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WELSH JOINT EDUCATION COMMITTEE
Y CYD-BWYLLGOR ADDYSG CYMREIG

General Certificate of Education

Summer Examination 1970

Advanced Level

(3 Hours) MATHEMATICS A 1

Answer seven questions only.

1. (*a*) When the expression ax^4+bx^3+8x-5 is divided by $2x^3+x-1$, the remainder is $2x-3$. Find the values of a and b.

(*b*) Show that the sum of a geometric progression whose first term is a, last term l and common ratio r, is $(a-rl)/(1-r)$.

Find the sum to n terms of the series

$$2+5x+8x^2+11x^3+\ldots.$$

2. (*a*) Prove that $\log_b a \,.\log_a b = 1$.

Solve the equation

$$\log_{10} x-\log_x 10 = 2,$$

giving the roots to two significant figures.

(*b*) The values of x and y satisfy the relation

$$x^2-xy+y^2+3x-2y+1 = 0.$$

Show that, for real values of x, the values of y must lie between -1 and $1\frac{2}{3}$ inclusive.

Turn over.

3. (*a*) If $(2+ax)^{11} = a_0+a_1x+a_2x^2+\ldots+a_{11}x^{11}$, and $a_7 = 8a_8$, find the value of the constant a. Find also the value of the largest coefficient a_r.

(*b*) Sketch the graph of $y = \sin x$ from $x = -3\pi$ to $x = 4\pi$. Hence show that the equation

$$10 \sin x = 2+mx$$

has three distinct real roots if $m = 4$ and find the number of real roots if $m = -2$.

(*accurate graphs are not required*)

4. Assuming the results for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\sin 3\theta \equiv 3 \sin\theta - 4\sin^3\theta \quad \text{and} \quad \cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta.$$

Hence, or otherwise,

(i) prove the identity

$$\frac{\sin^2 3\theta}{\sin^2\theta} - \frac{\cos^2 3\theta}{\cos^2\theta} \equiv 8\cos 2\theta,$$

(ii) solve, for $0 < \theta < 180°$, the equation

$$2 \sin 3\theta = 1+2\sin\theta.$$

5. Find the equation of the circle that passes through the point $(2, 1)$ and touches the line $4x-3y = 10$ at the point $(1, -2)$. Find also the equation to the tangent at the point $(2, 1)$ and the length intercepted by this circle on the y-axis.

6. Prove that the chord joining the points $A(at^2, 2at)$ and $B\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ on the parabola $y^2 = 4ax$ passes through the focus S.

The tangent at A and the normal at B meet the axis of the parabola at P and Q respectively. Show that AP is parallel to BQ, and find the two possible coordinates of A when $SP = 2SQ$ in length.

7. (*a*) The complex numbers $z = x+iy$ and $w = u+iv$ are connected by the relation $w = (2z-1)/(z+1)$. Find the value of w corresponding to $z = 1+2i$ and the value of z corresponding to $w = 1+2i$.

(*b*) When the vertices of a square $ABCD$ are taken anticlockwise in that order, the vertices A and B represent the points $1+i$ and $5+4i$ respectively. Find the complex numbers represented by the points C and D.

8. (*a*) If $y = e^{3x} \sin 4x$, show that $dy/dx = ae^{3x} \sin(4x+\lambda)$, where a and λ are constants to be determined. Hence without further differentiation show that

$$\frac{d^2y}{dx^2} = 25e^{3x} \sin(4x+\beta),$$

where $\tan\beta = -24/7$.

(*b*) It is proposed to construct a window in the shape of a rectangle surmounted by an equilateral triangle, the perimeter of the window to be 22 ft. If the area of the window is to be a maximum, prove that the width of the window must be two-thirds of the greatest height.

9. (*a*) Evaluate

$$\int_0^1 \frac{6+x}{4-x^2}\,dx \quad \text{and} \quad \int_0^1 \frac{6+x}{\sqrt{(4-x^2)}}\,dx.$$

(*b*) A sphere of radius a is divided into two segments by a plane distant $\frac{1}{2}a$ from the centre. Show by integration that the volume of the smaller segment is $\frac{5}{24}\pi a^3$.

10. (*a*) Show that for $0 < x < \pi$ the function

$$(\sin x - x \cos x)$$

increases steadily as x increases, and deduce that

$$x < \tan x \quad \text{for} \quad 0 < x < \tfrac{1}{2}\pi.$$

(*b*) Give a rough sketch of the graph of $y = \log(1+x)$ and show that the area enclosed between the curve, the x-axis and the ordinates $x = 1$ and $x = 2$, is $\log_e(27/4) - 1$.