

WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Tystysgrif Addysg Gyffredinol

Summer Examination 1975
Arholiad Haf 1975

Advanced Level
Safon Uwch

Marks

(3 hours)

MATHEMATICS

A 1

Answer seven questions only.

1. (a) Solve the equation

$$\frac{x^2}{x+2} + \frac{x+2}{x^2} = \frac{10}{3}$$

(b) Find the numerical value of the term independent of x in the binomial expansion of

$$\left(3x^3 + \frac{1}{9x^2}\right)^{10}$$

2. (a) Establish the result $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$ and hence deduce the values of

(i) $10^2 + 11^2 + 12^2 + \dots + 20^2$,

(ii) $1^2 - 2^2 + 3^2 - 4^2 + \dots - 16^2$.

(b) Without using graph paper or unnecessary calculations give a sketch of the graph of

$$y^2 = \frac{x-1}{x+1}$$

Turn over.

3. (a) If $x = \cos \theta + \cos \phi$ and $y = \sin \theta + \sin \phi$ find the value of $\cos(\theta - \phi)$ and show that

$$\cos(\theta + \phi) = (x^2 - y^2) / (x^2 + y^2). \quad 3, 4$$

(b) Solve the equations

(i) $\sec \theta + \tan \theta = 2$ for $0 < \theta < 180^\circ$; 4

(ii) $3 \sin \theta + 4 \cos \theta = 2$ for $0 < \theta < 360^\circ$. 4

4. An isosceles triangle ABC has $AB = AC = 6$ cm. The point D on AC is such that $CD = 2$ cm and $\widehat{DBC} = 36^\circ$. Show that $\cot C = \sqrt{3}/5$ and find the exact value of $\tan A$. 10, 5

5. (a) Determine the ratio in which the line

$$2x - 3y - 6 = 0$$

divides the line joining the points $(6, -5)$ and $(-2, 3)$. 6

(b) Find the equation of the circle passing through the point $(2, 1)$ and touching the line $4x - 3y - 10 = 0$ at the point $(1, -2)$. 9

6. Show that the tangents to the parabola $y^2 = 4ax$ at the points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ meet at the point T given by $x = at_1t_2, y = a(t_1 + t_2)$. 5

Show that the area of the triangle PQT is $\frac{1}{2}a^2|t_2 - t_1|^3$ and hence deduce that if the area of the triangle is $4a^2$, then T lies on the parabola

$$y^2 = 4a(x + a). \quad 5, 5$$

7. (a) Obtain the square roots of the complex number $\frac{1}{2}(1 - i\sqrt{3})$.

(b) Solve the equation

$$3x^3 - 4x^2 + 11x + 10 = 0$$

given that $x = 1 + 2i$ is a complex root. 5

(c) Give a diagram showing the region in the Argand plane within which the values of z satisfy all the inequalities

$$\Re(z) \geq 0, \quad |z| \leq |z - 2| \quad \text{and} \quad 0 \leq \arg z \leq \frac{1}{4}\pi. \quad 6$$

8/ (a) Differentiate the following with respect to x :

$$\frac{(x-2)^3}{\sqrt{2x+1}} \quad \text{and} \quad 2^x.$$

(b) Show that the volume of a right circular cone of height h inscribed in a sphere of radius a is equal to

$$\frac{1}{3}\pi h^2(2a-h).$$

Hence, or otherwise, find the maximum volume of a cone inscribed in this sphere.

9/ (a) Evaluate the following integrals (numerical answers are not required):

$$\int_1^2 x^3 \log x \, dx \quad \text{and} \quad \int_0^1 \frac{(2+x)}{\sqrt{4-x^2}} \, dx.$$

(b) Find the volume generated when the area under the curve $y = x^2$ bounded by the x -axis and the ordinates $x = 1$ and $x = 2$ is rotated about the line $x = 3$.

10/ Show that the expression

$$y = x/(x-1)(x-4)$$

does not possess any values between $-\frac{1}{3}$ and -1 . Give a rough sketch of its graph showing the essential features.

Find the area between this curve, the x -axis and the ordinates $x = 0$ and $x = -2$.