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1936-1943

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*John Hughes 548*

CENTRAL WELSH BOARD

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J.R. Lloyd

CENTRAL WELSH BOARD.

GEOMETRY.

SCHOOL CERTIFICATE, JULY, 1936.

(3 hours.)

1. The hypotenuse of a right-angled triangle is 3.5 inches and one side is 3 inches in length. Construct the triangle, using ruler and compasses only. [Construction lines should be shown, but explanation and proof are not required.]

State two sets of conditions, either of which would ensure that two right-angled triangles are congruent.

$ABC$  is an acute-angled triangle. The perpendiculars from  $B$  and  $C$  to the opposite sides meet these sides in  $P$  and  $Q$  respectively.  $BP = CQ$  and  $BP$  and  $CQ$  intersect in  $R$ . Prove that  $AQ = AP$  and that  $AR$  bisects the angle  $A$ .

2. [Formal proofs are not required in this question, but reasons should be stated, as shortly as possible.]

(i)  $ABCDE$  is a pentagon in which

$$A = \hat{C} = \hat{D} = 2\hat{B} = 2\hat{E}.$$

Calculate the angle  $A$ .

(ii)  $ABCD$  is a quadrilateral inscribed in a circle, centre  $O$ .  $\hat{BAD} = 130^\circ$ ,  $\hat{ACB} = 15^\circ$ . Calculate the angle  $AOD$ .

✓ X. G.D.

✓ G.D.



(iii)  $ABC$  is a triangle in which  $BC = 15$  inches.  $D$  is a point on  $AB$ , and  $DE$  is drawn parallel to  $BC$  to meet  $AC$  in  $E$ . If  $\frac{AD}{DB} = \frac{3}{2}$ , calculate  $DE$  and the ratio of the area  $ADE$  to the area  $ABC$ .

3. Show that the line joining the middle points of two sides of a triangle is parallel to the third side and equal to half of it in length.

$ABCD$  is a trapezium,  $AB$  being parallel to  $DC$ .  $X$  is the mid-point of the side  $AD$ , and  $Y$  the mid-point of  $BC$ . Prove that  $XY = \frac{AB + DC}{2}$ .

4. Prove that the sum of the squares on two sides of a triangle is equal to twice the square on the median bisecting the third side plus twice the square on half the third side.

$ABC$  is a triangle and  $P$  and  $Q$  are points trisecting the side  $BC$ . Prove that

$$AB^2 + AC^2 - \frac{1}{3}BC^2 = AP^2 + AQ^2 + PQ^2.$$

5. [No proofs are required in this question, but the steps of the constructions should be stated as shortly as possible and all construction lines shown.]

(i)  $AB$  is a line 3 inches in length.  $P$  is a point which moves on one side of  $AB$  so that the angle  $APB$  is  $50^\circ$ . Construct the locus of  $P$ . Measure and record the greatest distance of  $P$  from  $AB$ .

(ii) Draw a triangle  $ABC$  in which  $BC = 3$  inches,  $CA = 2\frac{3}{4}$  inches,  $AB = 3\frac{1}{2}$  inches. Construct *without calculation* a square equal in area to the triangle. Measure and record the length of the side of the square.

6. Prove that the angles between a tangent to a circle and a chord through the point of contact are equal respectively to the angles in the alternate segments.

Two circles intersect in  $A, B$ . The tangent at  $A$  to the first circle meets the second circle in  $C$ ; the tangent at  $B$  to the circle  $ACB$  meets the first circle in  $D$ .  $DA$  produced meets the circle  $ACB$  in  $E$ , and  $CB$  produced meets the circle  $ADB$  in  $F$ . Prove that (i)  $DE$  is parallel to  $FC$ , and (ii)  $DECF$  is a parallelogram.

7.  $ABC$  is a triangle right-angled at  $A$ .  $AD$  is the perpendicular from  $A$  to  $BC$ . Prove that  $BA^2 = BC \cdot BD$ .

$AB$  is a straight line and  $AC$  and  $BD$  are two perpendiculars to  $AB$ , erected on the same side of it, but unequal in length.  $AD$  and  $BC$  intersect at a point  $O$ . If  $AB^2 = AC \cdot BD$ , prove that the triangles  $COA, AOB, BOD$  are similar and that  $AO \cdot AD = BO \cdot BC$ .

8.  $ABC$  is a triangle in which  $AB = 16$  inches,  $BC = 20$  inches, and the angle  $ABC$  is  $50^\circ$ . Calculate the area of the triangle and the angle  $ACB$ .

## TRIGONOMETRY.

JULY, 1936.

(3 hours.)

1. (i)  $ABC$  is a triangle in which  $\angle B = 60^\circ$  and  $\angle A = 75^\circ$ .  $AD$  is drawn perpendicular to  $BC$  to cut  $BC$  in  $D$ . If  $AD$  is 3 inches long, find, without using tables, the lengths of the sides of the triangle  $ABC$ , in each case correct to two decimal places.



(ii) If  $\angle A = 36^\circ 35'$ , and  $\angle B = 86^\circ 8'$ , use your tables to find the values of  $\sin B$ ,  $\cos(A+B)$ , and  $\tan(A+2B)$ .

2. Prove that the area of a triangle  $ABC$  is  $\frac{1}{2}bc \sin A$ .

A triangle  $ABC$  has  $AB = 5.7$  inches,  $CA = 6.4$  inches, and an area of  $14.72$  square inches. Find the two possible values of  $\angle A$  and the length of the perpendicular drawn from  $B$  to  $AC$ .

3. At a given moment a ship  $B$  is 20 nautical miles east of another ship  $A$ .  $A$  travels at 11 nautical miles per hour in a direction  $N. 28^\circ E.$  and  $B$  travels at 18 nautical miles per hour in a direction  $N. 31^\circ W.$  Calculate the distances of  $B$  east and north of  $A$  three-quarters of an hour later, and find the bearing of  $B$  from  $A$ .

4. State the formula you would use to find the side  $c$  of a triangle  $ABC$  if you were given the sizes of  $a$ ,  $b$ ,  $\angle C$ .

Prove your formula is true in the case when  $\angle C$  is an obtuse angle.

$ABCD$  is a quadrilateral in which  $AB = 6.1$ ,  $BC = 10$ ,  $CD = 12$ ,  $DA = 8.2$ , and  $\angle DAB = 110^\circ$ . Calculate the size of  $\angle BCD$ .

5. Draw the graphs of

$$y = \cot x \text{ and } y = 8(\sin x - \cos x)$$

for the range  $10^\circ$  to  $80^\circ$  in the values of  $x$ . Hence find, as accurately as you can, the value of  $x$  in this range which satisfies the equation

$$\cot x = 8(\sin x - \cos x).$$

6. (i) If  $A$  is an angle between  $180^\circ$  and  $360^\circ$  and if  $\cos A = \frac{3}{5}$ , find, without using tables, the values of  $\sin 2A$  and  $\sin \frac{A}{2}$ .

(ii) Prove that  $\frac{\sin(A-B)}{\cos(A+B)} = \frac{\cot B - \cot A}{\cot A \cot B - 1}$ .

7. (i) What values of  $x$ , from  $0^\circ$  to  $180^\circ$  inclusive, satisfy the equation  $\sin x + \sin 2x = 0$ ?

(ii) The side  $AC$  of a triangle  $ABC$  is produced to  $Y$ .

If  $\tan BAY = \frac{1}{2}$  and  $\tan BCY = \frac{3}{4}$ , find, without using tables, the value of  $\tan ABC$ .

8. From an external point  $Y$ , two tangents  $YX$ ,  $YZ$  are drawn to a circle, centre  $O$ , to touch the circle at  $X$ ,  $Z$  respectively. If the radius is 6 cm. and  $OY$  is 20 cm., and if  $OY$  cuts the circle in  $M$ , find the area enclosed by  $XY$ ,  $YM$ , and the minor arc  $XM$ .

## GEOMETRY.

JULY, 1937.

(3 hours.)

1.  $ABC$  and  $DEF$  are two triangles in which  $AB = DE$  and  $AC = DF$ . State as many further conditions as you can, each of which would ensure that the triangles are congruent.

Do likewise for the case when in the triangles  $AB = DE$  and  $\hat{BAC} = \hat{EDF}$ .

$ABC$  is a triangle and  $AP$  is parallel to  $BC$ ;  $D$  is the mid-point of  $AB$  and  $E$  of  $AC$ .  $PD$  meets  $CB$  produced in  $Q$  and  $PE$  meets  $BC$  in  $R$ . Prove that  $QR = BC$ .



2. [Formal proofs are not required in this question, but reasons should be stated, as briefly as possible.]

(i)  $ABCDEFGH$  is a regular polygon of eight sides. Find the angles of the triangle  $ABD$ .

(ii) Two sides of a parallelogram are 7 cm. and 9 cm. respectively, and one diagonal is 14 cm. in length. Calculate the length of the other diagonal.

(iii) A line  $PAB$  meets a circle, centre  $C$ , in  $A$  and  $B$ .  $PA = 4$  inches,  $AB = 5$  inches,  $PC = 10$  inches. Calculate the radius of the circle.

3. Prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

$ABC$  is an isosceles triangle in which  $AB = AC$  and  $\frac{AB}{BC} = \frac{3}{2}$ .  $BP$  is the perpendicular from  $B$  to  $AC$ .

Show (by use of algebra or otherwise) that  $\frac{AP}{PC} = \frac{7}{2}$ .

4. [No proofs are required in this question, but all construction lines must be shown.]

(i) Draw (without using a protractor) a triangle with sides  $3\frac{1}{2}$  inches and 4 inches respectively in length, including an angle of  $60^\circ$ . Construct (without calculation) a second triangle of equal area, with an angle of  $60^\circ$  included by two sides, one of which is 5 inches in length. Measure and record the length of the second side.

(ii) [In this part ruler and compasses only are to be used.]

Draw two circles centres  $A$  and  $B$  of radii 4.5 cm. and 1.5 cm. respectively,  $AB$  being 9 cm.

Mark a point  $P$  on  $AB$  such that  $AP = 5.5$  cm. At  $P$  erect a perpendicular  $PQ$  to  $AB$  of length 6 cm. From  $Q$  draw a tangent to each of the circles. Measure and record the length of each tangent.

5. Show, with proof, how to inscribe in a triangle a circle touching each of the sides.

$P$  is the centre of the inscribed circle of the triangle  $ABC$ . Show that the difference between the angles  $BPC$  and  $\frac{A}{2}$  is a right angle.

The base of a triangle is fixed and the vertical angle is given in magnitude. State what is the locus of the centre of the inscribed circle, giving reasons for your answer.

6. If the opposite angles of a quadrilateral are supplementary, prove that the quadrilateral is cyclic.

$A$  and  $B$  are the points of contact of a common tangent to two intersecting circles, and  $C$  is the point of intersection of the circles further from  $AB$ . A line through  $A$  parallel to  $BC$  meets the circle  $ACD$  in  $D$ .  $DC$  produced meets the circle  $BCE$  in  $E$ . Prove that  $A, B, E, D$  are concyclic, and that  $BE$  is parallel to  $AC$ .

7. If a line is drawn parallel to the base of a triangle meeting the other two sides internally, prove that it divides these sides proportionally.

$P, Q$  are points on the same side of  $AB$  such that the triangles  $PAB$  and  $QAB$  are equal in area.  $PB$  and  $QA$  intersect in  $X$ , and a line through  $X$ , parallel to  $AB$ , meets  $PA$  in  $Y$  and  $QB$  in  $Z$ . Prove that  $XY = XZ$ .



8.  $\triangle ABC$  is a triangle in which  $\hat{B} = 53^\circ$  and  $\hat{C} = 71^\circ$ .  $AP$  is the perpendicular from  $A$  to  $BC$  and  $BQ$  the perpendicular from  $B$  to  $AC$ . If  $AP$  is 9 cm. in length, calculate the lengths of  $BC$  and  $BQ$ .

### TRIGONOMETRY.

JULY, 1937

(3 hours.)

1. (i) Without using tables, calculate the values, to two decimal places, of  $\sin 60^\circ$ ,  $\cos 150^\circ$ , and  $\tan 315^\circ$ .

Draw figures for each to illustrate how the sign is determined.

(ii) A triangle  $ABD$  is right-angled at  $D$ .  $C$  is a point on  $BD$  between  $B$  and  $D$ . If  $BC = 2''$ ,  $\angle ABC = 30^\circ$ , and  $\angle ACD = 45^\circ$ , calculate, without tables, the length of  $CD$  correct to two places of decimals.

(iii) Use your tables to find the angles between  $0^\circ$  and  $360^\circ$  (1) whose cosine is 0.7621, and (2) whose cotangent is  $-0.6261$ .

2. (i) Draw a right-angled triangle and mark one of its angles (not the right angle) as  $\theta$ . Using Pythagoras' Theorem prove that  $\sin^2 \theta + \cos^2 \theta = 1$ .

Without further reference to the figure, prove that

$$\sec^2 \theta - 1 = \tan^2 \theta$$

$$\text{and } \operatorname{cosec}^2 \theta - 1 = \cot^2 \theta.$$

(ii) Solve the equation, giving the values of  $\alpha$  from  $0^\circ$  to  $360^\circ$ ,

$$4 \sec \alpha + 1 = 6 \tan^2 \alpha.$$

3. Prove that, in an acute-angled triangle  $ABC$ ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In a triangle  $ABC$ ,  $a = 13.1$ ,  $b = 17.6$ , and  $\angle A = 41^\circ$ . Calculate the two possible lengths for  $AB$ .

4. A triangle  $PQR$  has  $\angle P = 100^\circ$ ,  $PQ = 4.23$  inches, and  $PR = 3.25$  inches. Find, by calculation, the size of  $\angle Q$ .

5. To measure the width of a river a length  $AB$  is pegged out along a straight part of one bank, and the bearings of an object  $C$  on the other bank are taken from  $A$  and  $B$ . It is found that  $AB = 108.3$  feet,  $\angle CAB = 62^\circ 20'$ , and  $\angle ABC = 51^\circ 40'$ . Calculate the width of the river at  $C$ .

6. Draw the graph of  $y = 4 \sin x + 3 \cos x$  for values of  $x$  at intervals of  $15^\circ$  from  $0^\circ$  to  $180^\circ$ .

Use your graph to find (i) the maximum value of  $4 \sin x + 3 \cos x$ , (ii) the values of  $x$  for which  $4 \sin x + 3 \cos x = 3\frac{1}{2}$ , in this range.

$$7. (i) \text{ Prove } \cot\left(\frac{\pi}{4} + x\right) = \frac{\cos x - \sin x}{\cos x + \sin x}.$$

(ii)  $A$  is an acute angle and  $\cos A = \frac{4}{5}$ ;  $B$  is an obtuse angle and  $\sin B = \frac{3}{5}$ . Without using tables find the values of  $\sin(A + B)$ ,  $\cos(A - B)$ , and  $\tan 2A$ .

8. In a circle centre  $O$ , radius 5 inches, is inscribed a triangle  $ABC$  in which  $\angle A = 43^\circ$  and  $\angle B = 76^\circ$ . Calculate (i) the area of  $\triangle ABC$ , (ii) the area of the minor segment cut off by the shortest side of  $\triangle ABC$ .



## GEOMETRY.

JULY, 1938.

(3 hours.)

1. Prove that a quadrilateral in which the diagonals bisect each other at right angles has all its sides equal.

State, *without proof*, what is the type of quadrilateral in which (i) the diagonals are equal and bisect each other, (ii) the diagonals are equal and bisect each other at right angles.

$AD$  and  $BC$  are parallel lines.  $AC$  bisects the angle  $BAD$  and  $BD$  bisects the angle  $ABC$ . Prove that the sides of the quadrilateral  $ABCD$  are all equal.

2. Prove that the square on a side of an acute-angled triangle is equal to the sum of the squares on the other two sides minus twice the rectangle contained by one of these sides and the projection of the other on it.

$AB$  and  $DC$  are the parallel sides of a trapezium with diagonals  $AC$  and  $BD$ . Prove that

$$AC^2 + BD^2 = AD^2 + BC^2 + 2AB \cdot CD.$$

3. [Formal proofs are not required in this question, but reasons should be stated, as briefly as possible.]

(i)  $ABC$  is a triangle in which  $\hat{B} = 75^\circ$  and  $\hat{C} = 53^\circ$ .  $P$  is a point in  $AC$  such that  $AP = AB$ . Calculate the angles  $ABP$  and  $CBP$ .

(ii)  $TA$  and  $TB$  are tangents touching a circle at  $A$  and  $B$ . A line  $TP$  meets the circle in  $P$ .  $\hat{TAP} = 34^\circ$ ,  $\hat{TBP} = 42^\circ$ . Calculate the angle  $ATB$ .

(iii)  $ABC$  is a right-angled triangle in which the hypotenuse  $AB = 25$  cm. and  $BC = 24$  cm. The bisector of the angle  $A$  meets  $BC$  in  $P$ . Calculate the length of  $AP$  without using trigonometrical tables.

4. Prove that the line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

$A$  is a fixed point and  $X$  a variable point, both on the circumference of a circle. Find, with proof, the locus of the mid-point of  $AX$  as  $X$  moves round the circle.

$P$  is a point on  $AX$  such that  $AP = \frac{X}{3A}$ . State, without proof, what is the locus of  $P$  as  $X$  moves round the circle.

5. [No proof is required in this question, but all construction lines must be shown.]

Draw a cyclic quadrilateral  $ABCD$  in which  $AB = 3.3$  inches,  $AD = 2.7$  inches,  $BC = 3.9$  inches, and  $\hat{BCD} = 70^\circ$ . Measure and state the radius of the circle through  $A, B, C, D$ , and the length of  $CD$ .

6.  $AOB$  and  $COD$  are two intersecting lines and  $AO \cdot OB = CO \cdot OD$ . Prove that  $A, B, C, D$  are concyclic.

$A$  and  $B$  are the points of contact of a common tangent to two circles intersecting in  $P$  and  $Q$ ,  $P$  being nearer than  $Q$  to  $AB$ .  $APXC$  is a line meeting the circle  $BPQ$  in  $X$ ,  $APX$  being produced to  $C$  where  $XC = AP$ ;  $BPYD$  is a line meeting the circle  $APQ$  in  $Y$ ,  $BPY$  being produced to  $D$  where  $YD = BP$ . Prove that  $ABCD$  is a cyclic quadrilateral.



7. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the areas of squares on corresponding sides.

$ABC$  is a triangle in which  $AB = AC = 13$  inches and  $BC = 10$  inches. The perpendicular from  $A$  to  $BC$  meets  $BC$  in  $P$ .  $S$  is the mid-point of  $AP$  and a line through  $S$  parallel to  $AC$  meets  $BA$  in  $E$  and  $BC$  in  $F$ . Calculate the area of the triangle  $BEF$ .

8.  $P$  is the top of a hill which slopes on one side at  $30^\circ$  to the horizontal and on the other side at  $27^\circ$  to the horizontal.  $A$  is a point on the  $30^\circ$  slope 50 yards from  $P$ , and  $B$  is a point on the  $27^\circ$  slope 40 yards from  $P$ . If a straight tunnel is cut from  $A$  to  $B$ , calculate its inclination to the horizontal.

### TRIGONOMETRY.

JULY, 1938.

(3 hours.)

1. (i) Use your tables to find
  - (1) the acute angle  $A$ , where  $\sin A = \sin 15^\circ + \sin 9^\circ$ ;
  - (2) the obtuse angle  $B$ , where  $\cos B = -2 \cos 70^\circ 5'$ ;

and

- (3) the acute angle  $C$ , where  $\tan C = \frac{1}{4} \sin^2 48^\circ$ .
- (ii)  $ABCD$  is a quadrilateral in which  $AB = 6$  inches,  $BC = 8$  inches,  $\angle A = 60^\circ$ ,  $\angle C = 45^\circ$ , and  $\angle D = 90^\circ$ .

Calculate, correct to two decimal places, without using tables, the lengths of  $AD$  and  $CD$ .

2. (i) Find the values of  $A$  between  $0^\circ$  and  $180^\circ$  which satisfy the equation

$$0.24 + 0.5 \cos A - \cos^2 A = 0.$$

(ii)  $ABCD$  is the horizontal top of a rectangular box and  $EFGH$  is the base,  $AE$ ,  $BF$ ,  $CG$ , and  $DH$  being vertical. If  $AB = 14$  inches,  $AD = 8$  inches,  $DH = 7.5$  inches, calculate the angles  $AFE$  and  $AGE$ .

3.  $A$ ,  $B$ , and  $C$  are three towns.  $B$  is  $N. 25^\circ E.$  of  $A$  and 45.2 miles away.  $C$  is  $N. 79^\circ E.$  of  $B$  and 21.6 miles from  $B$ . Calculate the distance of  $C$  from  $A$  to the nearest tenth of a mile, and the bearing of  $C$  from  $A$  to the nearest degree.

4. The circumscribing circle of an acute-angled triangle  $ABC$  has a radius  $R$ . By drawing the diameter through  $B$ , prove that  $\frac{a}{\sin A} = 2R$ .

Hence show that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Show that two non-congruent triangles  $PQR$  can be drawn such that  $PQ = 10.2$  inches,  $QR = 7.86$  inches,  $\angle P = 42^\circ$ . Calculate  $\angle R$  and  $PR$  in the smaller of the two triangles.

5. (i) In a triangle  $ABC$ ,  $\angle B = 73^\circ$ ,  $\angle A = 76^\circ$ , and  $AB = 5$  inches. A line is drawn from  $A$  to a point  $D$  in  $BC$  (between  $B$  and  $C$ ) so that  $\angle BAD = 20^\circ$ . Calculate the length of  $CD$ .

(ii) If  $\cos A = -\frac{3}{5}$  and  $A < 180^\circ$ , find, without using tables, the value of  $\sin \frac{1}{2}A$ .



6. Using the same axes and the same scales for both, plot the values of  $3 \cos x$  and of  $1 - 2 \sin x$  at intervals of  $15^\circ$  from  $0^\circ$  to  $180^\circ$ , and then draw the graphs of the two functions.

Find, as accurately as you can, from your graphs a solution of the equation

$$1 - 3 \cos x = 2 \sin x.$$

7. (i) Write down a single trigonometrical ratio for each of the following expressions:

(1)  $\sin 50^\circ \cos 10^\circ + \cos 50^\circ \sin 10^\circ,$

(2)  $\cos 50^\circ \cos 10^\circ - \sin 50^\circ \sin 10^\circ,$

(3)  $2 \sin 2\theta \cos 2\theta,$

(4)  $\frac{1 - \tan^2 \frac{1}{2}\theta}{1 + \tan^2 \frac{1}{2}\theta}.$

(ii) If  $R \cos a = 3$  and  $R \sin a = 4$ , show that

$$R \cos(x - a) = 3 \cos x + 4 \sin x.$$

Hence find the maximum value of

$$3 \cos x + 4 \sin x.$$

8. The arc  $BC$  of a circle, centre  $A$ , whose radius is 6.5 inches, is 11.31 inches in length.

Calculate—

(i) the size of  $\angle BAC$  in radians,

(ii) the area of the sector  $ABC$ ,

and

(iii) the length of the chord  $BC$ .

## GEOMETRY.

JULY, 1939.

(3 hours.)

1. In this question no proof is required, but neither protractor nor set square is to be used. All construction lines should be shown.

$ABC$  is an acute-angled triangle and  $AD$  the perpendicular from  $A$  to  $BC$ . Construct the triangle, given that

$AD = 2\frac{1}{4}$  inches,  $\angle DAB = 45^\circ$ ,  $\angle DAC = 30^\circ$ . Measure and state the lengths of  $AB$  and  $DC$ .

2. In this question no proofs are required, but sufficient reasons should be given to show that answers have not been obtained either by drawing or by guess-work.

(i)  $ABC$  is a triangle in which  $\angle B$  is an obtuse angle.  $AP$  is the perpendicular from  $A$  to  $CB$  produced.  $AB = 10$  inches,  $BC = 9$  inches and  $AC = 17$  inches. Calculate the lengths of  $BP$  and  $AP$ .

(ii)  $ABC$  is a triangle inscribed in a circle. The bisector of  $\angle A$  meets the circle in  $P$ ; the bisector of angle  $B$  meets the circle in  $Q$ .  $\angle B = 76^\circ$ ,  $\angle C = 64^\circ$ . Calculate the angles of the triangle  $APQ$ .

(iii)  $ABC$  is a triangle of base 12 cm. and height 9 cm.  $P$  divides  $AB$  and  $Q$  divides  $AC$  so that

$$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{2}{3}$$

Calculate the areas of the triangles  $APQ$  and  $BPQ$ .



3. Prove that a straight line drawn through the mid-points of one side of a triangle, parallel to a second side, bisects the third side.

$ABCD$  is a parallelogram.  $X$  is the mid-point of  $AD$  and  $Y$  the mid-point of  $BC$ . Prove that  $BX$  and  $YD$  are parallel and that they trisect the diagonal  $AC$ .

4. Prove that the sum of the squares on two sides of a triangle is equal to twice the square on the median bisecting the third side plus twice the square on half the third side.

$ABC$  is a triangle and  $X$  and  $Y$  are the points of trisection of  $BC$ ,  $X$  being nearer to  $B$ . Prove that  $2AB^2 + AC^2 = 3AX^2 + 6BX^2$ .

5. Prove that equal chords of a circle are equidistant from the centre.

*In the remainder of this question no proof is required, but neither protractor nor set square is to be used. All construction lines should be shown.*

Draw the locus of mid-points of chords of length 8 cm. in a fixed circle of radius 6 cm.  $C$  is a fixed point 10 cm. from the centre of the circle. Construct a secant  $CAB$  meeting the circle in  $A$  and  $B$  so that  $AB = 8$  cm.

6. Prove that the angles between a tangent to a circle and a chord through the point of contact are respectively equal to the angles in the alternate segments.

$P$  and  $Q$  are the points of contact of two tangents  $TP$ ,  $TQ$  to a circle.  $QR$  is a chord parallel to  $TP$ . Prove that  $\angle QPR = \angle PTQ$ .

7. Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the other two sides.

$AB$  is a diameter of a circle and  $CD$  a chord perpendicular to  $AB$ .  $P$  is a point on  $CD$ .  $AP$  produced meets the circle in  $Q$ . Prove that

$$\frac{PC}{PD} = \frac{QC}{QD}$$

8.  $ABC$  is a triangle in which  $AB = 15$  inches,  $AC = 13$  inches, and  $A = 54^\circ$ .  $BQ$  is the perpendicular from  $B$  to  $AC$ . Calculate the lengths of  $BQ$  and  $QC$ , and the angle  $ABC$ .

## TRIGONOMETRY.

JULY, 1939.

(3 hours.)

1.  $ABC$  is a triangle, right-angled at  $C$ .

$$AB = 3.9 \text{ in.}, BC = 3.6 \text{ in.}$$

$E$  is the middle point of  $AC$ . Calculate the size of  $\angle BEC$ .

2. In an equilateral triangle  $XYZ$ ,  $XP$  is the altitude drawn from  $X$  to  $YZ$ . On  $XZ$  as base, a triangle  $XQZ$  is drawn outside the triangle  $XYZ$ . Given that  $\angle XQZ = 90^\circ$ ,  $XQ = QZ$ , and  $XP = 3$  in., calculate, without using tables, the length of  $QZ$  correct to two decimal places.



3. (i) Without using tables, find the values of  $\sin 120^\circ$ ,  $\cos 150^\circ$ , and  $\sin \frac{7\pi}{6}$ . Explain *very briefly* or mark a figure adequately, in each case, to show how the sign is obtained.

(ii) Solve  $\tan A = \frac{0.2}{\tan A}$  giving all values of  $A$  from  $0^\circ$  to  $180^\circ$ .

4. Find a solution of the equation

$$\tan \frac{1}{2}x = \cos (x + 30^\circ)$$

by plotting with the same scales and same axes, the graphs of  $y = \tan \frac{1}{2}x$  and  $y = \cos (x + 30^\circ)$  for values of  $x$  from  $0^\circ$  to  $90^\circ$ .

5. In each of the following cases, find whether it is possible to draw a triangle  $ABC$  or not; label specially any case where two triangles  $ABC$  can be drawn:—

- (i)  $\angle A = 24^\circ 30'$ ,  $c = 52$  in.,  $a = 16.8$  in.
- (ii)  $\angle A = 110^\circ 30'$ ,  $c = 52$  in.,  $a = 64$  in.
- (iii)  $\angle A = 54^\circ$ ,  $c = 80$  cm.,  $a = 96$  cm.
- (iv)  $\angle A = 24^\circ$ ,  $c = 7$  cm.,  $a = 5$  cm.

(Accurate drawings are not required. If calculations are necessary they must be shown.)

6. In a triangle  $PQR$ ,  $PQ = 10.81$  in.,  $QR = 6.23$  in.,  $RP = 7.04$  in.

Calculate the angle  $PRQ$ .

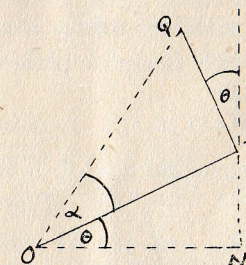
7. (i) Express each of the following as a single trigonometrical ratio:

- (1)  $\sin a \cos (\alpha - \beta) - \cos a \sin (\alpha - \beta)$ ;
- (2)  $2 \cos^2 \theta - 1$ . (3)  $2 \sin 50^\circ \cos 50^\circ$ .
- (4)  $\sqrt{\frac{1 + \cos 2\theta}{1 - \cos 2\theta}}$

(ii) Given that  $\tan x = \frac{5}{12}$ , calculate the value of  $17 \tan 2x$ .

8. A circle, centre  $O$ , radius  $15.5$  in., has a chord  $PQ$   $18$  in. long. Calculate (i) the length of the minor arc  $PQ$ , and (ii) the area of the sector  $OPQ$ . ( $\pi = 3.142$ ).

9. In the accompanying figure,  $OP = 5$  in.,  $PQ = 3$  in.,  $\angle ONP = 90^\circ$ . Prove that the perpendicular distance from  $Q$  to  $ON$  may be expressed as  $5 \sin \theta + 3 \cos \theta$ , or as  $\sqrt{34} \sin (\theta + \alpha)$ . Hence find the maximum value of  $5 \sin \theta + 3 \cos \theta$  as  $\theta$  varies.



10. The angular depressions of two buoys seen from the top of a cliff  $200$  ft. high are  $9^\circ 30'$ , and  $7^\circ 24'$ . The buoys are seen in directions S.  $47^\circ$  E. and S.  $39^\circ$  E. respectively. Calculate the distance between the buoys.



## TRIGONOMETRY.

JULY, 1940.

(3 hours.)

1.  $PQR$  is a triangle in which  $\angle R = 90^\circ$ ,  $\angle Q = 70^\circ 30'$ , and  $PQ = 20$  inches.  $QS$  ( $S$  being a point on  $PR$ ) bisects  $\angle Q$ . Calculate the length of  $SR$ .

2. Draw the graphs of  $y = \cos x$  and  $y = \frac{1}{2}(1 - \cos 2x)$  from  $x = 30^\circ$  to  $x = 90^\circ$ .

Read off, as accurately as you can, a solution of

$$1 - \cos 2x = 2 \cos x.$$

3. (i) Calculate without using the tables the values, correct to two decimal places, of  $\cos 45^\circ$ ,  $\cot 300^\circ$ , and  $\tan \frac{11\pi}{6}$ .

(ii) Solve the equation  $\cos^2 \theta + \sin \theta = \sin^2 \theta$ , giving all the values of  $\theta$  from  $0^\circ$  to  $360^\circ$ .

4. A triangle  $ABC$ , cut out of stiff cardboard, has angles of  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ , and its shortest side  $BC$  is 3 in. long.  $BC$  is placed on a horizontal table and  $A$  is held 2 in. above the table.

Find the angles of elevation of  $A$  as seen from  $B$  and  $C$ .

5. An obtuse-angled triangle  $ABC$  has  $b = 11.34$ ,  $c = 19.32$ , and  $\angle B = 32^\circ 7'$ . Calculate the angle  $C$ .

6. A chord of a circle 4 in. in radius cuts off a minor arc 3 in. long. Find, by calculation, the area of the minor segment cut off by this chord.

7. In the triangle  $ABC$ ,  $\angle A = 52^\circ 6'$ ,  $c = 27.1$ ,  $b = 18.4$ . Calculate the area of the triangle and the length of  $BC$ .

8. (i) Write down the simplest values of the following expressions (square roots need not be evaluated) :—

$$\begin{aligned} & \sin 49^\circ \cos 4^\circ - \cos 49^\circ \sin 4^\circ \\ & \cos 10^\circ \cos 20^\circ - \sin 10^\circ \sin 20^\circ, \\ & \frac{2 \tan 60^\circ}{1 - \tan^2 60^\circ} \quad 2 \cos^2 67\frac{1}{2}^\circ - 1. \end{aligned}$$

(ii) If  $\tan x = \frac{3}{4}$ , find, without using tables, the value of  $4 \cos 2x + 3 \sin 2x$ .

9.  $\angle BAC$  is a right angle, and  $AB = 10$  in. and  $AC = 8$  in.  $P$  is a point inside the angle  $BAC$  and is 5 in. from  $AB$ , and 4 in. from  $AC$ .

Without using tables, find the values of  $\tan APB$  and  $\tan APC$ .

10.  $C$  is a point on  $BD$  in the triangle  $ABD$ .

$$AB = 27.5 \text{ in.}, \angle BAD = 92^\circ 37',$$

$$\angle CAB = 51^\circ 31', \text{ and } \angle ACD = 84^\circ.$$

Calculate the length of  $AD$ .

## GEOMETRY.

JULY, 1940.

(3 hours.)

1. In this question, neither set-square nor protractor is to be used. No proof is required, but all construction lines must be shown.

$ABC$  is a triangle in which  $\angle A = 45^\circ$ ,  $AB = 4$  in., and  $AC = 3.5$  in. Construct the triangle, and also the circle passing through  $A$ ,  $B$ ,  $C$ . Measure and state the radius of the circle.

Measure  $\angle B$  & side  $BC$ .



2. In this question no proofs are required, but brief statements should be made showing that the results have not been obtained either by drawing or by guesswork.

(i)  $ABC$  is a triangle and  $M$  is the mid-point of  $BC$ .  $AB = 30$  cm.,  $AC = 16$  cm.,  $AM = 17$  cm. Calculate the length of  $BC$ .

(ii)  $ABC$  is a triangle inscribed in a circle, centre  $O$ . The tangent at  $A$  to the circle meets  $BC$  produced in  $P$ .  $\angle PAB = 125^\circ$ . Calculate  $\angle AOB$ .

(iii)  $ABC$  is a triangle in which the bisector of  $\angle A$  meets  $BC$  in  $P$ . Through  $P$  a line is drawn parallel to  $CA$  to meet  $AB$  in  $X$ .  $AB = 15$  cm.,  $AC = 10$  cm. Calculate the length of  $PX$ .

3.  $AB$  and  $DC$  are equal and parallel. Prove that  $ABCD$  is a parallelogram.

$ABCD$  is a parallelogram. The bisector of  $\angle A$  meets  $BC$  or  $BC$  produced in  $P$ ; the bisector of  $\angle B$  meets  $AD$  or  $AD$  produced in  $Q$ . Prove that  $ABPQ$  is a parallelogram in which all the sides are equal.

4. Prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

$O$  is a point within a triangle  $ABC$ .  $OP$ ,  $OQ$ , and  $OR$  are the perpendiculars to the sides  $BC$ ,  $CA$ , and  $AB$  respectively. Prove that

$$BP^2 + CQ^2 + AR^2 = CP^2 + AQ^2 + BR^2.$$

5. Prove that angles in the same segment of a circle are equal. (If a theorem stating the relation between an angle at the centre of a circle and an angle standing on the same arc is used, this theorem must be proved.)

In the remaining part of this question, neither set-square nor protractor is to be used. No proof is required, but all construction lines must be shown.

$ABC$  is an acute-angled triangle in which the base  $BC$  is a fixed line of length 7 cm. and  $\angle A = 60^\circ$ . The perpendiculars from  $B$  and  $C$  to the opposite sides meet in  $H$ . Calculate the angle  $BHC$  and construct the locus of  $H$ .

6. If from a point outside a circle two secants are drawn, prove that the rectangle contained by the segments of one secant is equal to the rectangle contained by the segments of the other.

Two non-intersecting circles are drawn. A third circle is drawn, which touches the first circle externally at  $A$  and the second circle externally at  $B$ ; the common tangent at  $A$  to the two circles touching at  $A$  cuts the remaining circle at  $C$  and  $D$ ; the common tangent at  $B$  to the two circles touching at  $B$  cuts the remaining circle at  $E$  and  $F$ .  $ACD$  and  $BEF$  intersect at  $P$ . Prove that

$$PC \cdot PD = PE \cdot PF.$$

7. State (without proof) three sets of conditions, each of which would ensure that two non-congruent triangles were similar.

$ABC$  is a triangle, and  $D$ ,  $E$ ,  $F$  are points on  $BC$ ,  $CA$ ,  $AB$  respectively such that  $EF = EA$  and



$DF = DB$ . The circle through  $D, E, F$  cuts  $AB$  again in  $G$ . Prove that  $\frac{DE}{DG} = \frac{AB}{AC}$ .

8.  $ABC$  is a triangle in which  $\angle A = 33^\circ$ ,  $AB = 12$  cm.,  $BC = 7$  cm., and  $\angle C$  is obtuse.  $BP$  is the perpendicular on  $AC$  produced. Calculate the lengths of  $BP$  and  $AC$  and the angle  $C$ .

### GEOMETRY.

JULY, 1941.

(2½ hours.)

[Answer SEVEN questions.]

1. In this question neither set-square nor protractor is to be used. No statement of construction or proof is required, but all construction lines should be shown.

$ABC$  is a triangle in which  $\angle A$  is a right angle,  $AB = 3$  in., and  $BC = 4.5$  in. Construct the triangle, and also the circle inside the triangle which touches each of the sides. Measure and state the radius of the circle.

2. In this question proofs are not required, but brief statements should be made showing that the results have not been obtained either by drawing or by guess-work.

(i)  $ABC$  is a triangle and  $M$  is the mid-point of  $BC$ . The bisector  $MP$  of the angle  $AMB$  meets  $AB$  in  $P$ .  $AB = 11$  cm.,  $AC = 7$  cm., and  $BC = 12$  cm. Calculate  $AM$  and  $AP$ .

(ii)  $PAR$  and  $PBS$  are tangents to a circle at the points  $A$  and  $B$  respectively, and the straight line  $PCD$  intersects the circle in  $C$  and  $D$ .  $\angle PAC = 28^\circ$ ,  $\angle RAD = 34^\circ$ , and  $\angle SBD = 72^\circ$ . Calculate the angles of the triangle  $PBC$ .

3. Prove that a rectangle and a parallelogram on the same base and between the same parallels are equal in area.

Construct (without proof) a parallelogram of area 27 sq. cm., with an angle of  $60^\circ$  and a pair of parallel sides each of length 4.5 cm. Measure and state the length of one of the other sides.

4.  $ABC$  is a triangle; equilateral triangles  $APB$  and  $AQC$  are described exterior to the triangle  $ABC$ . Prove that  $PC = QB$ .

If the triangle  $ABC$  is also equilateral, prove that the angle  $PBQ$  is a right-angle.

5. Prove that a tangent to a circle is perpendicular to the radius drawn to the point of contact.

$PTQ$  is a tangent to a circle of centre  $C$ ,  $T$  being the point of contact, and the angle  $PCQ$  is a right angle. If  $PX$ ,  $QY$  are the other tangents from  $P$  and  $Q$  to the circle, prove that  $PX$  and  $QY$  are parallel.

6. Prove that an angle at the centre of a circle is twice any angle at the circumference standing on the same arc; deduce that opposite angles of a cyclic quadrilateral are supplementary.



$P$  is a point inside a triangle  $ABC$  and  $X$  is a point on  $BC$ . The circle  $BXP$  cuts  $AB$  at  $Z$  and the circle  $CXP$  cuts  $AC$  at  $Y$ . Prove that  $AYPZ$  is a cyclic quadrilateral. ✓

7. Prove that the areas of similar triangles are proportional to the squares of corresponding sides.

$ABCD$  is a quadrilateral and  $P, Q, R, S$  are points on the sides  $AB, BC, CD, DA$  respectively such that

$$\frac{AP}{AB} = \frac{AS}{AD} = \frac{CQ}{CB} = \frac{CR}{CD} = \frac{1}{3}.$$

Express the sum of the areas  $APS$  and  $CQR$  as a fraction of the area  $ABCD$  and prove that the area  $PQRS = \frac{4}{9}$  of the area  $ABCD$ .

8.  $AC$  and  $BD$  are perpendicular to  $CD$ , and  $A$  and  $B$  are on the same side of  $CD$ .  $P$  is a point on  $CD$  and  $PA = PB$ .  $PC = 3$  cm.,  $PD = 7$  cm., and  $\angle APC = 70^\circ$ . Calculate the length of  $AB$ .

### TRIGONOMETRY

JULY, 1941.

(2½ hours.)

[Answer SEVEN questions, choosing NOT MORE than THREE of the first four questions.]

1. In a triangle  $ABC$ , the side  $b$  is 7.5 inches long, the perpendicular  $AD$  from  $A$  to  $BC$  is 5.5 inches long, and  $\angle BAD = 36^\circ 4'$ . Find the perimeter of the triangle  $ABC$ .

2. A door, whose corners are  $A, B, C, D$ , is hinged on the edge  $AD$ . If  $AB = 3$  feet and  $BC = 7\frac{1}{2}$  feet, and if the door is opened through an angle of  $30^\circ$ , find the angle between the old and new positions of the line  $AC$ .

3. Use your tables to find

(i) the values of  $\sin 2A$  and  $\tan^3 A$ , when

$$A = 55^\circ 22';$$

(ii) the values of  $\cos \frac{1}{2}B$  and  $\cot^3 \frac{1}{2}B$ , when

$$B = 220^\circ 20'.$$

4. In a triangle  $ABC$ ,  $b = 24.5$ ,  $c = 17.3$ , and  $B = 42^\circ 17'$ .

Calculate the size of the angle  $A$ , and explain quite clearly why you give one answer or two as the case may be.

5. Using the same scales and axes, draw the graphs of  $y = \sin x$  and  $y = 1.5 - \sec x$  from  $x = 0^\circ$  to  $x = 60^\circ$  (using intervals of  $10^\circ$ ).

Record the value of  $x$  at the intersection of the two graphs.

Show that the value you have recorded *must* be a solution of the equation

$$\sin 2x = 3 \cos x - 2.$$

6.  $AB$  is the diameter of a semi-circle  $ABC$ ;  $AB = 10$  cm., and the chord  $AC = 4.5$  cm. Calculate the area of the triangle  $ABC$ , and that of the minor segment cut off by the chord  $BC$ .

7. In a triangle  $ABC$ ,  $\angle A = 85^\circ$ ,  $\angle C = 15^\circ 20'$ ,  $b = 90.5$  cm.; calculate the perimeter of the triangle.

8. The lengths of the sides of a triangle are 103.5, 240, and 208.5 inches. Calculate the largest angle and the area of the triangle.



9. (i) Write down the formulae for  $\cos(A+B)$  and  $\cos 2A$ .

Prove that

$$\cos 3A = 4 \cos^3 A - 3 \cos A.$$

- (ii) Write each of the following in a form suitable for quick evaluation:—

$$\cos^2 35^\circ - \sin^2 35^\circ; \quad 2 \sin(45^\circ + \frac{1}{2}x) \cos(45^\circ + \frac{1}{2}x);$$

$$\frac{1 - \cos^2 18^\circ}{\sin 18^\circ}.$$

10. (i) Solve  $3 \sin \theta + 2 = \frac{1}{\sin \theta}$ , giving values of  $\theta$  from  $0^\circ$  to  $360^\circ$ .

- (ii) A triangle  $ABC$  has an obtuse angle at  $C$ .  $AD$  is the perpendicular from  $A$  to  $BC$  produced.  $BC = 16$  inches,  $CD = 20$  inches, and  $AD = 15$  inches.

Prove  $\cos BAC = \frac{63}{65}$ .

### GEOMETRY.

JULY, 1942.

( $2\frac{1}{2}$  hours.)

[Answer SEVEN questions]

1. In this question neither set-square nor protractor is to be used. No statement of proof or construction is required, but all construction lines should be shown.

Construct a quadrilateral  $ABCD$  in which  $AB = 3\frac{3}{4}$  inches,  $\angle ABC = 90^\circ$ ,  $BC = 2$  inches,  $\angle BAD = 60^\circ$ , and  $AD = 3$  inches. Draw the diagonal  $BD$  and measure  $BD$  and  $CD$ .

Construct and name a triangle equal in area to  $ABCD$ .

2. In this question proofs are not required, but brief statements should be made showing that the results have not been obtained either by drawing or by guesswork.

(i)  $ABCD$  is a quadrilateral inscribed in a circle, centre  $S$ .  $CB$  and  $DA$  meet when produced at  $E$  and the diagonals of  $ABCD$  meet at  $F$ . If  $\angle DSC = 70^\circ$  and  $\angle DEC = 20^\circ$ , calculate  $\angle DFC$ .

(ii) Two circles of radii 16 cm. and 4 cm. respectively touch externally. Their common tangent  $PQ$  touches the circles at  $P$  and  $Q$  respectively. Calculate the length of  $PQ$ .

3. (i)  $D$  is a point on the side  $AC$  of a triangle  $ABC$ .  $\angle ABD = \angle DBC = 40^\circ$ ,  $\angle ADB = 100^\circ$ . Prove  $AD > DC$ .

(ii)  $ABCD$  is a trapezium having  $AB$  parallel to  $DC$ . The diagonals meet at  $M$ . The areas of the triangles  $ABM$  and  $BMC$  are respectively 3 sq. in. and 5 sq. in.

Find the ratios  $\frac{AM}{MC}$  and  $\frac{BM}{MD}$ . Calculate the area of the triangle  $DMC$ .

4. Prove that, if two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram.

In a parallelogram  $ABCD$ , a point  $X$  is taken on  $AB$  and  $Y$  on  $DC$  such that  $AX = CY$ . Prove that  $AC$  and  $XY$  bisect each other where they meet at  $M$  and that  $BMD$  is a straight line.



5. Prove that the opposite angles of a quadrilateral inscribed in a circle are supplementary.

$ABC$  is a triangle inscribed in a circle and  $AB = AC$ . Any point  $D$  is taken on  $BC$  produced and  $AD$  is drawn cutting the circle at  $E$ .

Prove  $\angle ACE = \angle ADB$ .

6. From an external point, two tangents are drawn to a circle. Prove that they are equal in length and that they subtend equal angles at the centre of the circle.

$ABCD$  is a quadrilateral circumscribing a circle, centre  $S$  (i.e. each side of the quadrilateral touches the circle). Prove

(i)  $AB + CD = BC + DA$ ;

(ii)  $\angle ASB + \angle CSD = 180^\circ$ .

7. Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the bisected angle.

The base  $AB$  of a triangle  $ABC$  is 12 inches long and  $\frac{AC}{CB} = \frac{3}{2}$ . The internal and external bisectors of

$\angle ACB$  meet  $AB$  at  $P$  and  $Q$  respectively. Calculate the lengths of  $AP$  and  $BQ$ .

If  $A$  and  $B$  are fixed and  $C$  moves so that the ratio  $AC/CB$  is always equal to  $3/2$ , what is the locus of  $C$ ?

8.  $ABC$  is a triangle, right-angled at  $B$ ,  $\angle ACB = 48^\circ$ , and  $AC = 9$  inches. On  $AC$  is drawn, outside the triangle, a rectangle  $ACDE$  with  $AE = 4$  inches.

Calculate (i) the length of the perpendicular from  $E$  to  $BC$ ; (ii) the angle  $EBA$ .

## TRIGONOMETRY

JULY, 1942.

(2½ hours.)

[Answer Questions 1, 2, and 3, and any FOUR others (SEVEN questions altogether).]

1. A triangle  $ABD$  is right-angled at  $B$ , and  $C$  is a point in  $BD$ .  $AB = 7.2$  inches,  $AC = 9.4$  inches, and  $CD = 8.5$  inches. Calculate the angle  $CAD$ .

2. The lid of a school desk slopes at an angle of  $15^\circ$  with the horizontal. The horizontal edges of the lid are 20 inches long and the sloping edges 14 inches long. What angle will the diagonal line of the lid make with the horizontal?

3. (i) Find, without using tables, the values correct to two decimal places of  $\sin 135^\circ$  and  $\cos 240^\circ$ .

(ii) Use your tables to evaluate  $\frac{\sin A \cos 2A}{1 + \tan 2A}$  when  $A = 69^\circ 24'$ .

4. Draw, with the same scales and axes, (i) the graph of  $y = \cos x$  from  $x = 0^\circ$  to  $180^\circ$ , and (ii) the graph of  $y = \tan x$  from  $x = 0^\circ$  to  $60^\circ$  and from  $x = 120^\circ$  to  $180^\circ$ .

Record two solutions of the equation  $\cos x = \tan x$ .



5. Prove that the area of a triangle  $ABC$  is  $\frac{1}{2}bc \sin A$ .

If a triangle  $PQR$  has  $PQ=10.9$  cm.,  $PR=16.7$  cm., and has an area of 49 sq. cm., calculate two possible values of the angle  $QPR$ . Calculate the third side in one case only.

6. In the triangle  $ABC$ ,  $a=98.6$ ,  $b=79.1$ ,  $c=48.4$ . Calculate the angle  $A$  and the projection of  $BA$  on  $CA$ .

7. (i) If  $\sin \theta = \frac{4}{5}$  and if  $\theta$  lies between  $90^\circ$  and  $180^\circ$  find, without using tables,

$$\sin 2\theta, \quad \cos 2\theta, \quad \text{and} \quad \tan 2\theta.$$

(ii) Assuming that  $5 \cos x + 12 \sin x$  can be written in the form  $R \cos(x - \alpha)$ , show that  $R \cos \alpha = 5$  and  $R \sin \alpha = 12$ . Solve these two equations and hence show that the maximum value of

$$5 \cos x + 12 \sin x$$

is 13.

8.  $OA$  and  $OB$  are radii of a circle, centre  $O$ . The arc  $AB$  is  $7\frac{1}{2}$  cm. long and the angle  $AOB$  is  $\frac{3}{4}$  radian. Calculate the radius of the circle and the area of the smaller segment cut off by the chord  $AB$ .

9.  $A, B, C$  are three points in a horizontal plane.  $D$  is a point vertically above  $A$ .  $\angle ABC = 70^\circ 30'$ ,  $\angle ACB = 42^\circ 20'$ ,  $\angle ACD = 24^\circ 10'$ , and  $BC = 450$  feet. Calculate the height of  $D$  above  $A$ .

JULY, 1943

(2½ Hours)

# GEOMETRY

[Answer SEVEN questions.]

1. In this question neither set-square nor protractor is to be used. No statement of proof or construction is required, but all construction lines should be shown.

Construct a triangle  $ABC$  in which  $AB=5$  in.,  $AC=4\frac{1}{2}$  in., and  $\angle BAC=45^\circ$ . Measure as accurately as you can and record the length of  $BC$ .

Construct the perpendicular from  $C$  to  $AB$ ; measure and record its length.

Calculate the area of the triangle  $ABC$ .

2. (i) The sizes of the angles of a quadrilateral taken in order are  $5a$ ,  $7a$ ,  $9a$ , and  $11a$  degrees respectively. Prove that two sides of the quadrilateral are parallel.

(ii) From a point  $P$  outside a circle, two lines  $PAB$  and  $PCD$  are drawn to cut the circle in  $A, B$  and  $C, D$  respectively. If  $PA=3$  cm.,  $AB=9$  cm.,  $AC=4$  cm., and  $PC=4$  cm., calculate the length of the tangent from  $P$  to the circle. Calculate also the lengths of  $CD$  and  $BD$ .

3. In a parallelogram prove that the opposite sides are equal and that the diagonals bisect one another.

$ABCD$  is a quadrilateral. Parallelograms  $ADCM$  and  $BCDK$  are drawn. Prove that  $KM$  bisects  $AB$ .

4. Prove that, if the square on one side of a triangle is equal to the sum of the squares on the other two sides, the triangle is right-angled.

$ABCD$  is a square of side 12 in.  $E$  is a point in  $BC$  and  $\frac{BE}{EC} = \frac{1}{2}$ .  $F$  is a point in  $CD$  and  $\frac{CF}{FD} = \frac{2}{7}$ . Prove that the triangle  $AEF$  is right-angled.



5. If the line joining two points subtends equal angles at two other points on the same side of the line, prove that the four points lie on a circle.

$AB$  is a diameter of a circle, centre  $O$ , and  $OD$  is a radius of the circle perpendicular to  $AB$ . A chord  $DE$  is drawn, cutting  $AO$ , and meeting the circle again in  $E$ .  $AM$  is drawn perpendicular to  $DE$  to meet it at  $M$ . Prove that  $BE$  is parallel to  $OM$ .

6. (i) Two circles, centres  $P$  and  $Q$ , touch externally. Both circles touch a third circle, centre  $R$ , internally. If the radii of the circles, centres  $P$ ,  $Q$ , and  $R$ , are respectively 3, 4, and 9 in., calculate the perimeter of the triangle  $PQR$ .

(ii)  $A$ ,  $B$ ,  $C$  are points on a given circle, such that the tangent at  $A$  meets  $BC$  produced at a point  $D$ ; the angle  $BAC$  is  $120^\circ$ , and the angle  $ADC$  is  $20^\circ$ . Prove that  $AB=AD$ , and arc  $AB=2$  arc  $AC$ .

7. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on corresponding sides.

$M$  and  $N$  are points in the sides of  $AC$  and  $AB$  respectively of a triangle  $ABC$  and  $\frac{AN}{NB} = \frac{1}{2} = \frac{AM}{MC}$ .  $BM$  and  $CN$  intersect at  $P$ . Prove that the area of triangle  $BPC$  is nine times that of triangle  $MPN$ .

8. A triangle  $ABC$  is right-angled at  $B$ , and  $AC = 20$  cm. and  $\angle C = 24^\circ$ . Calculate the length of  $AB$ .

$P$  is a point in  $BC$  and  $PC = 5.74$  cm. Calculate the length of  $AP$  and the size of  $\angle APB$ .

JULY, 1943

(2½ Hours)

### TRIGONOMETRY.

[Answer Questions 1, 2, and 3, and any FOUR others (SEVEN questions altogether).]

1. A regular pentagon is inscribed in a circle of radius 15.8 cm. Calculate the length of a side of the pentagon.

If another circle is drawn to touch the sides of the pentagon, calculate the radius of this second circle. Calculate also the area of the pentagon.

2. A pyramid whose apex is  $M$  has a square base  $ABCD$ . The base is in a horizontal plane.  $AB=2$  ft.,  $AM=BM=CM=DM=3$  ft. Calculate the angle which  $AM$  makes with the vertical.

3. (i) Given that  $\theta$  is an acute angle and that  $\sin \theta = \frac{7}{25}$ , find, without using tables, the values of  $\cos \theta$  and  $\cot \theta$ . Write down the values of  $\cos(180^\circ - \theta)$ ,  $\sin(180^\circ + \theta)$ , and  $\tan(360^\circ - \theta)$ .

(ii) If  $\cos a = -0.6645$ , use your tables to find all the values of  $a$  between  $0^\circ$  and  $360^\circ$  which satisfy this equation.

4. Using the same scales and axes for both, draw the graphs of  $y = \sin 2x$  and  $y = \tan(x - 10^\circ)$  for values of  $x$  from  $10^\circ$  to  $60^\circ$ .

Record the value of  $x$  for which

$$\sin 2x = \tan(x - 10^\circ)$$

and the value of  $\sin 2x$  when  $x$  has this value.

5. In a triangle  $ABC$ ,  $b=15.40$  cm.,  $c=17.92$  cm., and  $\angle B=51^\circ 30'$ . Show clearly that two triangles are possible, and calculate the area of the smaller triangle.

6. Solve the triangle  $ABC$  in which  $a=17.90$ ,  $b=15.87$ , and  $c=13.85$ .

7. (i) Express each of the following as a single trigonometrical ratio:

(1)  $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ$ ;

(2)  $2 \sin 15^\circ \cos 15^\circ$ ;



(3)  $2 \cos^2 30^\circ - 1;$

(4)  $\frac{\sin 45^\circ}{\cos 45^\circ}.$

Give the numerical value of each result.

(ii) If  $\tan a = \frac{1}{2}$  and  $\tan \beta = \frac{1}{4}$ , show, without using tables, that  $\tan(2a - \beta) = \frac{13}{16}$ .

8. Three points  $A$ ,  $B$ , and  $C$  are on level ground and at  $B$  a post  $BD$  26 ft. high is erected vertically. The angles  $DAB$ ,  $DCB$ ,  $ABC$  are respectively  $10^\circ$ ,  $14^\circ$ , and  $95^\circ$ . Calculate the distance  $AC$ .







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