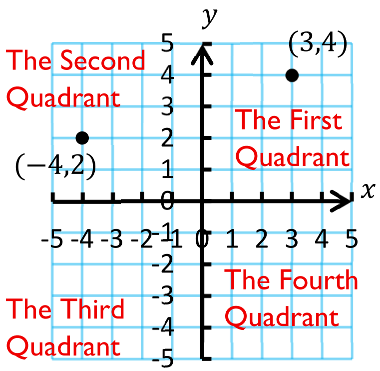
19a Co-ordinates in the Four Quadrants

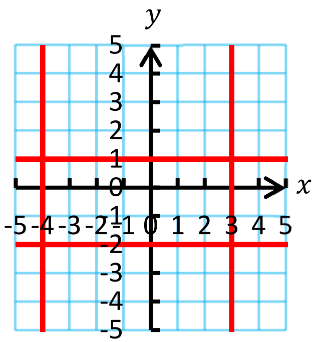
The **origin** is the **co-ordinate** (0, 0).

To plot (3, 4), start at the origin before moving 3 units to the right and 4 units up.

For the co-ordinate (3, 4), 3 is the x–coordinate and 4 is the y–coordinate.

The co-ordinate (3, 4) is in the first quadrant whilst (–3, –4) is in the third quadrant.





19b Vertical and Horizontal Lines

The equation $x = a$ represents a **vertical** line that goes through the point $(a, 0)$.

The line $y = b$ represents a **horizontal** line that goes through the point $(0, b)$.

Shown left are $x = 3$, $x = -4$, $y = 1$, $y = -2$.

21 Quadratic Graphs

For a **quadratic graph** of the form $y = ax^2 + bx + c$, the curve is ∪–shaped if a is positive; and ∩–shaped if a is negative. For example, the graph for $y = x^2 + 2x - 8$ is shown below. Often in an exam, you will need to plot such a graph using a partially completed table.

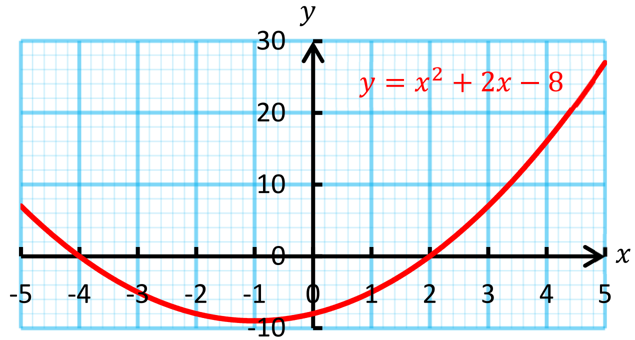
x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	7	0	−5		−9	−8	−5	0	7		27

If $x = 4$, then $y = 4^2 + 2 \times 4 - 8$, $y = 16 + 8 - 8$, $y = 16$.

If $x = -2$, then $y = (-2)^2 + 2 \times -2 - 8$, $y = 4 + -4 - 8$, $y = -8$.

To solve $x^2 + 2x - 8 = 0$, look to see where the graph has value 0, which here is at $x = -4$ and $x = 2$.

To solve $x^2 + 2x - 8 = 5$, look to see where the horizontal line $y = 5$ intersects the graph.

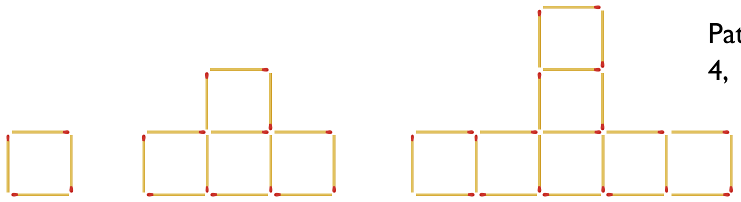


18 Linear Nth Term

Sequence	Rule in words	Rule in symbols
7, 11, 15, 19, 23, ...	Add four	+4
5, 15, 45, 135, 405, ...	Multiply by three	×3
200, 100, 50, 25, 12.5, ...	Divide by two	÷2

For the sequence 7, 11, 15, 19, 23, ..., the **nth term** is $4n + 3$, because the sequence goes up by 4 each time, and if there was another number at the start, it would have to be 7 – 4 = 3. We can find *any* term in the sequence using the nth term. For example, the 150th term in the sequence 7, 11, 15, 19, 23, ... with nth term $4n + 3$ is $4 \times 150 + 3 = 603$.

The first three terms of the sequence with the nth term $-2n + 30$:
[1st] $-2 \times 1 + 30 = 28$; [2nd] $-2 \times 2 + 30 = 26$; [3rd] $-2 \times 3 + 30 = 24$.



Pattern 1 Pattern 2 Pattern 3

Pattern of matches:
4, 13, 22, …

nth term
$9n - 5$

16a Changing the Subject

The purpose of **changing the subject** is to rearrange a formula so that a particular variable appears on its own on the left-hand side of the formula.

Make m the subject of the formula
$y = mx + c$.
$mx + c = y$ [swap sides]
$mx = y - c$ [subtract c]
$m = \frac{y-c}{x}$ [divide by x]

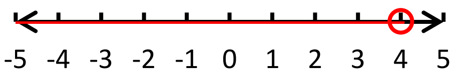
Make s the subject of the formula
$A = \frac{1}{2}su$.
$\frac{1}{2}su = A$ [swap sides]
$su = 2A$ [multiply by 2]
$s = \frac{2A}{u}$ [divide by u]

16b Inequalities

≥ 'greater than or equal to'. > 'greater than'. ≤ 'less than or equal to'. < 'less than'.

Solving an **inequality** is like solving an equation, but there is one important rule: the symbol in the middle of the inequality must be changed if we (a) swap sides; (b) multiply or divide by a negative number.

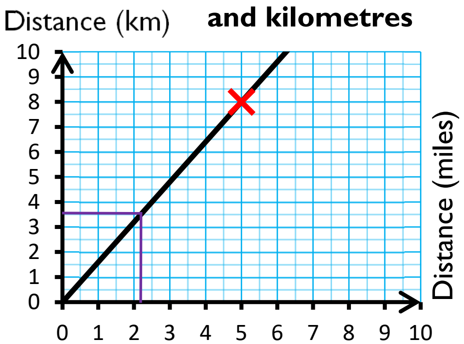
$5 > 3x - 7$
$3x - 7 < 5$ [swap sides]
$3x < 12$ [add 7]
$x < 4$ [divide by 3]



22a Conversion Graphs

A graph that changes between two measures, e.g. distance in miles and distance in kilometres.

Conversion graph between miles and kilometres

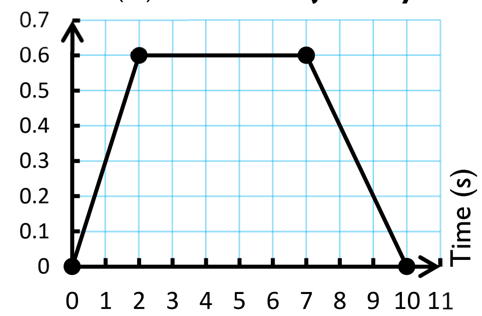


To plot the graph, use a point like 5 miles ≈ 8 km to begin.
Then use the graph to convert, e.g. 3.5 km ≈ 2.2 miles.

22b Travel Graphs

A graph that shows how distance or speed changes as time passes.

Distance-time graph for a robot's journey



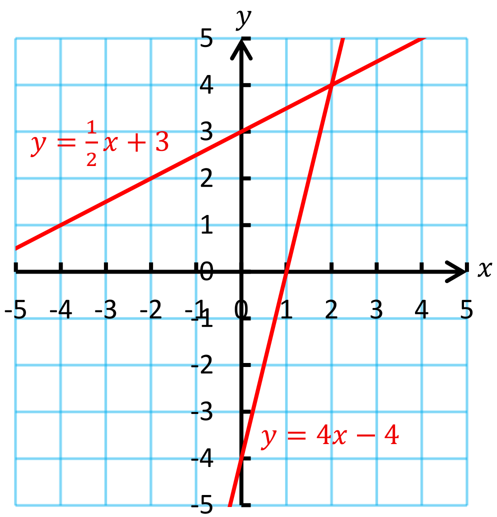
In the first 2 seconds, the robot's speed is 0.6 ÷ 2 = 0.3 metres per second.
The robot is stationary for 5 seconds.
In the final 3 seconds, the robot's speed is 0.6 ÷ 3 = 0.2 metres per second.

20 Equation of a Straight Line

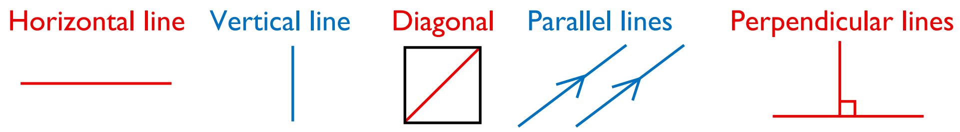
For any straight line not vertical or horizontal, we can write an equation for it in the form $y = mx + c$. The **gradient** is m; for each 1 unit we go to the right, we go m units up or down. The **y–intercept** is c; the line goes through $(0, c)$.

The graph on the right shows the lines $y = 4x - 4$ and $y = \frac{1}{2}x + 3$.
It is possible to use the table below to plot the graphs, or it is possible to use the gradient and intercept.
(For example, for $y = \frac{1}{2}x + 3$, the gradient is $\frac{1}{2}$ and the y–intercept is $(0, 3)$.)

x	0	1	2	3	4
$4x - 4$	-4	0	4	8	12
$\frac{1}{2}x + 3$	3	3.5	4	4.5	5

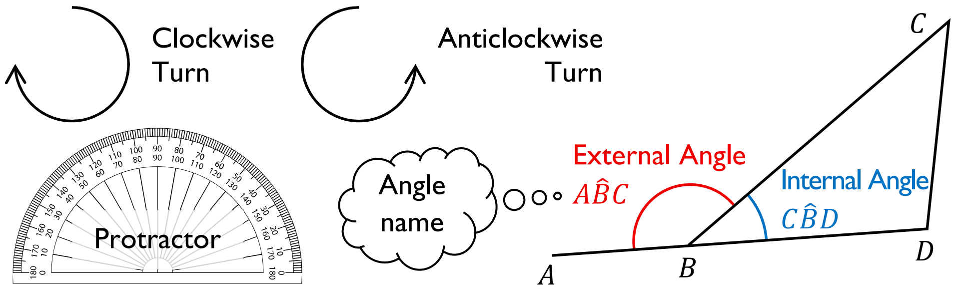


23a Line Vocabulary

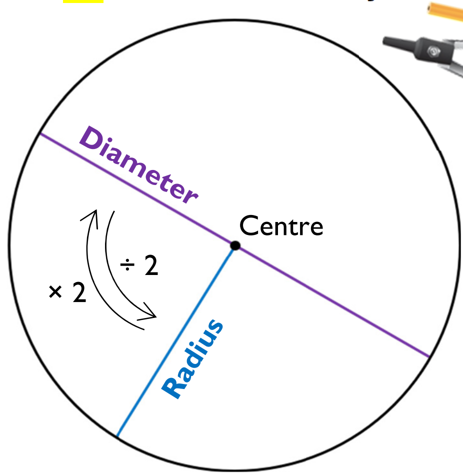


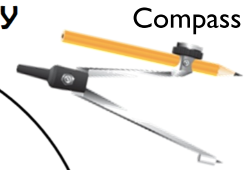
23b Angle Vocabulary

Angle	Acute	Right	Obtuse	Straight Line	Reflex	Full Turn
Size	Between 0° and 90°	90°	Between 90° and 180°	180°	Between 180° and 360°	360°

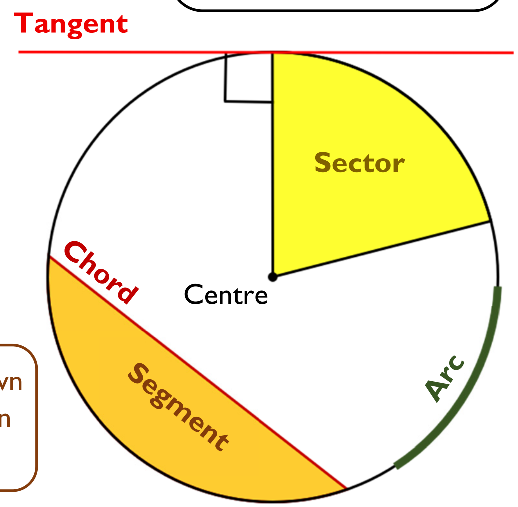


25 Circle Vocabulary





The **circumference** is the perimeter of the circle (the distance around the circle).



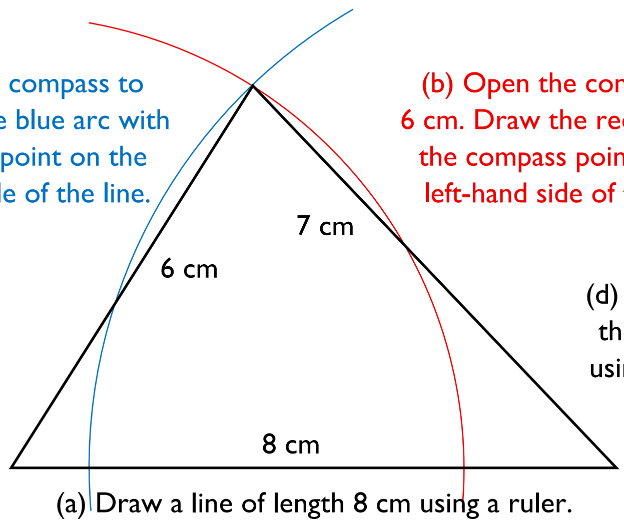
The **minor segment** is the one shown in orange. The **major segment** is on the other side of the chord.

27 Drawing a Triangle using a Compass

Draw a triangle with side lengths 6 cm, 7 cm and 8 cm using only a compass and ruler.

(c) Open the compass to 7 cm. Draw the blue arc with the compass point on the right-hand side of the line.

(b) Open the compass to 6 cm. Draw the red arc with the compass point on the left-hand side of the line.



(d) Complete the triangle using a ruler.

(a) Draw a line of length 8 cm using a ruler.

29a Using Map Ratio 29b Bearings

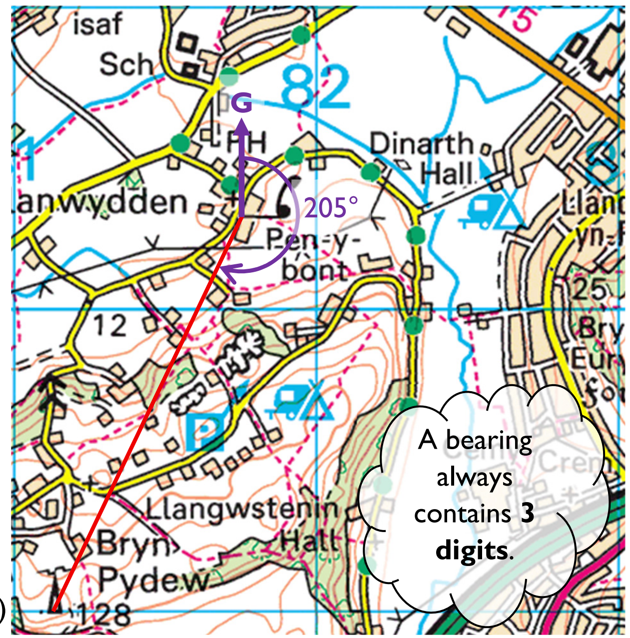
The **ratio** of the map shown on the right is 1 : 25,000.

(a) What is the true distance, in metres, between the TV mast and the phone box?

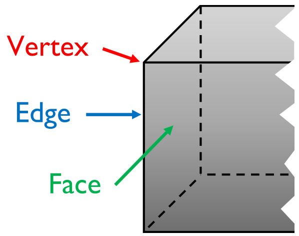
With a ruler, the distance on the map is 5.8 cm. The true distance is 5.8 × 25,000 cm = 145,000 cm = 1,450 m.

(b) What is the **bearing** of the TV mast from the phone box?

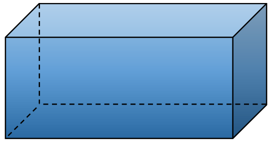
Bearings are measured clockwise from the North, in degrees. The bearing of the TV mast from the phone box is 205°. (Bearing of the phone box from the TV mast = 025°.)



26 Solid Vocabulary

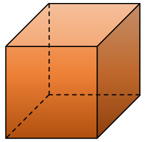


Cuboid



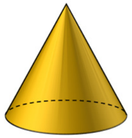
8 vertices, 12 edges, 6 faces

Cube



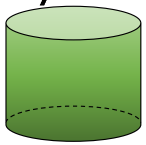
8 vertices, 12 edges, 6 faces

Cone



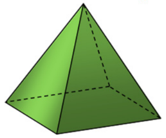
1 vertex, 1 edge, 2 faces

Cylinder



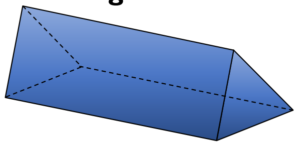
0 vertices, 2 edges, 3 faces

Square-Based Pyramid



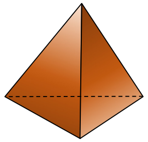
5 vertices, 8 edges, 5 faces

Triangular Prism



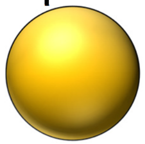
6 vertices, 9 edges, 5 faces

Tetrahedron



4 vertices, 6 edges, 4 faces

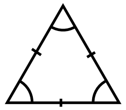
Sphere



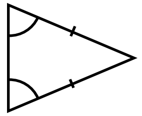
0 vertices, 0 edges, 1 face

24 Polygon Vocabulary

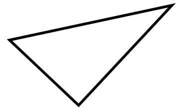
Equilateral triangle: 3 equal sides and 3 equal angles (60°).



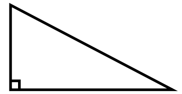
Isosceles triangle: 2 equal sides and 2 equal angles.



Scalene triangle: no equal sides nor equal angles.

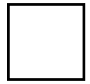


Right-angled triangle: One right angle equal to 90°.



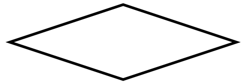
Rectangle

Square

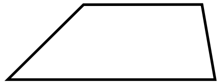


Parallelogram

Rhombus

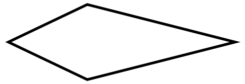


Trapezium

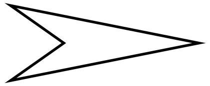


Isosceles trapezium

Kite



Arrowhead

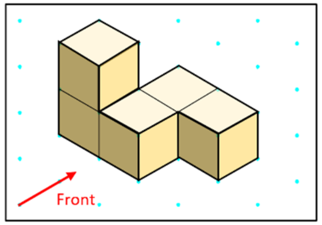


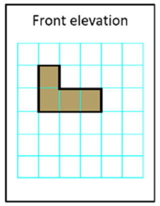
Name [edges]: triangle [3]; quadrilateral [4]; pentagon [5]; hexagon [6]; heptagon [7]; octagon [8]; nonagon [9]; decagon [10]; hendecagon [11]; dodecagon [12].
Regular polygon: all sides equal in length, **and** all angles equal in size.

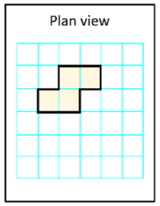
30a Plans and Elevations

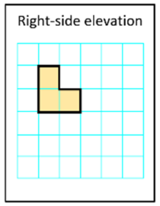
Given a solid drawn on isometric paper, it is possible to draw **plans and elevations** of the solid, namely what someone would see looking from different directions.

The volume of the solid is 5 cube units.



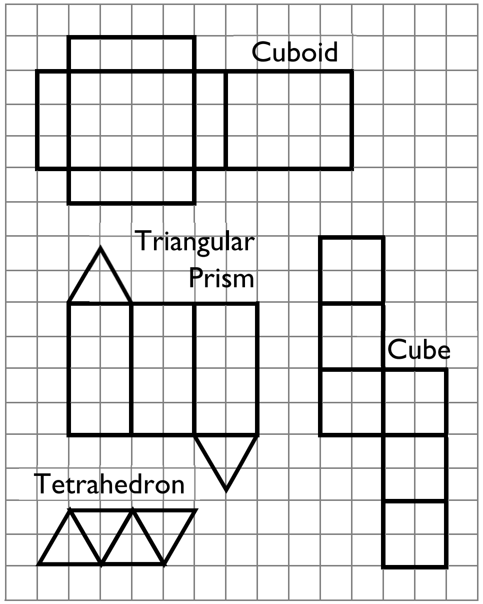






30b Nets

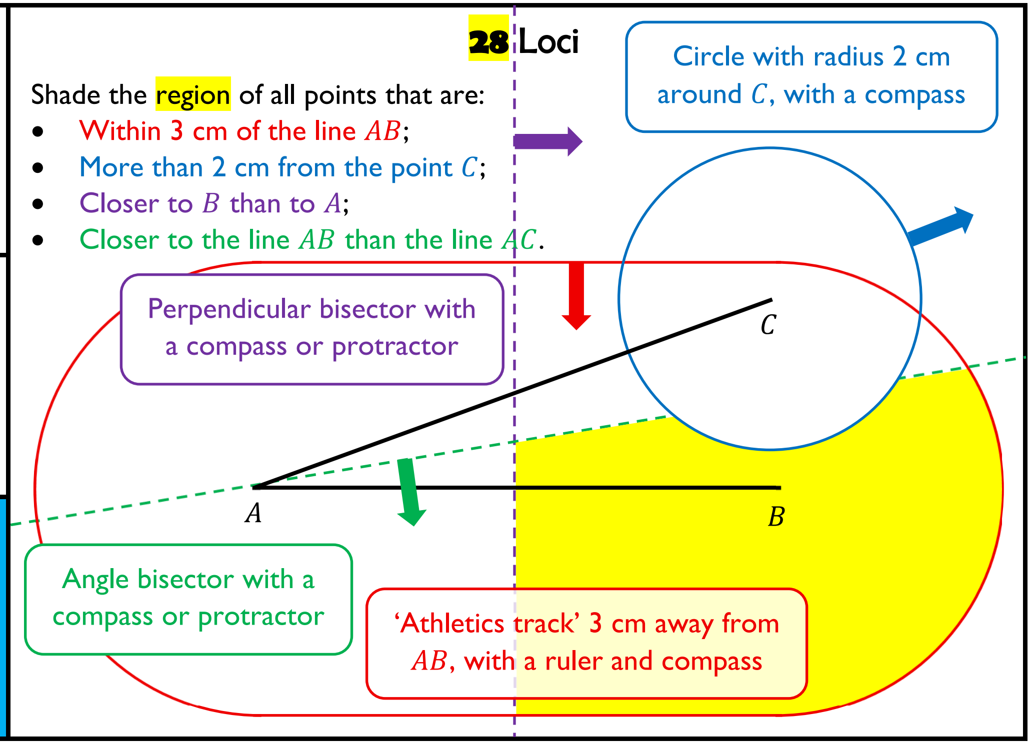
A 2-D shape that folds into a 3-D solid.



28 Loci

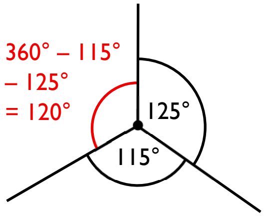
Shade the region of all points that are:

- Within 3 cm of the line AB;
- More than 2 cm from the point C;
- Closer to B than to A;
- Closer to the line AB than the line AC.

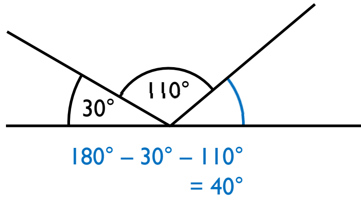


31 Angle Facts

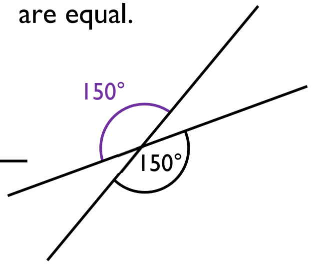
Angles around a point sum to 360°.



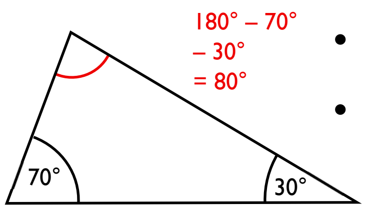
Angles on a straight line sum to 180°.



Vertically opposite angles are equal.

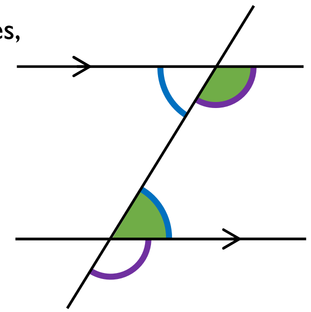


Angles in a triangle sum to 180°.



Given a pair of parallel lines,

- Alternate angles (Z-shaped) are equal;
- Corresponding angles (F-shaped) are equal;
- Internal angles (C-shaped) sum to 180°.



33 Units

Time

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days
1 year = 12 months
12-hour and 24-hour clock, e.g. 3:30 pm = 15:30.

0.1 hours = 6 minutes

Length

1 centimetre (cm) = 10 millimetres (mm)
1 metre (m) = 100 centimetres
1 kilometre (km) = 1,000 metres
[1 inch ≈ 2.5 cm]
[1 foot ≈ 0.3 m]
[1 mile ≈ 1.6 km, or 5 miles ≈ 8 km]

Mass

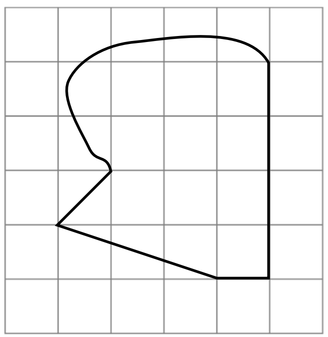
1 gram (g) = 1,000 milligrams (mg)
1 kilogram (kg) = 1,000 grams
1 metric ton (t) = 1,000 kilograms
[1 ounce ≈ 30 g]
[1 kg ≈ 2.2 pounds]

Capacity

1 litre (l) = 1,000 millilitres (ml)
[1 litre = 100 centilitres (cl)]
[1 pint ≈ 0.57 litres]
[1 litre ≈ 1.75 pints]
[1 gallon ≈ 4.5 litres]

You do not have to *learn* the facts in [square brackets], but you need to be able to *use* them in Unit 1, with the conversion always given.

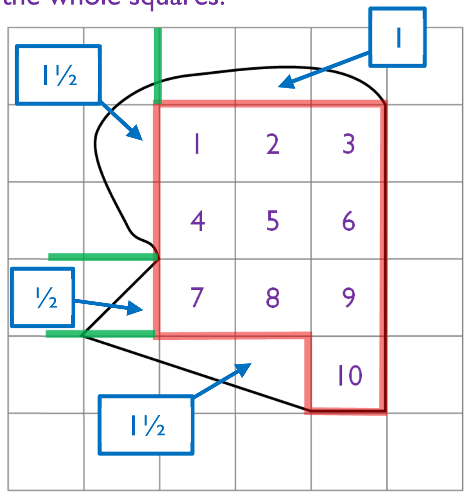
35 Estimating the Area of an Irregular Shape



To estimate the area of the shape on the left,

(1) Draw a grid inside the shape that contains all the whole squares.

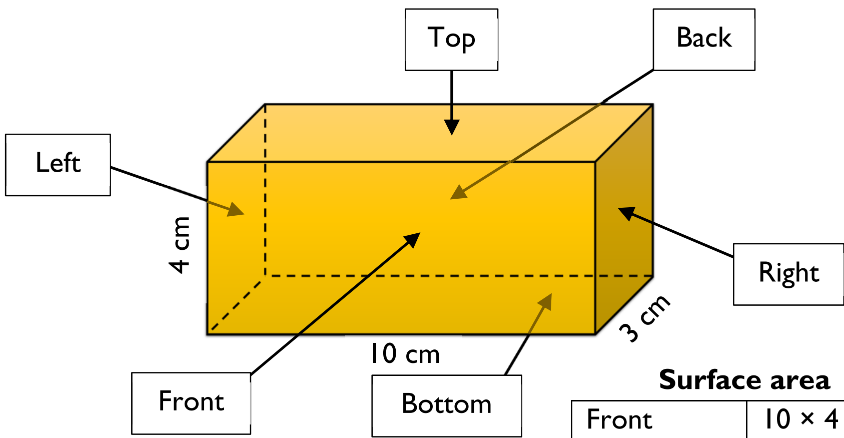
(2) Number the whole squares.



(3) Split the parts outside the grid into smaller pieces. Estimate the area of each piece.

(4) Add the number of whole squares to the total of pieces outside the grid:
10 + 4½ = 14½ square units.
(If each square is worth 10 cm²:
14½ × 10 = 145 cm².)

37 Volume and Surface Area of a Cuboid



Volume of a cuboid = length × width × height
= 10 × 3 × 4
= 120 cm³

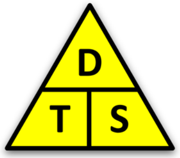
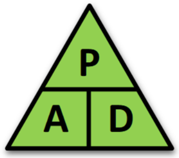
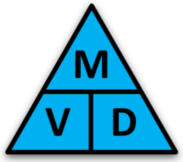
Capacity of the cuboid = 120 ml

A volume of 1 cm³ = 1 ml of water

Surface area

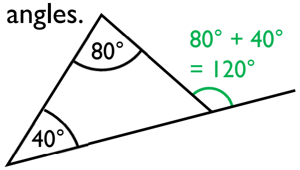
Front	10 × 4 = 40
Back	40
Left	3 × 4 = 12
Right	12
Top	10 × 3 = 30
Bottom	30
Total	**164 cm²**

34 Compound Measures

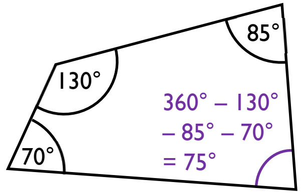
Units 1 and 3	Unit 3 only	
		
Speed = Distance ÷ Time Units: m/s, km/h, mph. **Fuel Consumption** = Distance Travelled ÷ Fuel Used. Units: miles per gallon (mpg), km per litre.	**Population Density** = Population ÷ Area Units: people per km², people per square mile.	**Density** = Mass ÷ Volume Units: g/cm³, kg/m³. **Rate of Flow** = Volume ÷ Time Units: m³ per hour, litres per second.

32 Angles in Polygons

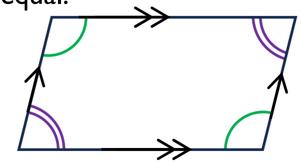
The external angle of a triangle is the sum of the two opposite interior angles.



The sum of the angles in a quadrilateral is 360°.

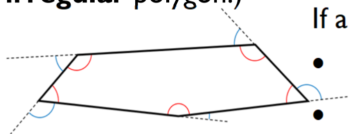


The opposite angles of some quadrilaterals are equal.



Unit 2 only

A polygon is **regular** if the length of all its sides is equal and the size of all its angles are also equal. (Otherwise: an **irregular** polygon.)



For a **polygon** with n sides,

- Total of the exterior angles $= 360°$.
- Total of the interior angles $= 180°(n-2)$.

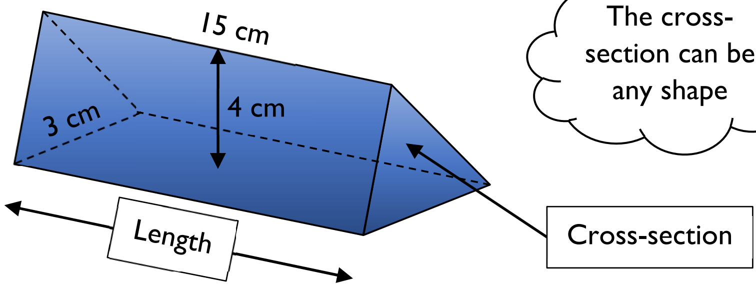
For any vertex in a polygon,

- Interior angle + exterior angle $= 180°$.

If a polygon is a regular polygon,

- One exterior angle $= \frac{360°}{n}$.
- One interior angle $= \frac{180°(n-2)}{n}$ or $180° - \frac{360°}{n}$.

38 Volume of a Prism



The cross-section can be any shape

Volume of a Prism = Area of Cross-section × Length

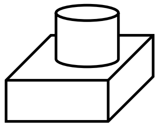
In the example, the cross-section is a triangle.

Area of the triangle $= \frac{\text{base} \times \text{height}}{2} = \frac{3 \times 4}{2} = 6$ cm².

Volume of the triangular prism = area of the triangle × length $= 6 \times 15 = 90$ cm³.

The capacity of the triangular prism is 90 ml.

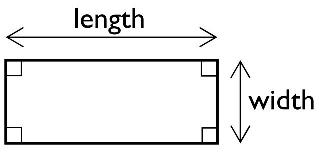
Split a composite solid into smaller pieces, e.g. split the one on the right into a cuboid and a cylinder.



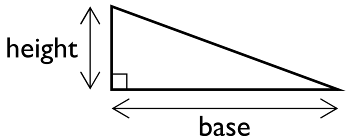
36 Perimeter and Area

The **perimeter** of a shape is the distance around the shape, obtained by adding all side lengths. The **area** of a shape is the size of the shape's surface. It is measured in square units.

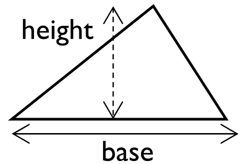
Area of a rectangle = length × width



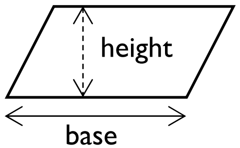
Area of a triangle $= \frac{\text{base} \times \text{height}}{2}$



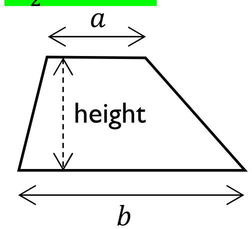
Split a **composite shape** into smaller pieces



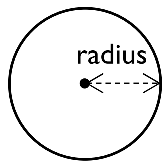
Area of a parallelogram = base × height



Area of a trapezium $= \frac{1}{2}(a+b)h$



Area of a circle $= \pi \times$ radius²

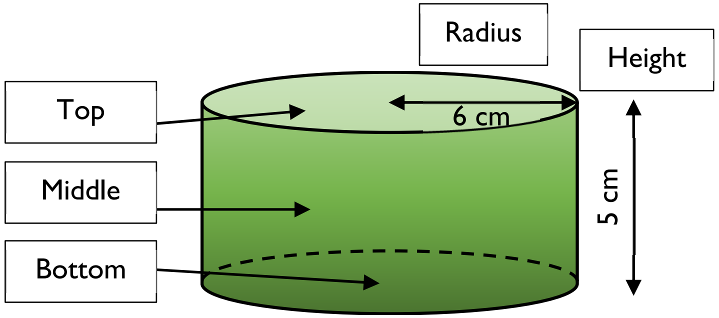


Halve to find the area of half a circle

Area of a square = length²

Circumference $= \pi \times$ diameter

39 Volume and Cross-Sectional Area of a Cylinder



Volume of a Cylinder
$= \pi \times \text{Radius}^2 \times \text{Height}$
$= \pi \times 6^2 \times 5$
$= 565.49$ cm^3, to 2 decimal places
Capacity of the cylinder
$= 565.49$ ml
$= 0.57$ litres, to 2 decimal places

Area of the top (or the cross-sectional area)
$= \pi \times \text{Radius}^2$
$= \pi \times 6^2$
$= 113.10$ cm^2, to 2 decimal places
Area of the bottom
$= 113.10$ cm^2, to 2 decimal places

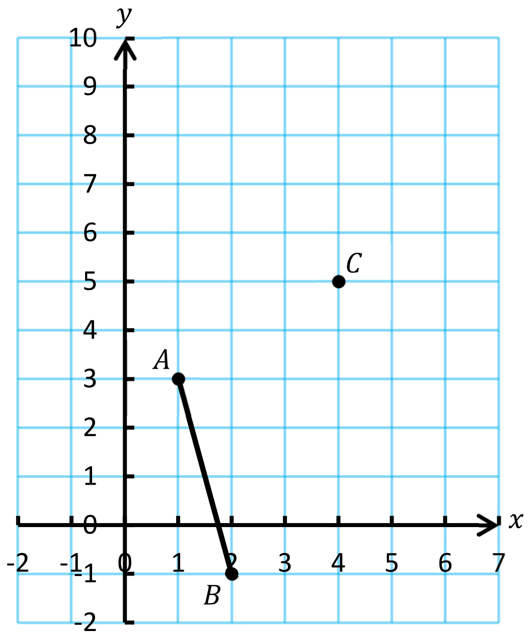
41 Finding Co-ordinates

What is the **mid-point** of line AB?

We can see on the graph that the answer is $(1.5, 1)$, or we can use the mean:
$A = (1, 3),\ B = (2, -1)$.
Mean of the x–coordinates:
$1 + 2 = 3,\ \ 3 \div 2 = 1.5$.
Mean of the y–coordinates:
$3 + -1 = 2,\ \ 2 \div 2 = 1$.
Mid-point $AB = (1.5, 1)$.

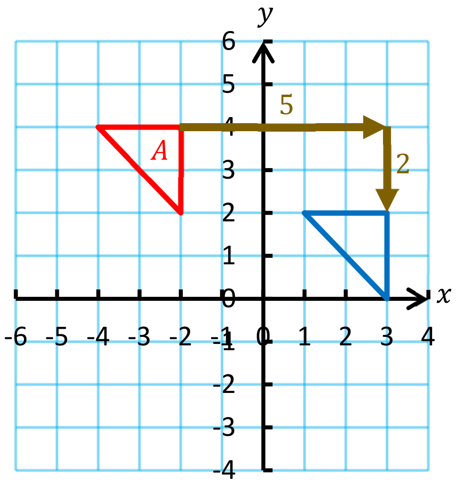
Find the co-ordinates of D such that $ABCD$ forms a parallelogram.
Answer: $D = (5, 1)$ or $D = (3, 9)$.

Note: this question can also be asked as: Find the co-ordinates of D so that CD is (a) parallel to AB; (b) the same length as AB.



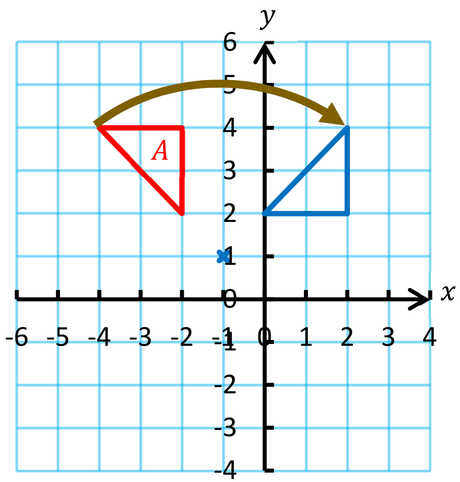
43 Transformations, Part 1

Translate the triangle A 5 units to the right, 2 units down.



To translate means to *move*

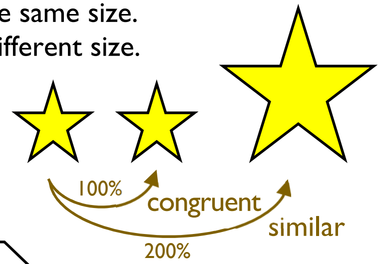
Rotate the triangle A by 90° clockwise around the point $(-1, 1)$.



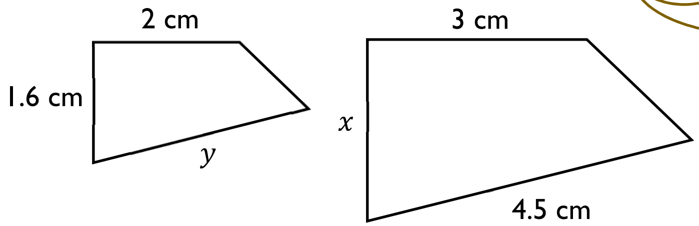
Need to ask for *tracing paper*

45 Congruent and Similar Shapes

Congruent shapes have the same shape and the same size.
Similar shapes have the same shape but have different size.



The two shapes below are similar shapes. Use the measurements on the shapes to find the lengths x and y.



By comparing the corresponding sides 2 cm and 3 cm, the **scale factor** is $3 \div 2 = 1.5$.

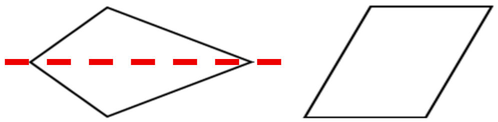
$x = 1.6 \times 1.5$
$x = 2.4$ cm

$y = 4.5 \div 1.5$
$y = 3$ cm

There may be a question on 3-D solids.

42 Symmetry

A **symmetry line** is an imaginary line that splits a shape into two halves that are reflections of each other.

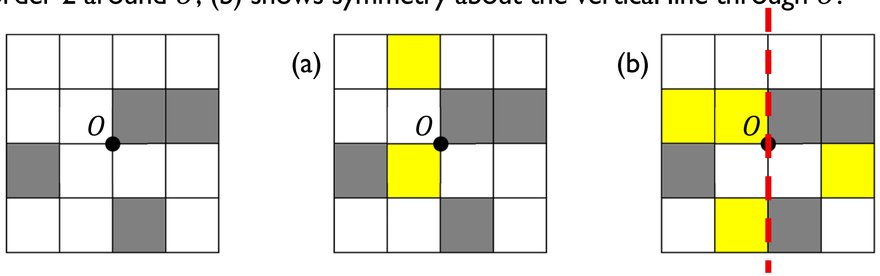


Kite
One symmetry line
No rotational symmetry

Parallelogram
No symmetry lines
Order of rotational symmetry 2

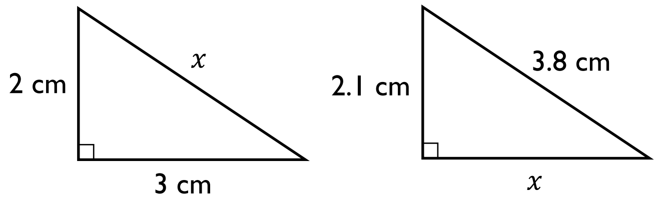
A shape has **rotational symmetry** if it comes to fit onto itself more than once during a complete rotation. The **order** of rotational symmetry is the number of times the shape comes to fit onto itself.

Shade the least number of squares so that the shape (a) shows rotational symmetry of order 2 around O; (b) shows symmetry about the vertical line through O.



40 Pythagoras' Theorem

For a right-angled triangle,
$$c^2 = a^2 + b^2$$
where c represents the length of the **hypotenuse** of the triangle (the longest side opposite the right angle), and a and b represent the other two lengths.

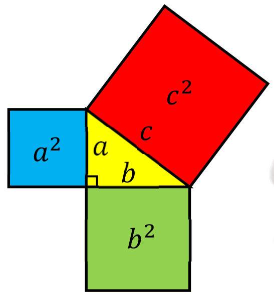


$$\begin{aligned} 2^2 &= \ 4 \\ 3^2 &= +\ 9 \\ & \quad\ \ \underline{\overline{13}} \end{aligned}$$

$\sqrt{13} = 3.61$ cm, correct to 2 d.p.

$$\begin{aligned} 3.8^2 &= \ 14.44 \\ 2.1^2 &= -\ \ \ 4.41 \\ & \quad\ \ \underline{\overline{10.03}} \end{aligned}$$

$\sqrt{10.03} = 3.17$ cm, correct to 2 d.p.



Want to **find** the hypotenuse?
You need to ADD.
Do you **know** the hypotenuse?
You need to SUBTRACT.

46 Questionnaires

A **hypothesis** is an idea to be proved, or disproved, using data that often comes from a **questionnaire**. The questions in a questionnaire need to

- avoid being leading;
- avoid overlapping options;
- be clear and concise;
- be relevant and suitable.

The bigger the sample, the more reliable the questionnaire's data is

We also need to be careful where, when and how a questionnaire is held.

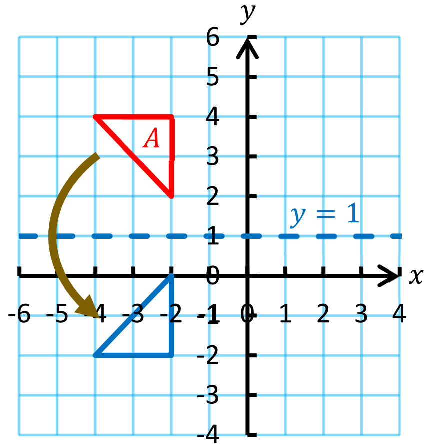
Taylor believes comedy films are more popular than horror films.
He writes the following questionnaire to test his hypothesis.

Taylor's Film Questionnaire
Question 1: What is your address? ________
Question 2: Which do you prefer? (Tick only one box)
Comedy films ☐ Horror films ☐
Question 3: How many films have you watched?
1–5 ☐ 5–10 ☐ 11–15 ☐ more than 15 ☐

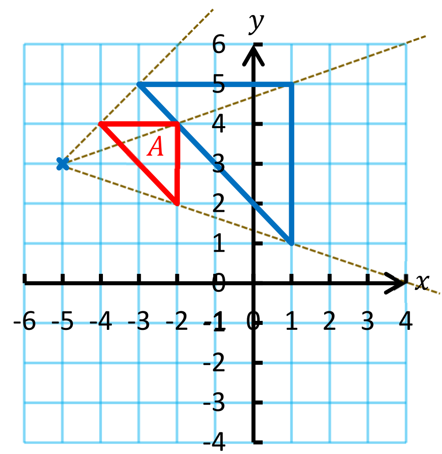
Criticisms: (a) Question 1 is not relevant. (b) Question 3 in not clear regarding the time under consideration. (c) In question 3, the option for 5 overlaps over two boxes. (d) There is no option for '0' in question 3.

44 Transformations, Part 2

Reflect the triangle A in the line $y = 1$.



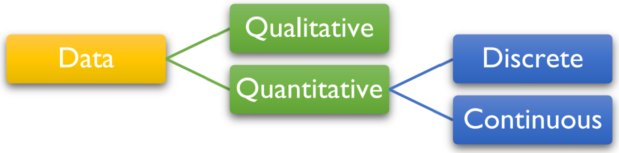
Enlarge the triangle A using the scale factor 2 and the centre of enlargement $(-5, 3)$.



A *fractional* scale factor is possible

47 Grouping Data

Types of data: **Qualitative** deals with descriptions, e.g. eye colour; favourite pet. **Quantitative** deals with numbers. **Discrete** qualitative data only takes specific values, e.g. shoe size; number of brothers. **Continuous** qualitative data can take any values within a specific range, e.g. length of your foot; time to run 100 m.



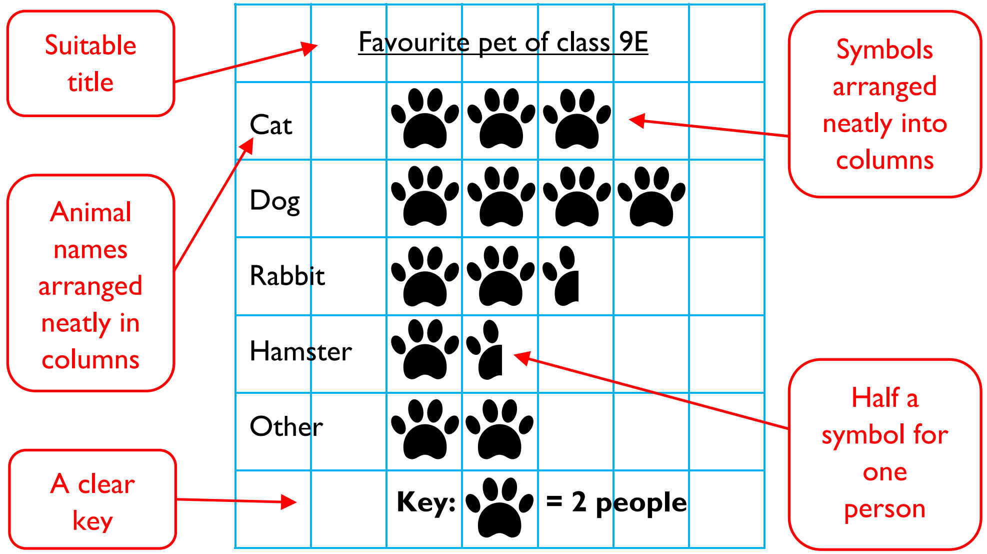
Here are the exam percentages of 40 year 7 learners:
56, 14, 61, 67, 90, 38, 47, 67, 72, 75, 80, 41, 21, 54, 92, 44, 68, 85, 43, 71, 69, 19, 39, 64, 83, 80, 65, 42, 20, 35, 27, 81, 70, 75, 39, 36, 71, 88, 50, 79. The table on the right **groups** the data into groups of equal width.

Group	Tally Marks	Frequency
1–20	𝍷𝍷𝍷	3
21–40	𝍸 𝍷𝍷	7
41–60	𝍸 𝍷𝍷𝍷	8
61–80	𝍸 𝍸 𝍸 𝍷	16
81–100	𝍸 𝍷	6
	Total	**40**

Tally marks in Unit 3 only

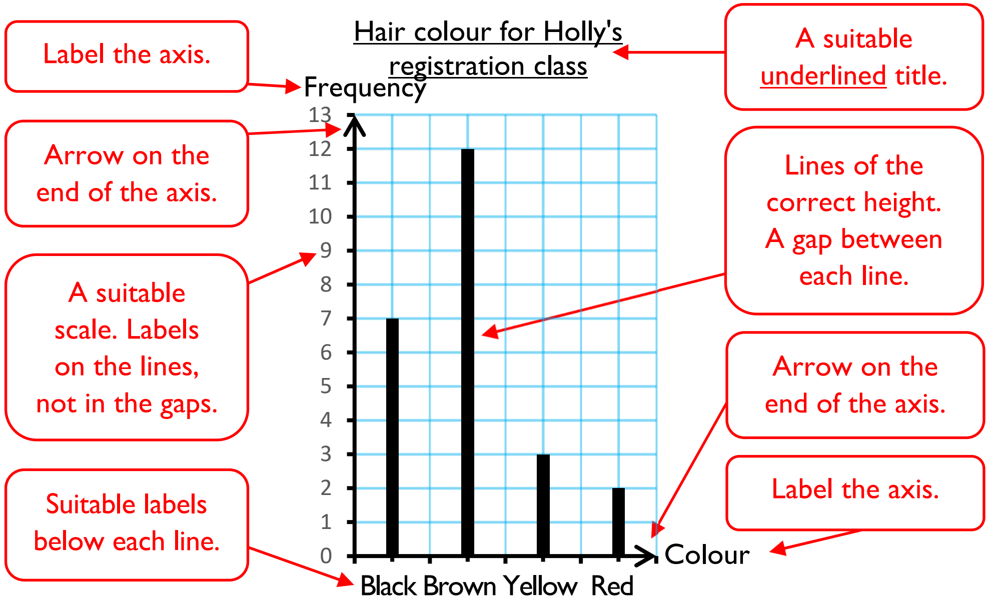
49 Pictogram

Uses a **symbol** to represent the data.



51 Vertical Line Diagrams

For qualitative data or discrete quantitative data that is not grouped.



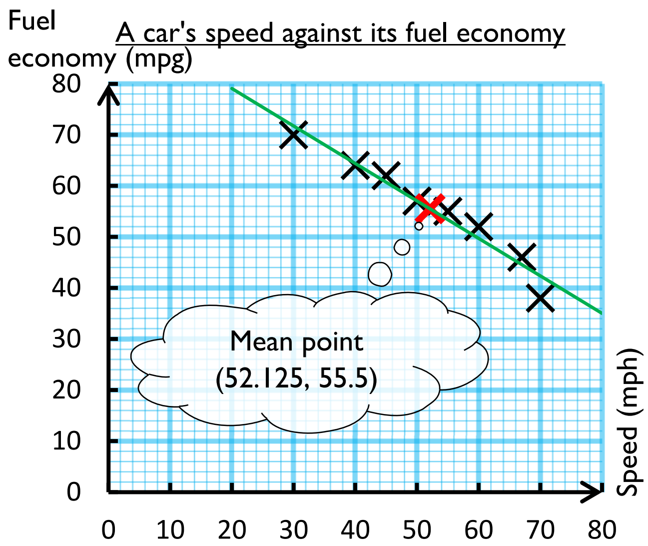
53 Scatter Diagrams, Part 1

Speed (miles per hour, mph)	30	40	45	50	55	60	67	70
Fuel economy (miles per gallon, mpg)	70	64	62	57	55	52	46	38

A **scatter diagram** plots two sets of paired data.

If there is **correlation** between the data, we can add a **line of best fit:**
(1) by eye;
(2) using the **mean point**.

For the above data, the mean of the speeds is (30 + 40 + 45 + 50 + 55 + 60 + 67 + 70) ÷ 8 = 52.125. Similarly, the mean of the fuel economies is 55.5.



50 Pie Chart

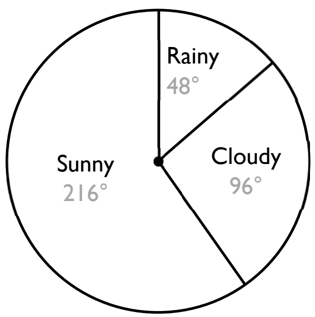
Here's the weather for the 30 days of June:
Rainy days: 4; Cloudy days: 8; Sunny days: 18.

Angle for each day: $360° \div 30 = 12$.
Angle for: *Rainy days:* $4 \times 12° = 48°$
Cloudy days: $8 \times 12° = 96°$
Sunny days: $18 \times 12° = 216°$

Start with 360° divided by the number of data items.

Use a protractor and ruler to draw the pie chart.

June Weather



Given the pie chart on the left, here's how to calculate the number of cloudy days in June (30 days)...

Step 1: measure (with a protractor) the angle for the cloudy sector: 96°.

Step 2: calculate $\frac{96}{360} \times 30 = \frac{48}{180} \times 30$
$= \frac{30}{180} \times 48 = \frac{1}{6} \times 48$
$= 8$ days.

48 Bar Chart

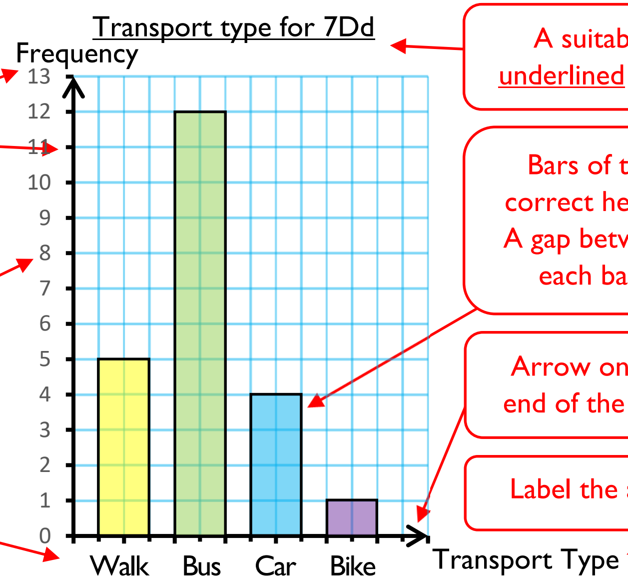
For qualitative data or discrete quantitative data that is not grouped.

Label the axis.

Arrow on the end of the axis.

A suitable scale. Labels on the lines, not in the gaps.

Suitable labels below each bar.



A suitable underlined title.

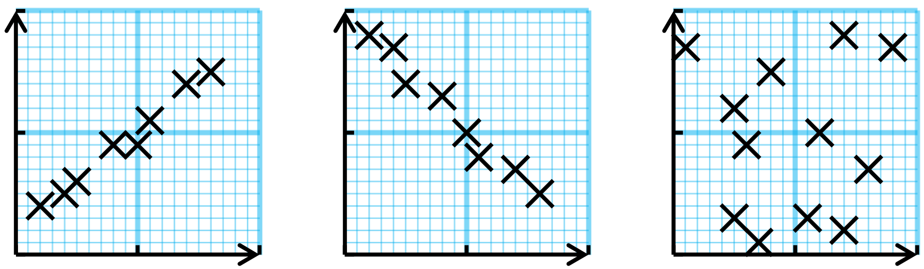
Bars of the correct height. A gap between each bar.

Arrow on the end of the axis.

Label the axis.

54 Scatter Diagram, Part 2

Positive correlation | Negative correlation | No correlation



- If a scatter diagram shows points close to each other in a line (like the first two above), then we can say that the correlation is **strong**.
- If a mean point is asked for or given, the line of best fit must go *through* the mean point. And – always remember to draw the line of best fit with a *ruler*.
- It is possible to use a line of best fit to estimate the value of one of the variables from the other, e.g. on card **53**, the fuel economy when travelling at 35 mph. Take care to remain within the range of the original data when doing this.
- Correlation does not always imply causation, i.e. if there is a pattern, it does not have to be a cause and effect relationship.

52 Line Graph

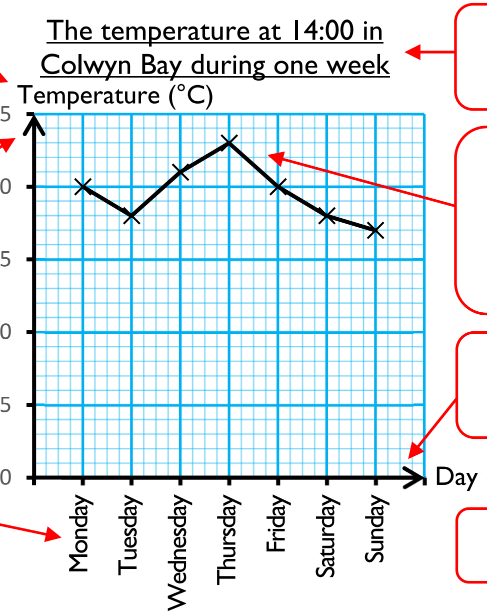
The median values in a line graph do not always have meaning.

Label the axis.

Arrow on the end of the axis.

A suitable scale. Labels on the lines, not in the gaps.

Suitable labels on the axis.



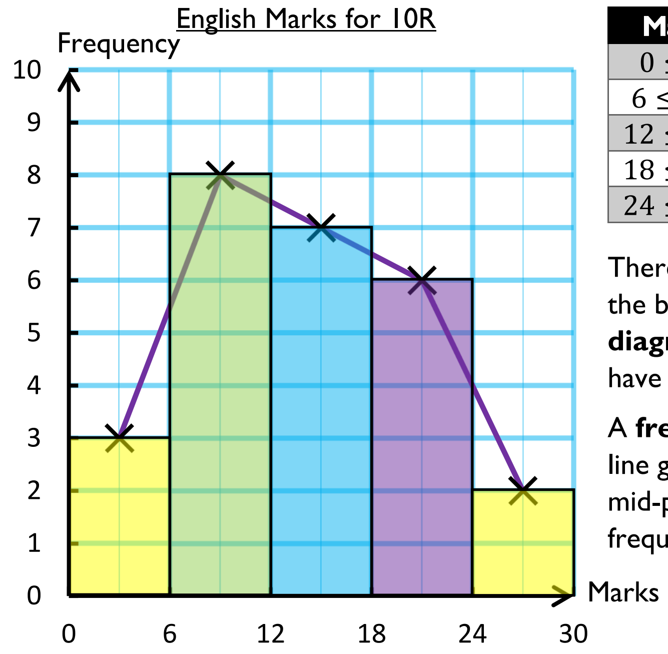
A suitable underlined title.

Plot the data using a cross or dot. Connect with a straight line.

Arrow on the end of the axis.

Label the axis.

55 Frequency Diagram and Frequency Polygon



Marks (m)	Frequency
$0 \le m < 6$	3
$6 \le m < 12$	8
$12 \le m < 18$	7
$18 \le m < 24$	6
$24 \le m < 30$	2

There are no gaps between the bars in a **frequency diagram**, but each bar must have equal width.

A **frequency polygon** is a line graph connecting the mid-point of each class to its frequency.

Ruler

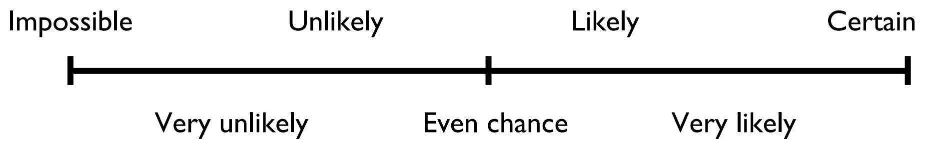
57 Averages and Range, Part 2

An **outlier** is a very large or very small number that does not fit the pattern of the rest of the data.

	Median	Mode	Mean
Advantages	Not affected by outliers.	Not affected by outliers. Can be used with qualitative data.	Uses all the data values.
Disadvantages	Does not use all the data values. Data needs to be arranged in order.	Does not use all the data values. There is no mode for some data sets.	Can be affected by outliers. It needs to be calculated.
Used for	Data with outliers.	Qualitative data. Data with outliers.	Data without outliers.

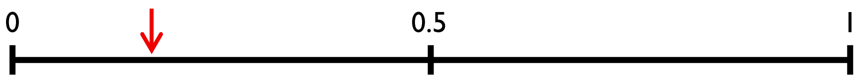
59 Probability, Part 1

Probability vocabulary:



What is the probability of rolling a 4 on a normal fair dice?

- In words (as a **chance**): Unlikely.
- On a probability scale:



- As a fraction: $\frac{1}{6}$. (Also, it can be written as a percentage or decimal.)

The total probability of all the outcomes of an experiment is 1.

The probability of *not* rolling a 4 on a normal fair dice is $1 - \frac{1}{6} = \frac{5}{6}$.

Apart from when a question asks specifically, you do not have to simplify the fraction in a probability question.

61 Relative Frequency

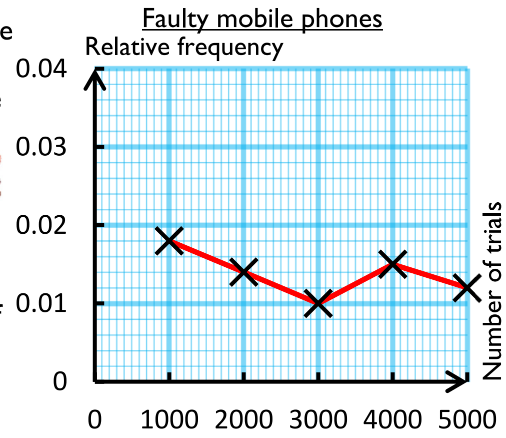
The **relative frequency** of an event compares the frequency to the number of trials.

$$\text{Relative frequency of an event} = \frac{\text{How many times the event has occurred}}{\text{Number of trials}}$$

For example, if a fair coin is flipped 10 times and 'heads' appears 3 times, then the relative frequency of obtaining heads is $\frac{3}{10} = 0.3$.

The more trials are held, the **better** the relative frequency is as an **estimate of the probability.** (In the long term, the relative frequency will stabilise.)

In the graph on the right, the best estimate that a factory produces a faulty mobile phone is 0.012 (the data point with the largest number of trials, 5,000). The number of faulty mobile phones in the first 5,000 produced is 5,000 × 0.012 = 60.



58 Grouped Data

For grouped data, averages and range must be **estimated**.

Marks (m)	Frequency	Cumulative Frequency
$0 \le m < 6$	3	3
$6 \le m < 12$	8	11
$12 \le m < 18$	7	18
$18 \le m < 24$	6	24
$24 \le m < 30$	2	26

Modal class: $6 \le m < 12$ (the most popular class with the largest frequency).

There are 3 + 8 + 7 + 6 + 2 = 26 data items. This is even, so on arranging the original data from least to greatest, there would be 2 numbers in the middle. These data items would be 26 ÷ 2 = 13, and 13 + 1 = 14. These are in the 3rd class. (The cumulative frequencies 3 and 3 + 8 = 11 are less than 13, and 11 + 7 = 18 is greater than 14.) So, the **median class** is $12 \le m < 18$.

To **estimate the mean**, multiply each mid-point with its frequency: The estimate of the mean is 366 ÷ 26 = 14.08, to 2 d.p.

$$\begin{aligned} 3 \times 3 &= 9 \\ 9 \times 8 &= 72 \\ 15 \times 7 &= 105 \\ 21 \times 6 &= 126 \\ 27 \times 2 &= \underline{54} \\ & \underline{366} \end{aligned}$$

The **estimate of the range** is the mid-point of the final class subtract the mid-point of the first class: 27 – 3 = 24.

The mid-point of $6 \le m < 12$ is 9 because 6 + 12 = 18, 18 ÷ 2 = 9.

56 Averages and Range, Part 1

The **median**, **mode** and **mean** are averages, whilst the **range** is a measure of spread.

'How many windows in your house'? 8, 12, 7, 9, 14, 9, 10, 16, 11, 9.

Median: *Step 1:* Arrange the data in order: 7, 8, 9, 9, 9, 10, 11, 12, 14, 16.
Step 2: See what is in the middle: 9 and 10. The median is halfway between 9 and 10: 9 + 10 = 19, 19 ÷ 2 = 9.5.

Mode: 9 (the most popular number, or the number that appears most often). Sometimes there is more than one more, or no mode at all (if the numbers are all different).

Mean: *Step 1:* Find the total of the data values: 8 + 12 + ⋯ + 9 = 105.
Step 2: Divide by the number of data items: 105 ÷ 8 = 13.125.

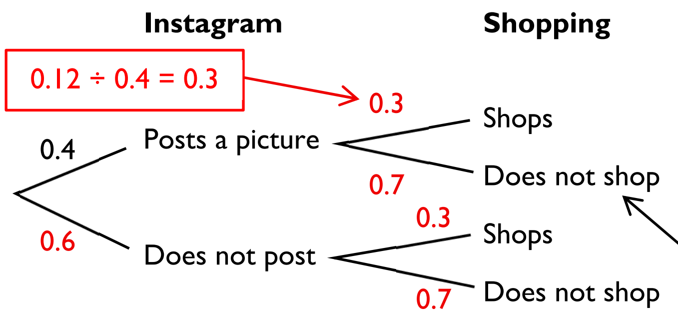
Range: 16 – 7 = 9 (greatest subtract least).

Deiniol has 5 numbers that have mode 5; median 6; mean 8 and range 9. What are Deiniol's numbers? The median is 6 so 6 is in the middle. 5 must appear twice to have a mode of 5. The range is 9 so the greatest number is 5 + 9 = 14.
The mean is 8 so the total is 5 × 8 = 40.
The final number is 40 – (5 + 5 + 6 + 14) = 10.

5	5	6	10	14

62 Tree Diagram

The probability that Elin posts a picture on Instagram over the weekend is 0.4. The probability that Elin goes shopping over the weekend is **independent** of her posting a picture on Instagram over the weekend. The probability of Elin posting a picture on Instagram over the weekend, and going shopping over the weekend, is 0.12. Use this information to complete the **tree diagram** below, and to find the probability that Elin posts a picture on Instagram over the weekend but does not go shopping over the weekend.



The probability of posting a picture but not going shopping is 0.4 × 0.7 = 0.28.

A question can have three consecutive events

60 Probability, Part 2

A red dice and a blue dice are labelled from 1 to 6.
Seren rolls both die and *multiples* the scores.
The following **sample space diagram** shows all the possible combinations.

		Number on the blue dice					
	×	1	2	3	4	5	6
Number on the red dice	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

Seren wins a game if the product of her scores is 4. The probability of winning is $\frac{3}{36}$ (the number 4 appears 3 times in the table). If Seren plays the game 72 times, she expects to win $\frac{3}{36} \times 72 = 3 \times 2 = 6$ times. (This is the **expected frequency**.) If the game costs 50p to play and the prize is £5, then over 72 games Seren pays 72 × 50c = £36 and expects to win 6 × £5 = £30. So, her expected loss is £36 – £30 = £6.