



The Mathematics Department

10

Accuracy of

Measurements

Foundation Tier

Version

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Name:

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Significant Figures

Sarah sees a television for sale on the internet, for £389.

Delyth, who is Sarah's friend, says that the television costs around £400. Greta, another one of Sarah's friends, says that the television costs around £390.

Sarah's two friends have both given an **estimate** for the price of the television. Delyth has given her estimate to **one significant figure**, whilst Greta has given her estimate to **two significant figures**.



When rounding off to **one significant figure**, we need to find an estimate that, at most, uses **one non-zero digit**.

When rounding off to **two significant figures**, we need to find an estimate that, at most, uses **two non-zero digits**.

(And so on for three significant figures; four significant figures; ...)



Example

Price	The price rounded off to one significant figure
£389	£400
£1,799	£2,000
£2.35	£2
£0.41	£0.40

Exercise 1

Round off the following prices to one significant figure.

- | | | | |
|------------|------------|-------------|--------------|
| (a) £799 | (b) £899 | (c) £999 | (d) £1,099 |
| (e) £1,846 | (f) £1,500 | (g) £28,381 | (h) £370,000 |
| (i) £4.19 | (j) £7.93 | (k) £16.70 | (l) £42.99 |
| (m) £0.84 | (n) £0.37 | (o) £0.13 | (p) £0.04 |



Exercise 2

Round off the following numbers to one significant figure.

- | | | | |
|------------|-------------|------------|-----------|
| (a) 329 | (b) 698.2 | (c) 450 | (d) 975 |
| (e) 19 | (f) 14 | (g) 7.3 | (h) 2.8 |
| (i) 0.54 | (j) 0.87 | (k) 1.56 | (l) 0.98 |
| (m) 24,528 | (n) 543,590 | (o) 19,829 | (p) 9,502 |



Example

Price	The price rounded off to two significant figures
£389	£390
£1,799	£1,800
£2.35	£2.40
£0.41	£0.41

Exercise 3



Round off the following prices to two significant figures.

- (a) £263 (b) £268 (c) £265 (d) £299
- (e) £4,368 (f) £2,809 (g) £54,238 (h) £876,000
- (i) £6.37 (j) £1.62 (k) £14.28 (l) £27.99
- (m) £1.95 (n) £19.99 (o) £0.13 (p) £0.04



Exercise 4

Round off the following numbers to two significant figures.

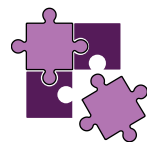
- (a) 836 (b) 278.5 (c) 913 (d) 976
- (e) 76.3 (f) 16.07 (g) 7.92 (h) 5.37
- (i) 0.283 (j) 0.927 (k) 1.8203 (l) 26.07
- (m) 82,573 (n) 903,400 (o) 49,732 (p) 997

Exercise 5

Complete the following table.

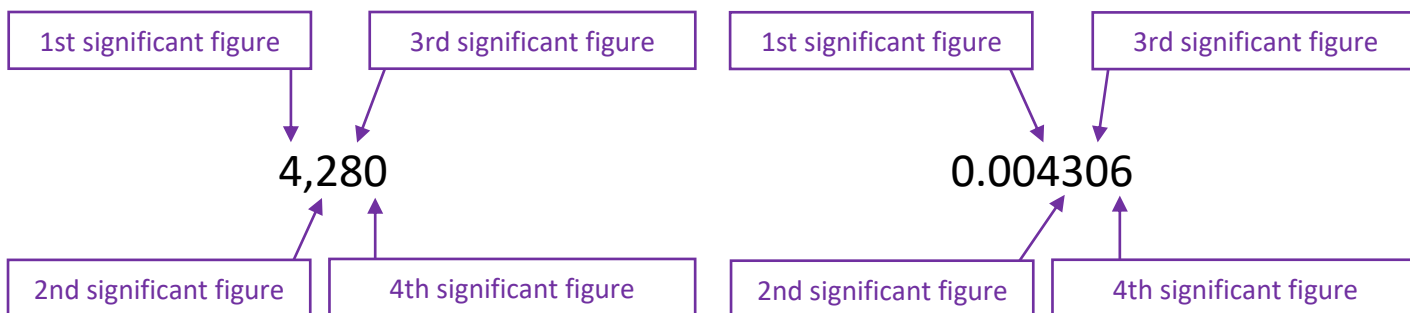
	Original number	Round off to one significant figure	Round off to two significant figures	Round off to three significant figures	Round off to four significant figures
(a)	28,539				
(b)	54.591				
(c)	3.0589				
(d)	1,339				
(e)	106,728				
(f)	4,525,925				
(g)	73,524,000				
(h)	183.078				
(i)	8,406.5				
(j)	0.87325				
(k)	0.018393				
(l)	0.0083736				

Labelling the Significant Figures of a Number



When reading a number from left to right, the **first** significant figure is the first digit encountered that is **not zero**. The other significant figures follow on immediately after the first significant figure.

Example



Exercise 6

For the following numbers, circle the **first** significant figure.

- (a) 8,205
- (b) 934.2
- (c) 387
- (d) 46
- (e) 0.0378
- (f) 0.982
- (g) 0.0400213
- (h) 0.000029002



Exercise 7

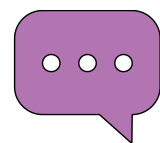
For the following numbers, circle the **second** significant figure.

- (a) 43,503
- (b) 2,043,293
- (c) 819
- (d) 8.32
- (e) 0.0729
- (f) 0.02083
- (g) 0.0004893
- (h) 0.000000072009

Sometimes, we need to add zeroes to the end of a number to write it correct to a specified number of significant figures. For example, to write 45 correct to three significant figures, we must write 45.0.

Exercise 8

- (a) Write 87 correct to three significant figures.
- (b) Write 987 correct to four significant figures.
- (c) Write 3,270 correct to six significant figures.
- (d) Write 7 correct to eight significant figures.

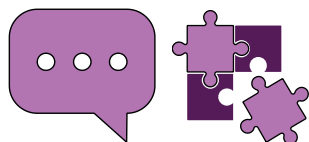


Key words	Corrections	I am happy with...	I need to revise...



Estimation

Dafydd wants to calculate the sum $\frac{482.6 \times 43.5}{2168.12}$, but unfortunately he has lost his calculator. In order to find an approximate value for the correct answer, it would be possible for Dafydd to **estimate** the size of the numbers in the calculation, and perform the calculation with these estimates.



$$\begin{aligned} \frac{482.6 \times 43.5}{2168.12} &\approx \frac{500 \times 40}{2000} \\ &= \frac{20000}{2000} \\ &= 10 \end{aligned}$$



Let us now look at the different parts of the above estimation, explaining each part in turn.

- 1) The first line contains the estimation symbol \approx . We say this symbol as 'is around' or 'is approximately', and we need to include the symbol (instead of the equals sign $=$) as the left-hand side is not *equal* to the right-hand side.
- 2) Unless common sense suggests otherwise, the convention for estimating numbers is to round them off to **one significant figure**. So, 482.6 rounds off to 500; 43.5 rounds off to 40; and 2168.12 rounds off to 2000.
- 3) Going from the first line to the second line, we perform the calculation $500 \times 40 = 20000$. To perform this sum quickly, note that $5 \times 4 = 20$, and that the **three zeroes** on the left-hand side lead to **three similar zeroes** on the right-hand side.
- 4) Going from the second line to the third line, we perform the calculation $\frac{20000}{2000} = 10$. To perform this sum quickly, note that three of the zeroes **cancel** on the top and bottom: $\frac{20000}{2000}$; leading to a much easier sum, $\frac{20}{2} = 10$. (The process of 'cancelling' zeroes comes from the fact that we can simplify the fraction by dividing the numerator and denominator by 10 each time.)

Exercise 9

Find an estimate for each of the following calculations.

- | | | |
|-----------------------------|------------------------|----------------------------|
| (a) 423×297 | (b) $19 \times 43,926$ | (c) $876 \times 6,875$ |
| (d) $97 \times 1,024$ | (e) 6.4×2.9 | (f) $0.98 \times 60,536.1$ |
| (g) $75,249 \times 342,192$ | (h) $198 \times 5,034$ | (i) $78 \times 452,138$ |

Exercise 10

Find an estimate for each of the following calculations.

- | | | |
|----------------------------|--------------------------------|------------------------------|
| (a) $\frac{7,924}{403}$ | (b) $\frac{324}{18.4}$ | (c) $\frac{879,054}{12,456}$ |
| (d) $\frac{5,783}{1,985}$ | (e) $\frac{98,160}{502}$ | (f) $\frac{87,204}{2,642}$ |
| (g) $\frac{82,457}{23.46}$ | (h) $\frac{7,304,539}{67,045}$ | (i) $\frac{614}{28}$ |

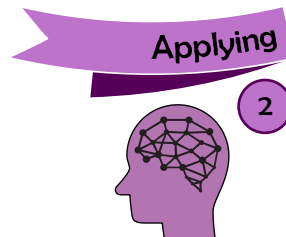
Exercise 11

Find an estimate for each of the following calculations.

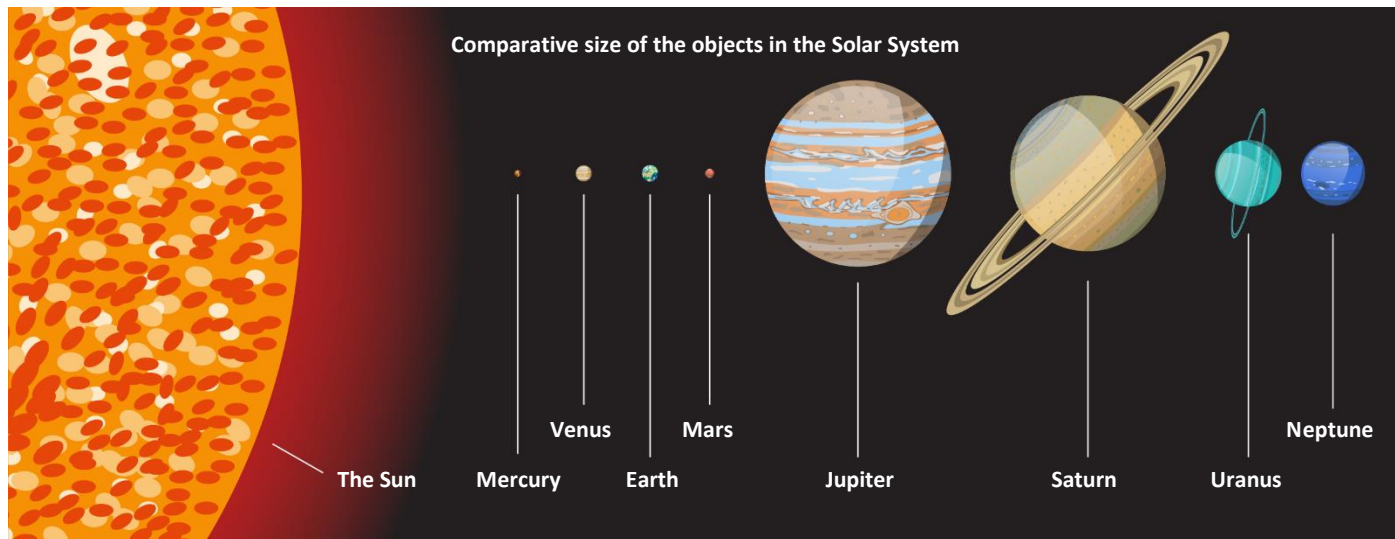
- | | | |
|-------------------------------------|-------------------------------------|---|
| (a) $\frac{798 \times 403}{1,980}$ | (b) $\frac{1,975 \times 435}{787}$ | (c) $\frac{61 \times 90,128}{2,975}$ |
| (d) $\frac{3.24 \times 648}{895}$ | (e) $\frac{78,420}{204 \times 4.8}$ | (f) $\frac{4.9 \times 26.4 \times 7.4}{2.4601}$ |
| (g) $\frac{94.2 \times 398}{552.4}$ | (h) $\frac{0.13 \times 783}{19}$ | (i) $\frac{4.15 \times 5.63}{0.53}$ |



Exercise 12



- (a) A t-shirt costs £5.99 and a skirt costs £12.99. Sophie wants to buy 2 t-shirts and 5 skirts. She has £50. Use an approximation to decide whether Sophie has enough money.
- (b) David saves £3.80 each week. Approximately how much does he save in a year?
- (c) Eleri has a mobile phone contract costing £29.99 a month. Approximately how much does Eleri pay over a year?



- (d) The diameter of the Sun is around 1,391,400 km, and the diameter of Earth is around 12,742 km. Approximately how many times bigger is the diameter of the Sun compared to the diameter of Earth?
- (e) John works in an office and earns £19.75 per hour. He works 39 hours a week. Find an estimate for John's weekly wage.
- (f) Someone has asked the pupils in year 9 to estimate the individual cost of a trip to watch a musical. There are 98 pupils in year 9. A packed lunch for each pupil costs £3.95. The total cost of the tickets to the musical is £915. The transport costs £290. Estimate the individual cost for each pupil to watch the musical, correct to the nearest pound. Show all of your method.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Upper and Lower Bounds



In order to understand how to find **upper and lower bounds**, it's a good idea to first revise different techniques of **rounding off** a number.

Exercise 13

Complete the following table.



Revision

1

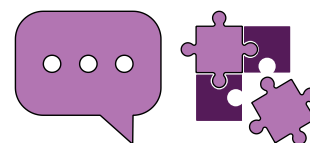
	Number	Round off to the nearest 10	Round off to 1 decimal place	Round off to 1 significant figure
E.g.	825.94	830	825.9	800
(a)	523.86			
(b)	49.15			
(c)	5,284.792			
(d)	3.67			
(e)	284.99			
(f)	43,704.75			
(g)	726			

Given a number rounded off in a particular way, we can consider what the original number was, before rounding occurred. For example, consider the number 470, which has been rounded off to the nearest 10. What could the original number have been? All the numbers between 465 and 474.9999... round off (to the nearest 10) to be 470. If x represents the original number, then we can say that

$$465 \leq x < 475.$$

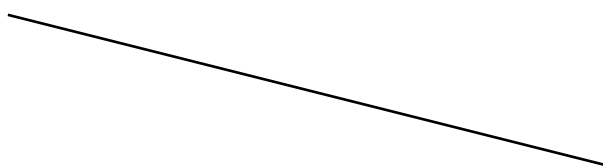
We can also write

$$x = 470 \pm 5$$



but we must remember that x cannot be exactly 475, since this number would round off to give 480. We say that the **lower bound** is 465, and the **upper bound** is 475.

One place where upper and lower bounds are used are when taking measurements, since **every measurement is an approximation**. For example, measure the line below with a ruler.



You should measure the line to be 8.2 cm, but is the line exactly 8.2 cm? How do you know that the line isn't truly 8.19 cm, or 8.21 cm? We would need a more accurate measuring instrument than a ruler to check this, therefore the best we can say (by using a ruler and our eyes) is that the line is 8.2 cm, correct to the nearest 0.1 cm (or mm).

The accuracy limit of a measurement, approximation or calculation uses upper and lower bounds. To measure the above line with a ruler, the lower bound is 8.15 cm and the upper bound is 8.25 cm. The true measurement lies between these two limits. If l denotes the true length of the line in cm, then we can say that

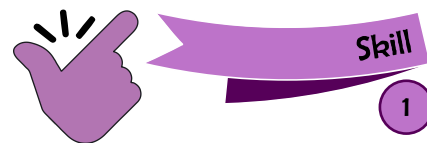
$$8.15 \leq l < 8.25$$

or

$$l = 8.2 \pm 0.05.$$

Example

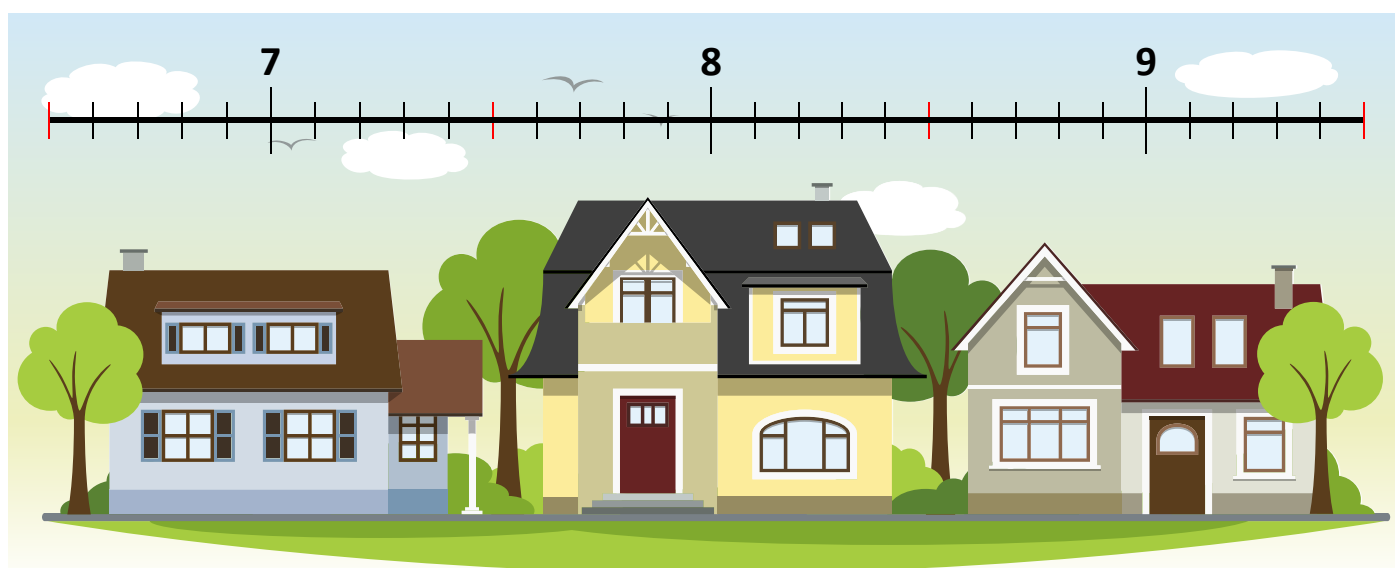
Measurement	Lower Bound	Upper Bound
42 cm (to the nearest cm)	41.5 cm	42.5 cm
85 km (to the nearest 5 km)	82.5 km	87.5 km
60 m (to the nearest 10 m)	55 m	65 m



Exercise 14

Complete the following table.

	Measurement	Lower Bound	Upper Bound
(a)	34 cm (to the nearest cm)		
(b)	5 m (to the nearest m)		
(c)	148 kg (to the nearest kg)		
(d)	25 cm (to the nearest 5 cm)		
(e)	420 ml (to the nearest 5 ml)		
(f)	1,825 km (to the nearest 5 km)		
(g)	40 m (to the nearest 10 m)		
(h)	80 cm (to the nearest 10 cm)		
(i)	180 g (to the nearest 10 g)		
(j)	400 cm (to the nearest 100 cm)		
(k)	5,400 m (to the nearest 100 m)		
(l)	27,800 litres (to the nearest 100 litres)		
(m)	8,000 km (to the nearest 1,000 km)		
(n)	45,000 miles (to the nearest 1,000 miles)		
(o)	3,000 tons (to the nearest 1,000 tons)		
(p)	6 cm (to the nearest even number)		
(q)	154 cl (to the nearest even number)		
(r)	4,250 ml (to the nearest even number)		
(s)	80 cm (to the nearest 20 cm)		
(t)	250 ml (to the nearest 50 ml)		
(u)	320 kg (to the nearest 40 kg)		

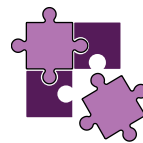


Challenge!

How does the above picture explain how to find upper and lower bounds?


 Appropriate Degree of Accuracy

Measurements and calculations should not be too precise for their purpose. For example, advertising a television of size 39.97 inches would be too precise (it would be better to say that the television was 40 inches in size).

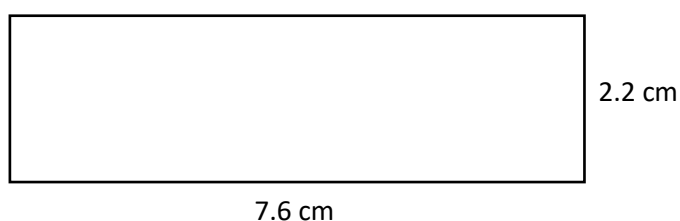


As a general rule, if a question asks you to give an answer to an **appropriate degree of accuracy**,

do not give an answer that is more precise than the values used in the calculation.

Example

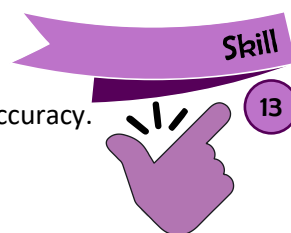
Calculate the area of the following rectangle. Give your answer to an appropriate degree of accuracy.



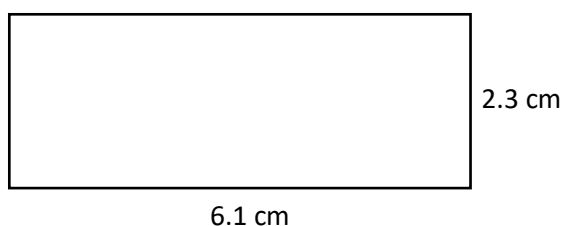
Answer: The area of the rectangle is $7.6 \times 2.2 = 16.72 \text{ cm}^2$. Because the numbers in the question have been rounded off to one decimal place, then our answer should also be rounded off to one decimal place. Therefore, the area of the rectangle (to an appropriate degree of accuracy) is 16.7 cm^2 .

Exercise 15

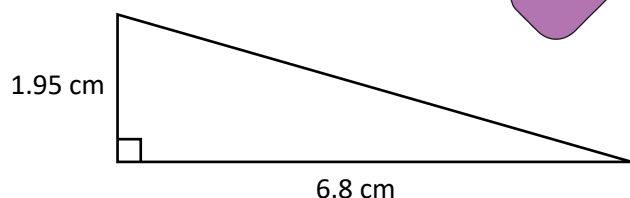
Calculate the area of the following shapes. Give your answers to an appropriate degree of accuracy.



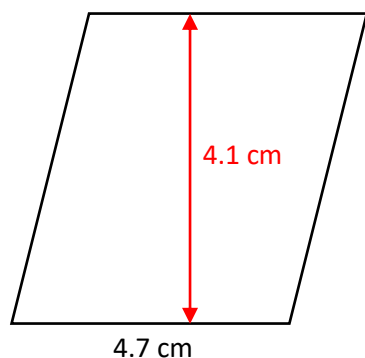
(a)



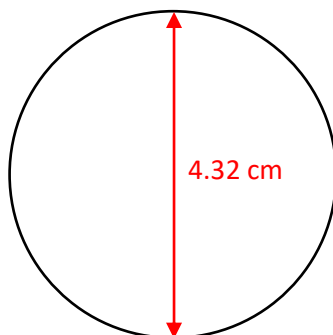
(b)



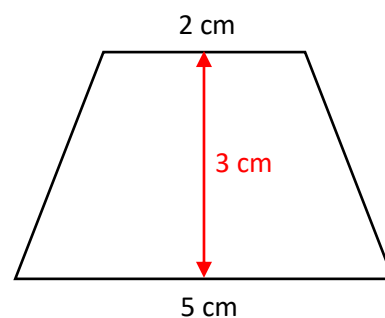
(c)



(d)



(e)


Exercise 16

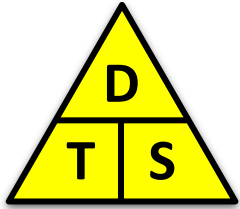
Mabli has a cuboid that measures 5.2 cm by 8.9 cm by 12.8 cm. Calculate, to an appropriate degree of accuracy, the volume of the cuboid.

Compound Measures



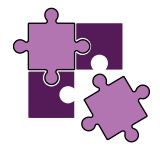
A **compound measure** is a type of measure that combines two different measures.

In the "Movement with Sphero" workbook, we saw an example of a compound measure, namely **speed**.



Distance = Time × Speed
Time = Distance ÷ Speed
Speed = Distance ÷ Time

To use the triangle, *hide* the letter that you want to find.



Example

(a) Iwan cycles a distance of 32 km in $2\frac{1}{2}$ hours.
 Calculate Iwan's average speed in km/hour.

Answer: Speed = Distance ÷ Time
 = $32 \div 2.5$
 = 12.8 km/hour

(b) Moli runs an 800 m race in 2 minutes 48 seconds.
 What is Moli's average speed in metres per second?

Answer: Speed = Distance ÷ Time
 = $800 \div 168$
 = 4.8 metres per second (to one decimal place)

Exercise 17

Revision
13

(a) Hannah cycles a distance of 49 km in $3\frac{1}{2}$ hours.
 Calculate Hannah's average speed in km/hour.

(b) Elin runs a 400 m race in 1 minute 12 seconds.
 What is Elin's average speed in metres per second?

(c) A bus travels the 12 miles from Llandudno to Abergele in 30 minutes.
 Find the average speed of the bus in miles per hour.

(d) A train travels at an average speed of 90 km/hour for 2 hours 15 minutes.
 How far has the train travelled during this time?

(e) In a 100 m race Dilwyn ran at an average speed of 7.1 metres per second.
 How much time did Dilwyn take to finish the race?

(f) An aeroplane flies at an average speed of 885 km/hour. How far did the aeroplane travel between the times 0820 and 0900?

(g) The following chart shows the travelling distance for a car, in miles, between some places in Wales.



	Swansea			
Aberystwyth	70	Aberystwyth		
Bangor	157	86	Bangor	
Cardiff	41	98	180	Cardiff
Wrexham	130	79	70	139

(i) What is the travelling distance for a car between Cardiff and Bangor?

(ii) Dewi travels between Cardiff and Bangor, and then between Bangor and Wrexham. How far has Dewi travelled in total?

(iii) Elis travels between Bangor and Aberystwyth in 2 hours and a half. What is Elis' average speed, in mph?

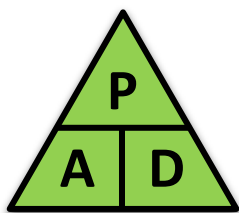
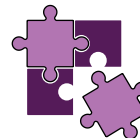
(iv) Eryllt travels between Wrexham and Swansea at an average speed of 32 miles per hour.

How much time did Eryllt take to travel between Wrexham and Swansea?



Population Density

Population density compares the population of a place to its area.



Population = Area × Population Density
Area = Population ÷ Population Density
Population Density = Population ÷ Area

Population density is (usually) measured in people per square kilometre, and is used to compare how developed different areas are.

Example

Paris has a population of 2,265,886 and an area of 105.4 km². What is Paris' population density?

Answer: Population Density = Population ÷ Area
 = 2,265,886 ÷ 105.4
 = 21,498 people per km² (correct to the nearest whole number)



Exercise 18

(a) Mumbai has a population of 12,478,447 and an area of 603 km². What is Mumbai's population density?

(b) San Francisco has a population of 805,816 and an area of 120.93 km². What is San Francisco's population density?

(c) Dublin has a population of 553,165 and an area of 114.99 km². What is Dublin's population density?

(d) The population density of London is 5,584 people per km². If London's population is 8,778,500, what is London's area?

(e) The population density of Chicago is 4,582 people per km². If Chicago's population is 2,695,598, what is Chicago's area?

(f) The population density of Cairo is 19,376 people per km². If Cairo's area is 606 km², what is Cairo's population?

(g) The population density of Miami is 4,324 people per km². If Miami's area is 92.38 km², what is Miami's population?

(h) The following table gives information about some Spanish cities.

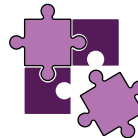
City	Madrid	Barcelona	Seville
Population	3,141,991	1,621,537	703,021
Area	604.3 km ²	101.9 km ²	140 km ²

- (i) Calculate the population density of the three Spanish cities.
- (ii) Which city is the one where people live closest to each other?

Challenge!

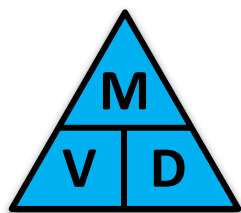
Try to calculate the population density of where you live.





Density

Density compares an object's mass to its volume.



$$\begin{aligned} \text{Mass} &= \text{Volume} \times \text{Density} \\ \text{Volume} &= \text{Mass} \div \text{Density} \\ \text{Density} &= \text{Mass} \div \text{Volume} \end{aligned}$$

To use the triangle, *hide* the letter that you want to find.

Density is (usually) measured in g/cm^3 or kg/m^3 , and is used to compare how much mass an object has per unit of volume.

Example

(a) The mass of a 200 cm^3 piece of metal is 1.2 kg . What is the metal's density, in g/cm^3 ?

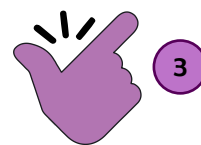
$$\begin{aligned} \text{Answer: Density} &= \text{Mass} \div \text{Volume} \\ &= 1,200 \div 200 \\ &= 6 \text{ g/cm}^3 \end{aligned}$$

(b) The density of a piece of aluminium is 2.7 g/cm^3 . Calculate the mass of a 20 cm^3 piece of aluminium.

$$\begin{aligned} \text{Answer: Mass} &= \text{Volume} \times \text{Density} \\ &= 20 \times 2.7 \\ &= 54 \text{ g} \end{aligned}$$

Exercise 19

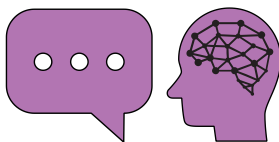
- (a) The mass of a stone is 4.4 kg .
- What is the mass of the stone in grams?
 - Find the density of the stone (in g/cm^3) if its volume is $2,000 \text{ cm}^3$.
- (b) A piece of cork weighs 10 kg . Its volume is 0.04 m^3 . Calculate the density of the cork, in kg/m^3 .
- (c) The mass of a $1,200 \text{ ml}$ piece of ice is $1,104 \text{ grams}$. Find its density. (Hint: $1 \text{ ml} = 1 \text{ cm}^3$.)
- (d) Gold is expensive and desirable not just for how it looks but also because it does not rust easily. Lewis wishes to build a car made out of gold but is worried about its mass. The density of gold is $19,300 \text{ kg/m}^3$ and the volume of Lewis' car is around 1.5 m^3 .
- Calculate the mass of Lewis' gold car.
 - Given that the mass of a normal car is around $1,500 \text{ kg}$, comment on the practicality of Lewis' plan.
- (e) Megan is mixing and pouring concrete. She mixes and then pours a total of 2.4 m^3 of concrete. Calculate the mass of this concrete if its density is $2,500 \text{ kg/m}^3$.
- (f) The density of water is 1 g/cm^3 . Calculate the mass of 1.5 litres of water.
- (g) A clay statue has density 1.4 g/cm^3 . Another statue is carved from wood and has a density of 0.8 g/cm^3 . The two statues are weighed in order to find their mass. The wood statue weighs $4,500 \text{ g}$. The clay statue weighs $7,700 \text{ g}$. Which statue has the greatest volume?
- (h) The dimensions of a metal cuboid are 8 cm , 6 cm and 5 cm . The mass of the cuboid is 0.9 kg . Calculate the density of the metal, noting your units clearly.



Other Compound Measures

Exercise 20

13



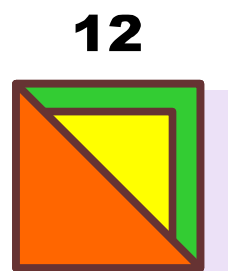
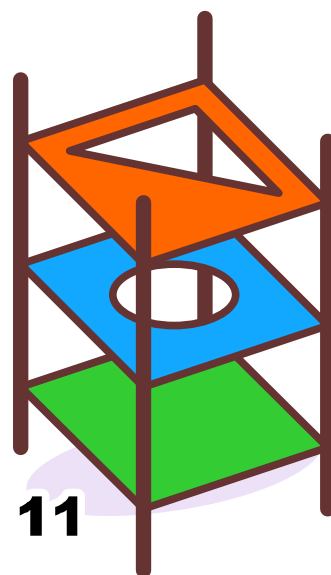
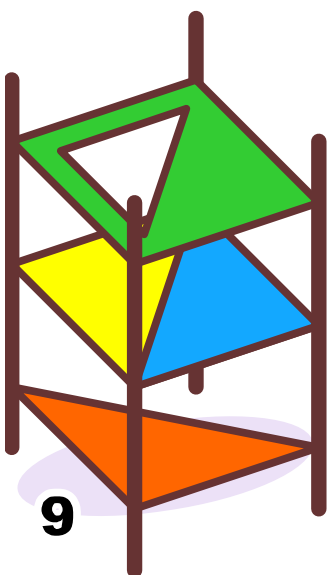
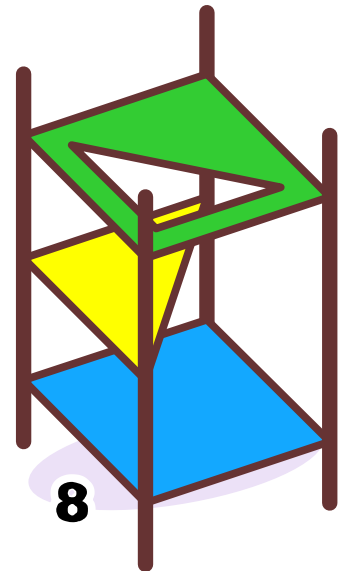
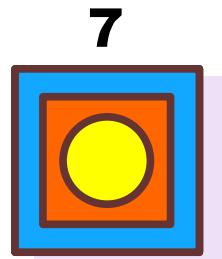
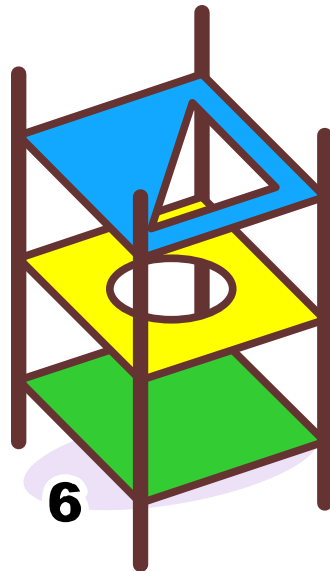
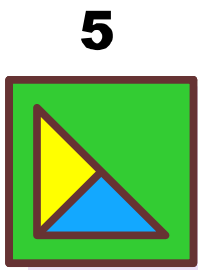
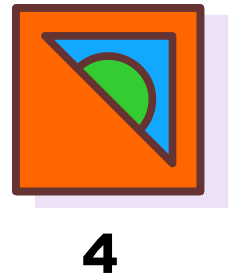
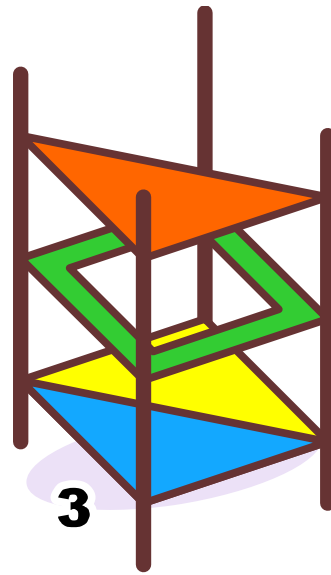
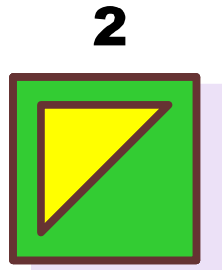
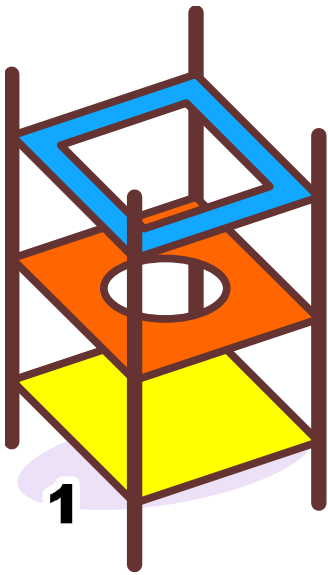
- (a) The fuel consumption of Bethan’s car is 50 miles per gallon.
 - (i) If Bethan travels 200 miles in her car, how many gallons of petrol does she use?
 - (ii) Today, petrol costs 117.9 pence per litre. Given that a gallon of petrol is around 4.5 litres, calculate the cost of the petrol for Bethan’s journey. Give your answer correct to the nearest penny.
- (b) John travels 126 miles in his car, using 2.25 gallons of petrol. What is the fuel consumption of John’s car, in miles per gallon?
- (c) Janet types at a rate of 70 words per minute. How much time would Janet need to type a report containing 3,500 words?
- (d) The average person can type at a rate of 40 words per minute. Bob types a 1,200-word report in 40 minutes. Does Bob type at a rate that is slower than the average person, or is quicker than the average person?
- (e) A mill produces 20 metric tons/hour of flour. How much flour is produced every 15 minutes?
- (f) A paint tin notes that it can cover an area of 13 m²/litre. If the paint tin contains 2.5 litres of paint, what total area can the tin cover?
- (g) A swimming pool is emptied in order to clean it. The pool (when full) can hold 375,000 litres of water. After it is cleaned, the pool is refilled by using a water pipe that can supply water at a rate of 200 litres per minute. How much time will it take to refill the pool? Give your answer in hours and minutes.
- (h) During the first 20 games of a football season, Gareth Bale scored an average of 0.7 goals per game. How many goals is it expected that Gareth Bale scores during the next 6 games?

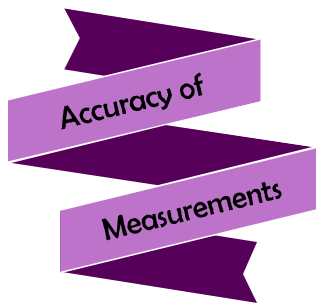


Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Puzzle: Can you find the top view for every stand?







Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I know how to round off a number to a specified number of significant figures , e.g. to 1 significant figure or to 2 significant figures.			1, 2	
I know how to recognise the significant figures in a given number.			3	
I know how to estimate the answer to a sum, by using rounding off to one significant figure.			4	
I know how to find the upper and lower bounds of a measurement.			5	
I know how to write answers to an appropriate degree of accuracy .			6	
I can use the formulae that are associated with the compound measure speed .			8	
I can use the formulae that are associated with the compound measure population density .			7	
I can use the formulae that are associated with the compound measure density .			9	
I am aware of how to work with other compound measures .			10	