WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Level/Advanced Supplementary

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MATHEMATICS (MODULAR) P1

Mathematical Methods 1

11.00 A.M. TUESDAY, 14 January 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

January 1997.

1. The second term of a geometric series is 72 and the fifth term is 9.

(*a*) Find the common ratio. [3]

(*b*) Find the sum to infinity. [2]

2. The points A, B, C have coordinates (3, 2), (1, –7) and (–2, 5) respectively. The perpendicular from A to BC cuts BC at the point D.

(*a*) Show that the coordinates of D are (–1, 1). [6]

(*b*) Find the area of triangle ABC. [3]

3. The time intervals between the instants at which successive vehicles pass a particular point on a one-way road were measured to the nearest second and the following grouped frequency distribution was calculated.

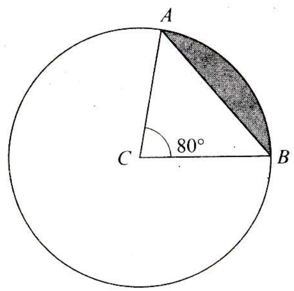
Time interval	1–2	3–4	5–6	7–8	9–10	11–15
Frequency	59	83	72	34	32	20

(*a*) Draw a cumulative frequency polygon. [3]

(*b*) Estimate the median time interval, giving your answer correct to one decimal place. [1]

4. In the binomial expansion of $\left(1+\frac{x}{2}\right)^n$ the coefficient of the term in x^2 is equal to the coefficient of the term in x^3. Find the value of n. [3]

5.



The diagram, which is not drawn to scale, shows a chord AB of a circle, centre C. The area of the shaded region is 77 cm^2 and $A\hat{C}B = 80°$. Find the radius of the circle. [3]

6. The curve C has equation $y = x^3 - 3x^2$.

(*a*) Find the coordinates of the stationary points of C and determine their nature. [5]

(*b*) Sketch the curve C. [1]

(*c*) Find the area of the region bounded by the curve C and the x-axis. [3]

7. The functions f and g are defined by $f(x) = 2x - 5$ for all x
and $g(x) = \sqrt{x-3}$ for $x \geqslant 3$.

The composite function fg is denoted by h.

(*a*) Find an expression for $h(x)$. [1]

(*b*) State the domain and range of h. [2]

8. (*a*) Integrate the following with respect to x.

(i) $\frac{1}{x^3}$.

(ii) e^{3x}. [2]

(*b*) Show that $\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{-4x}{(1+x^2)^2}$. [2]

Hence evaluate $\int_0^1 \frac{x}{(1+x^2)^2}\,dx$. [2]

9. Ann and John each toss a fair coin twice. Find the probability that

(i) Ann obtains two heads and John obtains no heads, [1]

(ii) Ann obtains more heads than John. [3]

10. (*a*) For $x > 0$, **sketch** the graphs of $y = \ln x$ and $y = \frac{1}{x}$ on the same diagram and hence show that there is only one value of x which satisfies

$$x \ln x = 1.$$ [3]

(*b*) Find the equation of the tangent to the curve $y = \ln x$ at the point $(a, \ln a)$. Given that this tangent passes through the point (–1, –1), show that

$$a \ln a = 1.$$ [4]

(*c*) Show that the value of a lies between 1·7 and 1·8. [2]

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MATHEMATICS (MODULAR) P2

Mathematical Methods 2

P.M. FRIDAY, 17 January 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each section or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The equations of the circles C_1 and C_2 are respectively

C_1: $x^2 + y^2 + 2x - 8y + 8 = 0$,

C_2: $(x + 5)^2 + (y - 1)^2 = 4$.

(*a*) Find the radius of C_1 and the radius of C_2. [2]

(*b*) Find the distance between the centres of C_1 and C_2. [3]

(*c*) Explain why C_1 and C_2 touch one another. [1]

2. The iterative formula

$$x_{n+1} = \frac{2x_n^{\;3} - 3}{3x_n^{\;2} - 8}$$

can be used to find a root of $x^3 - 8x + 3 = 0$. Use $x_0 = 0{\cdot}4$ to find a root of the equation, giving your answer correct to five decimal places. Record the values of $x_1, x_2, \ldots$, to as many decimal places as your calculator will allow. [3]

3. (*a*) Factorise $2y^3 + y^2 - 13y + 6$. [4]

(*b*) Use the substitution $y = e^x$ to solve the equation

$$2e^{3x} + e^{2x} - 13e^x + 6 = 0,$$

giving your answers correct to three decimal places. [2]

4. (*a*) Differentiate $x\sin^{-1} x + \sqrt{1 - x^2}$ with respect to x. [3]

(*b*) Given that

$$y^3 + xy = 2 + 3x,$$

find $\dfrac{dy}{dx}$ in terms of x and y. [3]

5. The points $P(p^2, 2p)$ and $Q(q^2, 2q)$ on the curve $y^2 = 4x$ are such that the line PQ passes through the point $S(1, 0)$.

(*a*) Show that $pq = -1$. [3]

(*b*) Show that the tangent to the curve at P has equation $py - x - p^2 = 0$. [3]

(*c*) The line through Q parallel to the x-axis meets the tangent to the curve at P at the point R. Show that the mid-point of PR lies on the line $x + 1 = 0$. [3]

6. (*a*) Express $\dfrac{2x^2 + x + 3}{(x + 1)(x^2 + 1)}$ in partial fractions. Hence find

$$\int \frac{2x^2 + x + 3}{(x + 1)(x^2 + 1)}\, dx.$$ [5]

(*b*) Use integration by parts to find

$$\int_0^{\frac{\pi}{4}} x \sin 2x \, dx.$$ [4]

7. A particle is moving along a straight line such that at time t its acceleration is $-kv$, where v is its velocity at that time and k is a positive constant.

(*a*) Write down a differential equation involving v and t. [1]

(*b*) Given that $v = V_0$ when $t = 0$, find, in terms of k, the time when $v = \dfrac{V_0}{10}$. [4]

8. Given that $y = \cos^6 x + \sin^6 x$ for $0 \leqslant x \leqslant \dfrac{\pi}{2}$,

(*a*) show that $\dfrac{d^2y}{dx^2} = 30\sin^2 x\cos^2 x - 6\cos^6 x - 6\sin^6 x$, [5]

(*b*) find the maximum and minimum values of y. [6]

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MATHEMATICS (MODULAR) M1

Mechanics 1

11.00 A.M. MONDAY, 20 January 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take $g = 9{\cdot}8\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle P is projected from a point O with a velocity which has horizontal and vertically upwards components of $25\ \text{ms}^{-1}$ and $55\ \text{ms}^{-1}$ respectively. Find

(*a*) the horizontal and vertical components of the displacement of P from O four seconds after projection. [3]

(*b*) the direction of the velocity of P four seconds after projection. [3]

2. A small smooth sphere A of mass $0{\cdot}2$ kg and moving with speed $10\ \text{ms}^{-1}$ catches up, and collides directly with a second smooth sphere B, of mass $0{\cdot}4$ kg and moving with speed $2\ \text{ms}^{-1}$. The coefficient of restitution between the spheres is $\frac{1}{4}$.
Find

(*a*) the speed of A immediately after collision,

(*b*) the magnitude of the impulse on A. [7]

3. Bethan and Ieuan have to lift and carry a heavy awkwardly shaped parcel of mass 50 kg and decide to do this by putting the parcel on a uniform plank, of mass 10 kg and length $2{\cdot}4$ m, and carry the plank by its ends. Bethan is not as strong as Ieuan and they therefore place the parcel in such a position that the force exerted by Bethan is two thirds that exerted by Ieuan. Modelling the plank as a uniform rod and the parcel as a particle find

(*a*) the force exerted by Ieuan,

(*b*) the distance of the parcel from Ieuan. [4]

4. A car of mass 1000 kg is travelling along a horizontal road with its engine working at a rate of 36 kW. When the speed of the car is $20\ \text{ms}^{-1}$ its acceleration is $0{\cdot}6\ \text{ms}^{-2}$. Find the resistance to the motion of the car at this speed. [3]

Assuming that the resistance is directly proportional to the square of the speed, find the maximum speed that the car can attain on a horizontal road with its engine working at a rate of 36 kW. [3]

5. The tension in the cable of a lift ascending with a constant acceleration of magnitude f is equal to the tension in the cable when the lift is descending with an acceleration of the same magnitude but with a 25% greater load (i.e. the mass of lift and passengers when descending is 25% greater than that when ascending). Find the value of f in terms of g. [5]

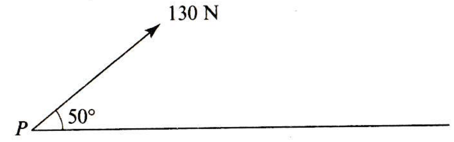
6. A small parcel of mass 4 kg is placed on a rough horizontal plane, the coefficient of friction between the parcel and the plane being $0{\cdot}3$. Modelling the parcel as a particle, find the horizontal force that would have to be applied so that the parcel would be on the point of moving. [2]

The parcel is then placed on a plane inclined at an angle of 15° to the horizontal, the coefficient of friction between the parcel and the plane being $0{\cdot}3$. The parcel is again to be modelled as a particle.

(*a*) Show that the parcel remains at rest on the plane. [2]

(*b*) A force applied to the parcel parallel to a line of greatest slope of the plane is just sufficient to move the parcel up the plane. Find the magnitude of this force. [2]

7. (*a*)



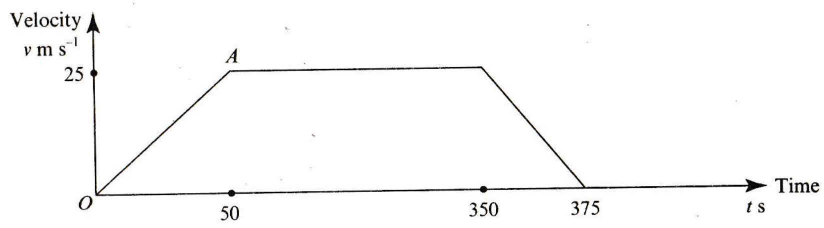
The diagram shows a particle P free to move along a straight wire and acted on by a force of magnitude 130 N acting at an angle of 50° to the wire. Find the work done by the force when P is moved a distance of 2·4 m to the right along the wire. [3]

(*b*) A particle Q of mass 0·4 kg moves on the x-axis under the action of a single force acting in the positive x direction and of variable magnitude $\frac{24}{x^4}$ N, where x m is the distance of Q from the origin O.

(i) Using the definition of work done as an integral, show that the work done by the force as x increases from 1 to 2 is 7 J. [2]

(ii) Given that the speed of Q when $x = 1$ is 2 ms^{-1}, find the speed of Q when $x = 2$. [3]

8.



(*a*) The diagram, which is not drawn to scale, is a velocity–time sketch modelling the motion of a train from rest between two stations.

(i) Find the acceleration of the train for $t = 25$ and for $t = 360$. [2]

(ii) Describe briefly the motion of the train as it travels from rest to rest. [2]

(iii) Find the distance between the two stations. [3]

(*b*) A more realistic model of the first fifty seconds of the motion should take account of the fact that the acceleration should vanish at the points O and A. One method of doing this is to assume that the straight line joining the points O and A should be replaced by the curve $v = at^3 + bt^2$, where a and b are constants.

Find, but do not attempt to solve, two independent equations which a and b have to satisfy. [6]

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MATHEMATICS (MODULAR) S1

Statistics 1

9.30 A.M. THURSDAY, 23 January 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

The only books of statistical tables that you may use in the examination are "Statistical Tables" by Murdoch and Barnes (Macmillan Press) **or** "Elementary Statistical Tables" (RND Publications).

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The number of radios sold per day by a shop can be modelled by a Poisson distribution with mean 2.

(*a*) Find the probability that at least 3 radios are sold on a particular day. [1]

(*b*) Assuming that the numbers of radios sold on different days are independent, find the probability that fewer than 16 radios are sold in a particular 6-day week. [2]

2. The events A, B are such that $P(A) = 0{\cdot}5$ and $P(A \cup B) = 0{\cdot}7$.

Find the value of $P(B)$ when

(*a*) A and B are mutually exclusive, [1]

(*b*) A and B are independent. [3]

3. The table below gives the frequency distribution of the number of occupants per flat in a large development.

Number of occupants per flat (x)	Frequency (f)
0	2
1	34
2	25
3	18
4	13
5	8

The following information was calculated from the above table.

$$\Sigma f = 100,\ \Sigma fx = 230,\ \Sigma fx^2 = 704.$$

(*a*) Calculate

(i) the mean

(ii) the variance

of the number of occupants per flat. [3]

(*b*) Someone suggests that a Poisson distribution could be used to model these data.

(i) Explain briefly why your results show that this is not an appropriate model.

(ii) Give another reason why the Poisson distribution does not provide a suitable model for these data. [2]

4. The weight of fish, X tonnes, landed per day by a fishing boat can be assumed to be a continuous random variable with probability density function f given by

$$f(x) = 20x^3(1-x), \quad \text{for } 0 \leqslant x \leqslant 1,$$
$$f(x) = 0, \quad \text{otherwise.}$$

(*a*) Find the mean weight of fish landed per day. [2]

(*b*) Obtain an expression, valid for $0 \leqslant x \leqslant 1$, for the cumulative distribution function of X. Use your expression to find the probability that X exceeds $0{\cdot}75$. [4]

5. A school committee consists of five boys and four girls. A sub-committee is to be formed by selecting three of these pupils at random, without replacement.

The number of girls on the sub-committee is denoted by X.

(*a*) Show that

$$P(X = 2) = \frac{5}{14}.$$ [2]

(*b*) Determine the probability distribution of X, displaying the probabilities in a table. [3]

(*c*) Calculate the expected number of girls on the sub-committee. [2]

6. A bag contains twelve balls, of which three are red, four are blue and five are yellow. Three balls are selected at random, without replacement, from the bag.

(*a*) Calculate the probability that one ball of each colour is selected. [2]

(*b*) Calculate the probability that the three balls selected are all of the same colour. [2]

(*c*) Given that the three balls selected are **not** all of the same colour, calculate the probability that none of the selected balls is red. [4]

7. A certain make of dog biscuit is sold in bags of nominal weight 20 kg. The bags are filled automatically. It is known from past experience that the actual weight of biscuits in a bag is normally distributed with mean 20·1 kg and standard deviation 0·2 kg.

(*a*) Calculate the probability that a randomly selected bag contains less than the nominal weight of biscuits. [3]

(*b*) A shopkeeper buys 10 bags of these biscuits. Calculate the probability that

(i) at least 1 of these bags contains less than the nominal weight of biscuits,

(ii) the combined weight of biscuits in the 10 bags is less than 200 kg.

What additional assumption did you make in order to solve (i) and (ii)? [7]

8. In a postal survey, n questionnaires are sent out. The number, X, of completed questionnaires returned is modelled by the binomial distribution $B(n, p)$ where p is the probability of a person completing and returning the questionnaire.

(*a*) Given that $n = 50$ and $p = 0{\cdot}85$, use the model to find $P(X \geqslant 40)$. [3]

(*b*) In another survey, $n = 200$ and $p = 0{\cdot}7$.

(i) Use a suitable distributional approximation to the model to evaluate $P(X \leqslant 150)$.

(ii) A student is employed to process the completed questionnaires and he is paid £50 plus £2 per questionnaire. Find the mean and standard deviation of the amount of money paid to the student. [8]

(*c*) Give **one** reason why the binomial model is unlikely to give accurate results. [1]

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MATHEMATICS (MODULAR) P1

Mathematical Methods 1

P.M. WEDNESDAY, 4 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

June 1997.

1. The fourth term of an arithmetic series is 20 and the ninth term is 40.
Find

(a) the common difference, [2]

(b) the first term, [1]

(c) the sum of the first twenty terms. [1]

2. Integrate the following with respect to x.

(a) $\frac{1}{4x+5}$, [1]

(b) e^{-3x}, [1]

(c) $\frac{1}{(5x-3)^2}$. [2]

3. Differentiate the following with respect to x, simplifying your answers where possible.

(a) $x^2 \ln x$, [1]

(b) $\frac{3x^2-5}{2x^2+7}$, [2]

(c) $(x^3+5)^{10}$. [2]

4. In a consignment of ten articles four are defective. If three articles are selected at random **without replacement**, find the probability that

(a) the first article drawn is defective and the second and third are non-defective, [2]

(b) no more than one article is defective. [3]

5. A, B, C are the points (4, 3), (2, 2) and (5, −4) respectively.

(a) Show that the lines AB and BC are perpendicular. [2]

(b) A point D is such that $ABCD$ is a rectangle. Find the equation of the line AD and the equation of the line CD. Hence, or otherwise, find the coordinates of D. [5]

(c) Find the area of rectangle $ABCD$. [2]

6. In the binomial expansion of $(1+x)^n$, where n is a positive integer, the coefficient of x^3 is twice the coefficient of x. Find n. [3]

7. AB is a chord of a circle of centre O and radius r, with $A\hat{O}B = \theta$ radians ($\theta < \pi$). The area of triangle ABO is one half the area of the sector ABO.

(a) Show that $2 \sin \theta = \theta$. [2]

(b) Show that θ lies between 1·8 and 1·9. Find the value of θ correct to one decimal place. [3]

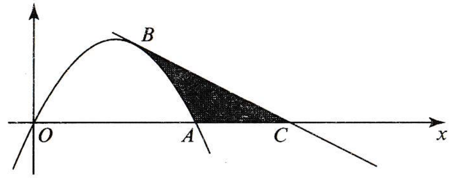
8. The equation of the curve C is

$$y = 4 + 12x - 3x^2 - 2x^3.$$

(a) Find, showing your working, the stationary points of C and determine their nature. [5]

(b) Sketch the curve C. [1]

9.



The diagram above shows the curve $y = 3x - x^2$. The curve meets the x-axis at the origin O and at the point A. The tangent to the curve at the point $B(2, 2)$ intersects the x-axis at C.

(a) Find the equation of the tangent to the curve at B. [2]

(b) Find the shaded area. [6]

10. The function f is defined by

$$f(x) = \frac{2}{\sqrt{x^2-4}} \qquad \text{for } x > 2.$$

(a) Write down the range of f. [1]

(b) Derive an expression for $f^{-1}(x)$ and verify that $ff^{-1}(x) = x$. [4]

(c) Write down the domain of ff^{-1}. [1]

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MATHEMATICS (MODULAR) P2

Mathematical Methods 2

A.M. FRIDAY, 13 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Differentiate $x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)$ with respect to x. [3]

(b) Given that $x = \cos t$, $y = \sin t$, find $\dfrac{dy}{dx}$ in terms of t. [2]

2. Find all values of θ in the range 0° to 360° satisfying

(a) $2\sin 2\theta = \sin\theta$, [3]

(b) $3\sec^2\theta + 5\tan\theta - 5 = 0$. [4]

3. Solve the equation

$$4x^3 + 12x^2 + 5x - 6 = 0.$$ [4]

4. The iterative formula

$$x_{n+1} = \tfrac{1}{5}(x_n^{\ 3} + 1)$$

may be used to find a root of

$$x^3 - 5x + 1 = 0.$$

Starting with $x_0 = 0{\cdot}2$, find a root of the equation, giving your answer correct to five decimal places. Record the value of $x_1, x_2, \ldots,$ to as many decimal places as your calculator will allow. [3]

5. A circle C has equation

$$x^2 + y^2 - 2x - 8y - 8 = 0.$$

(a) Find the coordinates of the centre of C. [1]

(b) Find the radius of C. [1]

(c) Find the equation of the tangent to C at the point (5, 1). [3]

6. (a) Express $\dfrac{3x^2 - 2x + 8}{(x-1)(x^2+8)}$ in partial fractions. [4]

(b) Show that $\displaystyle\int_2^4 \frac{3x^2 - 2x + 8}{(x-1)(x^2+8)}\,dx = \ln 6.$ [4]

7. (a) Use integration by parts to find

$$\int x^2 \ln x \, dx.$$ [3]

(b) Using the substitution $u = \sqrt{9 + x^2}$, show that

$$\int_0^4 \frac{x^3}{\sqrt{9+x^2}}\,dx = \frac{44}{3}.$$ [4]

8. Water leaks from a tank in such a way that the depth x metres of the water at time t seconds satisfies the differential equation

$$\frac{dx}{dt} = \frac{-2\sqrt{x}}{453 + 108x}.$$

(a) Find the general solution of this equation in the form $t = f(x)$. [4]

(b) Find the time for the depth of the water to fall from 4 metres to 1 metre. [2]

9. (a) Find the equation of the tangent to the curve $y = e^{-x}$ at the point $P(t, e^{-t})$. [2]

(b) The tangent at P meets the x and y axes at the points Q and R respectively. Show that A, the area of triangle OQR, where O is the origin, is given by

$$A = \tfrac{1}{2}(t+1)^2 e^{-t}.$$ [2]

(c) Find, showing your working, the stationary values of A and determine their nature. [6]

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MATHEMATICS (MODULAR) P3

Mathematical Methods 3

A.M. FRIDAY, 20 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's rule with 5 ordinates and an interval of $\frac{\pi}{12}$ to find an approximate value for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$$

Give your answer correct to five decimal places. [4]

2. Differentiate the following with respect to x.

(*a*) $\int_0^x \frac{1}{\sqrt{t^3+1}}\, dt.$ [1]

(*b*) $x^{\cos x}$. [3]

3. Given that α, β, γ are the roots of the equation

$$x^3 + x^2 + 4x - 5 = 0,$$

find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$ and $\alpha\beta$. [4]

4. Given that

$$z = \frac{1+i}{1-2i},$$

find

(*a*) z in the form $a + ib$, [2]

(*b*) the modulus and argument of z. [2]

5. (*a*) Show that the first three terms in the expansion in ascending powers of x of

$$(1+9x)^{\frac{1}{3}}$$

are the same as the first three terms in the expansion in ascending powers of x of

$$\frac{1+6x}{1+3x}.$$

For what values of x are both these expansions valid? [5]

(*b*) Use $(1+9x)^{\frac{1}{3}} \simeq \frac{1+6x}{1+3x}$ with $x = \frac{1}{64}$ to obtain an approximation to $(73)^{\frac{1}{3}}$ as a rational fraction in its lowest terms. [2]

6. The unknowns x, y, z satisfy the equations

$$\begin{aligned} x + \quad y + (\lambda + 1)z &= 0, \\ x + (\lambda - 2)y + \quad 2z &= 0, \\ x - \quad y + \quad z &= \lambda - 2, \end{aligned}$$

where λ is a constant. Use reduction to echelon form to solve these equations when

(*a*) $\lambda = 1$, [3]

(*b*) $\lambda = 2$. [3]

In each case give a geometrical interpretation of your result. [2]

7. The point $P(2\cos\theta, 3\sin\theta)$ lies on the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

(*a*) Find the equation of the tangent to the ellipse at the point $P(2\cos\theta, 3\sin\theta)$, where $\theta \neq 0$. [3]

(*b*) Given that the tangent in (*a*) passes through the point $(2, -6)$, show that

$$\cos\theta - 2\sin\theta = 1.$$ [1]

(*c*) Solve the equation in (*b*) for $0° \leqslant \theta \leqslant 360°$ and deduce the coordinates of P. [4]

8. Three matrices **D**, **E**, **F** are such that

$$\mathbf{EF} = \mathbf{FD}$$

and $\mathbf{F}^{-1}$ exists. Express **E** in terms of **F**, $\mathbf{F}^{-1}$ and **D**. [1]

(*a*) Show that $\mathbf{E}^9 = \mathbf{F}\mathbf{D}^9\mathbf{F}^{-1}$. [2]

(*b*) Given that

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & -1 \\ 4 & 5 & 3 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

find $\mathbf{E}^9$. [6]

9. The function f with domain $[0, 2]$ is defined by

$$f(x) = x^2, \quad \text{for } 0 \leqslant x < 1,$$
$$f(x) = 2 - x, \quad \text{for } 1 \leqslant x \leqslant 2.$$

(*a*) Sketch the graph of f. [2]

(*b*) The even function g has domain $[-2, 2]$ and

$$g(x) = f(x) \quad \text{for } 0 \leqslant x \leqslant 2.$$

(i) Sketch the graph of g. [1]

(ii) Evaluate $\int_{-2}^{1} g(x)\, dx$. [3]

(iii) Given $A = [-\frac{1}{2}, 2]$, find $g(A)$. [1]

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MATHEMATICS (MODULAR) P4

Mathematical Methods 4

A.M. TUESDAY, 24 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Show that the equation

$$e^x + x - 3 = 0$$

has a root between 0 and 1. Use the Newton-Raphson method to solve the equation, giving your answers correct to five decimal places. Record your values of x_0, x_1, x_2, . . . to as many decimal places as your calculator will allow. [4]

2. Show that

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} \equiv \frac{2}{r(r+1)(r+2)}.$$

Hence, or otherwise, find a simplified expression for

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}.$$ [4]

3. A rotation of the plane is given by

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} & 2 \\ \frac{3}{5} & \frac{4}{5} & -6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Find

(*a*) the angle of rotation, [1]

(*b*) the coordinates of the centre of rotation. [4]

4. Given the matrix $\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, show by induction that

$$\mathbf{A}^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix},$$

for all positive integers n. [6]

5. Given that $z = x + iy$ and $w = u + iv$ are complex numbers related by

$$w = \frac{1}{z} + 1,$$

obtain expressions for u and v in terms of x and y. [3]

The complex numbers z and w are represented by the points P and Q respectively in the Argand diagram. Given that P moves along the line $y = 2x$, show that Q moves along the line $2u + v - 2 = 0$. [2]

6. State De Moivre's Theorem. [1]

(*a*) Find, in polar form, all the values of z satisfying

$$z^4 = 16\text{i}.$$ [5]

(*b*) Write down the binomial expansion of $(\cos\theta + \text{i}\sin\theta)^4$ and obtain an expression for $\cos 4\theta$ in terms of $\cos\theta$ and $\sin\theta$. [3]

7. Given that $f(x) = (1 + x)\ln(1 + x)$,

(*a*) find the 5th derivative of $f(x)$, [3]

(*b*) show that the first five non-zero terms in the Maclaurin expansion for $f(x)$ are

$$x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20},$$ [2]

(*c*) find, in terms of r, an expression for the rth term $(r \geqslant 2)$ of the Maclaurin expansion for $f(x)$. [2]

8. (*a*) Using the definitions of the hyperbolic functions $\cosh x$ and $\sinh x$ given in the information booklet, show that

(i) $\cosh^2 x - \sinh^2 x = 1$, [1]

(ii) $2\cosh^2 x - 1 = \cosh 2x$, [1]

(iii) $2\sinh x\cosh x = \sinh 2x$. [1]

(*b*) Show that the length of the arc of the curve $y = x^2$ between the origin and the point (1, 1) is $\frac{1}{4}(2\sqrt{5} + \sinh^{-1} 2)$. [6]

9. Given that

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, \text{d}x,$$

show that

$$I_n = \frac{1}{n-1} - I_{n-2}, \qquad (n \geqslant 2).$$ [4]

Hence show that

$$I_4 = \frac{3\pi - 8}{12}.$$ [2]

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MATHEMATICS (MODULAR) M1

Mechanics 1

A.M. MONDAY, 9 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take $g = 9{\cdot}8$ m s^{-2}.

INFORMATION FOR CANDIDATES

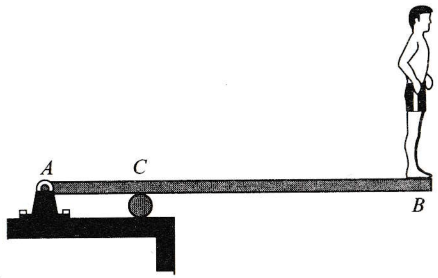
An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1.



The diagram shows a diver of weight 750 N pausing at the end B of a diving board AB of length 4·4 m and weight 350 N. The board is supported at A and at C where $AC = 1{\cdot}1$ m. The force exerted at A is vertically downwards and that at C is vertically upwards. Modelling the diving board as a uniform rod and the diver as a particle find the forces exerted on the diving board at A and C. [4]

2. A jogger of mass 80 kg runs at a steady speed of 4 m s^{-1} on a horizontal road and works at a constant rate of 140 W. Find the magnitude of the resistive forces acting. [2]

He then comes to a hill inclined at an angle α to the horizontal where $\sin\alpha = \frac{1}{14}$ and runs up this hill at a steady speed such that his rate of working will be increased by 30%. Assuming that the resistive forces are unchanged, find this speed. [2]

3. A balloon, of total mass 400 kg, including contents, has a downward acceleration of 0·3 m s^{-2}. Find the upward force on the balloon. [2]

Ballast of 20 kg is dropped from the balloon at the instant the balloon has a downward speed of 1·5 m s^{-1}. Assuming that the upward force remains constant in the subsequent motion, find the acceleration of the balloon and the time it takes to return to the position where the ballast was dropped. [4]

4. A particle P is moving along the x axis and its velocity v m s^{-1} in the positive x direction at time t s is given by

$$v = 3t^2 - 24t + 21.$$

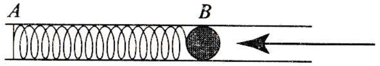
(*a*) Find the acceleration of P at time t s. [2]

(*b*) Find x at time t s given that $x = 0$ when $t = 0$. [2]

(*c*) Find the values of t for which the particle is instantaneously at rest and the distance between the points of instantaneous rest. [3]

5. (*a*) Assuming Hooke's law show, by integration, that the work in extending a spring of modulus λ and natural length l a distance x beyond its natural length is $\dfrac{\lambda x^2}{2l}$. [2]

(*b*)



The diagram shows a spring of natural length 0·15 m in a smooth horizontal tube with its end A fixed and a small bead B of mass 0·2 kg held in equilibrium by a force of magnitude 60 N pressing it against the free end of the spring. The compression of the spring in this position is 0·03 m.

(i) Find the modulus of elasticity of the spring. [2]

(ii) The bead is released. Find, using the conservation of energy, the speed of the bead just as the spring attains its natural length. [2]

6. A girl wishes to estimate the depth d m of a mine shaft. She drops a stone down the shaft and finds that there is an interval of 6 seconds between the instant she dropped the stone and the instant she heard the stone hit the bottom of the shaft.

(*a*) She decides to make a first estimate by assuming that the stone took 6 seconds to drop from the top to the bottom of the shaft. Calculate her first estimate for d. [2]

(*b*) She then used her first estimate for d, together with the fact that the speed of sound is 332 m s^{-1} to estimate the actual time taken by the stone to drop. She then used this time to find a second estimate for d. Calculate her second estimate for d. [3]

(*c*) Obtain, but do not attempt to solve, the equation satisfied by d when the time taken by sound to travel from the bottom to the top of the shaft is taken into account. [1]

7. A ball of mass 0·4 kg dropped vertically downwards onto a smooth horizontal floor has a speed of 8 m s^{-1} just as it is about to hit the floor. The coefficient of restitution between the ball and floor is $\dfrac{1}{4}$.

(*a*) Assuming that the time of contact between the ball and floor may be neglected find the magnitude of the impulse exerted by the floor on the ball. [3]

(*b*) Assuming that the ball is in contact with the floor for 0·03 s find the magnitude of the impulse exerted by the floor on the ball. (Hint: you need to take into account the impulse due to the force of gravity.)[2]

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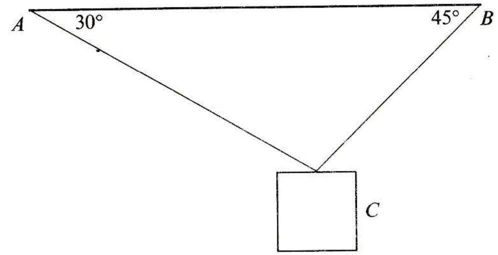
8.



The diagram shows a crate C of weight 2000 N suspended in equilibrium by two cables AC and BC attached to two fixed points A and B on the same horizontal level. The cables AC and BC are inclined at 30° and 45° to the horizontal respectively. Modelling the crate as a particle and the cables as light inextensible strings find the tension in the cable BC. [5]

State what modelling assumption the adjective 'light' allows you to make about the tensions in the cables. [1]

9. A tennis ball is projected, with speed 40 m s^{-1} at an angle of 5° below the horizontal, from a point at a height of 2·4 m above a tennis court. It is intended that the ball should pass over the net at a point whose horizontal displacement from the point of projection is 12 m. The net is 0·9 m high. The ball is to be modelled as a particle and air resistance is to be neglected.

(*a*) Show that the ball will take approximately 0·3 s to reach a point directly above the net. [2]

(*b*) Find the clearance of the ball above the net. [4]

(*c*) Find the magnitude and direction of the velocity of the ball on first impact with the ground. [4]

Comment briefly on whether or not your answer to part (*b*) suggests that the particle model gives a valid answer to part (*c*). [1]

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MATHEMATICS (MODULAR) M2

Mechanics 2

P.M. WEDNESDAY, 25 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take $g = 9{\cdot}8\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Answer **all** *questions*

1. Show that an appropriate integrating factor for the differential equation

$$\frac{dy}{dx} + \frac{5y}{x} = x^2$$

is x^5. Hence, or otherwise, find the solution of the differential equation for which $y = 1$ when $x = 1$. [5]

2. The work done by the force $\mathbf{F} = 3\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$ N in moving a particle from the point with position vector $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ m to the point with position vector $a\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$ m, where a is a constant, is 52 J. Find the value of a. [4]

3. The position vector $\mathbf{r}$ m of a particle P of mass 0·4 kg at time t s is given by

$$\mathbf{r} = t^3\,\mathbf{i} + t^2\,\mathbf{j} + t^4\,\mathbf{k}.$$

(*a*) Find the velocity of P at time t s. [2]

(*b*) Find the acceleration of P at time t s. [2]

(*c*) Find the kinetic energy of P at time t s. [2]

(*d*) Find the rate at which the force acting on P is working at time t s. [2]

(*e*) State the relation between the kinetic energy of a particle and the rate of working of the force acting on the particle. Verify that your answers to (*c*) and (*d*) satisfy this relationship. [2]

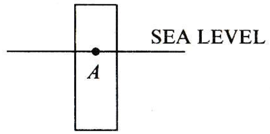
4. In some amusement parks there is a ride which is effectively a hollow cylinder which can rotate about its vertical axis. The riders stand on the horizontal base of the cylinder and in contact with the curved surface of the cylinder. When the angular speed reaches a certain value the floor is dropped but the riders remain in contact with the curved surface of the cylinder. The radius of the cylinder is 2·5 m and the speed of rotation is 30 revolutions per minute. Find the smallest possible coefficient of friction between the rider and the cylinder surface so that the ride works effectively. [6]

5. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 20y = 60x + 13. \qquad [7]$$

In this question you may quote, without proof, any SHM formulae.

6.



The diagram shows a cylindrical buoy of height 2 m and mass 440 kg floating vertically in equilibrium in a calm sea, the point marked A on the cylinder being at sea level. The upward buoyancy force due to the sea, when the length of the buoy beneath sea level is d m, is $2750d$ N. Find the height of A above the base of the buoy. [1]

(*a*) The top of the buoy is then moved downwards a distance of 0·2 m and released from rest at time $t = 0$ s. During the subsequent motion the downward displacement of A from sea level at time t s is denoted by x m. Assuming that

- the motion of the buoy can be modelled by the motion of a particle of mass 440 kg at the centre of gravity under the action of gravity and the buoyancy force,
- the motion of the buoy does not affect the sea level,

show that

$$\frac{d^2x}{dt^2} = -6{\cdot}25x. \qquad [2]$$

(i) Write down an expression for x in terms of t. [2]

(ii) Find the time taken before A first returns to sea level and the maximum speed of the buoy. [2]

(iii) Find the time taken until A is at a depth of 0·1 m below sea level. [2]

(*b*) A passing ship disturbs the sea level so that at time t s the displacement of the sea level below the original level of the calm sea is $0{\cdot}4 \sin 2t$. The downward displacement of A below the original level of the calm sea is again denoted by x m. Obtain, but do not attempt to solve, the differential equation satisfied by x. [2]

7. A particle of mass 0·4 kg is projected vertically upwards with a speed of 30 ms^{-1}. Neglecting air resistance, verify that the time taken to reach its greatest height is more than 3 s. [2]

In an experiment when the particle was projected as above it was found that the actual time to reach its greatest height was 2 s. Assuming that this difference in times is due to the existence of air resistance which is directly proportional to the speed of the particle, show that the speed v ms^{-1} of the particle at time t s after projection satisfies a differential equation of the form

$$\frac{dv}{dt} = -9{\cdot}8 - kv,$$

where k is a constant. Solve this differential equation to determine v in terms of k and t. Verify that a value of 0·4 for k produces good agreement with observation. Use this value of k to find the air resistance when the particle is moving at a speed of 12 ms^{-1}. [9]

Suggest a different modelling assumption about air resistance which could also account for the above time difference. [1]

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MATHEMATICS (MODULAR) S1

Statistics 1

P.M. TUESDAY, 17 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

The only books of statistical tables that you may use in the examination are "Statistical Tables" by Murdoch and Barnes (Macmillan Press) **or** "Elementary Statistical Tables" (RND Publications).

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Three pupils are to be chosen at random **without replacement** from a class of 15 girls and 12 boys. Calculate the probability that

   (*a*) 3 girls will be chosen, [1]

   (*b*) 2 girls and 1 boy will be chosen. [3]

2. Peter travels by bus to work each day. His arrival time at the bus-stop on any morning is equally likely to be any time from 07.15 to 07.45, and he takes the first bus to arrive at the bus-stop. Buses arrive punctually at the bus-stop at 07.20, 07.25, 07.32 and 07.45.

   (*a*) Write down an appropriate model for the distribution of Peter's arrival time at the bus-stop. [1]

   (*b*) Use your model to calculate the probability that, on any morning, Peter will not have to wait more than 5 minutes for a bus to arrive. [2]

3. The two events A and B are such that

   $$P(A) = 0{\cdot}6,\ P(B) = 0{\cdot}3 \text{ and } P(A \cup B) = 0{\cdot}72.$$

   (*a*) Show that A and B are independent. [3]

   (*b*) Find the value of $P(A \cup B')$. [2]

4. A box labelled A contains 5 red and 5 white balls; a second box labelled B contains 4 red and 6 white balls; a third box labelled C contains 2 red and 8 white balls. One of the boxes is chosen at random and two balls are drawn at random **with replacement** from the chosen box.

   (*a*) Show that the probability of both balls drawn being red is $\frac{3}{20}$. [2]

   (*b*) Calculate the probability that the two balls drawn are of the same colour. [2]

   (*c*) Given that the two balls drawn were of the same colour find the conditional probability that they were drawn from box A. [3]

5. Assume that independently for each game played, a team has probability 0·7 of winning a home game and probability 0·4 of winning an away game. The team participates in a competition in which it plays 5 home games and 5 away games.

   (*a*) Find, correct to three significant figures, the probability that in the competition the team will win

   (i) exactly 4 of its 5 home games, [2]

   (ii) exactly 3 of its home games and exactly 2 of its away games. [2]

   (*b*) Each player will be paid a bonus of £50 for every home win and a bonus of £100 for every away win. Find the mean and, to the nearest £, the standard deviation of the total in bonuses that will be paid to a player who plays in all 10 games. [4]

   (*c*) Comment on the validity of the assumption made in the opening sentence of this question. [1]

6. The time, X minutes, taken by Alec to complete a French translation may be modelled by a normal distribution having mean 27 and standard deviation 4.

(*a*) Find, correct to three decimal places, the probability that Alec will take longer than 34 minutes to complete a French translation. [2]

(*b*) Find the value of c if the probability of Alec completing a French translation in under c minutes is 0·65. Give your answer correct to two decimal places. [2]

The time, Y minutes, taken by Bethan to complete a French translation may be modelled by a normal distribution having mean 30 and standard deviation 3.

(*c*) Calculate, correct to three decimal places, the probability that Bethan will complete a French translation in less time than Alec. [4]

7. Along a particular stretch of motorway, accidents occur at an average rate of 1·25 per week.

(*a*) Write down an appropriate model for the distribution of the number of accidents in a period of w weeks. Use your model to calculate, correct to three decimal places, the probability that

(i) exactly 2 accidents will occur in a week,

(ii) 4 or more accidents will occur in a period of 4 weeks. [5]

(*b*) Find an approximate value for the probability that in a year (52 weeks) fewer than 70 accidents will occur. Give your answer correct to two decimal places. [4]

8. The continuous random variable X is distributed with probability density function f, where

$$f(x) = \frac{1}{5}(3 - x), \qquad \text{for } 0 \leqslant x < 1,$$

$$f(x) = \frac{1}{4}, \qquad \text{for } 1 \leqslant x \leqslant 3,$$

$$f(x) = 0, \qquad \text{otherwise.}$$

(*a*) Find the mean value of X, [3]

(*b*) Show that the cumulative distribution function F of X is such that

$$F(x) = \frac{1}{10}x(6 - x), \qquad \text{for } 0 \leqslant x < 1.$$

Determine an expression for $F(x)$ valid for $1 \leqslant x \leqslant 3$. Hence, or otherwise, evaluate

$$P(0 \cdot 5 \leqslant X \leqslant 2 \cdot 0).$$ [7]

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MATHEMATICS (MODULAR) S2

Statistics 2

P.M. MONDAY, 23 June 1997

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

The only books of statistical tables that you may use in the examination are "Statistical Tables" by Murdoch and Barnes (Macmillan Press) **or** "Elementary Statistical Tables" (RND Publications).

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. When an object is weighed repeatedly on a certain machine the readings obtained may be modelled by a normal distribution having mean equal to the true weight of the object and standard deviation 0·5 g.

(*a*) The readings obtained when a particular object was weighed 25 times on the machine had mean 20·6 g. Calculate a 95% confidence interval for the true weight of the object. [2]

(*b*) Find the smallest number of weighings of an object that should be made if the width of the 95% confidence interval for its true weight is to be less than 0·2 g. [3]

2. Let θ denote the probability that a new treatment will cure a patient suffering from a particular ailment. The treatment was administered to a random sample of 100 patients and it cured 80 of them.

(*a*) Write down an unbiased estimate of θ. [1]

(*b*) Calculate an estimate of the standard error of your answer to (*a*). [2]

(*c*) Hence find approximate 90% confidence limits for θ. [2]

3. Let p denote the probability that a drawing pin thrown onto a table will come to rest with its point upright. A mathematician studied the physical dimensions of the drawing pin and claimed that p was equal to 0·4. To test this claim the drawing pin is to be thrown onto a table 20 times. It is decided to reject the claim if $X \leqslant 3$ or $X \geqslant 12$, where X is the number of times that the drawing pin comes to rest with its point upright.

(*a*) Find the significance level of this decision rule. [3]

(*b*) Explain what is meant by the term "significance level" in the context of this question. [1]

4. Over a long period of time the number of accidents in a factory has averaged 1·5 per month. Additional safety devices were installed for a trial period of 6 months in the hope that they would reduce the accident rate. During the trial period there were 3 accidents. The factory manager consulted a statistician for advice as to whether or not the additional safety devices would lead to a reduction in the accident rate. The statistician assumed that a Poisson distribution was appropriate. Based on the information provided the statistician calculated the p-value to be 0·0212.

(*a*) Write down the null and alternative hypotheses which are appropriate here. [1]

(*b*) Show how the p-value was calculated. [2]

(*c*) State, giving a reason for your choice, which of the following recommendations you would report to the factory manager:

(i) the additional safety devices will almost certainly lead to a reduction in the accident rate;

(ii) the additional safety devices are unlikely to reduce the accident rate;

(iii) the evidence so far is inconclusive, so the safety devices should remain in place for a further period before reaching a conclusion. [1]

5. The following table gives a summary of the results obtained in an investigation of the heights (in inches) of wheat stalks grown using two fertilisers A and B.

	Fertiliser A	Fertiliser B
Number of stalks	50	60
Sum of the heights	780	894
Sum of the squares of the heights	12 364	13 651

(*a*) Calculate unbiased estimates of the mean and the variance of the heights of stalks grown using Fertiliser A. [2]

(*b*) Calculate an approximate 98% confidence interval for the mean height of stalks grown using Fertiliser A. [3]

(*c*) From the data in the above table the unbiased estimates of the mean and the variance of the heights of stalks grown using Fertiliser B are 14·9 and 5·6, respectively.

Use a 5% significance level to test whether the mean heights of stalks are equal for the two fertilisers. [6]

6. A chemist knows that a variable y associated with the output of a chemical process is linearly related to the operating temperature x°C in the form $y = \alpha + \beta x$. The process was run twice at each of the operating temperatures 50°C, 60°C and 70°C with the results given in the following table.

		x	50	60	70
y	First observed value		76	63	53
	Second observed value		75	66	57

The following calculations were made from the above six pairs of results:

$$\Sigma x = 360,\ \Sigma x^2 = 22\,000,\ \Sigma y = 390,\ \Sigma xy = 22\,990.$$

(*a*) Find the least squares estimates of α and β. [4]

(*b*) Observed y-values are subject to independent normally distributed errors having mean zero and standard deviation 1·5.

(i) Calculate 95% confidence limits for β. Deduce 95% confidence limits for the decrease in the true value of y when x is increased by 10°C.

(ii) Use a 5% significance level to test the null hypothesis that when $x = 65$ the true value of y is equal to 59 against the alternative hypothesis that it is greater than 59. [10]

7. (*a*) Let X denote a random variable having mean μ and standard deviation σ. Express $E(X^2)$ in terms of μ and σ.

Let X_1 and X_2 denote two independent observations of a random variable X whose mean μ and standard deviation σ are both unknown. Show that

$$W = \tfrac{1}{2}(X_1 - X_2)^2$$

is an unbiased estimator of σ^2. [4]

(*b*) Let X denote the number on a card drawn at random from a pack of four cards which are numbered 1, 2, 3 and 4, respectively. Show that the variance of X is 1·25.

Let X_1 and X_2 denote the numbers on two cards drawn at random **without replacement** from the above pack of four cards. List all possible pairs of values of X_1 and X_2. For each such pair, calculate the value of W as defined above and hence display the sampling distribution of W in a table. Show that this W is **not** an unbiased estimator of the variance of the four numbers that were in the pack. Offer a reason why the result in (*a*) is not valid here. [8]