Additional Mathematics Revision Sheet

UNIT 1 (WITHOUT A CALCULATOR)

Surds

$\sqrt{n} \times \sqrt{n} = n.$

$\left(\sqrt{n}\right)^2 = n.$

$\sqrt[a]{n} \times \sqrt[a]{m} = \sqrt[a]{nm}.$

$\sqrt[a]{n} \div \sqrt[a]{m} = \sqrt[a]{\frac{n}{m}} = \frac{\sqrt[a]{n}}{\sqrt[a]{m}}.$

Rationalising the denominator:

$\frac{a}{b\sqrt{c}} = \frac{a \times \sqrt{c}}{b\sqrt{c} \times \sqrt{c}} = \frac{a\sqrt{c}}{bc}.$

$\frac{a}{d+e\sqrt{f}} = \left(\frac{a}{d+e\sqrt{f}}\right) \times \left(\frac{d-e\sqrt{f}}{d-e\sqrt{f}}\right).$

E.g. $\frac{2}{3+5\sqrt{2}} = \frac{2(3-5\sqrt{2})}{(3+5\sqrt{2})(3-5\sqrt{2})}$

$= \frac{6-10\sqrt{2}}{9-15\sqrt{2}+15\sqrt{2}-25\times 2} = \frac{6-10\sqrt{2}}{-41}.$

Rules of Indices

$n^a \times n^b = n^{a+b}.$

$n^a \div n^b = n^{a-b} = \frac{n^a}{n^b}.$

$n^0 = 1.$

$(n^a)^b = n^{a \times b}.$

$(nm)^a = n^a m^a.$

$\left(\frac{n}{m}\right)^a = \frac{n^a}{m^a}.$

$n^{-a} = \frac{1}{n^a}.$

$n^{\frac{1}{a}} = \sqrt[a]{n}.$

$\left(\sqrt[b]{n}\right)^a = n^{\frac{a}{b}} = \sqrt[b]{n^a}.$

Proof

Proof by **deduction**.

Using **algebraic identities** (≡).

Quadratic Functions

$ax^2 + bx + c$ with $a \neq 0$.

'u'–shaped if $a > 0$;

'n'–shaped if $a < 0$.

Completing the Square

$x^2 + 2ax = (x + a)^2 - a^2.$

Finding maximum or minimum points.

Solving Equations and Inequalities

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

Solving equations with **higher powers**.

Simultaneous equations.

The Discriminant

Two distinct real roots:

$b^2 - 4ac > 0.$

Two real roots:

$b^2 - 4ac \geq 0.$

One repeated root:

$b^2 - 4ac = 0.$

Two complex roots / No real roots:

$b^2 - 4ac < 0.$

Arithmetic Series

First term a.

Common difference d.

Nth term $t_n = a + (n-1)d$.

Sum of the first n terms:

$S_n = \frac{1}{2}n(a + l).$

$S_n = \frac{1}{2}n(2a + (n-1)d).$

Geometric Series

First term a.

Common ratio r.

Nth term $t_n = ar^{n-1}$.

Sum of the first n terms:

$S_n = \frac{a(1-r^n)}{1-r}.$

UNIT 2 (WITH A CALCULATOR)

Differentiation

[From first principles:

$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.]

Quick differentiation: If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$.

Second derivative: $\frac{d^2y}{dx^2}$.

Finding the equation of the tangent or normal (perpendicular to the tangent):

$y - y_1 = m(x - x_1).$

Stationary Points

We must solve $\frac{dy}{dx} = 0$ and use the $\frac{d^2y}{dx^2}$ test to find the nature of the stationary point. If $\frac{d^2y}{dx^2}$ is **negative** the point is a **maximum** point; if it is positive we have a **minimum** point; if it is **zero** further investigation is necessary.

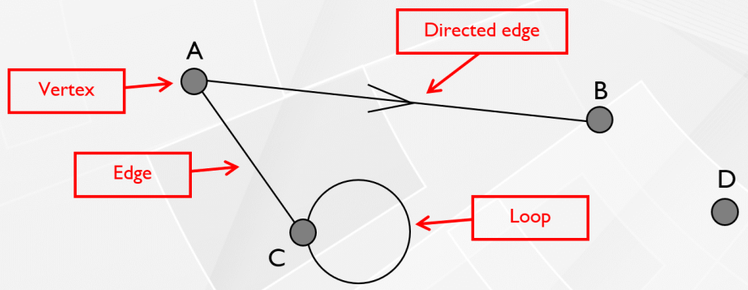
Integration

$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c.$

$\int_p^q ax^n dx = \left[\frac{ax^{n+1}}{n+1}\right]_p^q.$

UNIT 6 (WITH A CALCULATOR)

Graphs



A **walk** is a route through a graph along edges from one vertex to the next.

An **open walk** is a walk where the start vertex is not the same as the end vertex.

A **closed walk** is a walk where the start vertex is the same as the end vertex.

A **trail** is a walk in which no edge is visited more than once.

A **path** is a walk in which no vertex is visited more than once.

A **cycle** is a closed walk where no vertex (apart from the start vertex) is visited more than once

A graph is a **simple graph** if it contains

- no loops;
- no directed edges;
- only one edge connecting any pair of vertices.

A graph is a **connected graph** if every pair of vertices is connected by a walk.

A graph is a **complete graph** if every vertex is connected to every other vertex by a unique edge.

A graph is a **tree** if it is a connected graph with no cycles.

A **spanning tree** is a subgraph of a graph G that includes all vertices and is also a tree.

A graph is an **Eulerian graph** if it is a connected graph where every vertex has an even degree.

A graph is a **semi-Eulerian graph** if it is a connected graph where precisely two vertices have odd degree.

Algorithms

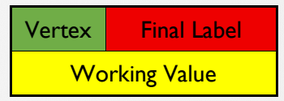
Choosing a minimal spanning tree:

Kruskal (form a path starting from a point).

Prim (choose edges in ascending order).

Dijkstra's algorithm is used to find the shortest distance between two points.

Key

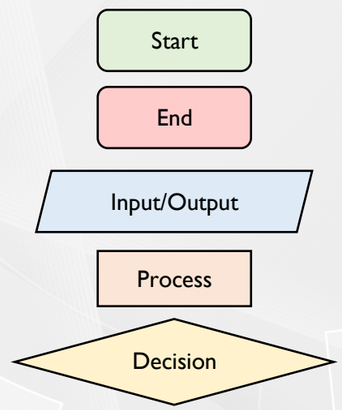


Sorting data:

Bubble sort (working left-to-right over one or more passes).

Quick sort (choosing a pivot and sorting each side of the pivot).

Flow chart symbols:



An algorithm can be **finite**, **definite** or **correct**.

Critical Paths

An **arc network** represents a project. Each **arc** (or arrow) represents an activity, with the label showing the letter of the activity and its completion time in brackets. The **nodes** (circles) show the start and end of activities and are considered to be **events**.

The **float** of an activity is

Late event time for event j – early event time for event i – length of the activity.

Any activity with a float of zero is considered to be a **critical** activity.

A **Gantt Chart** shows when different events run (or can run) in a project.

Linear Programming

The **decision variables** are used to form **constraints** for the problem. In the **feasible region**, the **objective function** must be maximised to find the **optimal solution**.

The Examination

Length of each paper: 50 minutes.

40 marks.

$33\frac{1}{3}$% of the qualification.

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