YSGOL CREUDDYN

DAWN DYSG DAIONI

Mathematics

Year 10

Cynnwys

Topic	Mathematics	Length	Tiers	Page Number
01 Data Handling and Statistics 4	(a) Comparing Averages. (b) Questionnaires. (c) Sampling. (d) Frequency Polygons. (e) Box and Whisker Diagrams.	5–7 weeks	F, I	5
02 Powers and Roots	(a) Rules of Indices. (b) Standard Form. (c) Graph Plotting.	5–6 weeks	I	35
03 Measuring Shapes 3	(a) Trigonometry of Right-Angled Triangles. (b) Transformations: Enlargement. (c) Tessellations. (d) Perimeter and Area of Composite Shapes.	4–5 weeks	F, I	57
04 Fractions, Percentages and Decimals	(a) Calculating. (b) Recurring Decimals. (c) Converting. (d) Exam Technique.	5 weeks	I	79
05 Developing Algebra 2	(a) Simple Factorising. (b) Factorising Quadratic Expressions. (c) Simultaneous Equations. (d) Changing the Subject. (e) Expression, Equation, Formula, Identity.	5–6 weeks	F, I	109
06 Measuring Shapes	(a) Solids. (b) Composite Solids. (c) Similar Shapes. (d) Dimensions.	3–5 weeks	F, I	133
07 Accuracy of Measurements	(a) Rounding Off: Significant Figures (b) Estimation (c) Upper and Lower Bounds. (d) Appropriate Degree of Accuracy. (e) Compound Measures.	3 weeks	F, I	153

The Mathematics Department

10

Data Handling and Statistics 4

Intermediate Tier

Name:

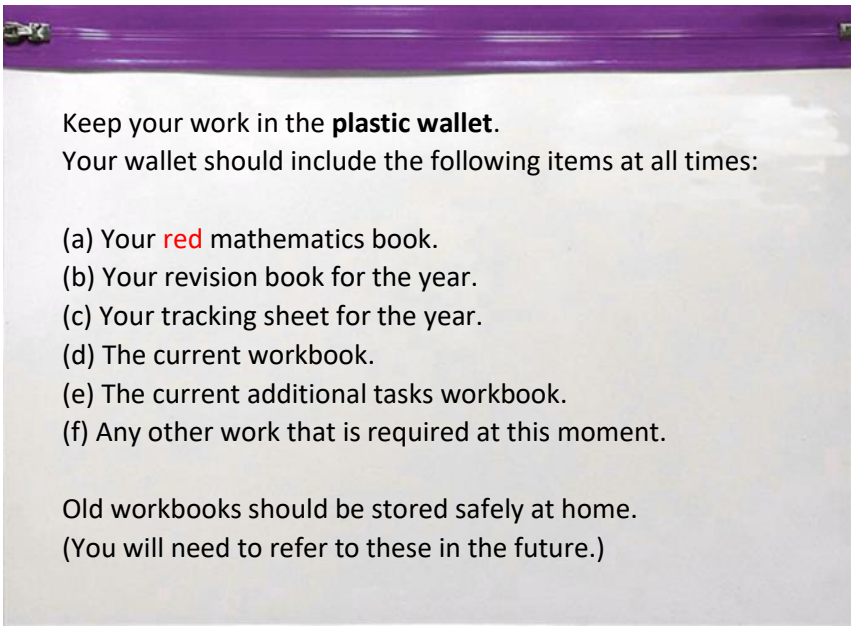
Contents

Chapter	Mathematics	Page Number
The Mathematics Department	Looking after your work. Content of the workbooks. Mathematics learning journey.	3
The Sequential Data Handling Cycle	An infographic explaining the sequential data handling cycle.	6
Comparing Averages	Revising averages. Choosing the most appropriate average. Comparing averages.	7
Questionnaires	Designing questionnaires. Criticising questions. Hypotheses.	12
Sampling	Simple random sampling. Systematic sampling.	15
Frequency Polygons	Drawing frequency polygons. Interpreting frequency polygons.	19
Box and Whisker Diagrams	Drawing box and whisker diagrams. The connection between box and whisker diagrams and cumulative frequency diagrams.	23

The Mathematics Department

Welcome back to year 10! Here is a reminder of the guidelines for looking after your work.

- At the start of every lesson, write "Classwork", the date and a suitable title for the work. Each of these should be underlined.
- No blank pages should be left in the red mathematics book.
- Draw diagrams using a pencil and, where appropriate, using the correct equipment, e.g. ruler, protractor, compass.
- Show your method fully.
- Remember to include relevant units in your answers, e.g. cm, £, ml.



Equipment

- Black, red and blue biros.
- HB Pencil.
- Ruler (a 30 cm one is better).
- Eraser.
- Protractor.
- Compass.
- Scientific calculator (Casio fx-83GTCW).
- Highlighter.

The Revision Book

This will form the basis for your GCSE revision.

- Complete at least 4 pages in your revision book for each unit of work.
- You should include material you will need in the future for remembering the work quickly. This can include notes on the work; examples; important facts; and revision posters.

Workbooks

You will receive 1 copy of the workbook and 1 copy of the additional tasks workbook at the start of each new unit of work. (If you lose the workbook, a new one will cost 50p.)

A Welsh copy of the workbook, and additional supporting materials, can be found on the department's website, www.mathemateg.com

Content of the workbooks

When you see a QR code (like the one on the left), scan it using your mobile device in order to reach a Welsh YouTube video hosted on the following channel.

www.youtube.com/adolygumathemateg

The letters in circles, for example (I), show the tier of the work in the GCSE specification.

Tier	Foundation	Intermediate	Higher
GCSE Grades	U, G, F, E, D	U, E, D, C, B	U, C, B, A, A*

All the workbooks contain a variety of exercises, labelled as follows.

Exercises on new topics.

Answering a question in context, or solving a problem.

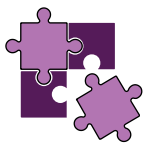
A more difficult question.

Revision of material from previous workbooks.

There are evaluation boxes at the end of each chapter to revise the completed work.

Key words	Corrections	I am happy with...	I need to revise...
Write down the new or important mathematical terms from the chapter.	What do you need to remember when completing this type of work in the future?	Write down the topics you had success with.	Write down the topics you need to look at again.

Curriculum for Wales Proficiencies



Conceptual understanding

Communication using symbols

Strategic competence

Logical reasoning

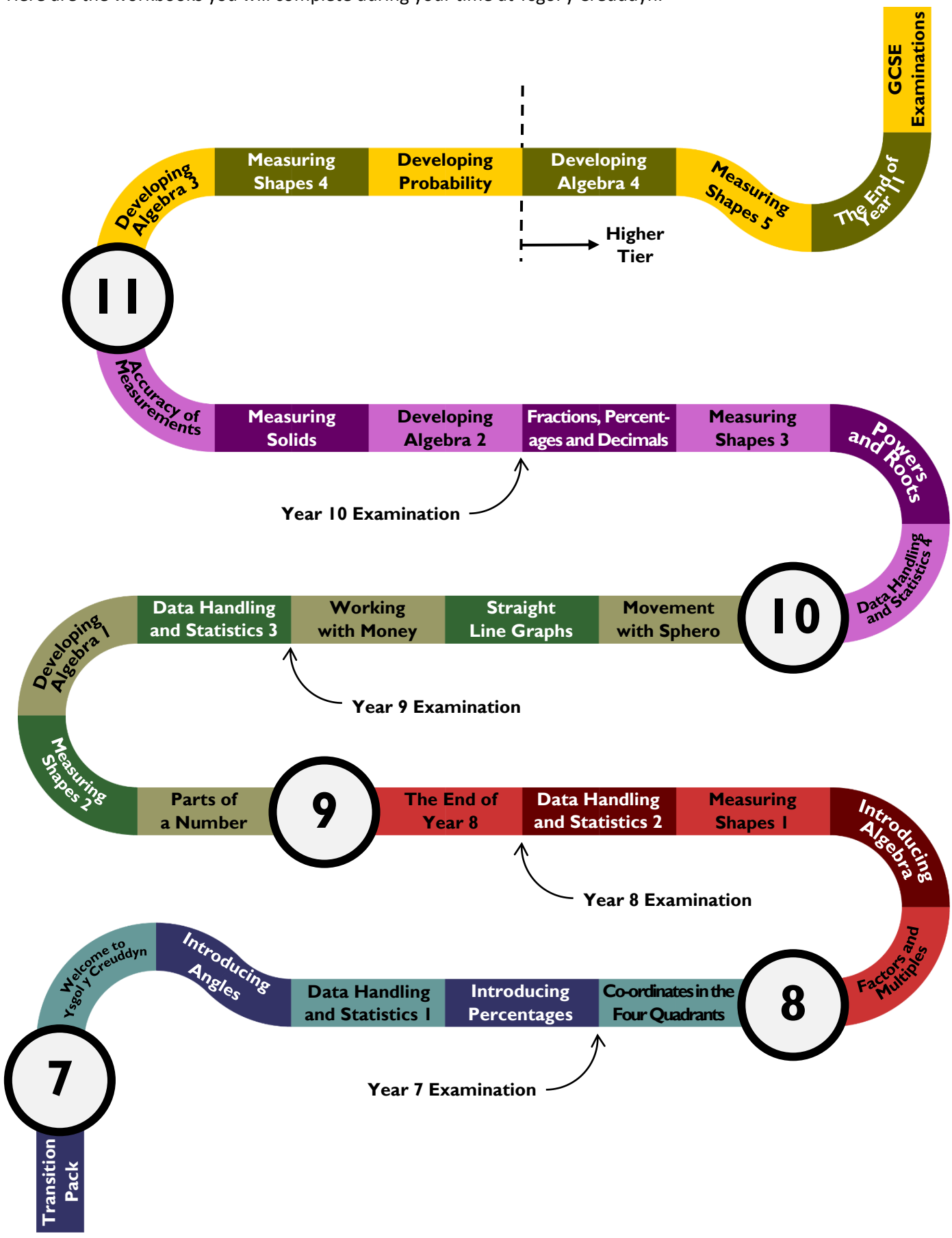
Fluency

Supporting Materials:

- Diagnostic Questions
  - A quiz for each workbook on the website www.diagnosticquestions.com.
- Reflection Sheet
  - An opportunity to assess your understanding of a workbook, and to see the test question order.
- Old WJEC examination questions; worksheets; investigations; puzzles.
  - Available for some topics.

Mathematics Learning Journey for Ysgol y Creuddyn

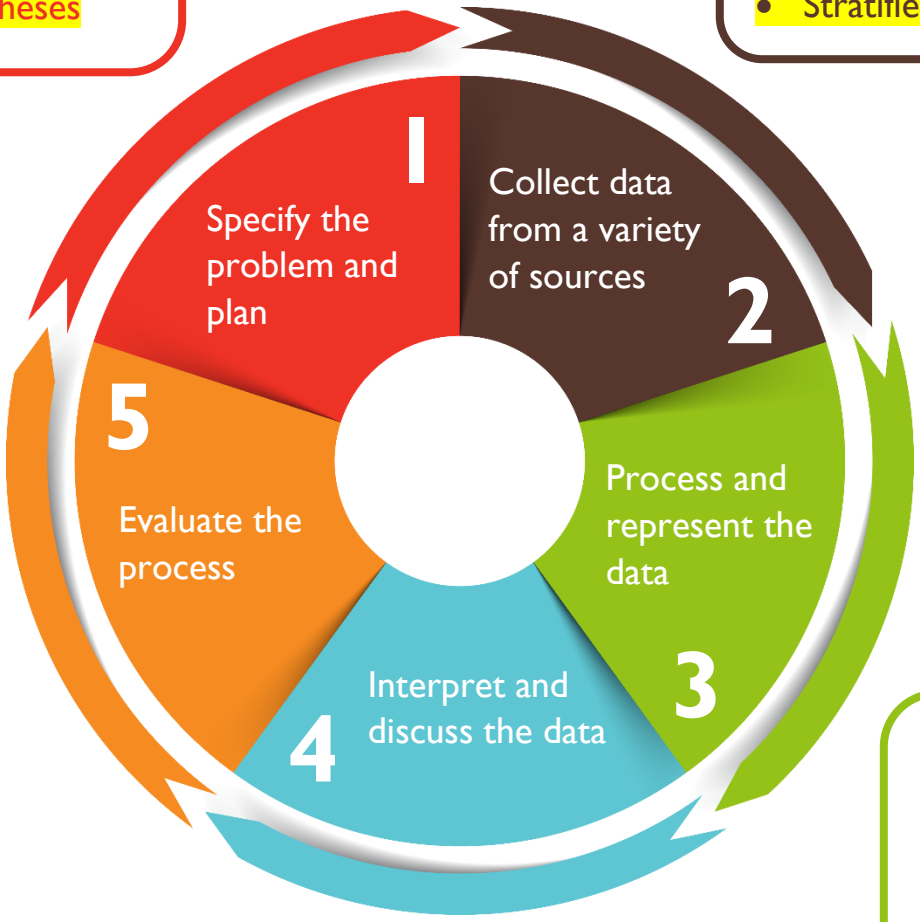
Here are the workbooks you will complete during your time at Ysgol y Creuddyn.



The Sequential Data Handling Cycle

- Designing questionnaires
- Criticising questions
- Hypotheses

- Types of data
- Simple random sampling
- Systematic sampling
- Stratified sampling



- The mean
- The range
- The mode
- The median
- The interquartile range
- Box and whisker diagrams
- Choosing the most appropriate average
- Comparing averages

- Bar charts
- Frequency diagrams
- Pie charts
- Pictograms
- Vertical line diagrams
- Line graphs
- Scatter diagrams
- Cumulative frequency diagrams
- Frequency polygons

Comparing Averages

Revising Averages

We have studied three different types of averages over the last few years.

Year 7	Year 8	Year 9	Year 10
Mean	Mode	Median	**Comparing Averages**

Exercise 1

Revision

F

Fill in the boxes below to explain how to calculate the mean, the mode and the median.

The Mean	The Mode	The Median

Exercise 2

Calculate the mean, the mode and the median of the following data.

12, 14, 14, 15, 16, 17, 17, 17, 19, 20.

I

Choosing the Most Appropriate Average

	The Mean	The Mode	The Median
Advantages	• Uses all the data values.	• Not affected by outliers. • Can be used for qualitative data.	• Not affected by outliers.
Disadvantages	• Can be affected by outliers. • Needs to be calculated.	• Doesn't use all the data values. • There is no mode for some data sets.	• Doesn't use all the data values. • Need to rearrange the data to find it.
Used for	• Data which doesn't include outliers.	• Qualitative data. • Data which includes outliers.	• Data which includes outliers.

Challenge!

Extension

Use the internet to investigate the term *skewness*.
Which average is the best to use when the data has a skewed distribution?

Exercise 3

Which average is most appropriate for the following data sets?

(a) Favourite football team:
Liverpool, Chelsea, Man City, Everton, Liverpool, Man Utd.

(b) Times in a 100 m race (in seconds):
9.81, 9.89, 9.91, 9.93, 9.94, 9.96, 10.04, 10.06.

(c) Price of Heinz Baked Beans in different shops:
75p, 60p, 74p, 80p, 70p, 95p, 85p.

(d) Age of players who start a football game:
28, 31, 19, 24, 25, 28, 30, 23, 20, 29, 26.

(e) Year 10 pupil heights:
162 cm, 160 cm, 161 cm, 148 cm, 163 cm, 161 cm.

(f) Favourite subject in school:
Science, Music, Drama, Mathematics, Music.

(g) Spelling test scores (out of 10):
5, 7, 8, 4, 5, 3, 6, 4, 5, 4, 7.

(h) Number of brothers:
0, 1, 2, 1, 0, 6, 1, 0, 1, 2.

Exercise 4

Are the following statements TRUE or FALSE?

(a) The mode is the most popular data item in a set of data.
(b) Half the values in a data set are greater than the mean.
(c) Half the values in a data set are greater than the median.
(d) When finding the median, it doesn't matter if you order the data from least to greatest or from greatest to least.
(e) It is always possible to find the mean of a data set.

Exercise 5

The following table shows the percentages of 10 pupils in Welsh and Mathematics tests.

Welsh	57	63	91	58	56	75	59	76	91	54
Mathematics	67	68	66	68	68	66	70	69	68	70

(a) Complete the following table.

	Welsh	**Mathematics**
The mean		
The median		
The mode		
The range		

(b) Which statistics from the table support the following newspaper headlines?

(i) **The Welsh results are very high this year.**

(ii) **Pupils do not perfom better in Mathematics than in Welsh.**

(iii) **There was lots of copying during the Mathematics test.**

(iv) **The Mathematics test was easier than the Welsh test.**

(v) **The Welsh results show that some people tried harder than others.**

Exercise 6

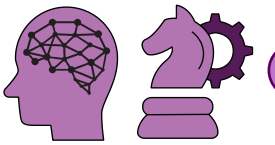
F

Three people were asked to estimate when 30 seconds had passed. This was done 5 times.

Osian	Siân	Shubnam
31	36	32
26	19	30
32	39	24
27	36	37
29	20	32

Who, in your opinion, is best at estimating the length of 30 seconds?

Exercise 7



F

Fill the following boxes with whole numbers.

(a) 2 numbers mode = 5	(b) 2 numbers mean = 5	(c) 2 numbers mean = 5 range = 6
(d) 2 numbers mean = 5 mode = no mode	(e) 2 numbers mean = 6 range = 8	(f) 2 numbers mean = 10 largest number = 12
(g) 3 numbers mean = 3 mode = 2	(h) 3 numbers mean = 7 mode = 10	(i) 3 numbers median = 4 range = 0
(j) 4 numbers mode = 5 range = 1	(k) 4 numbers median = 3 range = 1	(l) 4 numbers mean = 6 median = 6.5 range = 11

Exercise 8

F

During a skiing trip, the PE department recorded the daily snowfall for 5 consecutive days. Here is some information about the daily snowfall.

Mean	Mode	Median	Range
5.8 cm	3 cm	5.6 cm	6.6 cm

(a) Use these statistics to calculate what was the daily snowfall for these 5 consecutive days.

(b) If it had snowed exactly 2 cm more each day, what would the new statistics be?

Mean	Mode	Median	Range

Exercise 9

In a game, it is possible for every player to score between 1 and 10 points.
Lois and Beca play the game 5 times.

The table below shows the points scored by Lois in every game.

	Game 1	Game 2	Game 3	Game 4	Game 5
Lois	5	2	8	5	1
Beca					

Beca had a higher mean score than Lois.
Beca had a lower median score than Lois.
Beca had a lower range of scores than Lois.

Complete the table above with a set of possible scores for Beca.

Exercise 10

Jim and Andy play for their local cricket team.
They scored the following runs in their six most recent games.

Jim	42	71	39	62	70	40
Andy	115	6	84	36	10	85

(a) Calculate Jim's mean and Andy's mean.

(b) Calculate Jim's median and Andy's median.

(c) There is no room for both Jim and Andy on the team for the next game. The management team need to choose either Jim or Andy to play for the team. Use your answers to parts (a) and (b) to give advice to the management team.

Exercise 11

The table below shows the number of ice hockey season tickets sold by a team last season, together with the prices for the tickets.

Ticket Price (£)	Number of tickets sold
250	180
300	230
350	230
500	150

For the cost of the tickets sold last season, calculate

(a) The mode; (b) The median; (c) The range.

(d) The ice hockey team owner says that more than half of the tickets sold were greater in value than £300. Explain why the team owner is incorrect.

Exercise 12

50 people took part in a charity walk.
The table below shows the grouped frequency distribution of the sums of money raised, to the nearest £.

Sum, s, in £	Number of people
$10 \leqslant s \leqslant 19$	2
$20 \leqslant s \leqslant 29$	18
$30 \leqslant s \leqslant 39$	29
$40 \leqslant s \leqslant 49$	1

(a) Find the modal class of the data.

(b) Find the median class of the data.

(c) Calculate an estimate for the mean sum of money raised per person.

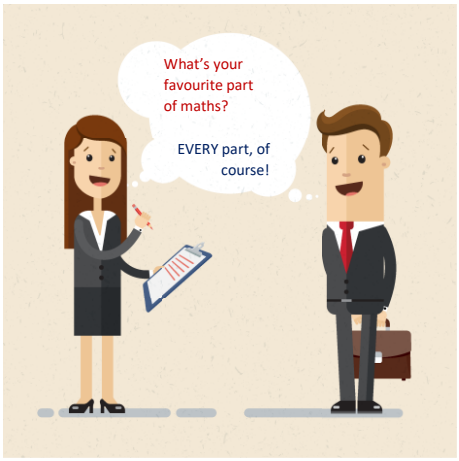
(d) Another 50 people took part in the same charity walk. The total of the money collected by these 50 extra people was £1,600. Is it possible to say that these 50 people raised more money than the original 50 people?

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Questionnaires

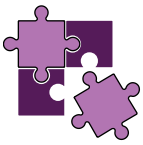
A **questionnaire** is a good way to collect data, but we must be careful when designing the questions.



(1) We must avoid asking **leading** questions which favour one answer over another. For example, the question "Do you agree that eating ice cream is bad for you?" leads people to agree with the statement. (Why?)

(2) We must avoid using answer boxes where the **options overlap**. For example, in the following question, people who are 20 years old can choose two different answer boxes.
How old are you?
Under 10 *10–20* *20–30* *Over 30*

(3) We must be careful **where, when and how** a questionnaire is conducted. For example, the following questionnaires would not be appropriate.
   a. Conducting a sports questionnaire outside of a football stadium.
   b. Conducting a questionnaire about night shift workers at mid-day.
   c. Conducting a questionnaire on mobile phone usage through a phone survey.



(4) We must use **clear** and **concise** questions. For example, the question "How often do you go to the gym?" is not appropriate without explaining the meaning of the word "often", i.e. daily, weekly, monthly…?

(5) We must use **appropriate** and **relevant** questions. For example,
   a. Some people would refuse to answer "How old are you?", however they might answer if options showing different age ranges were included.
   b. The question "What is your eye colour?" would not be appropriate in a questionnaire asking opinions about recycling.

Skill F

Exercise 13

Write a criticism of the following questions.

(a) Do you read books? Circle your answer.

Yes No Sometimes

Writing a criticism means finding something incorrect or out of place.

(b) Do you agree that the cruel sport of fox hunting should be made illegal?

(c) How often do you use the gym in a typical month? Circle your answer.

Never Once or Twice 2–5 Times More than 5 times

(d) How old are you? Circle your answer.

10–15 16–20 21–25 26–30 31–35

(e) In your current job, how much money do you earn?

(f) How often do you shop in a supermarket? Circle your answer.

Three times a week Twice a week Once a week Once a month

(g) Do you agree that the amazing Liverpool are the best football team in the world? Circle your answer.

Yes Of course Absolutely

Exercise 14

A survey is conducted to see how often teenagers buy trainers.

The following questions are included in the questionnaire.

Question 1: Where do you live?

Question 2: How often do you buy trainers?

Never	1–10 times	10–15 times	More than 15 times
☐	☐	☐	☐

(a) For each question state **one** reason why it is **not** appropriate.

(b) The survey is conducted by leaving copies of the questionnaire on the seats in a sports clothing shop. Give one criticism of how the survey was conducted.

Exercise 15

A survey was carried out to discover whether people would rather watch sports or detective programs on the television.

The following three questions were included.

Question 1: What is your address?

Question 2: Which type of TV program do you prefer? Tick one box.

Sports	Detective
☐	☐

Question 3: How many hours do you spend watching TV?

Less than 1 hour	1–5 hours	More than 5 hours
☐	☐	☐

(a) Give **one** reason why question 1 is not appropriate.

(b) Give **one** reason why question 3 is not appropriate.

(c) The survey was conducted by asking people who were leaving a football stadium one Saturday afternoon. Give **one** criticism of how the survey was carried out.

Exercise 16

Elen is conducting a survey in her school about the standard of food in the canteen. She asks every 20th person that goes to get hot food. Explain what is wrong with Elen's plan.

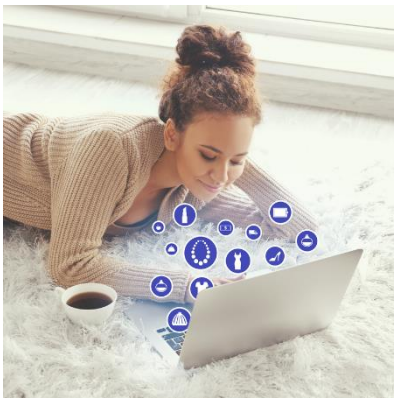
Hypotheses

A **hypothesis** is a statement like “children spend more time playing computer games than adults”.
It is possible to create a questionnaire and collect data to test a hypothesis.

Exercise 17

Steffan wants to prove the following hypothesis.

‘Most people spend more than two hours on the internet every night.’



He intends to

- give a short questionnaire to people in the local fitness centre,
- ask the following questions:
  - In your opinion, do people spend too much time on the internet?
  - How much time do you spend on the internet?
- ask each participant to post their completed questionnaire in an envelope with a stamp on it.

Write **three** unfavourable comments about this plan.

Exercise 18

Mari wants to prove the following hypothesis.

‘Older pupils in a secondary school are better at remembering their times tables than younger pupils.’

She intends to

- give a short questionnaire to 50 randomly chosen pupils in each year,
- ask the same 5 multiplication questions to everyone, to complete in a maths lesson,
- ask the pupils to mark each other’s work and return the questionnaires to her through the maths teacher.

Write **three** unfavourable comments about this plan.

Exercise 19

Iwan wants to prove the following hypothesis.

‘The children in Llandudno spend more time playing computer games than the adults’.

Write a questionnaire that Iwan could use to prove or disprove this hypothesis.

Key words	Corrections	I am happy with...	I need to revise...

Sampling

In the previous chapter, we discussed how to design an appropriate questionnaire in order to prove a specific hypothesis. Often, it is not possible to ask the opinion of **all** members of a **population**. For example, it would be laborious to ask all pupils in a secondary school their opinion on a matter. Instead, we often use a **sample** of the population, which is a smaller group, and attempt to come to a conclusion about the opinion of the whole population based on the information from the sample.

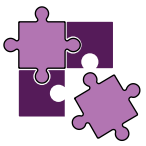
We must be careful when selecting a sample. It must be **large enough,** and **representative** of the population. Asking only 5 pupils would not represent the opinion of a whole school, neither would asking only the pupils in year 8.

At GCSE level, we must be familiar with the following methods of selecting a sample.

Simple random sampling	Systematic sampling	Stratified sampling
Intermediate and Higher Tier	Intermediate and Higher Tier	Higher Tier only

Simple Random Sampling

In a **simple random sample**, every member of a population has **the same chance** of being chosen.
There are two main methods of choosing a simple random sample:



- Using a table of random digits;
- Using the random number generator function on a calculator.

Example

A school hopes to change the start time for the school day and is eager to ask the opinion of the 600 pupils.
The head teacher decides to choose a simple random sample of 10 pupils to question.

- The head teacher numbers all the pupils from 001 to 600.
- Starting from a random starting point in a table of random digits, the head teacher reads the numbers in groups of three.
- The head teacher accepts any number between 001 and 600, and rejects the others.
  The head teacher also ignores any repeated numbers.

Here is part of a table of random digits.

7087	0858	0164	1769	3218	1467	1938	8093	7918	2814
7796	7080	7227	3140	0933	0181	2013	7918	1177	4715
3830	9523	3653	8514	6061	0674	6025	9834	0499	3668
1347	1225	1910	3621	9722	8482	6298	1957	3507	7209

By starting at the digit in red (chosen at random), the head teacher chooses the following pupils: 218, 146, 388, 093, 147, 072, 273, 140, 301, 013.

Exercise 20

Use the following table of random digits to choose a sample of 5 people out of 500, by
(a) starting at the first digit;
(b) starting at the red digit;
(c) starting at the blue digit.

0572	8836	4865	9430	8461	9978	1392	1166	7262	4438
8065	4455	5432	7323	9142	8933	4356	1767	0291	2037
9297	6827	1225	2158	8791	7847	6420	3726	1650	6365
3457	0248	5823	9512	1725	6247	0994	4066	8207	8813

Exercise 21

Use the following table of random digits to choose a sample of 8 people out of 75, by
(a) starting at the first digit;
(b) starting at the red digit;
(c) starting at the blue digit.

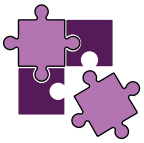
0003	3857	6162	2670	0883	5411	7163	3140	4505	6239
2415	1096	4182	7652	6254	5054	8743	2175	9256	8364
9570	0276	0303	6250	8236	3012	2980	7517	6803	1580
8478	6061	7948	2014	5047	0797	9177	3878	6272	5734

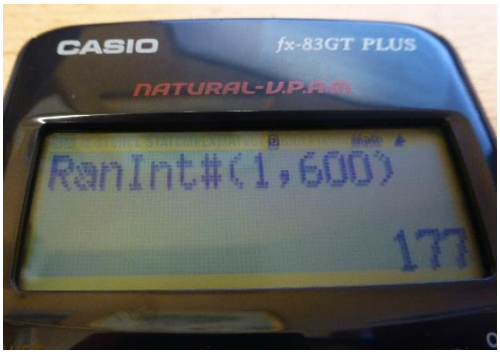
Exercise 22

Use the following table of random digits to choose a sample of 6 people out of 1600, by starting at the first digit.

3618	5991	8471	1714	0315	3185	2048	9874	5016	4707
5685	2304	2731	0092	7065	2428	0164	2798	1511	7259
9027	6444	9761	1197	5305	2910	3860	3490	7629	1963
2587	4167	6515	4516	0708	3449	5001	0437	6137	9031

Random Numbers on a Calculator





Instead of using a table of random digits to generate random numbers, it is possible to use a scientific calculator to generate random numbers.
For example, to choose a random number between 001 and 600 (like the school example from the previous page), it is possible to press the following buttons on a Casio calculator:

fx-83 GT PLUS: [ALPHA] [•] [1] [SHIFT] [)] [6] [0] [0] [)] [=]
fx-83 GT CW: Catalog, Probability, Random Integer, “1, 600”, =

We can generate further random numbers by pressing [=] again.

Exercise 23

Repeat Exercises 20 to 22, using the random number generator on your calculator to select the required samples.

Systematic Sampling

In a **systematic sample**, the sample is selected from the population in a **regular pattern**.

Example

A school hopes to change the start time for the school day and is eager to ask the opinion of the 600 pupils. The head teacher decides to choose a systematic sample of 10 pupils to question.

- The head teacher numbers all the pupils from 001 to 600.
- 600 ÷ 10 = 60, therefore we work through the list of pupils in intervals of 60. (60 is the **sampling interval**.)
- To choose the starting number, we must use a table of random digits or the random number generator on a calculator to choose a number between 01 and 60.

If we use the table of random digits from page 6, and read the digits in groups of two, the first number between 01 and 60 we see is 08. Therefore, the systematic sample is

008, 068, 128, 188, 248, 308, 368, 428, 488, 548.

Example

A young farmers' club is considering holding a fair and wishes to collect the opinion of the members on the content of the fair. The chairperson decides to choose a systematic sample of 10 members to question. The club has a total of 87 members.

- The chairperson numbers all the members of the club from 01 to 87.
- 87 ÷ 10 = 8.7, therefore we need to work through the list of members in intervals of 8. (Why doesn't working through the list in intervals of 9 work?)
- To choose the starting number, we must use a table of random digits or the random number generator on a calculator to choose a number between 01 and 08.

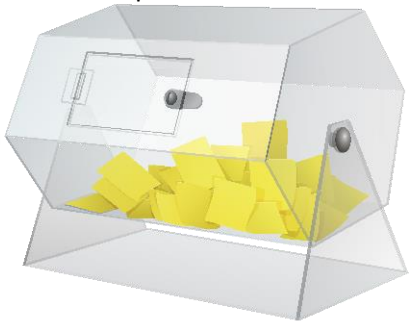
We must omit or ignore the digits after the decimal point.

The chairperson uses the random number generator function on a calculator to choose 03 as the starting number. Therefore, the systematic sample is

03, 11, 19, 27, 35, 43, 51, 59, 67, 75.

Notice, in the above example, that every member of the club doesn't have the same chance of being selected. The members between 01 and 80 have the same chance of being selected, $\frac{1}{8}$, but the members between 81 and 87 have **no** chance of being selected. Therefore, a systematic sample is not necessarily a random sample.

Would using a machine like this be a valid method of selecting a starting number?



Exercise 24

Find the sampling interval in the following systematic samples.

(a) Choosing 10 people out of 80.
(b) Choosing 5 people out of 45.
(c) Choosing 10 people out of 74.
(d) Choosing 4 people out of 18.
(e) Choosing 7 people out of 40.
(f) Choosing 9 people out of 63.
(g) Choosing 12 people out of 140.
(h) Choosing 20 people out of 1,500.

Exercise 25

Choose a systematic sample of 10 people out of 70, by starting with
(a) the second person (02);
(b) the fourth person (04);
(c) the seventh person (07).

Exercise 26

Choose a systematic sample of 10 people out of 140, by starting with
(a) the first person;
(b) the fifth person;
(c) the twelfth person.

Exercise 27

Choose a systematic sample of 8 people out of 100, by starting with
(a) the third person;
(b) the eighth person;
(c) the tenth person.

Exercise 28

Choose a systematic sample of 12 people out of 1,400, by starting with
(a) the sixth person;
(b) the twentieth person;
(c) the 89th person.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Frequency Polygons

Drawing Frequency Polygons

We draw **frequency polygons** for the following types of data.

- Grouped discrete quantitative data.
- Continuous quantitative data.

A frequency polygon is a **line graph** where we plot the **midpoint** for every class against the **frequency.**

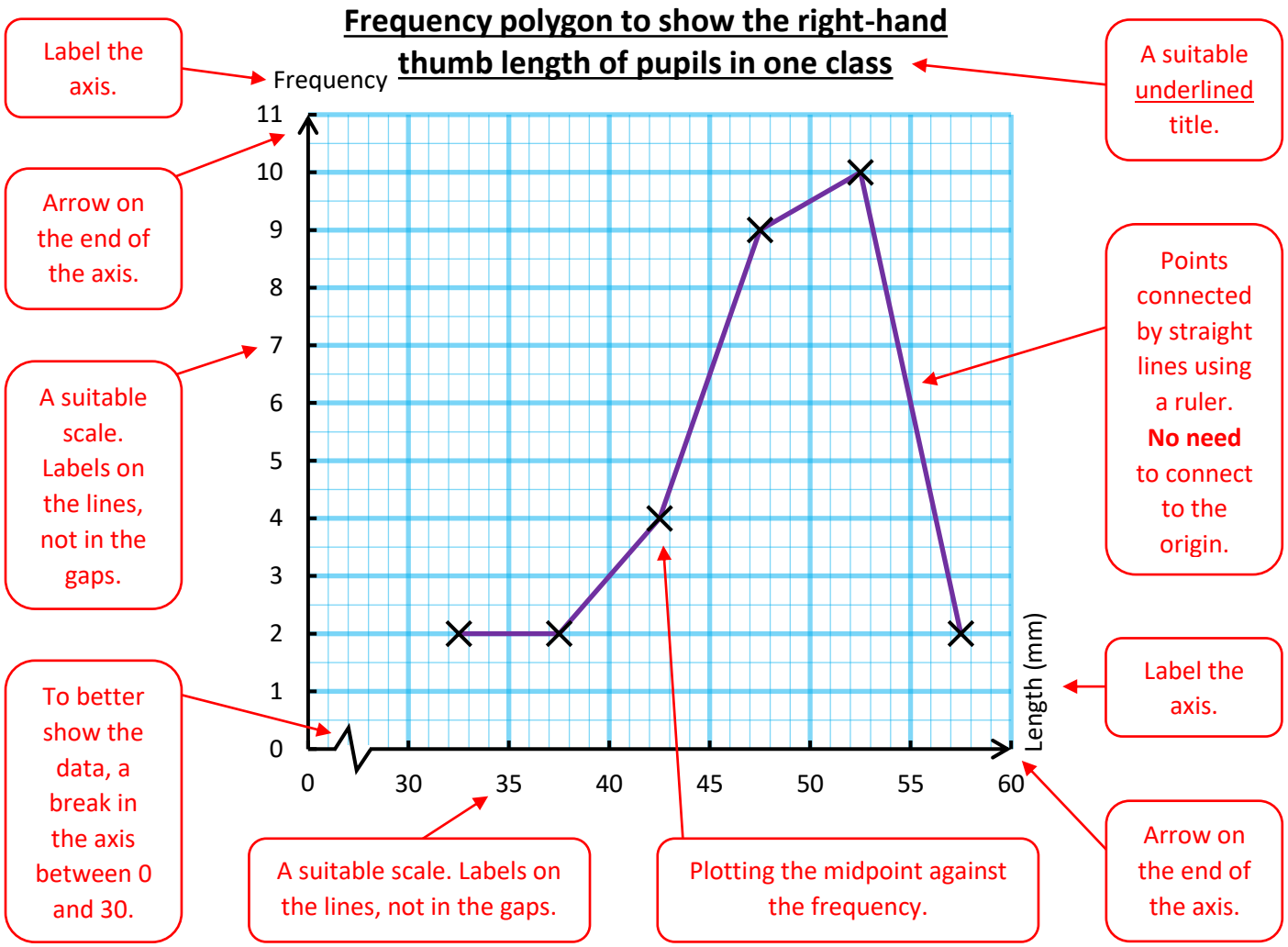
Example

The following frequency table shows the right-hand thumb length (l) for pupils in one class.

Class Interval (l mm)	Frequency
$30 \leqslant l < 35$	2
$35 \leqslant l < 40$	2
$40 \leqslant l < 45$	4
$45 \leqslant l < 50$	9
$50 \leqslant l < 55$	10
$55 \leqslant l < 60$	2

The midpoint for the class $30 \leqslant l < 35$ is 32.5.
Method 1: 32.5 is half way between 30 and 35.
Method 2: 30 + 35 = 65. 65 ÷ 2 = 32.5.

The frequency polygon below represents the data.



Exercise 29

Draw a **frequency polygon** for the following data on squared paper.

(a) Number of books bought by pupils in 7E during the last year.

None to four books: 12 Five to nine books: 5
Ten to fourteen books: 6 Fifteen to nineteen books: 1

(b) Number of minutes a dentist spends with each patient.

1–5 minutes: 2 6–10 minutes: 4 11–15 minutes: 9 16–20 minutes: 5
21–25 minutes: 3 26–30 minutes: 3 31–35 minutes: 0 36–40 minutes: 1

(c) Number of absent days from school for pupils in 7C last term.

0–4 days: 11 5–9 days: 8 10–14 days: 6 15–19 days: 0 20–24 days: 5

(d) Number of words in every sentence for the first 50 sentences of a book.

1–10 words: 2 11–20 words: 9 21–30 words: 14 31–40 words: 7
41–50 words: 4 51–60 words: 8 61–70 words: 6

Exercise 30

Draw a **frequency polygon** for the data in each of the following frequency tables.

(a) Weight was lost by people in a weight loss group over 6 months.

Weight (w kg)	Frequency
$0 \leqslant w < 6$	4
$6 \leqslant w < 12$	11
$12 \leqslant w < 18$	12
$18 \leqslant w < 24$	7
$24 \leqslant w < 30$	3

(b) Height of 60 pupils.

Height (h cm)	Frequency
$168 \leqslant h < 172$	2
$172 \leqslant h < 176$	6
$176 \leqslant h < 180$	17
$180 \leqslant h < 184$	22
$184 \leqslant h < 188$	10
$188 \leqslant h < 192$	3

(c) Sound of 60 electrical items.

Sound (s db)	Frequency
$15 \leqslant s < 20$	4
$20 \leqslant s < 25$	12
$25 \leqslant s < 30$	15
$30 \leqslant s < 35$	6
$35 \leqslant s < 40$	8
$40 \leqslant s < 45$	3
$45 \leqslant s < 50$	12

Exercise 31

The following data gives the time taken by 50 runners to complete a cross-country race, to the nearest minute.

30	37	43	55	52	47	49	36	44	40
41	49	52	53	39	41	46	42	50	49
39	53	54	57	43	59	34	38	40	42
48	53	50	52	37	36	45	53	48	42
52	39	41	46	50	52	38	58	57	46

Time (t minutes)	Tally Marks	Frequency
$30 \leqslant t < 35$		
$35 \leqslant t < 40$		
$40 \leqslant t < 45$		
$45 \leqslant t < 50$		
$50 \leqslant t < 55$		
$55 \leqslant t < 60$		

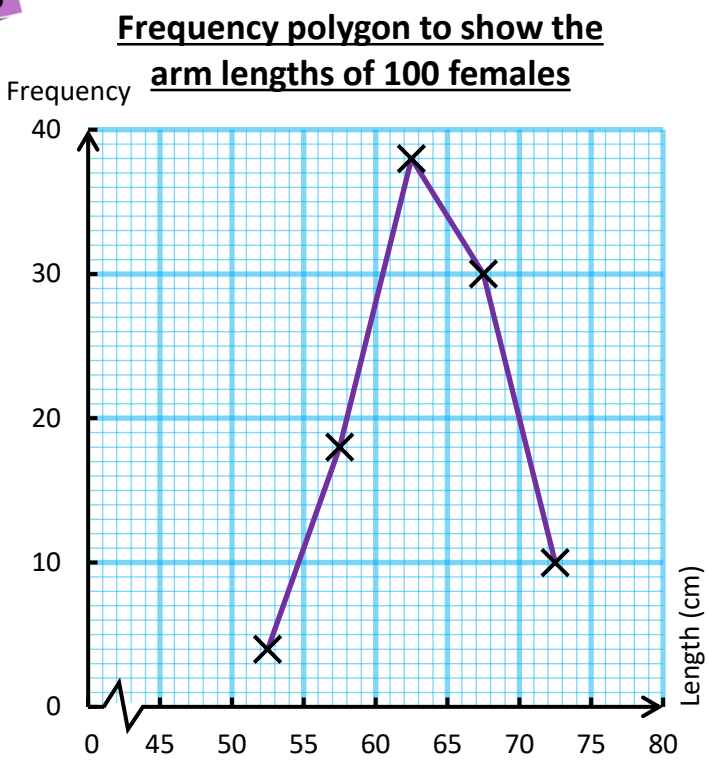
(a) Complete the **frequency table** for the data. (Remember that the data item 35 minutes would go into $35 \leqslant t < 40$, not $30 \leqslant t < 35$.)

(b) Draw a **frequency polygon** for the data.

Interpreting Frequency Polygons Applying I

Exercise 32

The frequency polygon on the right shows the arm lengths of 100 females.



(a) How many females have an arm length between 55 cm and 60 cm?

(b) How many more females have an arm length between 65 cm and 70 cm, compared to females with arm length between 70 cm and 75 cm?

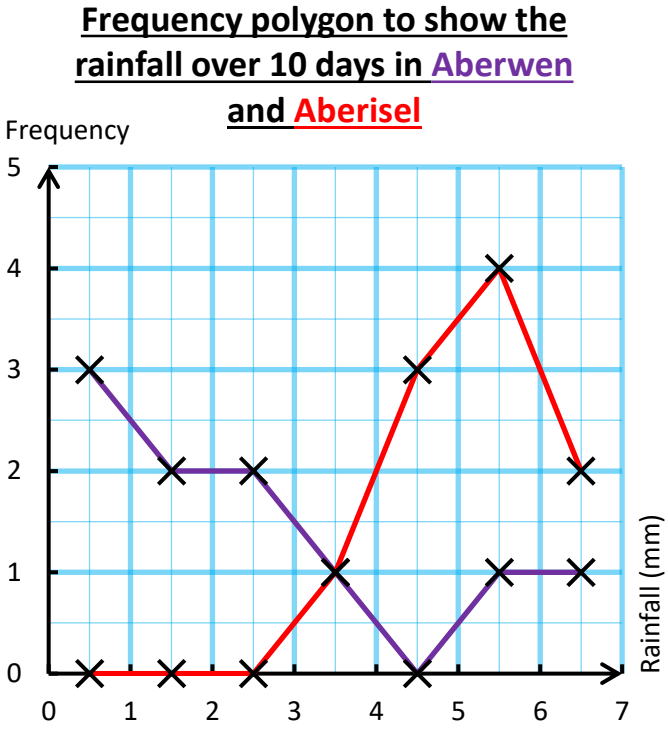
(c) Complete the frequency table below, using the information from the frequency polygon.

Arm length, l cm	Frequency
$50 < l \leqslant 55$	
$55 < l \leqslant 60$	
$60 < l \leqslant 65$	
$65 < l \leqslant 70$	
$70 < l \leqslant 75$	

(d) What is the modal class of the data?

Exercise 33

The daily rainfall for ten days was measured in Aberwen and Aberisel. The frequency polygon on the right shows the results. The purple line represents Aberwen, and the red line represents Aberisel.



(a) For how many days was the rainfall between 0 mm and 1 mm of rain in Aberwen?

(b) For how many days was the rainfall between 6 mm and 7 mm of rain in Aberisel?

(c) Complete this sentence: Considering the days where the rainfall was between 5 mm and 6 mm of rain, Aberisel had this rainfall on _____ more days than Aberwen.

(d) Over these 10 days, in your opinion where was the wettest place? Explain your answer.

(e) Deiniol says "The frequency polygon shows that the rainfall in Aberwen and Aberisel is the same on the fourth day". Is Deiniol telling the truth? Explain your answer.

Extension

Challenge!

Use the frequency table in Exercise 32 to calculate an estimated mean arm length for the 100 females.

Challenge!

Use the frequency polygon from Exercise 33 to calculate the estimated mean rainfall in Aberwen and in Aberisel. Does your answer agree with your conclusion to part (d) of Exercise 33?

Exercise 34 I

The table below shows the marks for 10R in their English test. (The test was out of 30.)

Marks (m)	Frequency
$0 \leqslant m < 6$	3
$6 \leqslant m < 12$	8
$12 \leqslant m < 18$	7
$18 \leqslant m < 24$	6
$24 \leqslant m < 30$	2

(a) Eric looks at the table and says: "*Three people got 0 out of 30 in this test!*". Is Eric telling the truth?

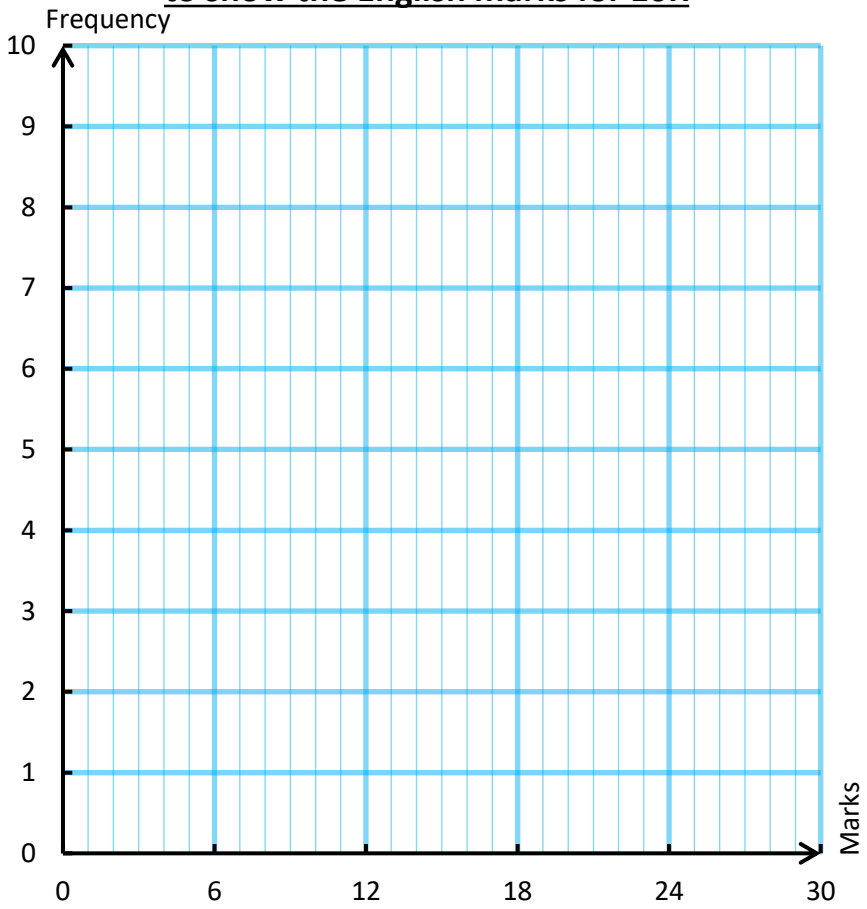
(b) Susan looks at the table and says: "*Nobody got full marks in this test!*". Is Susan telling the truth?

(c) On the graph paper on the right, draw a frequency polygon for the data.

(d) On the same graph paper, draw a frequency diagram for the data.

(e) What is the connection between any frequency polygon and frequency diagram drawn for the same data?

Frequency polygon and frequency diagram to show the English marks for 10R



(f) Calculate the following for the data from the English test.

(i) The modal class.
(ii) The median class.
(iii) The estimated mean.
(iv) The estimated range.

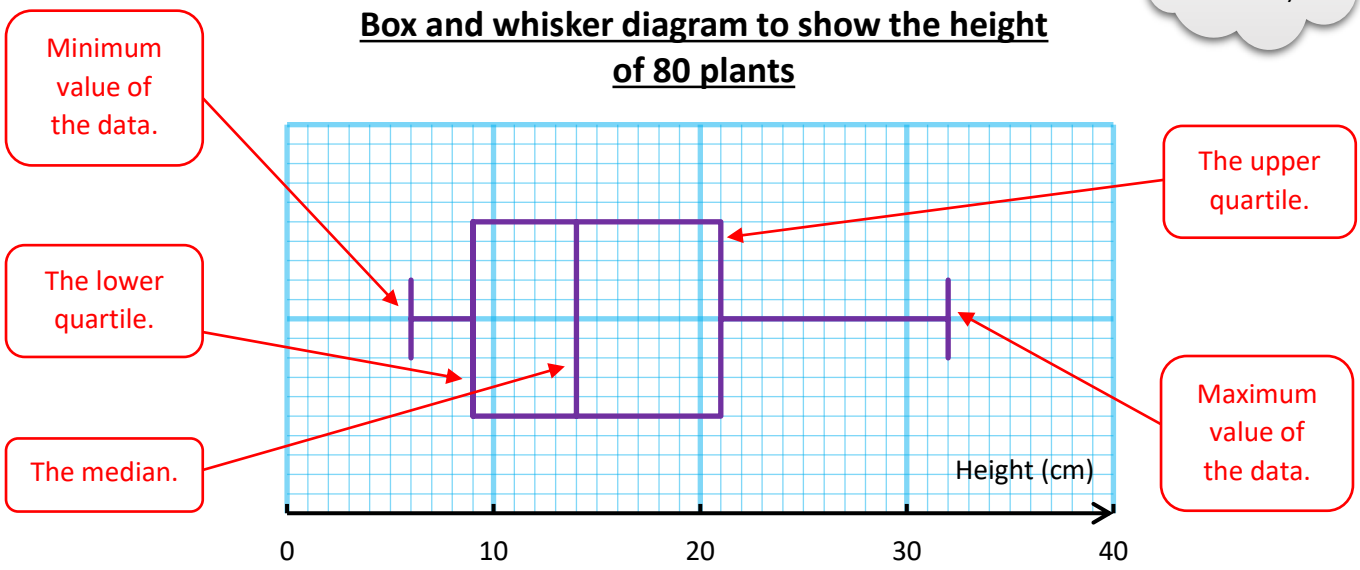
Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Box and Whisker Diagrams

(Or Box and Whisker Plots.)

A **box and whisker diagram** shows a number of statistics on one diagram.



Exercise 35

Skill I

For the box and whisker diagram shown above, write

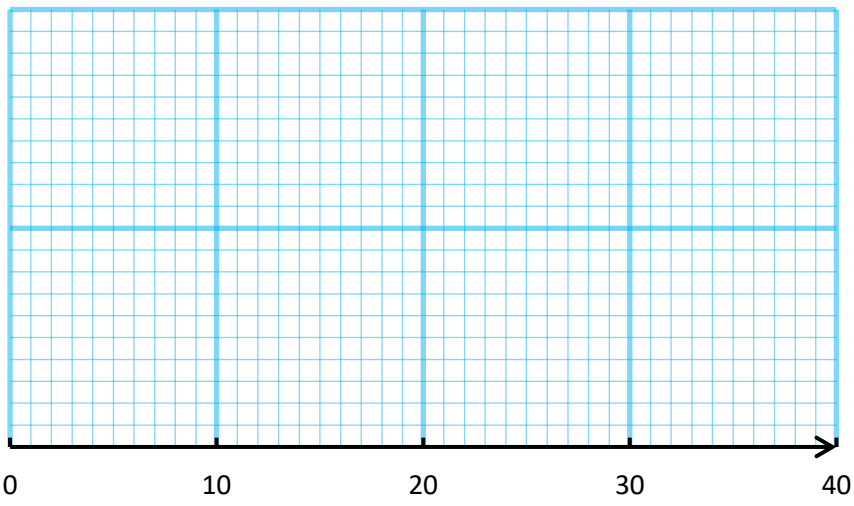
(a) The smallest plant height.
(b) The lower quartile.
(c) The median plant height.
(d) The upper quartile.
(e) The largest plant height.
(f) The range of plant heights.
(g) The interquartile range of plant heights.

Exercise 36

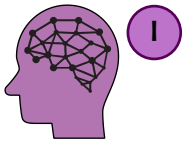
Moli calculated the following statistics for 50 numbers.
Use the statistics to draw a box and whisker diagram on the graph paper below.

Smallest number = 5 Lower quartile = 12 Median = 19 Upper quartile = 24 Largest number = 34

Box and whisker diagram for Moli's 50 numbers



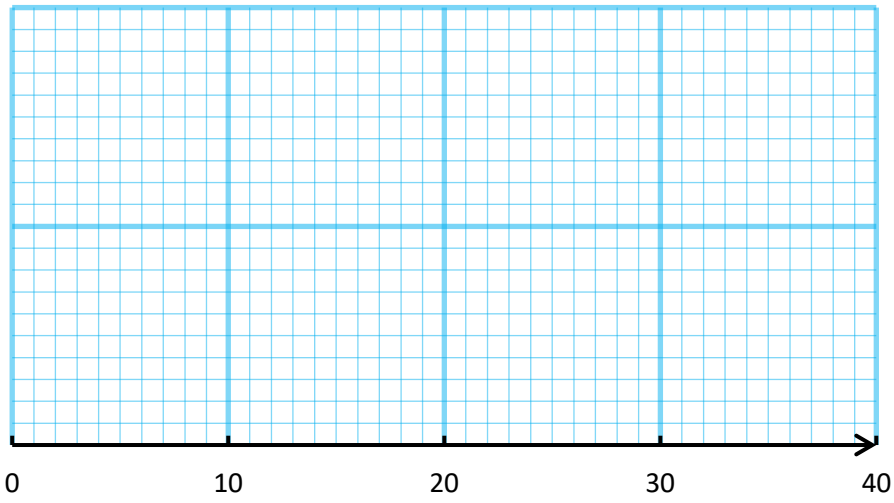
Exercise 37



Dafydd calculates the following statistics for 70 numbers.
Use the statistics to draw a box and whisker diagram on the graph paper below.

Smallest number = 8 Lower quartile = 15 Median = 20 Interquartile range = 12 Range = 27

Box and whisker diagram for Dafydd's 70 numbers



Exercise 38

Draw box and whisker diagrams for the following sets of data.

(a) 4, 11, 14, 15, 17, 19, 20, 22, 22, 26, 29, 34, 35, 35, 38.

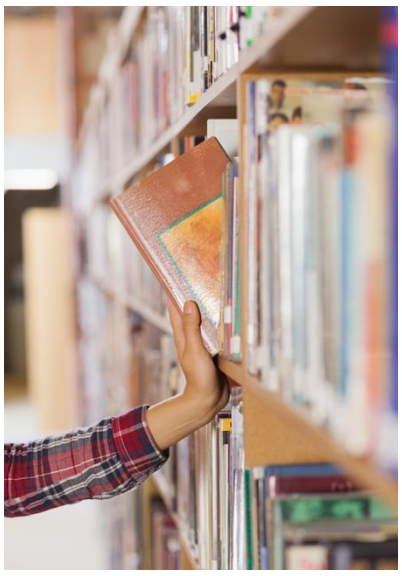
(b) 24, 13, 9, 35, 3, 17, 21, 30, 12, 28.

(c) 2, 6, 14, 18, 26, 27, 27, 30, 31.

The Connection between Box and Whisker diagrams and Cumulative Frequency Diagrams

There is a useful connection between cumulative frequency diagrams and box and whisker diagrams.

Example



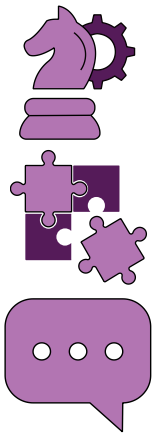
Consider the following data which shows the width of books sitting on a shelf in a library.

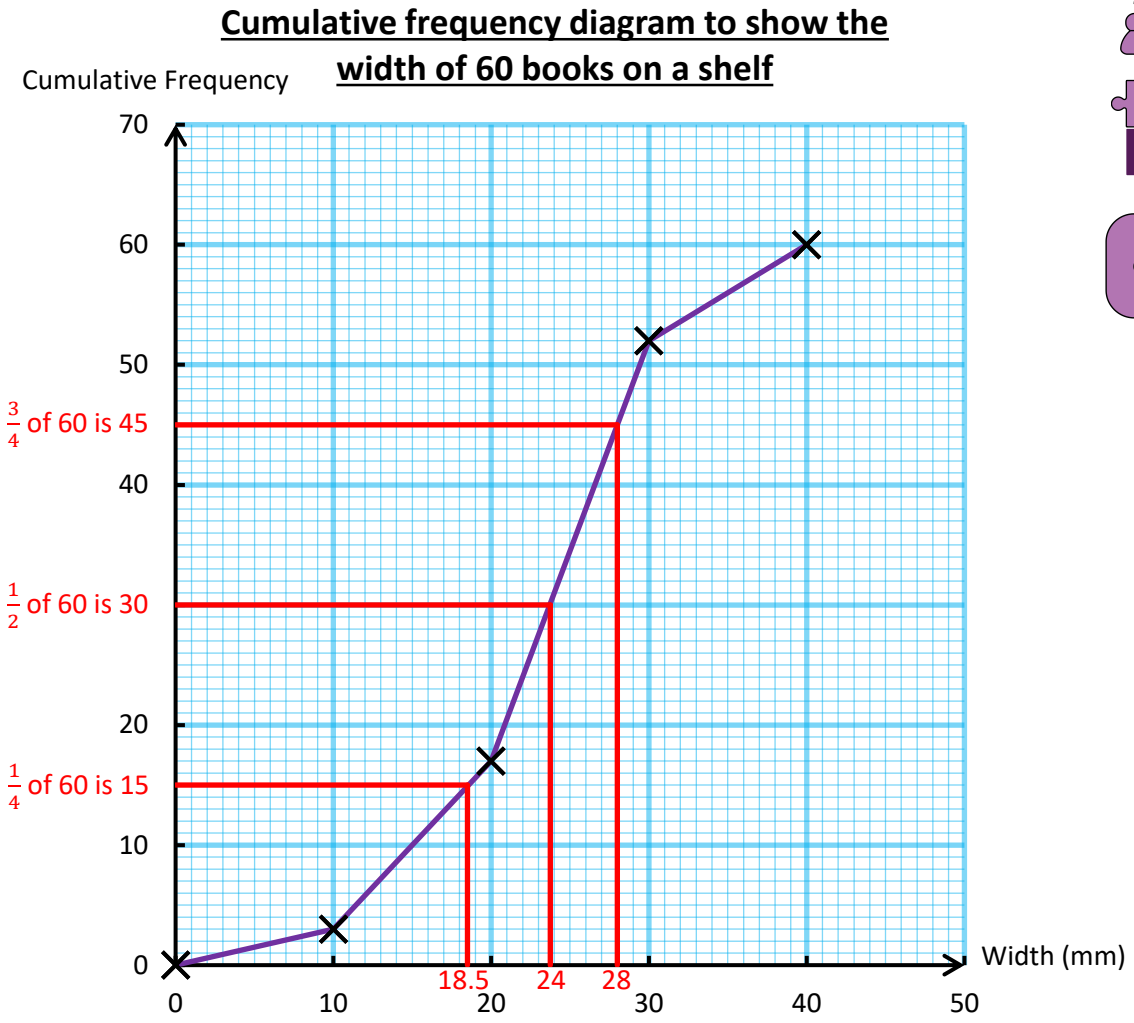
Book width (w mm)	Frequency
$0 < w \leqslant 10$	3
$10 < w \leqslant 20$	14
$20 < w \leqslant 30$	35
$30 < w \leqslant 40$	8

To draw a cumulative frequency diagram for the data we must first draw a cumulative frequency table.

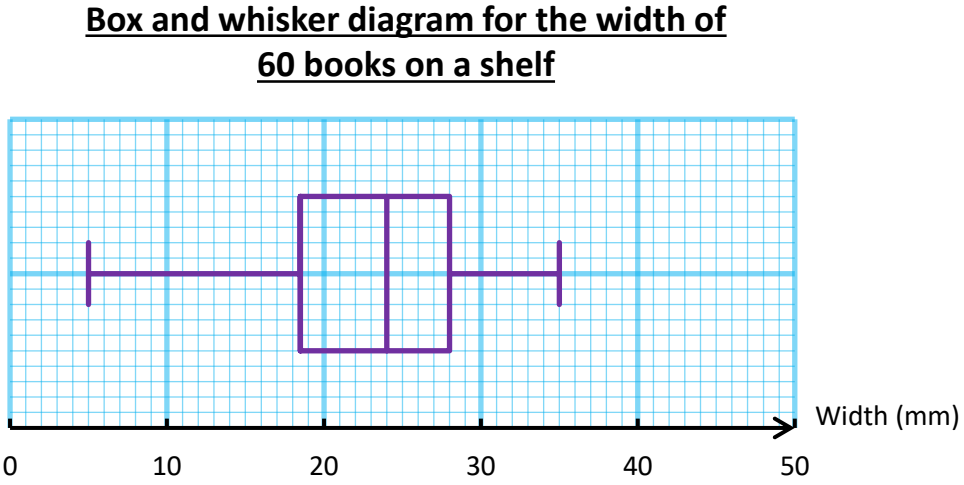
Book width (w mm)	Frequency	Cumulative Frequency	
$0 < w \leqslant 10$	3	3	
$10 < w \leqslant 20$	14	17	(3 + 14 = 17)
$20 < w \leqslant 30$	35	52	(17 + 35 = 52)
$30 < w \leqslant 40$	8	60	(52 + 8 = 60)

We can now draw a cumulative frequency diagram for the data.





To be able to draw the box and whisker diagram for the data, we need to find the quartiles. We can use the cumulative frequency diagram to estimate these. (These are the red lines on the above diagram). We also need to estimate the minimum value and the maximum value for the data. For the minimum value, we use the midpoint of the first class ($0 < w \leqslant 10$) which gives 5 mm. For the maximum value, we use the midpoint of the last class ($30 < w \leqslant 40$) which gives 35 mm. We can now draw the box and whisker diagram for the data.



Exercise 39

Applying

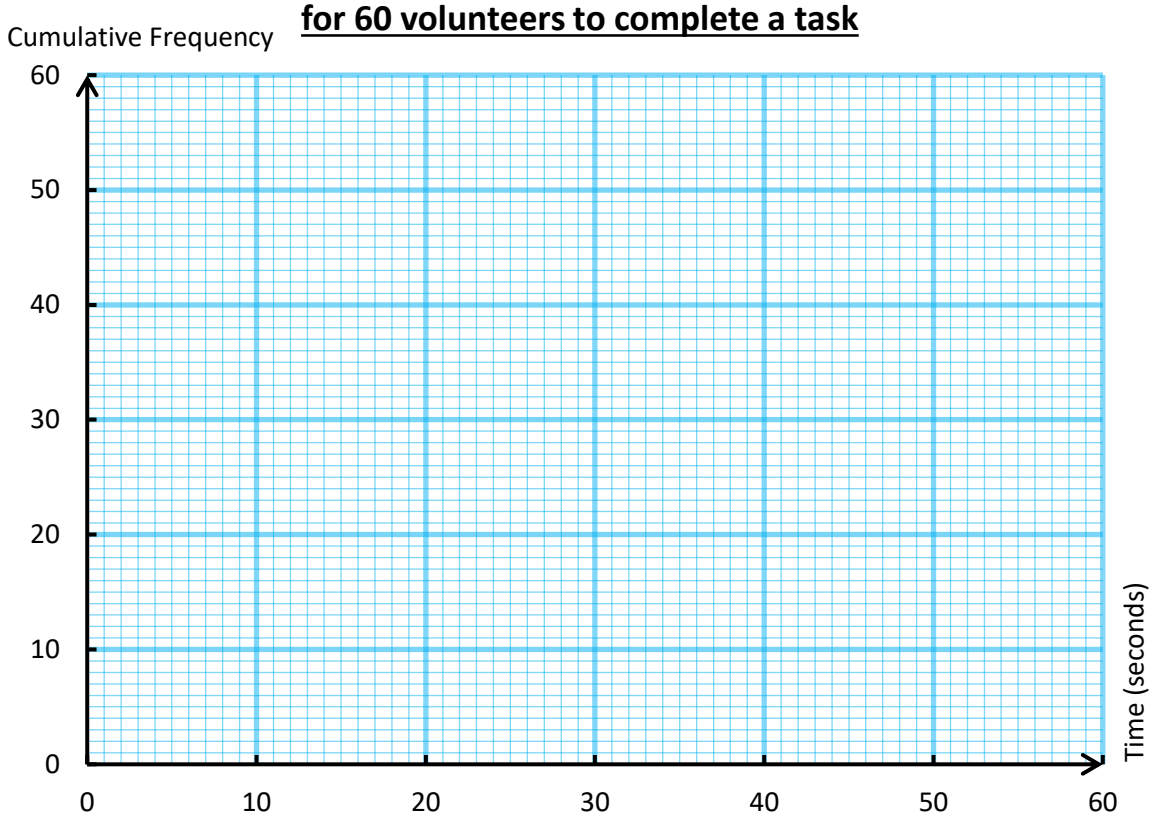
I

The times taken by 60 volunteers to complete a task were recorded, in seconds. The results are shown in the frequency table below. Complete the cumulative frequency table, the cumulative frequency diagram and the box and whisker diagram for the data.

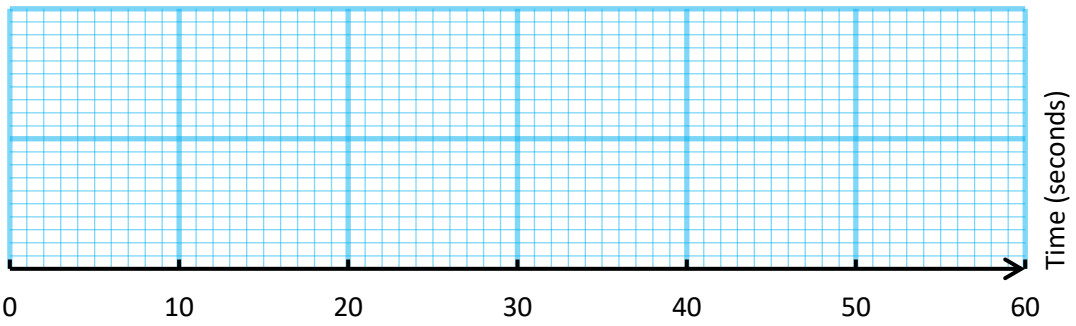
Time, t, to finish task (seconds)	Frequency
$15 < t \leqslant 20$	3
$20 < t \leqslant 25$	6
$25 < t \leqslant 30$	9
$30 < t \leqslant 35$	19
$35 < t \leqslant 40$	15
$40 < t \leqslant 45$	5
$45 < t \leqslant 50$	3

Time, t, to finish task (seconds)	Cumulative Frequency
$t \leqslant 15$	
$t \leqslant 20$	
$t \leqslant 25$	
$t \leqslant 30$	
$t \leqslant 35$	
$t \leqslant 40$	
$t \leqslant 45$	
$t \leqslant 50$	

Cumulative frequency diagram to show the times taken for 60 volunteers to complete a task



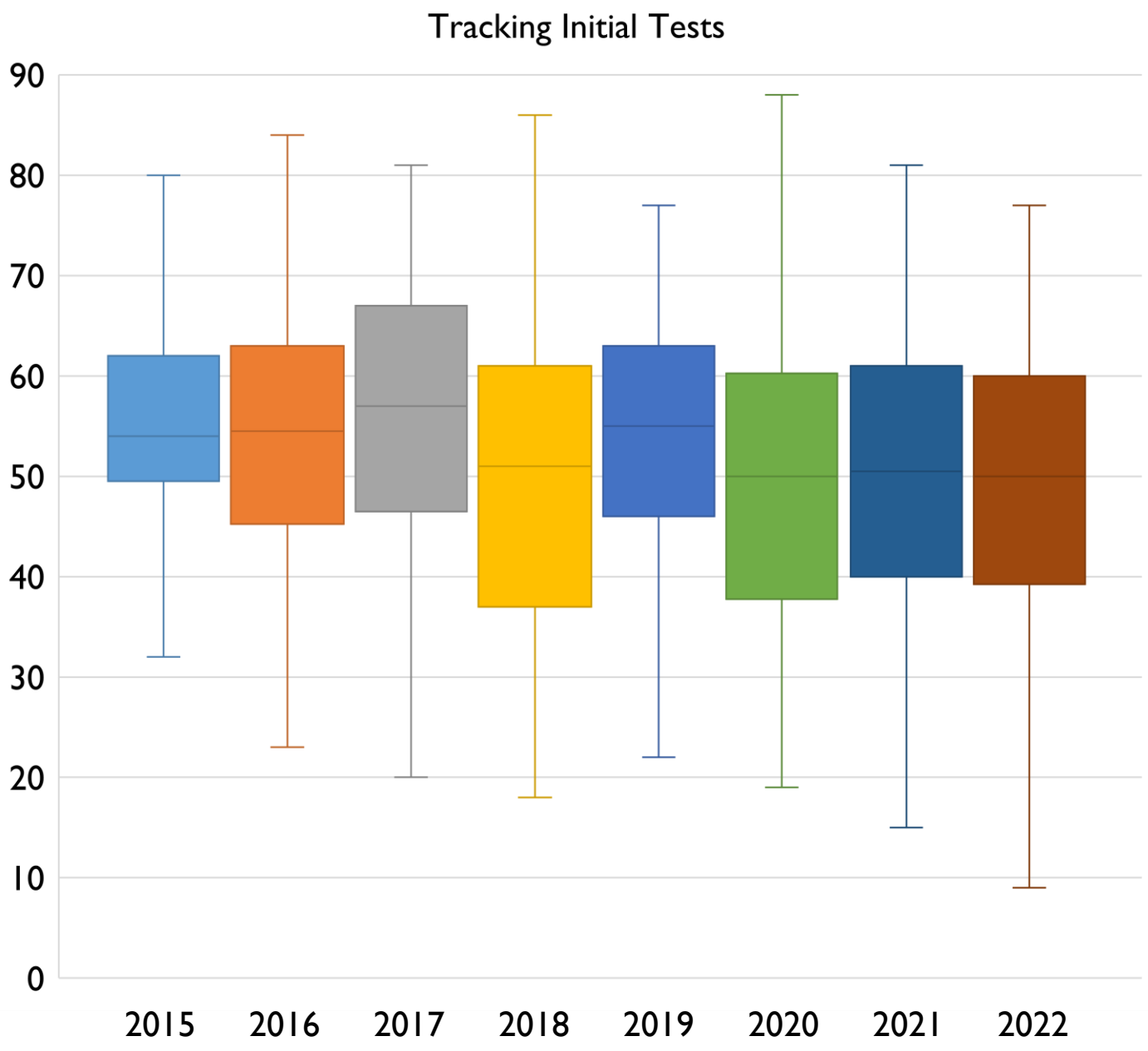
Box and whisker diagram to show the times taken for 60 volunteers to complete a task



Exercise 40

C

The following box and whisker diagrams show the marks (out of 90) gained by year 7 in their mathematics tests when starting school.



(a) What was the median mark in 2020?
(b) What was the lower quartile in 2021?
(c) What was the highest mark in 2015?
(d) In which year was the highest mark gained? What was this mark?
(e) What was the range in 2017?
(f) What was the interquartile range in 2021?
(g) In which year was the interquartile range at its lowest?
(h) In which year was the range at its lowest?
(i) Complete the following sentence: In 2016, 25% of the year gained a mark greater than _____.
(j) Complete the following sentence: In 2022, 75% of the year gained a mark greater than _____.
(k) Complete the following sentence: In _____, 50% of the year gained a mark greater than 55.
(l) In your opinion, which year performed best in the tests? Explain your answer.
(m) Between 2019 and 2022, the lowest mark decreases. What could be responsible for this?

Exercise 41

I

The frequency table below shows the time taken, in minutes, for company workers to travel to work every morning.

Time, t, in minutes	Frequency
$0 < t \leqslant 10$	3
$10 < t \leqslant 20$	8
$20 < t \leqslant 30$	14
$30 < t \leqslant 40$	6
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	2

(a) Draw a cumulative frequency table for the data.

(b) Draw a cumulative frequency diagram for the data.

(c) Draw a box and whisker diagram for the data.

(d) 5 years ago the median time was 22 minutes. How has the journey changed? Give a possible explanation.

(e) One day the median time was 24 minutes, the upper quartile was 50 minutes and the maximum time was 75 minutes. Explain what could have happened.

Exercise 42

Mr. Hughes and Mrs. Jones have bought the same type of seeds for planting a special plant. The two plant the seeds at the same time and measure the heights of the plants 6 months later. The box and whisker diagrams on the right show the results.

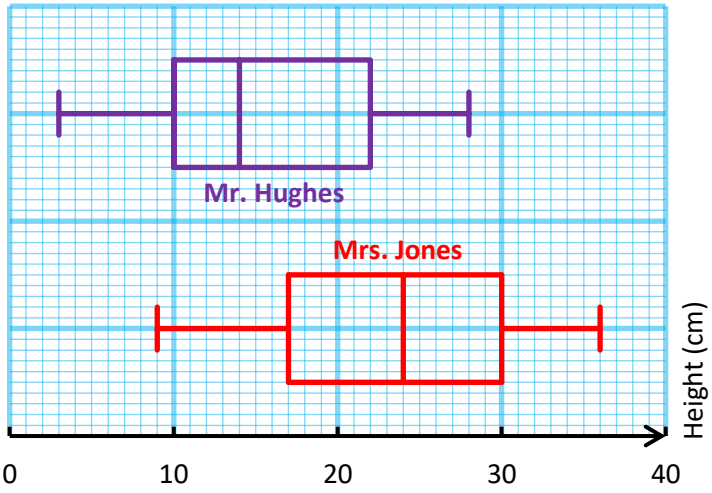
(a) What is the median height of Mr. Hughes' plants?

(b) What is the height of Mrs. Jones' tallest plant?

(c) Calculate the interquartile range of Mr. Hughes' plants.

(d) One of the people used fertiliser over the last 6 months. Who did this, in your opinion? Explain your answer.

Box and whisker diagrams for the heights of Mr. Hughes and Mrs. Jones' plants 6 months after planting

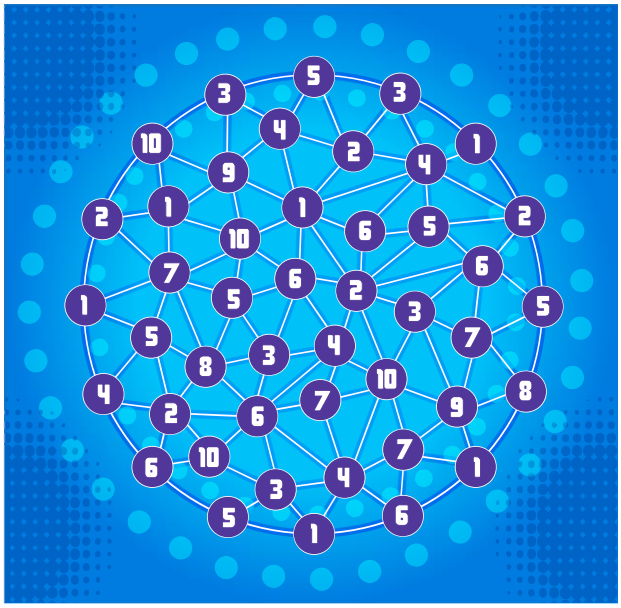


Evaluation

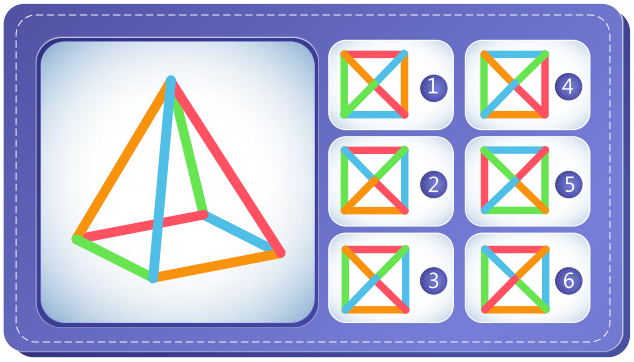
Key words	Corrections	I am happy with...	I need to revise...

Puzzles

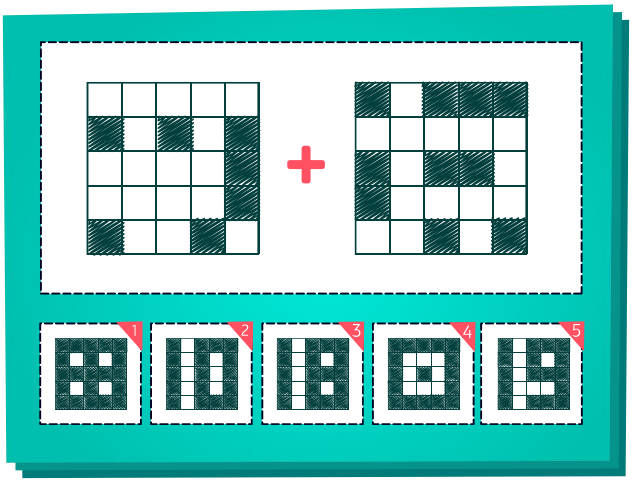
(a) Connect the numbers from 1 to 10.



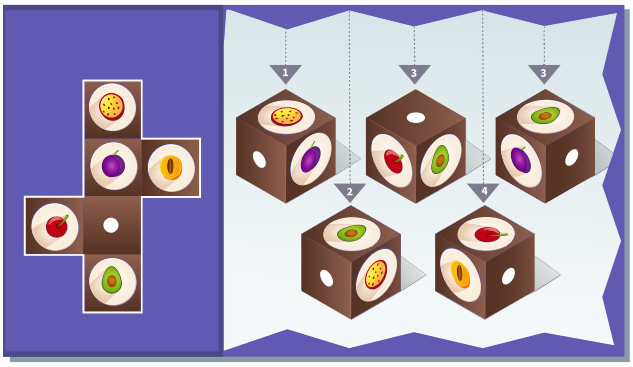
(b) Which is the correct plan view?



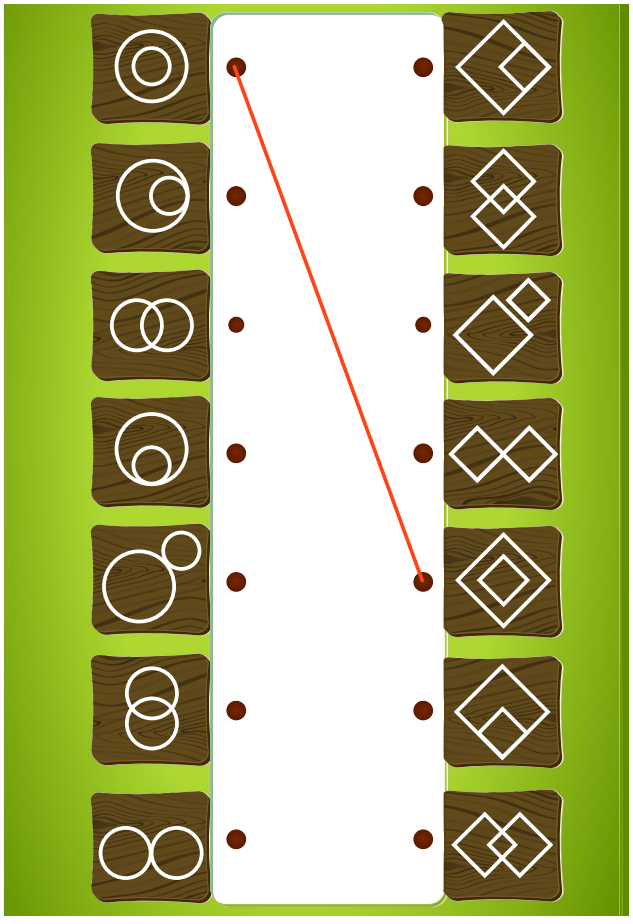
(c) Join the pictures to deduce which image is formed.



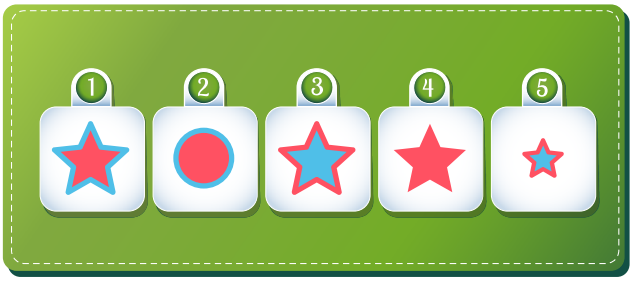
(d) Which cube is formed by folding the net?



(e) Pair the pictures.



(f) Which picture is the odd one out?



Data Handling and Statistics 4

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test:	Correct in the test?
I can calculate the **mode**, the **median**, the **mean** and the **range** for **discrete data**.			10	
I can calculate the **modal class**, the **median class**, an **estimate of the mean** and an **estimate of the range** for **grouped data**.			4, 8	
I know how to decide **which average is most suitable** for a set of data.			9	
I know how to use averages and measures of spread to **compare** two data sets.			9	
I can **find the original data set** given information about averages and the range.			6	
I know how to **criticise questions** in a **questionnaire**.			1	
I know how to write **unfavourable comments** about a plan to prove a specific **hypothesis**.			1	
I know how to choose a **simple random sample** using a **table of random digits** or using the **random number generator function on a calculator**.			2	
I know how to calculate the **sampling interval** for a systematic sample.			3	
I know how to choose a **systematic sample**.			3	
I know how to draw a **frequency polygon**.			4	
I know how to **interpret** a frequency polygon.				
I know how to draw a **box and whisker diagram**.			5	
I can use a **cumulative frequency diagram** in order to draw a box and whisker diagram.			7	

The Mathematics Department

10

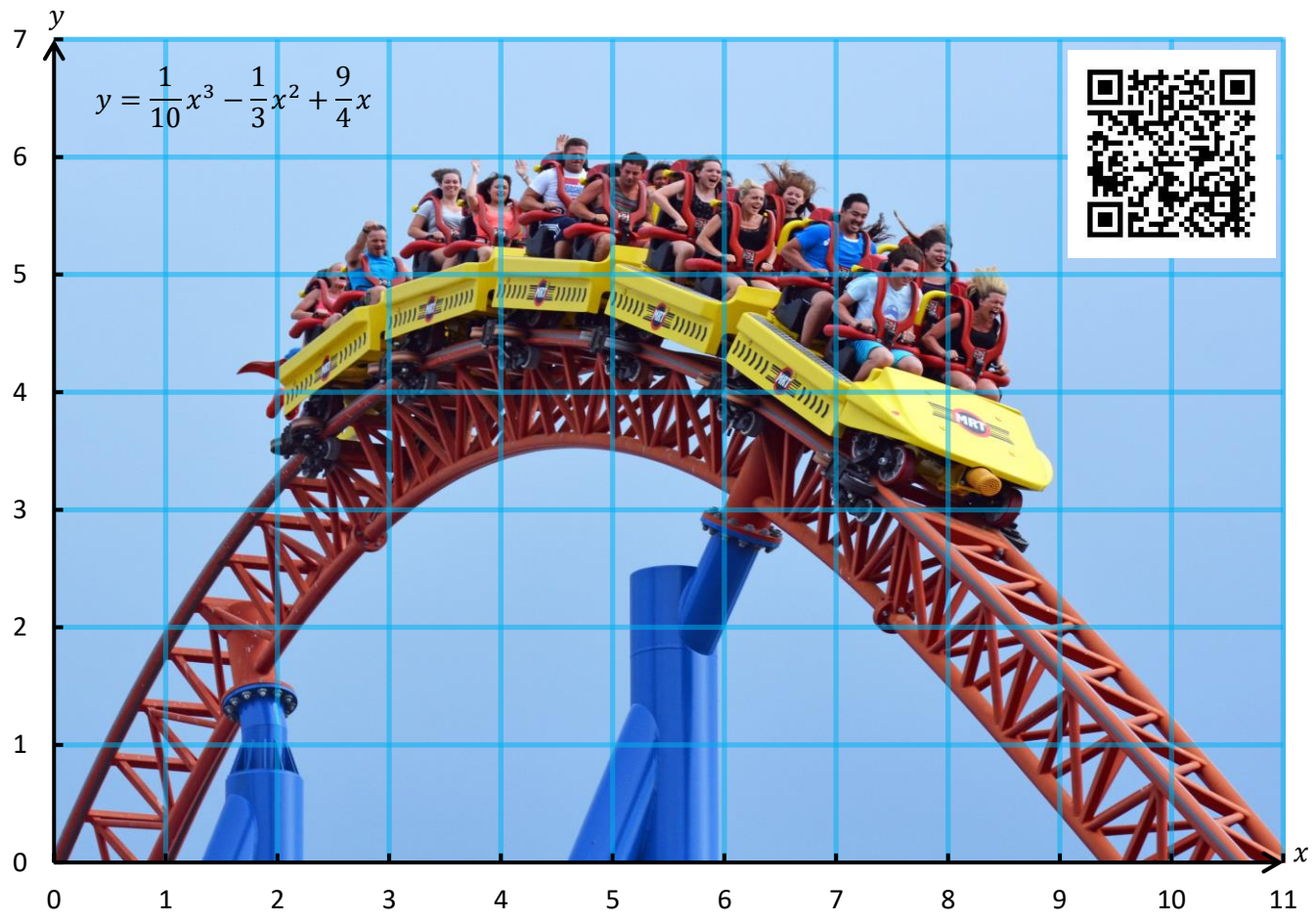
Powers and Roots

Intermediate Tier

Name:

Contents

Chapter	Mathematics	Page Number
Rules of Indices	The index form. Evaluating the index form. The multiplication rule. The division rule. The zeroth index rule. Raising a power to another index. The negative index rule. The reciprocal as a negative index. Unitary fraction index rule. Algebra and rules of indices.	3
Standard Form	Writing numbers $x \geq 1$ in standard form. Writing numbers $0 < x < 1$ in standard form. Changing from standard form to an ordinary number. Adding and subtracting in standard form. Almost in standard form. Multiplying and dividing in standard form.	11
Graph Plotting	Quadratic graphs. Recognising and sketching graphs of the form $y = ax^2 + b$. Graphical method of solving equations of the form $x^2 + ax + b = 0$.	17



Rules of Indices

The Index Form

In year 8, we considered how to write a number as a product of its prime factors, in **index form.**
For example, we can write 72 as a product of its prime factors, in index form, like this.

$$72 = 2^3 \times 3^2.$$

The index form is a product of terms of the form n^a. Each of these terms include a **base** and an **index**.

The base. This shows which number is getting multiplied in the term. → n^a ← **The index**. This shows how many times the number n appears in the multiplication sum.

For example, we can write $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ as 7^{10}.
The **base** is 7, since 7 is the number which is being multiplied. The **index** is 10, since 7 appears 10 times.

Other examples

$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$

The number **5** is being multiplied.
It appears **7** times.

$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

The number **4** is being multiplied.
It appears **6** times.

$2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

The numbers **2** and **3** are being multiplied. **2** appears **3** times, and **3** appears **2** times.

Exercise 1

Skill I

Write the following in index form.

(a) $3 \times 3 \times 3 \times 3 \times 3$
(b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
(c) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
(d) $3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5$
(e) $3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$
(f) $3 \times 3 \times 3 \times 3 \times 3 \times 5$
(g) $2 \times 2 \times 7 \times 7 \times 7 \times 7 \times 9 \times 9 \times 9$
(h) $8 \times 8 \times 8 \times 33 \times 33 \times 33 \times 33$
(i) $3 \times 8 \times 8 \times 3 \times 8 \times 3 \times 3 \times 8$
(j) $3 \times 5 \times 5 \times 5 \times 3 \times 3 \times 7 \times 7$
(k) $2 \times 5 \times 7 \times 7 \times 2 \times 2 \times 2 \times 5$
(l) $13 \times 11 \times 7 \times 7 \times 11 \times 7$

Exercise 2

Write the following as multiplication sums without indices.

(a) 2^5
(b) 2^3
(c) 2^1
(d) 4^6
(e) 17^8
(f) 256^5
(g) $\left(\frac{1}{3}\right)^4$
(h) $2^4 \times 5^3$
(i) $4^4 \times 5^5$
(j) $24^3 \times 45^4$
(k) $\left(\frac{1}{5}\right)^3 \times \left(\frac{3}{4}\right)^3$
(l) $5^3 \times 13^2 \times 27^4$
(m) $3^2 \times 5^4 \times 10^2 \times 14^3$
(n) $2^3 \times \left(\frac{1}{2}\right)^3 \times 4^3$
(o) $\left(\frac{3}{7}\right)^2 \times \left(\frac{3}{4}\right)^4 \times \left(\frac{7}{9}\right)^3$

Investigation

Who was **Pierre de Fermat**? What was his contribution to mathematics?
What was his last theorem? When was this theorem proved?
Are there other theorems connected to index form?

Evaluating the Index Form

Evaluating index form is the process of writing a number written in index form as an ordinary number. For example, we can write 3^4 as $3 \times 3 \times 3 \times 3$, which is equal to 81.

Other examples

$4^3 = 4 \times 4 \times 4$
$= 64$

$5^4 = 5 \times 5 \times 5 \times 5$
$= 625$

$2^4 + 7^3 = (2 \times 2 \times 2 \times 2) + (7 \times 7 \times 7)$
$= 16 + 343$
$= 359$

Exercise 3

Evaluate the following, without using a calculator.

(a) 3^4 (b) 6^3 (c) 10^5 (d) 2^9 (e) 20^4
(f) $3^2 + 2^5$ (g) $6^3 - 3^4$ (h) $6^3 \times 2^2$ (i) $10^4 \div 2^2$ (j) $5^4 + 4^4$

Exercise 4

Use your calculator to evaluate the following. If appropriate, write your answer correct to 2 decimal places.

(a) 125^2 (b) 17^4 (c) $29^3 + 5$ (d) $9^3 + 5$ (e) $12^4 - 5^6$
(f) $12^3 + 3^7$ (g) $3^4 \times 4^5$ (h) $2^3 \times 4^2 + 3^2$ (i) $(4^3)^4$ (j) $4^6 \div 2^6 + 10^3$
(k) $11^3 - 4^4$ (l) $4^5 - 5^6$ (m) $3^8 + 4^{10} - 5^6$ (n) $4^6 - 3^2 \times 8^3$ (o) $3^4 + 8^8 \div 4^{10}$
(p) $\frac{5^6}{3^7}$ (q) $\frac{4^4+3^6}{2^4}$ (r) $\frac{11^3}{2^5 \times 3^5}$ (s) $\frac{4^3+6^4\times2^3}{10^3-5^3}$ (t) $\left(\frac{4^3+6^4\times2^3}{10^3-5^3}\right)^3$

Exercise 5

Applying

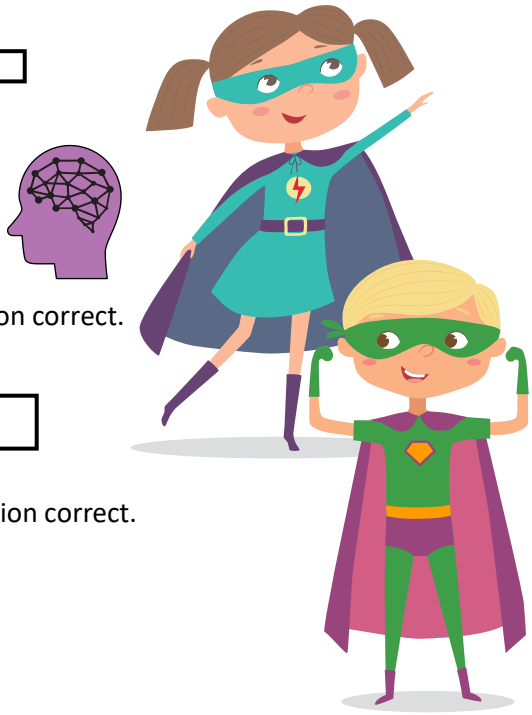
(a) Without using a calculator, calculate the numbers which fill the following spaces.

(i) $2^{12} = 4096$, $2^{11} =$ _____
(ii) $4^6 = 4096$, $4^7 =$ _____
(iii) $3^{11} = 177147$, $3^{12} =$ _____
(iv) $5^5 = 3125$, $5^4 =$ ____
(v) $8^3 = 512$, $8^4 =$ _____
(vi) $2^8 = 256$, $2^{10} =$ _____
(vii) $6^4 = 1296$, $6^6 =$ _____
(viii) $3^7 = 2187$, $3^9 =$ _____
(ix) $5^9 = 1953125$, $5^7 =$ _____
(x) $7^4 = 2401$, $7^6 =$ _____

(b) Put the numbers 1 to 6 into the following boxes to make:

□□ + □□ + □□

(i) The largest possible number;
(ii) The smallest possible number;
(iii) A total of 147.



(c) Put the numbers 1 to 6 into the following boxes to make the calculation correct.

□□ + □□ = □□

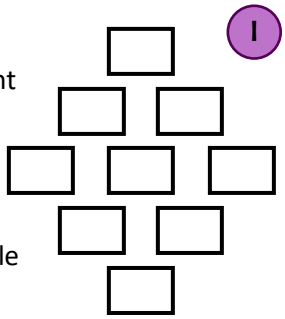
(d) Write any whole numbers in the following boxes to make the calculation correct.
How many possible solutions are there?

□□ = 64

Exercise 6

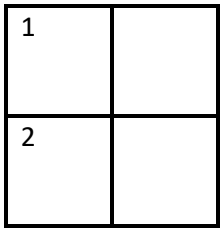
Write the digits 1 to 9 in the grid on the right so that each row (reading across) is a square number. You may use each digit once only.

Can you prove that there is only one possible solution?



Exercise 7

Complete the following crossnumber.



Clues:

Across
1. Cube number
2. Cube number

Down
1. One less than a cube number

Rules of Indices

When performing calculations in index form, we notice several different patterns.
The **rules of indices** write these patterns in a convenient way.

The Multiplication Rule

$$n^a \times n^b = n^{a+b}$$

When multiplying one number or variable to an index, by the **same** number or variable to another index, we must **add** the indices. We can see below why this is true.

$$8^4 \times 8^3 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$$
$$= 8^7.$$

Other examples

$4^3 \times 4^6 = 4^{3+6}$
$= 4^9$

$8^2 \times 8^9 = 8^{2+9}$
$= 8^{11}$

$a^5 \times a^6 \times a^4 = a^{5+6+4}$
$= a^{15}$

$7^9 \times 7^{-3} = 7^{9+-3}$
$= 7^6$

Exercise 8

Skill

Simplify each of the following expressions.

(a) $7^5 \times 7^3$ (b) $7^3 \times 7^5$ (c) $8^5 \times 8$ (d) $x^5 \times x^3$ (e) $7^4 \times 7^8$
(f) $a^5 \times a^3$ (g) $a^5 \times a^7 \times a^{10}$ (h) $3^2 \times 3^9 \times 3^4$ (i) $5^5 \times 5^2 \times 5^{12}$ (j) $y^3 \times y^{13} \times y^{16}$
(k) $7^{15} \times 7^{-4}$ (l) $14^9 \times 14^{-6}$ (m) $8^{-10} \times 8^3$ (n) $d^5 \times d^{-8}$ (o) $f^{-4} \times f^{-3}$
(p) $i^{-5} \times i^{11} \times i^{-3}$ (q) $p^{-9} \times p^{-2} \times p^5$ (r) $4^{-17} \times 4^{-7} \times 4^{31}$ (s) $(-5)^5 \times (-5)^3$ (t) $(-5)^5 \times (-5)^{-3}$
(u) $(-5)^{-5} \times (-5)^{-3}$ (v) $a^3 \times a^{\frac{1}{2}}$ (w) $a^{\frac{3}{5}} \times a^{\frac{1}{5}}$ (x) $a^{\frac{2}{3}} \times a^{\frac{4}{7}}$ (y) $a^{\frac{8}{3}} \times a^{\frac{5}{4}}$

Exercise 9

Find the missing numbers that go into the boxes in all of the following questions.

(a) $7^5 \times 7^{\square} = 7^8$ (b) $7^{\square} \times 7^4 = 7^6$ (c) $7^{13} \times 7^{\square} = 7^{11}$ (d) $7^8 \times 7^4 = 7^{\square}$ (e) $x^2 \times x^{\square} = x^{14}$
(f) $5^5 \times 5^{\square} = 5^6$ (g) $4^9 \times 4^{\square} = 4^8$ (h) $11^{\square} \times 11^{10} = 11^7$ (i) $2^{\square} \times 2^{-5} = 2^9$ (j) $8^{-2} \times 8^{\square} = 8^{-9}$

Challenge!

Extension

An **Armstrong Number** is a whole number where the sum of the digits, each raised to the number of digits, is equal to the original number.
For example, the number 371 is an Armstrong number since $3^3 + 7^3 + 1^3 = 371$.
1,634 is also an Armstrong number since $1^4 + 6^4 + 3^4 + 4^4 = 1{,}634$.
How many Armstrong numbers are there between 1 and 10,000?

The Division Rule

$$n^a \div n^b = n^{a-b} \text{ or } \frac{n^a}{n^b} = n^{a-b}$$

When dividing one number or variable to an index, by the **same** number or variable to another index, we must **subtract** the indices. We can see below why this is true.

$$\begin{aligned}\frac{7^8}{7^5} &= \frac{7\times7\times7\times7\times7\times7\times7\times7}{7\times7\times7\times7\times7} \\ &= \frac{7\times7\times7\times \cancel{7\times7\times7\times7\times7}}{\cancel{7\times7\times7\times7\times7}} \\ &= 7\times7\times7 \\ &= 7^3.\end{aligned}$$

Other Examples

$4^6 \div 4^3 = 4^{6-3}$
$= 4^3$

$8^9 \div 8^2 = 8^{9-2}$
$= 8^7$

$\frac{a^7}{a^4} = a^{7-4}$
$= a^3$

$6^5 \div 6^{-2} = 6^{5--2}$
$= 6^7$

Exercise 10

Skill I

Simplify each of the following expressions.

(a) $7^5 \div 7^3$ (b) $7^3 \div 7^5$ (c) $7^{11} \div 7$ (d) $7^8 \div 7^4$ (e) $x^5 \div x^3$

(f) $a^5 \div a^3$ (g) $a^5 \div a^7$ (h) $\frac{3^9}{3^2}$ (i) $\frac{5^{15}}{5^{12}}$ (j) $\frac{y^3}{y^{16}}$

(k) $7^{15} \div 7^{-4}$ (l) $14^9 \div 14^{-6}$ (m) $8^{-10} \div 8^3$ (n) $d \div d^{-8}$ (o) $f^{-4} \div f^{-3}$

(p) $i^{-5} \div i^{-3}$ (q) $p^{-9} \div p^{-2} \div p^5$ (r) $4^{-17} \div 4^{-7} \div 4^{31}$ (s) $(-5)^5 \div (-5)^3$ (t) $(-5)^5 \div (-5)^{-3}$

(u) $(-5)^{-5} \div (-5)^{-3}$ (v) $a^3 \div a^{\frac{1}{2}}$ (w) $a^{\frac{3}{5}} \div a^{\frac{1}{5}}$ (x) $a^{\frac{2}{3}} \div a^{\frac{4}{7}}$ (y) $a^{\frac{8}{3}} \div a^{\frac{5}{4}}$

Exercise 11

Find the missing numbers that go into the boxes in all of the following questions.

(a) $7^5 \div 7^{\square} = 7^2$ (b) $7^{\square} \div 7^4 = 7^6$ (c) $7^{13} \div 7^{\square} = 7^{11}$ (d) $7^6 \div 7^4 = 7^{\square}$ (e) $x^{15} \div x^{\square} = x^7$

(f) $5^5 \div 5^{\square} = 5^7$ (g) $\frac{4^9}{4^{\square}} = 4^5$ (h) $\frac{11^{30}}{11^{\square}} = 11^{14}$ (i) $2^{\square} \div 2^{-5} = 2^{-2}$ (j) $8^{-2} \div 8^{\square} = 8^{-9}$

Example

$3^7 \times 3^5 \div 3^2 = 3^{7+5} \div 3^2$
$= 3^{12} \div 3^2$
$= 3^{12-2}$
$= 3^{10}$

$3^9 \div 3^2 \times 3^5 = 3^{9-2} \times 3^5$
$= 3^7 \times 3^5$
$= 3^{7+5}$
$= 3^{12}$

$\frac{y^7}{y^4} \times y^{10} = y^{7-4} \times y^{10}$
$= y^3 \times y^{10}$
$= y^{3+10}$
$= y^{13}$

Exercise 12

Simplify each of the following expressions.

(a) $3^5 \times 3^6 \div 3^2$ (b) $6^8 \times 6^6 \div 6^7$ (c) $7^{11} \times 7^5 \div 7^8$ (d) $5^8 \times 5^3 \div 5^4$ (e) $x^9 \times x^3 \div x^4$

(f) $a^5 \times a^4 \div a^8$ (g) $a^{10} \times a^{-5} \div a^7$ (h) $2^{13} \div 2^3 \times 2^4$ (i) $5^7 \div 5^2 \times 5^4$ (j) $8^9 \div 8 \times 8^3$

(k) $5^8 \times 5^3 \times 5^7 \div 5^4$ (l) $4^9 \div 4^2 \times 4^3 \div 4^5$ (m) $8^{-4} \times 8^3 \times 8^6$ (n) $d^5 \div d^{-8} \times d^4$ (o) $u^{-3} \div u^{-3} \times u^{-3}$

(p) $\frac{10^6}{10^2} \times 10^8$ (q) $\frac{7^9}{7^3} \times 7^2$ (r) $19^4 \times \frac{19^{12}}{19^3}$ (s) $4^9 \div \frac{4^6}{4^2}$ (t) $e^8 \times \frac{e^6}{e^3}$

(u) $\frac{4^5 \times 4^7}{4^3}$ (v) $\frac{15^7 \times 15}{15^2}$ (w) $\frac{6^{10}}{6^2 \times 6^5}$ (x) $\frac{r^6 \times r^{-1}}{r^{-2}}$ (y) $\frac{q^{-3}}{q^4 \times q^{-8}} \times q^3$

The Zeroth Index Rule

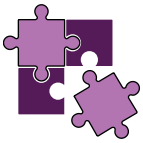
$$n^0 = 1$$

Any number or variable raised to a zero index gives the answer **1**. Why? Let us consider the following calculations.

$\frac{3^2}{3^2} = \frac{9}{9}$
$= 1$

$\frac{7^2}{7^2} = \frac{49}{49}$
$= 1$

$\frac{12^2}{12^2} = \frac{144}{144}$
$= 1$



But the division rule states that

$\frac{3^2}{3^2} = 3^{2-2}$
$= 3^0$

$\frac{7^2}{7^2} = 7^{2-2}$
$= 7^0$

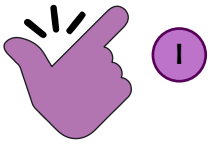
$\frac{12^2}{12^2} = 12^{2-2}$
$= 12^0$

So, we must have

$3^0 = 1$ $7^0 = 1$ $12^0 = 1$

This would work for any number or variable, not just 3 or 7 or 12.

Exercise 13



Evaluate the following.

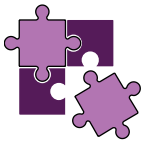
(a) 3^0 (b) 28^0 (c) 37648^0 (d) $19^0 \times 27^0$ (e) $19^0 + 27^0$

(f) x^0 (g) π^0 (h) $2^3 \times 2^0$ (i) $3^4 \div 3^0$ (j) $\frac{7^5}{7^5}$

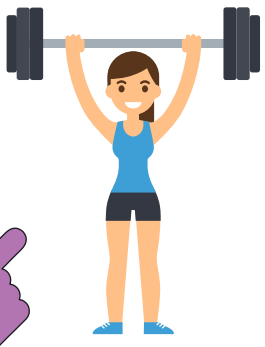
Raising a Power to Another Index

$$(n^a)^b = n^{a \times b}$$

If a power of the form n^a is raised to another index, then we need to **multiply** the indices. We can see below why this is true.



$$(5^3)^4 = \overbrace{5^3 \times 5^3 \times 5^3 \times 5^3}^{\text{4 times}}$$
$$= 5^{3+3+3+3}$$
$$= 5^{12}$$
$$= 5^{3\times4}.$$



Exercise 14

Simplify the following, giving your answers in index form.

(a) $(5^2)^4$ (b) $(5^4)^2$ (c) $(7^2)^4$ (d) $(x^2)^4$ (e) $((-5)^2)^4$

(f) $(5^{-2})^4$ (g) $(5^2)^{-4}$ (h) $(5^{-2})^{-4}$ (i) $\left(\left(\frac{1}{5}\right)^2\right)^4$ (j) $(5^2)^0$

(k) $(6^3)^6$ (l) $(11^{25})^3$ (m) $(2^7)^8$ (n) $(43^5)^{-7}$ (o) $(10^{-3})^{-9}$

(p) $(9^0)^6$ (q) $(0.3^6)^9$ (r) $(11^{-9})^7$ (s) $((-3)^{-4})^{-3}$ (t) $(y^{14})^3$

(u) $(5^2)^4 \times 5^6$ (v) $(3^6)^2 \div 3^4$ (w) $(8^5)^2 \div 8^{10}$ (x) $6 \times (6^5)^3$ (y) $(4^{12})^5 \times 4^0$

The Negative Index Rule

$$n^{-a} = \frac{1}{n^a}$$

Any number or variable raised to a **negative** index can be written as a fraction whose numerator is **1**.
Why? Let us consider the following sequences.

$7^3 = 7 \times 7 \times 7$ ÷ 7
$7^2 = 7 \times 7$ ÷ 7
$7^1 = 7$ ÷ 7
$7^0 = 1$

$15^3 = 15 \times 15 \times 15$ ÷ 15
$15^2 = 15 \times 15$ ÷ 15
$15^1 = 15$ ÷ 15
$15^0 = 1$

What will appear next in these sequences?

$7^0 = 1$ ÷ 7
$7^{-1} = \frac{1}{7}$ ÷ 7
$7^{-2} = \frac{1}{7\times7}$

$15^0 = 1$ ÷ 15
$15^{-1} = \frac{1}{15}$ ÷ 15
$15^{-2} = \frac{1}{15\times15}$

Exercise 15

Complete the following table.

n	5	4	3	2	1	0	–1	–2	–3	–4	–5
2^n	32										

Exercise 16

Write the following as ordinary fractions, without using indices.

(a) 3^{-2} (b) 4^{-2} (c) 5^{-2} (d) 3^{-3} (e) 4^{-3}
(f) 3^{-4} (g) 8^{-2} (h) 7^{-1} (i) 10^{-4} (j) 11^{-2}
(k) $2^{-2} \times 3$ (l) $6^{-2} \div 2$ (m) $8^{-1} \times 4$ (n) $9^{-2} \times 2^{-1}$ (o) $2^{-1} + 2^{-3}$

The Reciprocal as a Negative Index

In year 9, we defined the **reciprocal** of a number as follows:

The reciprocal of a number is $\frac{1}{\text{the number}}$.

For example, the reciprocal of 4 is $\frac{1}{4}$ and the reciprocal of 15 is $\frac{1}{15}$. Because $n^{-1} = \frac{1}{n^1} = \frac{1}{n}$, we can now define the reciprocal of a number in an alternative way:

The reciprocal of a number n is n^{-1}.

Exercise 17

Prove that multiplying a number n by its reciprocal n^{-1} always gives an answer of 1.
(Clue: You will need to use the multiplication rule from page 5.)

Unitary Fraction Index Rule

$$n^{\frac{1}{a}} = \sqrt[a]{n}$$

Any number or variable raised to an index that is a **unitary fraction** of the form $\frac{1}{a}$ can be written as the ***a*–th root** of n. For example, if $a = 4$ then n raised to a quarter $\left(n^{\frac{1}{4}}\right)$ can be written as the fourth root of n $\left(\sqrt[4]{n}\right)$.

Why is this true? Consider the following application of the rule $(n^a)^b = n^{a\times b}$ from page 7.

$\left(n^{\frac{1}{2}}\right)^2 = n^{\frac{1}{2}\times 2}$

$\left(n^{\frac{1}{2}}\right)^2 = n^1$

$\left(n^{\frac{1}{2}}\right)^2 = n$

Because the square root of a number is a number that squares to give the original number, we have $n^{\frac{1}{2}} = \sqrt{n}$.

$\left(n^{\frac{1}{3}}\right)^3 = n^{\frac{1}{3}\times 3}$

$\left(n^{\frac{1}{3}}\right)^3 = n^1$

$\left(n^{\frac{1}{3}}\right)^3 = n$

Because the cube root of a number is a number that cubes to give the original number, we have $n^{\frac{1}{3}} = \sqrt[3]{n}$.

Example

$9^{\frac{1}{2}} = \sqrt{9}$
$= 3$

$216^{\frac{1}{3}} = \sqrt[3]{216}$
$= 6$

$625^{\frac{1}{4}} = \sqrt[4]{625}$
$= 5$

$5^4 = 5\times 5\times 5\times 5$
$= 625$ therefore $\sqrt[4]{625} = 5$.

I

Exercise 18

Evaluate the following.

(a) $16^{\frac{1}{2}}$ (b) $25^{\frac{1}{2}}$ (c) $49^{\frac{1}{2}}$ (d) $8^{\frac{1}{3}}$ (e) $27^{\frac{1}{3}}$

(f) $16^{\frac{1}{4}}$ (g) $81^{\frac{1}{4}}$ (h) $64^{\frac{1}{2}}$ (i) $64^{\frac{1}{3}}$ (j) $64^{\frac{1}{6}}$

(k) $36^{\frac{1}{2}}\times 9^{\frac{1}{2}}$ (l) $125^{\frac{1}{3}} + 81^{\frac{1}{2}}$ (m) $100^{\frac{1}{2}} \div 4^{\frac{1}{2}}$ (n) $32^{\frac{1}{5}} - 1^{\frac{1}{3}}$ (o) $121^{\frac{1}{2}}\times 0^{\frac{1}{10}}$

Exercise 19 (Revision)

Simplify each of the following expressions.

(a) $2^{10}\times 2^5$ (b) $3^5\times 3$ (c) $15^6\times 15^{-2}$ (d) $x^{-4}\times x^9$ (e) $4^{-3}\times 4^{-2}$

(f) $2^{10}\div 2^5$ (g) $3^5\div 3$ (h) $15^6\div 15^{-2}$ (i) $x^{-4}\div x^9$ (j) $4^{-3}\div 4^{-2}$

(k) 2^0 (l) 45^0 (m) $(2^{10})^5$ (n) $(15^6)^{-2}$ (o) $(4^{-3})^{-2}$

(p) 7^{-2} (q) 5^{-3} (r) $81^{\frac{1}{2}}$ (s) $343^{\frac{1}{3}}$ (t) $\frac{16^{\frac{1}{2}}}{2^{-2}}$

Exercise 20

Applying

Complete the following pyramids, where each number is the product of the two numbers in the boxes underneath.

(a)

3	2	1	0

(b)

4^2	4^1	4^0	$4^{\frac{1}{2}}$

(c)

2^2	2^1	2^0	2^{-1}

Algebra and Rules of Indices

We can use the rules of indices with variables (such as x and y) as well as with numbers.

Example

$4x^2y^3 \times 3x^4y^5 = 12x^6y^8$ $20a^8b^6 \div 4a^2b^3 = 5a^6b^3$ $\frac{42p^{12}q^{15}}{6p^3q^5} = 7p^9q^{10}$

Multiply the numbers, add the indices.

Divide the numbers, subtract the indices.

Exercise 21

Skill I

Simplify the following algebraic expressions.

(a) $2x^3y^4 \times 3x^4y^2$ (b) $8a^5b^3 \times 4a^3b^6$ (c) $9p^5q^3 \times 3p^4q$

(d) $16x^{10}y^{12} \div 2x^2y^4$ (e) $24a^6b^{15} \div 4a^2b^3$ (f) $80p^{32}q^{20} \div 10p^4q^{10}$

(g) $\frac{8x^{14}y^{10}}{2x^2y^2}$ (h) $\frac{28a^{16}b^4}{7a^4b}$ (i) $\frac{100p^4q^8}{25p^2q^4}$

(j) $4g^5h^3 \times -2g^5h^3$ (k) $6s^4t^6 \times 5s^{-2}t^3$ (l) $-3u^{-5}v^7 \times -9u^3v^{-2}$

(m) $25c^8d^{-12} \div 5c^2d^3$ (n) $\frac{-32e^{-4}f^{10}}{2ef^2}$ (o) $\frac{84x^5y^{-14}z}{2x^{-2}y^2z^{-2}}$

Puzzle

Which option is the best?

Option 1: Receive £1 million today.

Option 2: Receive 1p today, then 2p tomorrow, then 4p the next day, and so on until 30 days have passed.

Challenge!

What is the answer to $\sqrt{9}$? One answer is 3, as $3^2 = 9$. But –3 is also an answer, as $(-3)^2 = 9$. How many different answers do the following calculations have?

(a) $\sqrt{16}$ (b) $\sqrt[3]{27}$ (c) $\sqrt[4]{16}$ (d) $9^{\frac{1}{2}}$ (e) $3125^{\frac{1}{5}}$

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Standard Form

The standard form is a special way to write numbers, usually very large or very small numbers.

A number is written in standard form if it has the form

$$a \times 10^n,$$

where a is a number between 1 and 10 ($1 \leq a < 10$) and n is an integer.

Example

Circle the numbers below that are written in standard form.

3.2×10^5 14.2×10^9 7.2×10^{-6} $6.21 \times 10^{0.2}$

0.2 is not an integer.

Not a number between 1 and 10.

Division, not multiplication.

0.34×10^{23} -3.2×10^{-5} 1×10^7 $8.7 \div 10^8$

Exercise 22

Skill I

Circle the numbers below that are written in standard form.

(a) 6.7×10^5 (b) $5.4 \div 10^9$ (c) 9×10^{-3} (d) 14.3×10^{12}

(e) 0.38×10^6 (f) 9.3×10^0 (g) -3.2×10^6 (h) 10×10^4

(i) 4.5×5^5 (j) 3×10^{189} (k) $6 \times 10^{1.4}$ (l) 5.6721×10^{-9}

Writing Numbers $x \geq 1$ in Standard Form

Given a number that is greater than or equal to one, this is how we write it in standard form.

a) Add a **decimal point** to the number, if there isn't already one present.

For example, the number 320 would change to be 320.0, and the number 73,000 would change to be 73,000.0.

b) Consider how many times we must **divide the number by 10** in order to reach a number a that is between 1 and 10 ($1 \leq a < 10$). We do this by counting how many times we "jump" the decimal point to the left.

4 3 2 1

73,000.0

c) Use the number between 1 and 10 and the number of times we divided by 10 in order to write the original number in standard form.

$$73{,}000 = 7.3 \times 10^4$$

Exercise 23

Write the following numbers in standard form.

(a) 54,000 (b) 234,000 (c) 8,000 (d) 3,000,000

(e) 340 (f) 43,000,000 (g) 4,328,000,000 (h) 7

(i) 98,000,000,000 (j) 823,240,000,000 (k) 10 (l) 1

Writing Numbers $0 < x < 1$ in Standard Form

Given a number between 0 and 1, this is how we write it in standard form.

a) Consider how many times we must **multiply the number by 10** in order to reach a number a that is between 1 and 10 $(1 \leq a < 10)$. We do this by counting how many times we "jump" the decimal point to the right.

1 2 3

0.00241

b) Use the number between 1 and 10 and the number of times we multiplied by 10 in order to write the original number in standard form.

$0.00241 = 2.41 \times 10^{-3}$

Remember that $10^{-3} = \frac{1}{10^3}$, therefore multiplying by 10^{-3} is the same as dividing by 10^3.

Exercise 24

I

Write the following numbers in standard form.

(a) 0.00428	(b) 0.000027	(c) 0.021	(d) 0.87
(e) 0.00000689	(f) 0.4	(g) 0.0009873	(h) 0.0901
(i) 0.00000000728	(j) 0.000000429	(k) 0.0000502	(l) 0.999999999

Exercise 25

Write the following numbers in standard form.

(a) 84,200	(b) 0.000647	(c) 5,000,000	(d) 0.005183
(e) 502,050	(f) 0.0000004	(g) 0.98	(h) 852,000,000,000
(i) 0.000201	(j) 2,384,900,000	(k) 1.03	(l) 0.03

Changing from Standard Form to an Ordinary Number

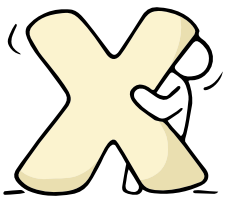
Example

(a) Write 6.962×10^6 as an ordinary number.

We must multiply 6.962 by 10 six times.

69.62	1 time
696.2	2 times
6962	3 times
69620	4 times
696200	5 times
6962000	6 times

The answer is 6,962,000.

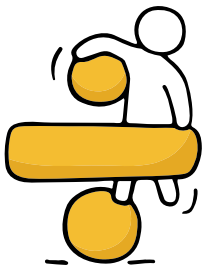


(b) Write 9.62×10^{-5} as an ordinary number.

We must divide 9.62 by 10 five times.

0.962	1 time
0.0962	2 times
0.00962	3 times
0.000962	4 times
0.0000962	5 times

The answer is 0.0000962.



Exercise 26

Write the following numbers, which are in standard form, as ordinary numbers.

(a) 8.243×10^6	(b) 4.2×10^4	(c) 8×10^5	(d) 3.704×10^8
(e) 6.25×10^{-5}	(f) 1.75×10^{-2}	(g) 8.02×10^{-3}	(h) 6.2829×10^{-7}
(i) 7×10^{-2}	(j) 9.2×10^1	(k) 3.504×10^{-1}	(l) 8.6284×10^{-6}
(m) 4×10^0	(n) 5.289×10^8	(o) 8.2×10^{-9}	(p) 8.28465×10^{10}

Almost in Standard Form

In order to multiply and divide numbers given in standard form, we must first learn how to change numbers that are almost in standard form to be in standard form.

Example

Change the following numbers, that are almost in standard form, to be in standard form.

(a) 45×10^7 ÷10 +1 $= 4.5 \times 10^8$

(b) 0.4×10^3 ×10 −1 $= 4 \times 10^2$

(c) 68×10^{-4} ÷10 +1 $= 6.8 \times 10^{-3}$

(d) 0.064×10^{-7} ×100 −2 $= 6.4 \times 10^{-9}$

Divide the number / move decimal point to the left: increase the index.
Multiply the number / move decimal point to the right: decrease the index.

Exercise 27

Change the following numbers, that are almost in standard form, to be in standard form.

(a) 61×10^7 (b) 532×10^7 (c) 0.61×10^7 (d) 0.54×10^7
(e) 61×10^{-7} (f) 532×10^{-7} (g) 0.61×10^{-7} (h) 0.54×10^{-7}
(i) 83×10^9 (j) 0.325×10^{14} (k) 7324×10^{-5} (l) 53×10^{-14}
(m) 0.025×10^8 (n) 0.0024×10^{-16} (o) 10×10^5 (p) 0.63×10^{-43}

Adding and Subtracting Numbers in Standard Form

Method 1: Equal Indices

Calculate the following, giving your answer in standard form.

(a) $(3 \times 10^{15}) + (8 \times 10^{15})$
$= 11 \times 10^{15}$ The indices are equal so we can add the 8 to the 3.
$= 1.1 \times 10^{16}$ Change the answer to be in standard form.

(b) $(7.2 \times 10^{24}) - (5 \times 10^{23})$
$= (7.2 \times 10^{24}) - (0.5 \times 10^{24})$ Change the second number so that its index is also 24.
$= 6.7 \times 10^{24}$ The indices are equal so we can subtract the 0.5 from the 7.2.

Exercise 28 C

Calculate the following, giving your answer in standard form.

(a) $(5 \times 10^{12}) + (2 \times 10^{12})$ (b) $(5 \times 10^{27}) + (2 \times 10^{27})$ (c) $(2 \times 10^{27}) + (5 \times 10^{27})$
(d) $(7 \times 10^{27}) + (5 \times 10^{27})$ (e) $(7 \times 10^{27}) - (5 \times 10^{27})$ (f) $(7 \times 10^{-7}) - (5 \times 10^{-7})$

(g) $(6 \times 10^8) + (2 \times 10^8)$ (h) $(6 \times 10^8) + (2 \times 10^7)$ (i) $(6 \times 10^7) + (2 \times 10^8)$
(j) $(6 \times 10^6) + (2 \times 10^8)$ (k) $(6 \times 10^6) - (2 \times 10^5)$ (l) $(6 \times 10^6) - (2 \times 10^4)$

(m) $(9 \times 10^{-5}) - (3 \times 10^{-5})$ (n) $(9 \times 10^{-5}) + (3 \times 10^{-5})$ (o) $(9 \times 10^{-5}) + (3 \times 10^{-6})$
(p) $(9 \times 10^{-6}) + (3 \times 10^{-5})$ (q) $(9 \times 10^{-5}) - (3 \times 10^{-7})$ (r) $(9 \times 10^{-4}) + (3 \times 10^{-6})$

(s) $(7.3 \times 10^8) + (2.6 \times 10^8)$ (t) $(7.4 \times 10^8) + (2.6 \times 10^8)$ (u) $(7.3 \times 10^8) + (2.6 \times 10^7)$
(v) $(7.3 \times 10^{108}) + (2.6 \times 10^{107})$ (w) $(7.3 \times 10^{-108}) + (2.6 \times 10^{-107})$ (x) $(7.3 \times 10^0) + (2.6 \times 10^{-1})$

Method 2: Change to ordinary numbers

Calculate the following, giving your answer in standard form.

(a) $(3.4 \times 10^5) + (7.18 \times 10^4)$

	3	4	0	0	0	0
+		7	1	8	0	0
	4	1	1	8	0	0
	1					

Answer: 4.118×10^5

(b) $(7.36 \times 10^{-3}) - (1.9 \times 10^{-4})$

	0	.	0	0	7	$^{2}\not{3}$ $^{1}6$
–	0	.	0	0	0	1 9
	0	.	0	0	7	1 7

Answer: 7.17×10^{-3}

Exercise 29

Calculate the following, giving your answers in standard form.

(a) $(2.7 \times 10^3) + (5.26 \times 10^2)$
(b) $(6.152 \times 10^5) + (7.64 \times 10^4)$
(c) $(2.09 \times 10^4) + (4 \times 10^3)$
(d) $(6.29 \times 10^6) + (3.283 \times 10^5)$
(e) $(5 \times 10^4) + (8.024 \times 10^6)$
(f) $(4.2 \times 10^7) + (1.59 \times 10^8)$

(g) $(2.7 \times 10^3) - (5.26 \times 10^2)$
(h) $(6.152 \times 10^5) - (7.64 \times 10^4)$
(i) $(2.09 \times 10^4) - (4 \times 10^3)$
(j) $(8 \times 10^6) - (4.6 \times 10^3)$
(k) $(2.07 \times 10^4) - (9.442 \times 10^3)$
(l) $(1.4 \times 10^2) - (4.6 \times 10^1)$

(m) $(2.7 \times 10^{-3}) + (5.26 \times 10^{-2})$
(n) $(6.152 \times 10^{-5}) + (7.64 \times 10^{-4})$
(o) $(2.09 \times 10^{-4}) + (4 \times 10^{-3})$
(p) $(6.4 \times 10^{-1}) + (7.28 \times 10^{-2})$
(q) $(8 \times 10^{-4}) + (7.4 \times 10^{-3})$
(r) $(1.02 \times 10^{-7}) + (7.32 \times 10^{-6})$

(s) $(5.26 \times 10^{-2}) - (2.7 \times 10^{-3})$
(t) $(7.64 \times 10^{-4}) - (6.152 \times 10^{-5})$
(u) $(4 \times 10^{-3}) - (2.09 \times 10^{-4})$
(v) $(6.43 \times 10^{-4}) - (3.82 \times 10^{-5})$
(w) $(4.6 \times 10^{-7}) - (6 \times 10^{-10})$
(x) $(3.814 \times 10^2) - (4.76 \times 10^{-2})$

Multiplying Numbers in Standard Form

Example

Calculate $(2.5 \times 10^5) \times (6 \times 10^3)$, giving your answer in standard form.

Answer: $(2.5 \times 10^5) \times (6 \times 10^3)$

$= (2.5 \times 6) \times (10^5 \times 10^3)$ Rearrange (the order in multiplication sums doesn't matter).
$= 15 \times 10^{5+3}$ Multiply the numbers; use rules of indices to add the indices.
$= 15 \times 10^8$ This is almost in standard form; we must divide the 15 by 10 to correct...
$= 1.5 \times 10^9$ Final answer (divide by 10 so add 1 to the index).

Exercise 30

Calculate the following, giving your answer in standard form.

(a) $(2 \times 10^5) \times (4 \times 10^3)$
(b) $(2 \times 10^5) \times (8 \times 10^3)$
(c) $(2 \times 10^5) \times (4 \times 10^{-3})$
(d) $(1.3 \times 10^6) \times (2 \times 10^8)$
(e) $(4 \times 10^9) \times (3 \times 10^4)$
(f) $(7 \times 10^{14}) \times (6 \times 10^2)$
(g) $(4.6 \times 10^7) \times (3 \times 10^4)$
(h) $(7.5 \times 10^{14}) \times (8 \times 10^{23})$
(i) $(7 \times 10^7) \times (3.8 \times 10^9)$
(j) $(6 \times 10^{-4}) \times (6 \times 10^{14})$
(k) $(3 \times 10^6) \times (2 \times 10^{-2})$
(l) $(1 \times 10^{-4}) \times (8 \times 10^{-3})$
(m) $(2.4 \times 10^4) \times (1.5 \times 10^7)$
(n) $(5.3 \times 10^{14}) \times (6.2 \times 10^3)$
(o) $(5.13 \times 10^{-6}) \times (7.4 \times 10^2)$

Dividing Numbers in Standard Form

Example

Calculate $(4 \times 10^8) \div (5 \times 10^2)$, giving your answer in standard form.

Answer: $(4 \times 10^8) \div (5 \times 10^2)$

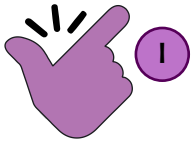
$= (4 \div 5) \times (10^8 \div 10^2)$ Rearrange

$= 0.8 \times 10^{8-2}$ Divide the numbers; use rules of indices to subtract the indices.

$= 0.8 \times 10^6$ This is almost in standard form; we must multiply the 0.8 by 10 to correct...

$= 8 \times 10^5$ Final answer (multiply by 10 so subtract 1 from the index).

Exercise 31



Calculate the following, giving your answer in standard form.

(a) $(8 \times 10^8) \div (4 \times 10^2)$ (b) $(4 \times 10^8) \div (8 \times 10^2)$ (c) $(8 \times 10^8) \div (4 \times 10^{-2})$

(d) $(3.6 \times 10^6) \div (3 \times 10^3)$ (e) $(6.4 \times 10^{12}) \div (4 \times 10^3)$ (f) $(9.3 \times 10^5) \div (3 \times 10^5)$

(g) $(8.6 \times 10^7) \div (2 \times 10^2)$ (h) $(7.5 \times 10^{14}) \div (5 \times 10^{20})$ (i) $(1 \times 10^8) \div (3 \times 10^4)$

(j) $(4.2 \times 10^{-3}) \div (3 \times 10^4)$ (k) $(2 \times 10^5) \div (5 \times 10^{-2})$ (l) $(1 \times 10^{-4}) \div (8 \times 10^{-2})$

(m) $(2.4 \times 10^8) \div (4 \times 10^3)$ (n) $(5.25 \times 10^{50}) \div (1.5 \times 10^{10})$ (o) $(2 \times 10^{-5}) \div (8 \times 10^{-15})$

Exercise 32

Calculate the following, giving your answer in standard form.

(a) $(6 \times 10^4) + (4 \times 10^3)$ (b) $(6 \times 10^4) - (4 \times 10^3)$ (c) $(6 \times 10^4) \times (4 \times 10^3)$

(d) $(6 \times 10^4) \div (4 \times 10^3)$ (e) $\frac{6 \times 10^4}{4 \times 10^3}$ (f) $\frac{(6 \times 10^4) + (4 \times 10^3)}{4 \times 10^3}$

(g) $(8.4 \times 10^6) + (2 \times 10^2)$ (h) $(8.4 \times 10^6) - (2 \times 10^2)$ (i) $(8.4 \times 10^6) \times (2 \times 10^2)$

(j) $(8.4 \times 10^6) \div (2 \times 10^2)$ (k) $(8.4 \times 10^6) \times 5$ (l) $(8.4 \times 10^6) + (2 \times 10^{-2})$

Challenge!

Use your calculator to check your answers to Exercise 44, making sure that your calculator shows the answer in standard form.

Exercise 33

The Earth is more or less spherical.

(a) The radius of the Earth is 6,378.1 km. Calculate the circumference of the Earth, writing your answer in standard form correct to 3 significant figures.

(b) The surface area of the whole Earth is approximately 5.112×10^8 km². The oceans cover approximately 3.618×10^8 km² of surface area and the rest of the surface area is covered by land. Calculate the amount of surface area of the Earth covered by land, giving your answer in standard form.

Investigation

In the standard form $a \times 10^n$, the number a must be between 1 and 10 ($1 \leq a < 10$) and n is an integer.
Is it therefore possible to write negative numbers using standard form? Use the internet to investigate the answer to this question.

Exercise 34

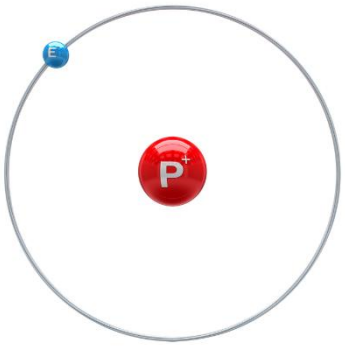
The Millennium Stadium in Cardiff has enough seats for 74,500 people.
The population of Wales would fill the Millennium Stadium 41 times.

Use this information to calculate the approximate population of Wales.
Give your answer in standard form correct to 3 significant figures.

Exercise 35

The mass of one hydrogen atom is about

$$1.66 \times 10^{-24} \text{ kg}.$$



One litre of air contains approximately 2.51×10^{22} hydrogen atoms.

(a) What is the mass of hydrogen in one litre of air?
Give your answer in standard form.

(b) Express your answer to (a) without using standard form.

Exercise 36

Currently, Mr. Ben's rice store contains 2,700 kg of basmati rice.

(a) Write 2,700 kg in grams.
Give your answer in standard form.

(b) One grain of rice has mass 0.03 g.
Write the mass of one grain of rice in standard form.

(c) How many grains of rice are in 2,700 kg of rice?
Write your answer in standard form.

Challenge!

Investigate using the internet the meaning of the word googol.
Use your findings to write the number 50 googol in standard form.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Plotting Graphs

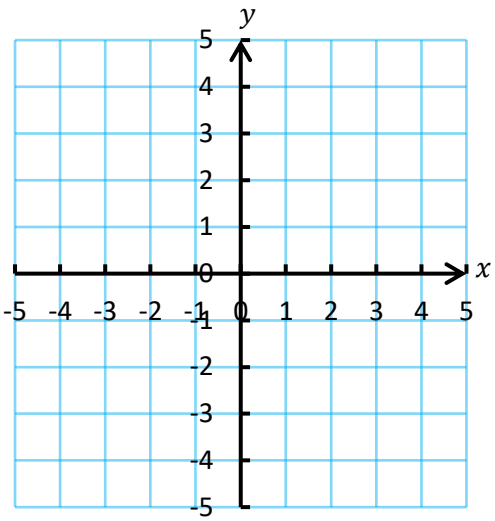
In year 9, you learnt how to plot linear equations of the form $y = mx + c$. In this chapter, we will learn how to plot equations that have different forms.

Revision F

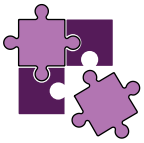
Exercise 37

On the graph paper on the right, plot the equation $y = 2x - 3$.

x	0	1	2	3
y				



Quadratic Graphs



Equations of the form $y = ax^2 + bx + c$ are **quadratic** equations.

The word quadratic refers to the x^2 term in the equation, a **squared** term.
Quadratic graphs have a U or ∩ shape.

Skill I

Exercise 38

Complete the following table to note whether each equation is linear or quadratic.

Equation	Type	Equation	Type
$y = 4x - 2$	Linear	$y = 3x^2 - 4x + 2$	Quadratic
$y = 2x^2 + 4x + 7$		$y = 5x^2 - 4x + 6$	
$y = 2x + 5$		$y = -4x + 2$	
$y = 4x + 7x^2 - 3$		$y = 2 + 6x + 3x^2$	
$y = 3 + 4x$		$y = 6x$	
$y = 5$		$y = 4 + 3x^2$	
$y = -3x^2 + 3$		$y = x(x + 2)$	

Exercise 39

Revision F

(a) 4 × 4	(b) –4 × –4	(c) 4 × –4	(d) –4 × 4
(e) 4 + 2	(f) 4 + –2	(g) –4 + 2	(h) –4 + –2
(i) 4 – 2	(j) –4 – 2	(k) 4 – –2	(l) –4 – –2
(m) 7^2	(n) $(-7)^2$	(o) $7^2 + 2 \times 7$	(p) $(-7)^2 + 2 \times -7$

Substitution

To plot a quadratic graph, we can **substitute** values into the associated equation.

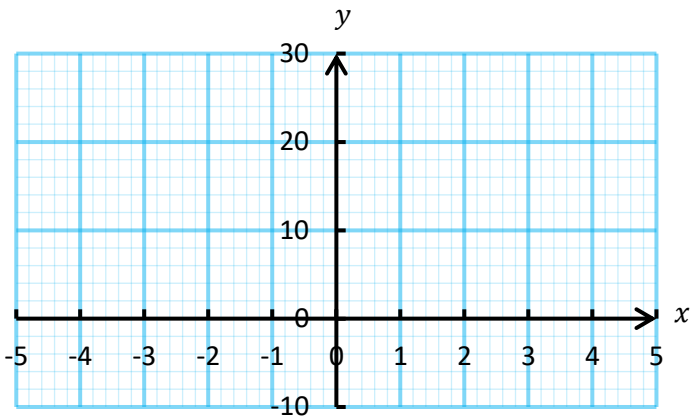
Exercise 40

C

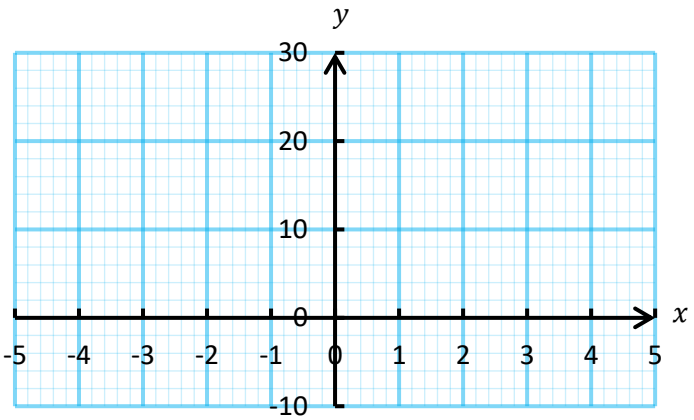
Complete the following tables by substitution, before plotting a smooth curve on the graph paper.
(No calculators are allowed.)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
x^2											
$x^2 - 5$											

(a) $y = x^2$

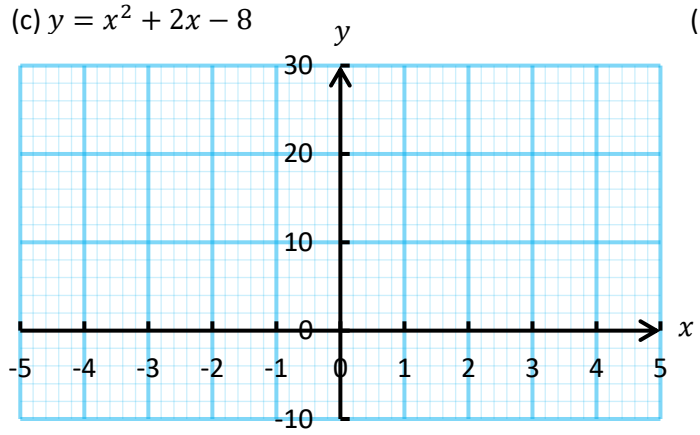


(b) $y = x^2 - 5$

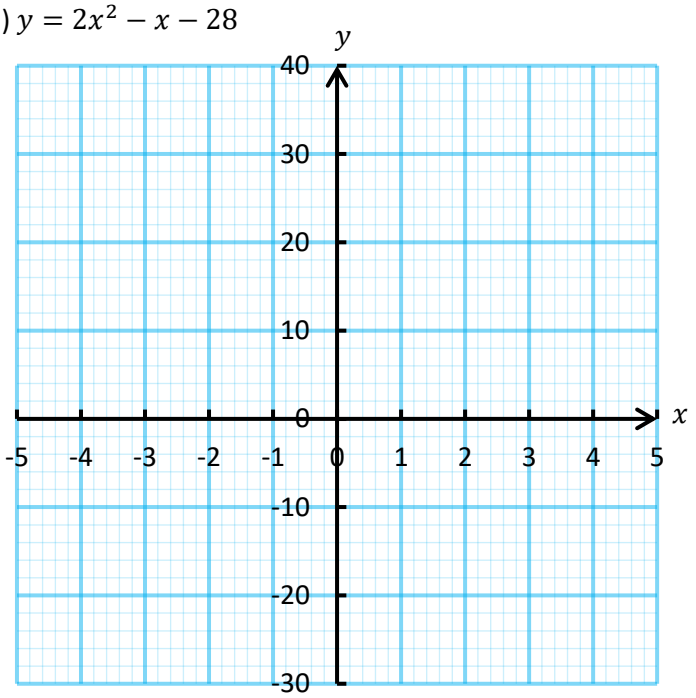


x	–5	–4	–3	–2	–1	0	1	2	3	4	5
x^2											
$2x$											
$x^2 + 2x - 8$											
x^2											
$2x^2$											
$2x^2 - x - 28$											

(c) $y = x^2 + 2x - 8$



(d) $y = 2x^2 - x - 28$



Exercise 41

Fill the blanks in the tables below. Then, in your books, plot suitable graphs for the equations.

(a)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = x^2 + 4$	29	20		8	5	4	5	8	13		29

(b)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = 3x^2 - 10$	65	38	17		–7	–10	–7		17	38	65

(c)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = 4x^2 + x - 7$	88		26	7	–4	–7	–2	11		61	98

Exercise 42

Plot appropriate graphs for the following equations.

(a) $y = x^2 - 4x$ (b) $y = x^2 - 3x + 4$ (c) $y = 3x - x^2$

(d) $y = x^2 - x - 5$ (e) $y = 5 - 2x^2$ (f) $y = 3x^2 + 4x + 2$

(g) $y = 15 - x^2 + 3x$ (h) $y = 4x^2 - x + 7$ (i) $y = -2x^2 + 5x - 6$

Recognising and sketching graphs of the form $y = ax^2 + b$

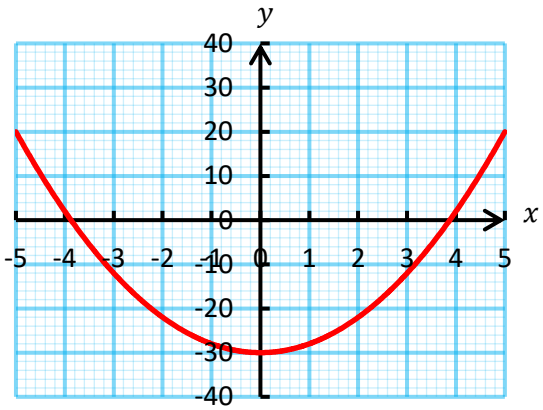
Exercise 43

You will need to use the website www.desmos.com/calculator to complete this exercise.

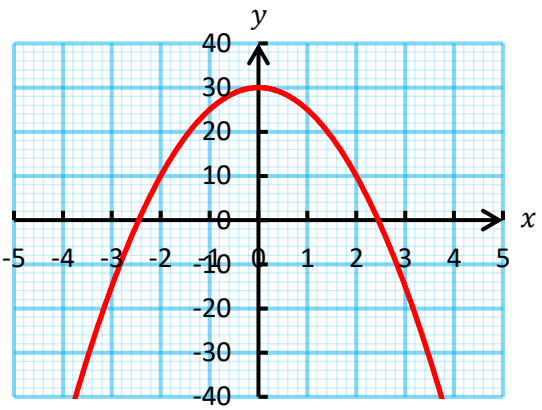
Type $y = ax^2 + b$ into the box. When the "add slider" option appears click "all".
What effect does changing the values of a and b have on the graph?

Exercise 44

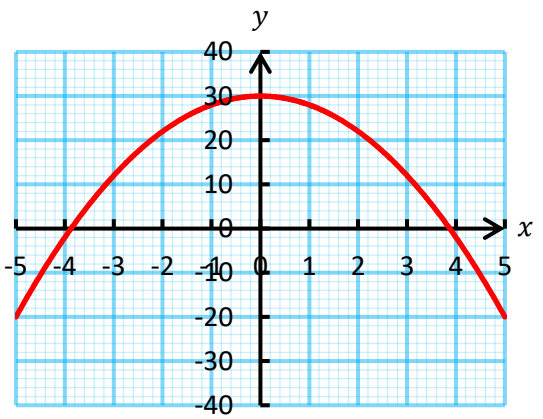
Pair the graphs and the equations.



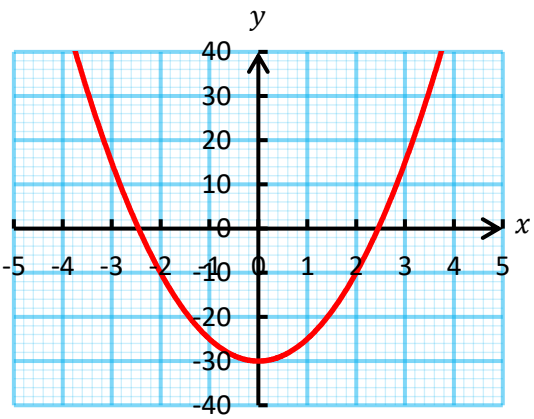
$y = -5x^2 + 30$



$y = 5x^2 - 30$



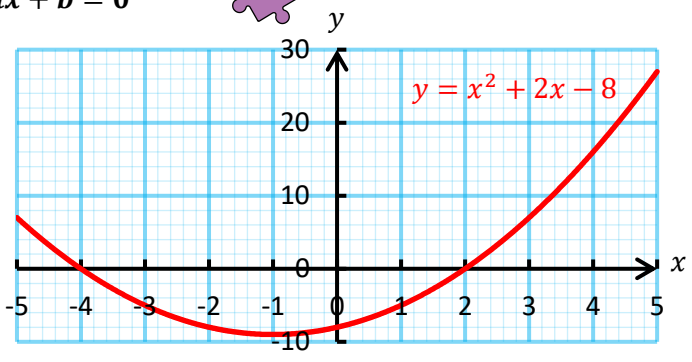
$y = -2x^2 + 30$



$y = 2x^2 - 30$

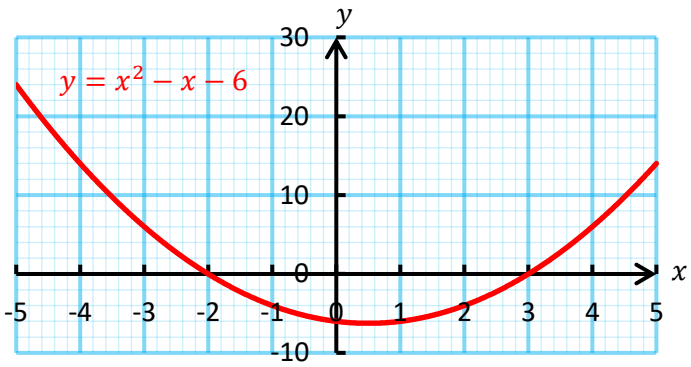
Graphical method of solving equations of the form $x^2 + ax + b = 0$

Consider the graph shown on the right for the equation $y = x^2 + 2x - 8$. To solve the equation $x^2 + 2x - 8 = 0$, we can use the graph to see where the graph crosses the x–axis (this is where the function $x^2 + 2x - 8$ is zero). We see that the graph crosses the x–axis at the points where $x = -4$ and $x = 2$, therefore the solutions to the equation $x^2 + 2x - 8 = 0$ are $x = -4$ and $x = 2$.

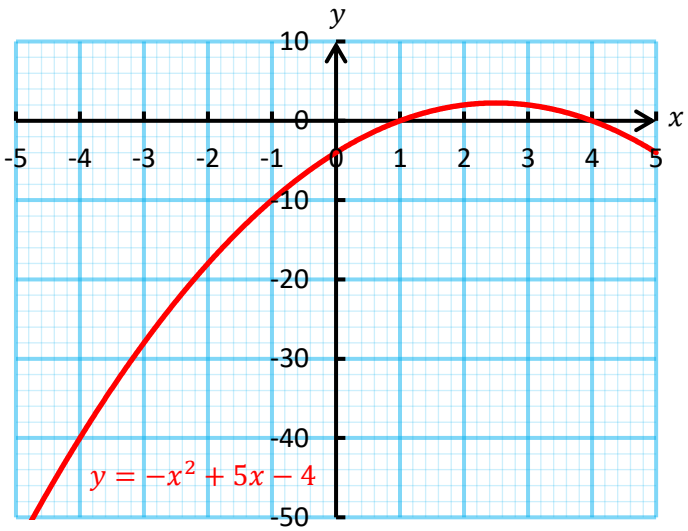


Exercise 45 I

(a) Use the graph below to solve the equation $x^2 - x - 6 = 0$.

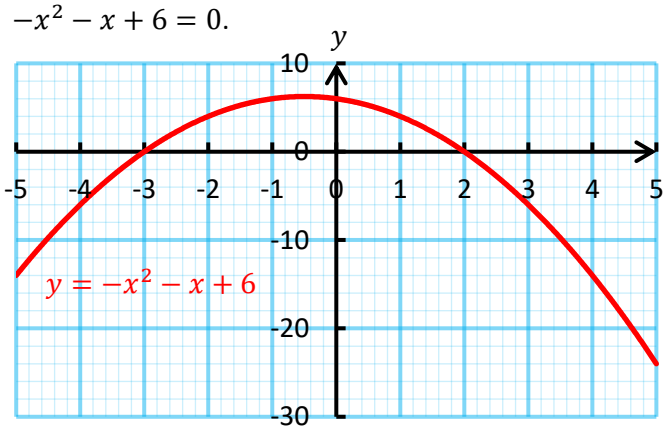


(b) Use the graph below to solve the equation $-x^2 + 5x - 4 = 0$.

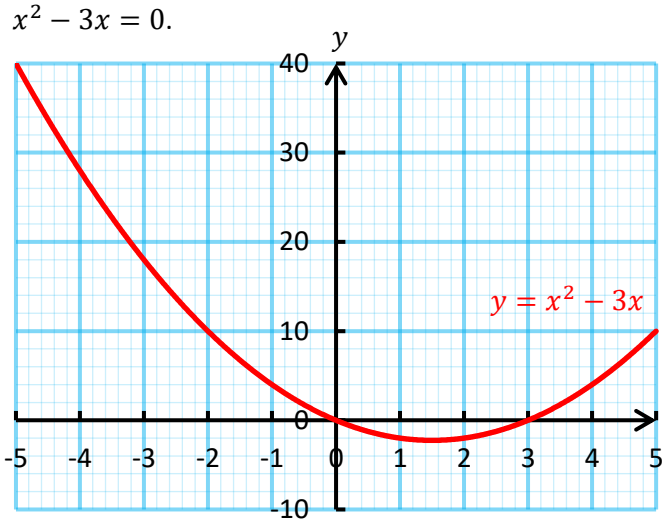


(c) Use the graph above to solve the equation $x^2 - x - 6 = 10$.
Give your answers correct to one decimal place.

(d) Use the graph below to solve the equation $-x^2 - x + 6 = 0$.



(e) Use the graph below to solve the equation $x^2 - 3x = 0$.



(f) Use the graph above to solve the equation $-x^2 - x + 6 = -10$.
Give your answers correct to one decimal place.

(g) Use the graph above to solve the equation $x^2 - 3x = 4$.

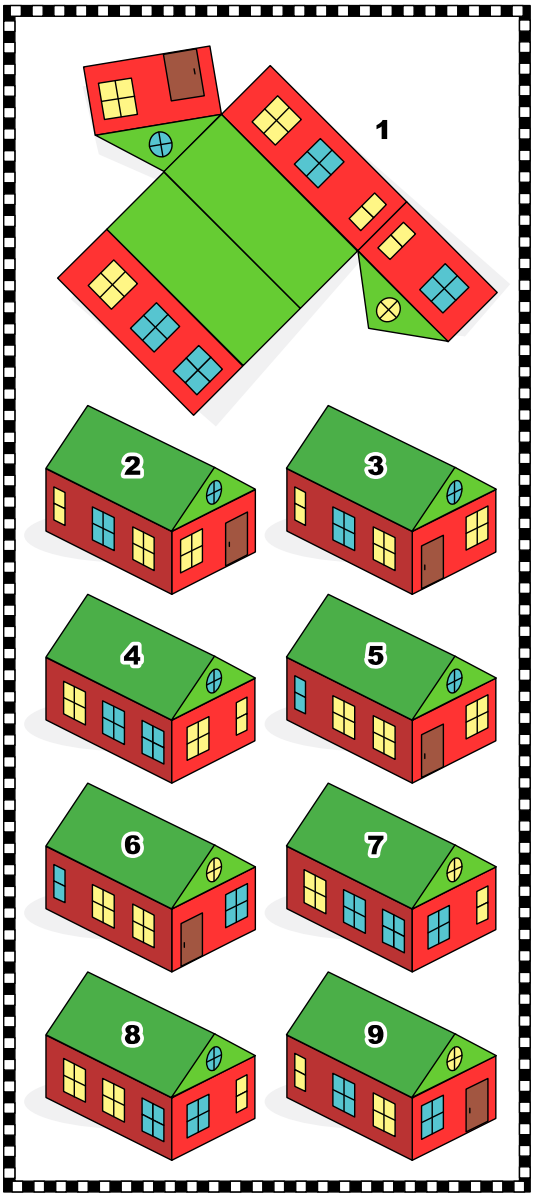
(h) (i) By drawing a suitable graph, solve the equation $x^2 + 3x - 4 = 0$.
(ii) Use your graph from part (i) to solve the equation $x^2 + 3x - 4 = 5$.
Give your answers correct to one decimal place.

Evaluation

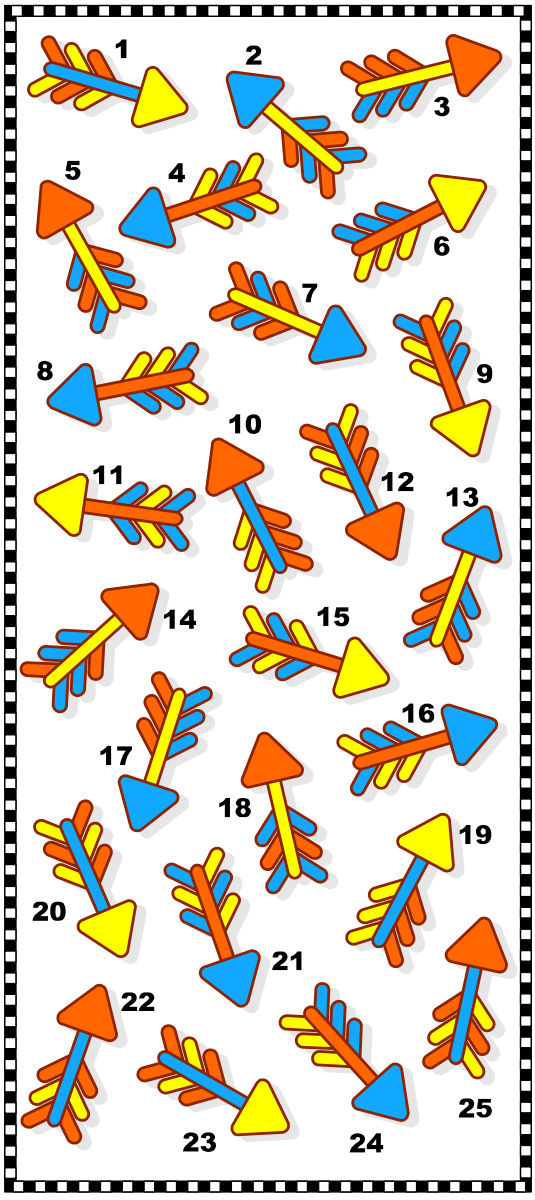
Key words	Corrections	I am happy with...	I need to revise...

Puzzles

If 1 is folded to make a house, which of 2–9 are formed?



Which two arrows are the same?



Powers and Roots

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test	Correct in the test?
I know how to change between **ordinary numbers** and numbers written in **index form**.			1, 8	
I know how to use the **multiplication rule** $n^a \times n^b = n^{a+b}$.			2	
I know how to use the **division rule** $n^a \div n^b = n^{a-b}$.			2, 8	
I know how to use the **zeroth index rule** $n^0 = 1$.			2, 8	
I know how to use the rule where **a power is raised to another index**, $(n^a)^b = n^{a\times b}$.			2	
I know how to use the **negative index rule** $n^{-a} = \frac{1}{n^a}$.			2, 8	
I know how to use the **unitary fraction index rule** $n^{\frac{1}{a}} = \sqrt[a]{n}$.			2, 8	
I can **simplify algebraic expressions** using the rules of indices.			3	
I can write numbers in **standard form**.			4	
I can **add and subtract** numbers written in standard form.			5	
I can **multiply and divide** numbers written in standard form.			5	
I can **solve problems** using standard form.			6, 7	
I can **plot quadratic graphs**.			9	
I know how to **recognise and sketch quadratic graphs**.			9, 10	
I know how to use a **graphical method** to solve quadratic equations.			9	

The Mathematics Department

10

Measuring Shapes 3

Name:

Contents

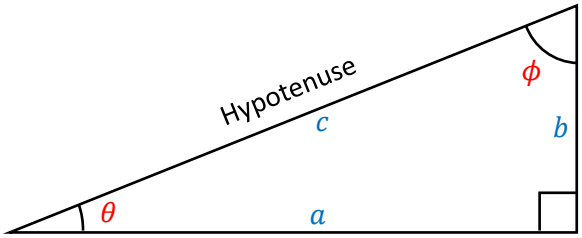
Chapter	Mathematics	Page Number
Trigonometry of Right-Angled Triangles	Labelling triangles. Calculating the length of a side. Calculating the size of an angle.	3
Transformations: Enlargement	Positive scale factor. Fractional scale factor. Centre of enlargement. Finding the centre of enlargement. **Negative scale factor**.	11
Perimeter and Area of Composite Shapes	Composite shapes. **Length of an arc and the area of a sector**.	17

Trigonometry

Right-Angled Triangles

Any right-angled triangle has:

- One angle that is a right angle, or 90°;
- Two acute angles θ and ϕ;
- A hypotenuse c, which is always opposite the right angle;
- Two sides a and b which are shorter than the hypotenuse.



In year 9, we introduced Pythagoras' Theorem, which connects the lengths a, b and c:

$$c^2 = a^2 + b^2$$

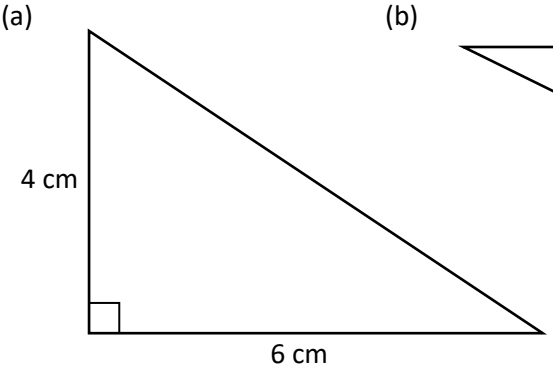
Given the length of any two sides in a triangle, we can use Pythagoras' Theorem to calculate the length of the third side.

Exercise 1

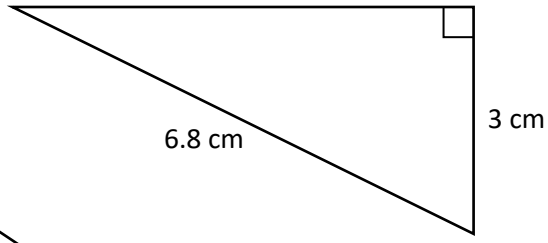
Revision

Use Pythagoras' Theorem to calculate the length of the third side in these right-angled triangles.
Round off your answers to two decimal places.

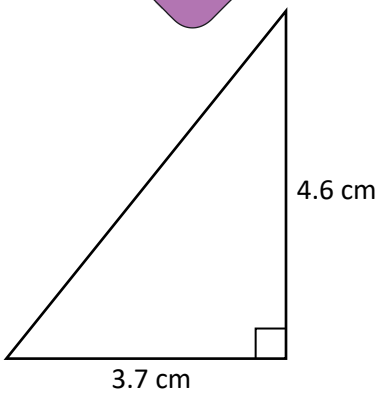
(a)



(b)



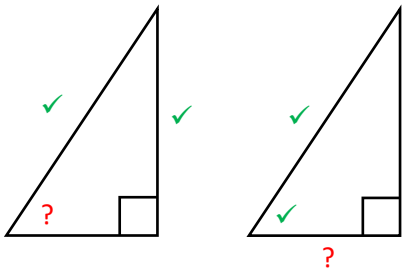
(c)



What is trigonometry?

Trigonometry is used for:

- Calculating the size of one of the acute angles in a right-angled triangle, given the length of any two sides;
- Calculating the length of one of the sides in a right-angled triangle given the length of one other side and the size of one acute angle.



How?

Trigonometry uses the relationship between the size of the angles and the lengths of the sides in any right-angled triangle.

Exercise 2

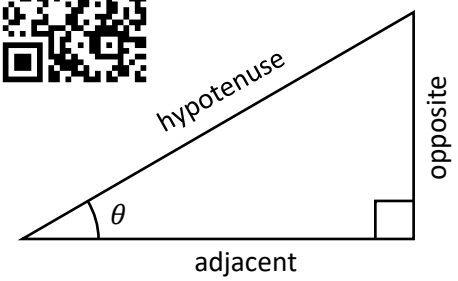
Applying

Draw any three right-angled triangles where one of the angles measures 30°.
Measure the length of the hypotenuse and the length of the side opposite the 30° angle.
What do you notice?

Labelling the sides of a right-angled triangle

Let θ represent the size of one of the acute angles in a right-angled triangle.
We follow these conventions when labelling the sides of the triangle.

- The **opposite** is the side opposite the angle θ.
- The **hypotenuse** is the side opposite the right angle.
- The **adjacent** is the side left over (it's close to the angle θ).

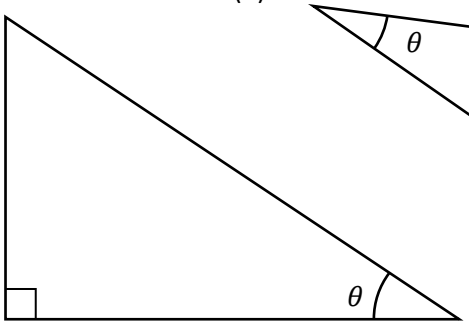


Exercise 3

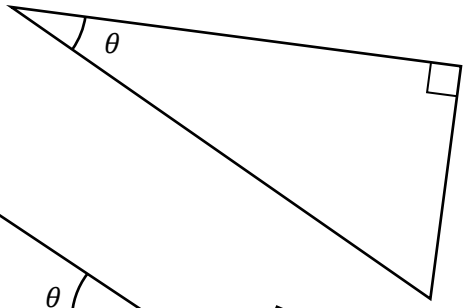
Skill I

Label the sides of these triangles using the words "opposite", "hypotenuse" and "adjacent".

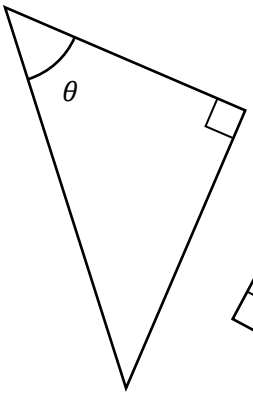
(a)



(b)

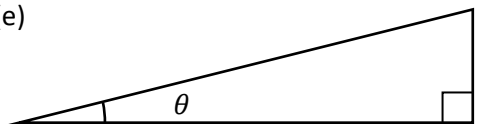


(c)

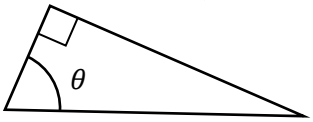


(d)

(e)



(f)



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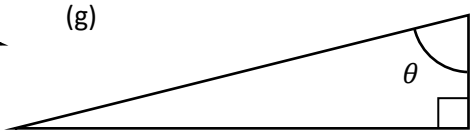


Sin, Cos, Tan

For a specific angle θ, we define the **sin**, **cos** and **tan** of the angle as follows.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

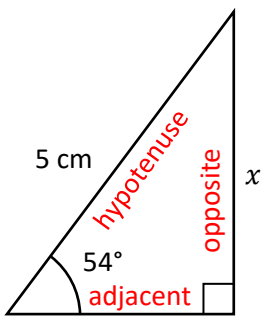
How to remember the formula...

In mathematics, we worship the King SOHCAHTOA.

$$S^{O}_{H}\ C^{A}_{H}\ T^{O}_{A}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Finding lengths using trigonometry



Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x.

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we want to calculate the length of the **opposite** (x) side, and we know the length of the **hypotenuse** (5 cm). The trigonometric ratio that uses the words **opposite** and **hypotenuse** is sin, therefore we must use the formula

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

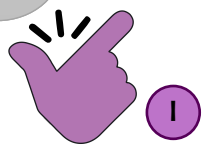
in this question. By substituting values into the formula, we obtain

$$\sin 54° = \frac{x}{5}.$$

By multiplying both sides of the equation by 5, we obtain

$$x = 5 \times \sin 54°.$$

x on **top** of the fraction leads to a **multiplication** sum in the answer.

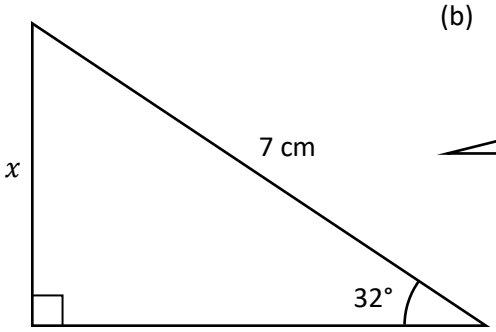


By typing this sum on a calculator, we find that $x = 4.05$ cm, correct to two decimal places.

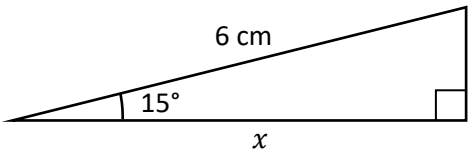
Exercise 4

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x.

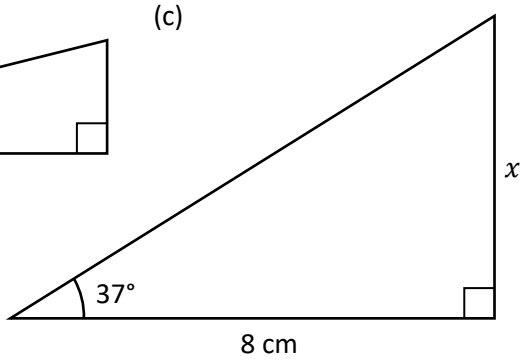
(a)



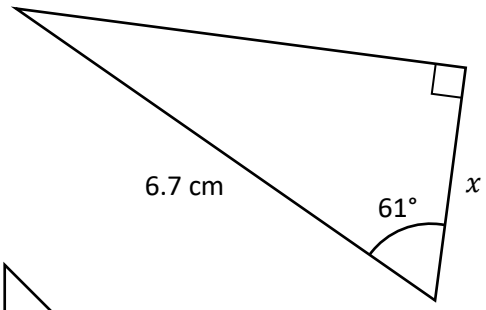
(b)



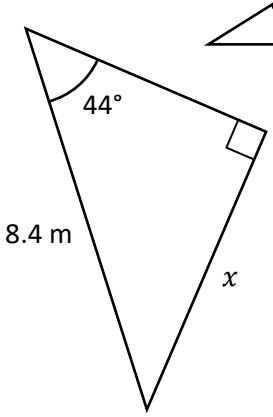
(c)



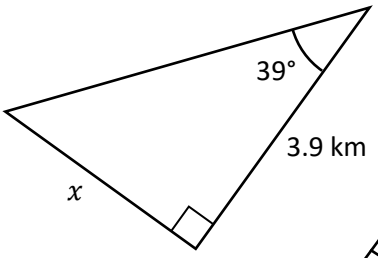
(d)



(e)



(f)



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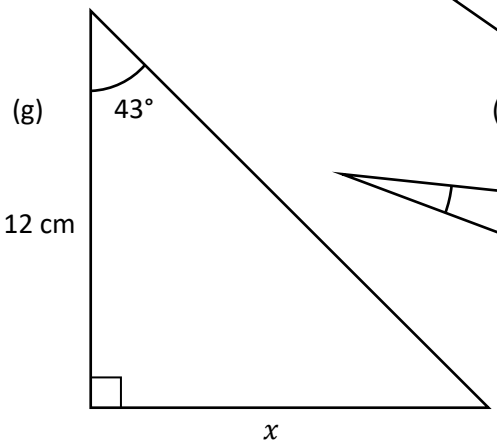

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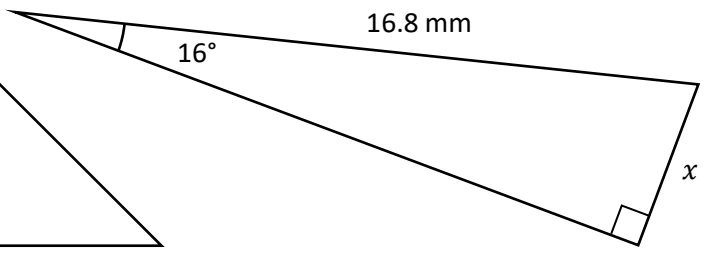
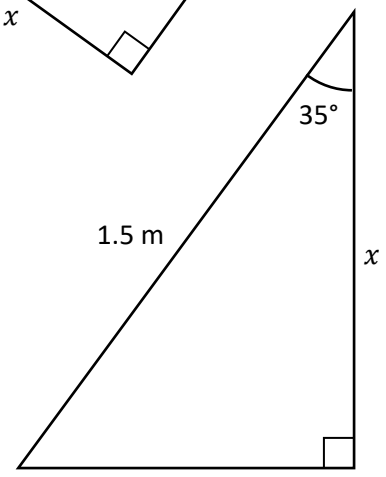
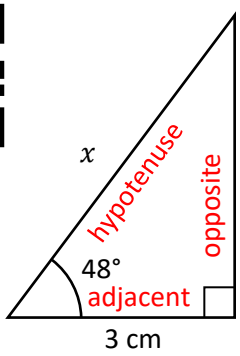


(i)



Example

Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x.



To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we know the length of the **adjacent** (3 cm), and we want to calculate the length of the **hypotenuse** (x). The trigonometric ratio that uses the words **adjacent** and **hypotenuse** is cos, therefore we must use the formula

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

in this question. By substituting values into the formula, we obtain

$$\cos 48° = \frac{3}{x}.$$

x on the **bottom** of the fraction leads to a **division** sum in the answer.

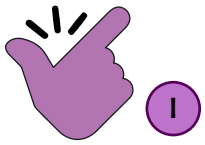
By multiplying both sides of the equation by x, we obtain

$$x \times \cos 48° = 3.$$

By dividing both sides of the equation by $\cos 48°$, we obtain

$$x = 3 \div \cos 48°.$$

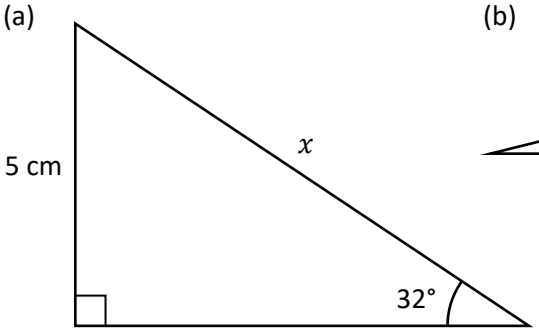
By typing this sum on a calculator, we find that $x = 4.48$ cm, correct to two decimal places.



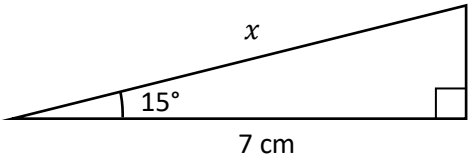
Exercise 5

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x.

(a)



(b)



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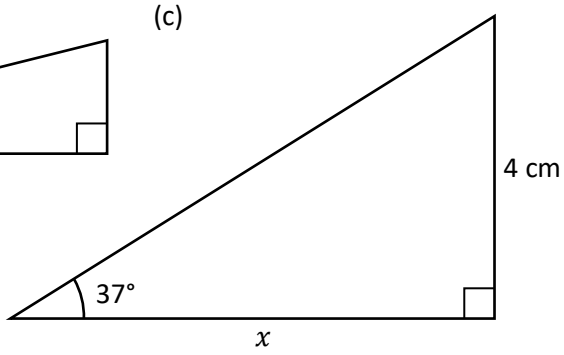

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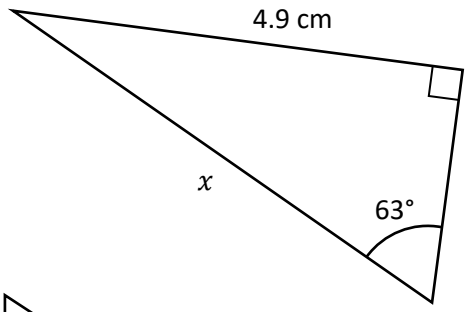

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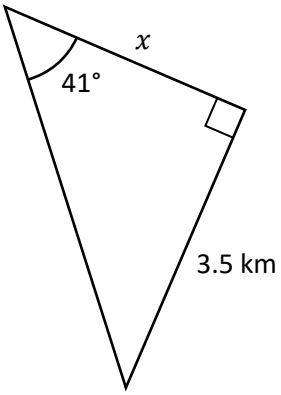
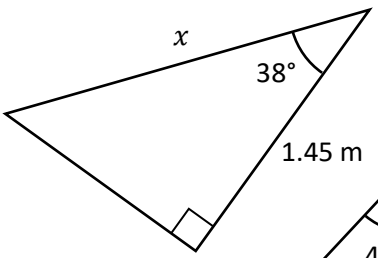
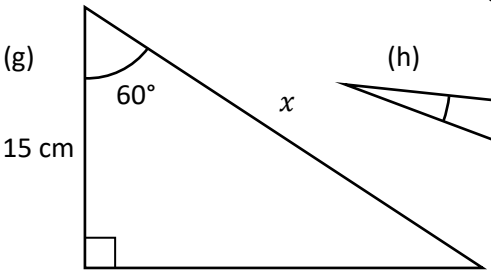


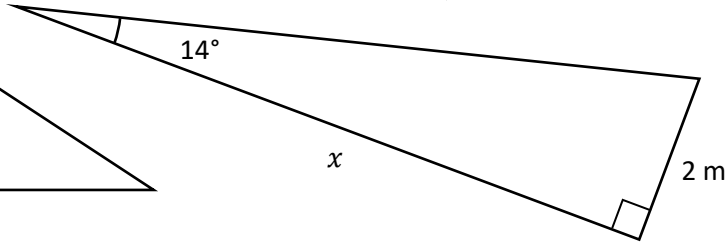
(f)



(g)



(h)



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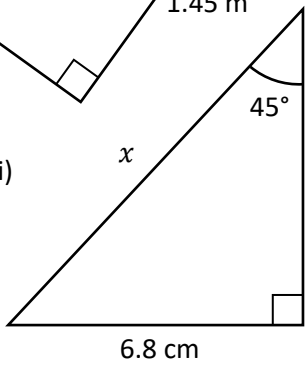
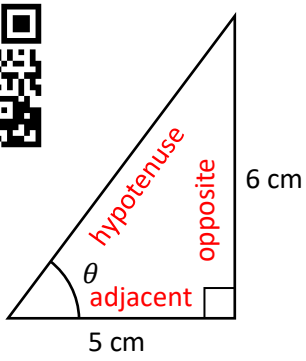


Finding angles using trigonometry

Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the size of the angle θ.



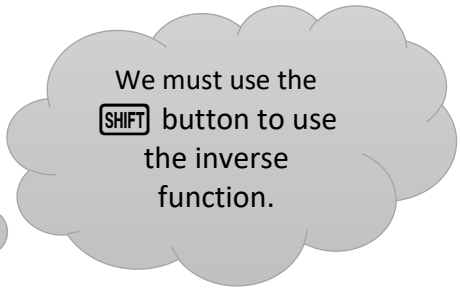
To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we know the length of the **opposite** (6 cm) and the length of the **adjacent** (5 cm). The trigonometric ratio that uses the words **opposite** and **adjacent** is tan, therefore we must use the formula

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

in this question. By substituting values into the equation, we obtain

$$\tan\theta = \frac{6}{5}.$$

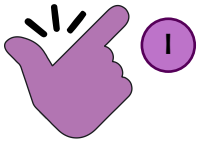


To find the size of the angle θ we must use the inverse tan function:

$$\theta = \tan^{-1}\left(\frac{6}{5}\right).$$

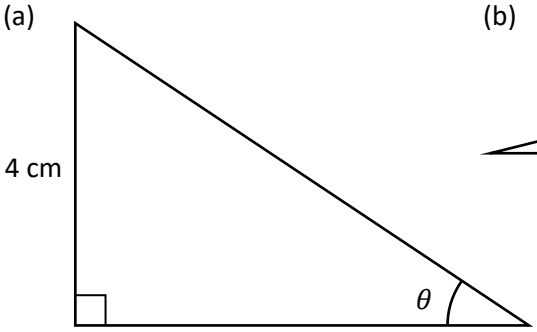
By typing this sum on a calculator, we find that $\theta = 50.19°$, correct to two decimal places.

Exercise 6

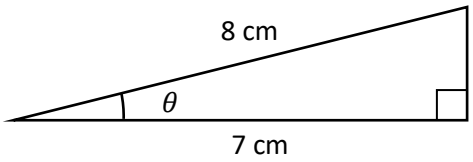


For the following right-angled triangles, calculate the size of the angle θ.

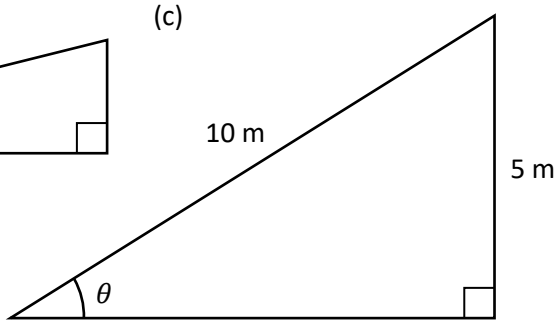
(a)



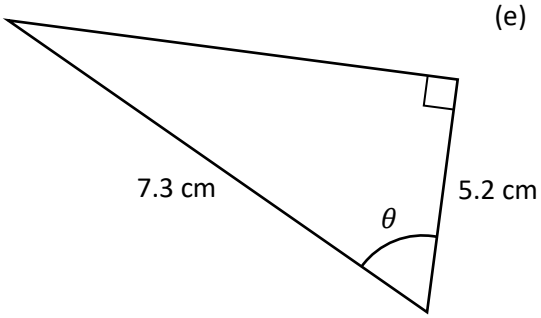
(b)



(c)



(d)



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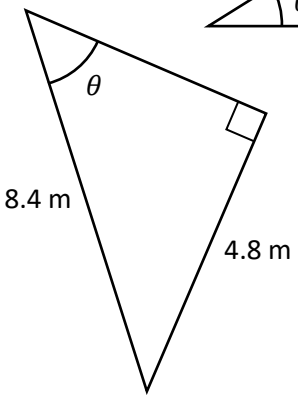

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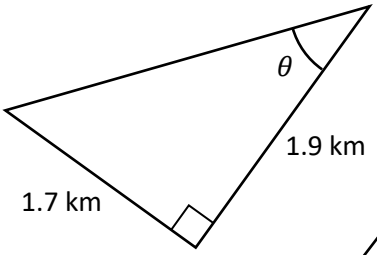
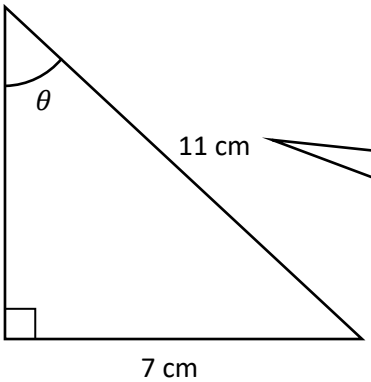
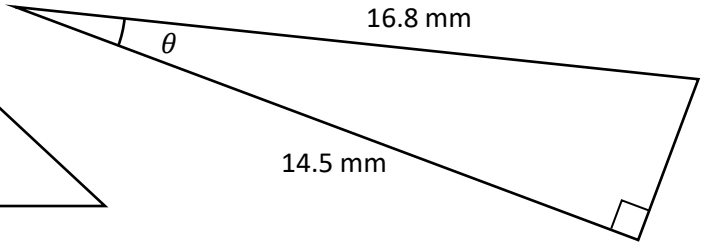


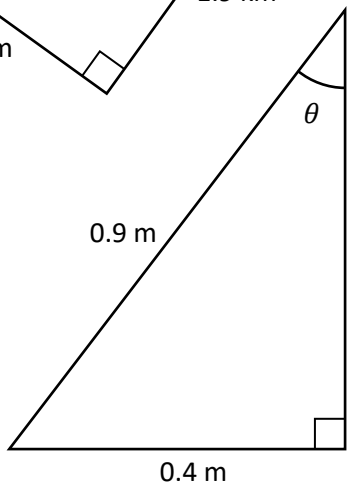
(g)



(h)



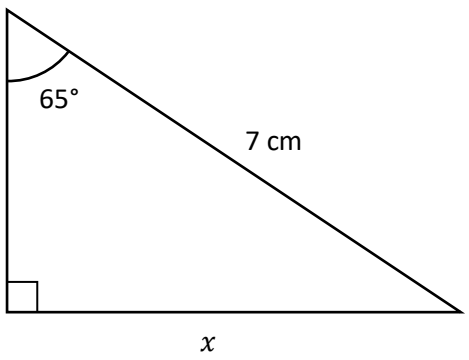
(i)



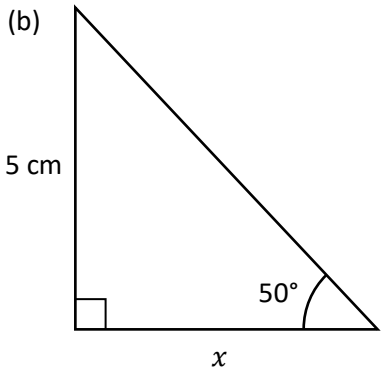
Exercise 7

For the following right-angled triangles, calculate the length of the side x, or the size of the angle θ. Round off your answers correct to two decimal places.

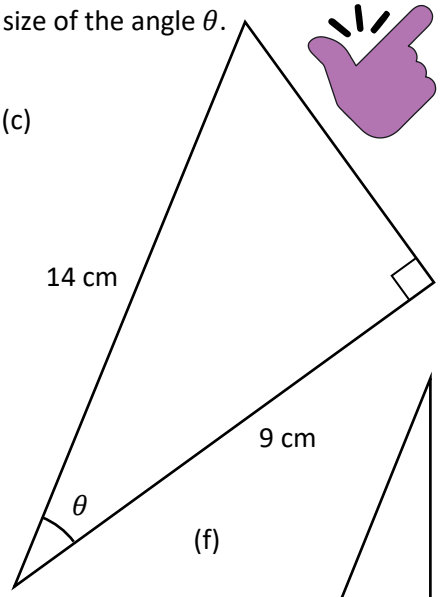
(a)



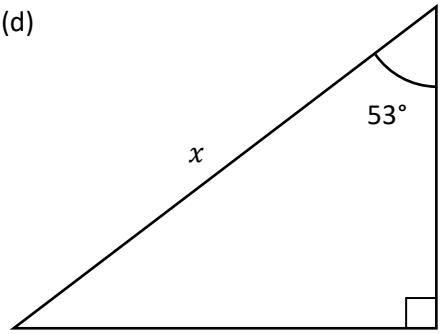
(b)



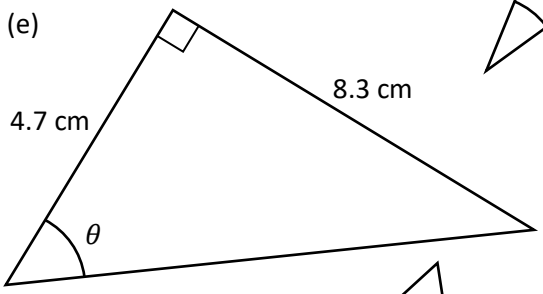
(c)



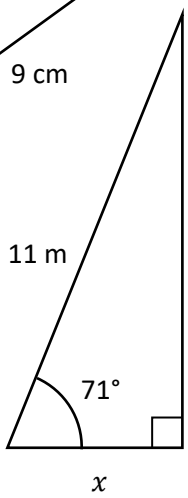
(d)



(e)



(f)



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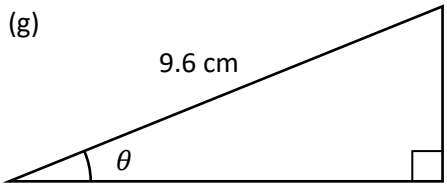

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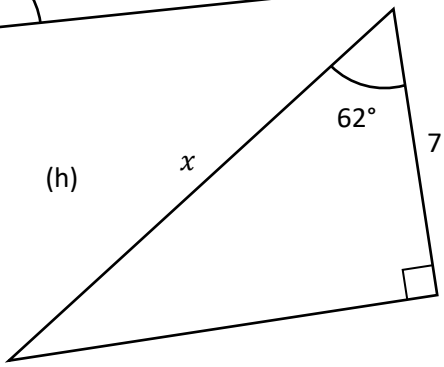
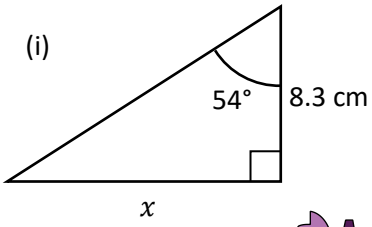


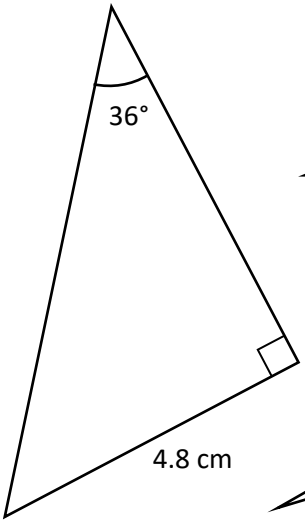
(i)



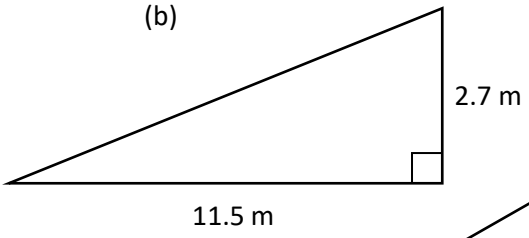
Exercise 8

For the following right-angled triangles, find the size of **every** missing angle and the length of **every** missing side.

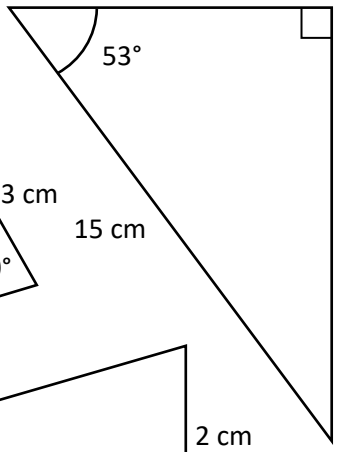
(a)



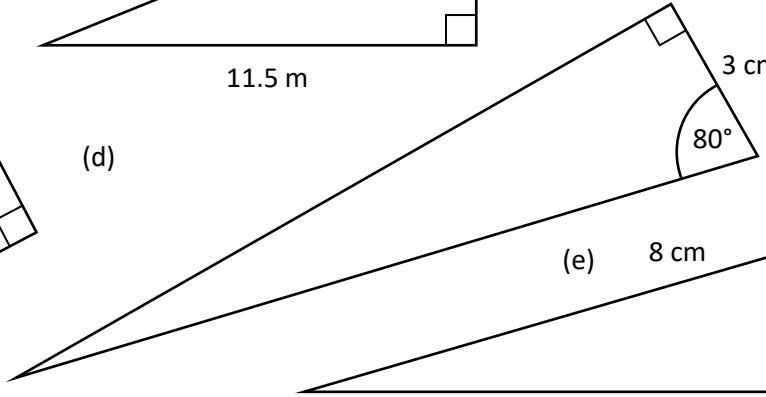
(b)



(c)



(d)

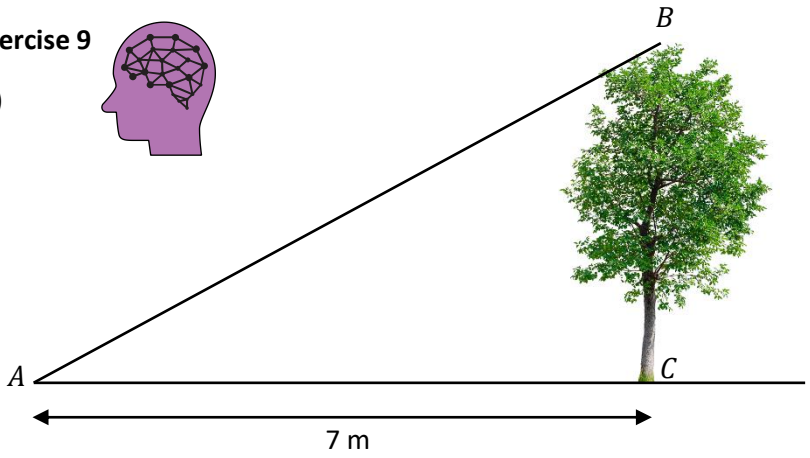


(e)

Exercise 9

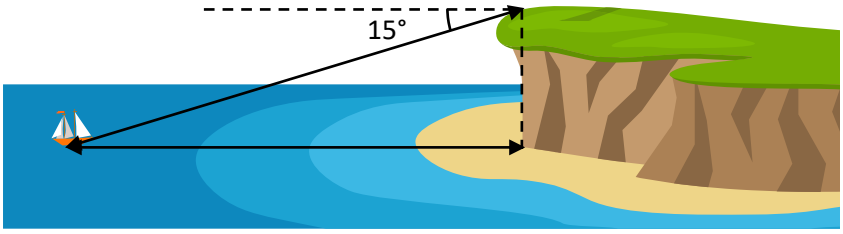
Applying 1

(a)



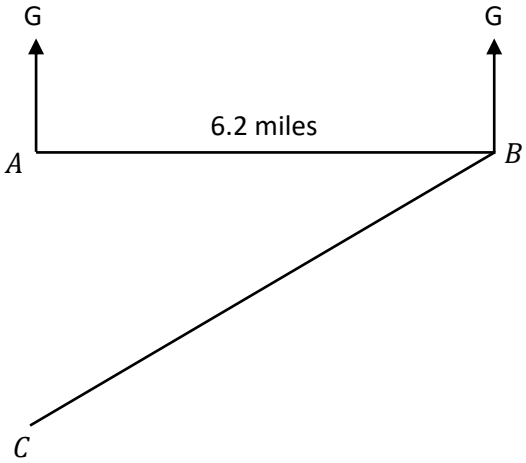
The vertical height of a tree is 3.2 metres. The horizontal distance from point A to the bottom of the tree is 7 metres. Calculate the angle of elevation of the top of the tree from the point A.

(b)



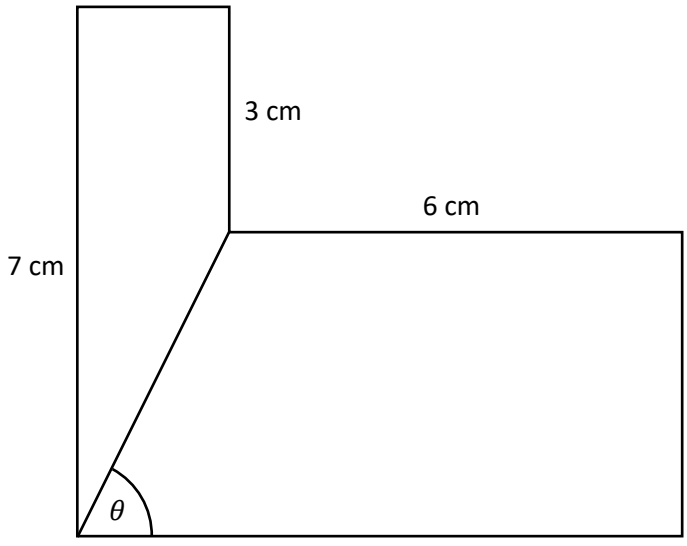
From the top of a vertical cliff, the angle of depression of a sailing boat is 15°. If the sailing boat is 200 m from the bottom of the cliff, calculate the height of the cliff above sea level.

(c)



A ship leaves a port and sails 6.2 miles at a bearing of 090° to reach B. Then it turns and sails at a bearing of 224° until it reaches a point C, which is south of the port A. Calculate the distance between the point C and the port A.

(d)

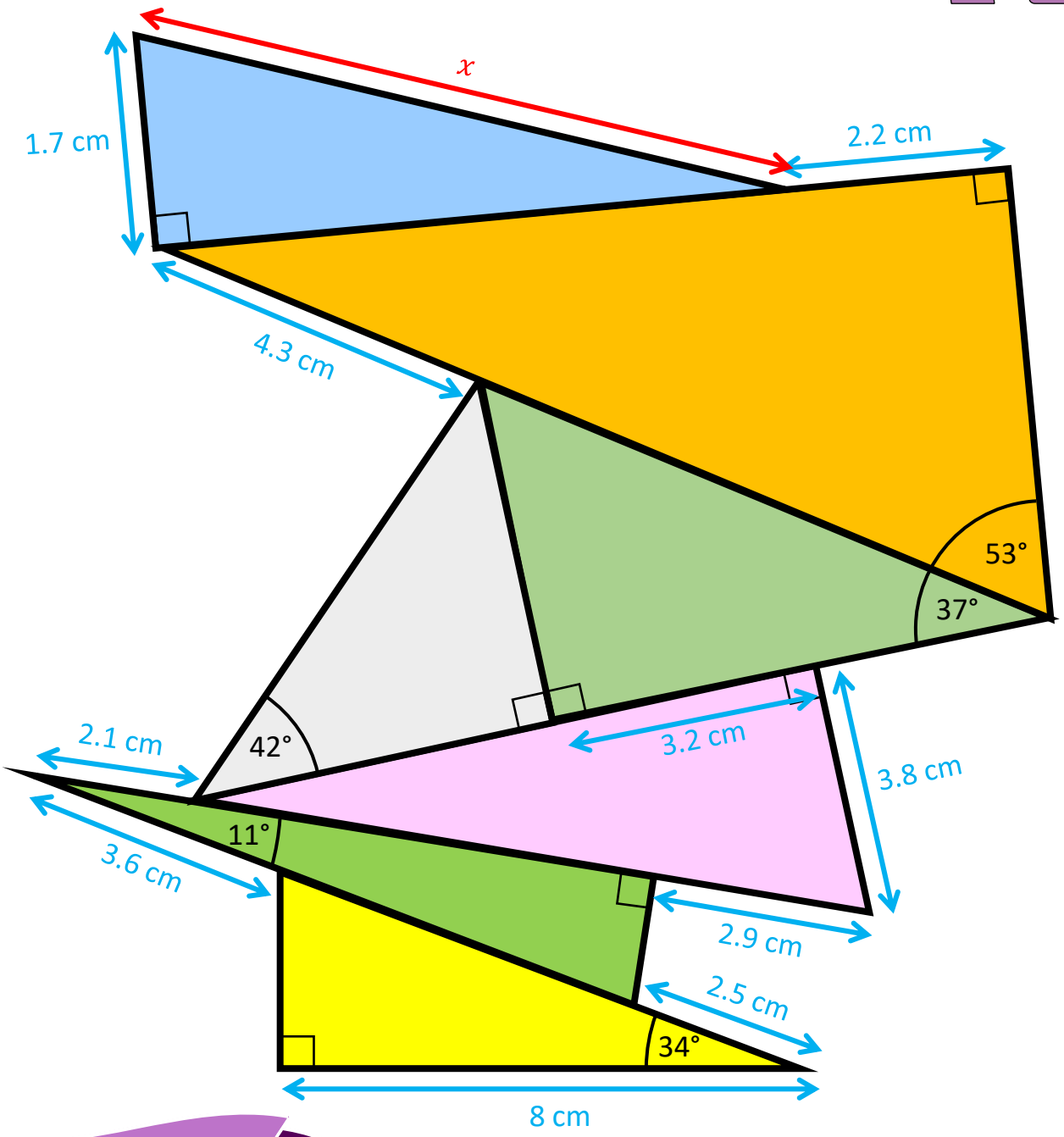


The diagram shows an irregular hexagon split into two trapezia. The edges of the hexagon are either vertical or horizontal. Calculate the size of the angle θ.

The word trigonometry comes from the Greek language: "*trigon*" means triangle and "*metry*" means measure.

Exercise 10: A Pile of Triangles

Use Trigonometry and Pythagoras' Theorem to find the length of x.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Enlargement

Enlargement is one of the four transformations.

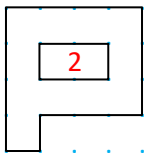
Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	**Enlargement**

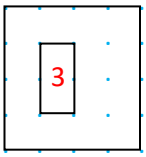
When a shape is enlarged, the **size** of the shape changes. The **scale factor** decides how the shape changes. For example, if the scale factor is 2, then the size of the shape doubles. If the scale factor is $\frac{1}{2}$, then the size of the shape halves.

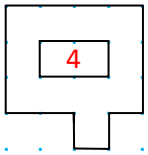
Skill

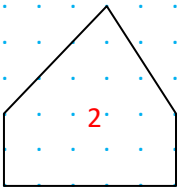
Exercise 11

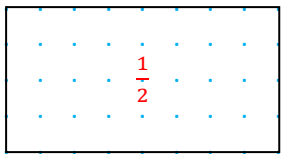
Enlarge the following shapes using the scale factor that is given in the centre of each shape. I

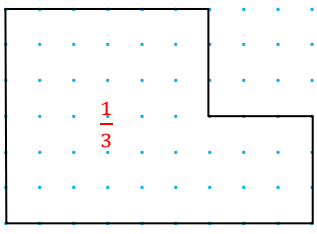






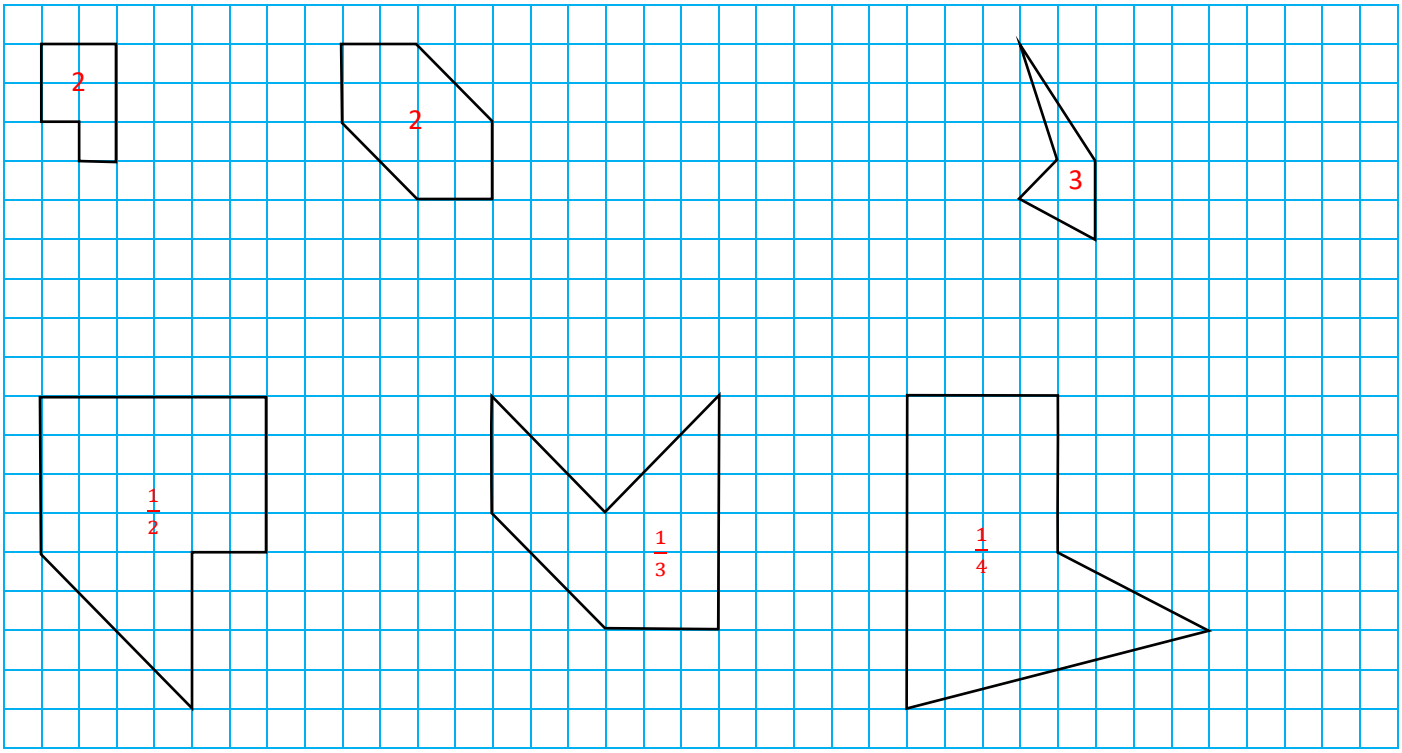






Exercise 12

Enlarge the following shapes using the scale factor that is given in the centre of each shape.

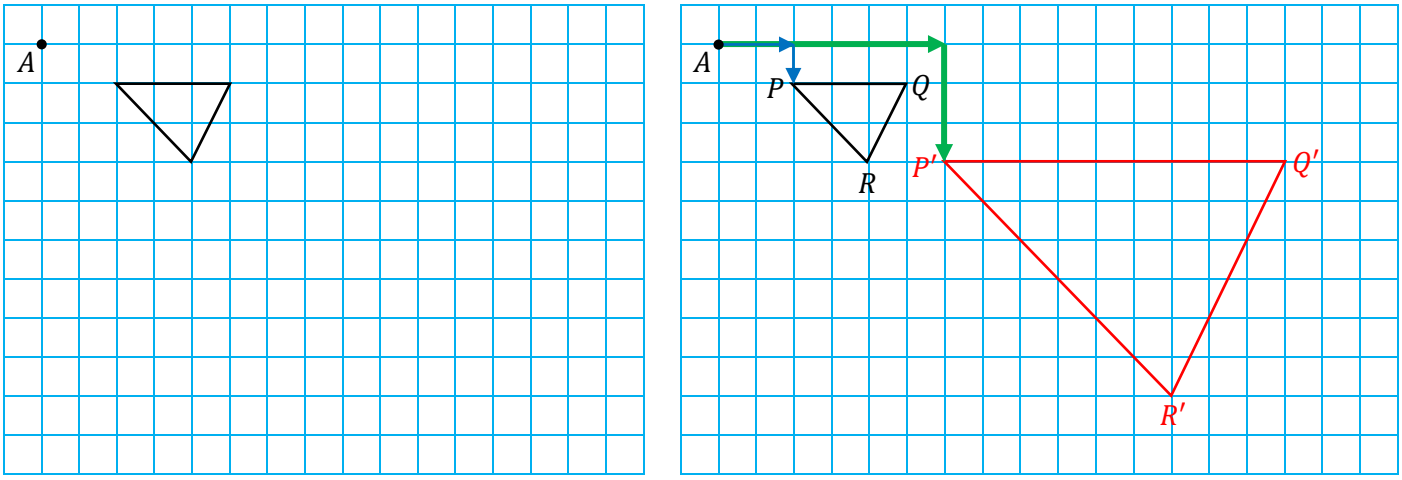


Centre of Enlargement

If a question states a point as the **centre of enlargement,** then the enlargement must appear in a certain location (it cannot appear *anywhere* like in Exercises 11 and 12).

Example

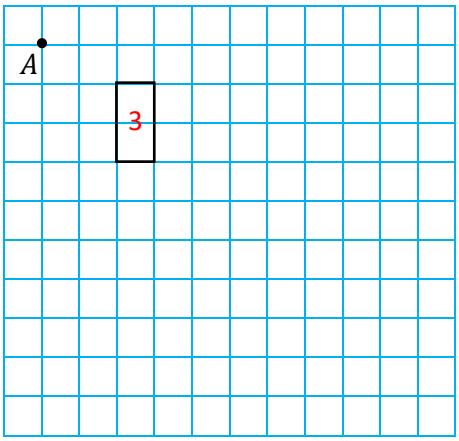
Enlarge the following triangle using a scale factor of 3 and the point A as the centre of enlargement.

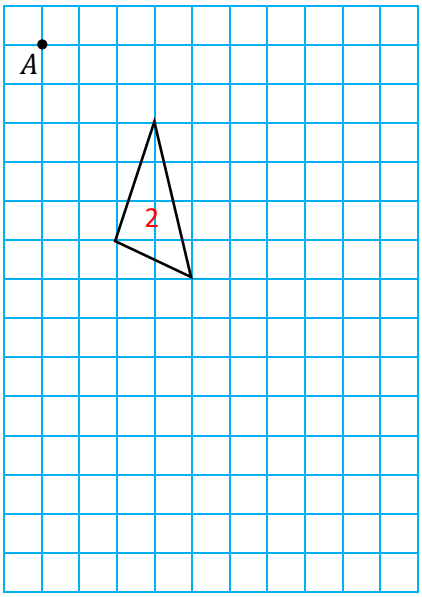


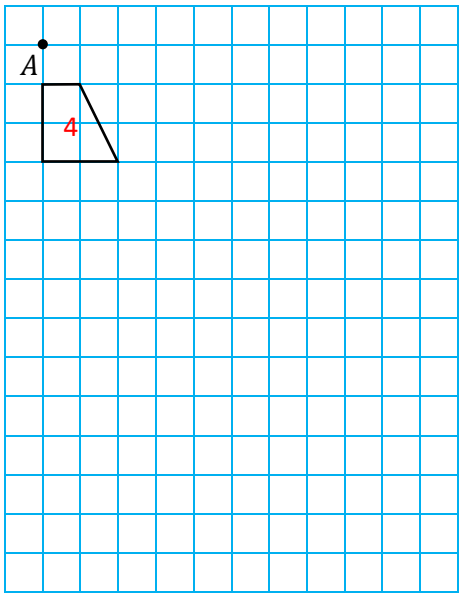
To go from the point A to the vertex P in the original triangle, we must go 2 units to the right and 1 unit down. Since the scale factor is 3, to go from the point A to the vertex P' in the new triangle, we must go $2 \times 3 = 6$ units to the right, and $1 \times 3 = 3$ units down. We can repeat this with the other vertices (Q and R), or you can start at the vertex P' and draw a triangle that is three times larger.

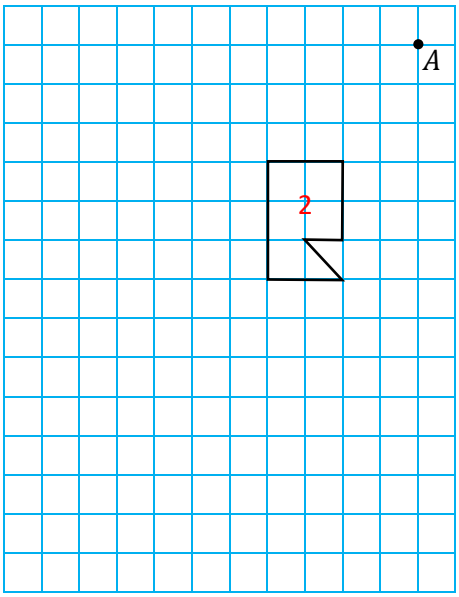
Exercise 13

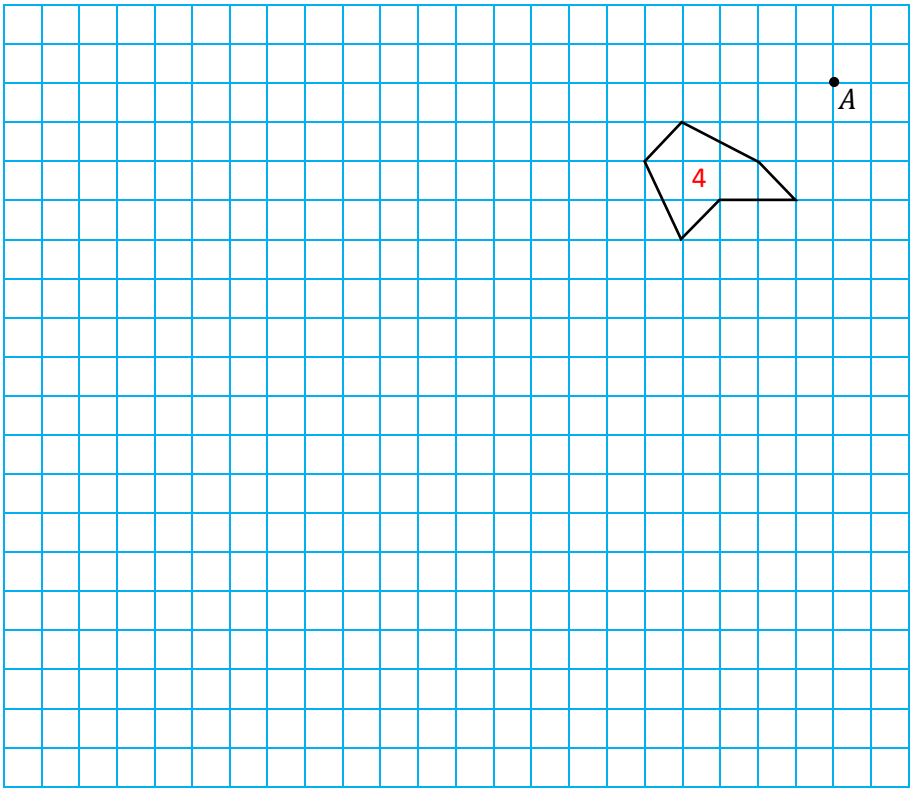
Enlarge the following shapes using the point A as the centre of enlargement and the number in the centre of each shape as the scale factor.

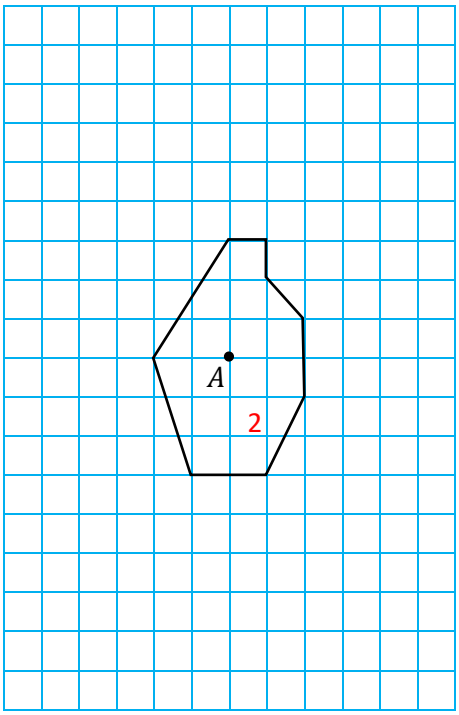


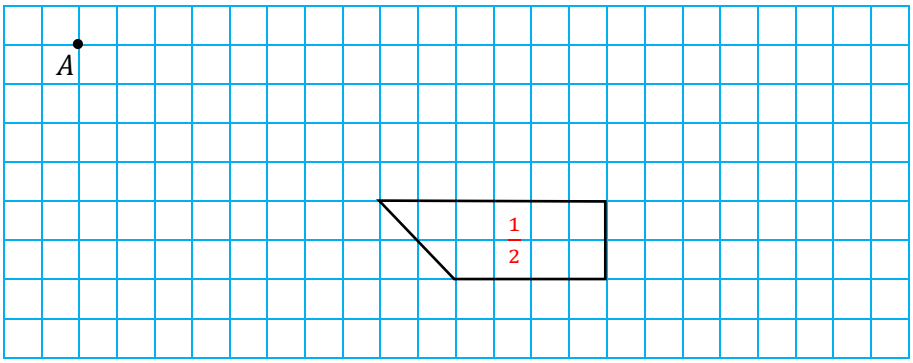






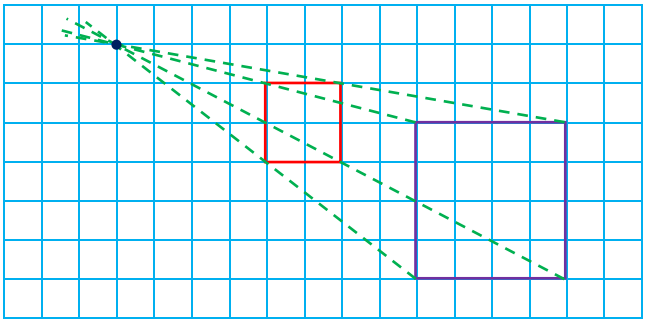






Finding the centre of enlargement

Given a shape and its enlargement, connect the corresponding vertices and extend the lines to find the location of the centre of enlargement.

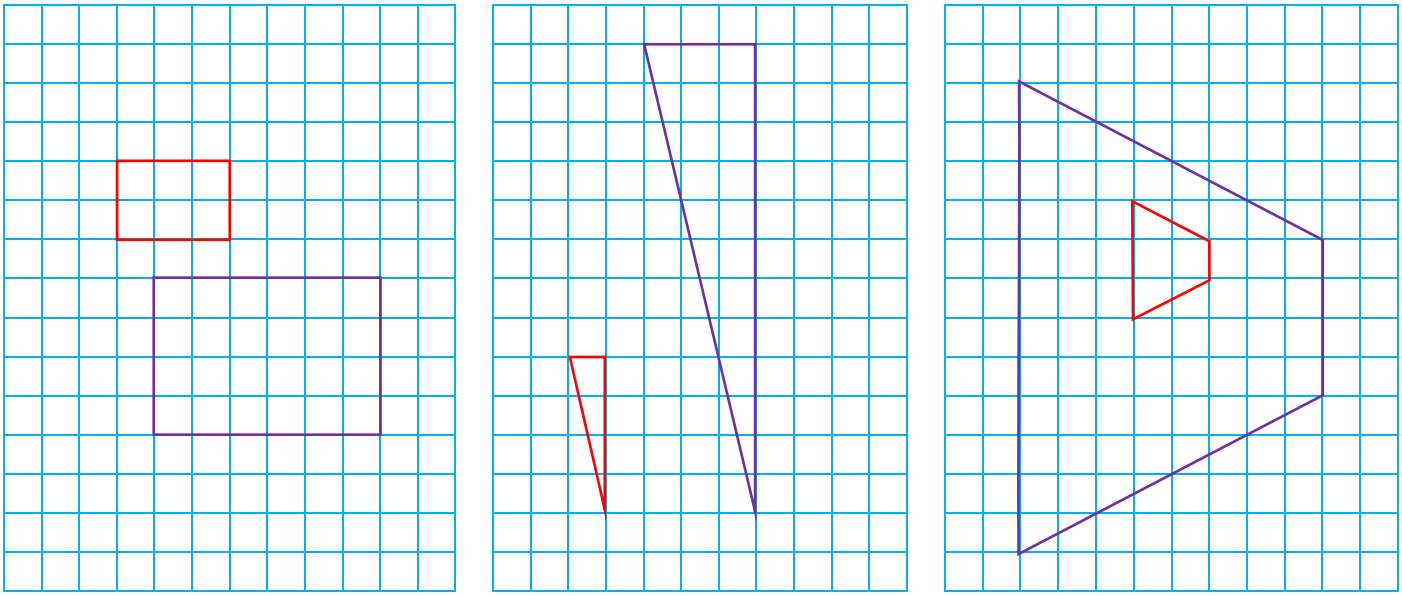


You can find the scale factor by comparing the sizes of the shapes.

Exercise 14

Find the scale factor and the centre of enlargement for the following enlargements.

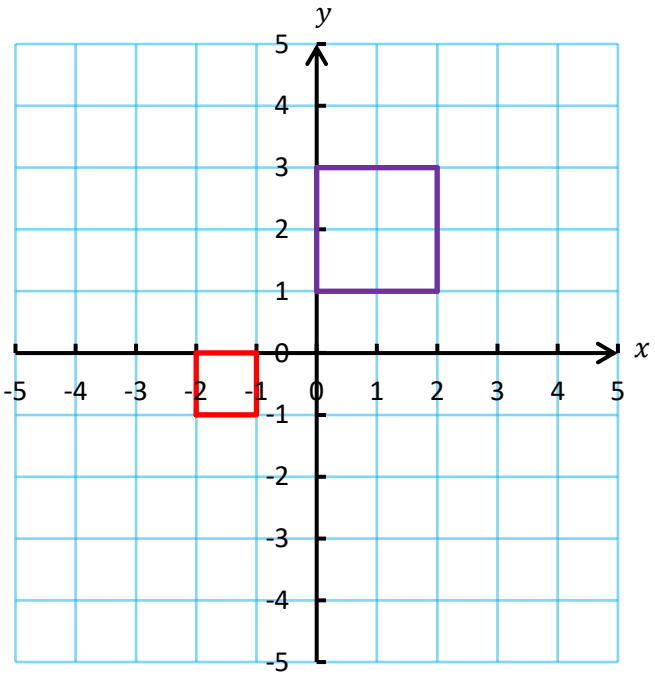
F



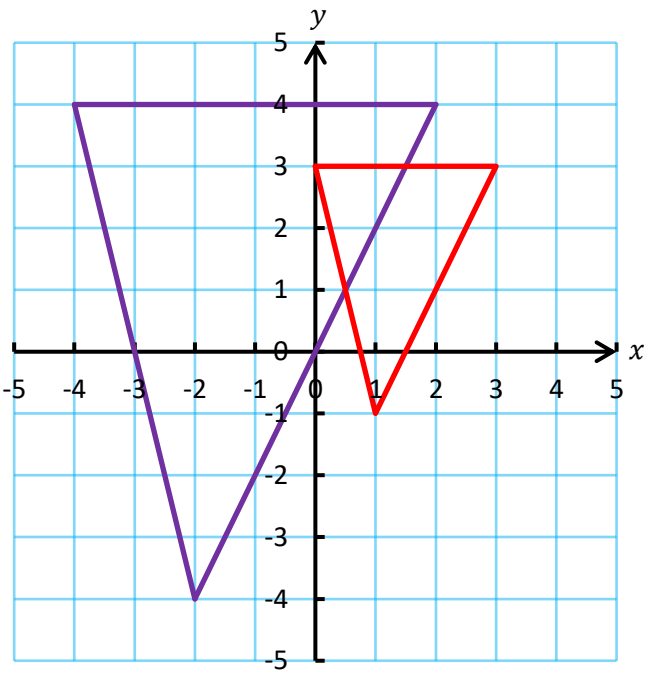
Exercise 15

Find the scale factor and the centre of enlargement for the following enlargements.

(a)



(b)



Negative Scale Factor

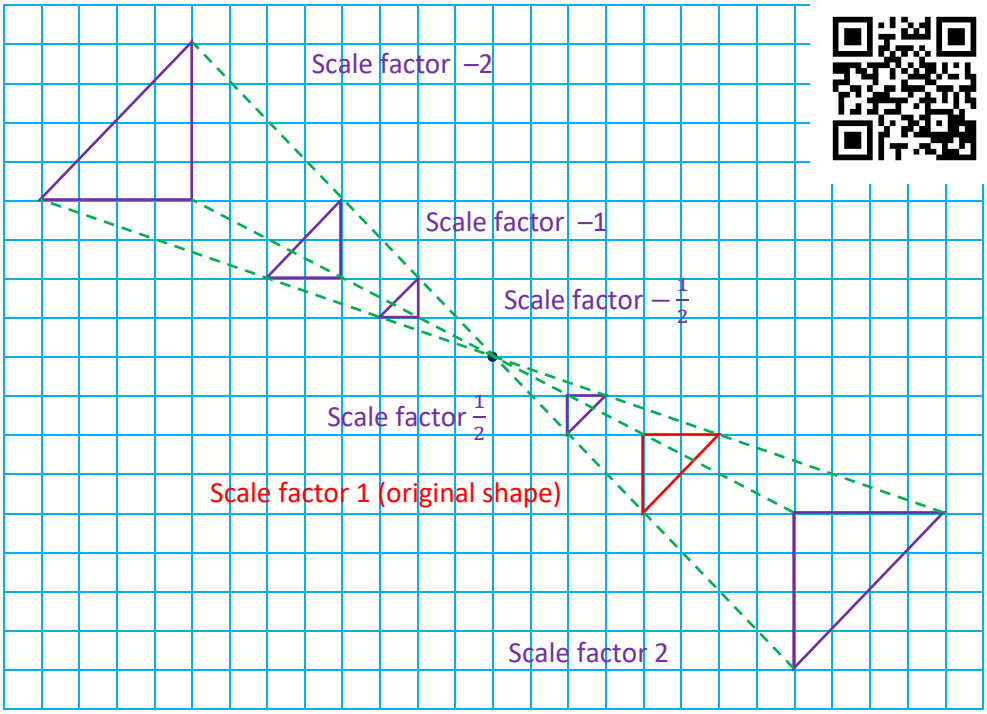
Higher Tier

A **negative** scale factor means working from the centre of enlargement to the opposite direction.

For example, consider the diagram on the right. To go from the centre of enlargement to the top left vertex of the original triangle, we must go 4 units right and 2 units down.

With a scale factor of 2, we must go 4 × 2 = 8 units right and 2 × 2 = 4 units down.

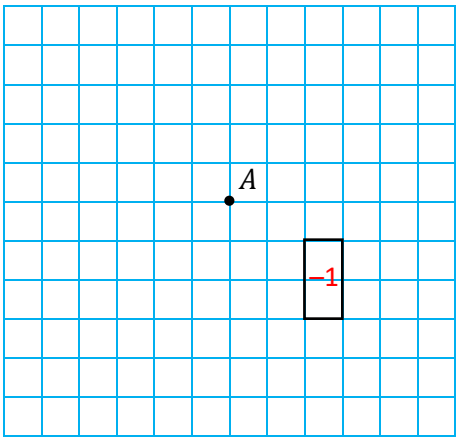
With a scale factor of –2, we must go 4 × 2 = 8 units **left** and 2 × 2 = 4 units **up**.

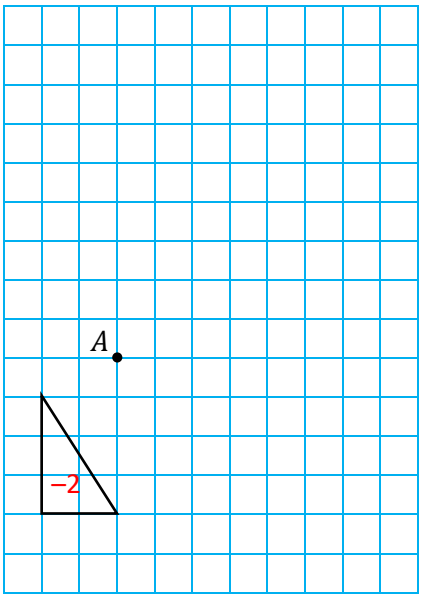


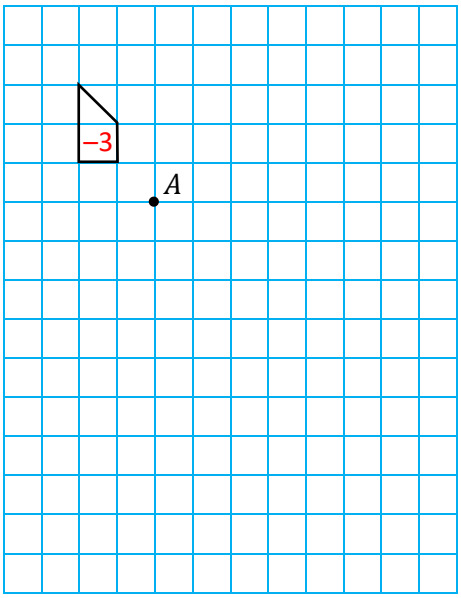
Exercise 16

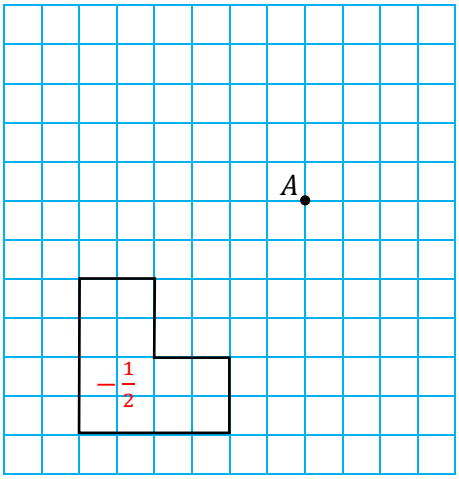
H

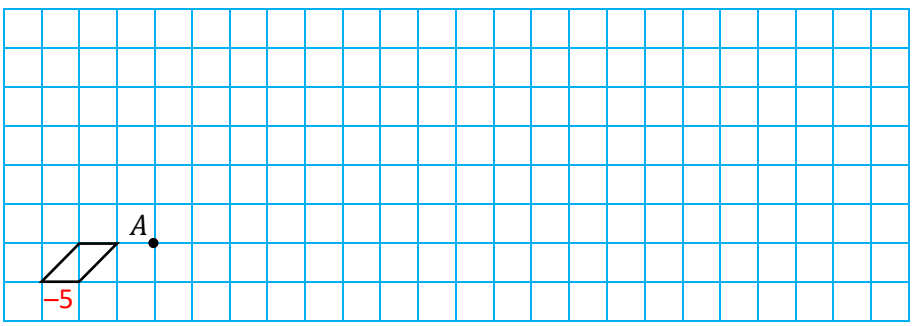
Enlarge the following shapes using the point A as the centre of enlargement and the number in the centre of each shape as the scale factor.









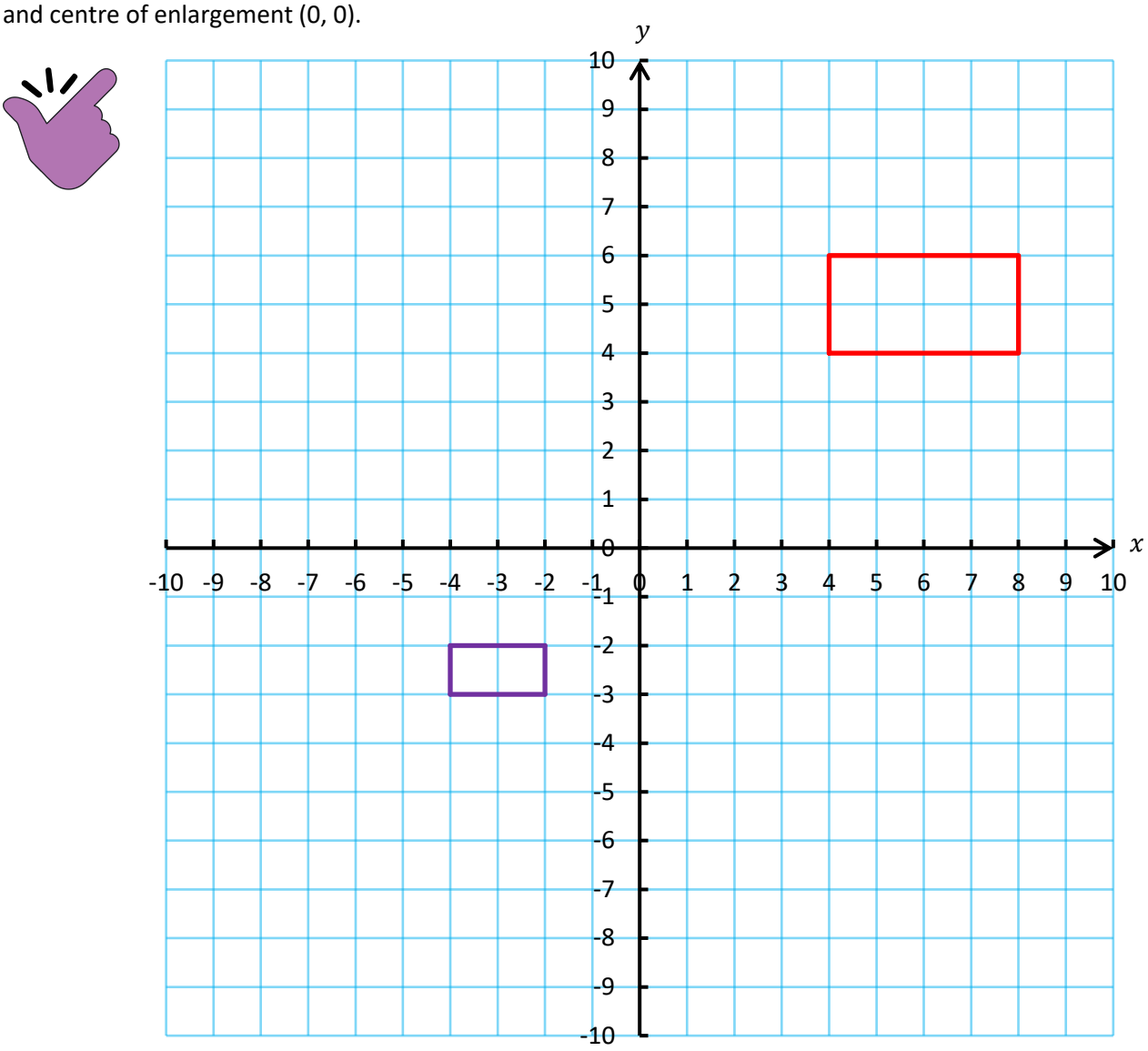


Exercise 17

H

The larger rectangle is transformed to the smaller rectangle. The co-ordinates of the centre of enlargement are (0, 0). Complete the following sentence to fully describe this transformation.

The transformation from the larger rectangle to the smaller rectangle is an enlargement using scale factor _____ and centre of enlargement (0, 0).



Evaluation

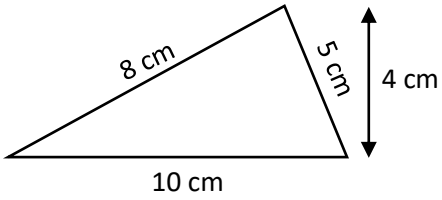
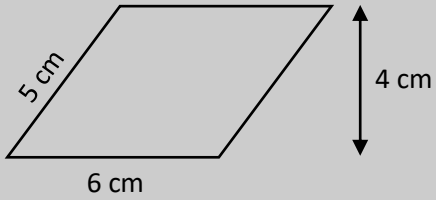
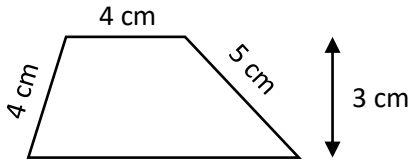
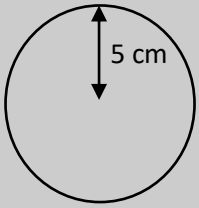
Key words	Corrections	I am happy with...	I need to revise...

Composite Shapes

Revision F

Exercise 18

Complete the following table.

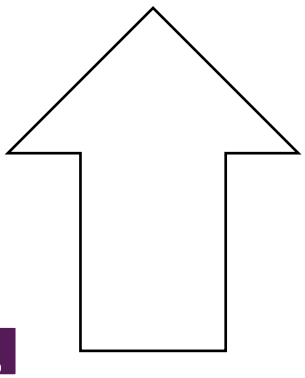
Shape	Name of the shape	Formula to find the area of the shape	Calculate the area of the shape
			
			
			
			

Exercise 19

Calculate the perimeter of each shape in the above table.

Composite Shapes

Composite shapes are shapes that you can split into simpler shapes, like the shapes in the above table. For example, the shape on the right is a composite shape – it is possible to split the shape into a rectangle (at the bottom) and a triangle (at the top). We can calculate the area of the composite shape by adding the area of the rectangle to the area of the triangle.

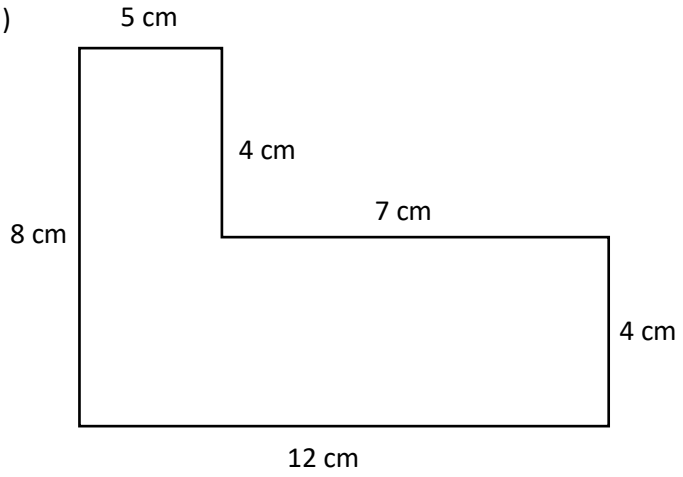


Exercise 20

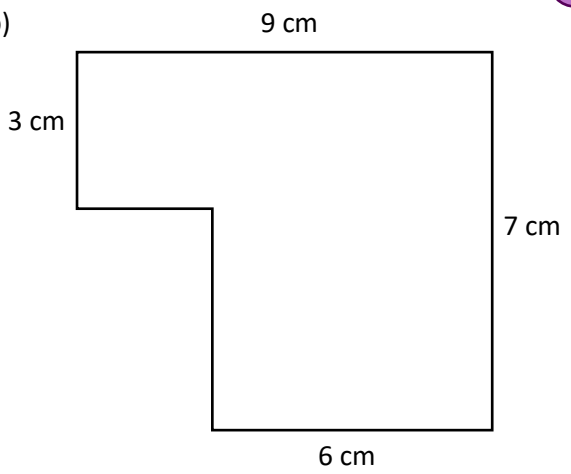
Skill F

Calculate the area of each of the following composite shapes.

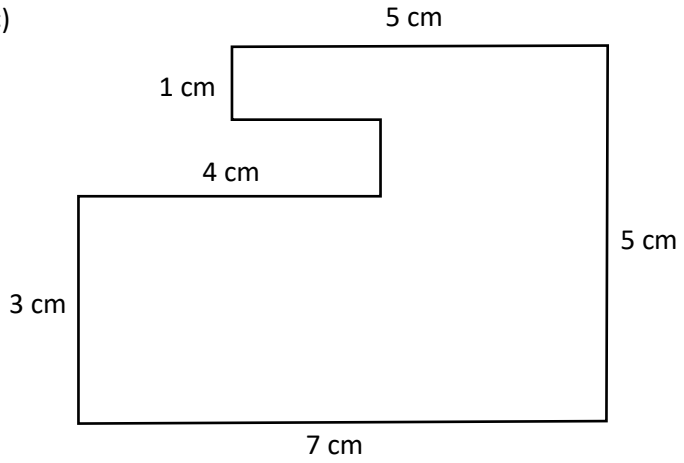
(a)



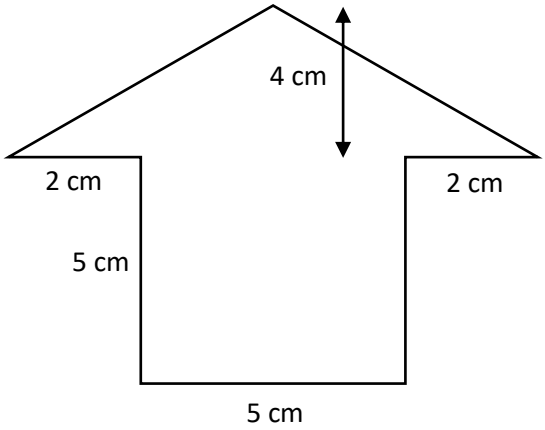
(b)



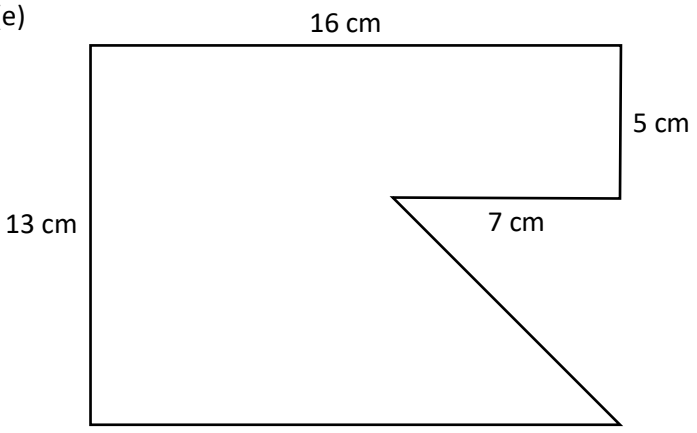
(c)



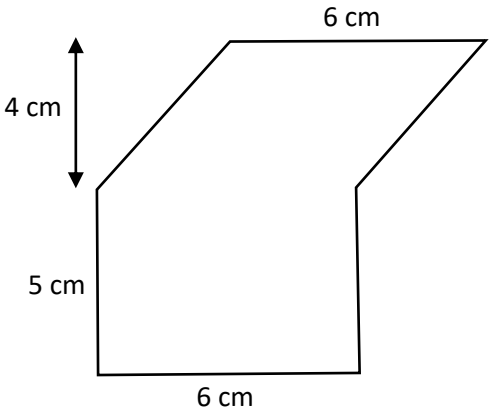
(d)



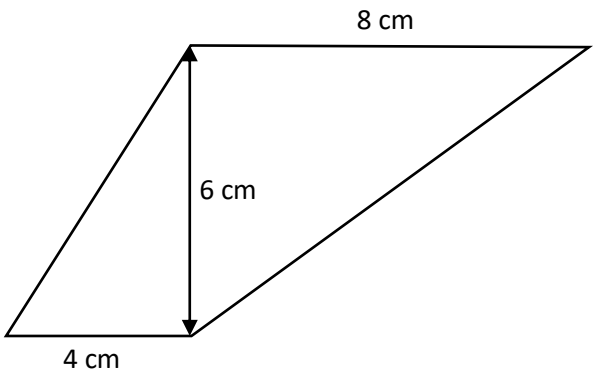
(e)



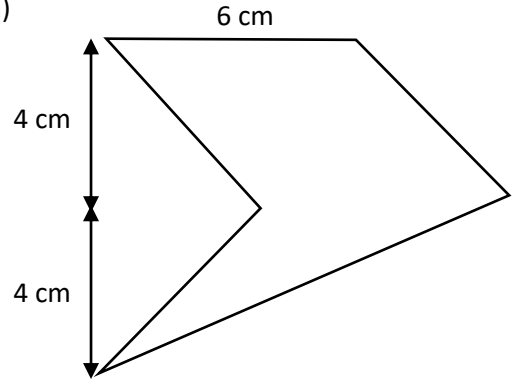
(f)



(g)



(h)

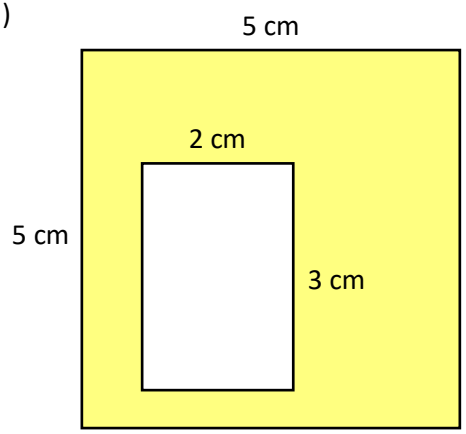


Exercise 21

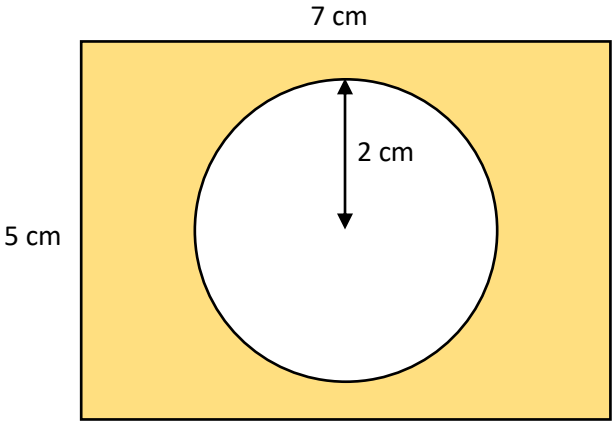
F

Calculate the area of the coloured region.

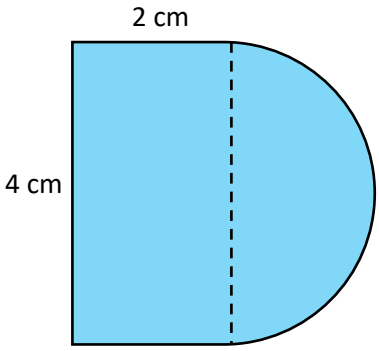
(a)



(b)

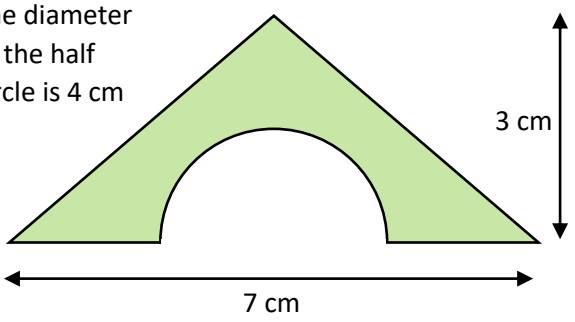


(c)

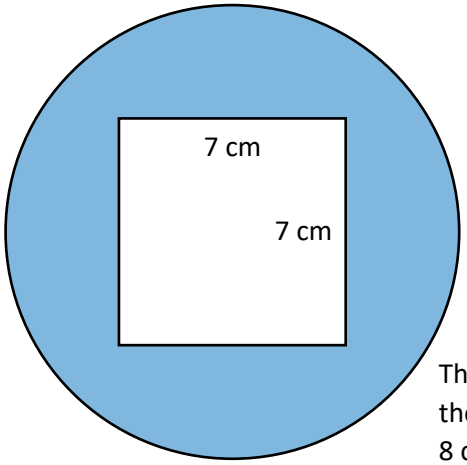


(d)

The diameter of the half circle is 4 cm

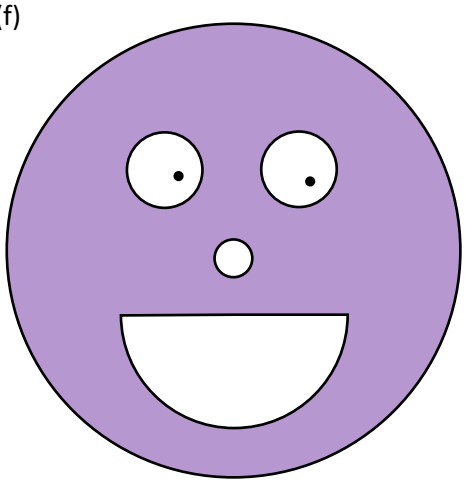


(e)



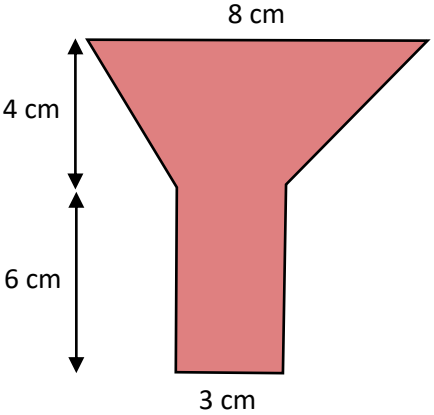
The radius of the circle is 8 cm

(f)

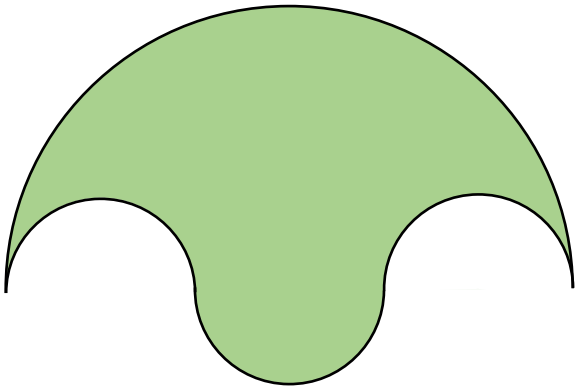


Circle	Radius
Face	6 cm
Eye	1 cm
Nose	0.5 cm
Mouth	3 cm

(g)



(h)

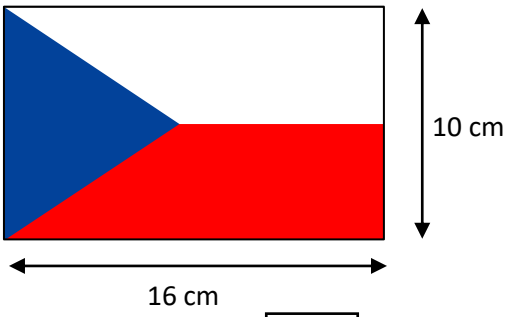


Circle	Radius
Large (1)	7.5 cm
Small (3)	2.5 cm

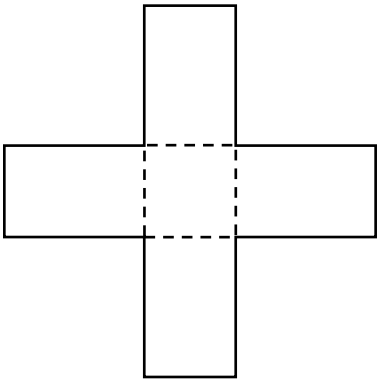
Exercise 22

C

(a) The flag of the Czech Republic is shown on the right.
Given that the vertex of the blue triangle appears in the middle of the flag, calculate the area of the red region.



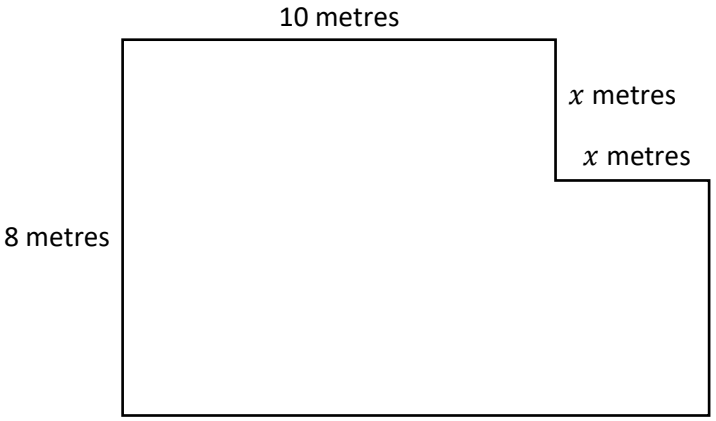
(b) A pattern in the form of a cross is formed by arranging four identical rectangles around the sides of a square, as shown in the diagram on the right.
(The diagram is not drawn to scale.)



The area of the square is 36 cm². The area of each rectangle is one and a half times the area of the square.

(i) Find the area of the pattern in the form of a cross.
(ii) Find the perimeter of the pattern in the form of a cross.

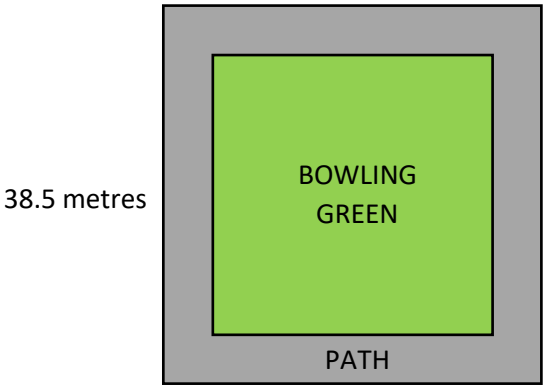
(c) Gerry has drawn a sketch of a room's floor plan. Each one of the corners of the room is either 90° or 270°. Gerry has forgotten some of the measurements but knows that two of the measurements are the same, and so labels them with x metres.



Show that the area of the floor, in square metres, is $80 + 8x - x^2$.

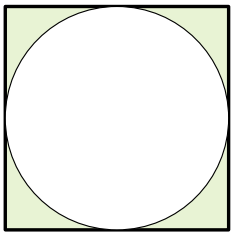
(d) The local council decides to place a concrete path of width $1\frac{1}{2}$ metres around a square bowling green, as shown in the diagram.

Calculate the area of the concrete path, in square metres.



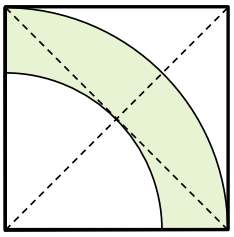
Challenge!

Which fraction of the square is shaded?



Challenge 2!

Which fraction of the square is shaded?

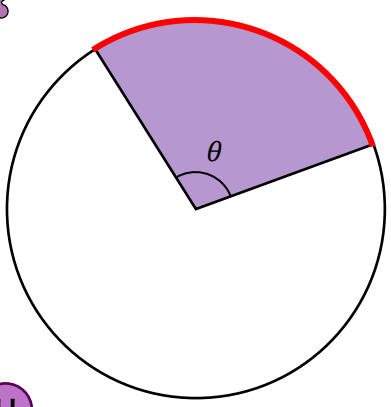


Length of an Arc and the Area of a Sector

Higher Tier

The length of an arc is a fraction of the circumference of a circle, whilst the area of a sector is a fraction of the area of the circle.

Arc Length = $\frac{\theta}{360°} \times$ circle circumference Area of a Sector = $\frac{\theta}{360°} \times$ circle area



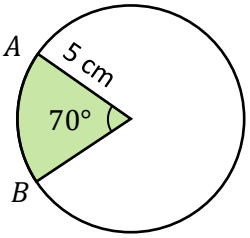
Arc Length = $\frac{\theta}{360°} \times \pi \times$ diameter

Area of a Sector = $\frac{\theta}{360°} \times \pi \times$ radius²

Exercise 23

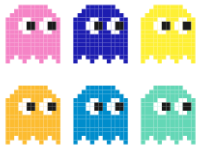
H

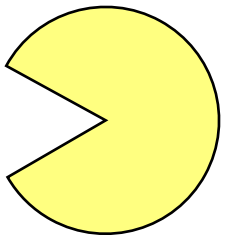
For the circle shown on the right,



(a) Calculate the length of the minor (*smaller*) arc AB.
(b) Calculate the area of the minor sector AB.
(c) Calculate the length of the major (*larger*) arc AB.
(d) Calculate the area of the major sector AB.
(e) What fraction of the circle is shaded in green? Give your answer in its simplest form.

Exercise 24

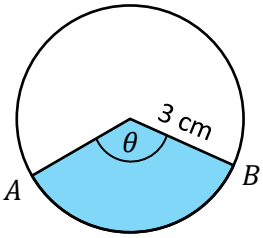




Pacman, the computer game character, is in the shape of a sector.
The angle in the centre of the shape is 300°. If the radius of Pacman is 6 cm,

(a) What is the area of Pacman?
(b) What is the perimeter of Pacman?

Exercise 25



The length of the minor arc AB in the diagram on the right is 7 cm.

(a) What is the size of the angle θ?
(b) Calculate the area of the blue sector.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Measuring Shapes 3

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test	Correct in the test?
I can recognise the **opposite**, the **adjacent** and the **hypotenuse** in a right-angled triangle.			1, 2, 11 (I)	
I can **calculate the lengths of edges** in a right-angled triangle using **trigonometry**.			1	
I can **calculate the angles** of a right-angled triangle using **trigonometry**.			2, 11 (I)	
I can **calculate the lengths of edges** in a right-angled triangle using **Pythagoras' Theorem**.			11 (I)	
I can **enlarge** shapes using a scale factor that is a **positive whole number**.			3	
I can **enlarge** shapes using a scale factor that is **fractional**.			5	
I know how to use the **centre of enlargement** whilst enlarging shapes.			4, 5	
Given a shape and its enlargement, I can **find the scale factor** and the **centre of enlargement**.			6	
I can **enlarge** shapes using a scale factor that is **negative**. **(Higher Tier only.)**			11 (H)	
I can calculate the **perimeter of a composite shape**.			7, 8	
I can calculate the **area of a composite shape**.			9, 10	
I can calculate the **length of an arc** in a circle. **(Higher Tier only.)**			12 (H)	
I can calculate the **area of a sector** in a circle. **(Higher Tier only.)**			12 (H)	

The Mathematics Department

10

Fractions, Percentages and Decimals

Name:

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Calculating

Mastering the Techniques

When working with fractions, decimals and percentages, we must develop fluency with many techniques.
Try the following questions, before going back and revising the techniques if you got the answer wrong.

Exercise 1

(a) 398 + 4829	(b) 693 – 246	(c) 372 × 68
(d) 925 ÷ 37	(e) 2.6 × 10	(f) 63 ÷ 100
(g) 0.0247 × 1000	(h) 46.27 × 8	(i) 29.3 + 2.43
(j) 52.6 × 7.84	(k) 14.7 ÷ 6	(l) 0.3 × 0.2

(m) $4 \div 0.5$

(n) 50% o 80

(o) 10% o 72

(p) 36% o £84

(q) $\frac{3}{5}$ o £35

(r) $1 - \frac{3}{7}$

(s) $\frac{5}{9} + \frac{2}{9}$

(t) $\frac{5}{9} - \frac{2}{9}$

(u) $\frac{2}{3} + \frac{3}{5}$

(v) $\frac{2}{3} - \frac{3}{5}$

(w) $\frac{2}{3} \times \frac{3}{5}$

(x) $\frac{2}{3} \div \frac{3}{5}$

Exercise 2

Check that you are able to use your calculator to find the correct answers to the questions in Exercise 1.

Exercise 3

Extension F

Use the clues on the cards to solve the problem.

(1) The wage of Number 6 is half the wage of Number 3.

(2) Number 9 has the highest wage in the team.

(3) The wage of Number 10 is $\frac{6}{7}$ of the wage of Number 7.

(4) There's a 15% bonus for every player who scores a goal in a game.

(5) The manager's wage is the mean wage of Number 8 and Number 11.

(6) Number 7 earns £8,000 per day.

(7) There are 11 players in a football team.

(8) If the goalie doesn't conceed a goal in a game, he gets a bonus of £5,000.

(9) The wage of Number 3 is 48% of the wage of Number 11.

(10) There's a bonus of 40% for any player who scores a hat-trick in a game.

(11) This week the team won 2–0.

(12) Your task is to calculate this week's wage for the manager and every player who started the game this week.

(13) Number 5 earns £200 per hour.

(14) Number 11 has the third highest wage in the team.

(15) Number 9's usual wage is double the goalie's usual wage.

(16) Number 4's usual wage is $\frac{5}{6}$ of Number 3's usual wage.

(17) One of the players in the team has the same wage as the goalie's usual wage.

(18) There's a difference of £2,000 between the wages of Number 10 and Number 11.

(19) The team only played one game this week.

(20) Number 2's wage is £6,000 higher than the lowest wage.

(21) The crowd for the recent game was 34,500.

(22) Number 1's wage is $\frac{5}{8}$ of Number 10's usual wage.

(23) The word "wage" means "weekly wage" in this task.

(24) During this week's game, Number 4 and Number 9 scored.

You will need to use the following timetable to answer Exercises 4 and 5…

Summary of North Wales to South Wales train services Crynodeb o wasanaethau rhwng Gogledd Cymru a De Cymru							Saturdays Dydd Sadwrn	
		◊ 🛒	◊ 🛒	◊ 🛒	R 🛒	◊ 🛒 L	◊ 🛒	◊ 🛒
Holyhead / Caergybi	**d**	0425	0635	0820	1033	1238	1423	1650
Valley / Y Fali	**d**	0432x	0641x	0826x	1039x			
Rhosneigr	**d**			0832x				
Ty Croes	**d**			0835x				
Bodorgan	**d**			0840x				
Llanfairpwll	**d**	0449x	0658x	0849x	1056x			
Bangor	**d**	0457	0707	0902	1105	1307	1453	1718
Llanfairfechan	**d**			0909x			1500x	
Penmaenmawr	**d**			0913x			1504x	
Conwy	**d**			0919x			1510x	
Llandudno Junction / Cyffordd Llandudno	**a**	0513	0723	0923	1121	1323	1515	1734
Llandudno Junction / Cyffordd Llandudno	**d**	0515	0725	0925	1125	1325	1516	1736
Colwyn Bay / Bae Colwyn	**d**	0521	0731	0931	1131	1331	1522	1742
Abergele & Pensarn	**d**							
Rhyl	**d**	0531	0741	0941	1141	1341	1533	1752
Prestatyn	**d**	0537	0747	0947	1147	1347	1538	1758
Flint / Y Fflint	**d**	0550	0800	1000	1200	1400	1552	1811
Shotton	**d**							
Chester / Caer	**a**	0604	0816	1016	1216	1414	1605	1825
Chester / Caer	**d**	0612	0819	1019	1219	1419	1619	1829
Wrexham General / Wrecsam Cyffredinol	**d**	0638	0834	1035	1234	1434	1635	1845
Ruabon / Rhiwabon	**d**	0645	0841	1042	1241	1441	1642	1852
Chirk / Y Waun	**d**	0651	0847	1048	1247	1447	1648	1858
Gobowen	**d**	0657	0853	1054	1253	1453	1654	1904
Shrewsbury / Yr Amwythig	**a**	0717	0913	1114	1313	1513	1714	1924
Shrewsbury / Yr Amwythig	**d**	0719	0915	1115	1315	1515	1716	1926
Church Stretton	**d**				1330			
Craven Arms	**d**				1338			
Ludlow	**d**	0745	0942	1141	1344	1541	1742	1951
Leominster	**d**	0755						
Hereford / Henffordd 7	**d**	0812	1007	1206	1410	1606	1807	2017
Abergavenny / Y Fenni	**d**	0835	1030	1229	1432	1629	1830	2040
Pontypool & New Inn / Pontypwl	**d**	0845	1040	1239	1443	1639	1840	
Cwmbran	**d**	0850	1045	1244	1448	1644	1845	2052
Newport / Casnewydd	**d**	0902	1057	1256	1506	1656	1857	2117
Cardiff Central / Caerdydd Canolog 7	**a**	0924	1115	1317	1526	1708	1915	2142

Notes

a	Arrival time
d	Departure time
x	Service stops on request
7	Recommended connecting time
L	To Maesteg
◊	Seat reservations available
R	Seat reservations recommended
🛒	At seat service

Nodiadau

a	Amser cyrraedd
d	Amser gadael
x	Yn aros ar gais
7	Amser cysylltu argymelledig
L	I Faesteg
◊	Seddi cadw ar gael
R	Argymhellir seddi cadw
🛒	Gwasanaeth troli

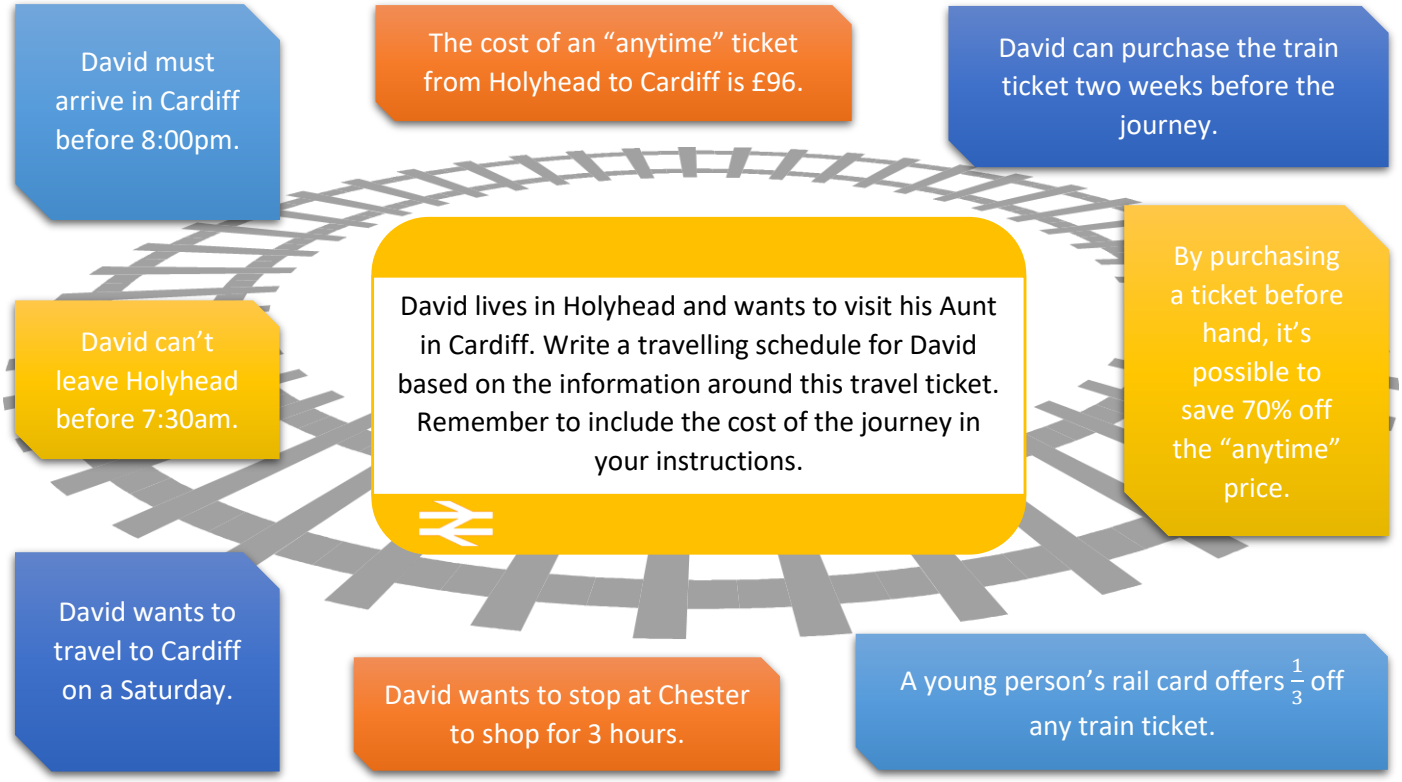
Exercise 4

Use the timetable on the previous page to answer the following questions.

(a) How much time does the 0425 train from Holyhead take to reach Llandudno Junction?
(b) For how many minutes does the 1423 train from Holyhead wait in Chester?
(c) In minutes, what is the fastest journey between Chester and Shrewsbury?
(d) Out of the seven trains, how many trains stop in Pontypool?
(e) On the train that recommends seat reservations, how long does it take to travel from Prestatyn to Cwmbran?
(f) Which train waits the longest in Llandudno Junction?

Exercise 5

Use the timetable on the previous page to solve the following problem.



Exercise 6

(a) A man gave $\frac{1}{4}$ of the money in his will to his eldest son; $\frac{1}{4}$ of the rest to his second son; and so on until there was only £40,500 left for his only daughter. The value of his will was £128,000. How many sons did he have?

(b) How much is 30% of 40% of 50% of £60?

(c) Gwen's necklace broke. $\frac{1}{3}$ of the beads went on the floor. $\frac{1}{5}$ went down the side of the chair. Gwen found $\frac{1}{6}$ of the beads on the table and her Mum found the rest, which was $\frac{1}{10}$. 12 were still on the string. How many beads were on the string before it broke?

(d) In the sequence $\left(1-\frac{1}{2}\right)$, $\left(\frac{1}{2}-\frac{1}{3}\right)$, $\left(\frac{1}{3}-\frac{1}{4}\right)$, ... what is the sum of the first hundred terms?

(e) I bought a number of apples from a local market. On the way home, I saw William and gave him $\frac{2}{3}$ of the apples. Then I ate one apple before visiting Beth. She had $\frac{3}{4}$ of the remaining apples. I arrived home with only 4 apples. How many apples did I have originally?

F

Exercise 7: Upside-down calculator story: the golf tournament.

Use the yellow and green clues to complete the story.
Key: 1 = I, 2 = Z, 3 = E, 4 = H, 5 = S, 6 = G, 7 = L, 8 = B, 9 = b, 0 = O.
Ignore decimal points in your answers.

It was the final of the $8^3 + 106$ golf tournament. $\frac{202\times4096}{16\times64}$ was in second place with only one $7^4 + 434 \times 3 + 1$ to go. $192\frac{3}{4} \times 2\frac{2}{3}$ bitter rival $\sin(30) + 0.201$ had already finished, and had $5^5 + 580$ possession of first place. The final $13719616^{0.5}$ was a par 3. A $170683\frac{1}{5} \div 5\frac{2}{5}$ would mean that $\sqrt{0.040401} + \frac{1}{2}$ was champion; a birdie would see $9^2 - \frac{\frac{1}{10}}{\frac{1}{2}}$ win. On the tee $7! \times \frac{1}{8} + 178$'s $7! + (9^3 - 132)$ were shaking like jelly and the club felt as slippery as an $\frac{38^2}{2} + 11$. "Here $\sqrt{281536.36}$ nothing", $(10^2 + 1) \times 8$ thought to himself, before striking the ball $\sqrt{\sqrt[3]{15625} \times 923 - 1}$ and straight onto the green – a great shot!

This year had been one $8^2 - 3 + \frac{\frac{2}{5}}{\frac{1}{2}}\frac{\cos(45)\times12150}{\sqrt{2}}$ for $\frac{16\times101}{9^2} \times 40\frac{1}{2}$. $2 \times e^0 \times 17$ hated to $5555 + (-2)^{11}$, but this year the feeling had become familiar; $405\frac{3}{4} + 402\frac{1}{4}$ had more $2142028 \times 25\%$ to $\left(\frac{1000}{701}\right)^{-1}$ than he could remember. Today was $\frac{20200\times4}{10^2}$'s final chance to $\frac{7\times2267}{2^2\times5^3}$ the critics and feel the $31610\frac{4}{5} + 23567\frac{1}{5}$ of victory. On reaching the green $8 + \frac{1}{10} - \frac{2}{100}$ tried not to notice the $3^6 \times 4 + (300 - 1)$ of the crowd watching on. He could $\tan(45) + 2 \times 167$ that the ball was around 10 feet from the $463 \times 2 \div \frac{1}{4}$, up a slight $6^5 - 5^4 + 4^3 - 3^2 + 2^1 - 1^0 + 507$. "$1032658225^{0.5}$ the day", $\sqrt{2^6 \times 101^2}$ thought, before putting the ball straight – and into the middle of the $7! - 167 \times 2^3$! Victory at last – $\frac{4\times211}{5^2}$ for $404 \times 20\%$, and at last a dent to $\sqrt{\frac{1}{2} - 0.008599}$'s $\frac{3^2\times7}{4\times25}$!

Challenge!

The name of the triangle on the right is the Leibniz Harmonic Triangle.

What patterns can you see?

Write down the next row in the pattern.

$$\begin{array}{ccccccccccccc}
 & & & & & & \frac{1}{1} & & & & & & \\
 & & & & & \frac{1}{2} & & \frac{1}{2} & & & & & \\
 & & & & \frac{1}{3} & & \frac{1}{6} & & \frac{1}{3} & & & & \\
 & & & \frac{1}{4} & & \frac{1}{12} & & \frac{1}{12} & & \frac{1}{4} & & & \\
 & & \frac{1}{5} & & \frac{1}{20} & & \frac{1}{30} & & \frac{1}{20} & & \frac{1}{5} & & \\
 & \frac{1}{6} & & \frac{1}{30} & & \frac{1}{60} & & \frac{1}{60} & & \frac{1}{30} & & \frac{1}{6} &
\end{array}$$

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Recurring Decimals

Changing Fractions to Decimals

If the denominator of a fraction is a power of 10, for example 10, 100 or 1,000, then it is easy to convert the fraction to a decimal.

Example

$\frac{3}{10} = 0.3$ $\frac{12}{100} = 0.12$ $\frac{6}{100} = 0.06$ $\frac{427}{1000} = 0.427$ $\frac{31}{1000} = 0.031$

Exercise 8

Skill F

Convert the following fractions to decimals.

(a) $\frac{7}{10}$ (b) $\frac{5}{10}$ (c) $\frac{17}{100}$ (d) $\frac{7}{100}$ (e) $\frac{95}{100}$ (f) $\frac{1}{10}$ (g) $\frac{93}{100}$

(h) $\frac{731}{1000}$ (i) $\frac{402}{1000}$ (j) $\frac{32}{1000}$ (k) $\frac{86}{1000}$ (l) $\frac{4}{1000}$ (m) $\frac{200}{1000}$ (n) $\frac{760}{1000}$

(o) $\frac{4237}{10000}$ (p) $\frac{29}{10000}$ (q) $\frac{3}{100000}$ (r) $\frac{17}{10}$ (s) $\frac{40}{100}$ (t) $\frac{329}{10}$ (u) $\frac{2053}{100}$

Using Equivalent Fractions

With some fractions, it is possible to find an equivalent fraction whose denominator is a power of 10, which then allows us to change the original fraction to a decimal.

Example

$\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10} = 0.6$ $\frac{11}{25} \xrightarrow{\times 4} \frac{44}{100} = 0.44$ $\frac{143}{200} \xrightarrow{\times 5} \frac{715}{1000} = 0.715$

Exercise 9

Convert the following fractions to decimals.

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{2}$ (e) $\frac{6}{25}$ (f) $\frac{21}{25}$ (g) $\frac{1}{25}$

(h) $\frac{3}{20}$ (i) $\frac{19}{20}$ (j) $\frac{11}{20}$ (k) $\frac{43}{50}$ (l) $\frac{9}{50}$ (m) $\frac{55}{50}$ (n) $\frac{304}{500}$

(o) $\frac{1}{200}$ (p) $\frac{64}{200}$ (q) $\frac{136}{200}$ (r) $\frac{31}{250}$ (s) $\frac{147}{250}$ (t) $\frac{9}{250}$ (u) $\frac{53}{5000}$

Terminating and Recurring Decimals

The fractions we have seen so far convert to **terminating** decimals, which means that the decimals have a **specific number** of digits after the decimal point. For example, the fraction $\frac{11}{25}$ from the above example converted to the decimal 0.44, which is a decimal that has **two digits** appearing after the decimal point.

As we will see on the next page, some fractions convert to **recurring** decimals, which are decimals with an **infinite number** of digits appearing after the decimal point. The decimal $0.222\ldots$ is an example of a recurring decimal, where the dots … show that the digit 2 repeats forever. In mathematics, we have a special notation to write recurring decimals, which is the **dot notation.** We write the decimal $0.222\ldots$ in dot notation as $0.\dot{2}$, where the dot on top of the digit 2 means the 2 repeats forever.

Example

$0.45222\ldots = 0.45\dot{2}$ $0.434343\ldots = 0.\dot{4}\dot{3}$ $0.243243243\ldots = 0.\dot{2}4\dot{3}$

Exercise 10

F

Write the following recurring decimals using dot notation.

(a) $0.777\ldots$ (b) $0.444\ldots$ (c) $0.5333\ldots$ (d) $5.222\ldots$ (e) $0.52888\ldots$

(f) $0.737373\ldots$ (g) $0.262626\ldots$ (h) $0.909090\ldots$ (i) $0.2454545\ldots$ (j) $0.0818181\ldots$

(k) $0.265265265\ldots$ (l) $0.405405405\ldots$ (m) $0.5274274274\ldots$ (n) $0.47812812812\ldots$ (o) $0.216737373\ldots$

Converting fractions to recurring decimals

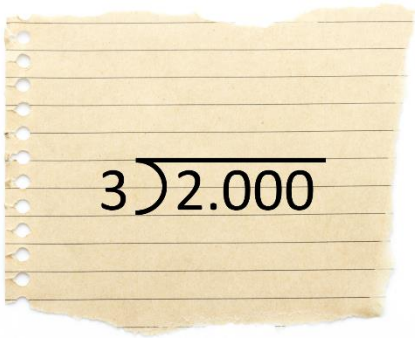
Not all fractions are equivalent to a fraction that has a denominator which is a power of 10.
With these fractions we need to use a division frame to convert the fraction to a recurring decimal.

Example

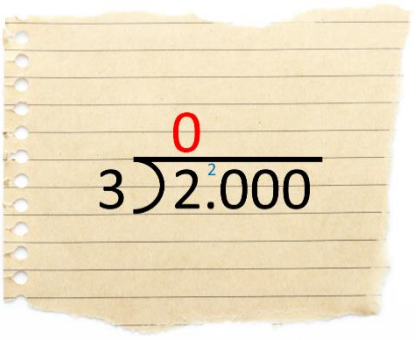
Convert the fraction $\frac{2}{3}$ to a recurring decimal.

We can add more zeroes to the 2.000, if needed.

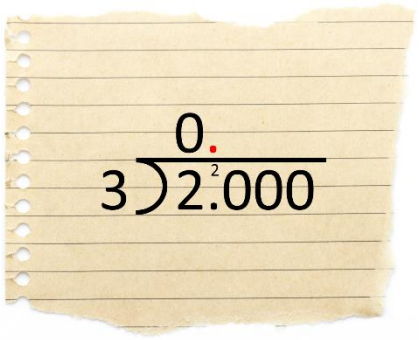
1. Put the sum 2 ÷ 3 into a division frame, writing 2 as the equivalent decimal 2.000.



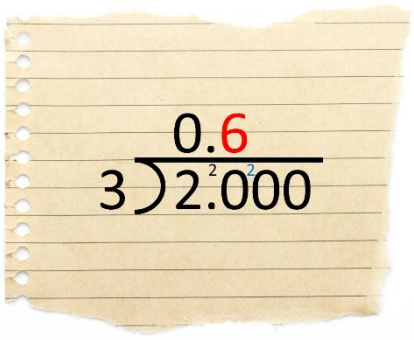
2. "How many times does 3 fit into 2?" It's too large, therefore it fits in 0 times, with 2 left over.



3. Remember to add the decimal point in the correct place.



4. "How many times does 3 fit into 20?" It fits in 6 times, with 2 left over.



5. "How many times does 3 fit into 20"? It fits in 6 times, with 2 left over.

6. We notice the pattern in the calculations, writing … to show that they repeat forever.

Answer: As a decimal, the fraction $\frac{2}{3}$ is equal to $0.666\ldots$ or, in dot notation, $0.\dot{6}$.

Exercise 11

Use a division frame to write the following fractions as recurring decimals, using dot notation.

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $\frac{5}{9}$ (d) $\frac{1}{9}$ (e) $\frac{3}{11}$ (f) $\frac{8}{11}$ (g) $\frac{10}{11}$

(h) $\frac{1}{6}$ (i) $\frac{2}{6}$ (j) $\frac{5}{6}$ (k) $\frac{1}{7}$ (l) $\frac{5}{7}$ (m) $\frac{6}{7}$ (n) $\frac{4}{13}$

Exercise 12 F

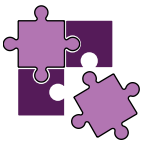
Use a calculator to check your answers to Exercise 11.

Challenge!

Use a division frame to write the following fractions as recurring decimals.

(a) $\frac{13}{17}$ (b) $\frac{11}{19}$ (c) $\frac{2}{23}$

Terminating or recurring decimals?

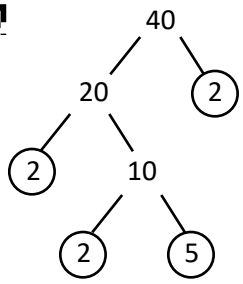


It is possible to determine whether a fraction converts to a terminating or a recurring decimal by following these steps.

1. **Simplify** the fraction, if possible.
2. Write the denominator of the fraction as a **product of its prime factors.**
3. If the product of prime factors includes the numbers 2 and/or 5 only, then the fraction converts to a **terminating decimal.** If any other prime number appears, then the fraction converts to a **recurring decimal**.

Example

Determine whether the fraction $\frac{46}{80}$ converts to a terminating decimal or to a recurring decimal.



1. We can simplify the fraction by halving: $\frac{46}{80} \xrightarrow{\div 2} \frac{23}{40}$.
2. By using a factor tree (on the right), we see that 40, as a product of its prime factors, is $40 = 2 \times 2 \times 2 \times 5$.
3. Since the product of prime factors for 40 includes the prime numbers 2 and 5 only (and no other primes), we can say that the fraction $\frac{46}{80}$ converts to a **terminating** decimal.

Exercise 13

Determine whether the following fractions convert to terminating decimals or to recurring decimals.

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{6}$ (d) $\frac{7}{8}$ (e) $\frac{4}{9}$ (f) $\frac{6}{12}$ (g) $\frac{8}{12}$

(h) $\frac{11}{15}$ (i) $\frac{12}{15}$ (j) $\frac{3}{25}$ (k) $\frac{12}{30}$ (l) $\frac{29}{30}$ (m) $\frac{1}{80}$ (n) $\frac{87}{125}$

Exercise 14

Convert the fractions from Exercise 13 to decimals.

Investigation

The series of images on the right attempts to show that $0.\dot{9} = 1$.

Can you think of a different way to show that $0.\dot{9} = 1$?

Suggestion: Use the decimal for $\frac{1}{3}$.

Converting recurring decimals to fractions

Higher Tier

We can use an algebraic method to convert recurring decimals to fractions.

Example

Convert the recurring decimal $0.5\dot{3}$ to a fraction.

Let the fraction for $0.5\dot{3}$ be represented by the letter a, so that $a = 0.5333\ldots$

In this recurring decimal, one digit is repeating (the digit 3), so we multiply a by 10. (If two digits repeat, we multiply a by 100. If three digits repeat, we multiply a by 1,000. And so on…)

$$\begin{array}{rl} & 10a = 5.3333\ldots \\ - & \quad a = 0.5333\ldots \\ \hline & 9a = 4.8 \end{array}$$

We subtract the original a from the $10a$. This leaves a terminating decimal since the digits that repeat cancel each other.

After subtracting a from $10a$, we see that $9a$ is equal to the terminating decimal 4.8.

This gives an equation that we can solve to give a value for our original number: $a = \frac{4.8}{9}$. But this is not quite a fraction, as a decimal appears as the numerator. We can deal with this by multiplying the top and bottom of the fraction by 10, to give $a = \frac{48}{90}$. To finish, we can simplify this fraction (by dividing by 6) which gives $a = \frac{8}{15}$. This is the simplest fraction that is equivalent to the recurring decimal $0.5\dot{3}$.

Exercise 15

H

Convert the following recurring decimals to fractions.

(a) $0.\dot{5}$	(b) $0.4\dot{7}$	(c) $0.7\dot{2}$	(d) $0.24\dot{6}$	(e) $0.0\dot{4}$
(f) $0.\dot{3}\dot{6}$	(g) $0.\dot{6}\dot{3}$	(h) $0.\dot{7}\dot{4}$	(i) $0.4\dot{6}\dot{7}$	(j) $0.41\dot{4}\dot{0}$
(k) $0.\dot{3}5\dot{7}$	(l) $0.\dot{7}1\dot{5}$	(m) $0.5\dot{2}4\dot{7}$	(n) $3.\dot{5}$	(o) $0.\dot{4}20\dot{7}$

Exercise 16

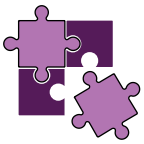
Check your answers to Exercise 15 by using the ( ) button on your calculator.

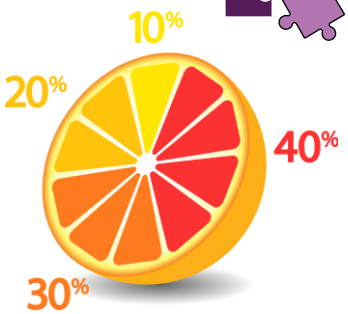
Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Converting

In this chapter, we will discuss how to convert between fractions, decimals and percentages.





Starting with a percentage

Percentage
e.g. 36%

Remove the % symbol.
Divide by 100.

Decimal
e.g. 0.36

Remove the % symbol.
Write as a fraction over 100.
Simplify the fraction.

Fraction
e.g. $\frac{36}{100} \xrightarrow{\div 4} \frac{9}{25}$

Skill F

Exercise 17

Complete the following table.

Fraction	Percentage	Decimal
	27%	
	68%	
	93%	
	4%	
	7%	
	100%	
	150%	
	400%	
	1%	
	0.5%	
	0.2%	
	0.05%	
	3.1%	
	10.2%	

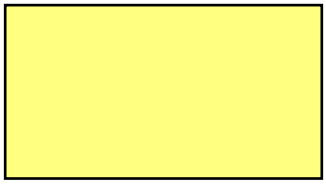
Exercise 18

Circle each number that has the same value as 40%.

0.04 $\frac{4}{10}$ 0.4 $\frac{10}{4}$ 0.40 $\frac{2}{5}$ 4%

Challenge!

If the length of a rectangle increases 10% but the area remains the same, by what fraction must the width be reduced by?



Starting with a fraction

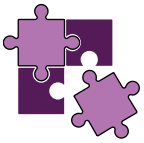
1) Try to find an equivalent fraction which has a denominator that is a power of 10. Not possible? 2) Use a division frame.

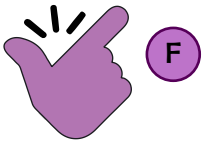
Fraction e.g. $\frac{2}{5}$

Decimal e.g. 0.4

Multiply by 100. Add a % symbol.

Percentage e.g. 40%





Exercise 19

Complete the following table.

Fraction	Percentage	Decimal
$\frac{87}{100}$		
$\frac{4}{100}$		
$\frac{7}{10}$		
$\frac{3}{5}$		
$\frac{13}{20}$		
$\frac{18}{25}$		
$\frac{184}{500}$		
$\frac{4}{9}$		
$\frac{3}{8}$		
$\frac{3}{2}$		
$\frac{4}{7}$		
$\frac{5}{11}$		
$\frac{12}{40}$		
$\frac{43}{200}$		

Exercise 20

(a) Write 13%, 0.2 and $\frac{3}{25}$ in ascending order.

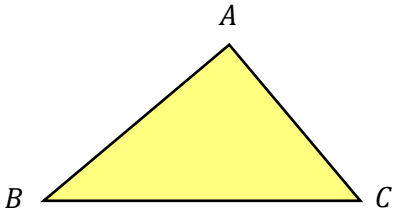
(b) Write $\frac{3}{4}$, 77% and 0.73 in ascending order.

(c) Write $\frac{32}{50}$, 0.63 and 67% in descending order.

(d) In a mathematics test, Rachel scored $\frac{3}{5}$ of the highest possible mark. Jimmy scored 62% and Susie's mark was 0.58 of the highest possible mark. State which student scored the most marks and which student scored the least marks.

Challenge!

In a triangle ABC, the angle B is $\frac{3}{4}$ of the angle C and $1\frac{1}{2}$ of the angle A. What is the size of the angle B?



Starting with a terminating decimal

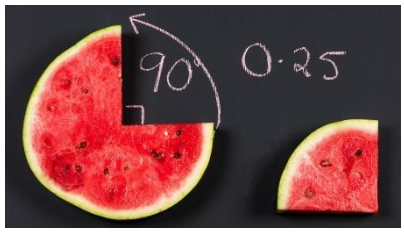
Decimal
e.g. 0.236

Multiply by 100.
Add a % symbol.

Percentage
e.g. 23.6%

Write as a fraction over 10^n, where n is the number of times you must multiply the decimal by 10 for it to become a whole number.
Simplify the fraction.

Fraction
e.g. $\frac{236}{1000} \xrightarrow{\div 4} \frac{59}{250}$



Exercise 21

Complete the following table.

Fraction	Percentage	Decimal
		0.99
		0.9
		0.5
		0.2
		0.14
		0.08
		0.01
		1.6
		2.4
		12.5
		1.09
		0.452
		1.452
		10.2

Exercise 22

A band hires a concert hall for two nights. They intend to hire the concert hall for a third night, but only if at least 0.9 of the tickets are sold, either for the first night or for the second night.

On the first night, 82% of the tickets were sold.
On the second night, $\frac{3}{4}$ of the tickets were sold.

Did the band hire the hall for the third night? You must show your method and explain how you made your decision.

Challenge!

Find the value of x, where $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots - \frac{1}{1024} = \frac{x}{1024}$.

Starting with a recurring decimal

Higher Tier

Decimal
e.g. $0.2\dot{4}$

Multiply by 100.
Add a % symbol.

Percentage
e.g. $24.\dot{4}\%$

Let a represent the fraction for the decimal. Multiply the decimal by 10^n, where n is the number of digits that repeat. Subtract a from $(10^n)a$ to leave an equation to solve for a.

Fraction
e.g. $\frac{11}{45}$

This is the techinque from the end of the previous chapter.

H

Exercise 23

Complete the following table.

Fraction	Percentage	Decimal
		$0.\dot{2}$
		$0.4\dot{3}$
		$0.\dot{2}\dot{9}$
		$3.\dot{4}\dot{8}$
		$0.\dot{5}2\dot{5}$
		$0.24\dot{3}\dot{7}$
		$6.\dot{3}2\dot{9}$

Challenge!

Here is an alternative method to convert the decimal $0.4\dot{7}$ to a fraction.
Can you use this method to convert $0.81\dot{4}$ to a fraction?

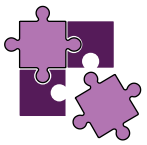
Step 1: Split into terminating and recurring parts: $0.4\dot{7} = 0.4 + 0.0\dot{7}$

Step 2: Convert the decimals to fractions: $0.4 = \frac{4}{10}$, $0.0\dot{7} = \frac{7}{90}$

Step 3: Add the fractions: $\frac{4}{10} + \frac{7}{90} = \frac{36}{90} + \frac{7}{90}$
$= \frac{43}{90}$

Evaluation

Key words	Corrections	I am happy with...	I need to revise...



Consider how you would answer the following question.

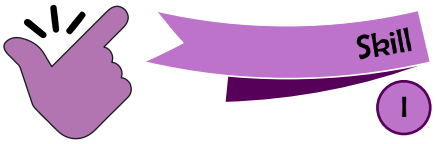
Chris' weekly wage is £480. If Chris is awarded a pay rise of 10%, what will his wage be next week?

One way to answer this question would be to calculate 10% of £480, which is £480 ÷ 10 = £48, and then add the £48 onto the original wage: £480 + £48 = £528. On a calculator, it would be possible to type the following two calculations to find the correct answer.

$$480 \times 10\% = 48$$
$$480 + 48 = 528$$

It is possible however to reach the correct answer in **one** calculation, and therefore in a **more efficient** way. Since Chris' wage is increasing 10%, it is growing from 100% of the original wage to 110% of the original wage. It would therefore be possible to find the new wage by finding 110% of the original wage. On a calculator, we can do this by typing the following calculation.

$$480 \times 110\% = 528$$



Exercise 24

Complete the following table.

Original Price	Original Percentage	Percentage Change	New Percentage	Sum to Find the New Price	New Price
£480	100%	Increase 10%	110%	£480 × 110%	£528
£54	100%	Decrease 20%	80%	£54 × 80%	£43.20
£250		Increase 25%			
£1,700		Increase 5%			
£20		Decrease 10%			
$800		Decrease 30%			
€460		Increase 2%			
£7.50		Decrease 30%			
£2,500		Increase 80%			
£100,000		Decrease 64%			
$650		Increase 16%			
£82.50		Decrease 26%			
£1.87		Increase 100%			
£74.50		Decrease 100%			
€2,925		Increase 250%			
£74,000		Decrease 99%			
£5.68		Increase 2.5%			
£100		Decrease 12.5%			
£276.40		Increase 0.4%			
€9.99		Decrease 60%			
£2,000,000		Decrease 1.4%			

Repeated Percentage Changes

Depreciation means the value **decreases**.

Example

Geraint buys a second hand car for **£6,500**.
Every year, the value of the car **depreciates 15%.**
What value will the car have after **three years?**

Inefficient Method

Year 1
£6,500 × 15% = £975
£6,500 – £975 = £5,525
Year 2
£5,525 × 15% = £828.75
£5,525 – £828.75 = £4,696.25
Year 3
£4,696.25 × 15% = £704.44, to the nearest penny.
£4,696.25 – £704.44 = £3,991.81

Better Method

Every year, the value of the car decreases from 100% to 85%.
Year 1
£6,500 × 85% = £5,525
Year 2
£5,525 × 85% = £4,696.25
Year 3
£4,696.25 × 85% = £3,991.81, to the nearest penny.

Efficient Method

Every year, the value of the car decreases from 100% to 85%. This happens three times.
Answer
£6,500 × 85%³ = £3,991.81, to the nearest penny.

Exercise 25

Applying I

Use an efficient method to answer the following questions.

(a) Megan buys a second-hand car for £8,000. Every year, the value of the car depreciates 12%. What value will the car have after three years?

(b) Aled buys a second-hand car for £14,500. Every year, the value of the car depreciates 20%. What value will the car have after five years?

(c) Ffion buys a pair of earrings for £400. Every year, the value of the earrings increases 15%. What value will the earrings have after four years?

(d) Steffan buys antique furniture for £1,500. Every year, the value of the furniture increases 3%. What value will the furniture have after 6 years?

(e) A football team buys a player for £75,000,000. The value of the player increases 24% every year. The club sells the player after three years. What was the sale price of the player after three years?

(f) Steven buys a field for £7,500. For every one of the next 5 years the price increases 10%.
(i) What value will the field have after two years?
(ii) How much does the price increase between the end of the third year and the end of the fifth year?

(g) A tropical rainforest loses 7% of its trees every year.
(i) What percentage of the rainforest will be left after a year?
(ii) What percentage of the rainforest will be left after two years?
(iii) After how many years will there be less than 50% of the current rainforest left?
(iv) How much of the rainforest will be left after 50 years?

(h) Five years ago, Mark bought a house for £180,000. For the first three years, the value of the house increased by 12% each year. For the next two years, the value of the house decreased by 7% each year. What is the value of the house today?

Calculating Compound Interest Efficiently

Example

Nia borrows **£8,500** from NatWest Bank at a compound interest rate of **4% per year**. Nia wants to pay back all the money after **three years.** How much money will Nia have to pay back after three years?

Inefficient method (Year 9)

Year 1
£8,500 × 4% = £340
£8,500 + £340 = £8,840
Year 2
£8,840 × 4% = £353.60
£8,840 + £353.60 = £9,193.60
Year 3
£9,193.60 × 4% = £367.74,
to the nearest penny.
£9,193.60 + £367.74 = £9,561.34

Better Method

Every year, the value of the money increases from 100% to 104%.
Year 1
£8,500 × 104% = £8,840
Year 2
£8,840 × 104% = £9,193.60
Year 3
£9,193.60 × 104% = £9,561.34,
to the nearest penny.

Efficient Method

Every year, the value of the money increases from 100% to 104%.
This happens three times.
Answer
£8,500 × 104%^{3} = £9,561.34,
to the nearest penny.

Exercise 26

Use an efficient method to calculate the answer to the following questions.

(a) Sophie borrows £11,000 from Barclays Bank at a compound interest rate of 3% per year. Sophie wants to pay back all the money after three years. How much money will Sophie have to pay back after three years?

(b) Bryn borrows £6,700 from HSBC Bank at a compound interest rate of 6% per year. Bryn wants to pay back all the money after five years. How much money will Bryn have to pay back after five years?

(c) Owen wants to invest £4,000 into Lloyds Bank at a compound interest rate of 5% per year. Owen wants to withdraw all the money from the bank after three years. How much money will Owen be able to take out after three years?

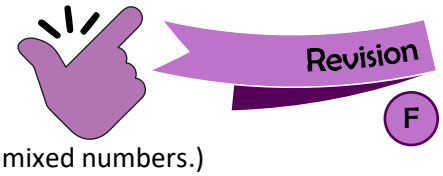
(d) Lorraine wants to invest £25,000 into Barclays Bank at a compound interest rate of 3.4% per year. Lorraine wants to withdraw all the money from the bank after eight years. How much money will Lorraine be able to take out after eight years?

Exercise 27

The following table shows the compound interest rates for different sums of money invested into a bank over a period of time. Complete the table.

	Sum to invest	Compound interest rate per year	Time period for the investment	Calculation to calculate the sum of money at the end	Sum of money at the end	Compound interest earnt
	£5,400	4%	3 years	£5,400 × 104%^{3}	£6,074.27	£674.27
(a)	£2,800	3%	5 years			
(b)	£19,000	2%	4 years			
(c)	£150,000	5%	2 years			
(d)	£8,500	7%	3 years			
(e)	£24,300	2.5%	4 years			
(f)	£100,000	4.8%	5 years			
(g)	£10,000	7.3%	15 years			

Fractional Changes



Exercise 28

Calculate the following. (It is not necessary to convert any improper fractions to mixed numbers.)

(a) $1-\frac{1}{3}$ (b) $1-\frac{1}{7}$ (c) $1-\frac{2}{5}$ (d) $1-\frac{3}{10}$

(e) $1+\frac{1}{3}$ (f) $1+\frac{1}{7}$ (g) $1+\frac{2}{5}$ (h) $1+\frac{3}{10}$

(i) $1-\frac{1}{45}$ (j) $1+\frac{3}{45}$ (k) $1-\frac{14}{25}$ (l) $1+\frac{3}{50}$

Example

A radio station measures how many listeners are listening at least once per week to their morning show between 7am and 10am. Over the past two months, the number of listeners has decreased $\frac{1}{8}$ per month. If the number of listeners was 32,000 two months ago, what is the number of listeners today?

Inefficient method

Month 1

32,000 × $\frac{1}{8}$ = 4,000

32,000 – 4,000 = 28,000

Month 2

28,000 × $\frac{1}{8}$ = 3,500

28,000 – 3,500 = 24,500

Better Method

Instead of calculating $\frac{1}{8}$ of the number and then taking away, we can multiply by $1-\frac{1}{8}=\frac{7}{8}$.

Month 1

32,000 × $\frac{7}{8}$ = 28,000

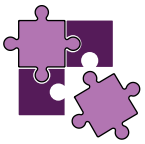
Month 2

28,000 × $\frac{7}{8}$ = 24,500

Efficient Method

We can multiply by $\frac{7}{8}$ twice.

$32{,}000\times\left(\frac{7}{8}\right)^2=24{,}500$



Exercise 29

Use an efficient method to answer the following questions.

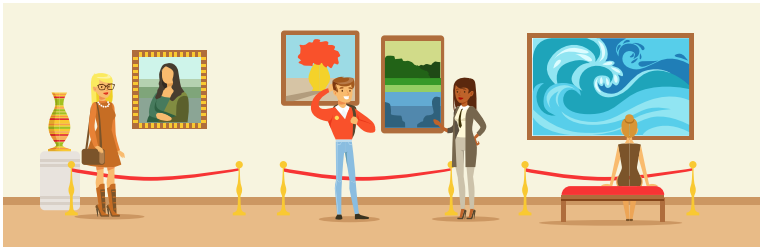
(a) Monthly sales for a magazine have dropped $\frac{1}{10}$ every month over the past three months. If the monthly sales were 60,000 three months ago, what are the monthly sales today?

(b) Boris decides to join a weight loss club. Boris accepts a target to lose $\frac{1}{80}$ of his weight every month, over a period of six months. If Boris weighs 90 kg today, how much weight does Boris need to lose over the next 6 months to reach his target? Write your answer to two decimal places.

(c) At the beginning of last year, 150,000 people visited a website every day. Over the year, the website saw a monthly increase of $\frac{1}{20}$ in their daily visitor numbers. How many people visited the website daily at the end of last year?

(d) A business saw a yearly increase of $\frac{2}{5}$ in product sales over a period of four years. If the yearly sales were £3,400,000 at the beginning of the period, what were the yearly sales at the end of the period?

(e) The number of people who visit a specific gallery in a month has reduced by $\frac{1}{200}$ every month over a period of 8 months. If 40,000 people visited the gallery eight months ago, how many people visited the gallery during the past month?

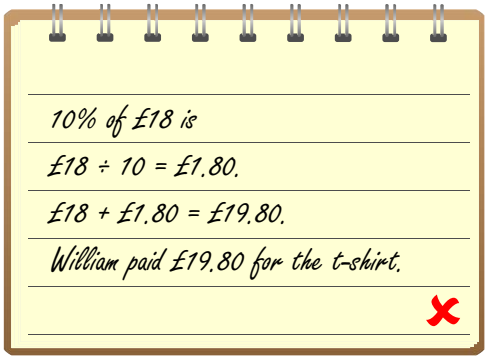


Reverse Percentages

There was a "10% off" sale in a clothes shop. William went into the shop and bought a t-shirt for £18.
If the sale *wasn't* on, how much would William have paid for the t-shirt?

Exercise 30 I

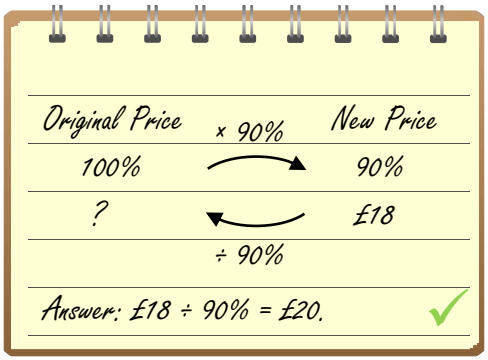
Clumsy Clive attempts to answer the question above on the note paper on the right. Explain why Clive is wrong.



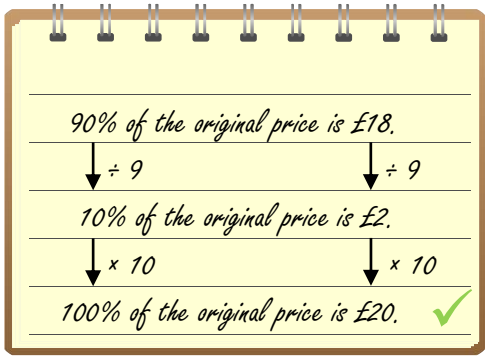
The correct answer

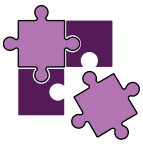
We must consider what happens to the **percentage** to find the correct answer. The price in the sale is 90% of the original price, since the sale subtracts 10% from the original price.

Method 1 (with a calculator)



Method 2 (no calculator)





Exercise 31

Answer the following questions using a calculator.

(a) There was a "10% off" sale in a book shop. Dafydd went into the shop and bought a book for £14.40. How much would Dafydd have paid for the book if the sale was not on?

(b) In a sale there was a discount of 5% off every item. The price of a laptop in the sale was £570. What was the price of the laptop before the sale?

(c) There was a "20% off" sale in a furniture shop. Heledd went into the shop and bought a set of chairs for £320. How much would Heledd have paid if the sale was not on?

Exercise 32

Answer the following questions without using a calculator.

(a) There was a "10% off" sale in a clothes shop. Siwan went into the shop and bought a skirt for £27. How much would Siwan have paid for the skirt if the sale was not on?

(b) In a sale there was a discount of 20% off all items. A set of dishes were on sale for £56. What was the price of the dishes before the sale?

(c) There was a "50% off" closing down sale in a clothes shop. Simon bought a shirt in the sale for £30. What was the price of the shirt before the sale?

Exercise 33 I

Complete the following table.

	Original Price	Percentage Increase	New Price
(a)	£70	15%	
(b)		12%	£67.20
(c)	£250	3%	
(d)		2.5%	$615
(e)	£900	150%	
(f)		0.4%	€251

Exercise 34

Complete the following table.

	Original Price	Percentage Decrease	New Price
(a)	£30	35%	
(b)		18%	£69.70
(c)	$200	9%	
(d)		98%	£40
(e)	€985	3.5%	
(f)		0.25%	£47.88

Exercise 35

(a) Abigail invested in a bond which was increasing at a rate of 8% per year. After one year the value of Abigail's investment was £972. How much was Abigail's original investment?

(b) The cost of a holiday was £600 including VAT at a rate of 20%. What was the price before VAT?

(c) Emily had a pay rise of 6%. Her salary after the pay rise was £25,970. What was her salary before the pay rise?

Exercise 36

(a) Over the last four years, the value of Mr. Davies' car has dropped 10% per year. If Mr. Davies' car is worth £8,000 today, what was it worth four years ago?

(b) Over the last nine years, the value of Mrs. Jones' house has increased 4% per year. If Mrs. Jones' house is worth £140,000 today, how much was her house worth 9 years ago?

Challenge!

There are 36 more girls than boys in a school. 54% of the pupils in the school are girls. How many girls are in the school?

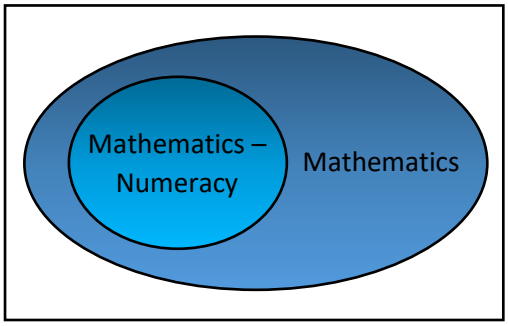
Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Exam Technique

There are two GCSE qualifications available in mathematics:

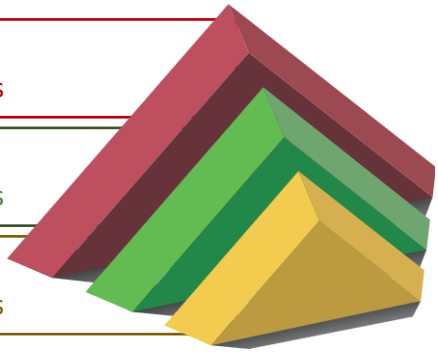
(1) GCSE Mathematics – Numeracy. (2) GCSE Mathematics.



Higher Tier: Grades A*–C
Exam: 1 hour 45 minutes / 80 marks

Intermediate Tier: Grades B–E
Exam: 1 hour 45 minutes / 80 marks

Foundation Tier: Grades D–G
Exam: 1 hour 30 minutes / 65 marks



As you can see in the blue diagram above, the Mathematics – Numeracy qualification is a subset of the Mathematics qualification, which means that the Mathematics exam contains all the topics from the Numeracy exam, and more.

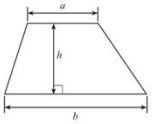
Page 2 of the exam paper always includes the following formulae.

Foundation Tier

2

Formula List - Foundation Tier

Area of trapezium $= \frac{1}{2}(a+b)h$

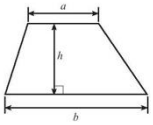


Intermediate Tier

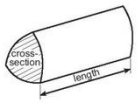
2

Formula List – Intermediate Tier

Area of trapezium $= \frac{1}{2}(a+b)h$



Volume of prism = area of cross-section × length

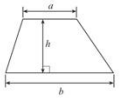


Higher Tier

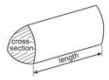
2

Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a+b)h$



Volume of prism = area of cross-section × length



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

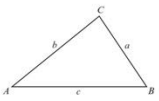
Curved surface area of cone $= \pi r l$

In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

The exams are taken in the following order.

(1) Unit 1 GCSE Mathematics – Numeracy (non-calculator).
(2) Unit 2 GCSE Mathematics – Numeracy (with a calculator).
(3) Unit 1 GCSE Mathematics (non-calculator).
(4) Unit 2 GCSE Mathematics (with a calculator).

Summary of exam content.

Assessment Objective	Numeracy	Mathematics
Recall and use their knowledge of the specified content	15% – 25%	50% – 60%
Select and use mathematical methods	50% – 60%	10% – 20%
Interpret and analyse problems and produce strategies to solve them	20% – 30%	25% – 35%

Assessing Written Communication

There is one question in every exam paper where "the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing". This means that you will get marks not just for the right answer, but for **how** you set out your answer.

For **organisation and communication** marks, you will be expected to:

- Present your answer in a structured way.
- Explain to the examiner what you are doing at each step of your answer.
- Set out your explanations and calculations in a way that is clear and logical.
- Write a conclusion that brings together the results and explains the meaning of the answer.

For **accuracy in writing** marks, you will be expected to:

- Show all of your calculation work.
- Make minimal spelling, punctuation and grammar mistakes.
- Use correct mathematical form in your calculations.
- Use appropriate terminology, appropriate units etc.

To make sure you obtain these marks (usually two marks in each examination paper), split the answer page in half, using the **Sum** and **Explanation** headings. On the left-hand side ("Sum"), write down any mathematical calculations you make. On the right-hand side ("Explanation"), explain why you have made that particular calculation. This needs to be done even if the sum is 1 + 1 = 2.

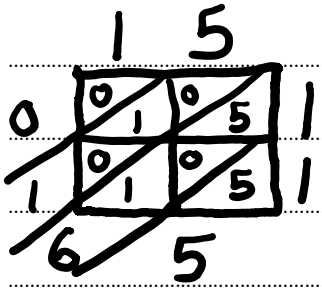
Example

Geraint has a Saturday job to save money for a holiday.
He earns £18 each week.

Geraint saves $\frac{5}{6}$ of the money that he earns each week and spends the rest.

How much money does he save in 11 weeks?
You must show all of your calculations.

Answer:

Sum	Explanation
£18 ÷ 6 = £3 £3 × 5 = £15	Calculate $\frac{1}{6}$ of the wage Calculate $\frac{5}{6}$ of the wage Geraint saves £15 every week.
	11 × £15 = £165, therefore Geraint saves £165 in 11 weeks

Exercise 37 (no calculator allowed)

Applying F

In all of the following questions, **you will be assessed on the quality of your organisation, communication and accuracy in your writing.**

(a) For her birthday, Casey received two gift vouchers for her favourite shop.

She bought a pair of jeans for £26 and two tops at £15.99 each.

Casey used her two gift vouchers to buy the jeans and tops. How much more did she have to pay?
You must show all of your calculations.

(b) A band was hired to play in the local hall.

The hall was hired for 4 hours at a rate of £20 per hour.
The band cost £150 to hire.
The price for a ticket to the event was £5 each. 128 tickets were sold.

Calculate how much money was spent, how much money was collected, and the profit or loss made on the event.

(c) A group of 14 Youth Club members want to go to a Theme Park.

Five of the members are 15 years old.
The rest of the members are younger than 15 years old.

How much money does the group save by going to the theme park on a weekend rather than on a weekday?

(d)

Small pot of daffodils

£1.20

Large pot of daffodils

£ [redacted]

Marjorie spent £20 on daffodils. She received 50p in change.
She bought 10 small pots of daffodils.
The large pots of daffodils were 25% more expensive to buy than the small pots of daffodils.
How many large pots of daffodils did she buy?
You must show all of your calculations.

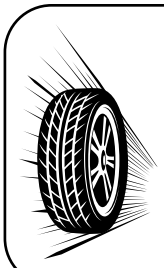
Exercise 38 (calculator allowed) F

In all of the following questions, **you will be assessed on the quality of your organisation, communication and accuracy in your writing.**

(a) Two car rental companies have the following deals available for renting the same model of car.

Just Go Cars

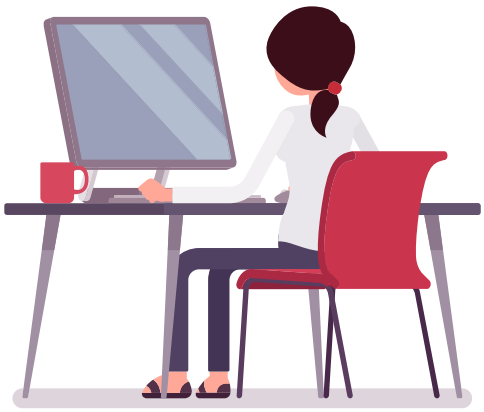
£32 per day
18p per mile
VAT of 20% to be added



Speedy Wheels

£68 per day
No charge for mileage
Includes VAT
Special offer: $\frac{1}{4}$ off

Dylan wants to rent a car for 7 days. He plans to drive 500 miles.
Which company should Dylan choose to get the cheapest bargain?



(b) Bethan's current annual salary is £30,000. After tax and other deductions, she receives 70% of this salary. Over one year, her job means that she travels 8,000 miles. Her car travels at 40 miles per gallon, and a gallon of petrol costs £6.25.

Bethan is considering a new job, working from home. Her new salary would be $\frac{2}{3}$ of her current salary, with the same percentage of deductions.

Calculate the difference, in monetary terms, that changing job would have. You must show all of your calculations.

(c) Here is the cost of buying electricity from *North Electricity.*

- A fixed charge of 28p per day.
- An energy charge of 14p for every kWh used.
- VAT of 5% to pay on the total cost.

Evan uses 850 kWh of electricity over a period of 90 days. Calculate the total bill for Evan to purchase electricity from *North Electricity.*

(d) A paint colour called ochre is made by using a recipe that includes white, red, blue and yellow paint. The percentages of each different colour used to create ochre paint is shown on the right.

Catrin has already bought 2.5 litres of blue paint.
She decides to buy white, red and yellow paint to use with **all** of her blue paint to make as much ochre paint as possible.

The sizes of paint tins available are 1 litre, 2.5 litres and 10 litres.

Only full tins of paint can be purchased.
Catrin only has a small shed to store the paint, therefore she wants minimal white, red and yellow paint left over.

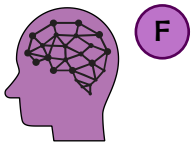
Calculate how much of each paint colour Catrin needs to buy. You must show all of your calculations.

Mark Schemes

After you complete your exam, the paper is sent away to be marked. The markers follow a mark scheme for the exam paper, which uses the following codes for the marks.

Code	Explanation
M	'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
m	'm' marks are dependent method marks. They are only given if the relevant previous 'M' mark has been earned.
A	'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
B	'B' marks are independent of method and are usually awarded for an accurate result or statement.
S	'S' marks are awarded for strategy.
E	'E' marks are awarded for explanation.
U	'U' marks are awarded for units.
P	'P' marks are awarded for plotting points.
C	'C' marks are awarded for drawing curves.

Exercise 39



Here is a past paper question together with its mark scheme.
Mark the four attempts using the mark scheme.

Question

Megan has some blocks. 10% of the blocks are white. $\frac{3}{5}$ of the blocks are red. The remaining blocks are green.
33 blocks are green. How many blocks are there in total? [5 marks]

Mark Scheme

White 10% Red 60% OR 0.6 OR $\frac{6}{10}$ Green 100% – 10% – 60% = 30% OR 0.3 OR $\frac{3}{10}$ 30% of the blocks is 33 (blocks). 100% of the blocks is 110 (blocks).	 B1 M1 A1 M1 A1	 Must use a common measure, i.e. percentages or fractions or decimals. Follow through 'their 30%'.

Candidate 1

Red $\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10} \xrightarrow{\times 10} \frac{60}{100} = 60\%$

Green 100% − 10% − 60%
= 30%

33 blocks is 30% ÷3
11 blocks is 10% ×10
110 blocks is 100%

Candidate 2

$5\overline{)3.0} = 0.6$ ← Red

Green 30%

33 blocks is 30% ÷3
11 blocks is 10% ×10
110 blocks is 100%

Candidate 3

White $\frac{1}{10}$ Red $\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10}$

Green $1 - \frac{7}{10} = \frac{3}{10}$

$33 \times 3 = 99$

$99 \div 10 = 9.9$

10 blocks in total

Candidate 4

$10\% + \frac{3}{5} = \frac{4}{5}$

Green $1 - \frac{4}{5} = \frac{1}{5}$

$= 20\%$

33 blocks is 20% $\times 5$

165 blocks is 100%

$33 \times 5 = 165$

Timing

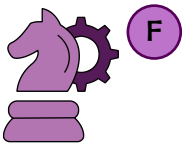
It's a good idea to complete any maths exam paper at a rate of **one mark per minute.** This way, you will have time left over at the end to check your answers.

Foundation Tier	Intermediate and Higher Tier
65 marks in 1 hour 30 minutes, therefore 65 marks in 90 minutes. Working at a rate of one mark per minute, there will be 25 minutes left over at the end to check your answers.	80 marks in 1 hour 45 minutes, therefore 80 marks in 105 minutes. Working at a rate of one mark per minute, there will be 25 minutes left over at the end to check your answers.

What should you do if you have time left over at the end of an exam?

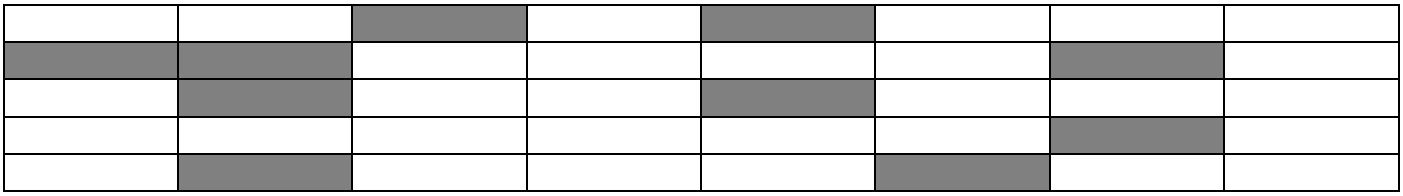
- Check there are no gaps in your exam paper.
- Make sure you have included the correct units, e.g. £, cm², ml.
- Make sure you have shown enough method.
- Make sure you have used the correct equipment, e.g. a ruler for drawing diagrams.
- Attempt to redo the question, perhaps using a different method, to check your answer.

Exercise 40



The following six questions are worth a total of 20 marks. Attempt to complete these in **only 25 minutes**. Mark the questions using the mark scheme provided by your teacher.

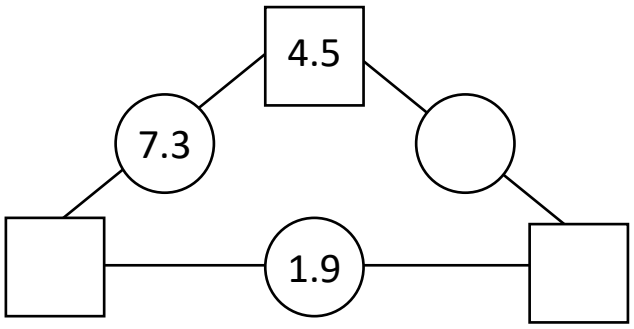
(1) What fraction of the following shape is shaded? Give your answer in its simplest form. [2]



(2) *You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.* [5]

Gwilym bought a camera for £150.
When he sold the camera, he made a loss of 6%.
How much did Gwilym sell his camera for?

(3) The number in the circle is equal to the two numbers in the squares on either side of the circle. [3]
Fill in the missing numbers.



(4) Evaluate $\frac{3}{8} + \frac{1}{2}$. [2]

(5) New pylons are needed in an area in Wales. [3]

- The pylons are in a straight line.
- The distance between the first pylon and the last pylon is 9 km.
- The pylons must be 0.5 km apart from each other.

How many pylons, in total, are needed for a 9 km piece of land?

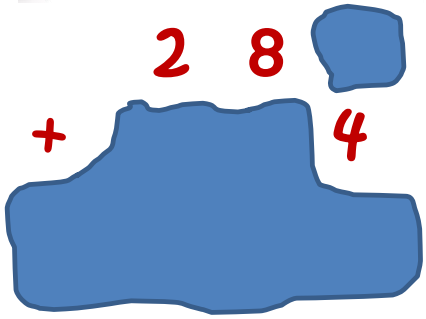
(6) Maria sells ribbon. [5]
The length of the ribbon she has is 400 cm.
Maria cuts off 30% of the ribbon and sells this piece to a customer.
She uses $\frac{2}{5}$ of the **leftover** ribbon to decorate a card.
Then, Maria cuts the leftover ribbon into three equal pieces.
What is the length of each of the three pieces that are left over?

Challenge!

Ink is spilled onto a sum in a textbook.

All of the digits from 0 to 9 have been used in the sum.

Find the correct locations for the other 7 digits.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Fractions, Percentages and Decimals

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test Without a calculator With a calculator	Correct in the test?
I can **calculate using fractions**, e.g. $\frac{3}{11}+\frac{5}{11}$, $\frac{7}{9}-\frac{1}{3}$, $\frac{2}{3}\times\frac{5}{7}$, $\frac{4}{5}\div\frac{2}{9}$.			1	
I can **calculate using percentages**, e.g. 10% of £75, 45% of 140.			5	
I can **calculate using decimals**, e.g. 6.5 + 2.79, 6 – 4.31, 0.4 × 0.3, 14.4 ÷ 6.			1	
I know how to use the **dot notation** for recurring decimals.			2	
I know how to decide whether a fraction is equivalent to a **terminating decimal** or to a **recurring decimal**.				
I can convert a **percentage** to a **decimal**.			3, 4	
I can convert a **percentage** to a **fraction**.			3, 4	
I can convert a **fraction** to a **decimal**.			2, 3, 4	
I can convert a **fraction** to a **percentage**.			3, 4	
I can convert a **decimal** to a **percentage**.			3, 4	
I can convert a **terminating decimal** to a **fraction**.			3, 4	
I can convert a **recurring decimal** to a **fraction**. **(Higher Tier only.)**			7 (H)	
I can calculate percentage changes **efficiently**.				
I can calculate **repeated percentage changes efficiently**.			2	
I can calculate **compound interest efficiently**.			3	
I can calculate **fractional changes efficiently**.				
I know how to answer questions involving **reverse percentages**.			1, 4	
I know how to answer a question where you are "**assessed on the quality of your organisation, communication and accuracy in your writing**".			6, 7 (I)	

The Mathematics Department

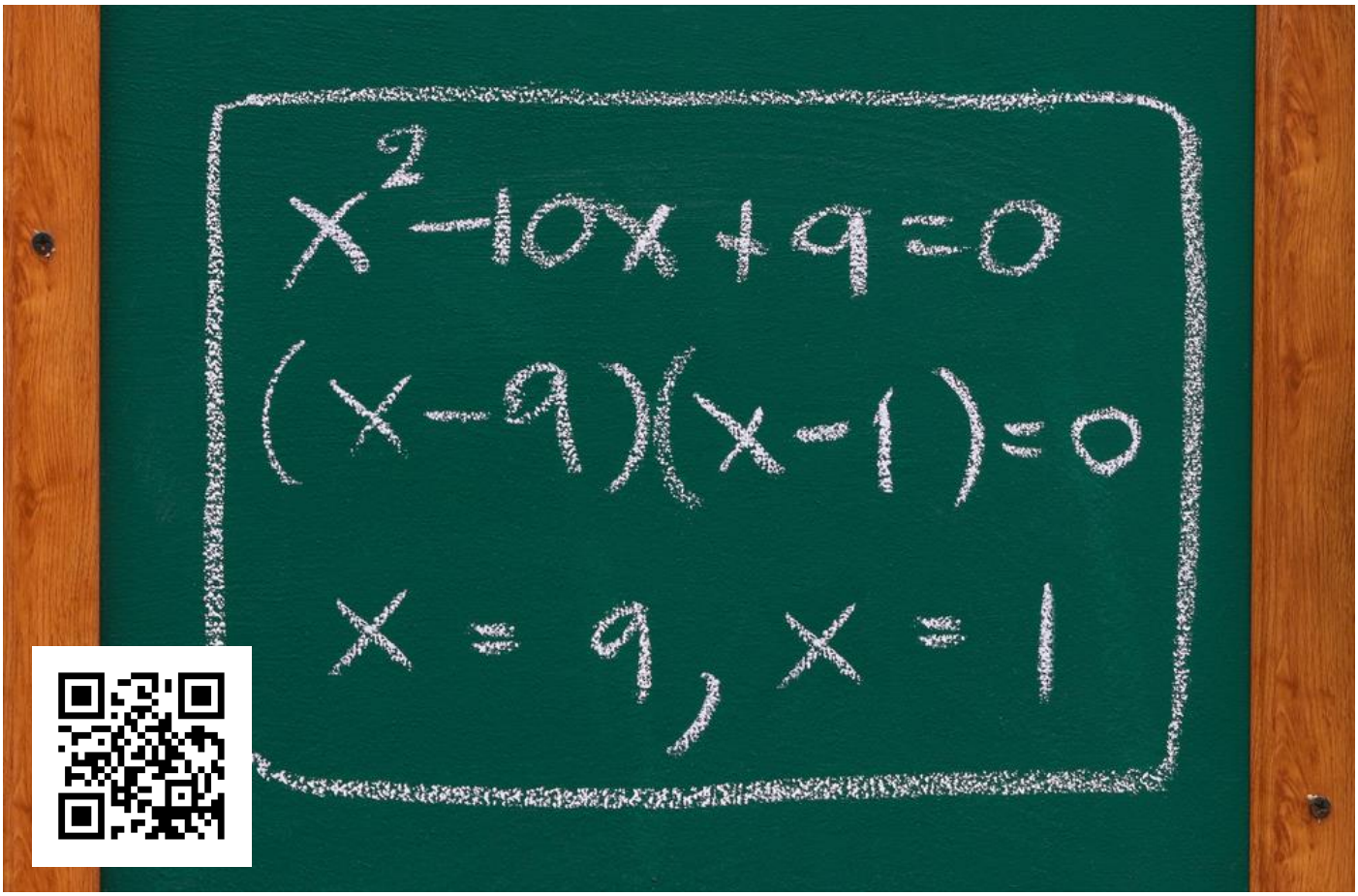
10

Developing Algebra 2

Name:

Contents

Chapter	Mathematics	Page Number
Simple Factorising	Reversing Expansion. Simple Factorising. More than one Variable.	3
Factorising Quadratic Expressions	Preparation. Factorising Quadratic Expressions of the form $x^2 + ax + b$. Solving Quadratic Equations through Factorising. **Factorising Quadratic Expressions of the form $ax^2 + bx + c$. The Difference of Two Squares.**	6
Simultaneous Equations	Multiplying Equations. Subtracting Equations. Multiplying Equations to obtain Equal Coefficients. Solving Linear Equations. Solving Simultaneous Equations.	14
Changing the Subject	Re-arranging formulae.	19
Expression, Equation, Formula, Identity	Recognising expressions, equations, formulae and **identities. Proving identities.**	21



Simple Factorising

Reversing Expansion

Like subtraction reverses addition, and division reverses multiplication, **factorising** reverses **expanding brackets,** a technique seen previously in the **Developing Algebra** workbook.

Exercise 1

Revision F

Expand the following algebraic expressions.

(a) $2(x+3)$ (b) $5(x+3)$ (c) $5(x-3)$

(d) $5(3-x)$ (e) $2(y+3)$ (f) $2(x+3+y)$

(g) $x(x+3)$ (h) $x(3+x)$ (i) $2x(x+3)$

(j) $5x(x+3)$ (k) $5x(2x+3)$ (l) $2x(5x-3)$

(m) $4(x-7)$ (n) $y(y+9)$ (o) $2z(z+4)$

(p) $7x(3x+4)$ (q) $-7x(3x+4)$ (r) $-7x(3x-4)$

Expanding gets rid of brackets, whereas factorising **reintroduces brackets.** Also, whilst expanding using multiplication sums, factorising uses **division** sums. To this intent, an important skill when factorising is to recognise the **highest common factor** of a set of numbers, which is the largest number that divides into the list of numbers.

Exercise 2

What is the highest common factor of the following numbers?

(a) 6 and 8 (b) 12 and 15 (c) 20 and 30 (d) 20 and 40 (e) 18 and 24

(f) 16 and 40 (g) 22 and 33 (h) 24 and 36 (i) 35 and 56 (j) 36 and 54

(k) 12, 14 and 16 (l) 12, 16 and 20 (m) 25, 35 and 45 (n) 30, 45 and 60 (o) 7, 11 and 13

In order to factorise an algebraic expression such as $12x+18$, we start by considering the highest common factor of the terms $12x$ and 18 in the expression.

Example

Factorise $12x+18$.

1. "What is the highest common factor of $12x$ and 18?" The answer is **6**, therefore we write 6 followed by a pair of brackets.

$$12x+18 = 6(\quad)$$

2. "6 multiplied by what gives $12x$?" 6 multiplied by $\mathbf{2x}$ is $12x$, therefore we write $2x$ inside the brackets.

$$12x+18 = 6(2x\quad)$$

3. "6 multiplied by what gives 18?" 6 multiplied by **3** gives 18, therefore we write $+3$ inside the brackets.

$$12x+18 = 6(2x+3)$$

Exercise 3

Factorise the following algebraic expressions.

(a) $4x + 6$	(b) $6 + 4x$	(c) $4x - 6$
(d) $6 - 4x$	(e) $4x + 8$	(f) $4z + 8$
(g) $6x + 8$	(h) $6x + 12$	(i) $12 - 6x$
(j) $9x + 21$	(k) $25x + 30$	(l) $18x + 30$
(m) $14x + 21$	(n) $28 + 14x$	(o) $30x - 40$
(p) $24y + 36$	(q) $60x + 80$	(r) $36x + 45$
(s) $36x + 54$	(t) $33y - 55$	(u) $33y - 66$
(v) $45 + 30x$	(w) $300x + 500$	(x) $80z - 240$
(y) $2x + 4y + 6$	(z) $5x + 10y + 20$	(α) $12x - 20y + 24$

Not all algebraic expressions can be factorised. For example, $5x + 7$ cannot be factorised since the highest common factor of $5x$ and 7 is 1. (We don't factorise $5x + 7$ as $1(5x + 7)$.)

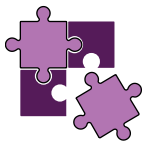
Exercise 4

Factorise all of the algebraic expressions that do factorise, and note which expressions do not factorise.

(a) $5x + 10$	(b) $5x + 11$	(c) $5x + 5$
(d) $16x$	(e) $16x + 2$	(f) $16x + 9$
(g) $8y - 12$	(h) $8y - 13$	(i) $8y - 14$

When factorising, it's not just numbers that can appear before the first bracket – we can include variables like x too.

Example



Factorise $6x^2 + 14x$.

1. "What is the highest common factor of $6x^2$ and $14x$?" The answer is $\mathbf{2x}$, therefore we write $2x$ followed by a pair of brackets.

$$6x^2 + 14x = 2x(\quad)$$

2. "$2x$ multiplied by what gives $6x^2$?" $2x$ multiplied by $\mathbf{3x}$ is $6x^2$, therefore we write $3x$ inside the brackets.

$$6x^2 + 14x = 2x(3x\quad)$$

3. "$2x$ multiplied by what gives $14x$?" $2x$ multiplied by $\mathbf{7}$ gives $14x$, therefore we write $+ 7$ inside the brackets.

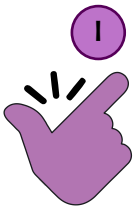
$$6x^2 + 14x = 2x(3x + 7)$$

Exercise 5

What is the highest common factor of the following terms?

(a) $4x^2$ and $18x$	(b) $4x^2$ and $16x$	(c) $5x^2$ and $20x$	(d) $5x^2$ and $7x$	(e) $18x^2$ and $24x$
(f) x^2 and x	(g) x^3 and x^2	(h) $4x^3$ and $18x^2$	(i) $9x^4$ and $3x^2$	(j) $12x^4$ and $15x^3$

Exercise 6



Factorise the following algebraic expressions.

(a) $3x^2 + 6x$ (b) $6x + 3x^2$ (c) $3x^2 - 6x$

(d) $x^2 + x$ (e) $x^3 + x$ (f) $x^3 + x^2$

(g) $4x^2 + 2x$ (h) $2x^3 - 6x$ (i) $2x^3 + 8x^2$

(j) $12x^2 + 4x$ (k) $4x - 12x^2$ (l) $10x^2 + 15x$

(m) $6x^4 + 9x^2$ (n) $24y^3 - 16$ (o) $21z^2 + 14z$

(p) $3x^2 + 5x$ (q) $7y - 11y^2$ (r) $4z^3 + 17z^2$

(s) $22x^2 + 33x^5$ (t) $12n - 4n^2$ (u) $2a + a^2$

(v) $x^6 + 6x$ (w) $x^6 + 6x^2$ (x) $6x^6 + 4x^4$

(y) $2x^3 + 4x^2 + 6x$ (z) $2x^2 + 4x + 6$ (α) $42x^4 - 30x^2 + 12x^3$

Exercise 7

What is the highest common factor of the following terms?

(a) x^2 and xy (b) $4x^2$ and $6xy$ (c) $12x^2y$ and $16xy$ (d) $5x^2y^2$ and $9y^2$ (e) $18x^3y$ and $12x^2y$

Example

$2x^2y + 4x = 2x(xy + 2)$ $30y^3z^2 - 24y^2z = 6y^2z(5yz - 4)$

Exercise 8

Factorise the following algebraic expressions.

(a) $xy + x$ (b) $4xy + 10y$ (c) $x^2y + 5xy$

(d) $2x^2y + 6xy$ (e) $10yz + 5yz^2$ (f) $12x^2y - 4x$

(g) $x^2 + xy^2$ (h) $16x^3z - 12z^2$ (i) $15x^4y + 25x^2y^2$

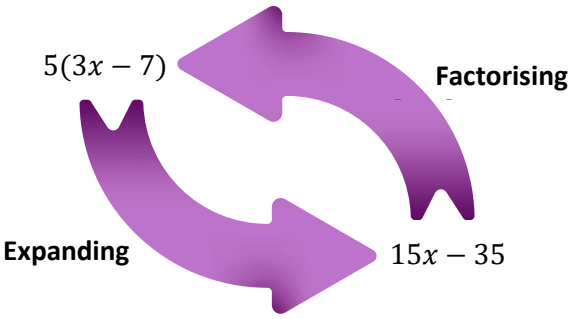
(j) $x^2yz + 4xyz$ (k) $8ab^2c^3 - 18abc^2$ (l) $26\pi x^2 + 65\pi x$

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Factorising Quadratic Expressions

As we saw in the previous chapter, factorising and expanding are two sides of the same coin.



Our aim in this chapter is to develop our understanding of factorising to be able to **factorise quadratic expressions.**

Preparation

To be able to factorise quadratic expressions, we must develop the following skill: given two numbers, for example 7 and 10, is it possible to find two numbers that **add to make 7** and **multiply to make 10**?

$$\square + \square = 7$$
$$\square \times \square = 10$$

In this case, the numbers we are looking for are **2 and 5**, since 2 + 5 = 7, and 2 × 5 = 10.

Exercise 9

Find a pair of numbers that satisfy the following calculations.

(a) $\square + \square = 10$, $\square \times \square = 24$
(b) $\square + \square = 11$, $\square \times \square = 24$
(c) $\square + \square = 14$, $\square \times \square = 24$
(d) $\square + \square = 25$, $\square \times \square = 24$

(e) $\square + \square = 9$, $\square \times \square = 18$
(f) $\square + \square = 11$, $\square \times \square = 18$
(g) $\square + \square = 8$, $\square \times \square = 16$
(h) $\square + \square = 17$, $\square \times \square = 16$

(i) $\square + \square = 13$, $\square \times \square = 30$
(j) $\square + \square = 17$, $\square \times \square = 30$
(k) $\square + \square = 8$, $\square \times \square = 12$
(l) $\square + \square = 7$, $\square \times \square = 12$

(m) $\square + \square = 15$, $\square \times \square = 14$
(n) $\square + \square = 11$, $\square \times \square = 28$
(o) $\square + \square = 5$, $\square \times \square = 4$
(p) $\square + \square = 2$, $\square \times \square = 1$

(q) $\square + \square = 13$, $\square \times \square = 42$
(r) $\square + \square = 23$, $\square \times \square = 42$
(s) $\square + \square = 17$, $\square \times \square = 42$
(t) $\square + \square = 43$, $\square \times \square = 42$

(u) $\square + \square = 32$, $\square \times \square = 60$
(v) $\square + \square = 16$, $\square \times \square = 60$
(w) $\square + \square = 19$, $\square \times \square = 60$
(x) $\square + \square = 17$, $\square \times \square = 60$

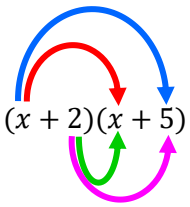
(y) $\square + \square = 16$, $\square \times \square = 55$
(z) $\square + \square = 24$, $\square \times \square = 80$
(α) $\square + \square = 29$, $\square \times \square = 100$
(β) $\square + \square = 36$, $\square \times \square = 99$

Challenge!

(a) $\square + \square = 27$, $\square \times \square = 72$
(b) $\square + \square = 19$, $\square \times \square = 84$
(c) $\square + \square = 22$, $\square \times \square = 96$
(d) $\square + \square = 39$, $\square \times \square = 360$

Factorising Quadratic Expressions of the form $x^2 + ax + b$

In the previous Developing Algebra workbook, we saw how to expand double brackets, for example $(x + 2)(x + 5)$, using the acronym **FOIL**, or by using a **multiplication grid**.



FIRST
OUTSIDE
INSIDE
LAST

$= x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$

Question: $(x + 2)(x + 5)$

	x	2
x	x^2	$2x$
5	$5x$	10

Answer: $x^2 + 2x + 5x + 10$
$= x^2 + 7x + 10$

Exercise 10

Revision

Expand the following algebraic expressions.

(a) $(x + 3)(x + 5)$ (b) $(x + 2)(x + 8)$ (c) $(x - 2)(x + 6)$

(d) $(x - 4)(x - 5)$ (e) $(y + 1)(y + 3)$ (f) $(x + 9)(x - 3)$

Let us now look at the **reverse** process of factorising the quadratic expression $x^2 + 7x + 10$, and finishing with the double bracket $(x + 2)(x + 5)$.

Example

Factorise the quadratic expression $x^2 + 7x + 10$.

Answer: We need to consider the following question: 'Which two numbers add to make 7 (the coefficient of the x term) and multiply to make 10 (the constant)?'.

Add	Multiply
7	10

The answer is 2 and 5, since 2 + 5 = 7, and 2 × 5 = 10. **Therefore**, $x^2 + 7x + 10$ factorises to give $(x + 2)(x + 5)$.

Exercise 11

Skill

Factorise the following algebraic expressions.

(a) $x^2 + 6x + 8$ (b) $x^2 + 7x + 12$ (c) $x^2 + 8x + 12$

(d) $x^2 + 8x + 15$ (e) $x^2 + 16x + 15$ (f) $x^2 + 2x + 1$

(g) $x^2 + 11x + 18$ (h) $x^2 + 9x + 18$ (i) $x^2 + 19x + 18$

(j) $x^2 + 12x + 20$ (k) $x^2 + 9x + 20$ (l) $x^2 + 21x + 20$

(m) $x^2 + 18x + 32$ (n) $x^2 + 12x + 32$ (o) $x^2 + 33x + 32$

(p) $x^2 + 11x + 24$ (q) $x^2 + 25x + 24$ (r) $x^2 + 10x + 24$

(s) $x^2 + 14x + 33$ (t) $x^2 + 15x + 36$ (u) $x^2 + 16x + 39$

(v) $x^2 + 17x + 42$ (w) $x^2 + 26x + 48$ (x) $x^2 + 15x + 50$

(y) $x^2 + 16x + 60$ (z) $x^2 + 19x + 60$ (α) $x^2 + 23x + 60$

Exercise 12

I

Factorise the following algebraic expressions.

(a) $x^2 + 10x + 25$ (b) $x^2 + 25 + 10x$ (c) $25 + 10x + x^2$

(d) $x^2 + 40 + 14x$ (e) $40 + 13x + x^2$ (f) $22x + 40 + x^2$

Example

Factorise the following algebraic expressions.

(a) $x^2 + 2x - 15$ (b) $x^2 - 2x - 15$ (c) $x^2 - 8x + 15$

Add	Multiply
2	–15

$\boxed{-3} + \boxed{5} = 2$

$\boxed{-3} \times \boxed{5} = -15$

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Add	Multiply
–2	–15

$\boxed{-5} + \boxed{3} = -2$

$\boxed{-5} \times \boxed{3} = -15$

$x^2 - 2x - 15 = (x - 5)(x + 3)$

Add	Multiply
–8	15

$\boxed{-3} + \boxed{-5} = -8$

$\boxed{-3} \times \boxed{-5} = 15$

$x^2 - 8x + 15 = (x - 3)(x - 5)$

Exercise 13

Factorise the following algebraic expressions.

(a) $x^2 + 4x - 12$ (b) $x^2 - 4x - 12$ (c) $x^2 + x - 12$

(d) $x^2 - x - 12$ (e) $x^2 + 23x - 24$ (f) $x^2 - 23x - 24$

(g) $x^2 + 10x - 24$ (h) $x^2 - 10x - 24$ (i) $x^2 + 5x - 24$

(j) $x^2 - 5x - 24$ (k) $x^2 + 2x - 24$ (l) $x^2 - 2x - 24$

(m) $x^2 + 39x - 40$ (n) $x^2 - 39x - 40$ (o) $x^2 + 18x - 40$

(p) $x^2 - 18x - 40$ (q) $x^2 + 6x - 40$ (r) $x^2 - 6x - 40$

(s) $x^2 + 3x - 40$ (t) $x^2 - 3x - 40$ (u) $x^2 + 4x - 32$

(v) $x^2 - 4x - 32$ (w) $x^2 - 14x - 32$ (x) $x^2 + 14x - 32$

(y) $x^2 - 31x - 32$ (z) $x^2 + 31x - 32$ (α) $x^2 - 2x - 8$

Exercise 14

Factorise the following algebraic expressions.

(a) $x^2 - 7x + 12$ (b) $x^2 - 8x + 12$ (c) $x^2 - 13x + 12$

(d) $x^2 - 10x + 24$ (e) $x^2 - 11x + 24$ (f) $x^2 - 25x + 24$

(g) $x^2 - 14x + 40$ (h) $x^2 - 13x + 40$ (i) $x^2 - 22x + 40$

(j) $x^2 - 33x + 32$ (k) $x^2 - 12x + 32$ (l) $x^2 - 18x + 32$

(m) $x^2 - 5x + 6$ (n) $x^2 - 6x + 9$ (o) $x^2 - 11x + 18$

Challenge!

Factorise the following algebraic expressions.

(a) $x^2 + 4x - 96$ (b) $x^2 - 5x - 84$ (c) $x^2 + x - 240$

Exercise 15

I

Factorise the following algebraic expressions.

(a) $x^2 + 8x + 16$ (b) $x^2 - 8x + 16$ (c) $x^2 + 10x + 16$

(d) $x^2 - 10x + 16$ (e) $x^2 + 6x - 16$ (f) $x^2 - 6x - 16$

(g) $x^2 + 17x + 16$ (h) $x^2 + 15x - 16$ (i) $x^2 - 17x + 16$

(j) $x^2 + 11x + 28$ (k) $x^2 + 16x + 28$ (l) $x^2 + 29x + 28$

(m) $x^2 - 11x + 28$ (n) $x^2 - 12x - 28$ (o) $x^2 + 27x - 28$

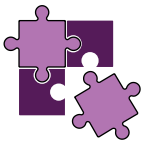
(p) $x^2 - 3x - 28$ (q) $x^2 - 16x + 28$ (r) $x^2 - 27x - 28$

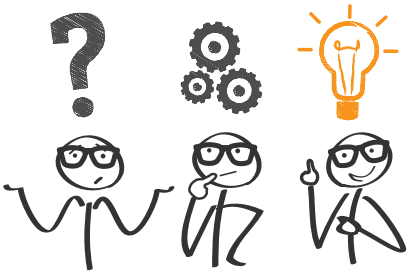
(s) $x^2 + 7x + 10$ (t) $x^2 + 11x + 10$ (u) $x^2 - 7x + 10$

(v) $x^2 + 9x - 10$ (w) $x^2 - 9x - 10$ (x) $x^2 - 11x + 10$

(y) $x^2 + x - 20$ (z) $x^2 - 8x - 20$ (α) $x^2 - 21x + 20$

Solving Quadratic Equations through Factorising





Example

Solve the quadratic equation $x^2 + 6x + 8 = 0$.

Step 1: Factorise.

Add	Multiply
6	8

$\boxed{2} + \boxed{4} = 6$

$\boxed{2} \times \boxed{4} = 8$

$x^2 + 6x + 8 = (x + 2)(x + 4)$

Step 2: Solve.

$x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

Either $x + 2 = 0$ or $x + 4 = 0$

$x = -2$ $x = -4$

Exercise 16

Solve the following quadratic equations.

(a) $x^2 + 5x + 6 = 0$ (b) $x^2 + 9x + 14 = 0$ (c) $x^2 + 12x + 27 = 0$

(d) $x^2 - 5x + 6 = 0$ (e) $x^2 - 9x + 14 = 0$ (f) $x^2 - 12x + 27 = 0$

(g) $x^2 + x - 6 = 0$ (h) $x^2 - 5x - 14 = 0$ (i) $x^2 + 6x - 27 = 0$

(j) $x^2 - x - 6 = 0$ (k) $x^2 + 5x - 14 = 0$ (l) $x^2 - 6x - 27 = 0$

(m) $x^2 + 7x + 6 = 0$ (n) $x^2 - 15x + 14 = 0$ (o) $x^2 - 26x - 27 = 0$

(p) $x^2 - 5x - 6 = 0$ (q) $x^2 + 13x - 14 = 0$ (r) $x^2 + 26x - 27 = 0$

Exercise 17

Applying

Wilf thinks of a number, x.

His sister says that if Wilf subtracts 6 from his number and then multiplies this new number with the number he first thought of, he will get an answer of –5.

Use this information to do the following.

(a) Form a quadratic equation of the form $x^2 + ax + b = 0$.

(b) Solve the equation to find the possible values of x.

Factorising Quadratic Expressions of the form $ax^2 + bx + c$

Higher Tier

Example

Factorise the quadratic expression $2x^2 + 11x + 12$.

Method 1: The Splitting Method

1. Multiply 2 (the coefficient of the x^2 term) by 12 (the constant) to obtain 24. We must look for a pair of numbers that add to give 11 (the coefficient of the x term) and multiply to give 24.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$\square + \square = 11$$
$$\square \times \square = 24$$

2. The numbers which work are 3 and 8. We rewrite the question by splitting the $11x$ to be $3x$ add $8x$.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$3 + 8 = 11$$
$$3 \times 8 = 24$$
$$2x^2 + 3x \quad + 8x + 12$$

3. We split the four terms into two halves and factorise the first half.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$3 + 8 = 11$$
$$3 \times 8 = 24$$
$$2x^2 + 3x \mid + 8x + 12$$
$$= x(2x + 3)$$

4. We copy the brackets, leaving a space between the old brackets and the new brackets.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$3 + 8 = 11$$
$$3 \times 8 = 24$$
$$2x^2 + 3x \mid + 8x + 12$$
$$= x(2x + 3) \quad (2x + 3)$$

5. $+ 4$ must appear in the space, since 4 multiplied by $2x$ is $8x$, and 4 multiplied by 3 is 12.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$3 + 8 = 11$$
$$3 \times 8 = 24$$
$$2x^2 + 3x \mid + 8x + 12$$
$$= x(2x + 3) + 4(2x + 3)$$

6. The expression has a common factor of $2x + 3$, therefore we factorise this out to leave the final answer.

$$2x^2 + 11x + 12$$
$$2 \times 12 = 24$$
$$3 + 8 = 11$$
$$3 \times 8 = 24$$
$$2x^2 + 3x \mid + 8x + 12$$
$$= x(2x + 3) + 4(2x + 3)$$
$$= (2x + 3)(x + 4)$$

Method 2: The Detective Method

1. Write a pair of brackets. At the start of the brackets, write a pair of terms that multiply to give $2x^2$.

$$2x^2 + 11x + 12$$
$$= (2x \quad)(x \quad)$$

2. At the end of the brackets, write a pair of terms that multiply to give 12. Use (e.g.) FOIL (in your head or on paper) to check your answer.

$$2x^2 + 11x + 12$$
$$= (2x + 2)(x + 6)$$

FOIL:
$$(2x + 2)(x + 6)$$
$$= 2x^2 + 12x + 2x + 12$$
$$= 2x^2 + 14x + 12 \quad ✘$$

3. If the answer is not correct, choose a different combination, repeating until you reach the correct answer.

$$2x^2 + 11x + 12$$
$$= (2x + 3)(x + 4)$$

FOIL:
$$(2x + 3)(x + 4)$$
$$= 2x^2 + 8x + 3x + 12$$
$$= 2x^2 + 11x + 12 \quad ✓$$

Exercise 18

Skill H

Factorise the following quadratic expressions.

(a) $2x^2 + 11x + 15$ (b) $2x^2 + 13x + 15$ (c) $2x^2 + 7x + 6$

(d) $3x^2 + 13x + 4$ (e) $3x^2 + 11x + 10$ (f) $3x^2 + 17x + 20$

(g) $4x^2 + 21x + 5$ (h) $4x^2 + 9x + 5$ (i) $4x^2 + 12x + 5$

(j) $5x^2 + 18x + 9$ (k) $5x^2 + 8x + 3$ (l) $6x^2 + 13x + 6$

(m) $2x^2 - x - 15$ (n) $3x^2 + x - 14$ (o) $5x^2 - 17x - 12$

(p) $3x^2 - 5x - 12$ (q) $4x^2 - 3x - 10$ (r) $2x^2 - 7x - 15$

(s) $4x^2 - 7x - 2$ (t) $3x^2 - 16x - 12$ (u) $4x^2 + 21x - 18$

(v) $3x^2 - 14x + 8$ (w) $5x^2 - 19x + 12$ (x) $3x^2 - 26x + 35$

(y) $2x^2 - 21x + 40$ (z) $2x^2 - 11x + 12$ (α) $4x^2 - 11x + 6$

Challenge!

Factorise the following quadratic expressions.

(a) $8x^2 - 2x - 15$ (b) $8x^2 - 19x + 6$ (c) $30x^2 - 42x + 12$

Example

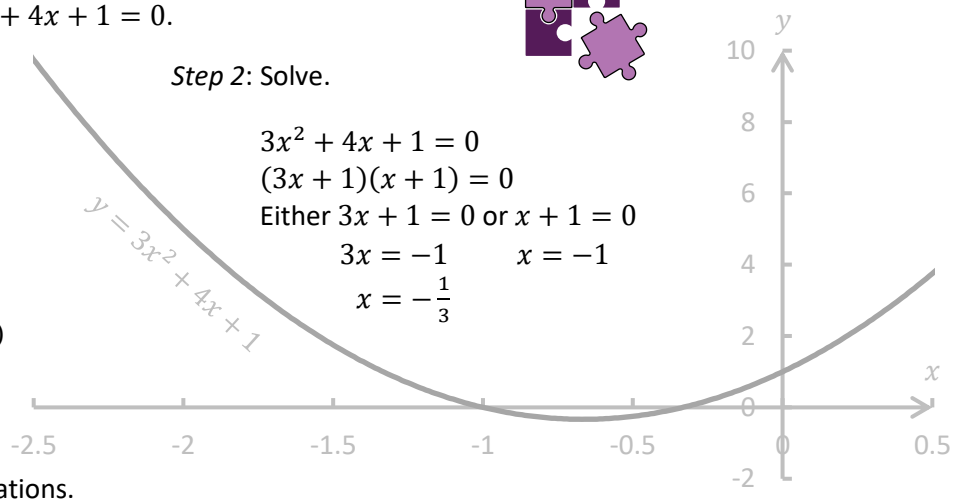
Solve the quadratic equation $3x^2 + 4x + 1 = 0$.

Step 1: Factorise.

$3 \times 1 = 3$.
$\boxed{1} + \boxed{3} = 4$
$\boxed{1} \times \boxed{3} = 3$
$3x^2 + 4x + 1$
$= 3x^2 + x + 3x + 1$
$= x(3x + 1) + 1(3x + 1)$
$= (3x + 1)(x + 1)$

Step 2: Solve.

$3x^2 + 4x + 1 = 0$
$(3x + 1)(x + 1) = 0$
Either $3x + 1 = 0$ or $x + 1 = 0$
$3x = -1$ $x = -1$
$x = -\frac{1}{3}$



Exercise 19

Solve the following quadratic equations.

(a) $2x^2 + 3x + 1 = 0$ (b) $2x^2 + 5x + 2 = 0$ (c) $2x^2 + 13x + 20 = 0$

(d) $2x^2 - 3x - 20 = 0$ (e) $2x^2 + 3x - 20 = 0$ (f) $2x^2 - 13x + 20 = 0$

(g) $3x^2 + 10x + 7 = 0$ (h) $3x^2 + 7x + 2 = 0$ (i) $3x^2 - 11x + 6 = 0$

(j) $2x^2 - 7x + 3 = 0$ (k) $2x^2 + 3x - 5 = 0$ (l) $2x^2 - 11x + 5 = 0$

(m) $4x^2 - 4x + 1 = 0$ (n) $4x^2 - 11x - 3 = 0$ (o) $5x^2 - 24x - 5 = 0$

(p) $6x^2 + x - 2 = 0$ (q) $6x^2 - 7x - 5 = 0$ (r) $15x^2 - 4x - 3 = 0$

Challenge!

Solve the following quadratic equations.

(a) $12x^2 + 28x - 5 = 0$ (b) $28x^2 + 15x + 2 = 0$ (c) $24x^2 - 2x - 15 = 0$

Example

Factorise the following quadratic expressions.

(a) $2x^2 + 20x + 42$

Answer: $2x^2 + 20x + 42$
$= 2(x^2 + 10x + 21)$
$= 2(x + 3)(x + 7)$

(b) $(x + 5)^2 + 8(x + 5)$

Answer: $(x + 5)^2 + 8(x + 5)$
$= (x + 5)((x + 5) + 8)$
$= (x + 5)(x + 13)$

(c) $2x^2 + 8x$

Answer: $2x^2 + 8x$
$= 2x(x + 4)$

Exercise 20 H

Factorise the following quadratic expressions.

(a) $2x^2 + 22x + 56$ (b) $(x + 3)^2 + 7(x + 3)$ (c) $2x^2 + 20x$

(d) $3x^2 + 18x + 24$ (e) $(x - 5)^2 + 8(x - 5)$ (f) $3x^2 - 12x$

(g) $4x^2 + 12x - 40$ (h) $(x - 2)^2 - 4(x - 2)$ (i) $4x^2 - 18x$

(j) $4x^2 + 26x + 30$ (k) $7(x + 4)^2 + 3(x + 4)$ (l) $5x^2 + 45x$

The Difference of Two Squares

An expression of the form $a^2 - b^2$ factorises in a special way.

$$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$$

Example

Factorise the following expressions.

(a) $x^2 - 9$

Answer: $x^2 - 9$
$= (x + 3)(x - 3)$

(b) $4x^2 - 49$

Answer: $4x^2 - 49$
$= (2x + 7)(2x - 7)$

(c) $27x^2 - 75y^2$

Answer: $27x^2 - 75y^2$
$= 3(9x^2 - 25y^2)$
$= 3(3x + 5y)(3x - 5y)$

Exercise 21 H

Factorise the following expressions.

(a) $x^2 - 4$ (b) $x^2 - 16$ (c) $x^2 - 1$

(d) $x^2 - 144$ (e) $y^2 - 100$ (f) $z^2 - 36$

(g) $4x^2 - 25$ (h) $9x^2 - 4$ (i) $49x^2 - 81$

(j) $64x^2 - 1$ (k) $100y^2 - 9$ (l) $16z^2 - 121$

(m) $2x^2 - 18$ (n) $2x^2 - 50$ (o) $3x^2 - 48$

(p) $8x^2 - 18$ (q) $6x^2 - 24$ (r) $5x^2 - 125$

(s) $x^2 - y^2$ (t) $4x^2 - z^2$ (u) $x^2y^2 - 1$

(v) $16x^2 - \pi^2$ (w) $8x^2 - 72z^2$ (x) $4x^2z^2 - 36y^2$

(y) $x^4 - 4$ (z) $9y^4 - 16$ (α) $32z^6 - 128y^2$

Exercise 22

H

Solve the following quadratic equations.

(a) $x^2 - 25 = 0$ (b) $y^2 - 64 = 0$ (c) $z^2 - 169 = 0$

(d) $4x^2 - 49 = 0$ (e) $9x^2 - 1 = 0$ (f) $4x^2 - 16 = 0$

Exercise 23

The area of the rectangle on the right is 45 cm². Use the difference of two squares method to calculate the height and width of the rectangle.

Exercise 24 (Revision)

Solve the following quadratic equations.

(a) $x^2 + 15x + 44 = 0$ (b) $x^2 - 15x + 44 = 0$ (c) $x^2 + 7x - 44 = 0$

(d) $4x^2 + 14x = 0$ (e) $4x^2 - 14x = 0$ (f) $14x - 4x^2 = 0$

(g) $2x^2 + 13x + 21 = 0$ (h) $2x^2 - 13x + 21 = 0$ (i) $2x^2 + x - 21 = 0$

(j) $x^2 - 36 = 0$ (k) $9x^2 - 100 = 0$ (l) $4x^2 - 36 = 0$

Challenge!

The picture below shows Menai Bridge. We can model the cable between the two towers using the quadratic equation $y = \frac{43}{7744}x^2 - \frac{43}{44}x$. Given that the origin is at the highest point of one of the towers, solve the equation $\frac{43}{7744}x^2 - \frac{43}{44}x = 0$ to calculate the horizontal distance (in metres) between the top of the two towers.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Simultaneous Equations

Our aim in this chapter is to solve problems similar to the one below.

"Deiniol buys 2 *fish* and 3 *chips* in the local *fish & chips* shop, and he pays £8. Awel buys 4 *fish* and 2 *chips* in the same shop, and pays £12. What is the cost of 1 *fish* and 1 *chips* in the shop?"

By using the variable f to represent the cost of 1 *fish*, and c to represent the cost of 1 *chips*, we can write the following equations to represent the problem.

$2f + 3c = 8$
$4f + 2c = 12$

To solve the above equations, which are **simultaneous equations,** we must develop a number of **algebraic techniques** for our algebraic *toolbox* …

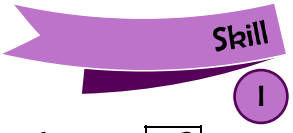
Multiplying Equations

Example

Multiply the equation $3x + 2y = 5$ by 4.

Answer: We multiply every term in the equation by four to obtain the equation $12x + 8y = 20$.

Exercise 25



Multiply the following equations by the numbers in the boxes.

(a) $2x + 6y = 4$	$\times 2$	(b) $3x + 4y = 5$	$\times 2$	(c) $7x + 2y = 6$	$\times 2$
(d) $4x + 3y = 7$	$\times 3$	(e) $8x + 11y = 3$	$\times 4$	(f) $6x + 3y = 11$	$\times 5$
(g) $2x - 5y = 3$	$\times 2$	(h) $-5x + 2y = 4$	$\times 2$	(i) $3x - 8y = -7$	$\times 2$
(j) $x + 7y = 3$	$\times 6$	(k) $8x + y = 9$	$\times 7$	(l) $3x - 4y = 1$	$\times 8$
(m) $3x + 6y = 7$	$\times -2$	(n) $5x - 2y = 10$	$\times -3$	(o) $-3x + 2y = -5$	$\times -4$

Subtracting Equations

Example

$$\begin{array}{rr} & 5x + 8y = 16 \\ - & 2x + 3y = 7 \\ \hline & 3x + 5y = 9 \end{array} \qquad \begin{array}{rr} & 7x + 9y = 25 \\ - & 3x + 9y = 4 \\ \hline & 4x \quad\;\; = 21 \end{array} \qquad \begin{array}{rr} & 5x + 6y = 10 \\ - & 5x - 2y = 3 \\ \hline & 8y = 7 \end{array}$$

Exercise 26

Subtract the second equation from the first equation.

(a) $7x + 8y = 20$ $3x + 4y = 12$	(b) $9x + 5y = 13$ $3x + 2y = 5$	(c) $15x + 9y = 14$ $12x + 4y = 9$
(d) $6x + 2y = 31$ $2x + 7y = 8$	(e) $8x + 3y = 15$ $2x + \;\; y = 10$	(f) $19x - 8y = 8$ $4x + 4y = 3$

Exercise 26 (continued)

(g) $8x + 7y = 15$
$2x + 7y = 6$

(h) $11x + 4y = 27$
$11x + 2y = 3$

(i) $20x + 5y = -8$
$18x - 5y = 4$

(j) $-4x + 8y = 18$
$4x + 2y = 4$

(k) $6x - 4y = 8$
$2x - 4y = 10$

(l) $8x - 4y = 15$
$8x - 6y = 2$

(m) $x + 18y = 12$
$x + 17y = 2$

(n) $18x - 2y = -5$
$11x - 9y = 3$

(o) $-4x + 10y = 8$
$-4x - 2y = -3$

Multiplying Equations to Obtain Equal Coefficients

The **coefficient** of an algebraic term is the **number** which is part of the term. For example, the coefficient of $16x$ is 16.

Example

Consider the following simultaneous equations.

$3x + 10y = 16$
$4x + 5y = 13$

By multiplying the first equation by 4, and the second equation by 3, we can ensure that the x **coefficients** are equal.

$3x + 10y = 16 \xrightarrow{\times 4} 12x + 40y = 64$
$4x + 5y = 13 \xrightarrow{\times 3} 12x + 15y = 39$

On the other hand, by leaving the first equation as it is, and multiplying the second equation by 2, we can ensure that the y **coefficients** are equal.

$3x + 10y = 16$ $\quad\quad$ $3x + 10y = 16$
$4x + 5y = 13 \xrightarrow{\times 2}$ $8x + 10y = 26$

Try to use the **smallest possible** numbers that work.

Exercise 27

Which numbers do we need to multiply the following equations by to obtain equal x coefficients?

(a) $2x + 4y = 4$
$3x + 5y = 7$

(b) $2x + 4y = 4$
$4x + 12y = 7$

(c) $2x + 4y = 4$
$5x + 8y = 7$

(d) $5x + 7y = 6$
$2x + 2y = 8$

(e) $5x + 7y = 6$
$6x + 3y = 8$

(f) $5x + 7y = 6$
$15x + 14y = 8$

(g) $4x + 8y = 12$
$5x + 4y = 5$

(h) $4x + 8y = 12$
$6x + 3y = 5$

(i) $4x + 8y = 12$
$12x + 12y = 5$

(j) $24x + 32y = 20$
$8x + 8y = 20$

(k) $24x + 32y = 20$
$12x + 64y = 20$

(l) $24x + 32y = 20$
$16x + 16y = 20$

Exercise 28

Which numbers do we need to multiply the equations from Exercise 27 by to obtain equal y coefficients?

Solving Linear Equations

The final part of the jigsaw is to be able to solve linear equations like the following ones.

Exercise 29

Solve the following equations.

(a) $4x = 8$

(b) $4x = 32$

(c) $7x = 35$

(d) $4x = 2$

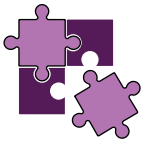
(e) $3x = 5$

(f) $8x = 7$

Solving Simultaneous Equations

We now have enough tools in our algebraic *toolbox* to return to the *fish & chips* problem from the beginning of this chapter.

"Deiniol buys 2 *fish* and 3 *chips* in the local *fish & chips* shop, and he pays £8. Awel buys 4 *fish* and 2 *chips* in the same shop, and pays £12. What is the cost of 1 *fish* and 1 *chips* in the shop?"



Step 1: Change the word problem into a pair of equations.

$2f + 3c = 8$
$4f + 2c = 12$

Step 2: Multiply the first equation by 2 so that the f coefficients are equal.

$$\begin{array}{ll} 2f + 3c = 8 & \xrightarrow{\times 2} \quad 4f + 6c = 16 \\ 4f + 2c = 12 & \qquad\qquad\quad 4f + 2c = 12 \end{array}$$

Step 3: Subtract the second equation from the first equation.

$$\begin{array}{lll} 2f + 3c = 8 & \xrightarrow{\times 2} & 4f + 6c = 16 \\ 4f + 2c = 12 & & -\ 4f + 2c = 12 \\ & & \hline & & 4c = 4 \end{array}$$

Step 4: Solve the equation $4c = 4$ to obtain $c = 1$.

Conclusion: The cost of 1 *chips* in the shop is £1.

To find the value of f (and thus the cost of 1 *fish*), we can use any of the following methods.

Method A: Repeat steps 2–4 above, but this time making sure that the c coefficients are equal.

$$\begin{array}{lll} 2f + 3c = 8 & \xrightarrow{\times 2} & 4f + 6c = 16 \\ 4f + 2c = 12 & \xrightarrow{\times 3} & -\ 12f + 6c = 36 \\ & & \hline & & -\ 8f \quad\ = -20 \end{array}$$

$$f = \frac{-20}{-8}$$
$$f = \frac{5}{2}$$

Method B: Substitute $c = 1$ into one of the original equations.

Substitute $c = 1$ into the equation $2f + 3c = 8$:

$2f + 3 \times 1 = 8$
$2f + 3 = 8$
$2f = 5$
$f = \frac{5}{2}$

Conclusion: The cost of 1 *fish* is £2.50.

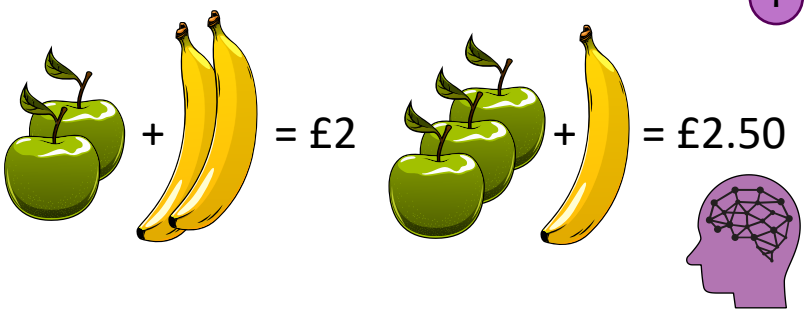
Note: You can check that the solutions are correct by substituting the values $c = 1$, $f = 2.5$ into the left-hand side of any of the original equations.

$2f + 3c$
$= 2 \times 2.5 + 3 \times 1$
$= 5 + 3$
$= £8$ ✓

$4f + 2c$
$= 4 \times 2.5 + 2 \times 1$
$= 10 + 2$
$= £12$ ✓

Exercise 30

Two apples and two bananas costs £2.
Three apples and one banana costs £2.50.
Find the cost of one apple and one banana.



Exercise 31

Skill 1

Solve the following simultaneous equations.

(a) $3x + 4y = 18$
$2x + 2y = 10$

(b) $2x + 3y = 9$
$4x + y = 13$

(c) $2x + 4y = 16$
$2x + 3y = 14$

(d) $5x - 2y = 6$
$2x + 2y = 8$

(e) $6x + 3y = 18$
$-2x + 2y = 6$

(f) $-2x + y = -2$
$4x - 3y = 0$

(g) $2x + 3y = 3$
$2x - y = 7$

(h) $3x + 2y = 7$
$3x - y = -8$

(i) $2x + 3y = 14$
$3x + 2y = 16$

(j) $-3x + 2y = 0$
$3x - 4y = 6$

(k) $x + 5y = 9$
$2x + 3y = 11$

(l) $5x - 2y = 19$
$3x + y = 18$

(m) $4x + y = 8$
$7x + 3y = 9$

(n) $2x - 3y = 8$
$x + 2y = -10$

(o) $2x + 6y = 34$
$4x - 2y = 5$

(p) $2x + 3y = 10$
$5x - 6y = 16$

(q) $2x + 3y = 0$
$8x + 9y = -1$

(r) $7x + 8y = 19$
$3x - 2y = -19$

(s) $3x + 4y = 15$
$x - 6y = -6$

(t) $3x - 4y = 14$
$5x - 8y = 30$

(u) $3x + 5y = 21$
$4x + 3y = 17$

(v) $3x - 2y = 17$
$2x + 7y = 3$

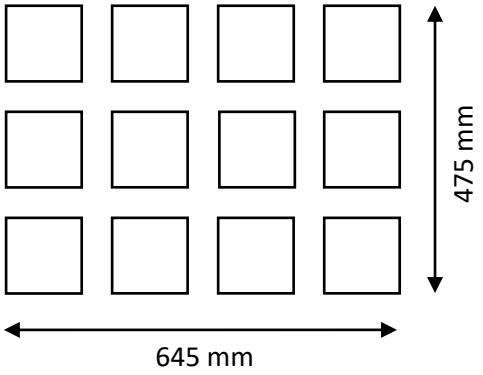
(w) $5x - 2y = 26$
$3x - 5y = 27$

(x) $2x + 4y = 5$
$5x + 7y = 8$

Exercise 32

(a) Aled buys 2 Cornish pasties and 3 sausage rolls in a shop, and pays £7. Ceinwen buys 4 Cornish pasties and 1 sausage roll in the same shop, and pays £9. What is the cost of 1 Cornish pasty and 1 sausage roll from the shop?

(b) A rectangle shape is made by using 12 square tiles with equal spaces between them. The total length of the rectangle is 645 mm and the total width of the rectangle is 475 mm. Find the dimensions of the tiles and the width of the space in mm.



(c) Glyn employs two people, Ben and Ceri. Ben and Ceri are paid at different hourly rates. Glyn has recorded how many hours both Ben and Ceri have worked on Monday and Tuesday. He has also noted the total he paid in wages.

Day	Number of hours worked		Total wages (£)
	Ben	Ceri	
Monday	6	5	116
Tuesday	4	8	138

Applying

Use an algebraic method to calculate how much Ben and Ceri are paid per hour.

(d) Ysgol Trefswm organised a concert to raise money for a charity.
All of the 120 tickets were sold for a total of £1,210.
The price of an adult ticket was £12.
The price of a child ticket was £7.
How many adult tickets and how many child tickets were sold?

Exercise 33: Old, Older, Oldest

Use the clues on the cards to solve the problem.

Extension 1

The information for these people has been shuffled. To begin you must form simultaneous equations and solve them. Every answer is a whole number.

(1) 18 is the sum of Arwyn's age and Bedwyr's age.

(2) 47 is the sum of Rhodri's age and Sali's age.

Your job is to find everyone's age and list them from oldest to youngest. There are two sets of twins.

(3) 38 is the sum of three times Gwen's age add two times Heledd's age.

(4) Three times Wil's age subtract four times Tesni's age is 33.

(5) Three times Ynyr's age add Penri's age is 27.

(6) Arwyn's age subtract Bedwyr's age is 8.

(7) Two times Wil's age add three times Tesni's age is 107.

(8) Three times Iolo's age add four times John's age is 147.

(9) 20 is two times Cai's age subtract Dewi's age.

(10) 78 is two times Fred's age subtract Elfyn's age.

(11) Four times Kate's age add three times Lora's age is 40.

(12) The sum of Cai's age and Dewi's age is 73.

(13) 91 is two times Mali's age add Nia's age.

(14) 43 is five times Kate's age add two times Lora's age.

(15) Fred is older than Elfyn by thirteen years.

(16) By doubling Ynyr's age and adding Penri's age you get 25.

(17) Two times Rhodri's age add Sali's age is 76.

(18) Seven times Iolo's age subtract three times John's age is 10.

(19) Double of Gwen's age add three times Heledd's age gives 42.

(20) Mali is eight years older than Nia.

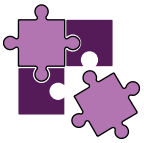
Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Changing the Subject

The purpose of **changing the subject** is to re-arrange formulae so that a particular **variable** appears on its own on the left-hand side of the formula. For example, consider the formula $p = 3w + d$, a formula to calculate the number of points (p) a football team has given how many games they have won (w) and how many drawn games (d) they have had. We can re-arrange the formula to give the number of games won by a football team.

$p = 3w + d$
$3w + d = p$ [Swap sides]
$3w = p - d$ [Subtract d from both sides]
$w = \frac{p-d}{3}$ [Divide both sides by 3]



After re-arranging the formula as shown above, we say that w is the **subject** of the formula.

There are a number of 'movements' we can perform to help re-arrange a formula to give a specific subject. Here are some of the most common movements.

Add a number to both sides of the formula

E.g. $y - 3 = x$
$y = x + 3$ [Add 3 to both sides]

Subtract a number from both sides of the formula

E.g. $y + 7 = x$
$y = x - 7$ [Subtract 7 from both sides]

Multiply both sides of the formula by a number

E.g. $\frac{y}{2} = 5x$
$y = 10x$ [Multiply both sides by 2]

Divide both sides of the formula by a number

E.g. $4y = x - 3$
$y = \frac{x-3}{4}$ [Divide both sides by 4]

Square both sides of the formula

E.g. $\sqrt{y} = 4x + 5$
$y = (4x + 5)^2$ [Square both sides]

Take the square root of both sides of the formula

E.g. $y^2 = 2x - 9$
$y = \sqrt{2x - 9}$ [Square root both sides]

Swap sides

E.g. $5x + 3 = y$
$y = 5x + 3$ [Swap sides]

Expand brackets

E.g. $4(y + 2) = 5x$
$4y + 8 = 5x$ [Expand brackets]

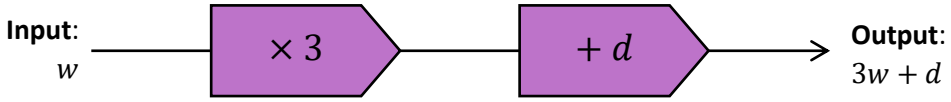
Let us reconsider the example at the top of this page. The question in the example can be set as follows.

Make w the subject of the formula $p = 3w + d$.

The aim in this question is to re-arrange the formula to leave only the variable w on the left-hand side of the formula. As the variable w initially appears on the right-hand side of the formula, it makes sense to start by swapping the sides of the formula, so that w appears on the left-hand side of the formula.

$p = 3w + d$
$3w + d = p$ [Swap sides]

There are several ways to proceed. You can think of the formula as an equation, and 'solve' to leave w on its own. Or you can think of how to calculate the left-hand side of the formula, if you start with the variable w.



By working backwards through the function machine, we can see the steps required to proceed, namely subtracting d from both sides of the formula, and then dividing both sides by 3.

Skill I

Exercise 34

(a) Make e the subject of the formula $p = 2e + c$.

(b) Make c the subject of the formula $p = 2e + c$.

(c) Make c the subject of the formula $y = mx + c$.

(d) Make m the subject of the formula $y = mx + c$.

(e) Make p the subject of the formula $c = p - 3t$.

(f) Make t the subject of the formula $c = p - 3t$.

(g) Make p the subject of the formula $A = p(q + r)$.

(h) Make q the subject of the formula $A = p(q + r)$.

(i) Make t the subject of the formula $F = \frac{m+4n}{t}$.

(j) Make m the subject of the formula $F = \frac{m+4n}{t}$.

(k) Make n the subject of the formula $F = \frac{m+4n}{t}$.

(l) Make r the subject of the formula $A = \pi r^2$.

(m) Make R the subject of the formula $I = \frac{PRT}{100}$.

(n) Make s the subject of the formula $A = \frac{su}{2}$.

(o) Make u the subject of the formula $C = \frac{1}{3}\pi r^2 u$.

(p) Make r the subject of the formula $C = \frac{1}{3}\pi r^2 u$.

(q) Make u the subject of the formula $A = \frac{1}{2}(a + b)u$.

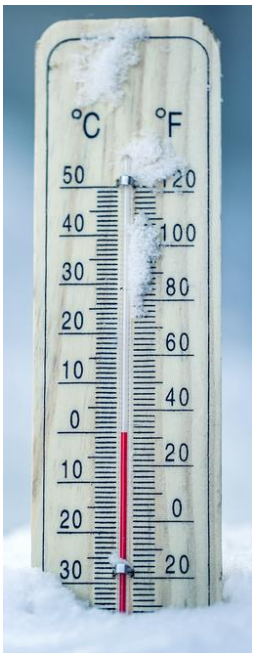
(r) Make b the subject of the formula $A = \frac{1}{2}(a + b)u$.

Exercise 35

The following formula was used by festival planners to calculate the parking fee for mini buses.

Parking Fee = Number of Passengers × 30p + £5

(a) What was the parking fee for a mini bus with 12 passengers?
(b) The parking fee for another mini bus was £7.40. How many passengers were on this mini bus?



Exercise 36

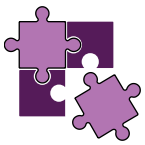
To change from degrees Celsius (°C) to degrees Fahrenheit (°F), you can use the following formula.

$$F = \frac{9}{5}(C + 40) - 40$$

(a) The temperature is 60°C. What is this in °F?
(b) Re-arrange the formula to find C in terms of F.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...



Expression, Equation, Formula, Identity

In algebra, what's the difference between expressions, equations, formulae and identities?

Expression

An expression is a collection of terms (e.g. $5x$ or 7) and operators (e.g. $+$ or $\times$).
$4x + 2$ and $\sqrt{6y - 4z}$ are examples of expressions.
There are no equals signs ($=$) in expressions.

Equation

An equation notes that two terms or expressions are equal. Two sides of an equation are separated by an equals sign ($=$). Sometimes, it is possible to solve an equation to find the value of a variable.

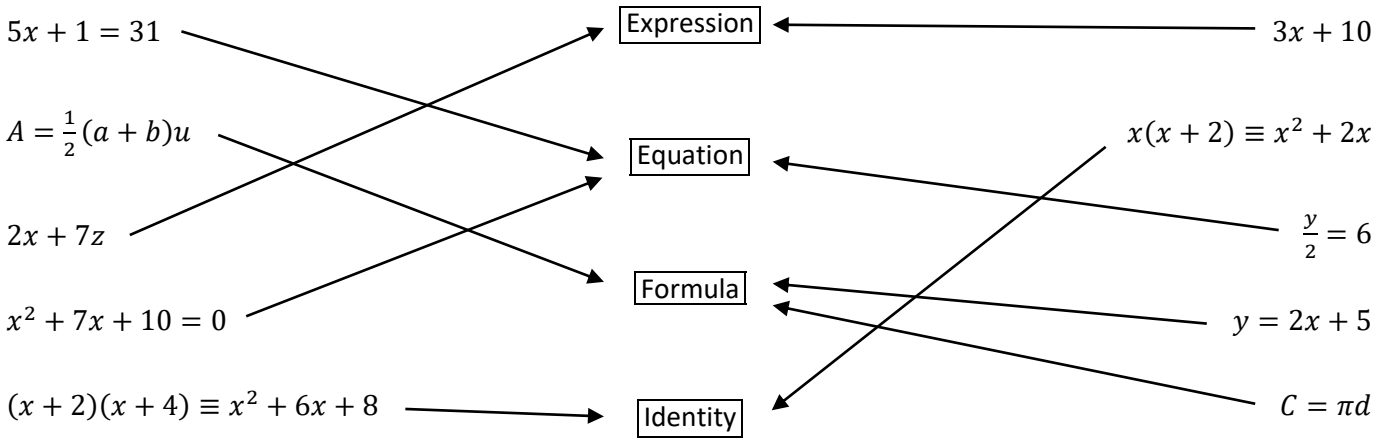
Formula

A formula is a special type of equation which shows the connection between different variables.
$P = 2a + 2b$ is an example of a formula, one which is used to calculate the perimeter of a rectangle with length a and width b.

Identity (Higher Tier Only)

An identity is an equation which is always true, no matter what the values of the variables are.
$2(x + 4) \equiv 2x + 8$ is an example of an identity. Two sides of an identity are separated by an equivalence sign ($\equiv$).

Example

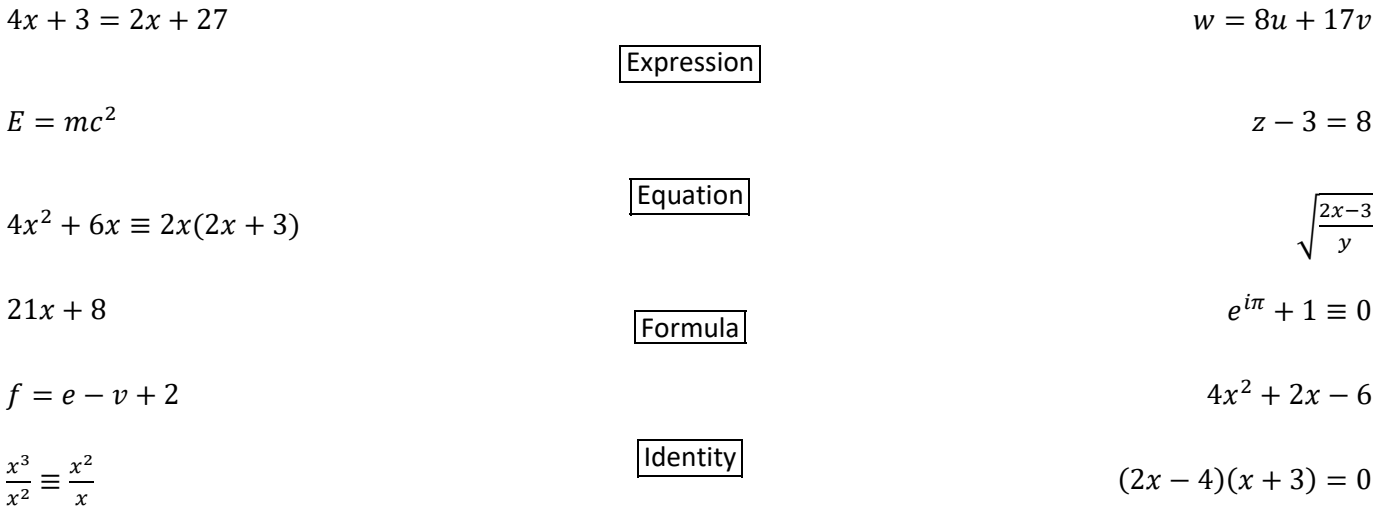


Exercise 37

Skill

I H

Add arrows pointing to the correct descriptions.



Proving Identities

Higher Tier

To prove an identity such as $(x+6)(x-2)-x(x+3) \equiv x-12$, we must use algebraic steps to change the left-hand side to be the right-hand side.

H

Left-hand side $= (x+6)(x-2)-x(x+3)$
$= x^2-2x+6x-12-(x^2+3x)$ [Expand brackets]
$= x^2+4x-12-x^2-3x$ [Collect like terms]
$= x-12$ [Simplify]
$=$ Right-hand side ✓

Exercise 38

Prove the following identities.

(a) $4(x+2) \equiv 4x+8$

(b) $2(x+4)+5(x+8) \equiv 7x+48$

(c) $(x+8)(x-3) \equiv x^2+5x-24$

(d) $6(x+8)-2(x-4) \equiv 4(x+14)$

(e) $(x+5)(x+2)+(x+8)(x+8) \equiv 2x^2+23x+74$

(f) $(x+6)(x-2)-(x+8)(x+2) \equiv -6x-28$

(g) $(y+4)(y-7)+3y(y-1) \equiv 4y^2-6y-28$

(h) $(2x+1)(x+2)-2x(x+4) \equiv -3x+2$

Exercise 39

Three of the following identities are incorrect. Which ones?

(a) $3(x-4) \equiv 3x-12$

(b) $(x+4)(x-2) \equiv x^2+2x-8$

(c) $(x+3)^2 \equiv x^2+9$

(d) $7(x+3)+2(x-2) \equiv 9x+17$

(e) $4(x+8)-2(x+8) \equiv 2(x+8)$

(f) $(x+2)(x-2) \equiv x^2+4$

(g) $5(y-2)-2(y-3) \equiv 3y-4$

(h) $\frac{x^2+6x+8}{x^2+5x+6} \equiv \frac{x+4}{x+3}$

(i) $(x+3)(x-3)-(x-4)(x+4) \equiv 7$

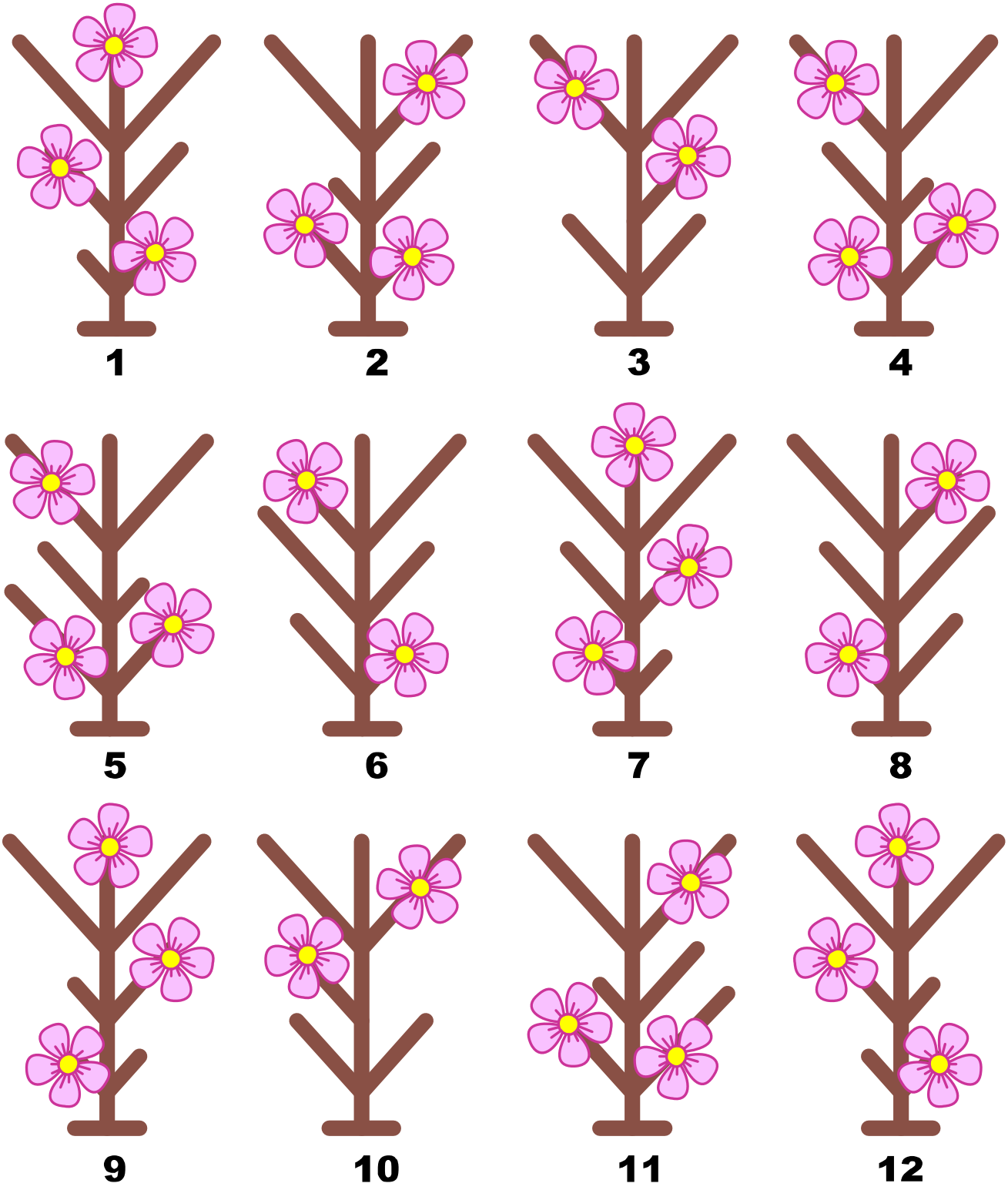
(j) $4(x+2)+(x-4)(x+7) \equiv x^2-20$

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Puzzle

Can you find the mirrored copy of each picture?



Developing Algebra 2

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test	Correct in the test?
I can **factorise** simple expressions such as $8x + 12$ or $15x^2 - 10$.			1	
I can **factorise quadratic expressions** of the form $x^2 + ax + b$.			1	
I can **factorise quadratic expressions** of the form $ax^2 + bx + c$. **(Higher Tier only.)**			2 (H), 3 (H)	
I can factorise quadratic expressions of the form $a^2 - b^2$ (a **difference of two squares**). **(Higher Tier only.)**			1 (H)	
I can **solve quadratic equations** through factorisation.			2, 3	
I can **solve simultaneous equations**.			4, 5	
I can **re-arrange a formula** in order to make a specific variable (e.g. x) the **subject** of the formula.			6, 7	
I can recognise **expressions, equations, formulae** and **identities**.			8	
I can **prove identities**. **(Higher Tier only.)**			9 (H)	

The Mathematics Department

10

Measuring Solids

Intermediate Tier

Name:

Contents

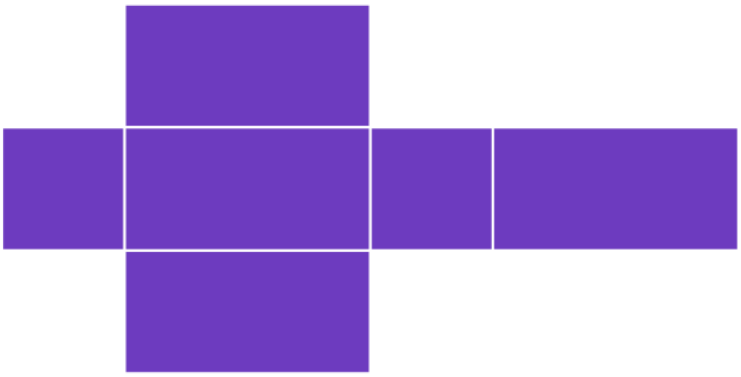
Chapter	Mathematics	Page Number
Solids	Volume and surface area of a cuboid. Volume of a prism. Volume and surface area of a cylinder.	3
Composite Solids	Volume of composite solids.	11
Similar Shapes	Calculating the scale factor. Calculating missing lengths. Similar or not?	13
Dimensions	Length, area and volume. Nonsensical formulae.	16

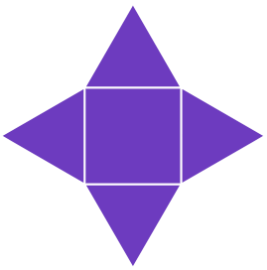
Solids

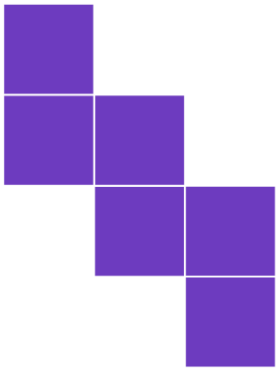
Skill F

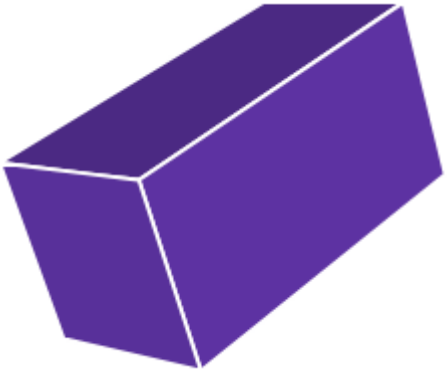
Exercise 1

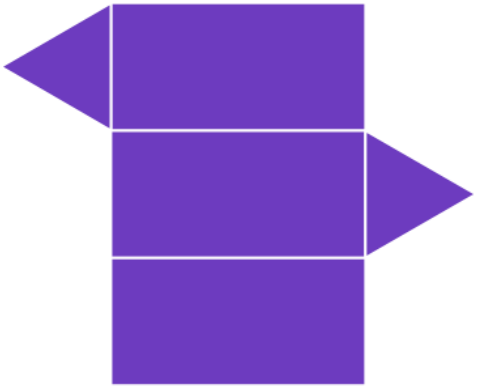
Pair the solid with its matching net. Write the name of the solid underneath it.











In this chapter, we will discuss how to calculate the volume and surface area of a variety of different solids.

The **volume** measures how much space a solid occupies or uses. It's measured in cube units, e.g. cm^3.

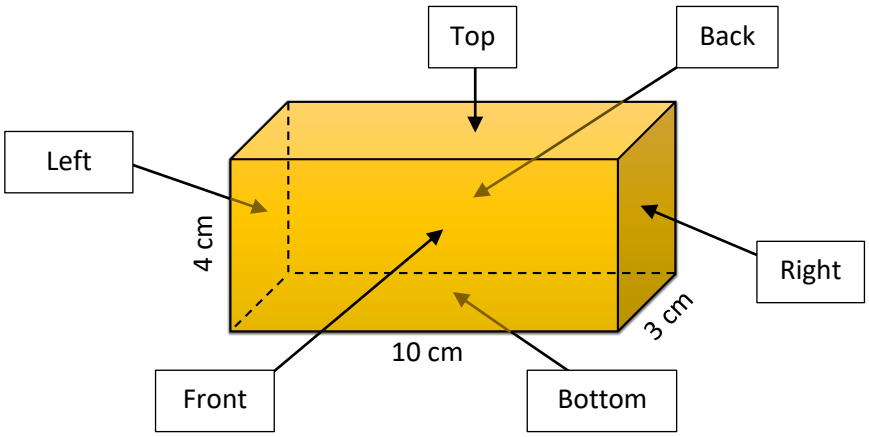
The **surface area** measures the area of the outside of the solid. We can think of the surface area as how much paper you would need to wrap the solid. Surface area is measured in square units, e.g. m^2.

Cuboid

We've previously seen the formula to **calculate** the volume of a cuboid in the "Measuring Shapes 1" workbook.

Volume of a Cuboid = Length × Width × Height

To calculate the **surface area** of a cuboid, we must add the areas of each of the six faces.



Example

The volume of the above cuboid is $10 \times 3 \times 4 = 120\ cm^3$.

The surface area of the cuboid is the total area of the six faces.

Front	$10 \times 4 = 40$
Back	40
Left	$3 \times 4 = 12$
Right	12
Top	$10 \times 3 = 30$
Bottom	30
Total	**164 cm^2**

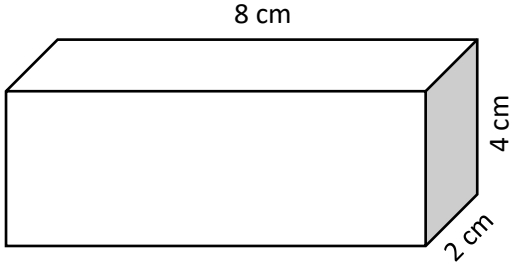
Notice that some of the areas are the same so we do not need to calculate them.

Exercise 2

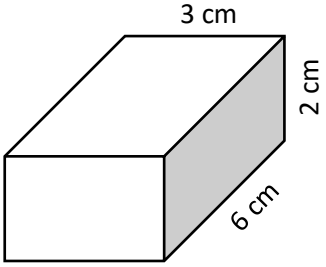
Skill F

Calculate the volume and surface area of the following cuboids.

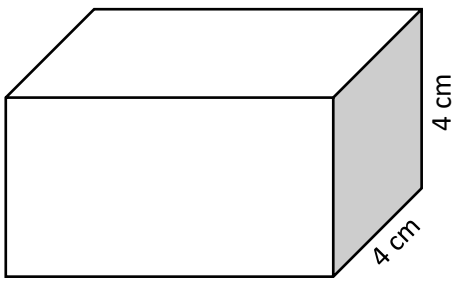
(a)



(b)

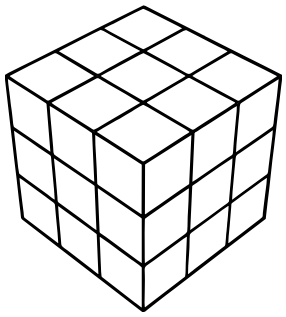


(c)



Exercise 3

The diagram shows a Rubik's cube before adding the coloured stickers.
The dimensions of one of the small cubes is 2 cm × 2 cm × 2 cm.



(a) What is the volume of one of the small cubes?

(b) How many small cubes form the Rubik's cube?

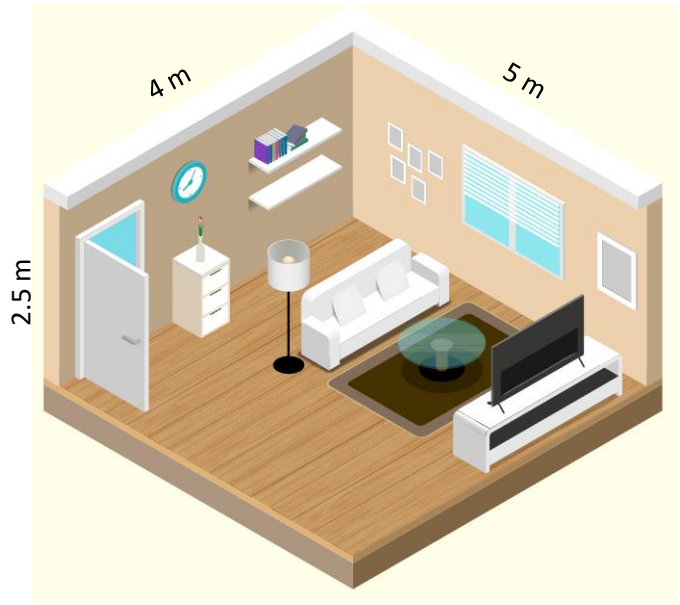
(c) What is the volume of the Rubik's cube?

(d) What is the surface area of the Rubik's cube?

(e) How many small 2 cm × 2 cm stickers are needed to be stuck on the Rubik's cube?

Exercise 4

The diagram on the right shows a floor plan for a living room.



(a) What is the volume of the living room?

(b) What is the area of the floor?

(c) The wood for the floor costs £14 per square metre. What was the cost of the wood for the whole floor?

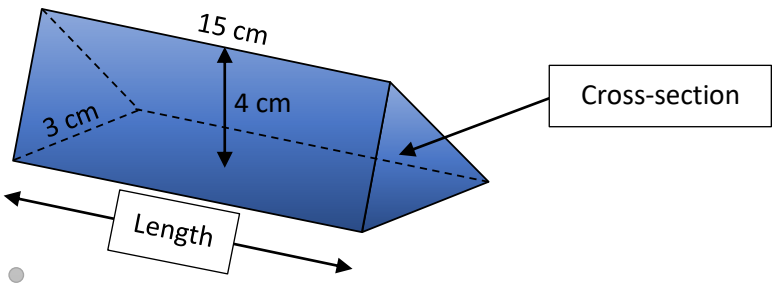
(d) Cerys wishes to repaint the wall on which the clock hangs. Given that the door measures 75 cm by 2 m, what area will need repainting?

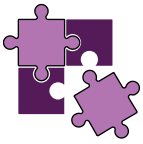
(e) To repaint the wall, Cerys buys a 2.5 litre tin of paint. The tin states that the paint covers up to 10 m² of area for each litre of paint. Will Cerys have enough paint to give the wall two coats of paint?

Prism

A prism is a solid in which both ends are identical and the cross-section at any point is identical to the two ends. The shape of the two ends gives the prism its name. For example, the diagram below shows a triangular prism.

This formula is on page 2 of every GCSE exam paper.





Volume of a Prism = Area of Cross-section × Length

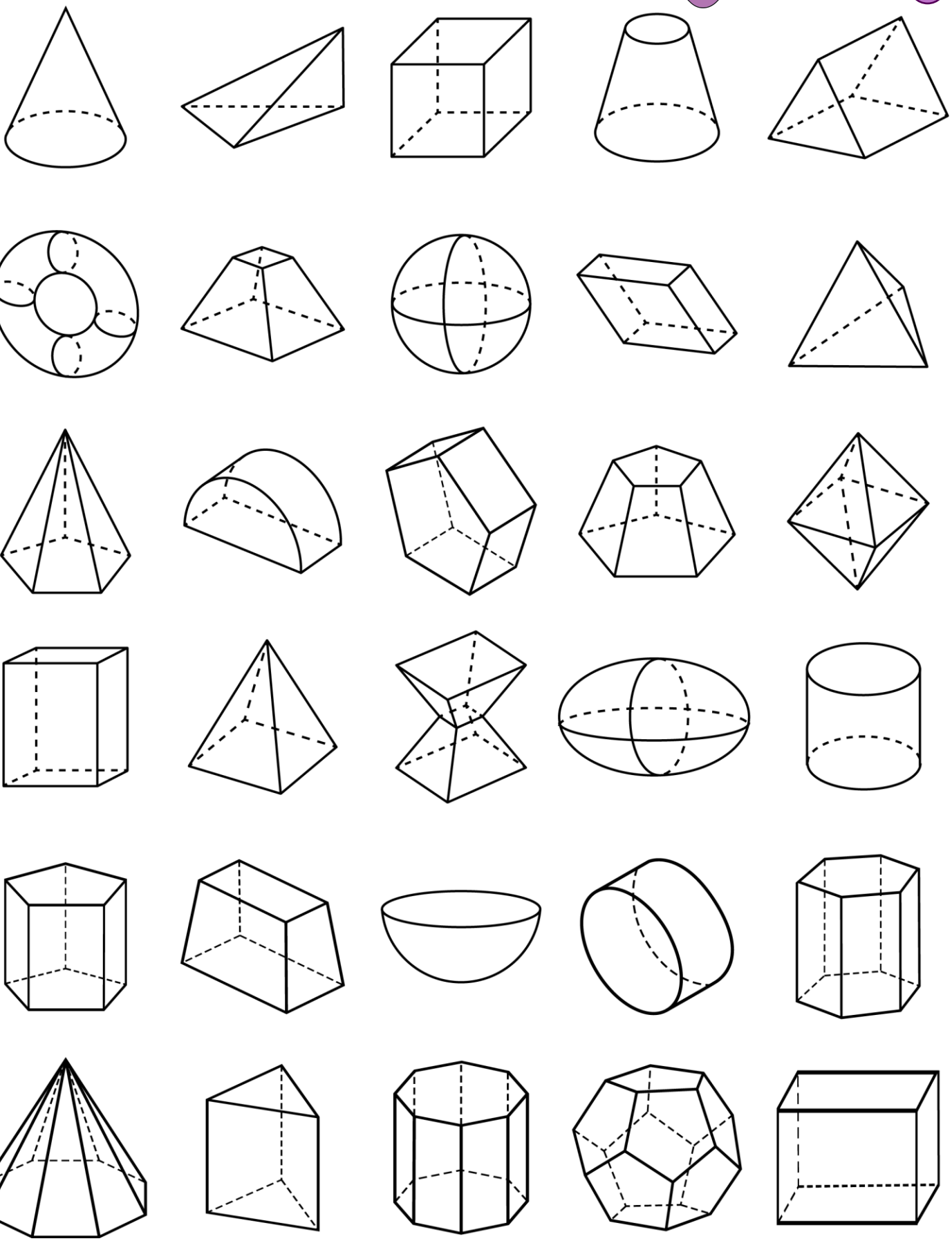
Example

For the above triangular prism, the area of the cross-section is the area of the triangle, which is $\frac{3\times4}{2} = 6$ cm².
Therefore the volume of the prism is 6 × 15 = 90 cm³.

Exercise 5

Skill F

Circle the solids below that are prisms.

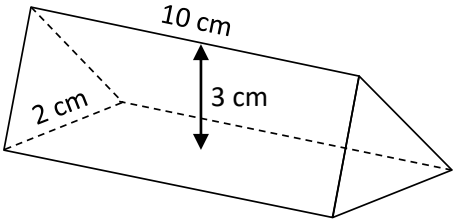


Exercise 6

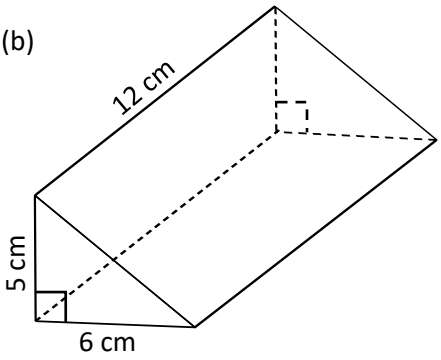
Skill I

Calculate the volume of the following prisms.

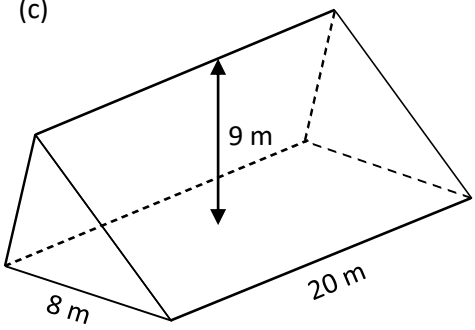
(a)



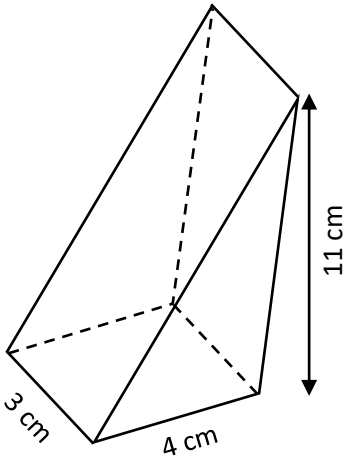
(b)



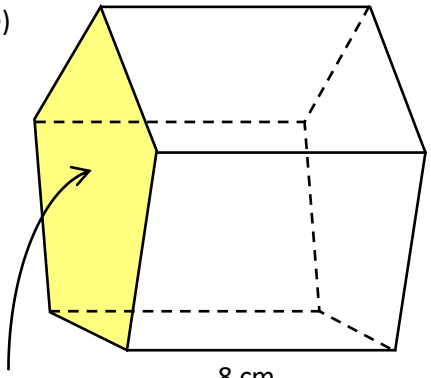
(c)



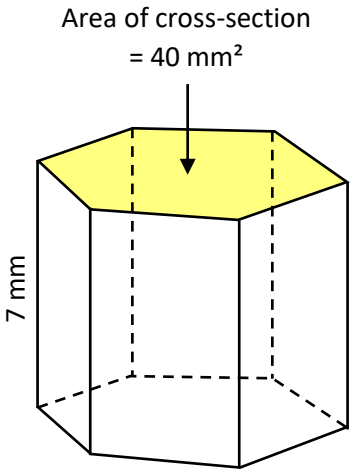
(d)



(e)



(f)



Challenge!

Calculate the surface area of the prism in question (b) above. Give your answer to 1 decimal place.

Exercise 7

Applying I

The picture on the right shows a water fountain.

The depth of the water in the lower part of the fountain is 40 cm.

The area of the cross-section of the water is 38,000 cm².

(a) What is the volume of the water in the fountain, in cm³?

(b) What is the volume of the water in the fountain, in ml?

(c) What is the volume of the water in the fountain, to the nearest litre?

(d) Buddug wants to empty the water fountain so that it can be cleaned. The water pump that Buddug uses to empty the fountain works at a rate of 100 litres per minute. To the nearest minute, how long will it take for the pump to empty the fountain?

Exercise 8

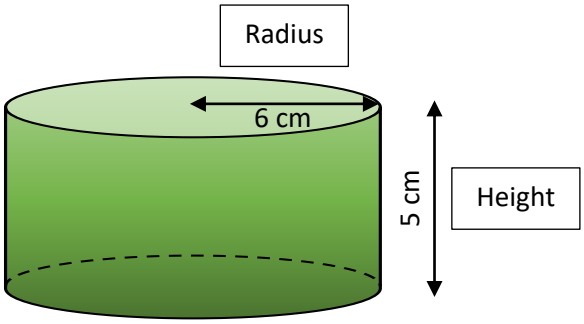
Skill I

What is the name of the prism that has the following shapes forming the cross-section?

(a) Rectangle (b) Square (c) Circle

Cylinder

A cylinder is a special type of prism where the cross-sectional shape is a circle.



Volume of a Cylinder = Area of the circle × Height or width of the cylinder

Volume of a Cylinder = $\pi \times \text{Radius}^2 \times \text{Height}$

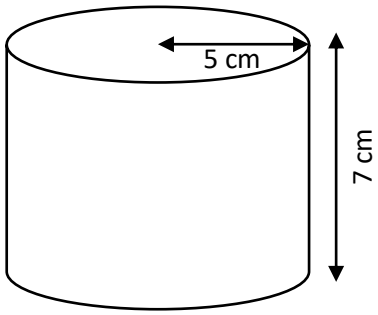
Example

The volume of the above cylinder is $\pi \times 6^2 \times 5 = 565.49$ cm^3, correct to two decimal places.

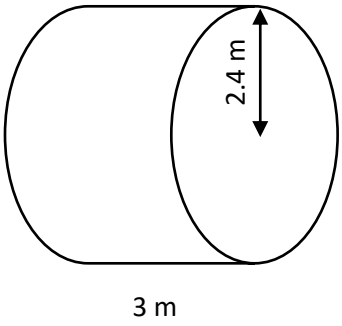
Exercise 9

Calculate the volume of the following cylinders.

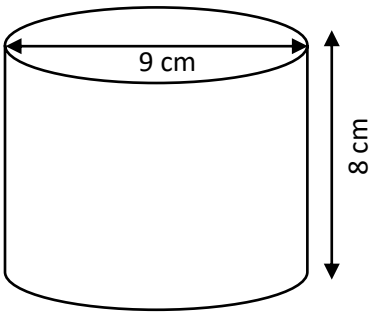
(a)



(b)



(c)



Exercise 10

Applying

The picture on the right shows an apple cake that has been baked in a baking tin.

(a) Given that the radius of the tin is 10 cm and its height is 6 cm, calculate the volume of the cake in the tin.

(b) The cake weighs 1.2 kg. Gwenda wants to cut the cake into equal pieces so that each piece weighs 150 g.
How many equal pieces will Gwenda need to cut?

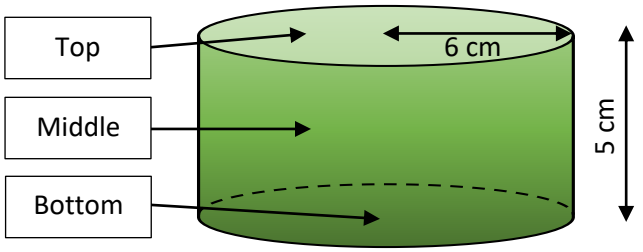
(c) What is the volume of each of the pieces of cake from part (b)?

Exercise 11

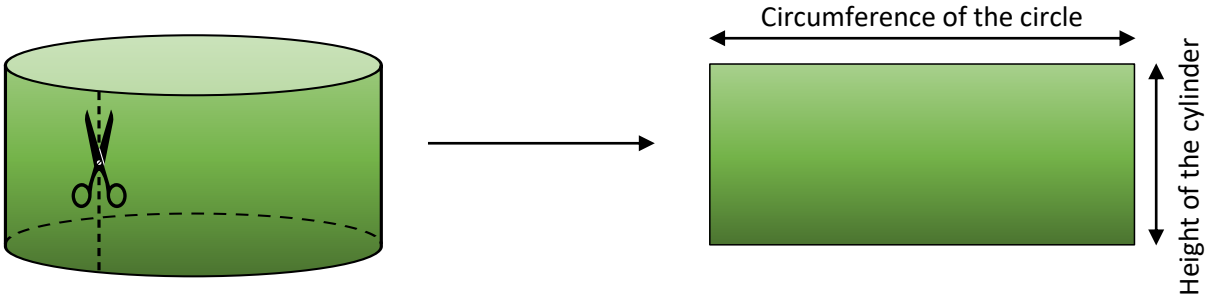
A hot water tank is in the shape of a cylinder. The height of the tank is 90 cm, and its diameter is 45 cm.
The manufacturer estimates that the tank holds 140 litres of water.
Has the manufacturer provided an overestimate or an underestimate?

Surface Area of a Cylinder

For a closed cylinder, we must add the areas of the top, middle and bottom faces in order to find the surface area of the cylinder.



The top and bottom are obviously circle shaped, but what is the shape of the middle face? Imagine cutting the middle face with scissors, vertically, and stretching the shape out. You would be left with a rectangle, with height the same as the cylinder's height and width equal to the circumference of the circle.



To find the area of this rectangle, we multiply the circumference of the circle ($\pi \times$ diameter of the circle) by the height of the cylinder.

Example

The surface area of the above cylinder is the total of the top, middle and bottom faces.

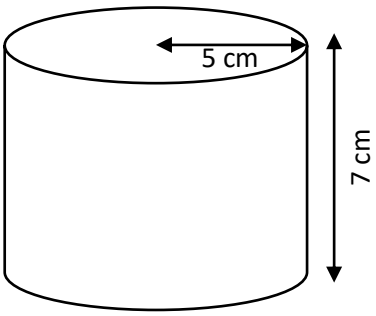
	Answer as a decimal	Answer in terms of π
Top	$\pi \times 6^2 = 113.10$ cm^2	$\pi \times 6^2 = 36\pi$ cm^2
Middle	$(\pi \times 12) \times 5 = 188.50$ cm^2	$(\pi \times 12) \times 5 = 60\pi$ cm^2
Bottom	113.10 cm^2	36π cm^2
Total	**414.7 cm^2, to one decimal place**	**132π cm^2**

Exercise 12

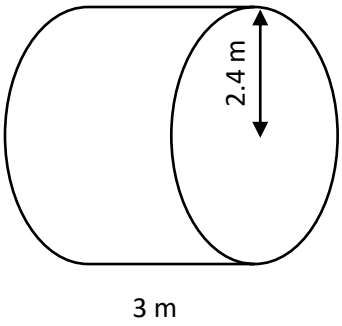
Skill I

Calculate the surface area of the following closed cylinders.

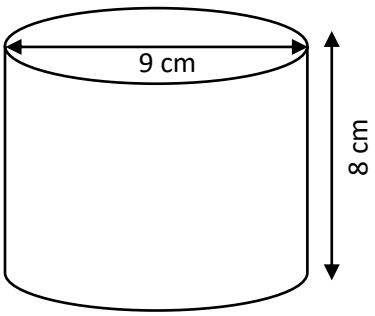
(a)



(b)



(c)



Exercise 13

Write your answers to Exercise 12 in terms of π.

Exercise 14 I

Complete the following table. State your answers correct to two decimal places.

Type of cylinder	Radius	Diameter	Height	Volume	Surface Area
Open	14 cm		6 cm		
Closed		6.8 m	2.4 m		
Closed	9.3 mm		12 mm		
Open		0.7 km	0.3 km		
Closed	18 cm		1.2 cm		

Exercise 15 Applying I

The cardboard tube of a toilet roll has the shape of a cylinder.
The diameter of the tube is 4.4 cm and the length of the tube is 11 cm.
Calculate the area of the cardboard used to create the tube.

Exercise 16 I

Pringles are sold in cylindrical packaging.

The height of the cylinder is 26 cm, and the diameter of the cylinder is 8 cm.

There is a layer of foil on the top along with a plastic lid.

The bottom is metal.

The centre is made from cardboard.

(a) What is the area of the metal bottom?

(b) What is the area of the top layer made of foil?

(c) Given that the plastic lid has a vertical edge of 0.8 cm, how much plastic is needed to create the lid?

(d) How much cardboard is needed to create one tube?

(e) How much cardboard is needed to create 10,000 tubes?

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Composite Solids

A **composite solid** is a solid we can split into simpler solids, like the solids considered in the first chapter of this workbook.

Example

It is possible to split the composite solid on the right into a cuboid (on the left) and a triangular prism (on the right).

Cuboid

Volume = Length × Width × Height

= 4 × 6 × 8

= 192 cm³

Triangular Prism

Volume = Area of Cross-section × Length

$= \left(\frac{4\times3}{2}\right) \times 6$

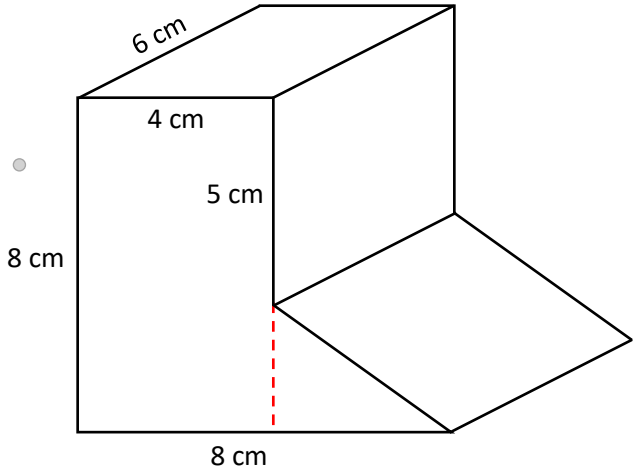
= 36 cm³

Composite Solid

Volume = 192 + 36

= 228 cm³

It is also possible to treat this solid as one large prism.

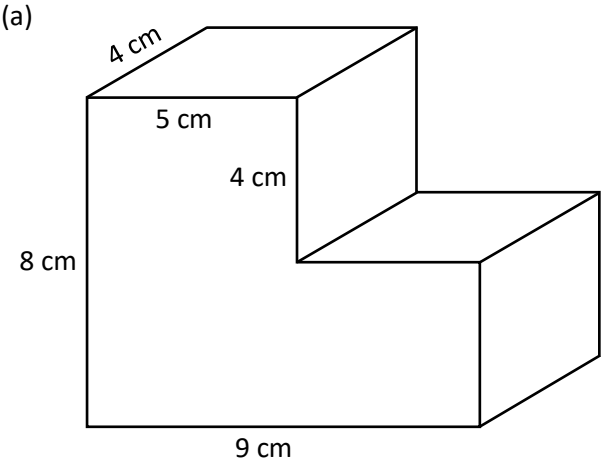


Exercise 17

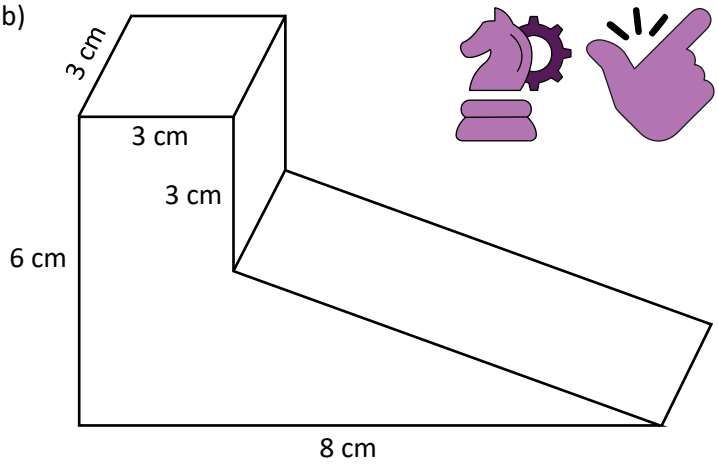
Skill I

Calculate the volume of the following composite solids.

(a)



(b)



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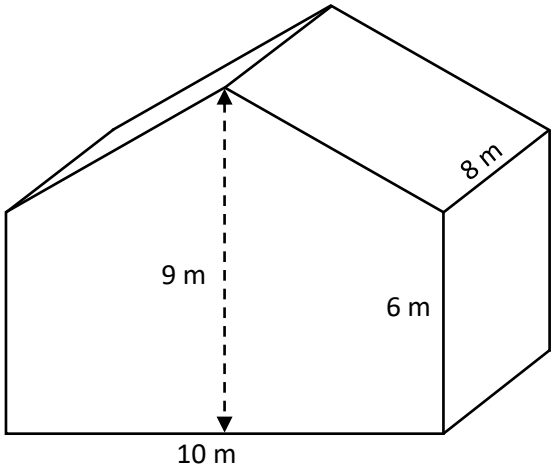
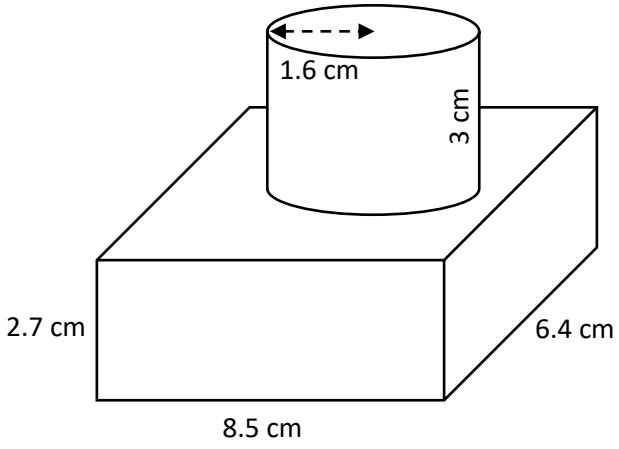


(d)

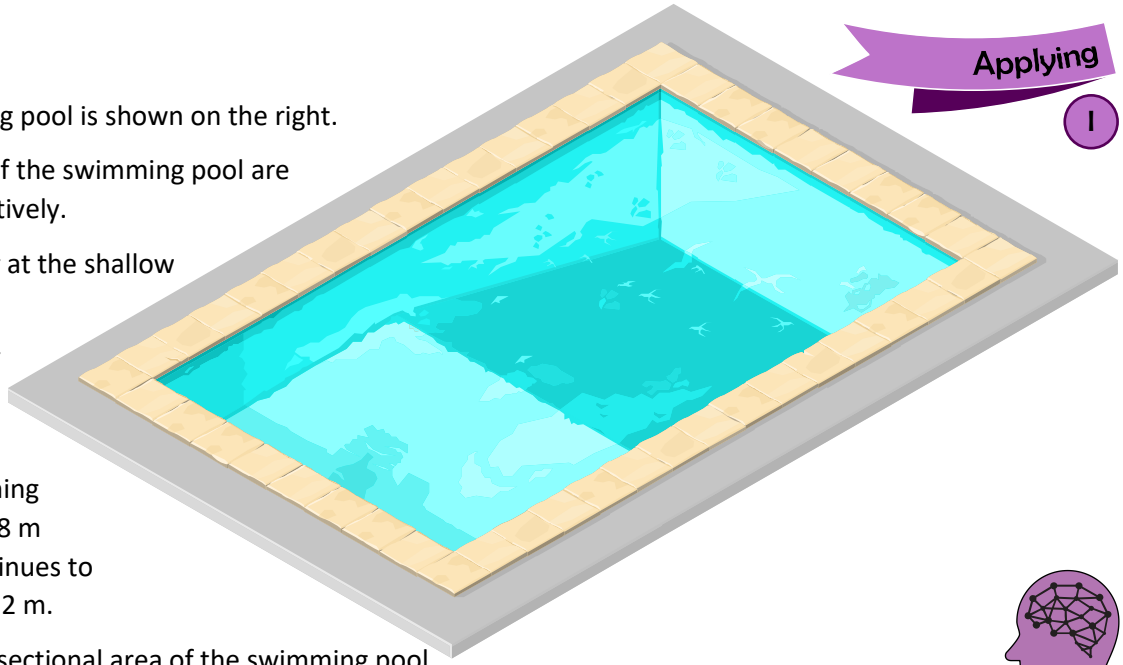


Applying

I

Exercise 18

A picture of a swimming pool is shown on the right.



The length and width of the swimming pool are 10 m and 20 m, respectively.

The depth of the water at the shallow end is 1 m.

The depth of the water at the deep end is 2.5 m.

The floor of the swimming pool starts to descend 8 m into the pool, and continues to descend for a total of 12 m.

(a) Calculate the cross-sectional area of the swimming pool.

(b) Calculate the volume of the swimming pool, in m^3.

(c) Given that there is 1,000 litres of water for every 1 m^3 of volume, calculate how much water is in the pool.

(d) An ordinary hose pipe supplies water at a rate of 10 litres a minute. How long would it take for one hose pipe to fill the swimming pool? Give your answer correct to the nearest day.

Exercise 19

The picture on the right shows some Polo Mint sweets.

- The diameter of each Polo Mint is 1.9 cm.
- The diameter of the hole in the middle is 0.8 cm.
- The thickness of each Polo Mint is 0.4 cm.

(a) Calculate the cross-sectional area of one Polo Mint.

(b) Calculate the volume of one Polo Mint.

(c) Usually, there are 23 Polo Mints in a packet.

(i) What is the volume of sweets in a whole packet?
(ii) What is the volume of the hole in the middle of a packet?

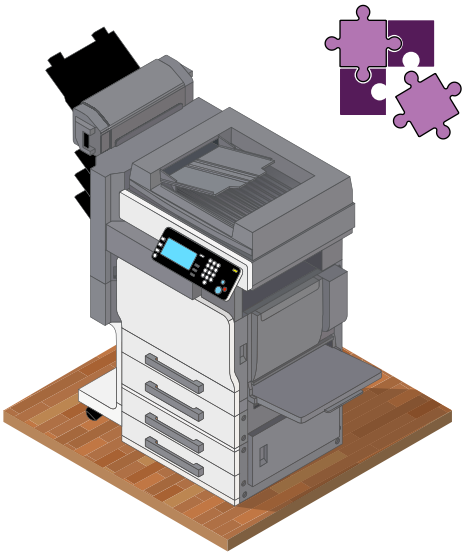
(d) What is the minimum amount of paper needed to make the label round the packet?

(e) What is the minimum amount of foil needed for each packet?

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Similar Shapes



Imagine using a photocopier to enlarge a diagram drawn on a piece of paper. If the original paper is A4 sized and the paper that comes out of the photocopier is A3 sized, then you have created a copy of the diagram twice the size.

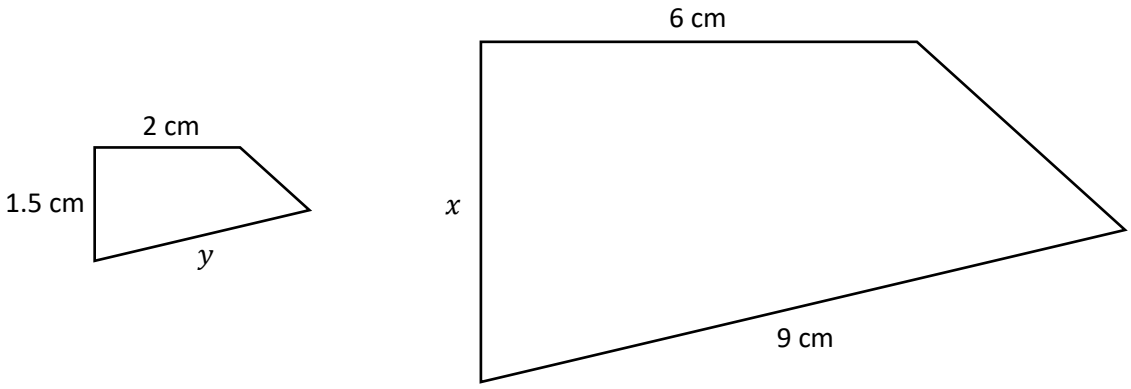
In mathematics, we say that the new diagram is **similar** to the original diagram. This means that the new diagram has the same **shape,** but the **size** has changed. Since the new diagram is twice as big, we can say that the **scale factor** is 2.

Similar shapes are the exact same shape, but of different sizes.

Given two shapes that are similar, we can use the measurements on the shapes either to find the scale factor or to find some missing lengths on the shapes.

Example

The two shapes shown below are similar shapes. Use the measurements on the shapes to find the lengths x and y.



Answer: The first step is to find the **scale factor.** To do this, we need to consider how much bigger the large shape is compared to the small shape. We see that the horizontal lengths at the top of both shapes are given. We can use these two lengths to find the scale factor, by calculating 6 ÷ 2 = 3. Therefore, the large shape is three times bigger than the small shape.

Having found the scale factor of 3, we can now use it to find the lengths x and y.

The edge that corresponds to the x edge in the small shape is 1.5 cm. We must **multiply** 1.5 cm by the scale factor to find the length of x, since we are going from the small shape to the large shape. Therefore

$$x = 1.5 \times 3$$
$$x = 4.5 \text{ cm}$$

The edge that corresponds to the y edge in the large shape is 9 cm. We must **divide** 9 cm by the scale factor to find the length of y, since we are going from the large shape to the small shape. Therefore

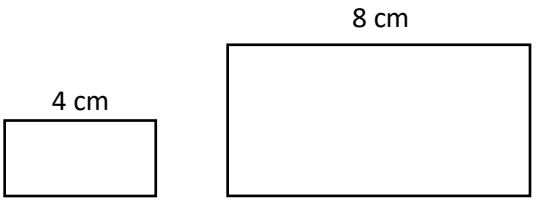
$$y = 9 \div 3$$
$$y = 3 \text{ cm}$$

Skill I

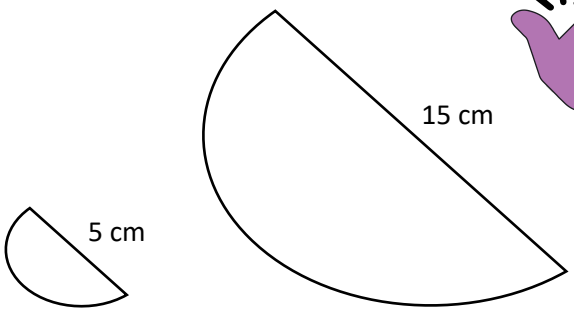
Exercise 20

The following pairs of shapes are similar shapes. Use the measurements on the shapes to find the scale factor.

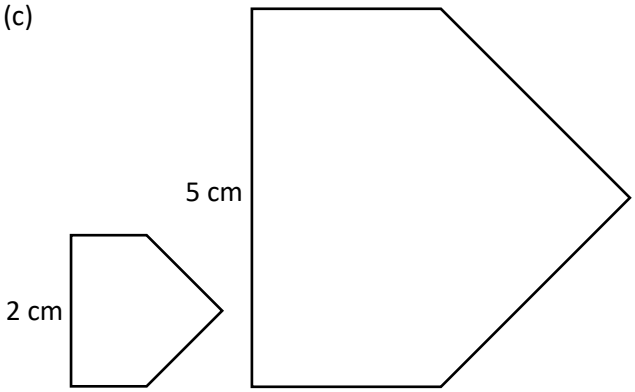
(a)



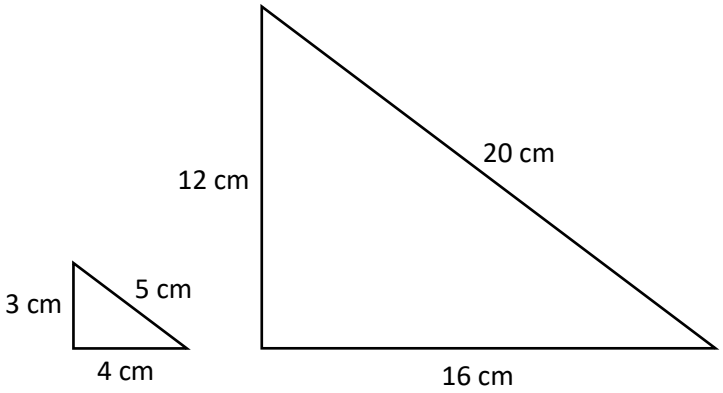
(b)



(c)



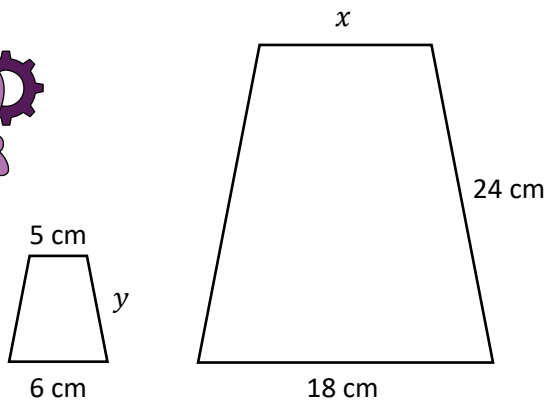
(d)



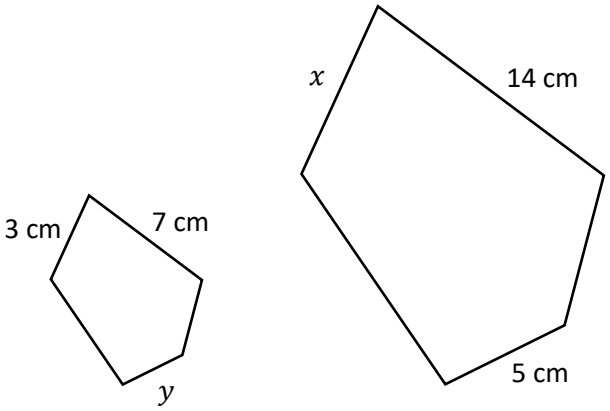
Exercise 21 I

The following pairs of shapes are similar shapes. Use the measurements on the shapes to find the lengths x and y.

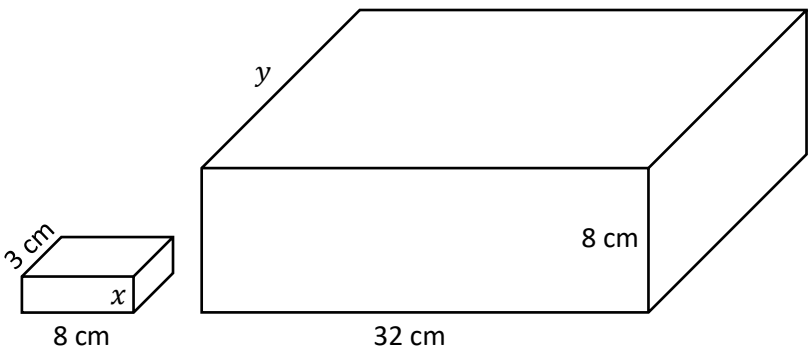
(a)



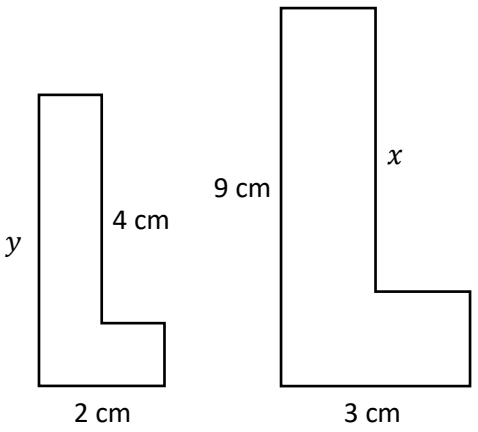
(b)



(c)



(d)

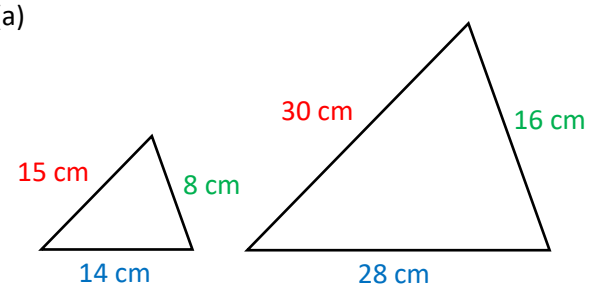


Similar or not?

If two shapes are similar, then the **corresponding edges are in the same ratio.**
This means that if we divide a pair of corresponding edges, we will always obtain the same answer.

Example

(a)

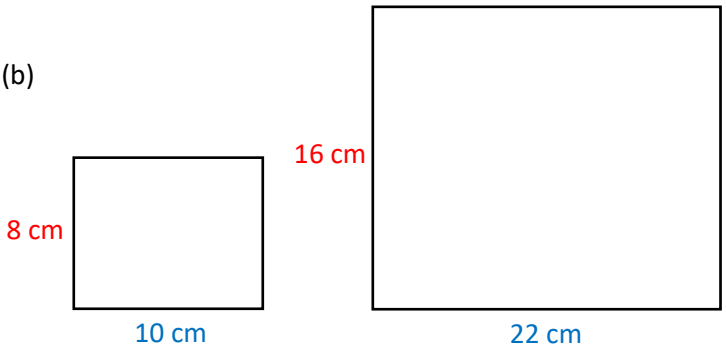


For the two triangles above, the corresponding edges are in the same ratio.

$30 \div 15 = 2$
$28 \div 14 = 2$
$16 \div 8 = 2$

Therefore the two triangles are similar.

(b)



For the two rectangles above, the corresponding edges are not in the same ratio.

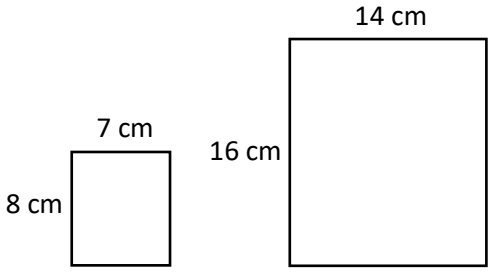
$16 \div 8 = 2$
$22 \div 10 = 2.2$

Therefore, the two rectangles are not similar.

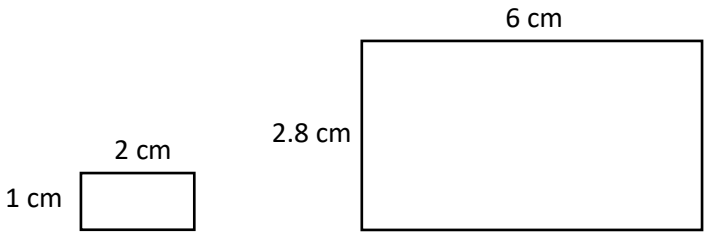
Exercise 22

Decide whether the following pairs of shapes are similar or not.

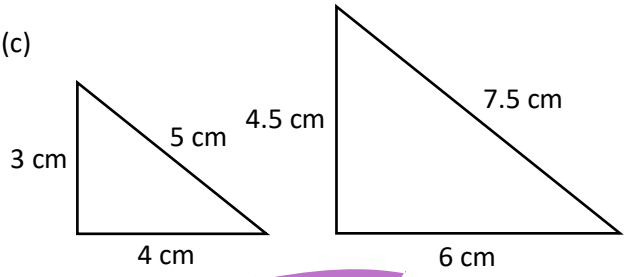
(a)



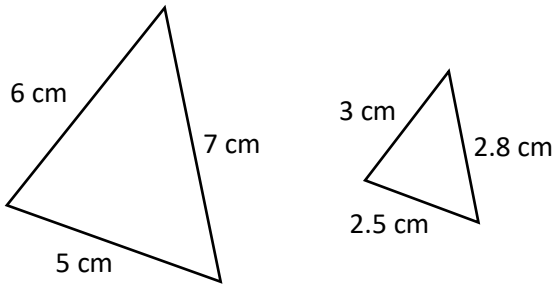
(b)



(c)



(d)



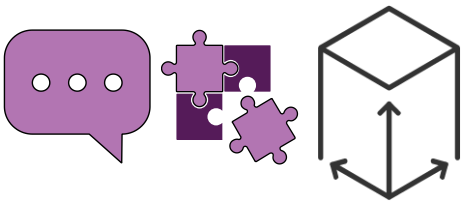
Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Dimensions

Length, Area and Volume

Given a particular formula, we need to be able to recognise, using **dimensions**, if the formula is for calculating a length, area, volume, or none of these.

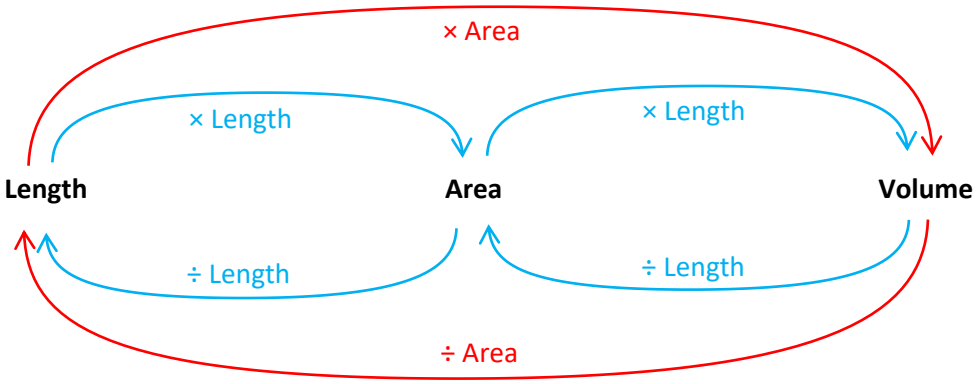


Any formula for a **length** is a **one-dimensional** formula.
Any formula for an **area** is a **two-dimensional** formula.
Any formula for a **volume** is a **three-dimensional** formula.

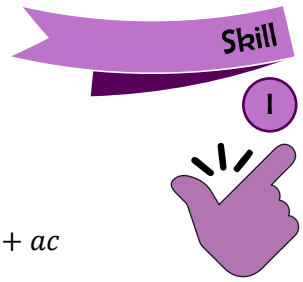
Example

Formula	Purpose	Number of Dimensions
$P = 2a + 2b$	Calculate the perimeter of a rectangle with length a and width b	1
$A = \pi r^2$	Calculate the area of a circle with radius r	2
$C = \pi r^2 h$	Calculate the volume of a cylinder of radius r and height h	3

It is possible to consider the following rules when deciding how many dimensions a formula has (and therefore to decide whether a formula is used to find length, area or volume).



Exercise 23



In the following formulae, a, b and c represent lengths.
Decide whether each formula represents a length, an area or a volume.

(a) $M = a + b$ (b) $M = a - b$ (c) $M = 3a$

(d) $M = ab$ (e) $M = 8bc$ (f) $M = ab + ac$

(g) $M = abc$ (h) $M = 4abc$ (i) $M = 4abc - cab$

(j) $M = a + 3c$ (k) $M = 3ab + 2ac$ (l) $M = 7a - 3c$

(m) $M = \frac{ab}{c}$ (n) $M = bc^2$ (o) $M = \frac{bc^2}{a}$

(p) $M = a(b + c)$ (q) $M = a^2(b + c)$ (r) $M = \frac{a^2}{b+c}$

(s) $M = \pi a$ (t) $M = 3\pi a$ (u) $M = 7\pi ab$

(v) $M = 4b^2 - 4ac$ (w) $M = ab - bc + ca$ (x) $M = \frac{abc}{4bc}$

(y) $M = 10\pi c^2$ (z) $M = \frac{ab^2}{a+b+c}$ (α) $M = ab(4 + \pi)$

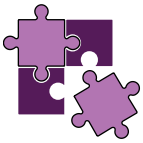
Exercise 24

I

Each of the following quantities has a specific number of dimensions.
Write the number of dimensions for each quantity. The first has been completed for you.

Quantity	Number of dimensions
Capacity of a swimming pool	3
Perimeter of a hexagon	
Volume of a cylinder	
Distance between Llandudno and Liverpool	
Area of a trapezium	
Length of the shoelaces in a pair of shoes	

Nonsensical Formulae



It is possible to write a formula that doesn't make sense. For example, it doesn't make sense to add a volume to an area (there would be no meaning to the answer), therefore the formula $M = ab + abc$ doesn't make sense. Here are some other combinations that lead to nonsensical formulae.

Length + Area	Length + Volume	Area + Volume
Length – Area	Length – Volume	Area – Volume
	Length × Volume	Area × Volume
Length ÷ Area	Length ÷ Volume	Area ÷ Volume

Exercise 25

I

In the following formulae, p, q and r represent lengths.
Decide which seven formulae do not make sense.

(a) $M = pq + rp$ (b) $M = 3p + qr$ (c) $M = p^2 - pq$

(d) $M = 2r + 3q$ (e) $M = pqr - 6pr$ (f) $M = q - r^3$

(g) $M = \frac{pq}{r}$ (h) $M = \frac{p}{qr}$ (i) $M = \frac{5q}{p^3}$

(j) $M = 2\pi r + q$ (k) $M = \pi r^2 + pqr$ (l) $M = \frac{3pq}{r} + 7\pi$

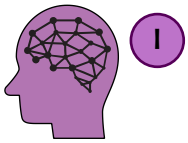
Exercise 26

I

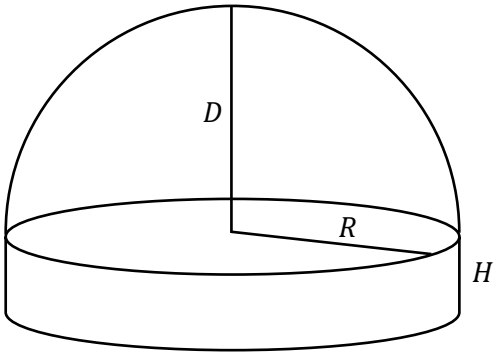
In each of the following expressions, every letter represents a length. By considering the dimensions of the expressions write, for each one, what the expression may be describing: length, area, volume or none of these.
The first one has been completed for you.

	The formula could be for
(a) $e^2 + df$	Area
(b) $5d + 8e + 2f$	.....
(c) $7de + 2d^2f$	.....
(d) $(d + e)f$	.....
(e) $5def - 2e^3$	.....
(f) $\pi de + fde$	.....

Exercise 27



The diagram on the right shows a solid.
The lengths D, R and H are noted on the diagram.



One of the following formulae can be used to estimate C, the volume of the solid.

$C = 3H + 2R + 5D$
$C = 3R + 5DR$
$C = 3R^2H + 2R^2D$
$C = 3R(4D + 5H)$

(a) Explain why we cannot use $C = 3H + 2R + 5D$ to estimate the volume of the solid.

(b) Note, stating your reasons, which of the above formulae can be used to estimate the volume of the solid.

Exercise 28

A factory uses wire to make the frame for a plant cover, as shown in the diagram on the right.
Every frame has a width L, depth D and upright height U.

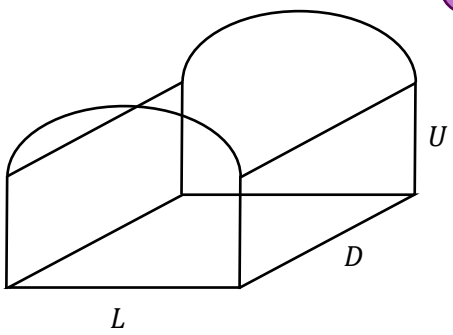


Diagram not drawn to scale.

One of the following formulae can be used to estimate C, the total length of wire needed to make the frame.

$C = 5L + 4D + 4U$
$C = 5L + 4DU$
$C = 5L(4D + 4U)$
$C = 5LDU$

(a) Explain why we cannot use the formula $C = 5LDU$ to estimate the total length of wire required.

(b) Note, stating your reasons, which of the above formulae can be used to estimate the total length of wire required.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Puzzle: Write the numbers between 1 and 9 in each row, column and 3×3 square.

SUDOKU

8	1						6	
7					4			2
		9						
6	8				9			
						7	9	
4				8	7			1
5						4		7
					2	9		
			4		5	2	1	6

	9			8				5
	5	3	2					
	2			4		1		3
		4			1	2		8
					2		7	4
	3					6		
	7							
5			9			8		
			6				9	1

Measuring Solids

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test	Correct in the test?
I know how to calculate the **volume of a cuboid**.			1, 9	
I know how to calculate the **surface area of a cuboid**.			1	
I know how to calculate the **volume of a prism**.			2	
I know how to calculate the **volume of a cylinder**.			4, 9	
I know how to calculate the **surface area of a cylinder**.			4	
I can calculate the **volume of composite solids**.			6	
Given two similar shapes, I can calculate the **scale factor**.			7	
Given two similar shapes, I can **calculate missing lengths**.			7	
I can decide **whether two shapes are similar**.			8	
I know how to recognise whether a formula represents **a length, an area, a volume or none of these**.			5	
I can recognise the **number of dimensions** for specific quantities.			3	

The Mathematics Department

10

Accuracy of Measurements

Name:

Contents

Chapter	Mathematics	Page Number
Rounding Off: Significant Figures	Rounding a number off to a specified number of significant figures. Labelling the significant figures of a number.	3
Estimation	Estimating the answer to calculations.	6
Upper and Lower Bounds	Revision of how to round off. Upper and lower bounds. Problem solving (intermediate tier). **Problem solving (higher tier).**	8
Appropriate Degree of Accuracy	Calculating answers to an appropriate degree of accuracy.	13
Compound Measures	Distance, time and speed. Population, area and population density. Mass, volume and density. Other compound measures.	15

Significant Figures

Sarah sees a television for sale on the internet, for £389.

Delyth, who is Sarah's friend, says that the television costs around £400. Greta, another one of Sarah's friends, says that the television costs around £390.

Sarah's two friends have both given an **estimate** for the price of the television. Delyth has given her estimate to **one significant figure**, whilst Greta has given her estimate to **two significant figures**.

When rounding off to **one significant figure**, we need to find an **estimate** that, **at most**, uses **one non-zero digit**.

When rounding off to **two significant figures**, we need to find an **estimate** that, **at most**, uses **two non-zero digits**.

(And so on for three significant figures; four significant figures; …)

Example

Price	The price rounded off to one significant figure
£389	£400
£1,799	£2,000
£2.35	£2
£0.41	£0.40

Exercise 1

Round off the following prices to one significant figure.

(a) £799 (b) £899 (c) £999 (d) £1,099

(e) £1,846 (f) £1,500 (g) £28,381 (h) £370,000

(i) £4.19 (j) £7.93 (k) £16.70 (l) £42.99

(m) £0.84 (n) £0.37 (o) £0.13 (p) £0.04

Exercise 2

Round off the following numbers to one significant figure.

(a) 329 (b) 698.2 (c) 450 (d) 975

(e) 19 (f) 14 (g) 7.3 (h) 2.8

(i) 0.54 (j) 0.87 (k) 1.56 (l) 0.98

(m) 24,528 (n) 543,590 (o) 19,829 (p) 9,502

Example

Price	The price rounded off to two significant figures
£389	£390
£1,799	£1,800
£2.35	£2.40
£0.41	£0.41

Exercise 3

Round off the following prices to two significant figures.

(a) £263 (b) £268 (c) £265 (d) £299

(e) £4,368 (f) £2,809 (g) £54,238 (h) £876,000

(i) £6.37 (j) £1.62 (k) £14.28 (l) £27.99

(m) £1.95 (n) £19.99 (o) £0.13 (p) £0.04

Exercise 4

Round off the following numbers to two significant figures.

(a) 836 (b) 278.5 (c) 913 (d) 976

(e) 76.3 (f) 16.07 (g) 7.92 (h) 5.37

(i) 0.283 (j) 0.927 (k) 1.8203 (l) 26.07

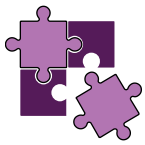
(m) 82,573 (n) 903,400 (o) 49,732 (p) 997

Exercise 5

Complete the following table.

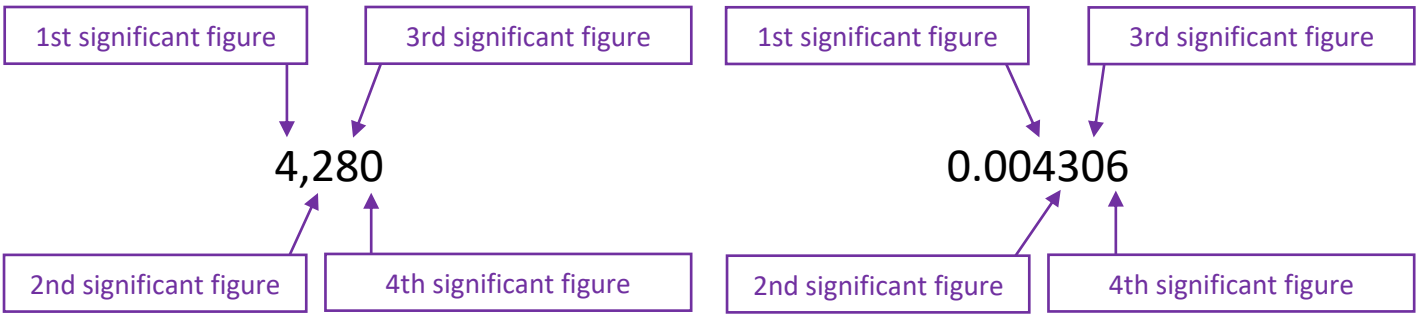
	Original number	Round off to one significant figure	Round off to two significant figures	Round off to three significant figures	Round off to four significant figures
(a)	**28,539**				
(b)	**54.591**				
(c)	**3.0589**				
(d)	**1,339**				
(e)	**106,728**				
(f)	**4,525,925**				
(g)	**73,524,000**				
(h)	**183.078**				
(i)	**8,406.5**				
(j)	**0.87325**				
(k)	**0.018393**				
(l)	**0.0083736**				

Labelling the Significant Figures of a Number

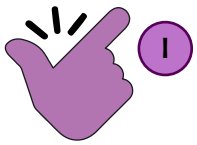


When reading a number from left to right, the **first** significant figure is the first digit encountered that is **not zero**. The other significant figures follow on immediately after the first significant figure.

Example



Exercise 6



For the following numbers, circle the **first** significant figure.

(a) 8,205	(b) 934.2	(c) 387	(d) 46
(e) 0.0378	(f) 0.982	(g) 0.0400213	(h) 0.000029002

Exercise 7

For the following numbers, circle the **second** significant figure.

(a) 43,503	(b) 2,043,293	(c) 819	(d) 8.32
(e) 0.0729	(f) 0.02083	(g) 0.0004893	(h) 0.000000072009

Sometimes, we need to add zeroes to the end of a number to write it correct to a specified number of significant figures. For example, to write 45 correct to three significant figures, we must write 45.0.

Exercise 8

(a) Write 87 correct to three significant figures.

(b) Write 987 correct to four significant figures.

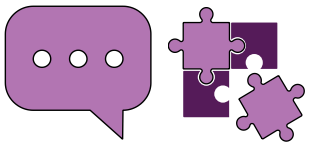
(c) Write 3,270 correct to six significant figures.

(d) Write 7 correct to eight significant figures.

Key words	Corrections	I am happy with...	I need to revise...

Estimation

Dafydd wants to calculate the sum $\frac{482.6\times43.5}{2168.12}$, but unfortunately he has lost his calculator. In order to find an approximate value for the correct answer, it would be possible for Dafydd to **estimate** the size of the numbers in the calculation, and perform the calculation with these estimates.



$$\begin{aligned}\frac{482.6\times43.5}{2168.12} &\approx \frac{500\times40}{2000}\\ &= \frac{20000}{2000}\\ &= 10\end{aligned}$$

Let us now look at the different parts of the above estimation, explaining each part in turn.

1) The first line contains the estimation symbol $\approx$. We say this symbol as 'is around' or 'is approximately', and we need to include the symbol (instead of the equals sign $=$) as the left-hand side is not *equal* to the right-hand side.
2) Unless the question tells us otherwise, the convention for estimating numbers is to round them off to **one significant figure**. So, 482.6 rounds off to 500; 43.5 rounds off to 40; and 2168.12 rounds off to 2000.
3) Going from the first line to the second line, we perform the calculation $500\times40=20000$. To perform this sum quickly, note that $5\times4=20$, and that the three zeroes on the left-hand side lead to three similar zeroes on the right-hand side.
4) Going from the second line to the third line, we perform the calculation $\frac{20000}{2000}=10$. To perform this sum quickly, note that three of the zeroes **cancel** on the top and bottom: $\frac{20000}{2000}$; leading to a much easier sum, $\frac{20}{2}=10$. (The process of 'cancelling' zeroes comes from the fact that we can simplify the fraction by dividing the numerator and denominator by 10 each time.)

Exercise 9

Find an estimate for each of the following calculations.

(a) 423 × 297 (b) 19 × 43,926 (c) 876 × 6,875

(d) 97 × 1,024 (e) 6.4 × 2.9 (f) 0.98 × 60,536.1

(g) 75,249 × 342,192 (h) 198 × 5,034 (i) 78 × 452,138

Exercise 10

Find an estimate for each of the following calculations.

(a) $\frac{7,924}{403}$ (b) $\frac{324}{18.4}$ (c) $\frac{879,054}{12,456}$

(d) $\frac{5,783}{1,985}$ (e) $\frac{98,160}{502}$ (f) $\frac{87,204}{2,642}$

(g) $\frac{82,457}{23.46}$ (h) $\frac{7,304,539}{67,045}$ (i) $\frac{614}{28}$

Exercise 11

Find an estimate for each of the following calculations.

(a) $\frac{798\times403}{1,980}$ (b) $\frac{1,975\times435}{787}$ (c) $\frac{61\times90,128}{2,975}$

(d) $\frac{3.24\times648}{895}$ (e) $\frac{78,420}{204\times4.8}$ (f) $\frac{4.9\times26.4\times7.4}{2.4601}$

(g) $\frac{94.2\times398}{552.4}$ (h) $\frac{0.13\times783}{19}$ (i) $\frac{4.15\times5.63}{0.53}$

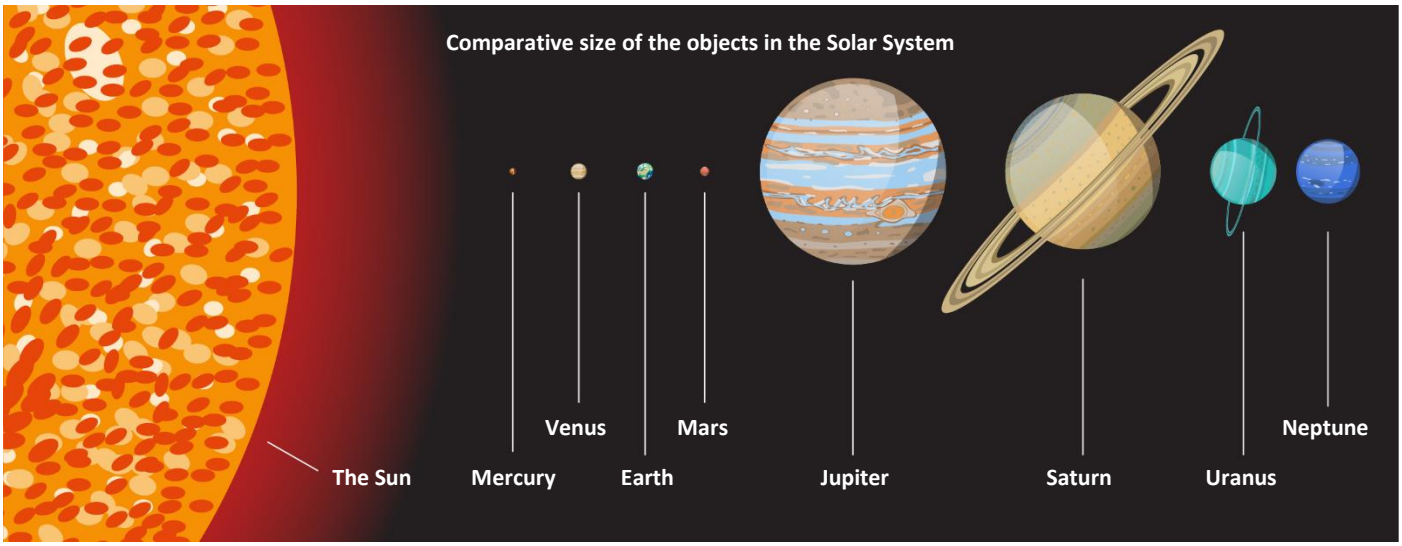
Exercise 12

Applying F

(a) A t-shirt costs £5.99 and a skirt costs £12.99. Sophie wants to buy 2 t-shirts and 5 skirts. She has £50. Use an approximation to decide whether Sophie has enough money.

(b) David saves £3.80 each week. Approximately how much does he save in a year?

(c) Eleri has a mobile phone contract costing £29.99 a month. Approximately how much does Eleri pay over a year?



(d) The diameter of the Sun is around 1,391,400 km, and the diameter of Earth is around 12,742 km. Approximately how many times bigger is the diameter of the Sun compared to the diameter of Earth?

(e) John works in an office and earns £19.75 per hour. He works 39 hours a week. Find an estimate for John's weekly wage.

(f) Someone has asked the pupils in year 9 to estimate the individual cost of a trip to watch a musical. There are 98 pupils in year 9.
A packed lunch for each pupil costs £3.95.
The total cost of the tickets to the musical is £915.
The transport costs £290.
Estimate the individual cost for each pupil to watch the musical, correct to the nearest pound. Show all of your method.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Upper and Lower Bounds

In order to understand how to find **upper and lower bounds**, it's a good idea to first revise different techniques of **rounding off** a number.

Exercise 13

Revision I

Complete the following table.

	Number	Round off to the nearest 10	Round off to 1 decimal place	Round off to 1 significant figure
E.g.	825.94	830	825.9	800
(a)	523.86			
(b)	49.15			
(c)	5,284.792			
(d)	3.67			
(e)	284.99			
(f)	43,704.75			
(g)	726			

Given a number rounded off in a particular way, we can consider what the original number was, before rounding occurred. For example, consider the number 470, which has been rounded off to the nearest 10. What could the original number have been? All the numbers between 465 and 474.9999⋯ round off (to the nearest 10) to be 470. If x represents the original number, then we can say that

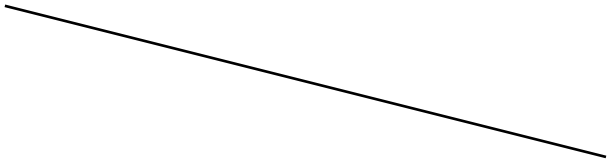
$$465 \leq x < 475.$$

We can also write

$$x = 470 \pm 5$$

but we must remember that x cannot be exactly 475, since this number would round off to give 480. We say that the **lower bound** is 465, and the **upper bound** is 475.

One place where upper and lower bounds are used are when taking measurements, since **every measurement is an approximation.** For example, measure the line below with a ruler.



You should measure the line to be 8.2 cm, but is the line exactly 8.2 cm? How do you know that the line isn't truly 8.19 cm, or 8.21 cm? We would need a more accurate measuring instrument than a ruler to check this, therefore the best we can say (by using a ruler and our eyes) is that the line is 8.2 cm, correct to the nearest 0.1 cm (or mm).

The accuracy limit of a measurement, approximation or calculation uses upper and lower bounds. To measure the above line with a ruler, the lower bound is 8.15 cm and the upper bound is 8.25 cm. The true measurement lies between these two limits. If l denotes the true length of the line in cm, then we can say that

$$8.15 \leq l < 8.25$$

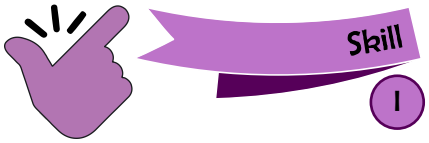
or

$$l = 8.2 \pm 0.05.$$

Example

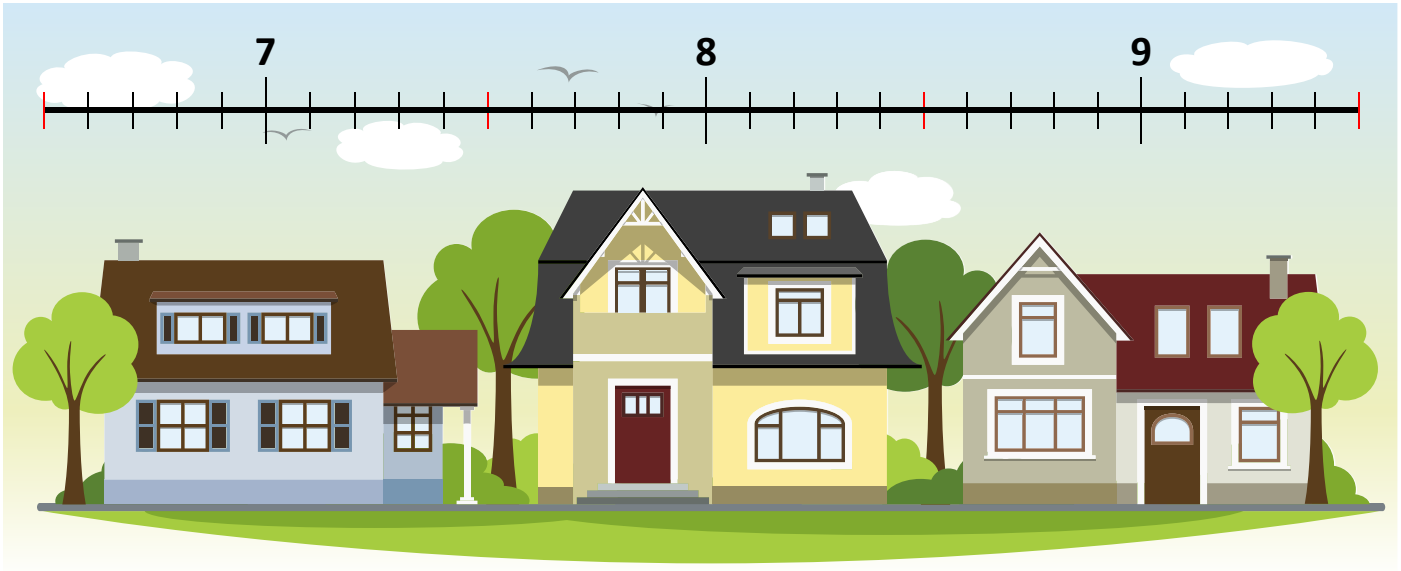
Measurement	Lower Bound	Upper Bound
42 cm (to the nearest cm)	41.5 cm	42.5 cm
85 km (to the nearest 5 km)	82.5 km	87.5 km
60 m (to the nearest 10 m)	55 m	65 m

Exercise 14



Complete the following table.

	Measurement	Lower Bound	Upper Bound
(a)	34 cm (to the nearest cm)		
(b)	5 m (to the nearest m)		
(c)	148 kg (to the nearest kg)		
(d)	25 cm (to the nearest 5 cm)		
(e)	420 ml (to the nearest 5 ml)		
(f)	1,825 km (to the nearest 5 km)		
(g)	40 m (to the nearest 10 m)		
(h)	80 cm (to the nearest 10 cm)		
(i)	180 g (to the nearest 10 g)		
(j)	400 cm (to the nearest 100 cm)		
(k)	5,400 m (to the nearest 100 m)		
(l)	27,800 litres (to the nearest 100 litres)		
(m)	8,000 km (to the nearest 1,000 km)		
(n)	45,000 miles (to the nearest 1,000 miles)		
(o)	3,000 tons (to the nearest 1,000 tons)		
(p)	6 cm (to the nearest even number)		
(q)	154 cl (to the nearest even number)		
(r)	4,250 ml (to the nearest even number)		
(s)	80 cm (to the nearest 20 cm)		
(t)	250 ml (to the nearest 50 ml)		
(u)	320 kg (to the nearest 40 kg)		



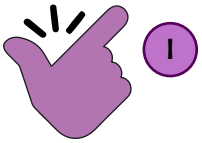
Challenge!

How does the above picture explain how to find upper and lower bounds?

Example

Measurement	Lower Bound	Upper Bound
7.6 kg (to one decimal place)	7.55 kg	7.65 kg
50 cm (to one significant figure)	45 cm	55 cm
740 ml (to two significant figures)	735 ml	745 ml

Exercise 15



Complete the following table.

	Measurement	Lower Bound	Upper Bound
(a)	5.2 cm (to one decimal place)		
(b)	6.7 m (to one decimal place)		
(c)	92.0 inches (to one decimal place)		
(d)	8.24 m (to 2 decimal places)		
(e)	15.28 km (to 2 decimal places)		
(f)	104.09 m (to 2 decimal places)		
(g)	9.258 km (to 3 decimal places)		
(h)	435.205 miles (to 3 decimal places)		
(i)	9.984 seconds (to 3 decimal places)		
(j)	40 m (to one significant figure)		
(k)	400 cm (to one significant figure)		
(l)	4,000 cl (to one significant figure)		
(m)	430 cm (to two significant figures)		
(n)	6,500 ml (to two significant figures)		
(o)	5.2 cm (to two significant figures)		
(p)	500 cm (to two significant figures)		
(q)	7,450 kg (to three significant figures)		
(r)	7,300 kg (to three significant figures)		
(s)	7,000 kg (to three significant figures)		

Exercise 16

Usain Bolt's 100 m record is 9.58 seconds, correct to 2 decimal places.
How fast could Bolt have run the race, in reality?

Exercise 17

Square tiles are to be laid onto the floor of a room.
The side length of each of these tiles is 45 cm, measured to the nearest cm.

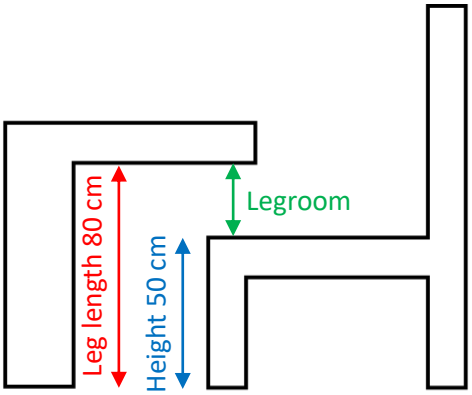
(a) Write down the least possible value and the greatest possible value for the length of a tile in cm.

(b) Calculate the least possible value and the greatest possible value for the perimeter of a tile in cm.

Exercise 18

Two boxes are stacked on top of each other.
The height of one box is 57 cm, correct to the nearest cm.
The height of the other box is 28 cm, correct to the nearest cm.
Calculate the least possible height and the greatest possible height of the boxes stacked on top of each other.

Exercise 19



The legroom between a table and a chair is calculated by finding the difference between the leg length of the table and the height of the seat of the chair. In the diagram, the height of the seat of the chair and the leg length of the table are given to the nearest cm. Find, in centimetres, the least and greatest possible values of the legroom.

Exercise 20

A DIY shop sells lengths of kitchen worktops. There are two different suppliers for the worktops. One supplier, *Worktop Magic*, notes that the length of each worktop is 4,000 mm, measured to the nearest 5 mm. The other supplier, *Worktops 4 U*, notes that the length of each worktop is 4,000 mm ± 3 mm.

(a) Complete the following table.

	Shortest Possible Length	Longest Possible Length
Worktop Magic		
Worktops 4 U		

(b) A customer wants a worktop measuring at least 4.02 m. Would *Worktop Magic* or *Worktops 4 U* be able to supply a suitable worktop? Give a reason for your answer.

Exercise 21

Steffan measures the width of a table. He does this correct to the nearest cm.
Meinir measures the width of the same table. She does this correct to the nearest mm.

(a) What is the smallest possible difference between Steffan and Meinir's measurements?

(b) What is the largest possible difference between Steffan and Meinir's measurements?

Exercise 22

The diameter of the alloy wheel shown on the right is 15 inches ± 0.1 inches.

(a) Write down the smallest possible diameter of the alloy wheel.

(b) Write down the largest possible diameter of the alloy wheel.

(c) Calculate the smallest possible circumference of the alloy wheel.

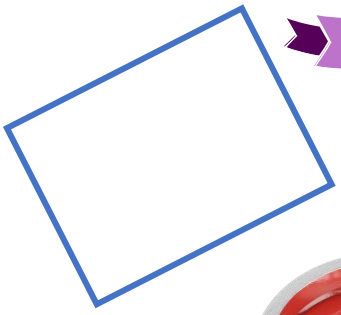
(d) Calculate the largest possible circumference of the alloy wheel.

(e) Copy and complete the following sentence:
The circumference of the alloy wheel is ☐ inches ± ☐ inches.

Exercise 23

H

Higher Tier



The length of a rectangle is 20 cm, correct to the nearest cm.
The width of the rectangle is 15 cm, correct to the nearest 5 cm.

(a) What is the longest possible length of the rectangle?
(b) What is the widest possible width of the rectangle?
(c) What is the smallest possible perimeter of the rectangle?
(d) What is the smallest possible area of the rectangle?

Exercise 24

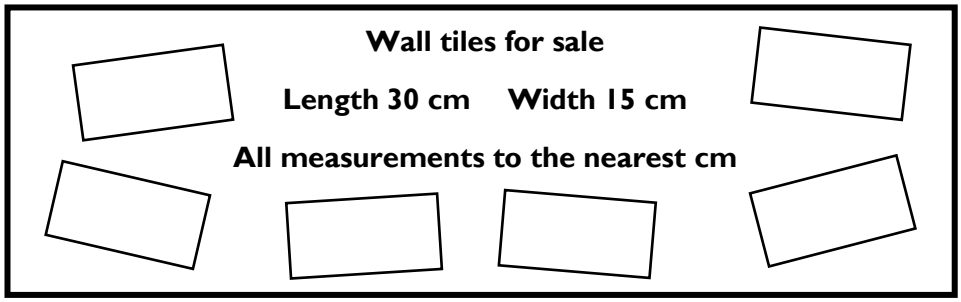
The capacity of a paint pot is 250 ml, correct to the nearest 10 ml.

(a) What is the lowest possible capacity of the paint pot?
(b) What is the largest possible capacity of the paint pot?
(c) Dewi purchases three of these paint pots. The total volume of the paint is V ml.
Copy and complete this statement: $\square \leq V < \square$.

Exercise 25

A car travels 84 miles in 2.4 hours. The distance is measured correct to the nearest mile and the time is measured correct to the nearest 0.1 hours.
Calculate the smallest possible average speed for the car over this distance.
Give your answer in m.p.h. correct to one decimal place.

Exercise 26



Is it always possible to tile an area up to 8,500 cm² using 20 of these tiles?
You must give a reason for your answer and show your method.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Appropriate Degree of Accuracy

Measurements and calculations should not be too precise for their purpose. For example, advertising a television of size 39.97 inches would be too precise (it would be better to say that the television was 40 inches in size).

As a general rule, if a question asks you to give an answer to an **appropriate degree of accuracy**,

do not give an answer that is more precise than the values used in the calculation.

Example

Calculate the area of the following rectangle. Give your answer to an appropriate degree of accuracy.

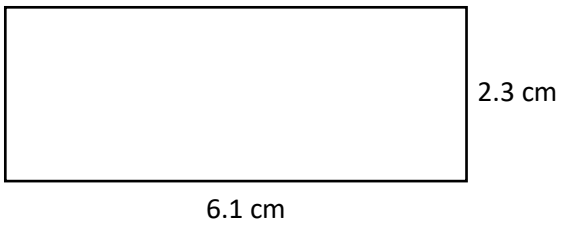
Answer: The area of the rectangle is $7.6 \times 2.2 = 16.72$ cm². Because the numbers in the question have been rounded off to one decimal place, then our answer should also be rounded off to one decimal place. Therefore, the area of the rectangle (to an appropriate degree of accuracy) is 16.7 cm².

Exercise 27

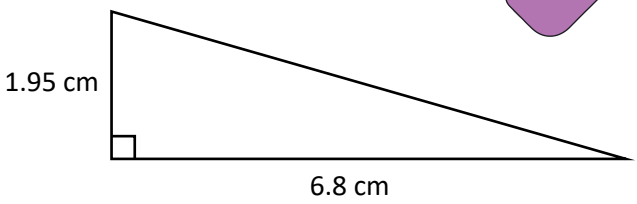
Skill I

Calculate the area of the following shapes. Give your answers to an appropriate degree of accuracy.

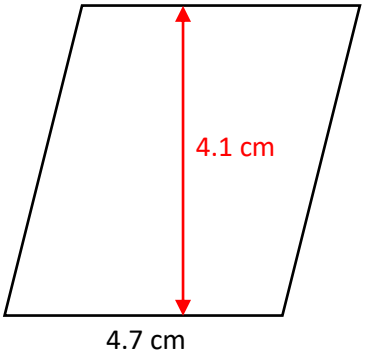
(a)



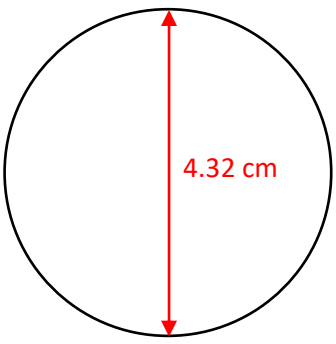
(b)



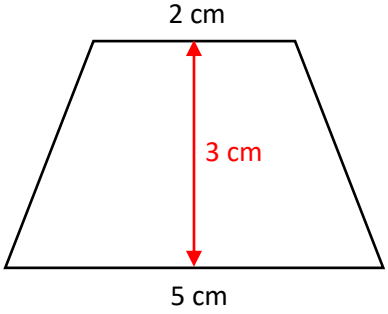
(c)



(d)



(e)



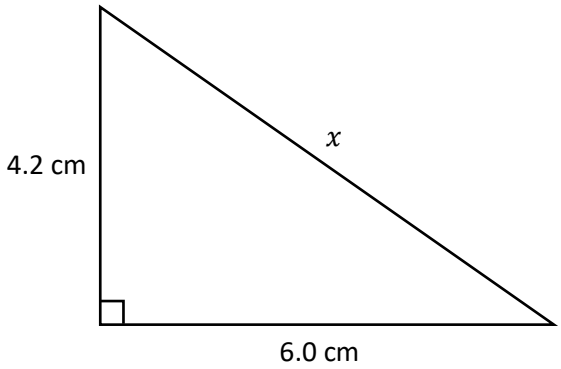
Exercise 28

Mabli has a cuboid that measures 5.2 cm by 8.9 cm by 12.8 cm. Calculate, to an appropriate degree of accuracy, the volume of the cuboid.

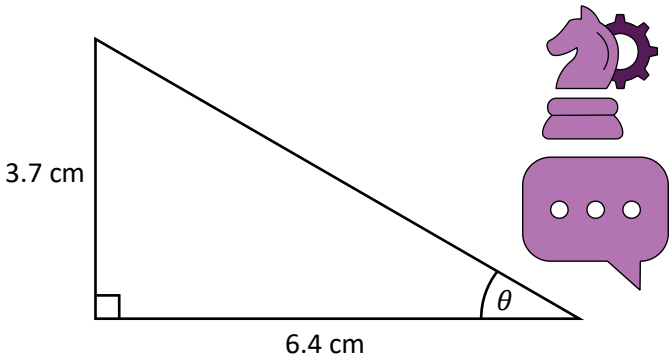
Exercise 29

I

(a) Calculate x to an appropriate degree of accuracy.



(b) Calculate θ to an appropriate degree of accuracy.



Exercise 30

Applying

Lisa wants to invest £6,000 into Barclays Bank at a compound interest rate of 3% per year. Lisa wants to withdraw all the money from the bank after four years. How much money will Lisa be able to withdraw after four years? Give your answer to an appropriate degree of accuracy.

Exercise 31

Rhys is going on holiday to China. The exchange rate for changing money in British pounds (£) to money in Chinese yuan (CYN) is shown below.

£1 = 9.28 CYN

Whilst in China, Rhys catches a taxi that costs 200 CYN. How much is this in pounds? Give your answer to an appropriate degree of accuracy.

Exercise 32

During an experiment, a scientist notices that the number of bacteria halves each second. There were 2.3×10^{30} bacteria at the start of the experiment. Calculate how many bacteria were present after 5 seconds. Give your answer to an appropriate degree of accuracy.

Exercise 33

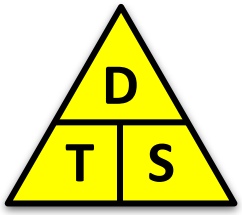
Cynan has a cylinder with diameter 5.3 cm and height 14 cm.
Calculate, to an appropriate degree of accuracy, the volume of the cylinder.

Evaluation

Key words	Corrections	I am happy with...	I need to revise...

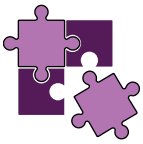
Compound Measures

A **compound measure** is a type of measure that combines two different measures.
In the "Movement with Sphero" workbook, we saw an example of a compound measure, namely **speed**.



Distance = Time × Speed
Time = Distance ÷ Speed
Speed = Distance ÷ Time

To use the triangle, *hide* the letter that you want to find.



Example

(a) Iwan cycles a distance of 32 km in $2\frac{1}{2}$ hours.
Calculate Iwan's average speed in km/hour.

Answer: Speed = Distance ÷ Time
= 32 ÷ 2.5
= 12.8 km/hour

(b) Moli runs an 800 m race in 2 minutes 48 seconds.
What is Moli's average speed in metres per second?

Answer: Speed = Distance ÷ Time
= 800 ÷ 168
= 4.8 metres per second (to one decimal place)

Exercise 34

Revision F

(a) Hannah cycles a distance of 49 km in $3\frac{1}{2}$ hours.
Calculate Hannah's average speed in km/hour.

(b) Elin runs a 400 m race in 1 minute 12 seconds.
What is Elin's average speed in metres per second?

(c) A bus travels the 12 miles from Llandudno to Abergele in 30 minutes.
Find the average speed of the bus in miles per hour.

(d) A train travels at an average speed of 90 km/hour for 2 hours 15 minutes.
How far has the train travelled during this time?

(e) In a 100 m race Dilwyn ran at an average speed of 7.1 metres per second.
How much time did Dilwyn take to finish the race?

(f) An aeroplane flies at an average speed of 885 km/hour. How far did the aeroplane travel between the times 0820 and 0900?

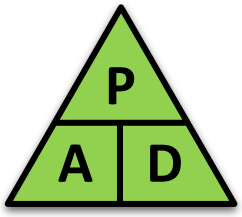
(g) The following chart shows the travelling distance for a car, in miles, between some places in Wales.

	Swansea			
Aberystwyth	70	Aberystwyth		
Bangor	157	86	Bangor	
Cardiff	41	98	180	Cardiff
Wrexham	130	79	70	139

(i) What is the travelling distance for a car between Cardiff and Bangor?
(ii) Dewi travels between Cardiff and Bangor, and then between Bangor and Wrexham. How far has Dewi travelled in total?
(iii) Elis travels between Bangor and Aberystwyth in 2 hours and a half. What is Elis' average speed, in mph?
(iv) Esyllt travels between Wrexham and Swansea at an average speed of 32 miles per hour.
How much time did Esyllt take to travel between Wrexham and Swansea?

Population Density

Population density compares the population of a place to its area.



Population = Area × Population Density
Area = Population ÷ Population Density
Population Density = Population ÷ Area

Population density is (usually) measured in people per square kilometre, and is used to compare how developed different areas are.

Example

Paris has a population of 2,265,886 and an area of 105.4 km². What is Paris' population density?

Answer: Population Density = Population ÷ Area
= 2,265,886 ÷ 105.4
= 21,498 people per km² (correct to the nearest whole number)

Exercise 35

(a) Mumbai has a population of 12,478,447 and an area of 603 km².
What is Mumbai's population density?

(b) San Francisco has a population of 805,816 and an area of 120.93 km². What is San Francisco's population density?

(c) Dublin has a population of 553,165 and an area of 114.99 km².
What is Dublin's population density?

(d) The population density of London is 5,584 people per km².
If London's population is 8,778,500, what is London's area?

(e) The population density of Chicago is 4,582 people per km².
Is Chicago's population is 2,695,598, what is Chicago's area?

(f) The population density of Cairo is 19,376 people per km².
If Cairo's area is 606 km², what is Cairo's population?

(g) The population density of Miami is 4,324 people per km².
If Miami's area is 92.38 km², what is Miami's population?

(h) The following table gives information about some Spanish cities.

City	*Madrid*	*Barcelona*	*Seville*
Population	3,141,991	1,621,537	703,021
Area	604.3 km²	101.9 km²	140 km²

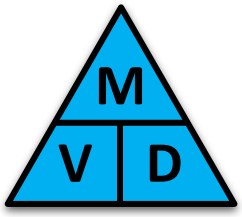
(i) Calculate the population density of the three Spanish cities.
(ii) Which city is the one where people live closest to each other?

Challenge!

Try to calculate the population density of where you live.

Density

Density compares an object's mass to its volume.



Mass = Volume × Density
Volume = Mass ÷ Density
Density = Mass ÷ Volume

To use the triangle, *hide* the letter that you want to find.

Density is (usually) measured in g/cm³ or kg/m³, and is used to compare how much mass an object has per unit of volume.

Example

(a) The mass of a 200 cm³ piece of metal is 1.2 kg.
What is the metal's density, in g/cm³?

Answer: Density = Mass ÷ Volume
= 1,200 ÷ 200
= 6 g/cm³

(b) The density of a piece of aluminium is 2.7 g/cm³.
Calculate the mass of a 20 cm³ piece of aluminium.

Answer: Mass = Volume × Density
= 20 × 2.7
= 54 g

Exercise 36

(a) The mass of a stone is 4.4 kg.
(i) What is the mass of the stone in grams?
(ii) Find the density of the stone (in g/cm³) if its volume is 2,000 cm³.

(b) A piece of cork weighs 10 kg. Its volume is 0.04 m³.
Calculate the density of the cork, in kg/m³.

(c) The mass of a 1,200 ml piece of ice is 1,104 grams. Find its density.
(Hint: 1 ml = 1 cm³.)

(d) Gold is expensive and desirable not just for how it looks but also because it does not rust easily. Lewis wishes to build a car made out of gold but is worried about its mass. The density of gold is 19,300 kg/m³ and the volume of Lewis' car is around 1.5 m³.
(i) Calculate the mass of Lewis' gold car.
(ii) Given that the mass of a normal car is around 1,500 kg, comment on the practicality of Lewis' plan.

(e) Megan is mixing and pouring concrete. She mixes and then pours a total of 2.4 m³ of concrete.
Calculate the mass of this concrete if its density is 2,500 kg/m³.

(f) The density of water is 1 g/cm³. Calculate the mass of 1.5 litres of water.

(g) A clay statue has density 1.4 g/cm³. Another statue is carved from wood and has a density of 0.8 g/cm³. The two statues are weighed in order to find their mass. The wood statue weighs 4,500 g. The clay statue weighs 7,700 g. Which statue has the greatest volume?

(h) The dimensions of a metal cuboid are 8 cm, 6 cm and 5 cm.
The mass of the cuboid is 0.9 kg. Calculate the density of the metal, noting your units clearly.

Other Compound Measures

Exercise 37

(a) The fuel consumption of Bethan's car is 50 miles per gallon.
(i) If Bethan travels 200 miles in her car, how many gallons of petrol does she use?
(ii) Today, petrol costs 117.9 pence per litre. Given that a gallon of petrol is around 4.5 litres, calculate the cost of the petrol for Bethan's journey. Give your answer correct to the nearest penny.

(b) John travels 126 miles in his car, using 2.25 gallons of petrol. What is the fuel consumption of John's car, in miles per gallon?

(c) Janet types at a rate of 70 words per minute. How much time would Janet need to type a report containing 3,500 words?

(d) The average person can type at a rate of 40 words per minute. Bob types a 1,200-word report in 40 minutes. Does Bob type at a rate that is slower than the average person, or is quicker than the average person?

(e) A mill produces 20 metric tons/hour of flour.
How much flour is produced every 15 minutes?

(f) A paint tin notes that it can cover an area of 13 m^2/litre.
If the paint tin contains 2.5 litres of paint, what total area can the tin cover?

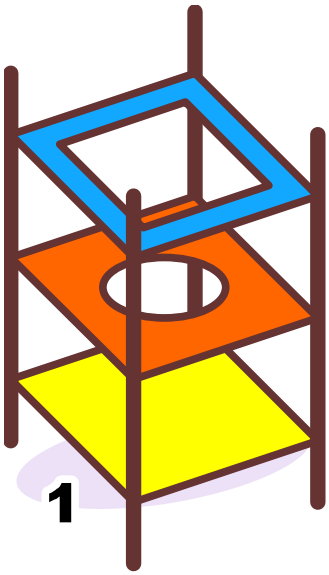
(g) A swimming pool is emptied in order to clean it. The pool (when full) can hold 375,000 litres of water. After it is cleaned, the pool is refilled by using a water pipe that can supply water at a rate of 200 litres per minute. How much time will it take to refill the pool? Give your answer in hours and minutes.

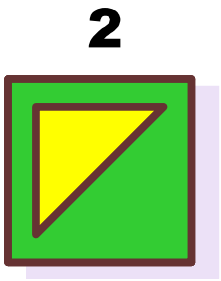
(h) During the first 20 games of a football season, Gareth Bale scored an average of 0.7 goals per game.
How many goals is it expected that Gareth Bale scores during the next 6 games?

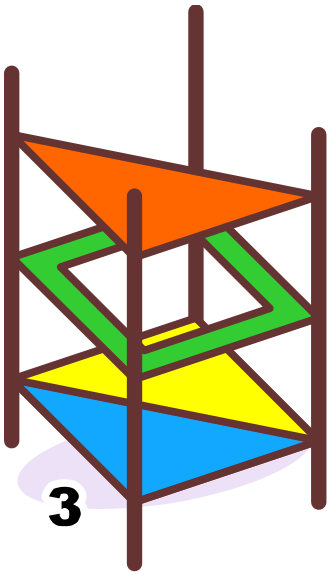
Evaluation

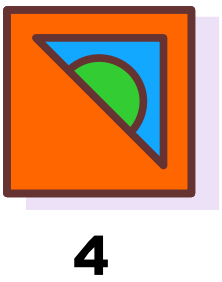
Key words	Corrections	I am happy with...	I need to revise...

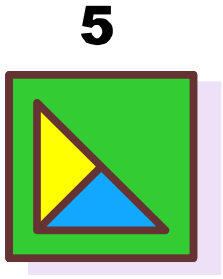
Puzzle: Can you find the top view for every stand?

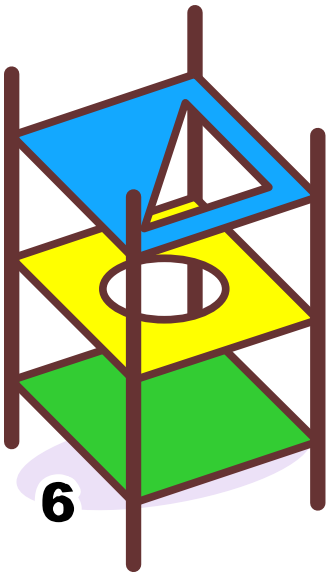


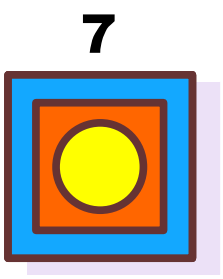


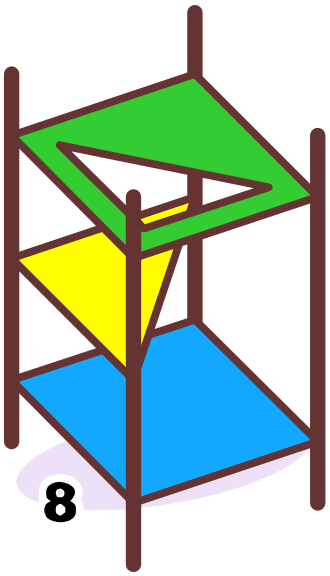


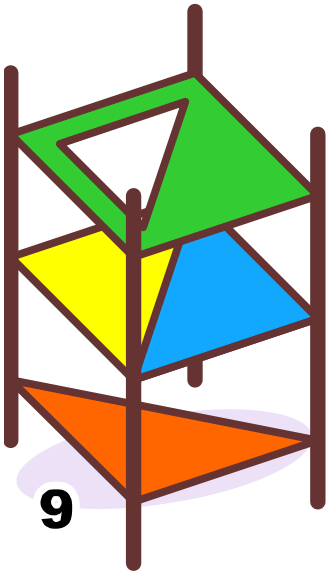


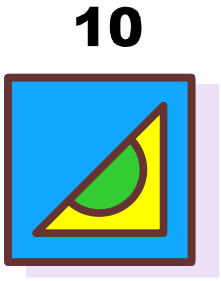


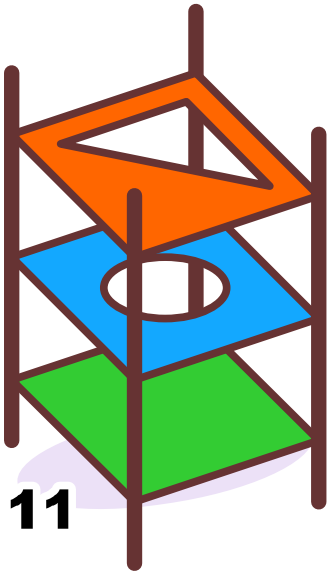


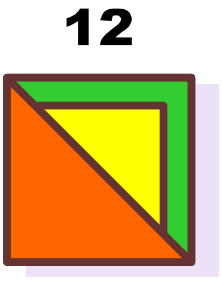












Accuracy of Measurements

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this.	I need to revise this.	Question in the test	Correct in the test?
I know how to **round off** a number to a specified number of **significant figures**, e.g. to 1 significant figure or to 2 significant figures.			1, 2	
I know how to **recognise the significant figures** in a given number.			3	
I know how to **estimate** the answer to a sum, by using rounding off to one significant figure.			4	
I know how to find the **upper and lower bounds** of a measurement.			5, 6, 7, 13, 14	
I know how to interpret measurements that are given using the symbol **±**, for example 32 cm ± 0.5 cm.			7	
I can use upper and lower bounds to **solve problems in context**.			6, 7, 13, 14	
I know how to write answers to an **appropriate degree of accuracy**.			8, 14	
I can use the formulae that are associated with the compound measure **speed**.			10	
I can use the formulae that are associated with the compound measure **population density**.			9	
I can use the formulae that are associated with the compound measure **density**.			11	
I am aware of how to work with **other compound measures**.			12, 13	

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