



Mathematics

Year 10

Foundation Tier

Contents

Topic	Mathematics	Length	Page Number
01 Data Handling and Statistics 4	(a) Comparing Averages. (b) Questionnaires. (c) Frequency Polygons.	5–7 weeks	5
02 Powers and Roots	(a) Rules of Indices. (b) Graph Plotting.	5–6 weeks	25
03 Measuring Shapes 3	(a) Transformations: Enlargement. (b) Perimeter and Area of Composite Shapes.	4–5 weeks	37
04 Fractions, Percentages and Decimals	(a) Calculating. (b) Recurring Decimals. (c) Converting. (d) Efficient Proportional Changes. (e) Exam Technique.	5 weeks	49
05 Developing Algebra 2	(a) Simple Factorising. (b) Changing the Subject. (c) Expression, Equation, Formula.	5–6 weeks	75
06 Measuring Shapes	(a) Solids. (b) Composite Solids. (c) Similar Shapes.	3–5 weeks	85
07 Accuracy of Measurements	(a) Rounding Off: Significant Figures. (b) Estimation. (c) Upper and Lower Bounds. (d) Appropriate Degree of Accuracy. (e) Compound Measures.	3 weeks	99



The Mathematics Department

10

Data Handling

and Statistics 4

Foundation Tier

Version
3.0
08/2025

Name:

Contents

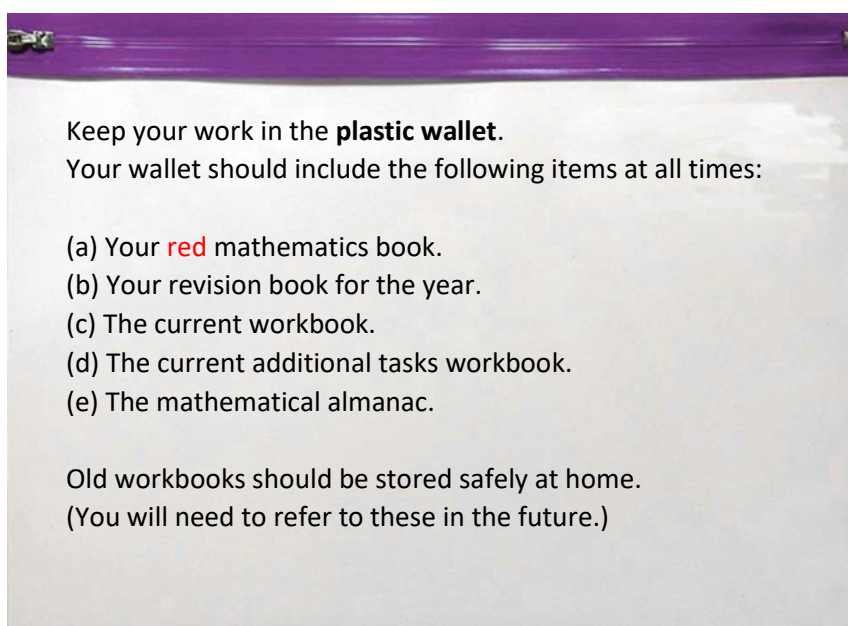
Chapter	Mathematics	Page Number
The Mathematics Department	Looking after your work. Content of the workbooks. Mathematics learning journey.	3
The Sequential Data Handling Cycle	An infographic explaining the sequential data handling cycle.	6
Comparing Averages	Revising averages. Choosing the most appropriate average. Comparing averages.	7
Questionnaires	Designing questionnaires. Criticising questions. Hypotheses.	12
Frequency Polygons	Drawing frequency polygons. Interpreting frequency polygons.	15





Welcome back to year 10! Here is a reminder of the guidelines for looking after your work.

- At the start of every lesson, write "Classwork", the date and a suitable title for the work. Each of these should be underlined.
- No blank pages should be left in the red mathematics book.
- Draw diagrams using a pencil and, where appropriate, using the correct equipment, e.g. ruler, protractor, compass.
- Show your method fully.
- Remember to include relevant units in your answers, e.g. cm, £, ml.



Equipment

- Black, **red** and **blue** biros.
- HB Pencil.
- Ruler (a 30 cm one is better).
- Eraser.
- Protractor.
- Compass.
- Scientific calculator (Casio fx-83GTCW).
- Highlighter.

The Revision Book

This will form the basis for your GCSE revision.

- Complete at least 4 pages in your revision book for each unit of work.
- You should include material you will need in the future for remembering the work quickly. This can include notes on the work; examples; important facts; and revision posters.

Workbooks

You will receive 1 copy of the workbook and 1 copy of the additional tasks workbook at the start of each new unit of work. (If you lose the workbook, a new one will cost 50p.)

A Welsh copy of the workbook, and additional supporting materials, can be found on the department's website, www.mathemateg.com



Content of the workbooks

When you see a QR code (like the one on the left), scan it using your mobile device in order to reach a Welsh YouTube video hosted on the following channel.

www.youtube.com/adolygumathemateg

The numbers in circles, for example **3**, show which GCSE unit the work appears in.

Circle	1	2	3	12	13	23	A
Units	1	2	3	1 and 2	1 and 3	2 and 3	1, 2 and 3

All the workbooks contain a variety of exercises, labelled as follows.



Exercises on new topics.



Answering a question in context, or solving a problem.



A more difficult question.



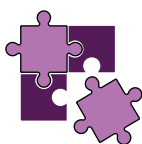
Revision of material from previous workbooks.



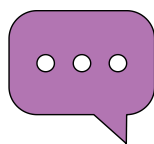
There are evaluation boxes at the end of each chapter to revise the completed work.

Key words	Corrections	I am happy with...	I need to revise...
Write down the new or important mathematical terms from the chapter.	What do you need to remember when completing this type of work in the future?	Write down the topics you had success with.	Write down the topics you need to look at again.

Curriculum for Wales Proficiencies



Conceptual understanding



Communication using symbols



Strategic competence



Logical reasoning



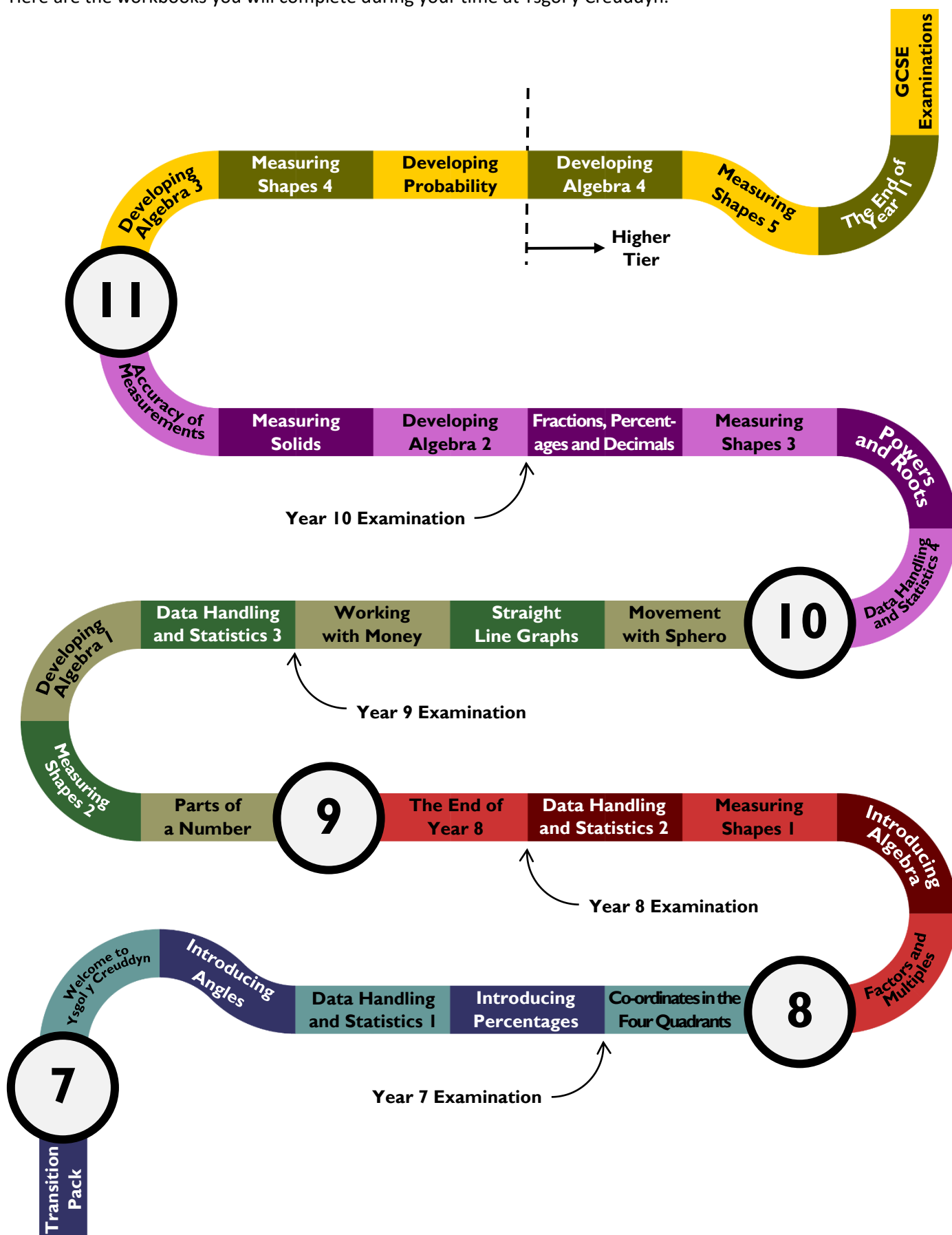
Fluency

Supporting Materials:

- Diagnostic Questions
 - A quiz for each workbook on the website www.diagnosticquestions.com.
- Reflection Sheet
 - An opportunity to assess your understanding of a workbook, and to see the test question order.
- Old WJEC examination questions; worksheets; investigations; puzzles.
 - Available for some topics.

Mathematics Learning Journey for Ysgol y Creuddyn

Here are the workbooks you will complete during your time at Ysgol y Creuddyn.



 The Sequential Data Handling Cycle

- Designing questionnaires
- Criticising questions
- Hypotheses



- Types of data
- Simple random sampling
- Systematic sampling
- Stratified sampling



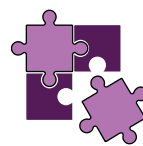
- The mean
- The range
- The mode
- The median
- The interquartile range
- Box and whisker diagrams
- Choosing the most appropriate average
- Comparing averages

- Bar charts
- Frequency diagrams
- Pie charts
- Pictograms
- Vertical line diagrams
- Line graphs
- Scatter diagrams
- Cumulative frequency diagrams
- Frequency polygons



Comparing Averages

Revising Averages



We have studied three different types of averages over the last few years.

Year 7	Year 8	Year 9	Year 10
Mean	Mode	Median	Comparing Averages

Exercise 1



Fill in the boxes below to explain how to calculate the mean, the mode and the median.

The Mean	The Mode	The Median

Exercise 2

Calculate the mean, the mode and the median of the following data.

12, 14, 14, 15, 16, 17, 17, 17, 19, 20.



Choosing the Most Appropriate Average



	The Mean	The Mode	The Median
Advantages	Uses all the data values.	- Not affected by outliers. - Can be used for qualitative data.	Not affected by outliers.
Disadvantages	- Can be affected by outliers. - Needs to be calculated.	- Doesn't use all the data values. - There is no mode for some data sets.	- Doesn't use all the data values. - Need to rearrange the data to find it.
Used for	Data which doesn't include outliers.	- Qualitative data. - Data which includes outliers.	Data which includes outliers.

Challenge!

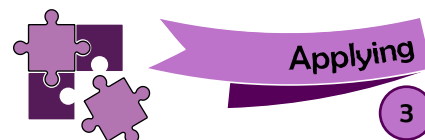
Use the internet to investigate the term *skewness*.

Which average is the best to use when the data has a skewed distribution?



Exercise 3

Which average is most appropriate for the following data sets?



- (a) Favourite football team:
Liverpool, Chelsea, Man City, Everton, Liverpool, Man Utd.
- (b) Times in a 100 m race (in seconds):
9.81, 9.89, 9.91, 9.93, 9.94, 9.96, 10.04, 10.06.
- (c) Price of Heinz Baked Beans in different shops:
75p, 60p, 74p, 80p, 70p, 95p, 85p.
- (d) Age of players who start a football game:
28, 31, 19, 24, 25, 28, 30, 23, 20, 29, 26.
- (e) Year 10 pupil heights:
162 cm, 160 cm, 161 cm, 148 cm, 163 cm, 161 cm.
- (f) Favourite subject in school:
Science, Music, Drama, Mathematics, Music.
- (g) Spelling test scores (out of 10):
5, 7, 8, 4, 5, 3, 6, 4, 5, 4, 7.
- (h) Number of brothers:
0, 1, 2, 1, 0, 6, 1, 0, 1, 2.

Exercise 4

Are the following statements TRUE or FALSE?



- (a) The mode is the most popular data item in a set of data.
- (b) Half the values in a data set are greater than the mean.
- (c) Half the values in a data set are greater than the median.
- (d) When finding the median, it doesn't matter if you order the data from least to greatest or from greatest to least.
- (e) It is always possible to find the mean of a data set.

**Exercise 5**

The following table shows the percentages of 10 pupils in Welsh and Mathematics tests.

Welsh	57	63	91	58	56	75	59	76	91	54
Mathematics	67	68	66	68	68	66	70	69	68	70

(a) Complete the following table.

	Welsh	Mathematics
The mean		
The median		
The mode		
The range		

(b) Which statistics from the table support the following newspaper headlines?

(i) **The Welsh results are very high this year.**

(ii) **Pupils do not perform better in Mathematics than in Welsh.**

(iii) **There was lots of copying during the Mathematics test.**

(iv) **The Mathematics test was easier than the Welsh test.**

(v) **The Welsh results show that some people tried harder than others.**

Exercise 6

3

Three people were asked to estimate when 30 seconds had passed. This was done 5 times.

Osian	Siân	Shubnam
31	36	32
26	19	30
32	39	24
27	36	37
29	20	32

Who, in your opinion, is best at estimating the length of 30 seconds?

Exercise 7

3

Fill the following boxes with whole numbers.

(a) 2 numbers mode = 5	(b) 2 numbers mean = 5	(c) 2 numbers mean = 5 range = 6
(d) 2 numbers mean = 5 mode = no mode	(e) 2 numbers mean = 6 range = 8	(f) 2 numbers mean = 10 largest number = 12
(g) 3 numbers mean = 3 mode = 2	(h) 3 numbers mean = 7 mode = 10	(i) 3 numbers median = 4 range = 0
(j) 4 numbers mode = 5 range = 1	(k) 4 numbers median = 3 range = 1	(l) 4 numbers mean = 6 median = 6.5 range = 11

Exercise 8

3

During a skiing trip, the PE department recorded the daily snowfall for 5 consecutive days. Here is some information about the daily snowfall.

Mean	Mode	Median	Range
5.8 cm	3 cm	5.6 cm	6.6 cm

(a) Use these statistics to calculate what was the daily snowfall for these 5 consecutive days.

(b) If it had snowed exactly 2 cm more each day, what would the new statistics be?

Mean	Mode	Median	Range

Exercise 9

In a game, it is possible for every player to score between 1 and 10 points.
Lois and Beca play the game 5 times.



The table below shows the points scored by Lois in every game.

	Game 1	Game 2	Game 3	Game 4	Game 5
Lois	5	2	8	5	1
Beca					

Beca had a higher mean score than Lois.
Beca had a lower median score than Lois.
Beca had a lower range of scores than Lois.

Complete the table above with a set of possible scores for Beca.

Exercise 10

Jim and Andy play for their local cricket team.
They scored the following runs in their six most recent games.

Jim	42	71	39	62	70	40
Andy	115	6	84	36	10	85

- (a) Calculate Jim's mean and Andy's mean.
- (b) Calculate Jim's median and Andy's median.
- (c) There is no room for both Jim and Andy on the team for the next game. The management team need to choose either Jim or Andy to play for the team. Use your answers to parts (a) and (b) to give advice to the management team.



Exercise 11

The table below shows the number of ice hockey season tickets sold by a team last season, together with the prices for the tickets.

Ticket Price (£)	Number of tickets sold
250	180
300	230
350	230
500	150

For the cost of the tickets sold last season, calculate

- (a) The mode; (b) The median; (c) The range.
- (d) The ice hockey team owner says that more than half of the tickets sold were greater in value than £300. Explain why the team owner is incorrect.

Exercise 12

50 people took part in a charity walk.

The table below shows the grouped frequency distribution of the sums of money raised, to the nearest £.

Sum, s , in £	Number of people
$10 \leq s < 19$	2
$20 \leq s < 29$	18
$30 \leq s < 39$	29
$40 \leq s < 49$	1

- (a) Find the modal class of the data.
- (b) Find the median class of the data.
- (c) Calculate an estimate for the mean sum of money raised per person.
- (d) Another 50 people took part in the same charity walk. The total of the money collected by these 50 extra people was £1,600. Is it possible to say that these 50 people raised more money than the original 50 people?

Evaluation

Key words	Corrections	I am happy with...	I need to revise...



Questionnaires

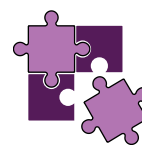
A **questionnaire** is a good way to collect data, but we must be careful when designing the questions.

- (1) We must avoid asking **leading** questions which favour one answer over another. For example, the question “Do you agree that eating ice cream is bad for you?” leads people to agree with the statement. (Why?)
- (2) We must avoid using answer boxes where the **options overlap**. For example, in the following question, people who are 20 years old can choose two different answer boxes.

How old are you?

Under 10 10–20 20–30 Over 30

- (3) We must be careful **where, when and how** a questionnaire is conducted. For example, the following questionnaires would not be appropriate.
 - a. Conducting a sports questionnaire outside of a football stadium.
 - b. Conducting a questionnaire about night shift workers at mid-day.
 - c. Conducting a questionnaire on mobile phone usage through a phone survey.
- (4) We must use **clear** and **concise** questions. For example, the question “How often do you go to the gym?” is not appropriate without explaining the meaning of the word “often”, i.e. daily, weekly, monthly...?
- (5) We must use **appropriate** and **relevant** questions. For example,
 - a. Some people would refuse to answer “How old are you?”, however they might answer if options showing different age ranges were included.
 - b. The question “What is your eye colour?” would not be appropriate in a questionnaire asking opinions about recycling.



Exercise 13

Write a criticism of the following questions.

- (a) Do you read books? Circle your answer.

Yes No Sometimes

- (b) Do you agree that the cruel sport of fox hunting should be made illegal?

- (c) How often do you use the gym in a typical month? Circle your answer.

Never Once or Twice 2–5 Times More than 5 times

- (d) How old are you? Circle your answer.

10–15 16–20 21–25 26–30 31–35

- (e) In your current job, how much money do you earn?

- (f) How often do you shop in a supermarket? Circle your answer.

Three times a week Twice a week Once a week Once a month

- (g) Do you agree that the amazing Liverpool are the best football team in the world? Circle your answer.

Yes Of course Absolutely



Skill

13

Writing a criticism means finding something incorrect or out of place.



Exercise 14

A survey is conducted to see how often teenagers buy trainers.

The following questions are included in the questionnaire.



Applying

13

Question 1: Where do you live?

Question 2: How often do you buy trainers?

Never

☐

1–10 times

☐

10–15 times

☐

More than 15 times

☐


(a) For each question state **one** reason why it is **not** appropriate.

(b) The survey is conducted by leaving copies of the questionnaire on the seats in a sports clothing shop. Give one criticism of how the survey was conducted.

Exercise 15

A survey was carried out to discover whether people would rather watch sports or detective programs on the television.

The following three questions were included.

Question 1: What is your address?

Question 2: Which type of TV program do you prefer? Tick one box.

Sports

☐

Detective

☐

Question 3: How many hours do you spend watching TV?

Less than 1 hour

☐

1–5 hours

☐

More than 5 hours

☐


(a) Give **one** reason why question 1 is not appropriate.

(b) Give **one** reason why question 3 is not appropriate.

(c) The survey was conducted by asking people who were leaving a football stadium one Saturday afternoon. Give **one** criticism of how the survey was carried out.

Exercise 16

Elen is conducting a survey in her school about the standard of food in the canteen. She asks every 20th person that goes to get hot food. Explain what is wrong with Elen's plan.

Hypotheses

A **hypothesis** is a statement like “children spend more time playing computer games than adults”. It is possible to create a questionnaire and collect data to test a hypothesis.



Exercise 17

Steffan wants to prove the following hypothesis.

‘Most people spend more than two hours on the internet every night.’

He intends to

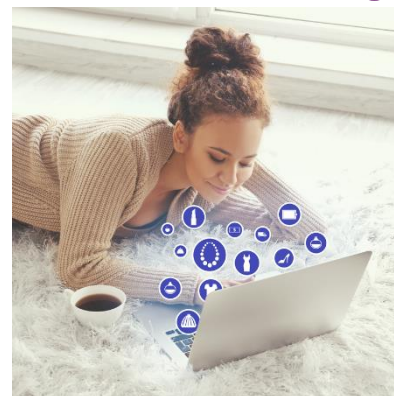
- give a short questionnaire to people in the local fitness centre,
- ask the following questions:
 - In your opinion, do people spend too much time on the internet?
 - How much time do you spend on the internet?
- ask each participant to post their completed questionnaire in an envelope with a stamp on it.

Write **three** unfavourable comments about this plan.



Applying

13



Exercise 18

Mari wants to prove the following hypothesis.

‘Older pupils in a secondary school are better at remembering their times tables than younger pupils.’

She intends to

- give a short questionnaire to 50 randomly chosen pupils in each year,
- ask the same 5 multiplication questions to everyone, to complete in a maths lesson,
- ask the pupils to mark each other’s work and return the questionnaires to her through the maths teacher.

Write **three** unfavourable comments about this plan.

Exercise 19

Iwan wants to prove the following hypothesis.

‘The children in Llandudno spend more time playing computer games than the adults.’

Write a questionnaire that Iwan could use to prove or disprove this hypothesis.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Frequency Polygons

Drawing Frequency Polygons

We draw **frequency polygons** for the following types of data.

- Grouped discrete quantitative data.
- Continuous quantitative data.



A frequency polygon is a **line graph** where we plot the **midpoint** for every class against the **frequency**.

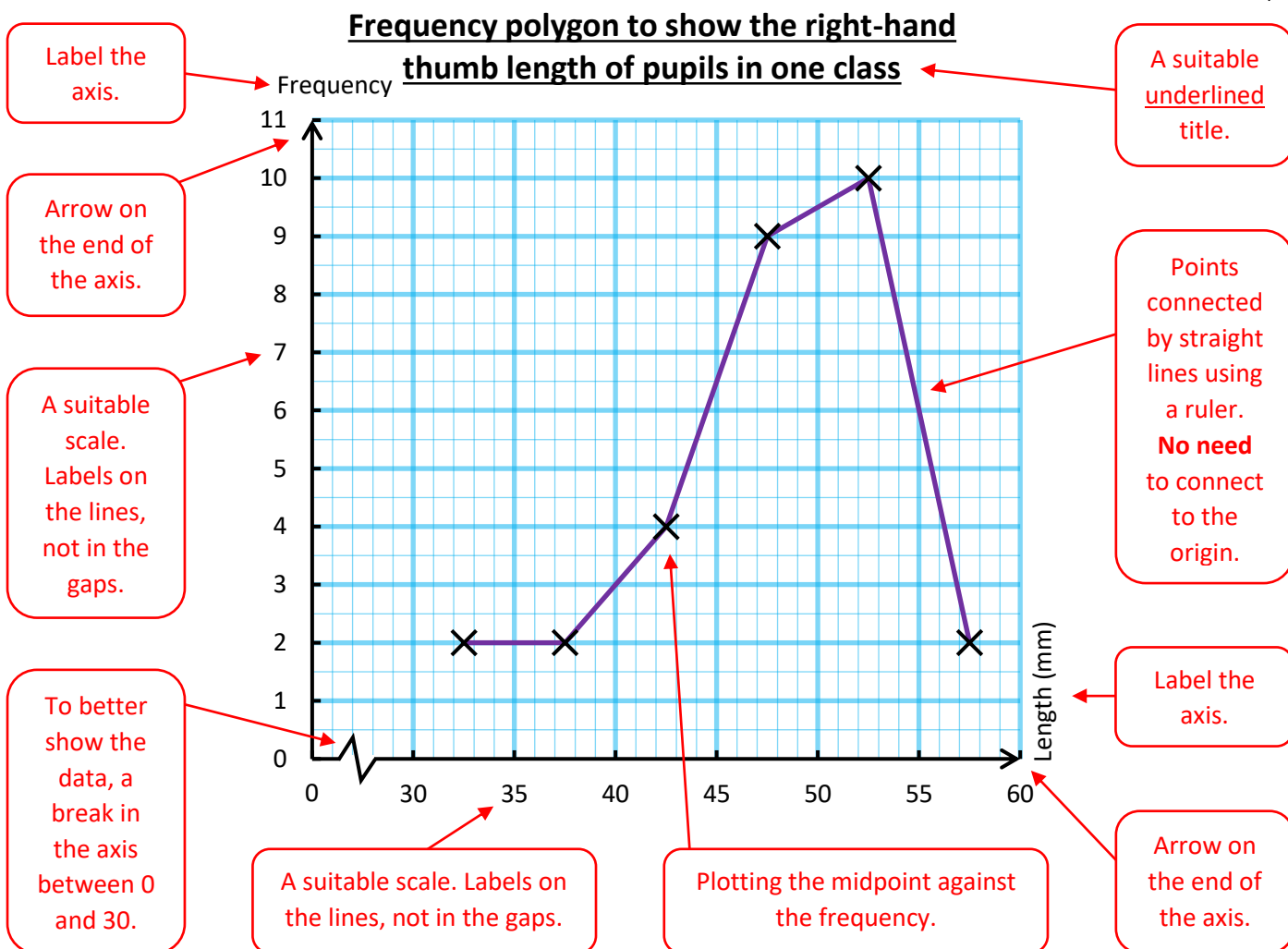
Example

The following frequency table shows the right-hand thumb length (l) for pupils in one class.

Class Interval (l mm)	Frequency
$30 \leq l < 35$	2
$35 \leq l < 40$	2
$40 \leq l < 45$	4
$45 \leq l < 50$	9
$50 \leq l < 55$	10
$55 \leq l < 60$	2

The midpoint for the class $30 \leq l < 35$ is 32.5.
Method 1: 32.5 is half way between 30 and 35.
Method 2: $30 + 35 = 65$. $65 \div 2 = 32.5$.

The frequency polygon below represents the data.



Exercise 20

Draw a **frequency polygon** for the following data on squared paper.

(a) Number of books bought by pupils in 7E during the last year.

None to four books: 12

Five to nine books: 5

Ten to fourteen books: 6

Fifteen to nineteen books: 1

(b) Number of minutes a dentist spends with each patient.

1–5 minutes: 2

6–10 minutes: 4

11–15 minutes: 9

16–20 minutes: 5

21–25 minutes: 3

26–30 minutes: 3

31–35 minutes: 0

36–40 minutes: 1

(c) Number of absent days from school for pupils in 7C last term.

0–4 days: 11

5–9 days: 8

10–14 days: 6

15–19 days: 0

20–24 days: 5

(d) Number of words in every sentence for the first 50 sentences of a book.

1–10 words: 2

11–20 words: 9

21–30 words: 14

31–40 words: 7

41–50 words: 4

51–60 words: 8

61–70 words: 6

Exercise 21

Draw a **frequency polygon** for the data in each of the following frequency tables.

(a) Weight was lost by people in a weight loss group over 6 months.

Weight (w kg)	Frequency
$0 \leq w < 6$	4
$6 \leq w < 12$	11
$12 \leq w < 18$	12
$18 \leq w < 24$	7
$24 \leq w < 30$	3

(b) Height of 60 pupils.

Height (h cm)	Frequency
$168 \leq h < 172$	2
$172 \leq h < 176$	6
$176 \leq h < 180$	17
$180 \leq h < 184$	22
$184 \leq h < 188$	10
$188 \leq h < 192$	3

(c) Sound of 60 electrical items.

Sound (s db)	Frequency
$15 \leq s < 20$	4
$20 \leq s < 25$	12
$25 \leq s < 30$	15
$30 \leq s < 35$	6
$35 \leq s < 40$	8
$40 \leq s < 45$	3
$45 \leq s < 50$	12

Exercise 22

The following data gives the time taken by 50 runners to complete a cross-country race, to the nearest minute.

30	37	43	55	52	47	49	36	44	40
41	49	52	53	39	41	46	42	50	49
39	53	54	57	43	59	34	38	40	42
48	53	50	52	37	36	45	53	48	42
52	39	41	46	50	52	38	58	57	46

Time (t minutes)	Tally Marks	Frequency
$30 \leq t < 35$		
$35 \leq t < 40$		
$40 \leq t < 45$		
$45 \leq t < 50$		
$50 \leq t < 55$		
$55 \leq t < 60$		



(a) Complete the **frequency table** for the data. (Remember that the data item 35 minutes would go into $35 \leq t < 40$, not $30 \leq t < 35$.)

(b) Draw a **frequency polygon** for the data.

Interpreting Frequency Polygons

Applying

Exercise 23

The frequency polygon on the right shows the arm lengths of 100 females.

- (a) How many females have an arm length between 55 cm and 60 cm?
- (b) How many more females have an arm length between 65 cm and 70 cm, compared to females with arm length between 70 cm and 75 cm?
- (c) Complete the frequency table below, using the information from the frequency polygon.

Arm length, / cm	Frequency
$50 < l \leq 55$	
$55 < l \leq 60$	
$60 < l \leq 65$	
$65 < l \leq 70$	
$70 < l \leq 75$	

- (d) What is the modal class of the data?

Exercise 24

The daily rainfall for ten days was measured in Aberwen and Aberisel. The frequency polygon on the right shows the results. The purple line represents Aberwen, and the red line represents Aberisel.

- (a) For how many days was the rainfall between 0 mm and 1 mm of rain in Aberwen?
- (b) For how many days was the rainfall between 6 mm and 7 mm of rain in Aberisel?
- (c) Complete this sentence: Considering the days where the rainfall was between 5 mm and 6 mm of rain, Aberisel had this rainfall on _____ more days than Aberwen.
- (d) Over these 10 days, in your opinion where was the wettest place? Explain your answer.
- (e) Deiniol says "The frequency polygon shows that the rainfall in Aberwen and Aberisel is the same on the fourth day". Is Deiniol telling the truth? Explain your answer.

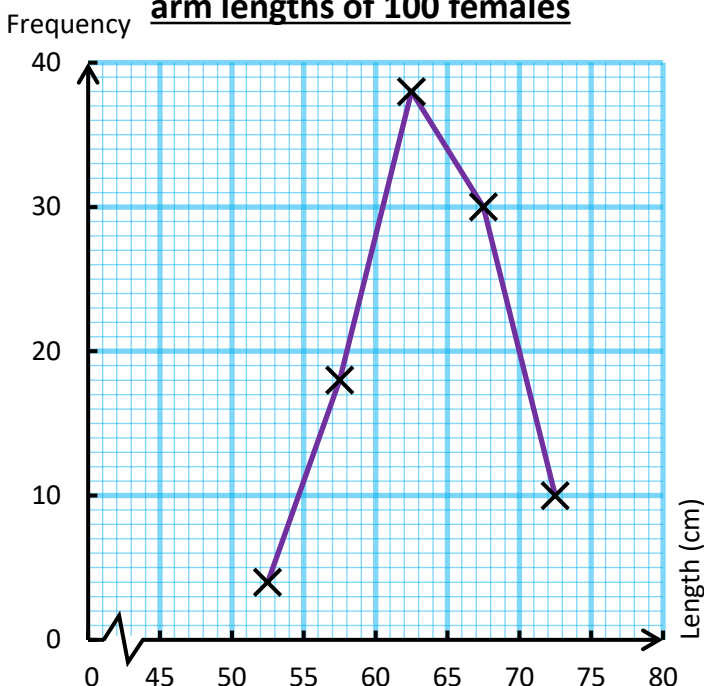
Challenge!

Use the frequency table in Exercise 41 to calculate an estimated mean arm length for the 100 females.

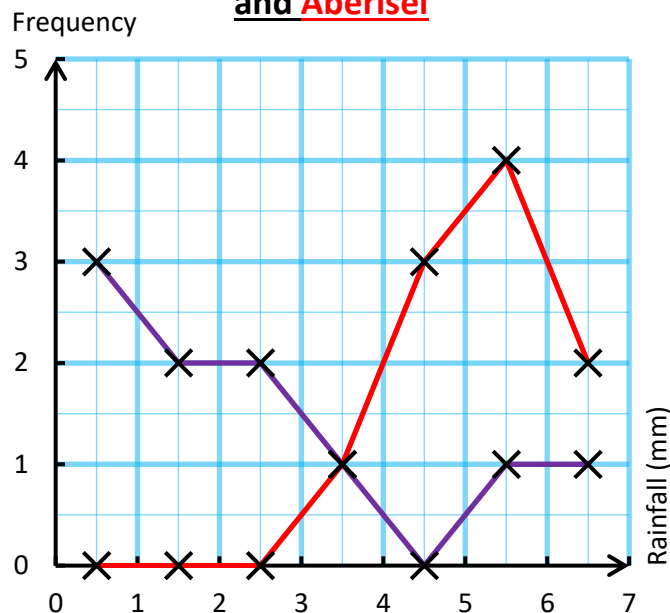
Challenge!

Use the frequency polygon from Exercise 42 to calculate the estimated mean rainfall in Aberwen and in Aberisel. Does your answer agree with your conclusion to part (d) of Exercise 42?

Frequency polygon to show the arm lengths of 100 females



Frequency polygon to show the rainfall over 10 days in Aberwen and Aberisel



Extension

Exercise 25**3**

The table below shows the marks for 10R in their English test. (The test was out of 30.)

Marks (m)	Frequency
$0 \leq m < 6$	3
$6 \leq m < 12$	8
$12 \leq m < 18$	7
$18 \leq m < 24$	6
$24 \leq m < 30$	2

(a) Eric looks at the table and says:

"Three people got 0 out of 30 in this test!". Is Eric telling the truth?

(b) Susan looks at the table and says:

"Nobody got full marks in this test!". Is Susan telling the truth?

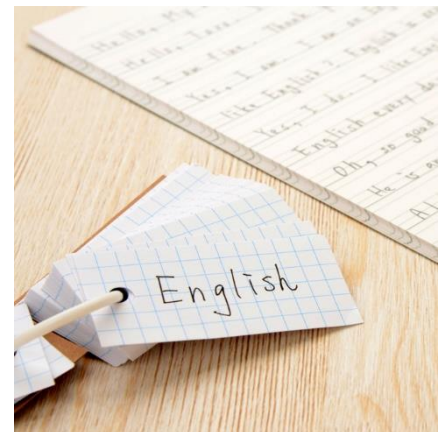
(c) On the graph paper on the right, draw a frequency polygon for the data.

(d) On the same graph paper, draw a frequency diagram for the data.

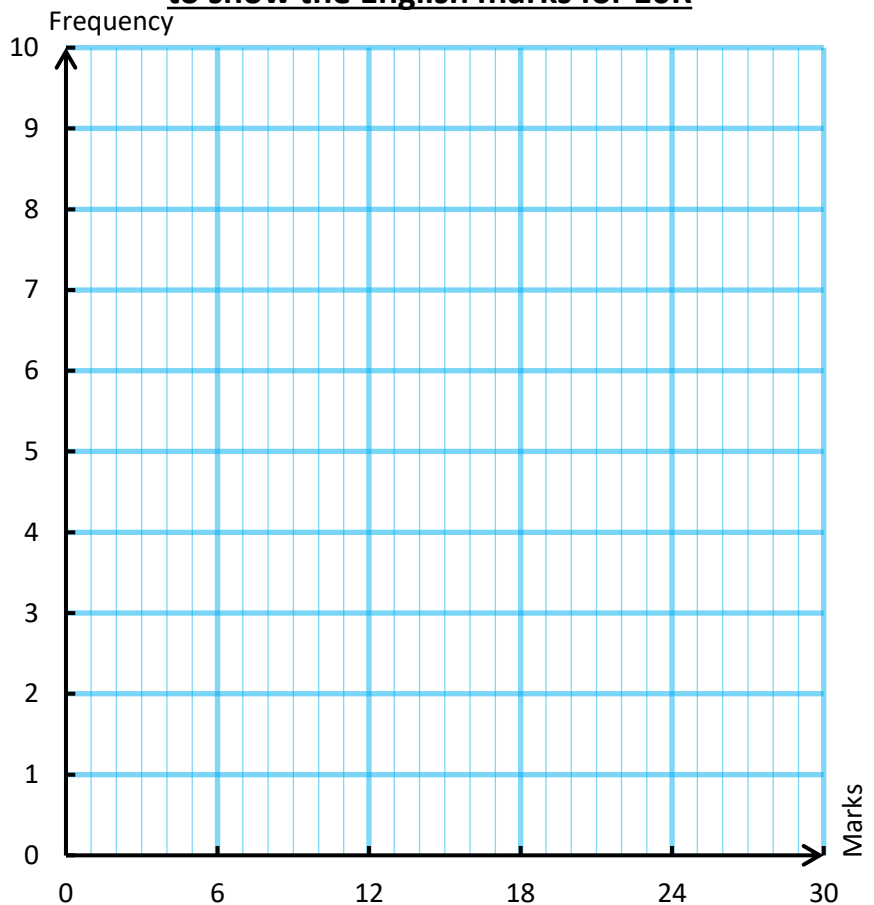
(e) What is the connection between any frequency polygon and frequency diagram drawn for the same data?

(f) Calculate the following for the data from the English test.

- The modal class.
- The median class.
- The estimated mean.
- The estimated range.



Frequency polygon and frequency diagram to show the English marks for 10R

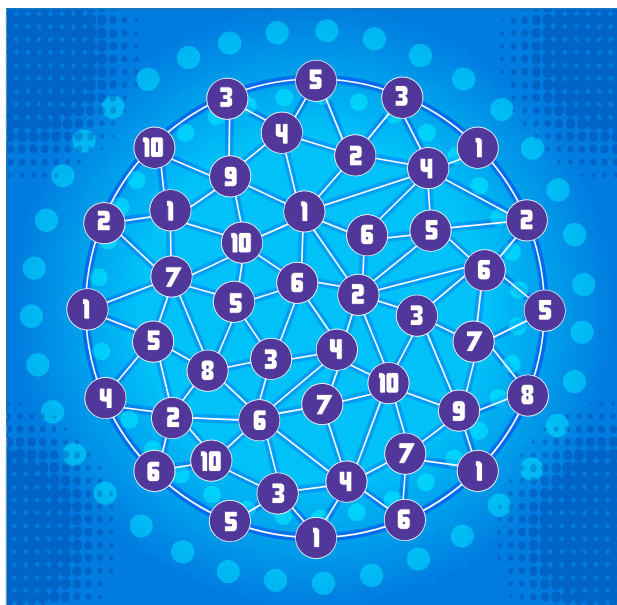


Evaluation

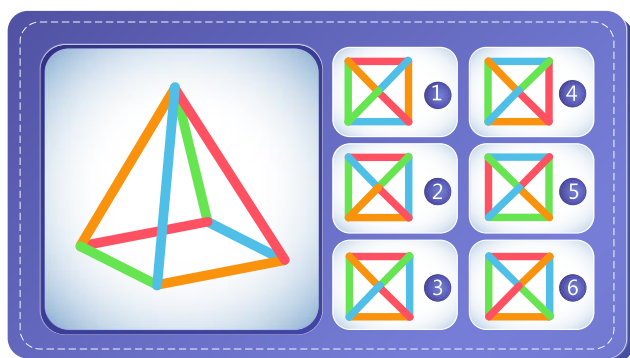
Key words	Corrections	I am happy with...	I need to revise...

Puzzles

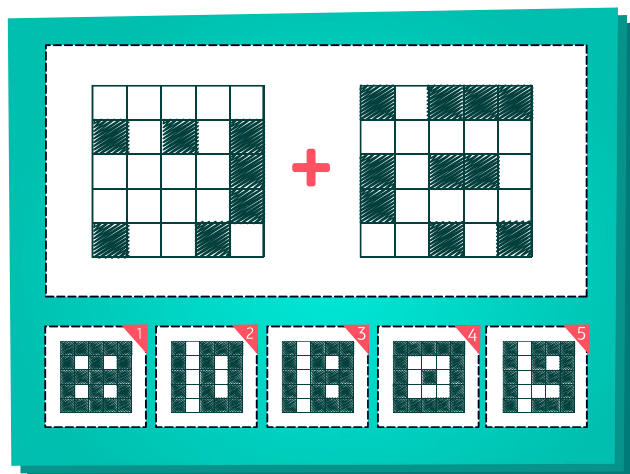
(a) Connect the numbers from 1 to 10.



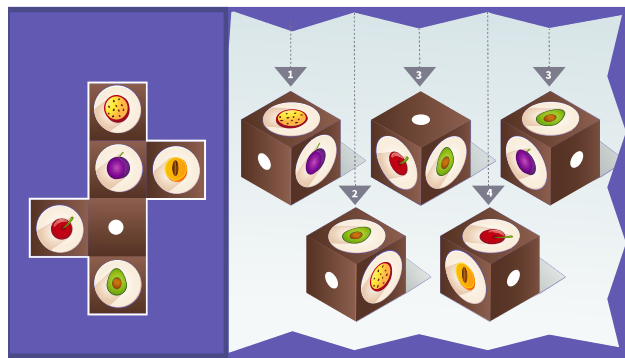
(b) Which is the correct plan view?



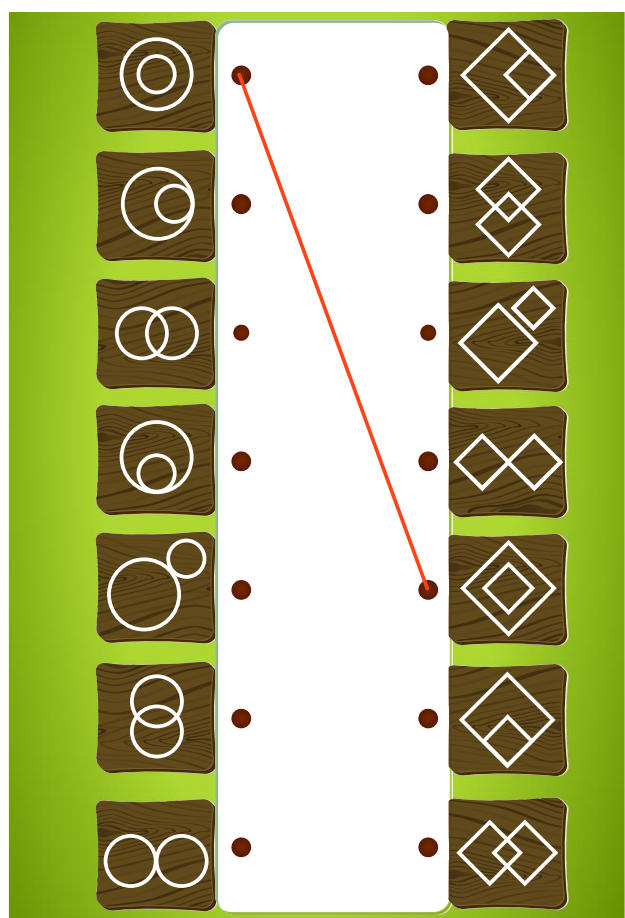
(c) Join the pictures to deduce which image is formed.



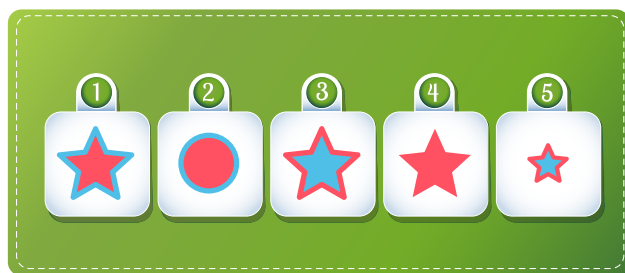
(d) Which cube is formed by folding the net?



(e) Pair the pictures.



(f) Which picture is the odd one out?



Data Handling

and Statistics 4

Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?I can calculate the **mode**, the **median**, the **mean** and the **range** for **discrete data**.

3

I can calculate the **modal class**, the **median class**, an **estimate of the mean** and an **estimate of the range** for **grouped data**.

2, 5

I know how to decide **which average is most suitable** for a set of data.

6

I know how to use averages and measures of spread to **compare** two data sets.

5, 6

I can **find the original data set** given information about averages and the range.

4, 7

I know how to **criticise questions** in a **questionnaire**.

1

I know how to write **unfavourable comments** about a plan to prove a specific **hypothesis**.

1

I know how to draw a **frequency polygon**.

2

I know how to **interpret** a frequency polygon.



The Mathematics Department

10

Powers

and Roots

Foundation Tier

Version

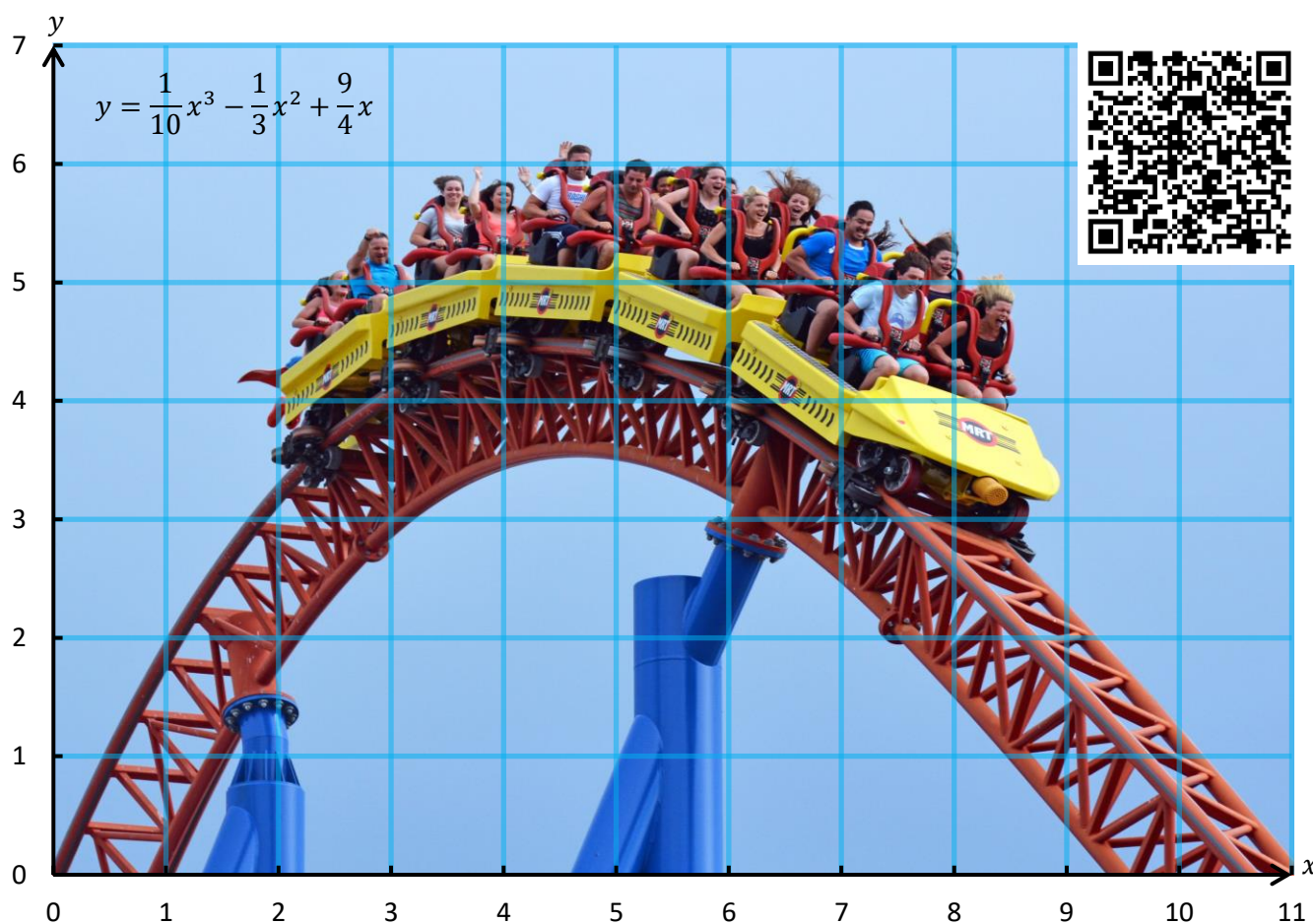
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08/2025

Name:

Contents

Chapter	Mathematics	Page Number
Rules of Indices	The index form. Evaluating the index form. The multiplication rule. The division rule. Raising a power to another index. Algebra and rules of indices.	3
Graph Plotting	Quadratic graphs. Recognising and sketching graphs of the form $y = ax^2 + b$. Graphical method of solving equations of the form $x^2 + ax + b = 0$.	8



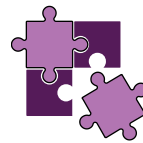
Rules of Indices

The Index Form

In year 8, we considered how to write a number as a product of its prime factors, in **index form**. For example, we can write 72 as a product of its prime factors, in index form, like this.

$$72 = 2^3 \times 3^2.$$

The index form is a product of terms of the form n^a . Each of these terms include a **base** and an **index**.



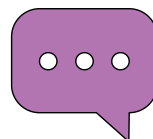
The base. This shows which number is getting multiplied in the term.

n^a

The index. This shows how many times the number n appears in the multiplication sum.

For example, we can write $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ as 7^{10} .

The **base** is 7, since 7 is the number which is being multiplied. The **index** is 10, since 7 appears 10 times.



Other examples

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$$

The number **5** is being multiplied. It appears **7** times.

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$$

The number **4** is being multiplied. It appears **6** times.

$$2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

The numbers **2** and **3** are being multiplied. **2** appears **3** times, and **3** appears **2** times.

Exercise 1

Write the following in index form.

(a) $3 \times 3 \times 3 \times 3 \times 3$

(b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

(c) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

(d) $3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5$

(e) $3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

(f) $3 \times 3 \times 3 \times 3 \times 3 \times 5$

(g) $2 \times 2 \times 7 \times 7 \times 7 \times 7 \times 9 \times 9 \times 9$

(h) $8 \times 8 \times 8 \times 33 \times 33 \times 33 \times 33$

(i) $3 \times 8 \times 8 \times 3 \times 8 \times 3 \times 3 \times 8$

(j) $3 \times 5 \times 5 \times 5 \times 3 \times 3 \times 7 \times 7$

(k) $2 \times 5 \times 7 \times 7 \times 2 \times 2 \times 2 \times 5$

(l) $13 \times 11 \times 7 \times 7 \times 11 \times 7$

Exercise 2

Write the following as multiplication sums without indices.

(a) 2^5

(b) 2^3

(c) 2^1

(d) 4^6

(e) 17^8

(f) 256^5

(g) $\left(\frac{1}{3}\right)^4$

(h) $2^4 \times 5^3$

(i) $4^4 \times 5^5$

(j) $24^3 \times 45^4$

(k) $\left(\frac{1}{5}\right)^3 \times \left(\frac{3}{4}\right)^3$

(l) $5^3 \times 13^2 \times 27^4$

(m) $3^2 \times 5^4 \times 10^2 \times 14^3$

(n) $2^3 \times \left(\frac{1}{2}\right)^3 \times 4^3$

(o) $\left(\frac{3}{7}\right)^2 \times \left(\frac{3}{4}\right)^4 \times \left(\frac{7}{9}\right)^3$

Investigation

Who was **Pierre de Fermat**? What was his contribution to mathematics? What was his last theorem? When was this theorem proved? Are there other theorems connected to index form?



Evaluating the Index Form

Evaluating index form is the process of writing a number written in index form as an ordinary number. For example, we can write 3^4 as $3 \times 3 \times 3 \times 3$, which is equal to 81.

**Other examples**

$$4^3 = 4 \times 4 \times 4 \\ = 64$$

$$5^4 = 5 \times 5 \times 5 \times 5 \\ = 625$$

$$2^4 + 7^3 = (2 \times 2 \times 2 \times 2) + (7 \times 7 \times 7) \\ = 16 + 343 \\ = 359$$

Exercise 3

Evaluate the following, without using a calculator.

(a) 3^4

(b) 6^3

(c) 10^5

(d) 2^9

(e) 20^4

(f) $3^2 + 2^5$

(g) $6^3 - 3^4$

(h) $6^3 \times 2^2$

(i) $10^4 \div 2^2$

(j) $5^4 + 4^4$

**Exercise 4**

Use your calculator to evaluate the following. If appropriate, write your answer correct to 2 decimal places.

(a) 125^2

(b) 17^4

(c) $29^3 + 5$

(d) $9^3 + 5$

(e) $12^4 - 5^6$

(f) $12^3 + 3^7$

(g) $3^4 \times 4^5$

(h) $2^3 \times 4^2 + 3^2$

(i) $(4^3)^4$

(j) $4^6 \div 2^6 + 10^3$

(k) $11^3 - 4^4$

(l) $4^5 - 5^6$

(m) $3^8 + 4^{10} - 5^6$

(n) $4^6 - 3^2 \times 8^3$

(o) $3^4 + 8^8 \div 4^{10}$

(p) $\frac{5^6}{3^7}$

(q) $\frac{4^4 + 3^6}{2^4}$

(r) $\frac{11^3}{2^5 \times 3^5}$

(s) $\frac{4^3 + 6^4 \times 2^3}{10^3 - 5^3}$

(t) $\left(\frac{4^3 + 6^4 \times 2^3}{10^3 - 5^3}\right)^3$

Exercise 5**Applying**

(a) Without using a calculator, calculate the numbers which fill the following spaces.

(i) $2^{12} = 4096$
 $2^{11} = \underline{\hspace{2cm}}$

(ii) $4^6 = 4096$
 $4^7 = \underline{\hspace{2cm}}$

(iii) $3^{11} = 177147$
 $3^{12} = \underline{\hspace{2cm}}$

(iv) $5^5 = 3125$
 $5^4 = \underline{\hspace{2cm}}$

(v) $8^3 = 512$
 $8^4 = \underline{\hspace{2cm}}$

(vi) $2^8 = 256$
 $2^{10} = \underline{\hspace{2cm}}$

(vii) $6^4 = 1296$
 $6^6 = \underline{\hspace{2cm}}$

(viii) $3^7 = 2187$
 $3^9 = \underline{\hspace{2cm}}$

(ix) $5^9 = 1953125$
 $5^7 = \underline{\hspace{2cm}}$

(x) $7^4 = 2401$
 $7^6 = \underline{\hspace{2cm}}$

(b) Put the numbers 1 to 6 into the following boxes to make:

$$\begin{array}{|c|} \hline \square \\ \hline \square \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \end{array}$$

- (i) The largest possible number;
(ii) The smallest possible number;
(iii) A total of 147.



(c) Put the numbers 1 to 6 into the following boxes to make the calculation correct.

$$\begin{array}{|c|} \hline \square \\ \hline \square \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}$$

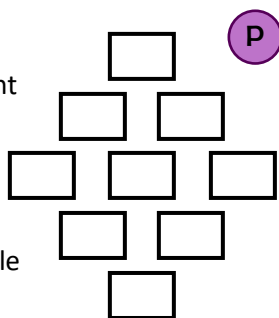
(d) Write any whole numbers in the following boxes to make the calculation correct. How many possible solutions are there?

$$\begin{array}{|c|} \hline \square \\ \hline \square \end{array} = 64$$



Exercise 6

Write the digits 1 to 9 in the grid on the right so that each row (reading across) is a square number. You may use each digit once only.



Can you prove that there is only one possible solution?

Rules of Indices

When performing calculations in index form, we notice several different patterns. The **rules of indices** write these patterns in a convenient way.

The Multiplication Rule

$$n^a \times n^b = n^{a+b}$$

When multiplying one number or variable to an index, by the **same** number or variable to another index, we must **add** the indices. We can see below why this is true.

$$8^4 \times 8^3 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \\ = 8^7.$$

Other examples

$$4^3 \times 4^6 = 4^{3+6} \\ = 4^9$$

$$8^2 \times 8^9 = 8^{2+9} \\ = 8^{11}$$

$$a^5 \times a^6 \times a^4 = a^{5+6+4} \\ = a^{15}$$

$$7^9 \times 7^{-3} = 7^{9+(-3)} \\ = 7^6$$

Exercise 8

Simplify each of the following expressions.

(a) $7^5 \times 7^3$

(b) $7^3 \times 7^5$

(c) $8^5 \times 8$

(d) $x^5 \times x^3$

(e) $7^4 \times 7^8$

(f) $a^5 \times a^3$

(g) $a^5 \times a^7 \times a^{10}$

(h) $3^2 \times 3^9 \times 3^4$

(i) $5^5 \times 5^2 \times 5^{12}$

(j) $y^3 \times y^{13} \times y^{16}$

(k) $7^{15} \times 7^{-4}$

(l) $14^9 \times 14^{-6}$

(m) $8^{-10} \times 8^3$

(n) $d^5 \times d^{-8}$

(o) $f^{-4} \times f^{-3}$

(p) $i^{-5} \times i^{11} \times i^{-3}$

(q) $p^{-9} \times p^{-2} \times p^5$

(r) $4^{-17} \times 4^{-7} \times 4^{31}$

(s) $(-5)^5 \times (-5)^3$

(t) $(-5)^5 \times (-5)^{-3}$

(u) $(-5)^{-5} \times (-5)^{-3}$

(v) $a^3 \times a^{\frac{1}{2}}$

(w) $a^{\frac{3}{5}} \times a^{\frac{1}{5}}$

(x) $a^{\frac{2}{3}} \times a^{\frac{4}{7}}$

(y) $a^{\frac{8}{3}} \times a^{\frac{5}{4}}$

Exercise 9

Find the missing numbers that go into the boxes in all of the following questions.

(a) $7^5 \times 7^{\square} = 7^8$

(b) $7^{\square} \times 7^4 = 7^6$

(c) $7^{13} \times 7^{\square} = 7^{11}$

(d) $7^8 \times 7^4 = 7^{\square}$

(e) $x^2 \times x^{\square} = x^{14}$

(f) $5^5 \times 5^{\square} = 5^6$

(g) $4^9 \times 4^{\square} = 4^8$

(h) $11^{\square} \times 11^{10} = 11^7$

(i) $2^{\square} \times 2^{-5} = 2^9$

(j) $8^{-2} \times 8^{\square} = 8^{-9}$

Challenge!

An **Armstrong Number** is a whole number where the sum of the digits, each raised to the number of digits, is equal to the original number.

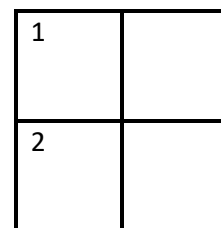
For example, the number 371 is an Armstrong number since $3^3 + 7^3 + 1^3 = 371$.

1,634 is also an Armstrong number since $1^4 + 6^4 + 3^4 + 4^4 = 1,634$.

How many Armstrong numbers are there between 1 and 10,000?

Exercise 7

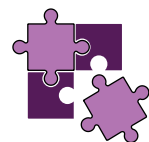
Complete the following crossnumber.

**Clues:****Across**

1. Cube number
2. Cube number

Down

1. One less than a cube number



2

Skill



Extension



The Division Rule



$$n^a \div n^b = n^{a-b} \text{ or } \frac{n^a}{n^b} = n^{a-b}$$

When dividing one number or variable to an index, by the **same** number or variable to another index, we must **subtract** the indices. We can see below why this is true.

$$\begin{aligned} \frac{7^8}{7^5} &= \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7} \\ &= \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7} \\ &= 7 \times 7 \times 7 \\ &= 7^3. \end{aligned}$$



Other Examples

$$\begin{aligned} 4^6 \div 4^3 &= 4^{6-3} \\ &= 4^3 \end{aligned}$$

$$\begin{aligned} 8^9 \div 8^2 &= 8^{9-2} \\ &= 8^7 \end{aligned}$$

$$\begin{aligned} \frac{a^7}{a^4} &= a^{7-4} \\ &= a^3 \end{aligned}$$

$$\begin{aligned} 6^5 \div 6^{-2} &= 6^{5-(-2)} \\ &= 6^7 \end{aligned}$$

Exercise 10

Simplify each of the following expressions.

(a) $7^5 \div 7^3$

(b) $7^3 \div 7^5$

(c) $7^{11} \div 7$

(d) $7^8 \div 7^4$

(e) $x^5 \div x^3$

(f) $a^5 \div a^3$

(g) $a^5 \div a^7$

(h) $\frac{3^9}{3^2}$

(i) $\frac{5^{15}}{5^{12}}$

(j) $\frac{y^3}{y^{16}}$

(k) $7^{15} \div 7^{-4}$

(l) $14^9 \div 14^{-6}$

(m) $8^{-10} \div 8^3$

(n) $d \div d^{-8}$

(o) $f^{-4} \div f^{-3}$

(p) $i^{-5} \div i^{-3}$

(q) $p^{-9} \div p^{-2} \div p^5$

(r) $4^{-17} \div 4^{-7} \div 4^{31}$

(s) $(-5)^5 \div (-5)^3$

(t) $(-5)^5 \div (-5)^{-3}$

(u) $(-5)^{-5} \div (-5)^{-3}$

(v) $a^3 \div a^{\frac{1}{2}}$

(w) $a^{\frac{3}{5}} \div a^{\frac{1}{5}}$

(x) $a^{\frac{2}{3}} \div a^{\frac{4}{7}}$

(y) $a^{\frac{8}{3}} \div a^{\frac{5}{4}}$

Exercise 11

Find the missing numbers that go into the boxes in all of the following questions.

(a) $7^5 \div 7^{\square} = 7^2$

(b) $7^{\square} \div 7^4 = 7^6$

(c) $7^{13} \div 7^{\square} = 7^{11}$

(d) $7^6 \div 7^4 = 7^{\square}$

(e) $x^{15} \div x^{\square} = x^7$

(f) $5^5 \div 5^{\square} = 5^7$

(g) $\frac{4^9}{4^{\square}} = 4^5$

(h) $\frac{11^{30}}{11^{\square}} = 11^{14}$

(i) $2^{\square} \div 2^{-5} = 2^{-2}$

(j) $8^{-2} \div 8^{\square} = 8^{-9}$

Example

$$\begin{aligned} 3^7 \times 3^5 \div 3^2 &= 3^{7+5} \div 3^2 \\ &= 3^{12} \div 3^2 \\ &= 3^{12-2} \\ &= 3^{10} \end{aligned}$$

$$\begin{aligned} 3^9 \div 3^2 \times 3^5 &= 3^{9-2} \times 3^5 \\ &= 3^7 \times 3^5 \\ &= 3^{7+5} \\ &= 3^{12} \end{aligned}$$

$$\begin{aligned} \frac{y^7}{y^4} \times y^{10} &= y^{7-4} \times y^{10} \\ &= y^3 \times y^{10} \\ &= y^{3+10} \\ &= y^{13} \end{aligned}$$

Exercise 12

Simplify each of the following expressions.

(a) $3^5 \times 3^6 \div 3^2$

(b) $6^8 \times 6^6 \div 6^7$

(c) $7^{11} \times 7^5 \div 7^8$

(d) $5^8 \times 5^3 \div 5^4$

(e) $x^9 \times x^3 \div x^4$

(f) $a^5 \times a^4 \div a^8$

(g) $a^{10} \times a^{-5} \div a^7$

(h) $2^{13} \div 2^3 \times 2^4$

(i) $5^7 \div 5^2 \times 5^4$

(j) $8^9 \div 8 \times 8^3$

(k) $5^8 \times 5^3 \times 5^7 \div 5^4$

(l) $4^9 \div 4^2 \times 4^3 \div 4^5$

(m) $8^{-4} \times 8^3 \times 8^6$

(n) $d^5 \div d^{-8} \times d^4$

(o) $u^{-3} \div u^{-3} \times u^{-3}$

(p) $\frac{10^6}{10^2} \times 10^8$

(q) $\frac{7^9}{7^3} \times 7^2$

(r) $19^4 \times \frac{19^{12}}{19^3}$

(s) $4^9 \div \frac{4^6}{4^2}$

(t) $e^8 \times \frac{e^6}{e^3}$

(u) $\frac{4^5 \times 4^7}{4^3}$

(v) $\frac{15^7 \times 15}{15^2}$

(w) $\frac{6^{10}}{6^2 \times 6^5}$

(x) $\frac{r^6 \times r^{-1}}{r^{-2}}$

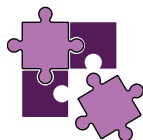
(y) $\frac{q^{-3}}{q^4 \times q^{-8}} \times q^3$

Raising a Power to Another Index

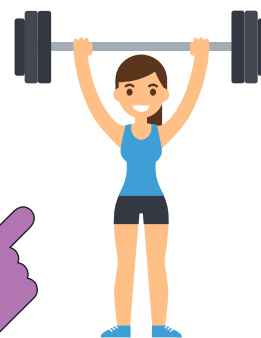
$$(n^a)^b = n^{a \times b}$$



If a power of the form n^a is raised to another index, then we need to **multiply** the indices. We can see below why this is true.



$$\begin{aligned}(5^3)^4 &= \overbrace{5^3 \times 5^3 \times 5^3 \times 5^3}^{4 \text{ times}} \\ &= 5^{3+3+3+3} \\ &= 5^{12} \\ &= 5^{3 \times 4}\end{aligned}$$



Exercise 13

2

Simplify the following, giving your answers in index form.

- | | | | | |
|--------------------------|------------------------|---------------------------|---|-----------------------------|
| (a) $(5^2)^4$ | (b) $(5^4)^2$ | (c) $(7^2)^4$ | (d) $(x^2)^4$ | (e) $((-5)^2)^4$ |
| (f) $(5^{-2})^4$ | (g) $(5^2)^{-4}$ | (h) $(5^{-2})^{-4}$ | (i) $\left(\left(\frac{1}{5}\right)^2\right)^4$ | (j) $(5^2)^0$ |
| (k) $(6^3)^6$ | (l) $(11^{25})^3$ | (m) $(2^7)^8$ | (n) $(43^5)^{-7}$ | (o) $(10^{-3})^{-9}$ |
| (p) $(9^0)^6$ | (q) $(0.3^6)^9$ | (r) $(11^{-9})^7$ | (s) $((-3)^{-4})^{-3}$ | (t) $(y^{14})^3$ |
| (u) $(5^2)^4 \times 5^6$ | (v) $(3^6)^2 \div 3^4$ | (w) $(8^5)^2 \div 8^{10}$ | (x) $6 \times (6^5)^3$ | (y) $(4^{12})^5 \times 4^0$ |

Algebra and Rules of Indices

Multiply the numbers, add the indices.

Divide the numbers, subtract the indices.

Example

$$4x^2y^3 \times 3x^4y^5 = 12x^6y^8$$

$$20a^8b^6 \div 4a^2b^3 = 5a^6b^3$$

$$\frac{42p^{12}q^{15}}{6p^3q^5} = 7p^9q^{10}$$



Exercise 14

Simplify the following algebraic expressions.

- | | | |
|-------------------------------------|-------------------------------------|--|
| (a) $2x^3y^4 \times 3x^4y^2$ | (b) $8a^5b^3 \times 4a^3b^6$ | (c) $9p^5q^3 \times 3p^4q$ |
| (d) $16x^{10}y^{12} \div 2x^2y^4$ | (e) $24a^6b^{15} \div 4a^2b^3$ | (f) $80p^{32}q^{20} \div 10p^4q^{10}$ |
| (g) $\frac{8x^{14}y^{10}}{2x^2y^2}$ | (h) $\frac{28a^{16}b^4}{7a^4b}$ | (i) $\frac{100p^4q^8}{25p^2q^4}$ |
| (j) $4g^5h^3 \times -2g^5h^3$ | (k) $6s^4t^6 \times 5s^{-2}t^3$ | (l) $-3u^{-5}v^7 \times -9u^3v^{-2}$ |
| (m) $25c^8d^{-12} \div 5c^2d^3$ | (n) $\frac{-32e^{-4}f^{10}}{2ef^2}$ | (o) $\frac{84x^5y^{-14}z}{2x^{-2}y^2z^{-2}}$ |

Evaluation

Skill

23

Key words	Corrections	I am happy with...	I need to revise...

Plotting Graphs

In year 9, you learnt how to plot linear equations of the form $y = mx + c$. In this chapter, we will learn how to plot equations that have different forms.

Exercise 15

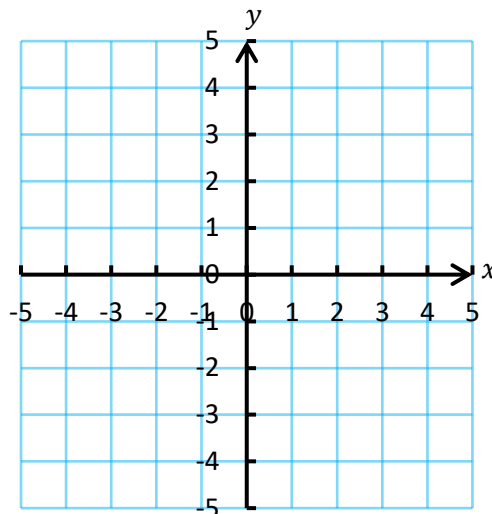
On the graph paper on the right, plot the equation $y = 2x - 3$.

x	0	1	2	3
y				



Revision

2

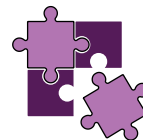


Quadratic Graphs

Equations of the form $y = ax^2 + bx + c$ are **quadratic** equations.

The word quadratic refers to the x^2 term in the equation, a **squared** term.

Quadratic graphs have a U or $\cap$ shape.



Skill

2

Exercise 16

Complete the following table to note whether each equation is linear or quadratic.

Equation	Type	Equation	Type
$y = 4x - 2$	Linear	$y = 3x^2 - 4x + 2$	Quadratic
$y = 2x^2 + 4x + 7$		$y = 5x^2 - 4x + 6$	
$y = 2x + 5$		$y = -4x + 2$	
$y = 4x + 7x^2 - 3$		$y = 2 + 6x + 3x^2$	
$y = 3 + 4x$		$y = 6x$	
$y = 5$		$y = 4 + 3x^2$	
$y = -3x^2 + 3$		$y = x(x + 2)$	

Exercise 17

- | | | | |
|------------------|--------------------|------------------------|----------------------------|
| (a) 4×4 | (b) -4×-4 | (c) 4×-4 | (d) -4×4 |
| (e) $4 + 2$ | (f) $4 + -2$ | (g) $-4 + 2$ | (h) $-4 + -2$ |
| (i) $4 - 2$ | (j) $-4 - 2$ | (k) $4 - -2$ | (l) $-4 - -2$ |
| (m) 7^2 | (n) $(-7)^2$ | (o) $7^2 + 2 \times 7$ | (p) $(-7)^2 + 2 \times -7$ |

Revision

A



Substitution

To plot a quadratic graph, we can **substitute** values into the associated equation.



2

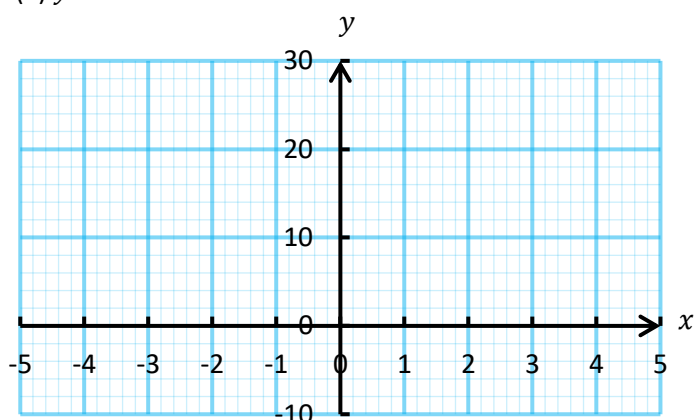
Exercise 18

Complete the following tables by substitution, before plotting a smooth curve on the graph paper.

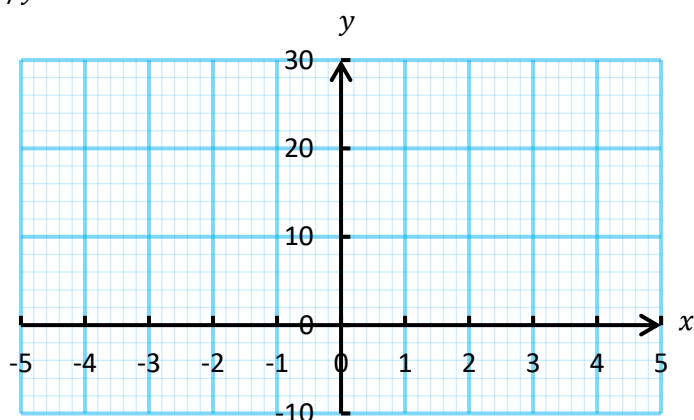
(No calculators are allowed.)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
x^2											
$x^2 - 5$											

(a) $y = x^2$

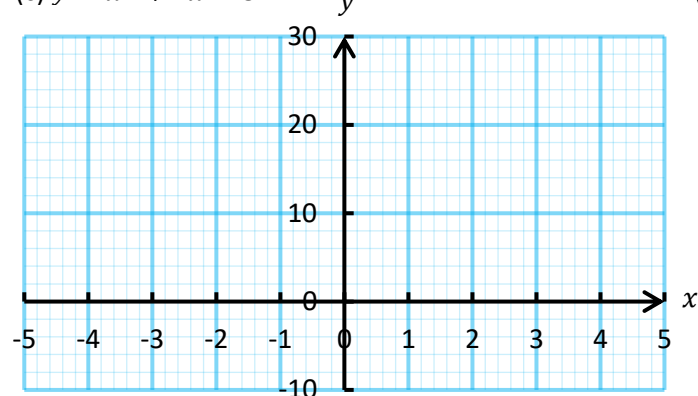


(b) $y = x^2 - 5$

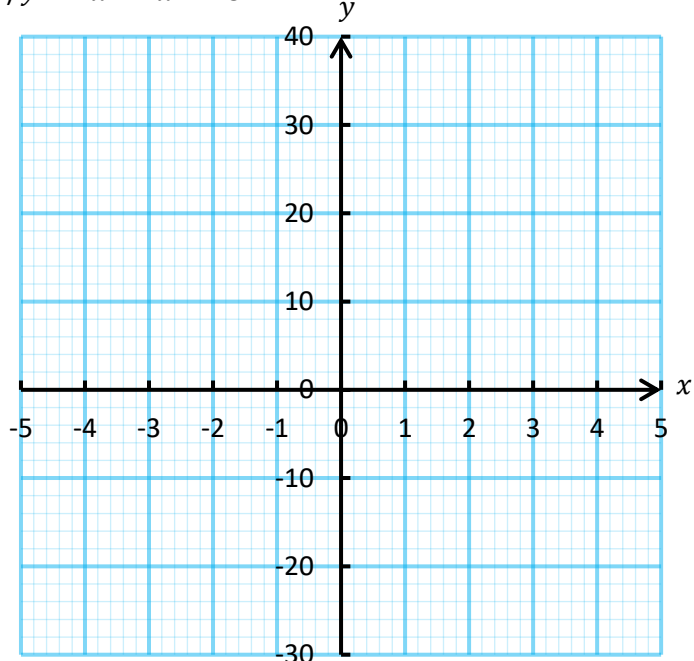


x	-5	-4	-3	-2	-1	0	1	2	3	4	5
x^2											
$2x$											
$x^2 + 2x - 8$											
x^2											
$2x^2$											
$2x^2 - x - 28$											

(c) $y = x^2 + 2x - 8$



(d) $y = 2x^2 - x - 28$



Exercise 19

Fill the blanks in the tables below. Then, in your books, plot suitable graphs for the equations.

(a)	x	-5	-4	-3	-2	-1	0	1	2	3	4	5
	$y = x^2 + 4$	29	20		8	5	4	5	8	13		29

(b)	x	-5	-4	-3	-2	-1	0	1	2	3	4	5
	$y = 3x^2 - 10$	65	38	17		-7	-10	-7		17	38	65

(c)	x	-5	-4	-3	-2	-1	0	1	2	3	4	5
	$y = 4x^2 + x - 7$	88		26	7	-4	-7	-2	11		61	98

Exercise 20

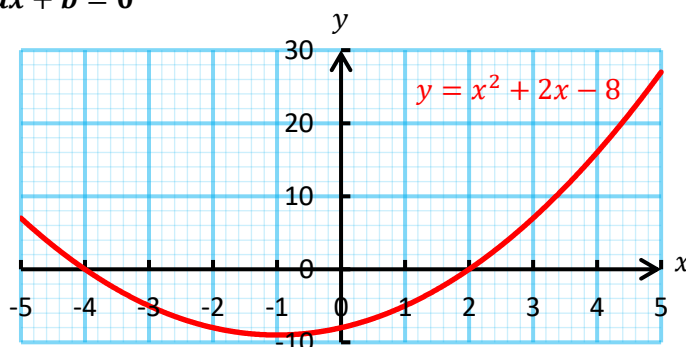
2

Plot appropriate graphs for the following equations.

- | | | |
|-------------------------|------------------------|--------------------------|
| (a) $y = x^2 - 4x$ | (b) $y = x^2 - 3x + 4$ | (c) $y = 3x - x^2$ |
| (d) $y = x^2 - x - 5$ | (e) $y = 5 - 2x^2$ | (f) $y = 3x^2 + 4x + 2$ |
| (g) $y = 15 - x^2 + 3x$ | (h) $y = 4x^2 - x + 7$ | (i) $y = -2x^2 + 5x - 6$ |

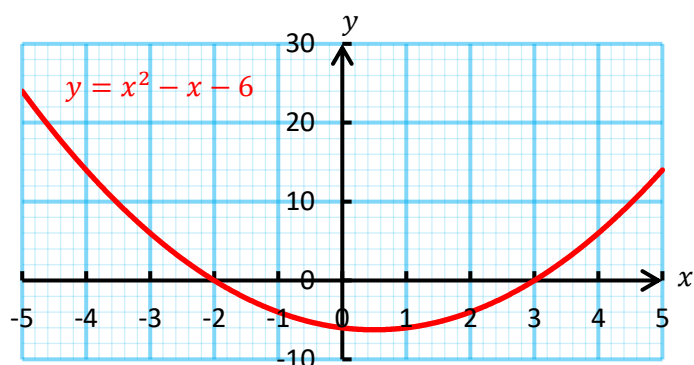
Graphical method of solving equations of the form $x^2 + ax + b = 0$

Consider the graph shown on the right for the equation $y = x^2 + 2x - 8$. To solve the equation $x^2 + 2x - 8 = 0$, we can use the graph to see where the graph crosses the x -axis (this is where the function $x^2 + 2x - 8$ is zero). We see that the graph crosses the x -axis at the points where $x = -4$ and $x = 2$, therefore the solutions to the equation $x^2 + 2x - 8 = 0$ are $x = -4$ and $x = 2$.

**Exercise 21**

2

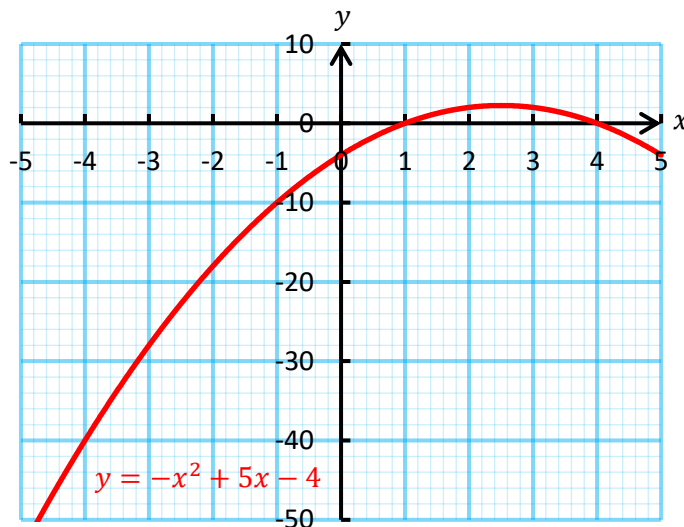
(a) Use the graph below to solve the equation $x^2 - x - 6 = 0$.



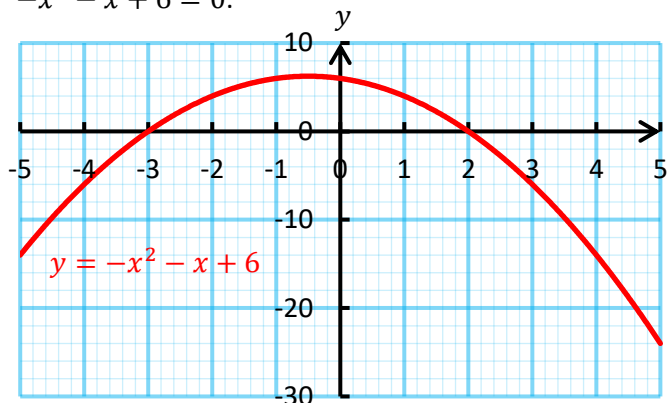
(c) Use the graph above to solve the equation $x^2 - x - 6 = 10$.

Give your answers correct to one decimal place.

(b) Use the graph below to solve the equation $-x^2 + 5x - 4 = 0$.



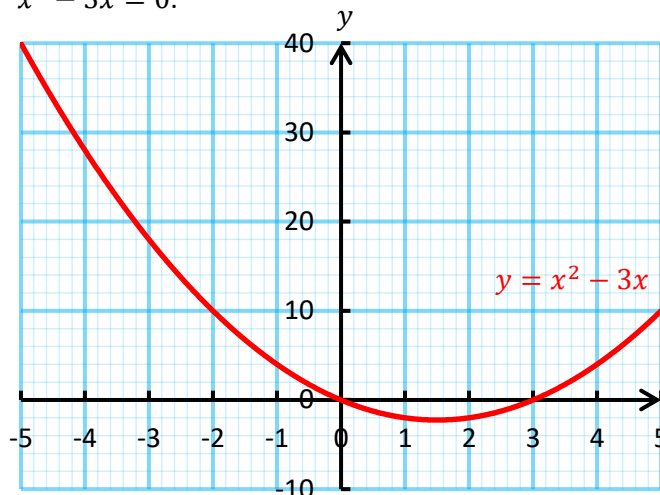
(d) Use the graph below to solve the equation $-x^2 - x + 6 = 0$.



(f) Use the graph above to solve the equation $-x^2 - x + 6 = -10$.
Give your answers correct to one decimal place.

- (h) (i) By drawing a suitable graph, solve the equation $x^2 + 3x - 4 = 0$.
(ii) Use your graph from part (i) to solve the equation $x^2 + 3x - 4 = 5$.
Give your answers correct to one decimal place.

(e) Use the graph below to solve the equation $x^2 - 3x = 0$.

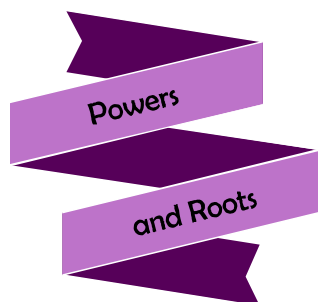


(g) Use the graph above to solve the equation $x^2 - 3x = 4$.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...



Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I know how to change between ordinary numbers and numbers written in index form .			1	
I know how to use the multiplication rule $n^a \times n^b = n^{a+b}$.			2	
I know how to use the division rule $n^a \div n^b = n^{a-b}$.			2	
I know how to use the rule where a power is raised to another index , $(n^a)^b = n^{a \times b}$.			2	
I can simplify algebraic expressions using the rules of indices.			3	
I can plot quadratic graphs .			4	
I know how to use a graphical method to solve quadratic equations.			4	



The Mathematics Department

10

Measuring

Shapes 3

Foundation Tier

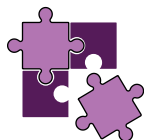
Version
3.0
08/2025

Name:

Contents

Chapter	Mathematics	Page Number
Transformations: Enlargement	Positive scale factor. Fractional scale factor. Centre of enlargement. Finding the centre of enlargement.	3
Perimeter and Area of Composite Shapes	Composite shapes.	7





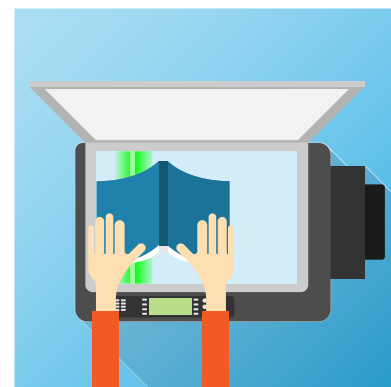
Enlargement



Enlargement is one of the four transformations.

Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	Enlargement

When a shape is enlarged, the **size** of the shape changes. The **scale factor** decides how the shape changes. For example, if the scale factor is 2, then the size of the shape doubles. If the scale factor is $\frac{1}{2}$, then the size of the shape halves.

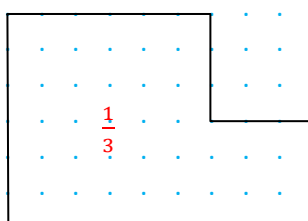
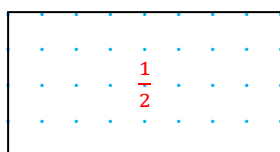
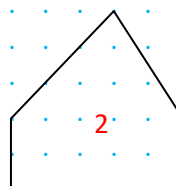
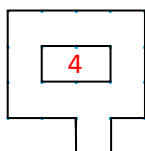
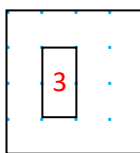
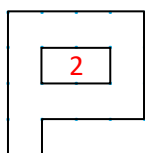


Skill

Exercise 1

Enlarge the following shapes using the scale factor that is given in the centre of each shape.

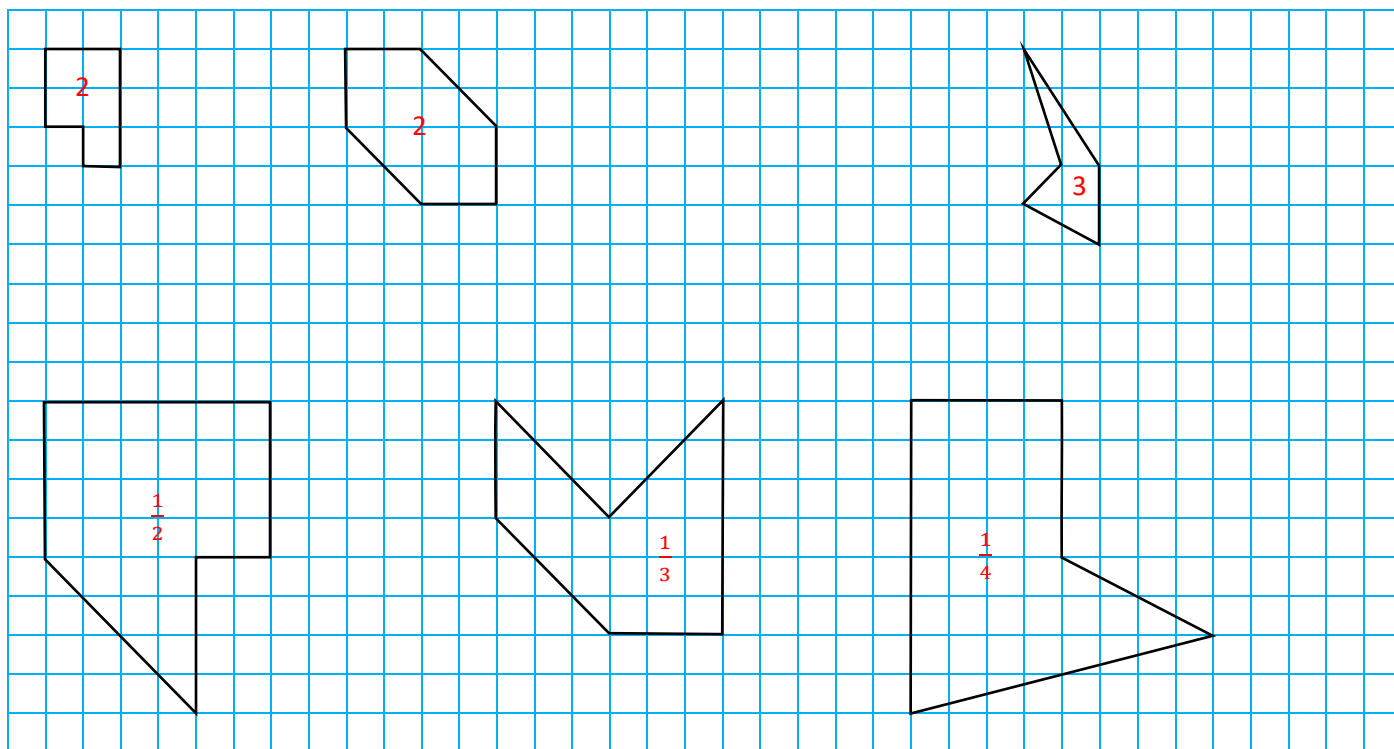
2



Exercise 2



Enlarge the following shapes using the scale factor that is given in the centre of each shape.



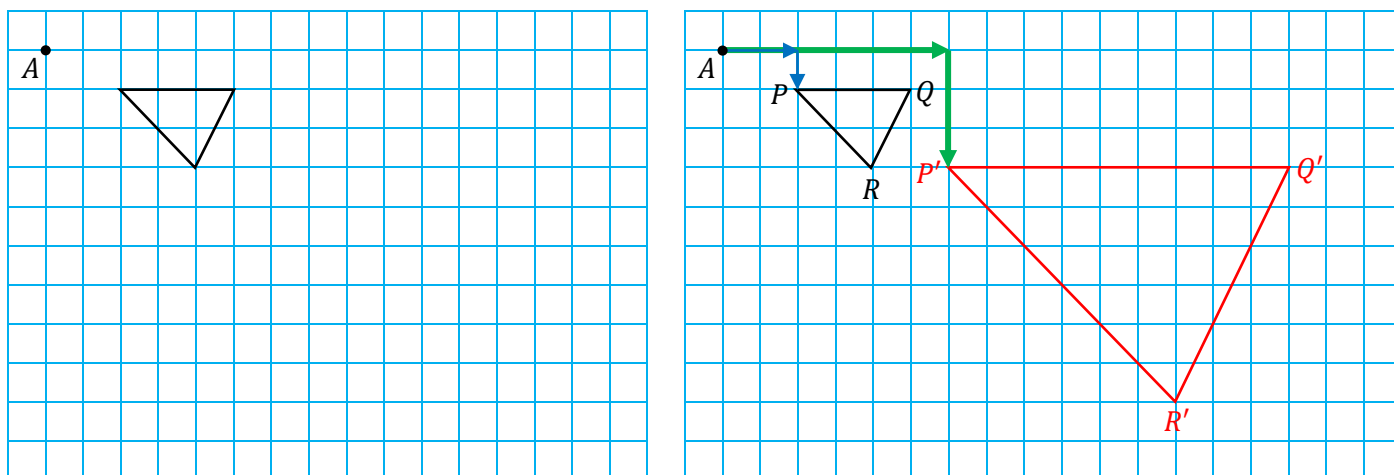
Centre of Enlargement

If a question states a point as the **centre of enlargement**, then the enlargement must appear in a certain location (it cannot appear *anywhere* like in Exercises 11 and 12).

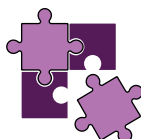


Example

Enlarge the following triangle using a scale factor of 3 and the point A as the centre of enlargement.

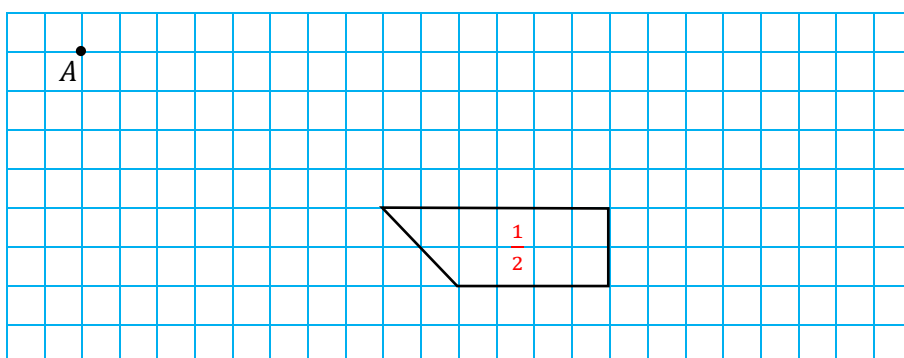
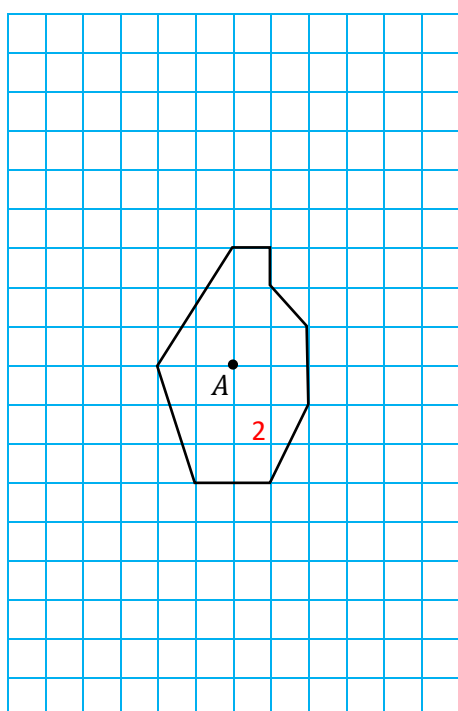
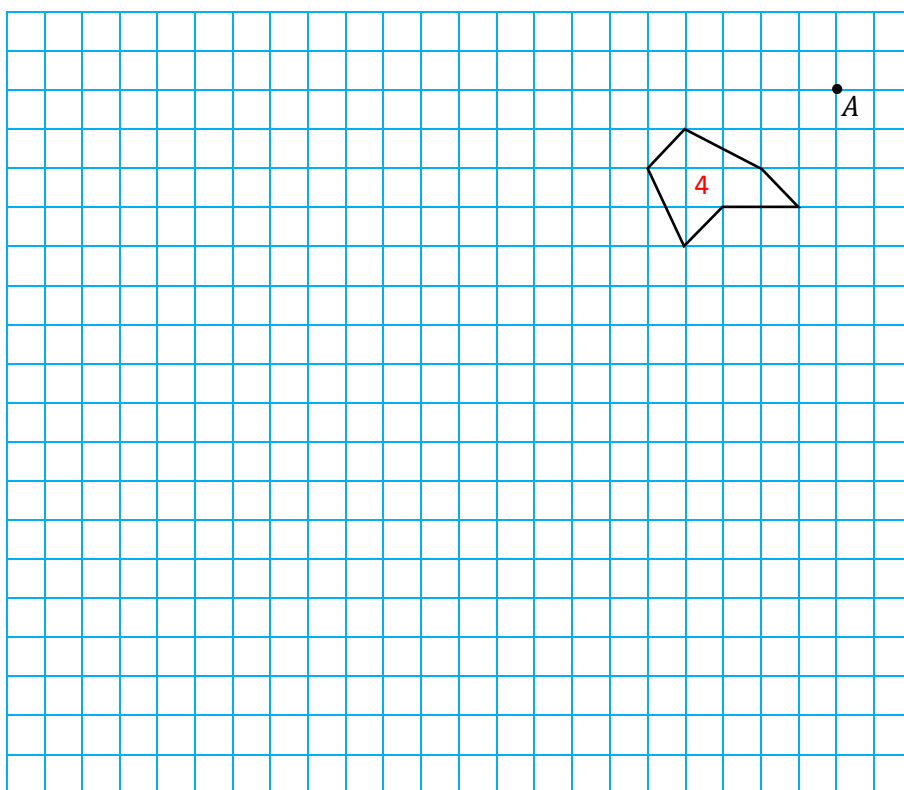
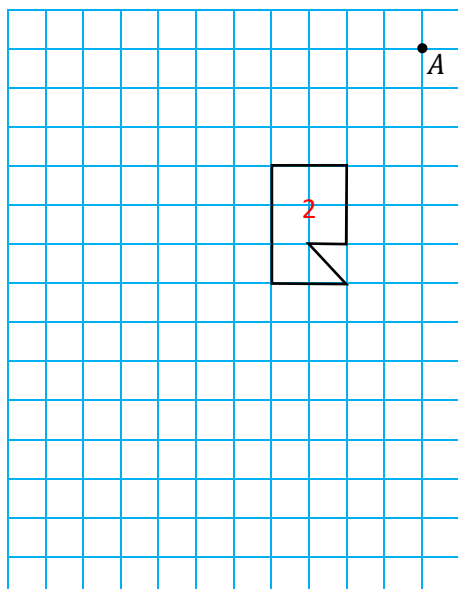
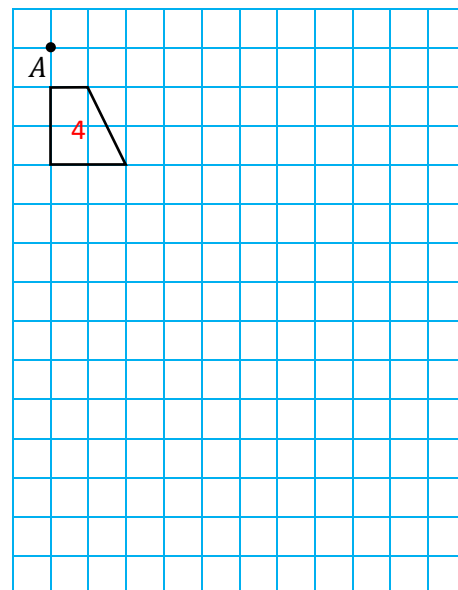
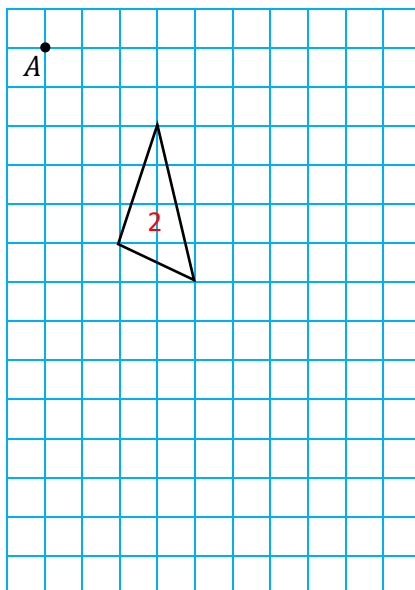
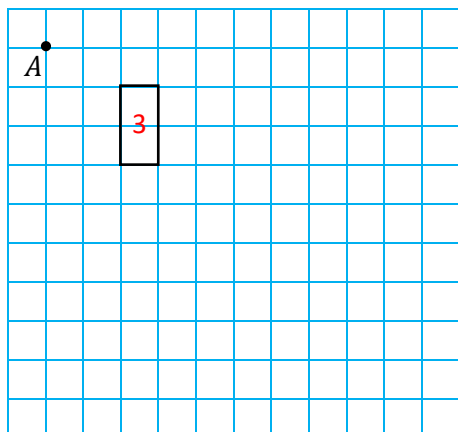


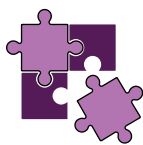
To go from the point A to the vertex P in the original triangle, we must go **2 units to the right and 1 unit down**. Since the scale factor is 3, to go from the point A to the vertex P' in the new triangle, we must go **$2 \times 3 = 6$ units to the right, and $1 \times 3 = 3$ units down**. We can repeat this with the other vertices (Q and R), or you can start at the vertex P' and draw a triangle that is three times larger.



Exercise 3

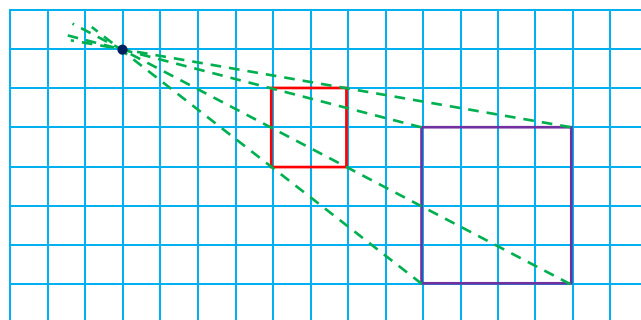
Enlarge the following shapes using the point A as the centre of enlargement and the number in the centre of each shape as the scale factor.



Finding the centre of enlargement

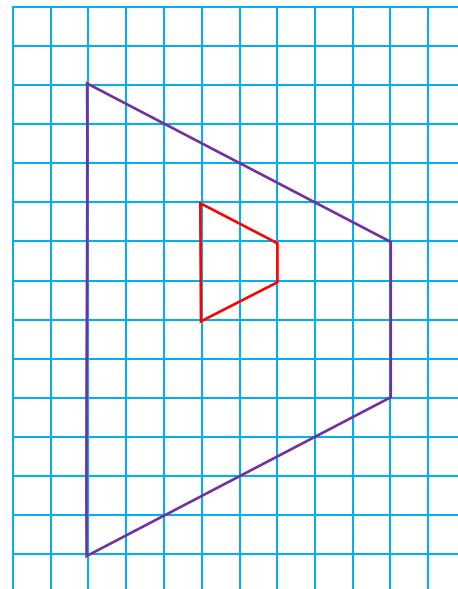
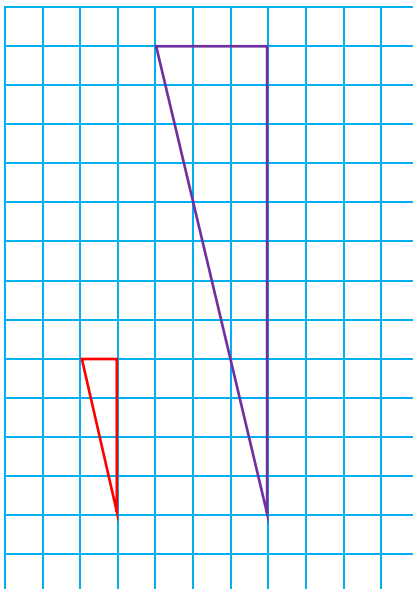
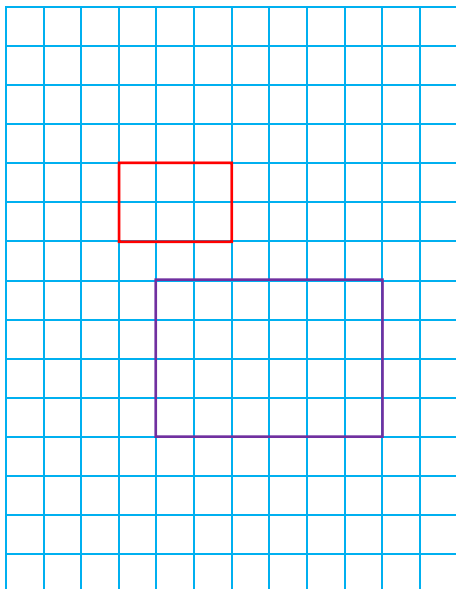
Given a **shape** and its **enlargement**, **connect the corresponding vertices** and **extend the lines** to find the location of the **centre of enlargement**.

You can find the scale factor by comparing the sizes of the shapes.

**Exercise 4**

Find the scale factor and the centre of enlargement for the following enlargements.

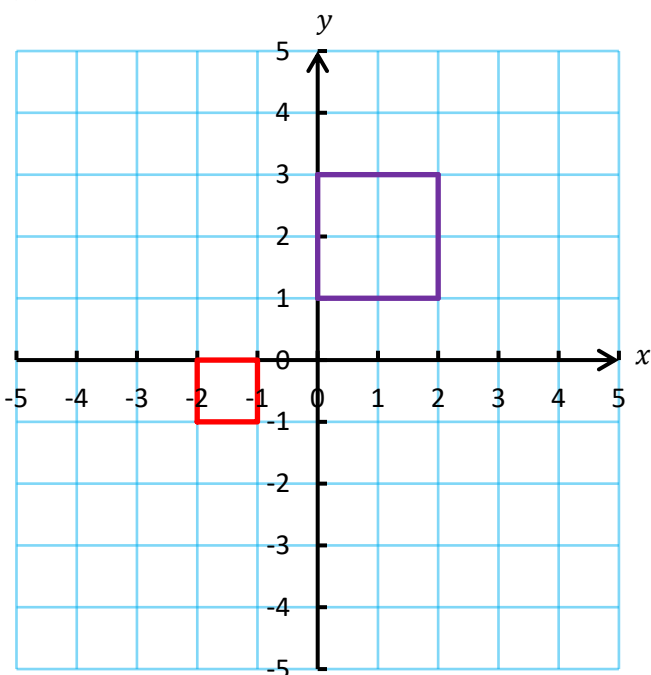
2

**Exercise 5**

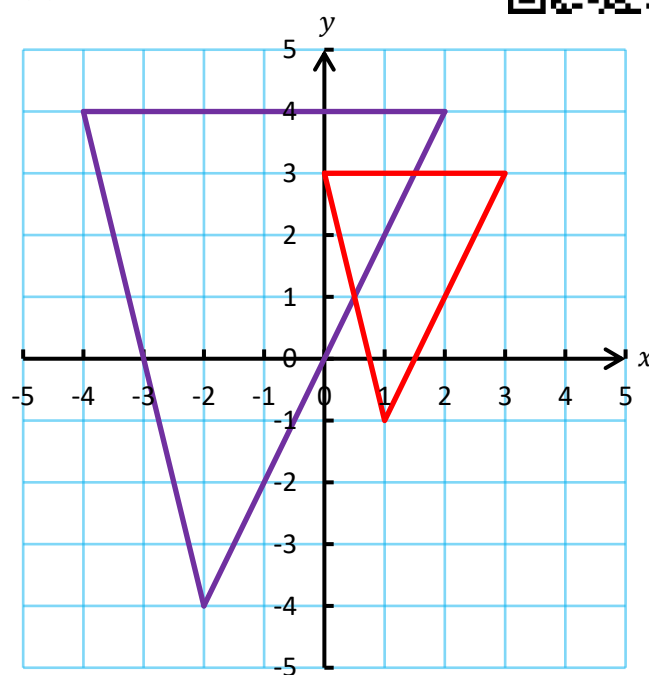
Find the scale factor and the centre of enlargement for the following enlargements.



(a)



(b)



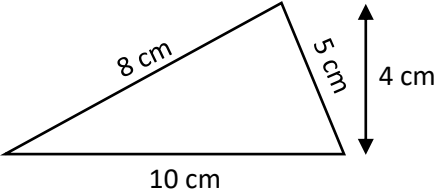
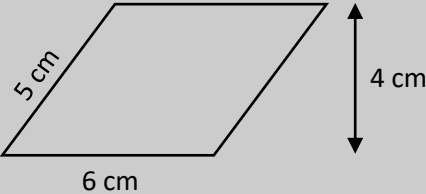
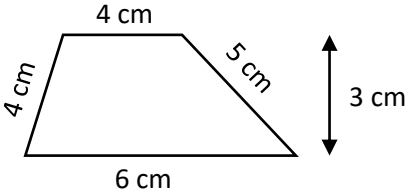
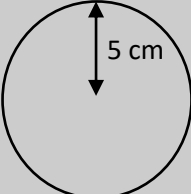
Composite Shapes

Revision

13

Exercise 6

Complete the following table.

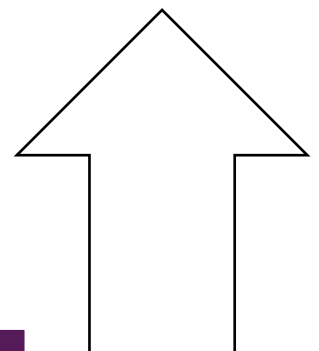
Shape	Name of the shape	Formula to find the area of the shape	Calculate the area of the shape
			
			
			
			
			

Exercise 7

Calculate the perimeter of each shape in the above table.

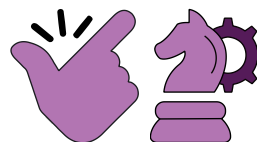
Composite Shapes

Composite shapes are shapes that you can split into simpler shapes, like the shapes in the above table. For example, the shape on the right is a composite shape – it is possible to split the shape into a rectangle (at the bottom) and a triangle (at the top). We can calculate the area of the composite shape by adding the area of the rectangle to the area of the triangle.

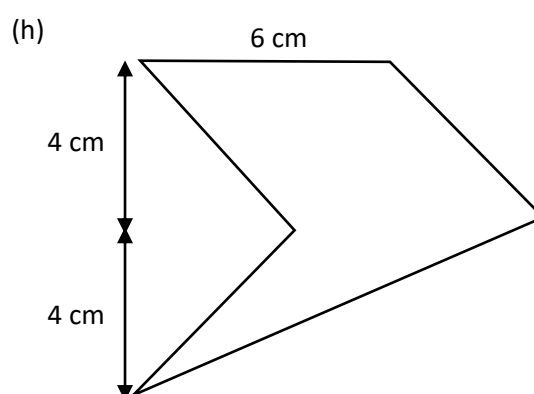
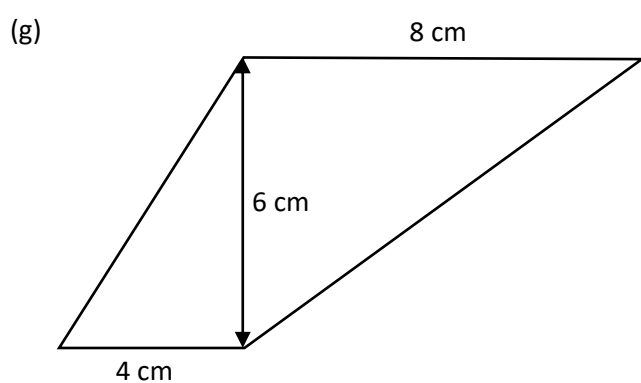
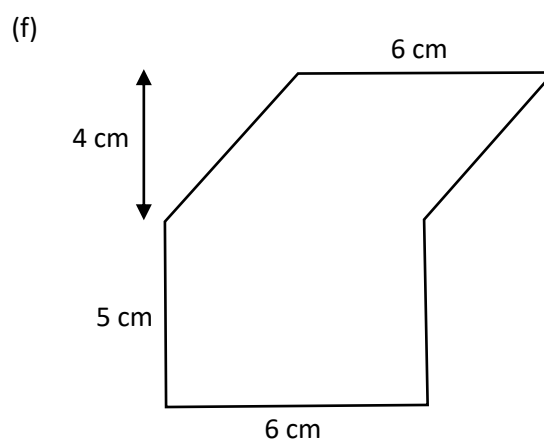
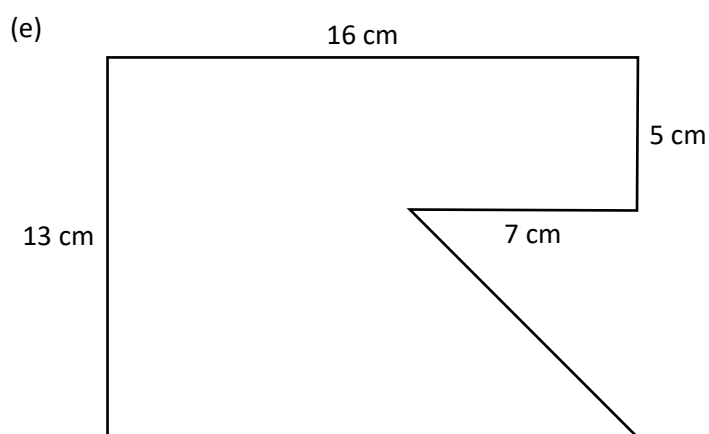
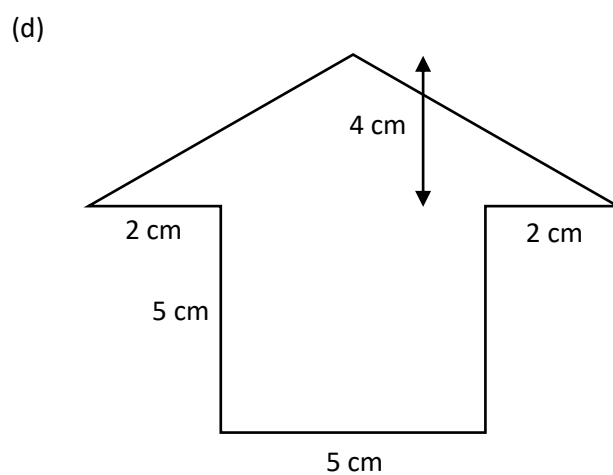
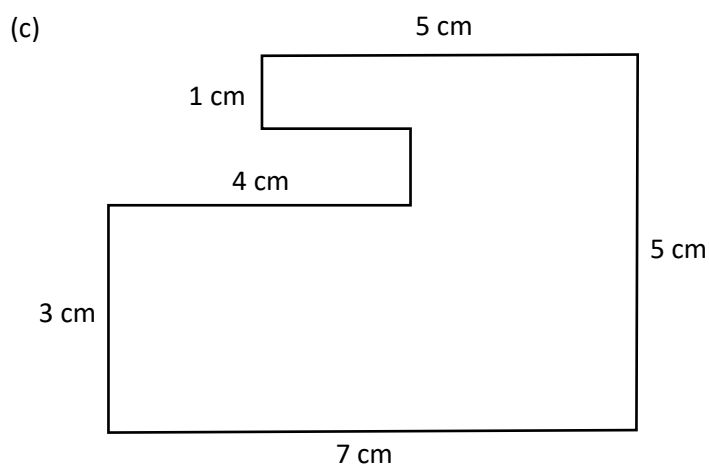
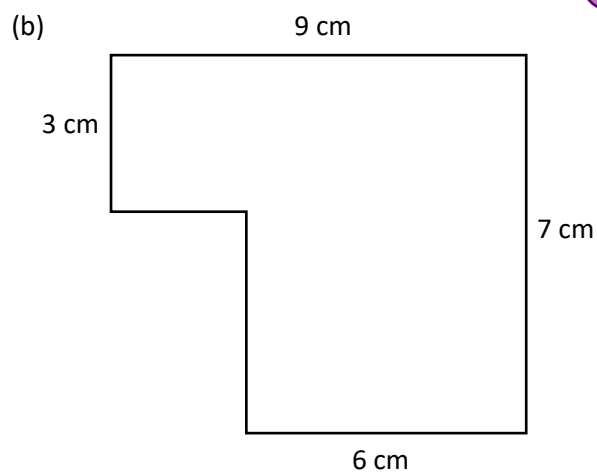
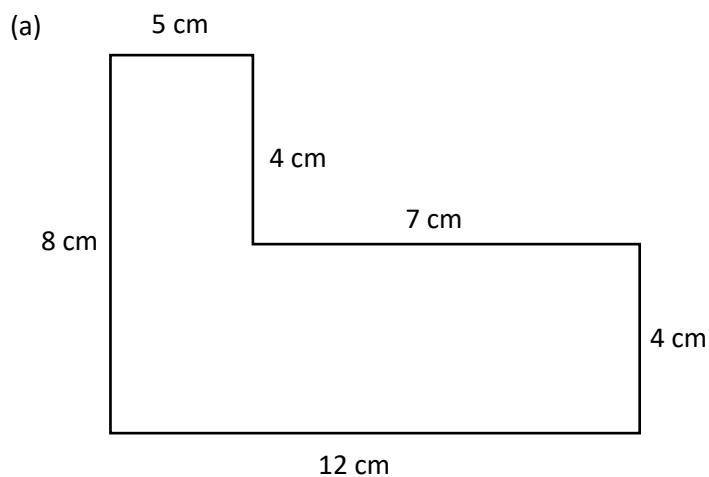


Exercise 8

Calculate the area of each of the following composite shapes.

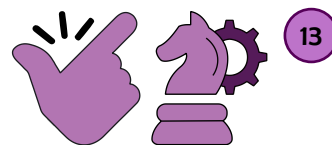


13

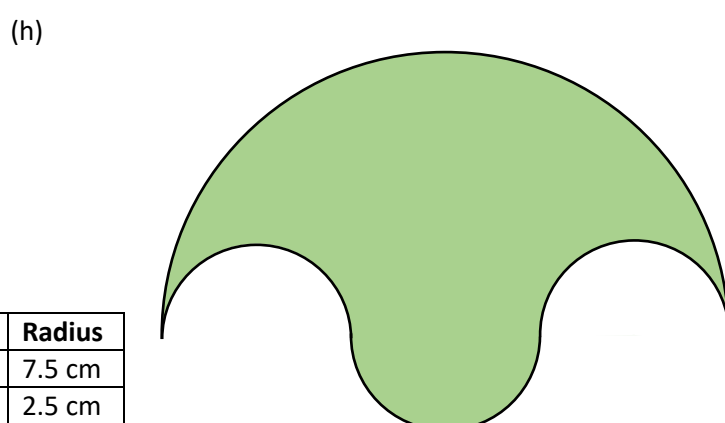
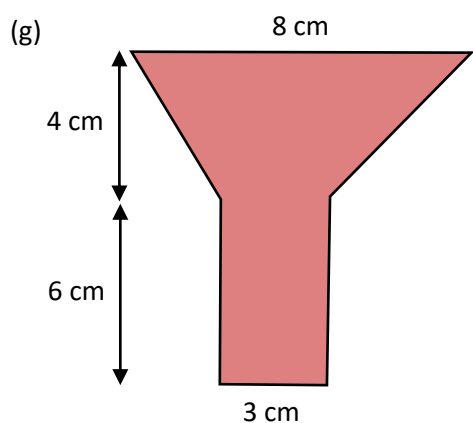
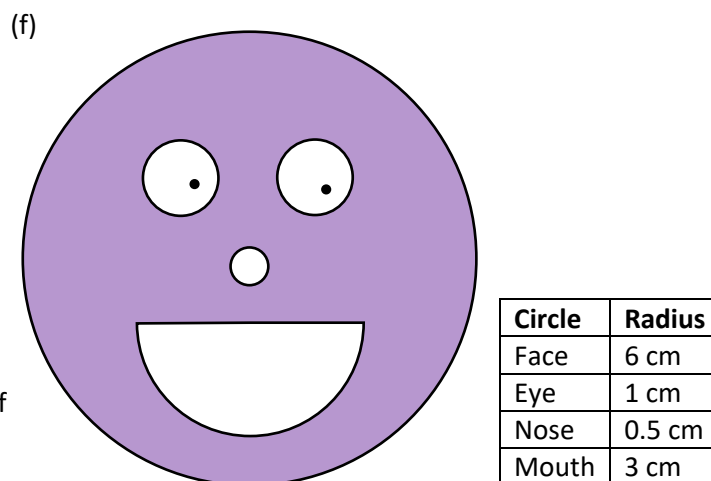
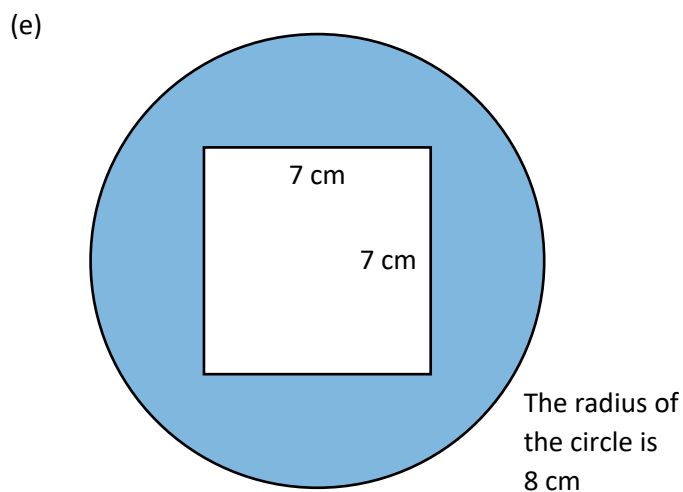
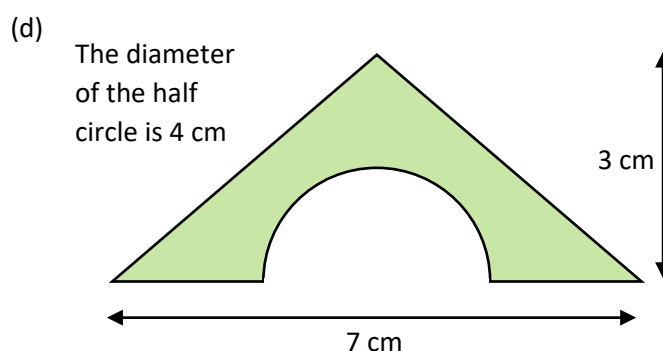
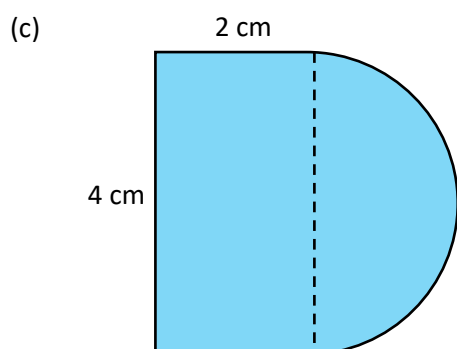
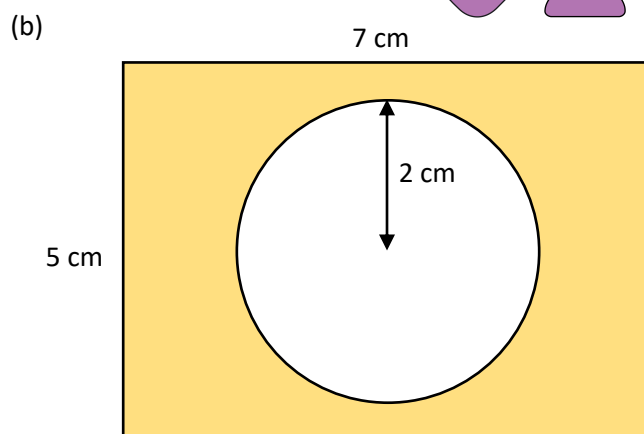
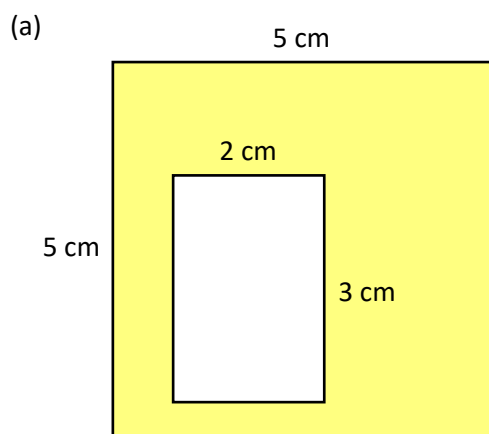


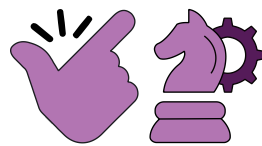
Exercise 9

Calculate the area of the coloured region.

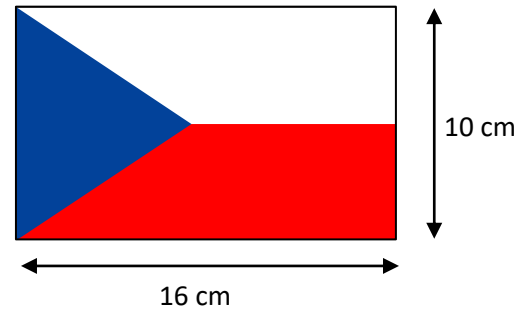


13



**Exercise 10**

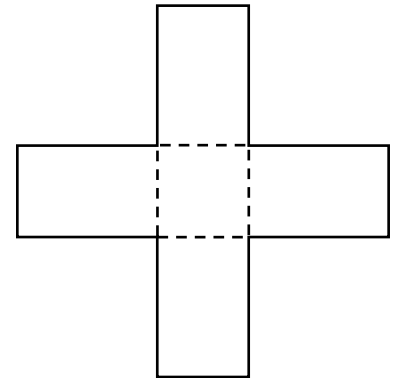
- (a) The flag of the Czech Republic is shown on the right. Given that the vertex of the blue triangle appears in the middle of the flag, calculate the area of the red region.



- (b) A pattern in the form of a cross is formed by arranging four identical rectangles around the sides of a square, as shown in the diagram on the right. (The diagram is not drawn to scale.)

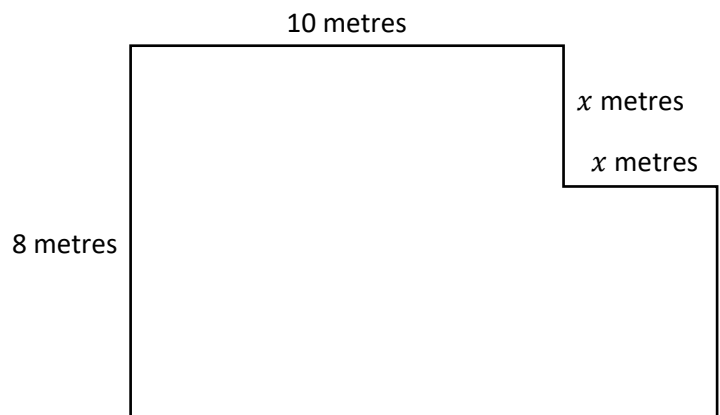
The area of the square is 36 cm^2 . The area of each rectangle is one and a half times the area of the square.

- (i) Find the area of the pattern in the form of a cross.
(ii) Find the perimeter of the pattern in the form of a cross.



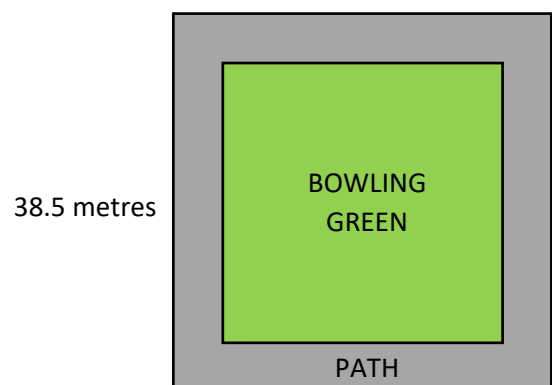
- (c) Gerry has drawn a sketch of a room's floor plan. Each one of the corners of the room is either 90° or 270° . Gerry has forgotten some of the measurements but knows that two of the measurements are the same, and so labels them with x metres.

Show that the area of the floor, in square metres, is $80 + 8x - x^2$.

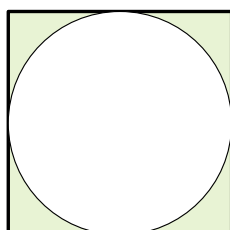


- (d) The local council decides to place a concrete path of width $1\frac{1}{2}$ metres around a square bowling green, as shown in the diagram.

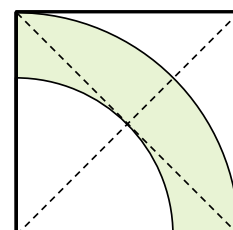
Calculate the area of the concrete path, in square metres.

**Challenge!**

Which fraction of the square is shaded?

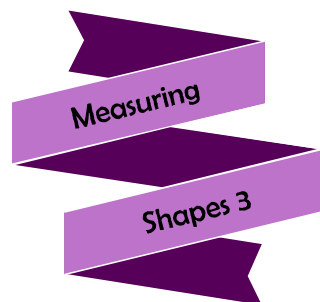
**Challenge 2!**

Which fraction of the square is shaded?



Puzzle: Find the seven differences between the pictures below.





Reflection Sheet

Name:

Percentage in the test:

I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I can enlarge shapes using a scale factor that is a positive whole number .		1	
I can enlarge shapes using a scale factor that is fractional .		3	
I know how to use the centre of enlargement whilst enlarging shapes.		2	
Given a shape and its enlargement, I can find the scale factor and the centre of enlargement .		4	
I can calculate the perimeter of a composite shape .		5, 6	
I can calculate the area of a composite shape .		7, 8	



The Mathematics Department

10

Fractions,

Percentages and

Decimals

Foundation Tier

Version

3.0

08/2025

Name:

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Calculating

Mastering the Techniques

When working with fractions, decimals and percentages, we must develop fluency with many techniques. Try the following questions, before going back and revising the techniques if you got the answer wrong.

Exercise 1



Revision

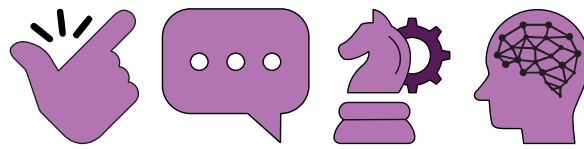
A

(a) $398 + 4829$	(b) $693 - 246$	(c) 372×68
(d) $925 \div 37$	(e) 2.6×10	(f) $63 \div 100$
(g) 0.0247×1000	(h) 46.27×8	(i) $29.3 + 2.43$
(j) 52.6×7.84	(k) $14.7 \div 6$	(l) 0.3×0.2

(m) $4 \div 0.5$	(n) $50\% \text{ o } 80$	(o) $10\% \text{ o } 72$
(p) $36\% \text{ o } £84$	(q) $\frac{3}{5} \text{ o } £35$	(r) $1 - \frac{3}{7}$
	(s) $\frac{5}{9} + \frac{2}{9}$	(t) $\frac{5}{9} - \frac{2}{9}$
	(u) $\frac{2}{3} + \frac{3}{5}$	(v) $\frac{2}{3} - \frac{3}{5}$
	(w) $\frac{2}{3} \times \frac{3}{5}$	(x) $\frac{2}{3} \div \frac{3}{5}$

Exercise 2

Check that you are able to use your calculator to find the correct answers to the questions in Exercise 1.

**Exercise 3****Extension****A**

Use the clues on the cards to solve the problem.

(1) The wage of Number 6 is half the wage of Number 3.

(2) Number 9 has the highest wage in the team.

(3) The wage of Number 10 is $\frac{6}{7}$ of the wage of Number 7.

(4) There's a 15% bonus for every player who scores a goal in a game.

(5) The manager's wage is the mean wage of Number 8 and Number 11.

(6) Number 7 earns £8,000 per day.

(7) There are 11 players in a football team.

(8) If the goalie doesn't concede a goal in a game, he gets a bonus of £5,000.

(9) The wage of Number 3 is 48% of the wage of Number 11.

(10) There's a bonus of 40% for any player who scores a hat-trick in a game.

(11) This week the team won 2–0.

(12) Your task is to calculate this week's wage for the manager and every player who started the game this week.



(13) Number 5 earns £200 per hour.

(14) Number 11 has the third highest wage in the team.

(15) Number 9's usual wage is double the goalie's usual wage.

(16) Number 4's usual wage is $\frac{5}{6}$ of Number 3's usual wage.

(17) One of the players in the team has the same wage as the goalie's usual wage.

(18) There's a difference of £2,000 between the wages of Number 10 and Number 11.

(19) The team only played one game this week.

(20) Number 2's wage is £6,000 higher than the lowest wage.

(21) The crowd for the recent game was 34,500.

(22) Number 1's wage is $\frac{5}{8}$ of Number 10's usual wage.

(23) The word "wage" means "weekly wage" in this task.

(24) During this week's game, Number 4 and Number 9 scored.

You will need to use the following timetable to answer Exercises 4 and 5...

Summary of North Wales to South Wales train services Crynodeb o wasanaethau rhwng Gogledd Cymru a De Cymru						Saturdays Dydd Sadwrn	
		◇	◇	◇	R	◇ L	◇
Holyhead / Caergybi	d	0425	0635	0820	1033	1238	1423
Valley / Y Fali	d	0432x	0641x	0826x	1039x		
Rhosneigr	d			0832x			
Ty Croes	d			0835x			
Bodorgan	d			0840x			
Llanfairpwll	d	0449x	0658x	0849x	1056x		
Bangor	d	0457	0707	0902	1105	1307	1453
Llanfairfechan	d			0909x			1500x
Penmaenmawr	d			0913x			1504x
Conwy	d			0919x			1510x
Llandudno Junction / Cyffordd Llandudno	a	0513	0723	0923	1121	1323	1515
Llandudno Junction / Cyffordd Llandudno	d	0515	0725	0925	1125	1325	1516
Colwyn Bay / Bae Colwyn	d	0521	0731	0931	1131	1331	1522
Abergele & Pensarn	d						
Rhyl	d	0531	0741	0941	1141	1341	1533
Prestatyn	d	0537	0747	0947	1147	1347	1538
Flint / Y Fflint	d	0550	0800	1000	1200	1400	1552
Shotton	d						
Chester / Caer	a	0604	0816	1016	1216	1414	1605
Chester / Caer	d	0612	0819	1019	1219	1419	1619
Wrexham General / Wrecsam Cyffredinol	d	0638	0834	1035	1234	1434	1635
Ruabon / Rhiwabon	d	0645	0841	1042	1241	1441	1642
Chirk / Y Waun	d	0651	0847	1048	1247	1447	1648
Gobowen	d	0657	0853	1054	1253	1453	1654
Shrewsbury / Yr Amwythig	a	0717	0913	1114	1313	1513	1714
Shrewsbury / Yr Amwythig	d	0719	0915	1115	1315	1515	1716
Church Stretton	d				1330		
Craven Arms	d				1338		
Ludlow	d	0745	0942	1141	1344	1541	1742
Leominster	d	0755					
Hereford / Henffordd 7	d	0812	1007	1206	1410	1606	1807
Abergavenny / Y Fenni	d	0835	1030	1229	1432	1629	1830
Pontypool & New Inn / Pontypwl	d	0845	1040	1239	1443	1639	1840
Cwmbran	d	0850	1045	1244	1448	1644	1845
Newport / Casnewydd	d	0902	1057	1256	1506	1656	1857
Cardiff Central / Caerdydd Canolog 7	a	0924	1115	1317	1526	1708	1915

Notes

a	Arrival time
d	Departure time
x	Service stops on request
7	Recommended connecting time
L	To Maesteg
◇	Seat reservations available
R	Seat reservations recommended
⚡	At seat service

Nodiadau

a	Amser cyrraedd
d	Amser gadael
x	Yn aros ar gais
7	Amser cysylltu argymhelig
L	I Faesteg
◇	Seddi cadw ar gael
R	Argymhellir seddi cadw
⚡	Gwasanaeth troli

Exercise 4

Use the timetable on the previous page to answer the following questions.

- How much time does the 0425 train from Holyhead take to reach Llandudno Junction?
- For how many minutes does the 1423 train from Holyhead wait in Chester?
- In minutes, what is the fastest journey between Chester and Shrewsbury?
- Out of the seven trains, how many trains stop in Pontypool?
- On the train that recommends seat reservations, how long does it take to travel from Prestatyn to Cwmbran?
- Which train waits the longest in Llandudno Junction?

**Exercise 5**

Use the timetable on the previous page to solve the following problem.



David must arrive in Cardiff before 8:00pm.

The cost of an "anytime" ticket from Holyhead to Cardiff is £96.

David can purchase the train ticket two weeks before the journey.

David lives in Holyhead and wants to visit his Aunt in Cardiff. Write a travelling schedule for David based on the information around this travel ticket. Remember to include the cost of the journey in your instructions.

By purchasing a ticket before hand, it's possible to save 70% off the "anytime" price.

David can't leave Holyhead before 7:30am.

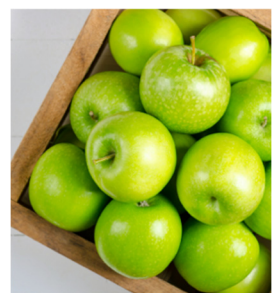
David wants to travel to Cardiff on a Saturday.

David wants to stop at Chester to shop for 3 hours.

A young person's rail card offers $\frac{1}{3}$ off any train ticket.

Exercise 6

- A man gave $\frac{1}{4}$ of the money in his will to his eldest son; $\frac{1}{4}$ of the rest to his second son; and so on until there was only £40,500 left for his only daughter. The value of his will was £128,000. How many sons did he have?
- How much is 30% of 40% of 50% of £60?
- Gwen's necklace broke. $\frac{1}{3}$ of the beads went on the floor. $\frac{1}{5}$ went down the side of the chair. Gwen found $\frac{1}{6}$ of the beads on the table and her Mum found the rest, which was $\frac{1}{10}$. 12 were still on the string. How many beads were on the string before it broke?
- In the sequence $\left(1 - \frac{1}{2}\right), \left(\frac{1}{2} - \frac{1}{3}\right), \left(\frac{1}{3} - \frac{1}{4}\right), \dots$ what is the sum of the first hundred terms?
- I bought a number of apples from a local market. On the way home, I saw William and gave him $\frac{2}{3}$ of the apples. Then I ate one apple before visiting Beth. She had $\frac{3}{4}$ of the remaining apples. I arrived home with only 4 apples. How many apples did I have originally?

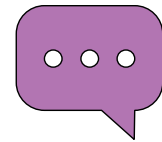


Exercise 7: Upside-down calculator story: the golf tournament.

Use the yellow and green clues to complete the story.

Key: 1 = I, 2 = Z, 3 = E, 4 = H, 5 = S, 6 = G, 7 = L, 8 = B, 9 = b, 0 = O.

Ignore decimal points in your answers.



13

It was the final of the $8^3 + 106$ golf tournament. $\frac{202 \times 4096}{16 \times 64}$ was in second place with only one $7^4 + 434 \times 3 + 1$ to go. $192 \frac{3}{4} \times 2 \frac{2}{3}$ bitter rival $\sin(30) + 0.201$ had already finished, and had $5^5 + 580$ possession of first place. The final $13719616^{0.5}$ was a par 3. A $170683 \frac{1}{5} \div 5 \frac{2}{5}$ would mean that $\sqrt{0.040401} + \frac{1}{2}$ was champion; a birdie would see $9^2 - \frac{1}{10} - \frac{1}{2}$ win. On the tee $7! \times \frac{1}{8} + 178$'s $7! + (9^3 - 132)$ were shaking like jelly and the club felt as slippery as an $\frac{38^2}{2} + 11$. "Here $\sqrt{281536.36}$ nothing", $(10^2 + 1) \times 8$ thought to himself, before striking the ball $\sqrt[3]{15625} \times 923 - 1$ and straight onto the green – a great shot!

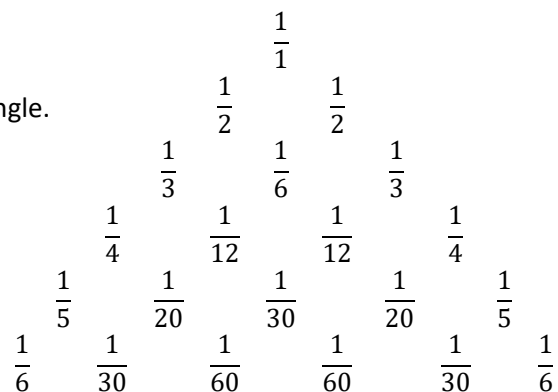
This year had been one $8^2 - 3 + \frac{2}{1-1/2}$ for $\frac{\cos(45) \times 12150}{\sqrt{2}}$ $\frac{16 \times 101}{9^2} \times 40 \frac{1}{2}$. $2 \times e^0 \times 17$ hated to $5555 + (-2)^{11}$, but this year the feeling had become familiar; $405 \frac{3}{4} + 402 \frac{1}{4}$ had more $2142028 \times 25\%$ to $\left(\frac{1000}{701}\right)^{-1}$ than he could remember. Today was $\frac{20200 \times 4}{10^2}$'s final chance to $\frac{7 \times 2267}{2^2 \times 5^3}$ the critics and feel the $31610 \frac{4}{5} + 23567 \frac{1}{5}$ of victory. On reaching the green $8 + \frac{1}{10} - \frac{2}{100}$ tried not to notice the $3^6 \times 4 + (300 - 1)$ of the crowd watching on. He could $\tan(45) + 2 \times 167$ that the ball was around 10 feet from the $463 \times 2 \div \frac{1}{4}$, up a slight $6^5 - 5^4 + 4^3 - 3^2 + 2^1 - 1^0 + 507$. "1032658225^{0.5} the day", $\sqrt{2^6 \times 101^2}$ thought, before putting the ball straight – and into the middle of the $7! - 167 \times 2^3$! Victory at last – $\frac{4 \times 211}{5^2}$ for $404 \times 20\%$, and at last a dent to $\sqrt{\frac{1}{2} - 0.008599}$'s $\frac{3^2 \times 7}{4 \times 25}$!

Challenge!

The name of the triangle on the right is the Leibniz Harmonic Triangle.

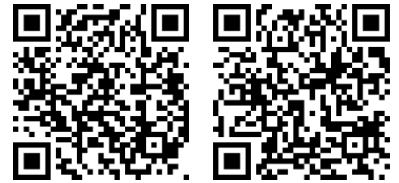
What patterns can you see?

Write down the next row in the pattern.

**Evaluation**

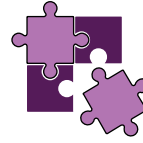
Key words	Corrections	I am happy with...	I need to revise...

Recurring Decimals



Changing Fractions to Decimals

If the denominator of a fraction is a power of 10, for example 10, 100 or 1,000, then it is easy to convert the fraction to a decimal.



Example

$$\frac{3}{10} = 0.3$$

$$\frac{12}{100} = 0.12$$

$$\frac{6}{100} = 0.06$$

$$\frac{427}{1000} = 0.427$$

$$\frac{31}{1000} = 0.031$$

Exercise 8

Convert the following fractions to decimals.

(a) $\frac{7}{10}$

(b) $\frac{5}{10}$

(c) $\frac{17}{100}$

(d) $\frac{7}{100}$

(e) $\frac{95}{100}$

(f) $\frac{1}{10}$

(g) $\frac{93}{100}$

(h) $\frac{731}{1000}$

(i) $\frac{402}{1000}$

(j) $\frac{32}{1000}$

(k) $\frac{86}{1000}$

(l) $\frac{4}{1000}$

(m) $\frac{200}{1000}$

(n) $\frac{760}{1000}$

(o) $\frac{4237}{10000}$

(p) $\frac{29}{10000}$

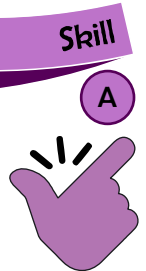
(q) $\frac{3}{100000}$

(r) $\frac{17}{10}$

(s) $\frac{40}{100}$

(t) $\frac{329}{10}$

(u) $\frac{2053}{100}$



Using Equivalent Fractions

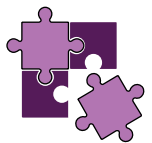
With some fractions, it is possible to find an equivalent fraction whose denominator is a power of 10, which then allows us to change the original fraction to a decimal.

Example

$$\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10} = 0.6$$

$$\frac{11}{25} \xrightarrow{\times 4} \frac{44}{100} = 0.44$$

$$\frac{143}{200} \xrightarrow{\times 5} \frac{715}{1000} = 0.715$$



Exercise 9

Convert the following fractions to decimals.

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) $\frac{1}{5}$

(d) $\frac{1}{2}$

(e) $\frac{6}{25}$

(f) $\frac{21}{25}$

(g) $\frac{1}{25}$

(h) $\frac{3}{20}$

(i) $\frac{19}{20}$

(j) $\frac{11}{20}$

(k) $\frac{43}{50}$

(l) $\frac{9}{50}$

(m) $\frac{55}{50}$

(n) $\frac{304}{500}$

(o) $\frac{1}{200}$

(p) $\frac{64}{200}$

(q) $\frac{136}{200}$

(r) $\frac{31}{250}$

(s) $\frac{147}{250}$

(t) $\frac{9}{250}$

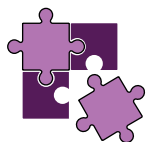
(u) $\frac{53}{5000}$



Terminating and Recurring Decimals

The fractions we have seen so far convert to **terminating** decimals, which means that the decimals have a **specific number** of digits after the decimal point. For example, the fraction $\frac{11}{25}$ from the above example converted to the decimal 0.44, which is a decimal that has **two digits** appearing after the decimal point.

As we will see on the next page, some fractions convert to **recurring** decimals, which are decimals with an **infinite number** of digits appearing after the decimal point. The decimal 0.222 ... is an example of a recurring decimal, where the dots ... show that the digit 2 repeats forever. In mathematics, we have a special notation to write recurring decimals, which is the **dot notation**. We write the decimal 0.222 ... in dot notation as $0.\dot{2}$, where the dot on top of the digit 2 means the 2 repeats forever.



Example

$0.45222 \dots = 0.45\dot{2}$

$0.434343 \dots = 0.4\dot{3}$

$0.243243243 \dots = 0.\dot{2}4\dot{3}$



2

Exercise 10

Write the following recurring decimals using dot notation.

(a) $0.777 \dots$

(b) $0.444 \dots$

(c) $0.5333 \dots$

(d) $5.222 \dots$

(e) $0.52888 \dots$

(f) $0.737373 \dots$

(g) $0.262626 \dots$

(h) $0.909090 \dots$

(i) $0.2454545 \dots$

(j) $0.0818181 \dots$

(k) $0.265265265 \dots$

(l) $0.405405405 \dots$

(m) $0.5274274274 \dots$

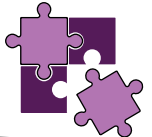
(n) $0.47812812812 \dots$

(o) $0.216737373 \dots$

Converting fractions to recurring decimals

Not all fractions are equivalent to a fraction that has a denominator which is a power of 10.

With these fractions we need to use a division frame to convert the fraction to a recurring decimal.

**Example**Convert the fraction $\frac{2}{3}$ to a recurring decimal.

We can add more zeroes to the 2.000, if needed.

1. Put the sum $2 \div 3$ into a division frame, writing 2 as the equivalent decimal 2.000.

2. "How many times does 3 fit into 2?" It's too large, therefore it fits in 0 times, with 2 left over.

3. Remember to add the decimal point in the correct place.

$$3 \overline{)2.000}$$

$$3 \overline{)2.000} \quad \begin{matrix} 0 \\ 2 \end{matrix}$$

$$3 \overline{)2.000} \quad \begin{matrix} 0. \\ 2 \end{matrix}$$

4. "How many times does 3 fit into 20?" It fits in 6 times, with 2 left over.

5. "How many times does 3 fit into 20?" It fits in 6 times, with 2 left over.

6. We notice the pattern in the calculations, writing ... to show that they repeat forever.

$$3 \overline{)2.000} \quad \begin{matrix} 0.6 \\ 2 \end{matrix}$$

$$3 \overline{)2.000} \quad \begin{matrix} 0.66 \\ 2 \end{matrix}$$

$$3 \overline{)2.000} \quad \begin{matrix} 0.666 \dots \\ 2 \end{matrix}$$

Answer: As a decimal, the fraction $\frac{2}{3}$ is equal to $0.666 \dots$ or, in dot notation, $0.\dot{6}$.**Exercise 11**

Use a division frame to write the following fractions as recurring decimals, using dot notation.

(a) $\frac{1}{3}$

(b) $\frac{2}{9}$

(c) $\frac{5}{9}$

(d) $\frac{1}{9}$

(e) $\frac{3}{11}$

(f) $\frac{8}{11}$

(g) $\frac{10}{11}$

(h) $\frac{1}{6}$

(i) $\frac{2}{6}$

(j) $\frac{5}{6}$

(k) $\frac{1}{7}$

(l) $\frac{5}{7}$

(m) $\frac{6}{7}$

(n) $\frac{4}{13}$



Exercise 12

Use a calculator to check your answers to Exercise 11.

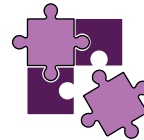
Challenge!

Use a division frame to write the following fractions as recurring decimals.

(a) $\frac{13}{17}$ (b) $\frac{11}{19}$ (c) $\frac{2}{23}$

Terminating or recurring decimals?

It is possible to determine whether a fraction converts to a terminating or a recurring decimal by following these steps.



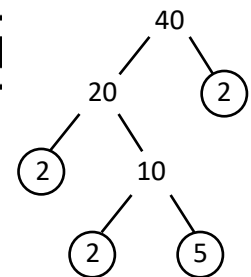
1. **Simplify** the fraction, if possible.
2. Write the denominator of the fraction as a **product of its prime factors**.
3. If the product of prime factors includes the numbers 2 and/or 5 only, then the fraction converts to a **terminating decimal**. If any other prime number appears, then the fraction converts to a **recurring decimal**.

Example

Determine whether the fraction $\frac{46}{80}$ converts to a terminating decimal or to a recurring decimal.



1. We can simplify the fraction by halving: $\frac{46}{80} \xrightarrow{\div 2} \frac{23}{40}$.
2. By using a factor tree (on the right), we see that 40, as a product of its prime factors, is $40 = 2 \times 2 \times 2 \times 5$.
3. Since the product of prime factors for 40 includes the prime numbers 2 and 5 only (and no other primes), we can say that the fraction $\frac{46}{80}$ converts to a **terminating decimal**.

**Exercise 13**

Determine whether the following fractions convert to terminating decimals or to recurring decimals.

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{6}$ (d) $\frac{7}{8}$ (e) $\frac{4}{9}$ (f) $\frac{6}{12}$ (g) $\frac{8}{12}$
 (h) $\frac{11}{15}$ (i) $\frac{12}{15}$ (j) $\frac{3}{25}$ (k) $\frac{12}{30}$ (l) $\frac{29}{30}$ (m) $\frac{1}{80}$ (n) $\frac{87}{125}$

**Exercise 14**

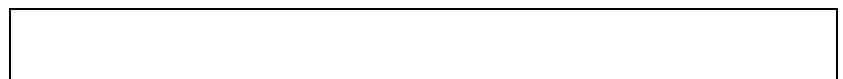
Convert the fractions from Exercise 13 to decimals.

Investigation

The series of images on the right attempts to show that $0.\dot{9} = 1$.

Can you think of a different way to show that $0.\dot{9} = 1$?

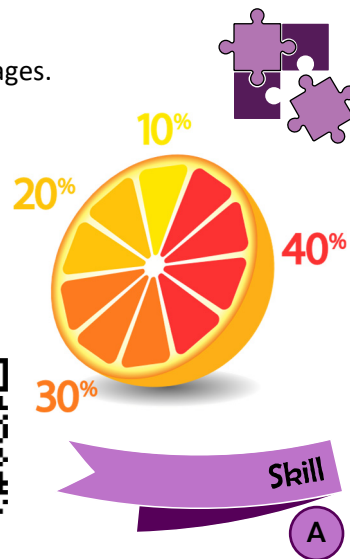
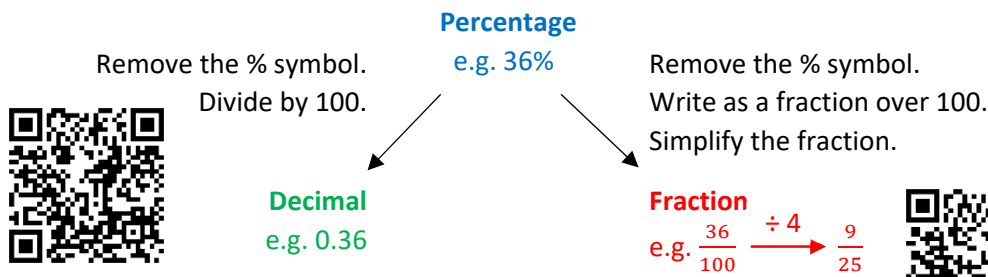
Suggestion: Use the decimal for $\frac{1}{3}$.



Converting

In this chapter, we will discuss how to convert between fractions, decimals and percentages.

Starting with a percentage



Exercise 15

Complete the following table.

Fraction	Percentage	Decimal
	27%	
	68%	
	93%	
	4%	
	7%	
	100%	
	150%	
	400%	
	1%	
	0.5%	
	0.2%	
	0.05%	
	3.1%	
	10.2%	

Exercise 16

Circle each number that has the same value as 40%.

0.04

 $\frac{4}{10}$

0.4

 $\frac{10}{4}$

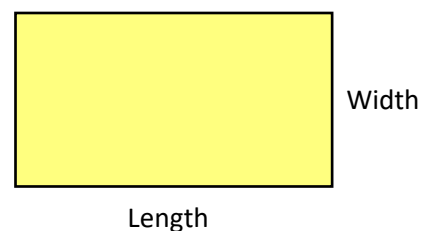
0.40

 $\frac{2}{5}$

4%

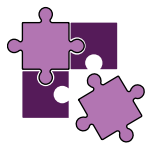
Challenge!

If the length of a rectangle increases 10% but the area remains the same, by what fraction must the width be reduced by?



Starting with a fraction

- 1) Try to find an equivalent fraction which has a denominator that is a power of 10.
Not possible? 2) Use a division frame.



Exercise 17

Complete the following table.

Fraction

e.g. $\frac{2}{5}$

Decimal

e.g. 0.4

Percentage

e.g. 40%

Multiply by 100.

Add a % symbol.



Fraction	Percentage	Decimal
$\frac{87}{100}$		
$\frac{4}{100}$		
$\frac{7}{10}$		
$\frac{3}{5}$		
$\frac{13}{20}$		
$\frac{18}{25}$		
$\frac{184}{500}$		
$\frac{4}{9}$		
$\frac{3}{8}$		
$\frac{3}{2}$		
$\frac{4}{7}$		
$\frac{5}{11}$		
$\frac{12}{40}$		
$\frac{43}{200}$		

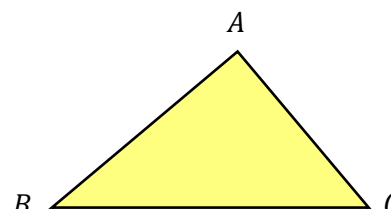
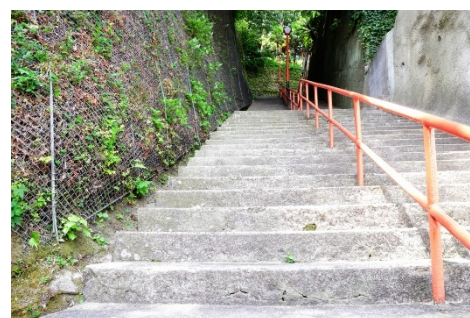
Exercise 18

- (a) Write 13%, 0.2 and $\frac{3}{25}$ in ascending order.
 (b) Write $\frac{3}{4}$, 77% and 0.73 in ascending order.
 (c) Write $\frac{32}{50}$, 0.63 and 67% in descending order.

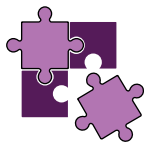
- (d) In a mathematics test, Rachel scored $\frac{3}{5}$ of the highest possible mark. Jimmy scored 62% and Susie's mark was 0.58 of the highest possible mark. State which student scored the most marks and which student scored the least marks.

Challenge!

In a triangle ABC , the angle B is $\frac{3}{4}$ of the angle C and $1\frac{1}{2}$ of the angle A . What is the size of the angle B ?



Starting with a terminating decimal



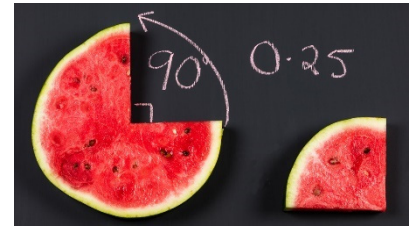
Multiply by 100.
Add a % symbol.

Percentage
e.g. 23.6%

Decimal
e.g. 0.236

Write as a fraction over 10^n , where n is the number of times you must multiply the decimal by 10 for it to become a whole number.
Simplify the fraction.

Fraction
e.g. $\frac{236}{1000} \xrightarrow{\div 4} \frac{59}{250}$



Exercise 19

Complete the following table.

Fraction	Percentage	Decimal
		0.99
		0.9
		0.5
		0.2
		0.14
		0.08
		0.01
		1.6
		2.4
		12.5
		1.09
		0.452
		1.452
		10.2

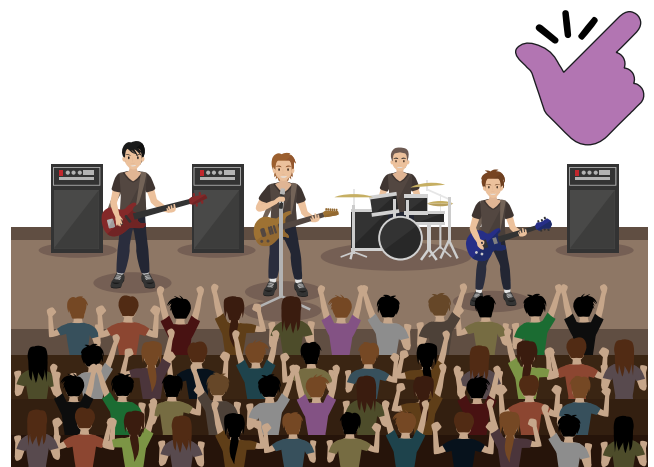
Exercise 20

A band hires a concert hall for two nights. They intend to hire the concert hall for a third night, but only if at least 0.9 of the tickets are sold, either for the first night or for the second night.

On the first night, 82% of the tickets were sold.

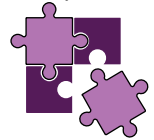
On the second night, $\frac{3}{4}$ of the tickets were sold.

Did the band hire the hall for the third night? You must show your method and explain how you made your decision.

Challenge!

Find the value of x , where $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots - \frac{1}{1024} = \frac{x}{1024}$.





Consider how you would answer the following question.

Chris' weekly wage is £480. If Chris is awarded a pay rise of 10%, what will his wage be next week?

One way to answer this question would be to calculate 10% of £480, which is $£480 \div 10 = £48$, and then add the £48 onto the original wage: $£480 + £48 = £528$. On a calculator, it would be possible to type the following two calculations to find the correct answer.

$$480 \times 10\% = 48$$

$$480 + 48 = 528$$

It is possible however to reach the correct answer in **one** calculation, and therefore in a **more efficient** way. Since Chris' wage is increasing 10%, it is growing from 100% of the original wage to 110% of the original wage. It would therefore be possible to find the new wage by finding 110% of the original wage. On a calculator, we can do this by typing the following calculation.

$$480 \times 110\% = 528$$

Exercise 21

Complete the following table.



Original Price	Original Percentage	Percentage Change	New Percentage	Sum to Find the New Price	New Price
£480	100%	Increase 10%	110%	$£480 \times 110\%$	£528
£54	100%	Decrease 20%	80%	$£54 \times 80\%$	£43.20
£250		Increase 25%			
£1,700		Increase 5%			
£20		Decrease 10%			
\$800		Decrease 30%			
€460		Increase 2%			
£7.50		Decrease 30%			
£2,500		Increase 80%			
£100,000		Decrease 64%			
\$650		Increase 16%			
£82.50		Decrease 26%			
£1.87		Increase 100%			
£74.50		Decrease 100%			
€2,925		Increase 250%			
£74,000		Decrease 99%			
£5.68		Increase 2.5%			
£100		Decrease 12.5%			
£276.40		Increase 0.4%			
€9.99		Decrease 60%			
£2,000,000		Decrease 1.4%			

Repeated Percentage Changes

Example

Geraint buys a second hand car for **£6,500**.
Every year, the value of the car **depreciates 15%**.
What value will the car have after **three years**?

Depreciation means the value **decreases**.



Inefficient Method

Year 1

$$£6,500 \times 15\% = £975$$

$$£6,500 - £975 = £5,525$$

Year 2

$$£5,525 \times 15\% = £828.75$$

$$£5,525 - £828.75 = £4,696.25$$

Year 3

$$£4,696.25 \times 15\% = £704.44, \text{ to the nearest penny.}$$

$$£4,696.25 - £704.44 = £3,991.81$$

Better Method

Every year, the value of the car decreases from 100% to 85%.

Year 1

$$£6,500 \times 85\% = £5,525$$

Year 2

$$£5,525 \times 85\% = £4,696.25$$

Year 3

$$£4,696.25 \times 85\% = £3,991.81, \text{ to the nearest penny.}$$

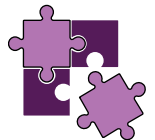
Efficient Method

Every year, the value of the car decreases from 100% to 85%.

This happens three times.

Answer

$$£6,500 \times 85\%^3 = £3,991.81, \text{ to the nearest penny.}$$



Applying

1

Exercise 22

Use an efficient method to answer the following questions.

(a) Megan buys a second-hand car for £8,000. Every year, the value of the car depreciates 12%. What value will the car have after three years?

(b) Aled buys a second-hand car for £14,500. Every year, the value of the car depreciates 20%. What value will the car have after five years?

(c) Ffion buys a pair of earrings for £400. Every year, the value of the earrings increases 15%. What value will the earrings have after four years?

(d) Steffan buys antique furniture for £1,500. Every year, the value of the furniture increases 3%. What value will the furniture have after 6 years?

(e) A football team buys a player for £75,000,000. The value of the player increases 24% every year. The club sells the player after three years. What was the sale price of the player after three years?

(f) Steven buys a field for £7,500. For every one of the next 5 years the price increases 10%.

(i) What value will the field have after two years?

(ii) How much does the price increase between the end of the third year and the end of the fifth year?

(g) A tropical rainforest loses 7% of its trees every year.

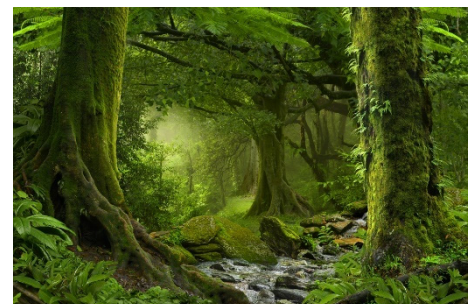
(i) What percentage of the rainforest will be left after a year?

(ii) What percentage of the rainforest will be left after two years?

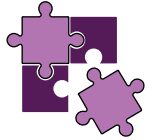
(iii) After how many years will there be less than 50% of the current rainforest left?

(iv) How much of the rainforest will be left after 50 years?

(h) Five years ago, Mark bought a house for £180,000. For the first three years, the value of the house increased by 12% each year. For the next two years, the value of the house decreased by 7% each year. What is the value of the house today?



Calculating Compound Interest Efficiently



Example

Nia borrows **£8,500** from NatWest Bank at a compound interest rate of **4% per year**. Nia wants to pay back all the money after **three years**. How much money will Nia have to pay back after three years?

Inefficient method (Year 9)

Year 1

$$£8,500 \times 4\% = £340$$

$$£8,500 + £340 = £8,840$$

Year 2

$$£8,840 \times 4\% = £353.60$$

$$£8,840 + £353.60 = £9,193.60$$

Year 3

$$£9,193.60 \times 4\% = £367.74,$$

to the nearest penny.

$$£9,193.60 + £367.74 = £9,561.34$$

Better Method

Every year, the value of the money increases from 100% to 104%.

Year 1

$$£8,500 \times 104\% = £8,840$$

Year 2

$$£8,840 \times 104\% = £9,193.60$$

Year 3

$$£9,193.60 \times 104\% = £9,561.34,$$

to the nearest penny.

Efficient Method

Every year, the value of the money increases from 100% to 104%.

This happens three times.

Answer

$$£8,500 \times 104\%^3 = £9,561.34,$$

to the nearest penny.



1

Exercise 23

Use an efficient method to calculate the answer to the following questions.

(a) Sophie borrows £11,000 from Barclays Bank at a compound interest rate of 3% per year. Sophie wants to pay back all the money after three years. How much money will Sophie have to pay back after three years?

(b) Bryn borrows £6,700 from HSBC Bank at a compound interest rate of 6% per year. Bryn wants to pay back all the money after five years. How much money will Bryn have to pay back after five years?

(c) Owen wants to invest £4,000 into Lloyds Bank at a compound interest rate of 5% per year. Owen wants to withdraw all the money from the bank after three years. How much money will Owen be able to take out after three years?

(d) Lorraine wants to invest £25,000 into Barclays Bank at a compound interest rate of 3.4% per year. Lorraine wants to withdraw all the money from the bank after eight years. How much money will Lorraine be able to take out after eight years?



Exercise 24

The following table shows the compound interest rates for different sums of money invested into a bank over a period of time. Complete the table.



	Sum to invest	Compound interest rate per year	Time period for the investment	Calculation to calculate the sum of money at the end	Sum of money at the end	Compound interest earned
	£5,400	4%	3 years	$£5,400 \times 104\%^3$	£6,074.27	£674.27
(a)	£2,800	3%	5 years			
(b)	£19,000	2%	4 years			
(c)	£150,000	5%	2 years			
(d)	£8,500	7%	3 years			
(e)	£24,300	2.5%	4 years			
(f)	£100,000	4.8%	5 years			
(g)	£10,000	7.3%	15 years			

Fractional Changes

Exercise 25

Calculate the following. (It is not necessary to convert any improper fractions to mixed numbers.)

(a) $1 - \frac{1}{3}$

(b) $1 - \frac{1}{7}$

(c) $1 - \frac{2}{5}$

(d) $1 - \frac{3}{10}$

(e) $1 + \frac{1}{3}$

(f) $1 + \frac{1}{7}$

(g) $1 + \frac{2}{5}$

(h) $1 + \frac{3}{10}$

(i) $1 - \frac{1}{45}$

(j) $1 + \frac{3}{45}$

(k) $1 - \frac{14}{25}$

(l) $1 + \frac{3}{50}$



Example

A radio station measures how many listeners are listening at least once per week to their morning show between 7am and 10am. Over the past two months, the number of listeners has decreased $\frac{1}{8}$ per month. If the number of listeners was 32,000 two months ago, what is the number of listeners today?

*Inefficient method**Month 1*

$$32,000 \times \frac{1}{8} = 4,000$$

$$32,000 - 4,000 = 28,000$$

Month 2

$$28,000 \times \frac{1}{8} = 3,500$$

$$28,000 - 3,500 = 24,500$$

Better Method

Instead of calculating $\frac{1}{8}$ of the number and then taking away, we can multiply by $1 - \frac{1}{8} = \frac{7}{8}$.

Month 1

$$32,000 \times \frac{7}{8} = 28,000$$

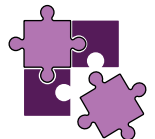
Month 2

$$28,000 \times \frac{7}{8} = 24,500$$

Efficient Method

We can multiply by $\frac{7}{8}$ twice.

$$32,000 \times \left(\frac{7}{8}\right)^2 = 24,500$$



Exercise 26

Use an efficient method to answer the following questions.

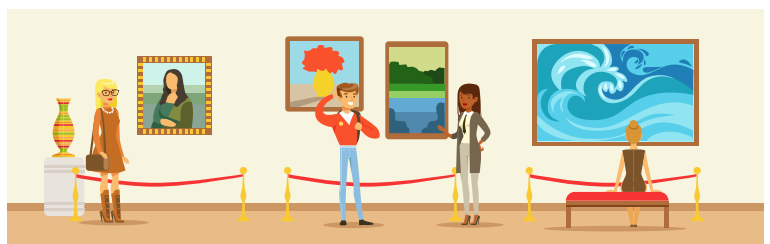
(a) Monthly sales for a magazine have dropped $\frac{1}{10}$ every month over the past three months. If the monthly sales were 60,000 three months ago, what are the monthly sales today?

(b) Boris decides to join a weight loss club. Boris accepts a target to lose $\frac{1}{80}$ of his weight every month, over a period of six months. If Boris weighs 90 kg today, how much weight does Boris need to lose over the next 6 months to reach his target? Write your answer to two decimal places.

(c) At the beginning of last year, 150,000 people visited a website every day. Over the year, the website saw a monthly increase of $\frac{1}{20}$ in their daily visitor numbers. How many people visited the website daily at the end of last year?

(d) A business saw a yearly increase of $\frac{2}{5}$ in product sales over a period of four years. If the yearly sales were £3,400,000 at the beginning of the period, what were the yearly sales at the end of the period?

(e) The number of people who visit a specific gallery in a month has reduced by $\frac{1}{200}$ every month over a period of 8 months. If 40,000 people visited the gallery eight months ago, how many people visited the gallery during the past month?



Applying

1



There are **two tiers** and **three units** in a GCSE mathematics examination:

Higher Tier: Grades A*–D
Foundation Tier: Grades C–G

Unit 1

Financial Mathematics and Other Applications of Numeracy

Higher Tier: 1 hour 45 minutes (80 marks)

Foundation Tier: 1 hour 30 minutes (65 marks)

30% of qualification

The purpose of this unit is to:

- Introduce and develop learners' understanding of topics and concepts relating to finance and to develop their financial literacy.
- Allow learners to use their knowledge and apply mathematical methods to personal and other real-world contexts, including those related to money and the workplace.

Calculator will be allowed in this examination.

Unit 2

Non-calculator

Higher Tier: 1 hour 45 minutes (80 marks)

Foundation Tier: 1 hour 30 minutes (65 marks)

30% of qualification

The purpose of this unit is to explore mathematical topics and concepts that don't require the use of a calculator. This unit contains all aspects of probability and has a significant focus on geometry.

Calculator will **not** be allowed in this examination.

Unit 3

Calculator-allowed

Higher Tier: 2 hours (90 marks)

Foundation Tier: 1 hour 45 minutes (75 marks)

40% of qualification

The purpose of this unit is to explore topics and concepts that:

- are more appropriately assessed with a calculator, or
- form the foundations for, or link to, topics that are more appropriately assessed with a calculator.

This unit contains the majority of topics covering the data handling aspects of statistics and has a significant focus on measures.

Calculator will be allowed in this examination.



Assessing Written Communication



There is one question in every exam paper where "the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing". This means that you will get marks not just for the right answer, but for **how** you set out your answer.

For **organisation and communication** marks, you will be expected to:

- Present your answer in a structured way.
- Explain to the examiner what you are doing at each step of your answer.
- Set out your explanations and calculations in a way that is clear and logical.
- Write a conclusion that brings together the results and explains the meaning of the answer.

For **accuracy in writing** marks, you will be expected to:

- Show all of your calculation work.
- Make minimal spelling, punctuation and grammar mistakes.
- Use correct mathematical form in your calculations.
- Use appropriate terminology, appropriate units etc.

To make sure you obtain these marks (usually two marks in each examination paper), split the answer page in half, using the **Sum** and **Explanation** headings. On the left-hand side ("Sum"), write down any mathematical calculations you make. On the right-hand side ("Explanation"), explain why you have made that particular calculation. This needs to be done even if the sum is $1 + 1 = 2$.

Example

Geraint has a Saturday job to save money for a holiday. He earns £18 each week.

Geraint saves $\frac{5}{6}$ of the money that he earns each week and spends the rest.

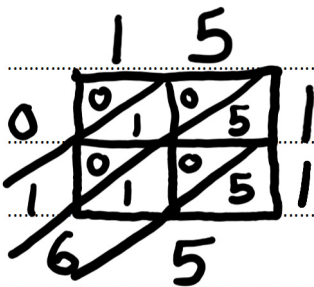
How much money does he save in 11 weeks?

You must show all of your calculations.

Answer:



Sum	Explanation
$£18 \div 6 = £3$	Calculate $\frac{1}{6}$ of the wage
$£3 \times 5 = £15$	Calculate $\frac{5}{6}$ of the wage
	Geraint saves £15 every week.
	$11 \times £15 = £165$, therefore Geraint saves £165 in 11 weeks



Exercise 27 (no calculator allowed)

In all of the following questions, **you will be assessed on the quality of your organisation, communication and accuracy in your writing.**



Applying

A

(a) For her birthday, Casey received two gift vouchers for her favourite shop.



She bought a pair of jeans for £26 and two tops at £15.99 each.

Casey used her two gift vouchers to buy the jeans and tops. How much more did she have to pay?
You must show all of your calculations.

(b) A band was hired to play in the local hall.

The hall was hired for 4 hours at a rate of £20 per hour.

The band cost £150 to hire.

The price for a ticket to the event was £5 each. 128 tickets were sold.

Calculate how much money was spent, how much money was collected, and the profit or loss made on the event.



(c) A group of 14 Youth Club members want to go to a Theme Park.

Five of the members are 15 years old.

The rest of the members are younger than 15 years old.

How much money does the group save by going to the theme park on a weekend rather than on a weekday?



(d)



Small pot of daffodils

£1.20



Large pot of daffodils

£

Marjorie spent £20 on daffodils. She received 50p in change.

She bought 10 small pots of daffodils.

The large pots of daffodils were 25% more expensive to buy than the small pots of daffodils.

How many large pots of daffodils did she buy?

You must show all of your calculations.

Exercise 28 (calculator allowed)

13

In all of the following questions, **you will be assessed on the quality of your organisation, communication and accuracy in your writing.**

(a) Two car rental companies have the following deals available for renting the same model of car.



Just Go Cars



£32 per day
18p per mile
VAT of 20% to be added

Speedy Wheels



£68 per day
No charge for mileage
Includes VAT
Special offer: $\frac{1}{4}$ off

Dylan wants to rent a car for 7 days. He plans to drive 500 miles. Which company should Dylan choose to get the cheapest bargain?

(b) Bethan's current annual salary is £30,000. After tax and other deductions, she receives 70% of this salary. Over one year, her job means that she travels 8,000 miles. Her car travels at 40 miles per gallon, and a gallon of petrol costs £6.25.

Bethan is considering a new job, working from home. Her new salary would be $\frac{2}{3}$ of her current salary, with the same percentage of deductions.

Calculate the difference, in monetary terms, that changing job would have. You must show all of your calculations.



(c) Here is the cost of buying electricity from *North Electricity*.

- A fixed charge of 28p per day.
- An energy charge of 14p for every kWh used.
- VAT of 5% to pay on the total cost.

Evan uses 850 kWh of electricity over a period of 90 days. Calculate the total bill for Evan to purchase electricity from *North Electricity*.



(d) A paint colour called ochre is made by using a recipe that includes white, red, blue and yellow paint. The percentages of each different colour used to create ochre paint is shown on the right.

Catrin has already bought 2.5 litres of blue paint.

She decides to buy white, red and yellow paint to use with **all** of her blue paint to make as much ochre paint as possible.

The sizes of paint tins available are 1 litre, 2.5 litres and 10 litres.

Only full tins of paint can be purchased.

Catrin only has a small shed to store the paint, therefore she wants minimal white, red and yellow paint left over.

Calculate how much of each paint colour Catrin needs to buy. You must show all of your calculations.



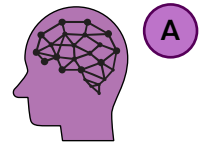
Mark Schemes

After you complete your exam, the paper is sent away to be marked. The markers follow a mark scheme for the exam paper, which uses the following codes for the marks.

Code	Explanation
M	'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
m	'm' marks are dependent method marks. They are only given if the relevant previous 'M' mark has been earned.
A	'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
B	'B' marks are independent of method and are usually awarded for an accurate result or statement.
S	'S' marks are awarded for strategy.
E	'E' marks are awarded for explanation.
U	'U' marks are awarded for units.
P	'P' marks are awarded for plotting points.
C	'C' marks are awarded for drawing curves.

Exercise 29

Here is a past paper question together with its mark scheme.
Mark the four attempts using the mark scheme.



Question

Megan has some blocks. 10% of the blocks are white. $\frac{3}{5}$ of the blocks are red. The remaining blocks are green.
33 blocks are green. How many blocks are there in total? [5 marks]

Mark Scheme

White 10%	B1	
Red 60% OR 0.6 OR $\frac{6}{10}$	M1	Must use a common measure, i.e. percentages or fractions or decimals.
Green 100% – 10% – 60%	A1	
= 30% OR 0.3 OR $\frac{3}{10}$	M1	Follow through 'their 30%'.
30% of the blocks is 33 (blocks).	A1	
100% of the blocks is 110 (blocks).		

Candidate 1

Red $\frac{3 \times 2}{5} \rightarrow \frac{6 \times 10}{100} \rightarrow \frac{60}{100} = 60\%$

Green $100\% - 10\% - 60\%$
 $= 30\%$

33 blocks is 30% $\rightarrow \div 3$
 11 blocks is 10% $\rightarrow \times 10$
110 blocks is 100%

Candidate 2

0.6 $\leftarrow$ Red

$5 \overline{) 3.0}$

Green 30%

33 blocks is 30% $\rightarrow \div 3$
 11 blocks is 10% $\rightarrow \times 10$
110 blocks is 100%

Candidate 3

$$\text{White } \frac{1}{10} \quad \text{Red } \frac{3 \times 2}{5} = \frac{6}{10}$$

$$\text{Green } 1 - \frac{7}{10} = \frac{3}{10}$$

$$33 \times 3 = 99$$

$$99 \div 10 = 9.9$$

10 blocks in total

Candidate 4

$$10\% + \frac{3}{5} = \frac{4}{5}$$

$$\text{Green } 1 - \frac{4}{5} = \frac{1}{5} = 20\%$$

$$33 \text{ blocks is } 20\% \rightarrow \times 5$$

$$\underline{165 \text{ blocks is } 100\%}$$

$$\begin{array}{r} 33 \\ \times 5 \\ \hline 165 \\ \hline \end{array}$$

Timing

It's a good idea to complete any maths exam paper at a rate of **one mark per minute**. This way, you will have time left over at the end to check your answers.

Foundation Tier	Higher Tier
Units 1 and 2: 65 marks in 90 minutes. Unit 3: 75 marks in 105 minutes. Working at a rate of one mark per minute, there will be 25-30 minutes left over at the end to check your answers.	Units 1 and 2: 80 marks in 105 minutes. Unit 3: 90 marks in 120 minutes. Working at a rate of one mark per minute, there will be 25-30 minutes left over at the end to check your answers.

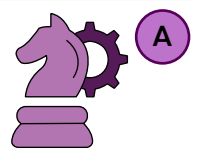
What should you do if you have time left over at the end of an exam?

- Check there are no gaps in your exam paper.
- Make sure you have included the correct units, e.g. £, cm², ml.
- Make sure you have shown enough method.
- Make sure you have used the correct equipment, e.g. a ruler for drawing diagrams.
- Attempt to redo the question, perhaps using a different method, to check your answer.



Exercise 30

The following six questions are worth a total of 20 marks. Attempt to complete these in **only 25 minutes**. Mark the questions using the mark scheme provided by your teacher.



(1) What fraction of the following shape is shaded? Give your answer in its simplest form.

[2]

(2) You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

[5]

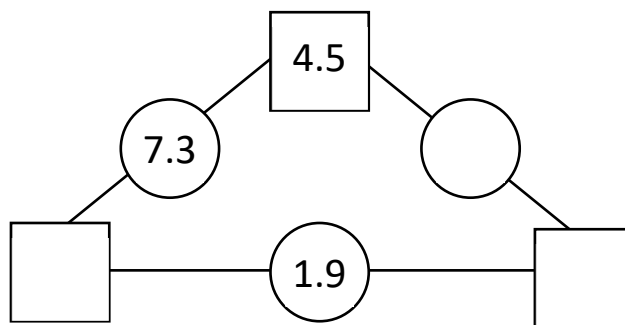
Gwilym bought a camera for £150.

When he sold the camera, he made a loss of 6%.

How much did Gwilym sell his camera for?

(3) The number in the circle is equal to the two numbers in the squares on either side of the circle. [3]

Fill in the missing numbers.



(4) Evaluate $\frac{3}{8} + \frac{1}{2}$. [2]

(5) New pylons are needed in an area in Wales. [3]

- The pylons are in a straight line.
- The distance between the first pylon and the last pylon is 9 km.
- The pylons must be 0.5 km apart from each other.



How many pylons, in total, are needed for a 9 km piece of land?

(6) Maria sells ribbon. [5]

The length of the ribbon she has is 400 cm.

Maria cuts off 30% of the ribbon and sells this piece to a customer.

She uses $\frac{2}{5}$ of the **leftover** ribbon to decorate a card.

Then, Maria cuts the leftover ribbon into three equal pieces.

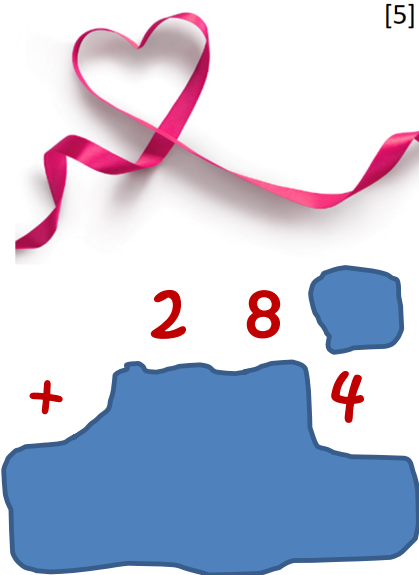
What is the length of each of the three pieces that are left over?

Challenge!

Ink is spilled onto a sum in a textbook.

All of the digits from 0 to 9 have been used in the sum.

Find the correct locations for the other 7 digits.



Key words	Corrections	I am happy with...	I need to revise...

Fractions,

Percentages and

Decimals

Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the testWithout a calculator
With a calculatorCorrect
in the
test?

I can calculate using fractions , e.g. $\frac{3}{11} + \frac{5}{11}$, $\frac{7}{9} - \frac{1}{3}$, $\frac{2}{3} \times \frac{5}{7}$, $\frac{4}{5} \div \frac{2}{9}$.			1	
I can calculate using percentages , e.g. 10% of £75, 45% of 140.			5	
I can calculate using decimals , e.g. $6.5 + 2.79$, $6 - 4.31$, 0.4×0.3 , $14.4 \div 6$.			1	
I know how to use the dot notation for recurring decimals.			2	
I know how to decide whether a fraction is equivalent to a terminating decimal or to a recurring decimal .				
I can convert a percentage to a decimal .			3, 4	
I can convert a percentage to a fraction .			3, 4	
I can convert a fraction to a decimal .			2, 3, 4	
I can convert a fraction to a percentage .			3, 4	
I can convert a decimal to a percentage .			3, 4	
I can convert a terminating decimal to a fraction .			3, 4	
I can calculate percentage changes efficiently .				
I can calculate repeated percentage changes efficiently .				
I can calculate compound interest efficiently .				
I can calculate fractional changes efficiently .				
I know how to answer a question where you are " assessed on the quality of your organisation, communication and accuracy in your writing ".			6, 7	



The Mathematics Department

10

Developing

Algebra 2

Foundation Tier

Version

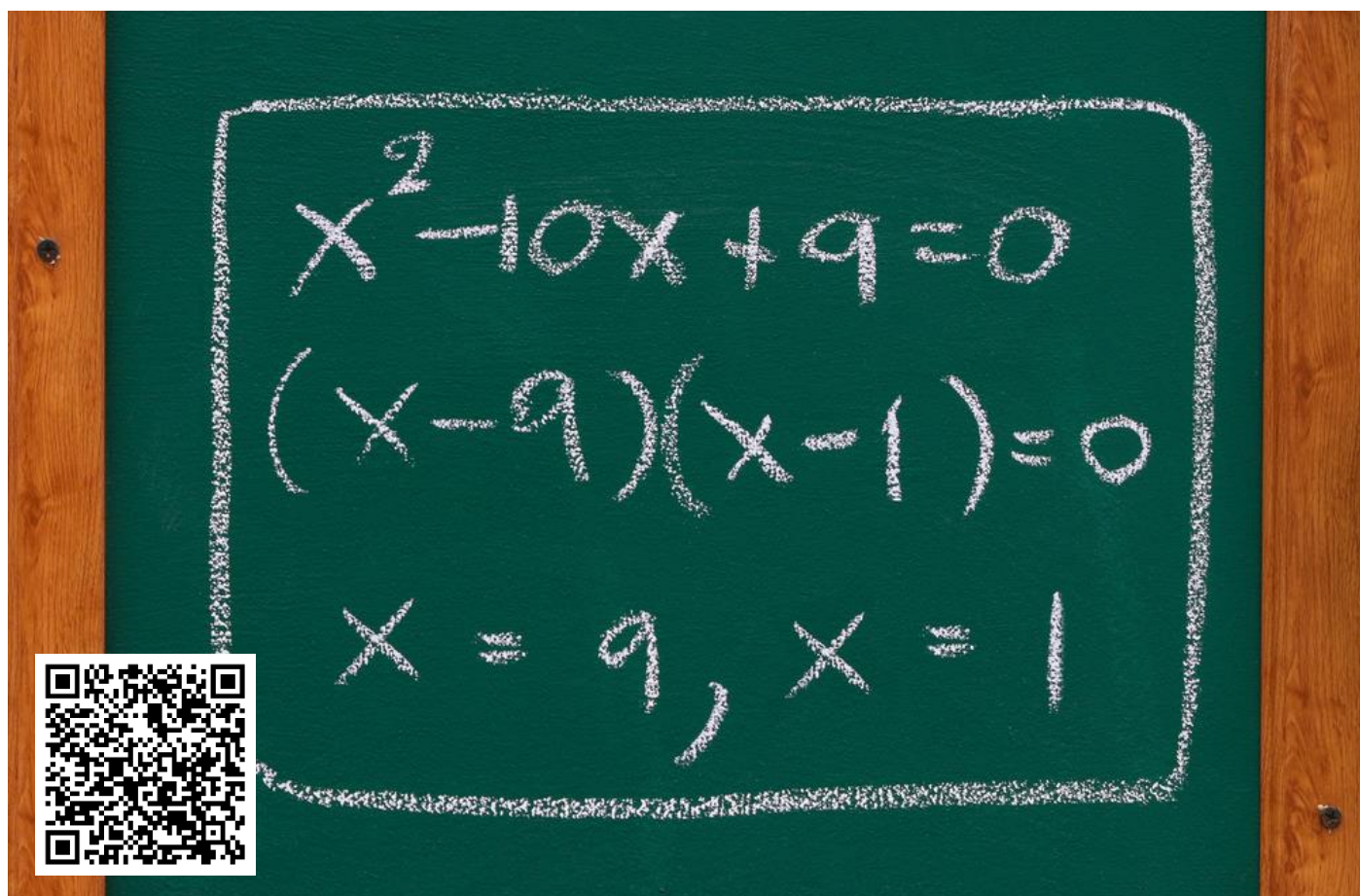
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Name:

Contents

Chapter	Mathematics	Page Number
Simple Factorising	Reversing Expansion. Simple Factorising. More than one Variable.	3
Changing the Subject	Re-arranging formulae.	6
Expression, Equation, Formula	Recognising expressions, equations and formulae.	8



Simple Factorising



Reversing Expansion

Like subtraction reverses addition, and division reverses multiplication, **factorising** reverses **expanding brackets**, a technique seen previously in the **Developing Algebra** workbook.

Exercise 1

Expand the following algebraic expressions.

(a) $2(x + 3)$

(b) $5(x + 3)$

(c) $5(x - 3)$

(d) $5(3 - x)$

(e) $2(y + 3)$

(f) $2(x + 3 + y)$

(g) $x(x + 3)$

(h) $x(3 + x)$

(i) $2x(x + 3)$

(j) $5x(x + 3)$

(k) $5x(2x + 3)$

(l) $2x(5x - 3)$

(m) $4(x - 7)$

(n) $y(y + 9)$

(o) $2z(z + 4)$

(p) $7x(3x + 4)$

(q) $-7x(3x + 4)$

(r) $-7x(3x - 4)$

Expanding gets rid of brackets, whereas factorising **reintroduces brackets**. Also, whilst expanding using multiplication sums, factorising uses **division** sums. To this intent, an important skill when factorising is to recognise the **highest common factor** of a set of numbers, which is the largest number that divides into the list of numbers.

Exercise 2

What is the highest common factor of the following numbers?

(a) 6 and 8

(b) 12 and 15

(c) 20 and 30

(d) 20 and 40

(e) 18 and 24

(f) 16 and 40

(g) 22 and 33

(h) 24 and 36

(i) 35 and 56

(j) 36 and 54

(k) 12, 14 and 16

(l) 12, 16 and 20

(m) 25, 35 and 45

(n) 30, 45 and 60

(o) 7, 11 and 13

In order to factorise an algebraic expression such as $12x + 18$, we start by considering the highest common factor of the terms $12x$ and 18 in the expression.

Example

Factorise $12x + 18$.

1. "What is the highest common factor of $12x$ and 18?" The answer is **6**, therefore we write 6 followed by a pair of brackets.

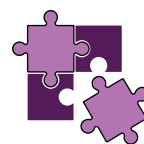
$$12x + 18 = 6(\quad)$$

2. "6 multiplied by what gives $12x$?" 6 multiplied by **$2x$** is $12x$, therefore we write $2x$ inside the brackets.

$$12x + 18 = 6(2x \quad)$$

3. "6 multiplied by what gives 18?" 6 multiplied by **3** gives 18, therefore we write **+ 3** inside the brackets.

$$12x + 18 = 6(2x + 3)$$



Exercise 3

Factorise the following algebraic expressions.

(a) $4x + 6$

(b) $6 + 4x$

(c) $4x - 6$

(d) $6 - 4x$

(e) $4x + 8$

(f) $4z + 8$

(g) $6x + 8$

(h) $6x + 12$

(i) $12 - 6x$

(j) $9x + 21$

(k) $25x + 30$

(l) $18x + 30$

(m) $14x + 21$

(n) $28 + 14x$

(o) $30x - 40$

(p) $24y + 36$

(q) $60x + 80$

(r) $36x + 45$

(s) $36x + 54$

(t) $33y - 55$

(u) $33y - 66$

(v) $45 + 30x$

(w) $300x + 500$

(x) $80z - 240$

(y) $2x + 4y + 6$

(z) $5x + 10y + 20$

(α) $12x - 20y + 24$

Not all algebraic expressions can be factorised. For example, $5x + 7$ cannot be factorised since the highest common factor of $5x$ and 7 is 1 . (We don't factorise $5x + 7$ as $1(5x + 7)$.)

Exercise 4

Factorise all of the algebraic expressions that do factorise, and note which expressions do not factorise.

(a) $5x + 10$

(b) $5x + 11$

(c) $5x + 5$

(d) $16x$

(e) $16x + 2$

(f) $16x + 9$

(g) $8y - 12$

(h) $8y - 13$

(i) $8y - 14$

When factorising, it's not just numbers that can appear before the first bracket – we can include variables like x too.

ExampleFactorise $6x^2 + 14x$.

1. "What is the highest common factor of $6x^2$ and $14x$?" The answer is $2x$, therefore we write $2x$ followed by a pair of brackets.

2. " $2x$ multiplied by what gives $6x^2$?" $2x$ multiplied by $3x$ is $6x^2$, therefore we write $3x$ inside the brackets.

3. " $2x$ multiplied by what gives $14x$?" $2x$ multiplied by 7 gives $14x$, therefore we write $+ 7$ inside the brackets.

$$6x^2 + 14x = 2x(\quad)$$

$$6x^2 + 14x = 2x(3x \quad)$$

$$6x^2 + 14x = 2x(3x + 7)$$

Exercise 5

What is the highest common factor of the following terms?

(a) $4x^2$ and $18x$

(b) $4x^2$ and $16x$

(c) $5x^2$ and $20x$

(d) $5x^2$ and $7x$

(e) $18x^2$ and $24x$

(f) x^2 and x

(g) x^3 and x^2

(h) $4x^3$ and $18x^2$

(i) $9x^4$ and $3x^2$

(j) $12x^4$ and $15x^3$

Exercise 6

Factorise the following algebraic expressions.

(a) $3x^2 + 6x$

(b) $6x + 3x^2$

(c) $3x^2 - 6x$

(d) $x^2 + x$

(e) $x^3 + x$

(f) $x^3 + x^2$

(g) $4x^2 + 2x$

(h) $2x^3 - 6x$

(i) $2x^3 + 8x^2$

(j) $12x^2 + 4x$

(k) $4x - 12x^2$

(l) $10x^2 + 15x$

(m) $6x^4 + 9x^2$

(n) $24y^3 - 16$

(o) $21z^2 + 14z$

(p) $3x^2 + 5x$

(q) $7y - 11y^2$

(r) $4z^3 + 17z^2$

(s) $22x^2 + 33x^5$

(t) $12n - 4n^2$

(u) $2a + a^2$

(v) $x^6 + 6x$

(w) $x^6 + 6x^2$

(x) $6x^6 + 4x^4$

(y) $2x^3 + 4x^2 + 6x$

(z) $2x^2 + 4x + 6$

(α) $42x^4 - 30x^2 + 12x^3$

**Exercise 7**

What is the highest common factor of the following terms?

(a) x^2 and xy

(b) $4x^2$ and $6xy$

(c) $12x^2y$ and $16xy$

(d) $5x^2y^2$ and $9y^2$

(e) $18x^3y$ and $12x^2y$

Example

$$2x^2y + 4x = 2x(xy + 2)$$

$$30y^3z^2 - 24y^2z = 6y^2z(5yz - 4)$$

Exercise 8

Factorise the following algebraic expressions.

(a) $xy + x$

(b) $4xy + 10y$

(c) $x^2y + 5xy$

(d) $2x^2y + 6xy$

(e) $10yz + 5yz^2$

(f) $12x^2y - 4x$

(g) $x^2 + xy^2$

(h) $16x^3z - 12z^2$

(i) $15x^4y + 25x^2y^2$

(j) $x^2yz + 4xyz$

(k) $8ab^2c^3 - 18abc^2$

(l) $26\pi x^2 + 65\pi x$



Key words	Corrections	I am happy with...	I need to revise...

Changing the Subject



The purpose of **changing the subject** is to re-arrange formulae so that a particular **variable** appears on its own on the left-hand side of the formula. For example, consider the formula $p = 3w + d$, a formula to calculate the number of points (p) a football team has given how many games they have won (w) and how many drawn games (d) they have had. We can re-arrange the formula to give the number of games won by a football team.

$$p = 3w + d$$

$$3w + d = p$$

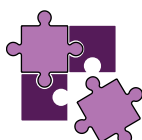
$$3w = p - d$$

$$w = \frac{p-d}{3}$$

[Swap sides]

[Subtract d from both sides]

[Divide both sides by 3]



After re-arranging the formula as shown above, we say that w is the **subject** of the formula.

There are a number of 'movements' we can perform to help re-arrange a formula to give a specific subject. Here are some of the most common movements.

Add a number to both sides of the formula

E.g. $y - 3 = x$

$$y = x + 3$$

[Add 3 to both sides]

Subtract a number from both sides of the formula

E.g. $y + 7 = x$

$$y = x - 7$$

[Subtract 7 from both sides]

Multiply both sides of the formula by a number

E.g. $\frac{y}{2} = 5x$

$$y = 10x$$

[Multiply both sides by 2]

Divide both sides of the formula by a number

E.g. $4y = x - 3$

$$y = \frac{x-3}{4}$$

[Divide both sides by 4]

Square both sides of the formula

E.g. $\sqrt{y} = 4x + 5$

$$y = (4x + 5)^2$$

[Square both sides]

Take the square root of both sides of the formula

E.g. $y^2 = 2x - 9$

$$y = \sqrt{2x - 9}$$

[Square root both sides]

Swap sides

E.g. $5x + 3 = y$

$$y = 5x + 3$$

[Swap sides]

Expand brackets

E.g. $4(y + 2) = 5x$

$$4y + 8 = 5x$$

[Expand brackets]

Let us reconsider the example at the top of this page. The question in the example can be set as follows.

Make w the subject of the formula $p = 3w + d$.

The aim in this question is to re-arrange the formula to leave only the variable w on the left-hand side of the formula. As the variable w initially appears on the right-hand side of the formula, it makes sense to start by swapping the sides of the formula, so that w appears on the left-hand side of the formula.

$$p = 3w + d$$

$$3w + d = p$$

[Swap sides]

There are several ways to proceed. You can think of the formula as an equation, and 'solve' to leave w on its own. Or you can think of how to calculate the left-hand side of the formula, if you start with the variable w .



By working backwards through the function machine, we can see the steps required to proceed, namely subtracting d from both sides of the formula, and then dividing both sides by 3.

**Exercise 9**

- (a) Make e the subject of the formula $p = 2e + c$.
 (c) Make c the subject of the formula $y = mx + c$.
 (e) Make p the subject of the formula $c = p - 3t$.
 (g) Make p the subject of the formula $A = p(q + r)$.
 (i) Make t the subject of the formula $F = \frac{m+4n}{t}$.
 (k) Make n the subject of the formula $F = \frac{m+4n}{t}$.
 (m) Make R the subject of the formula $I = \frac{PRT}{100}$.
 (o) Make u the subject of the formula $C = \frac{1}{3}\pi r^2 u$.
 (q) Make u the subject of the formula $A = \frac{1}{2}(a + b)u$.
- (b) Make c the subject of the formula $p = 2e + c$.
 (d) Make m the subject of the formula $y = mx + c$.
 (f) Make t the subject of the formula $c = p - 3t$.
 (h) Make q the subject of the formula $A = p(q + r)$.
 (j) Make m the subject of the formula $F = \frac{m+4n}{t}$.
 (l) Make r the subject of the formula $A = \pi r^2$.
 (n) Make s the subject of the formula $A = \frac{su}{2}$.
 (p) Make r the subject of the formula $C = \frac{1}{3}\pi r^2 u$.
 (r) Make b the subject of the formula $A = \frac{1}{2}(a + b)u$.

Exercise 10

The following formula was used by festival planners to calculate the parking fee for mini buses.

$$\text{Parking Fee} = \text{Number of Passengers} \times 30p + \text{£}5$$

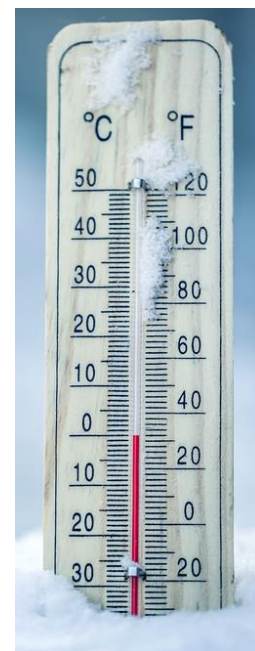
- (a) What was the parking fee for a mini bus with 12 passengers?
 (b) The parking fee for another mini bus was £7.40. How many passengers were on this mini bus?

Exercise 11

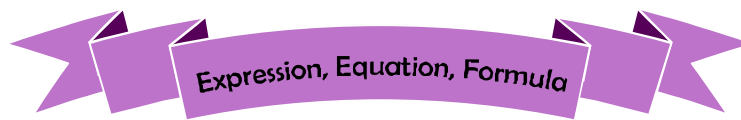
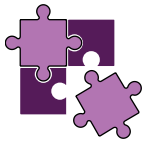
To change from degrees Celsius ($^{\circ}\text{C}$) to degrees Fahrenheit ($^{\circ}\text{F}$), you can use the following formula.

$$F = \frac{9}{5}(C + 40) - 40$$

- (a) The temperature is 60°C . What is this in $^{\circ}\text{F}$?
 (b) Re-arrange the formula to find C in terms of F .


Evaluation

Key words	Corrections	I am happy with...	I need to revise...



In algebra, what's the difference between expressions, equations and formulae?

Expression

An expression is a collection of terms (e.g. $5x$ or 7) and operators (e.g. $+$ or $\times$).
 $4x + 2$ and $\sqrt{6y - 4z}$ are examples of expressions.
 There are no equal signs ($=$) in expressions.

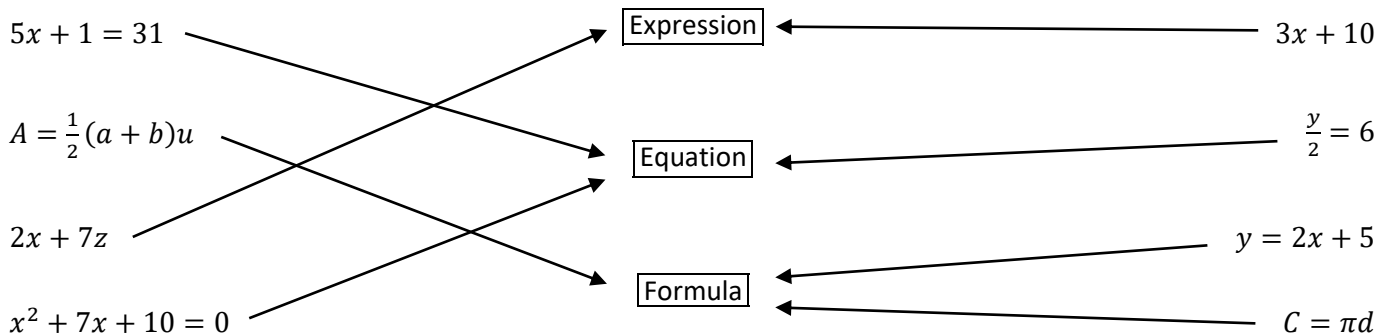
Equation

An equation notes that two terms or expressions are equal. Two sides of an equation are separated by an equal sign ($=$). Sometimes, it is possible to solve an equation to find the value of a variable.

Formula

A formula is a special type of equation which shows the connection between different variables.
 $P = 2a + 2b$ is an example of a formula, one which is used to calculate the perimeter of a rectangle with length a and width b .

Example



Exercise 12

Add arrows pointing to the correct descriptions.



$$4x + 3 = 2x + 27$$

Expression

$$w = 8u + 17v$$

$$E = mc^2$$

Equation

$$z - 3 = 8$$

$$21x + 8$$

Formula

$$\sqrt{\frac{2x-3}{y}}$$

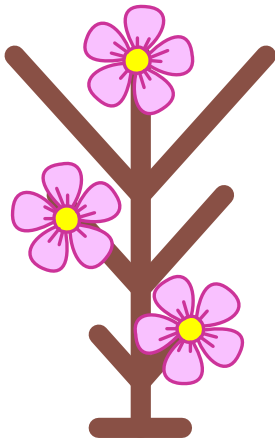
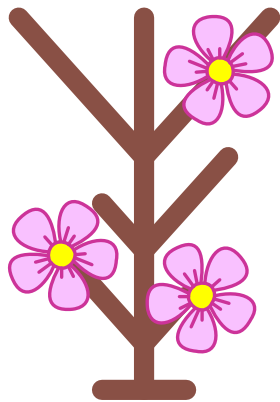
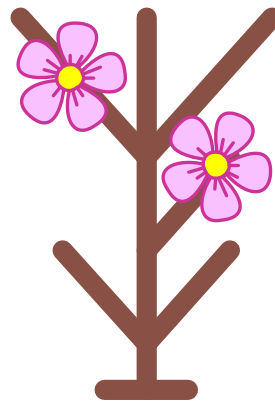
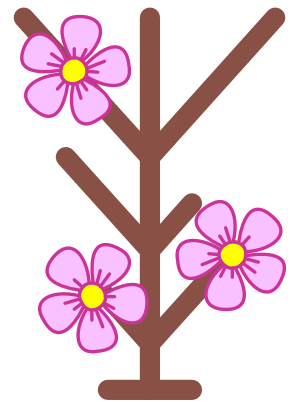
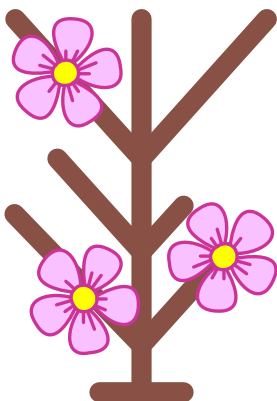
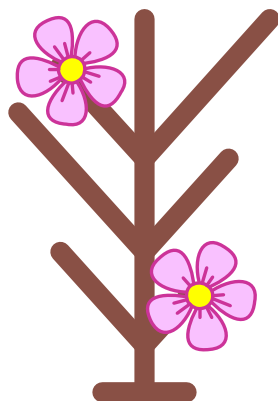
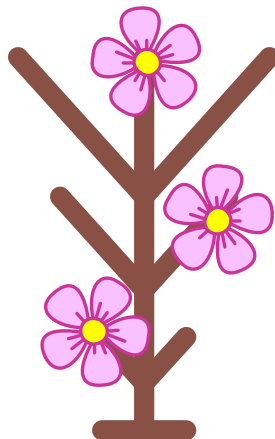
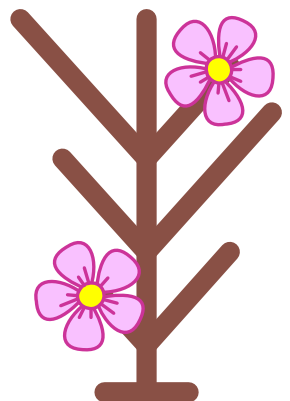
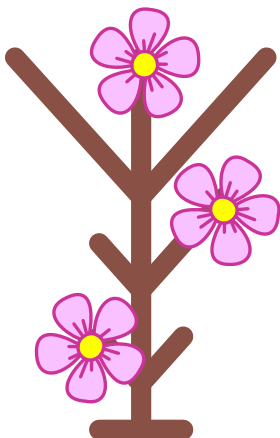
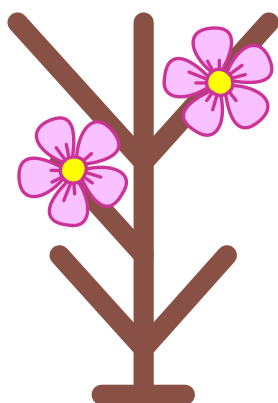
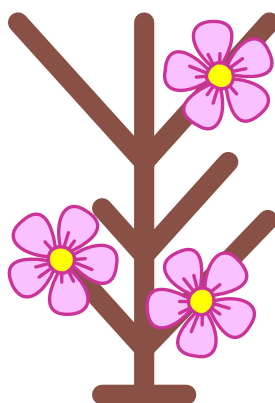
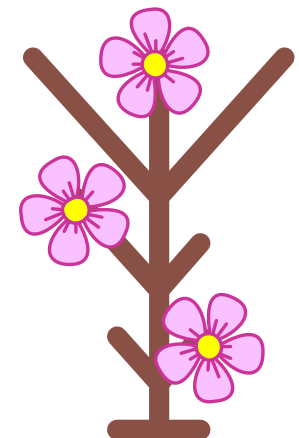
$$f = e - v + 2$$

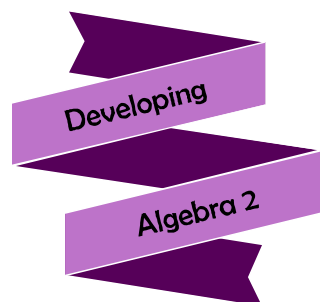
$$4x^2 + 2x - 6$$

$$(2x - 4)(x + 3) = 0$$

Puzzle

Can you find the mirrored copy of each picture?

**1****2****3****4****5****6****7****8****9****10****11****12**



Reflection Sheet

Name:

Percentage in the test:

I know this. 	I need to revise this. 	Question in the test	Correct in the test?
		1	
		3, 4	
		2	

I can **factorise** simple expressions such as $8x + 12$ or $15x^2 - 10$.

I can **re-arrange a formula** in order to make a specific variable (e.g. x) the **subject** of the formula.

I can recognise **expressions, equations** and **formulae**.



The Mathematics Department

10

Measuring

Solids

Foundation Tier

Version
3.0
08/2025

Name:

Contents

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Solids	Volume and surface area of a cuboid. Volume of a prism. Volume of a cylinder.	3
Composite Solids	Volume of composite solids.	9
Similar Shapes	Calculating the scale factor. Calculating missing lengths. Similar or not?	11



Solids

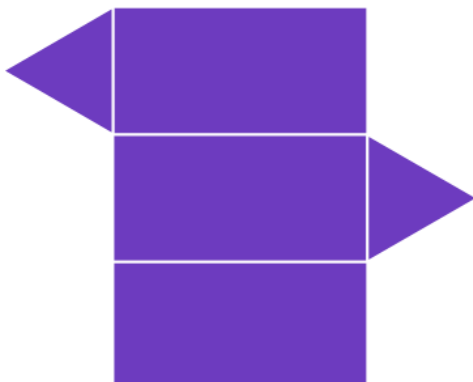
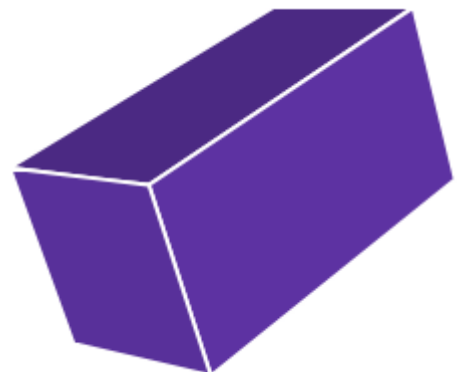
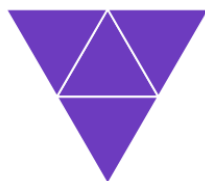
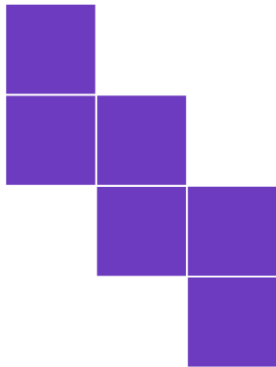
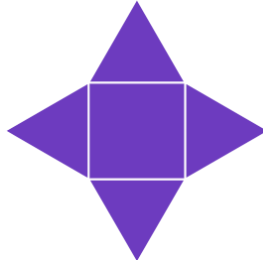
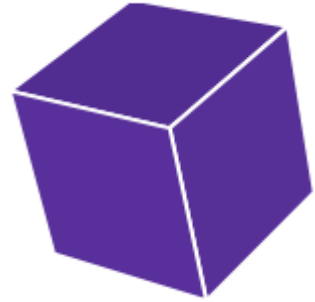
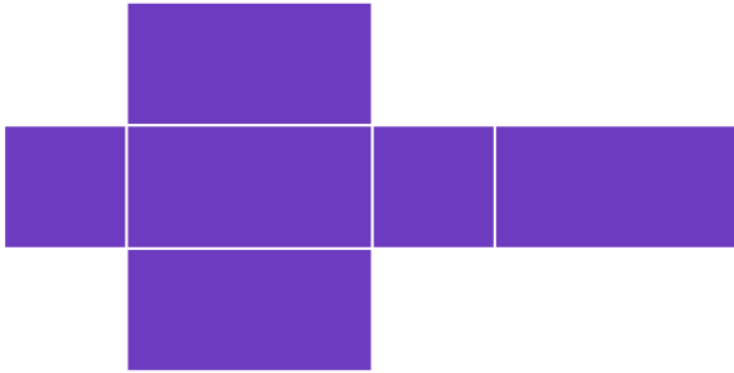
Exercise 1

Pair the solid with its matching net. Write the name of the solid underneath it.



Skill

3



In this chapter, we will discuss how to calculate the volume and surface area of a variety of different solids.

The **volume** measures how much space a solid occupies or uses. It's measured in cube units, e.g. cm^3 .

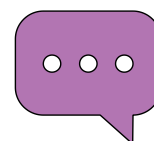
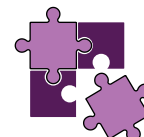
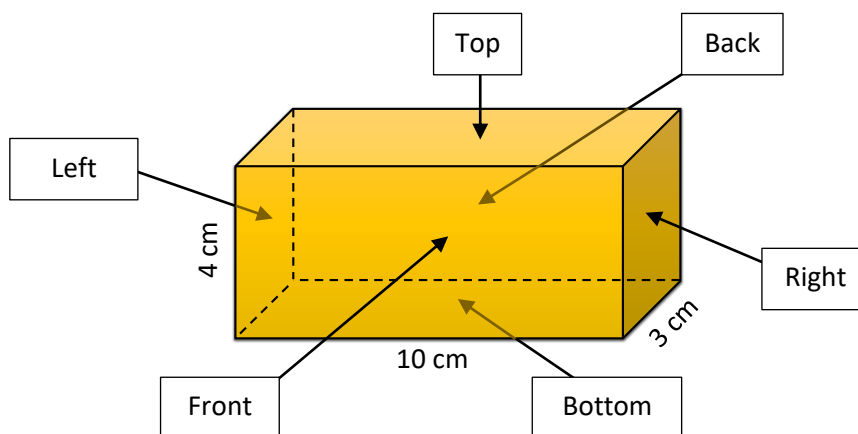
The **surface area** measures the area of the outside of the solid. We can think of the surface area as how much paper you would need to wrap the solid. Surface area is measured in square units, e.g. m^2 .

Cuboid

We've previously seen the formula to **calculate** the volume of a cuboid in the "Measuring Shapes 1" workbook.

Volume of a Cuboid = Length $\times$ Width $\times$ Height

To calculate the **surface area** of a cuboid, we must add the areas of each of the six faces.



Example

The volume of the above cuboid is $10 \times 3 \times 4 = 120 \text{ cm}^3$.

The surface area of the cuboid is the total area of the six faces.

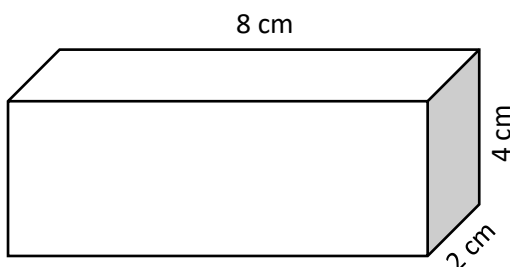
Front	$10 \times 4 = 40$
Back	40
Left	$3 \times 4 = 12$
Right	12
Top	$10 \times 3 = 30$
Bottom	30
Total	164 cm^2

Notice that some of the areas are the same so we do not need to calculate them.

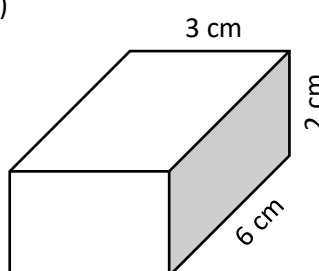
Exercise 2

Calculate the volume and surface area of the following cuboids.

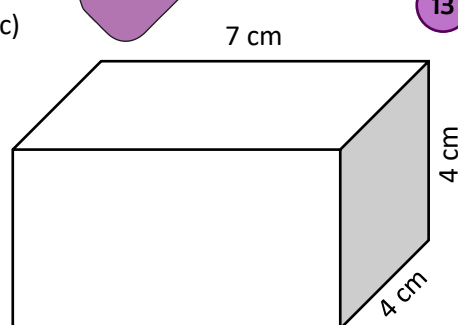
(a)



(b)



(c)



Skill

13

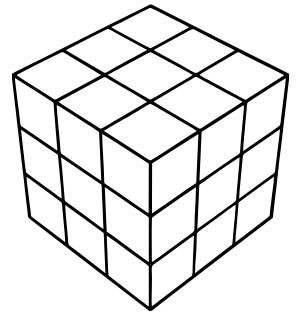
Exercise 3

The diagram shows a Rubik's cube before adding the coloured stickers. The dimensions of one of the small cubes is $2\text{ cm} \times 2\text{ cm} \times 2\text{ cm}$.

- What is the volume of one of the small cubes?
- How many small cubes form the Rubik's cube?
- What is the volume of the Rubik's cube?
- What is the surface area of the Rubik's cube?
- How many small $2\text{ cm} \times 2\text{ cm}$ stickers are needed to be stuck on the Rubik's cube?

**Applying**

13

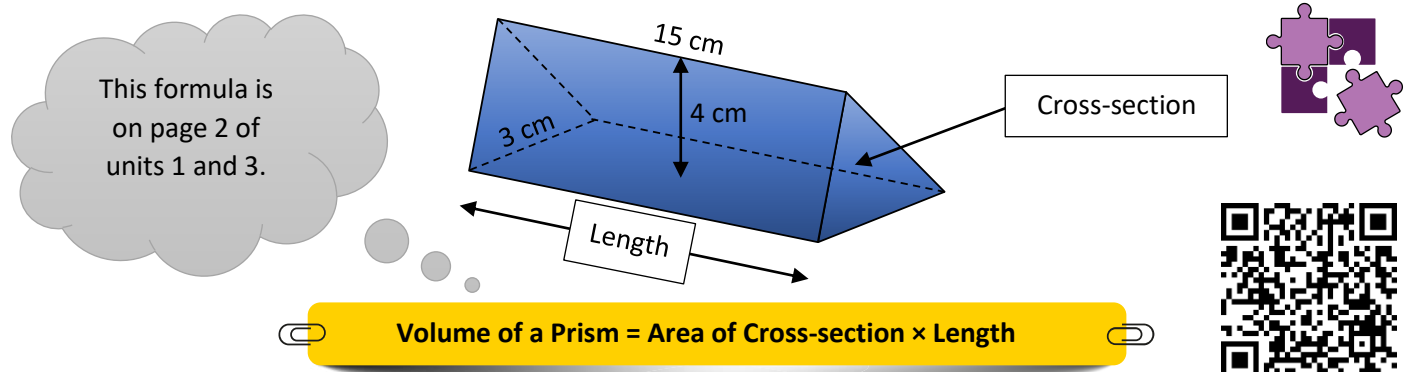
**Exercise 4**

The diagram on the right shows a floor plan for a living room.

- What is the volume of the living room?
- What is the area of the floor?
- The wood for the floor costs £14 per square metre. What was the cost of the wood for the whole floor?
- Cerys wishes to repaint the wall on which the clock hangs. Given that the door measures 75 cm by 2 m , what area will need repainting?
- To repaint the wall, Cerys buys a 2.5 litre tin of paint. The tin states that the paint covers up to 10 m^2 of area for each litre of paint. Will Cerys have enough paint to give the wall two coats of paint?

**Prism**

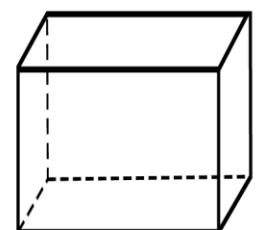
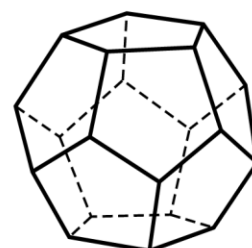
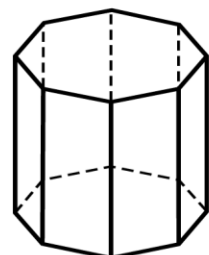
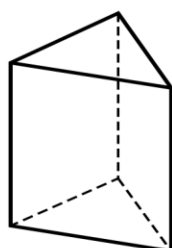
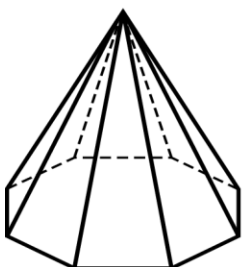
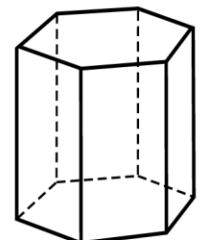
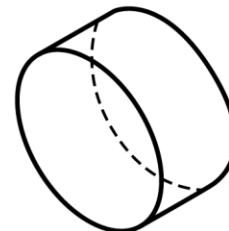
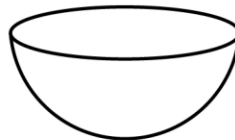
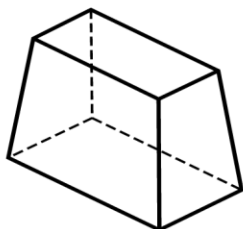
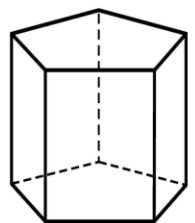
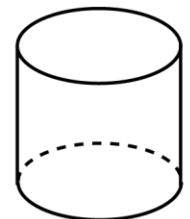
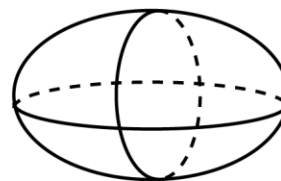
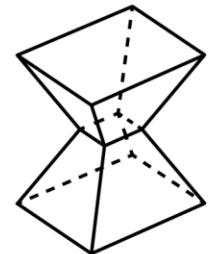
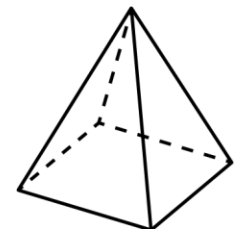
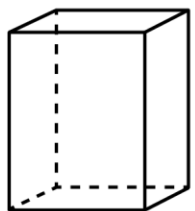
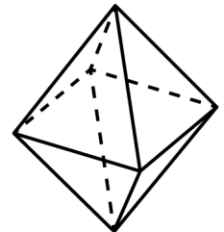
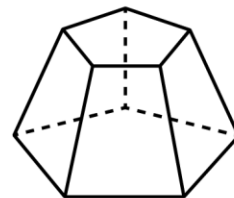
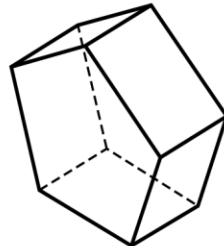
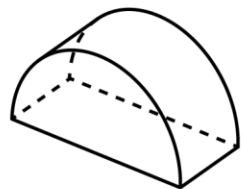
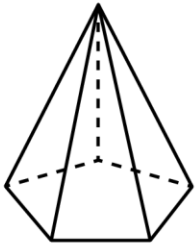
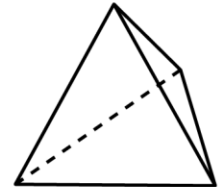
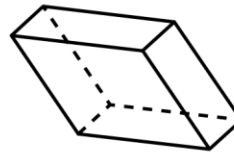
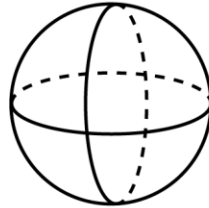
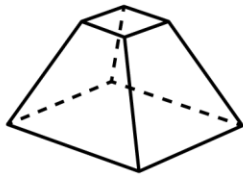
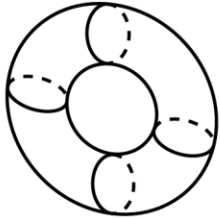
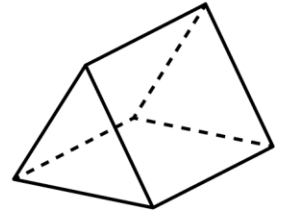
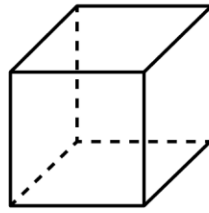
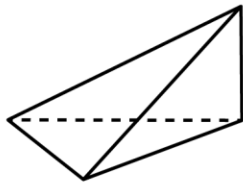
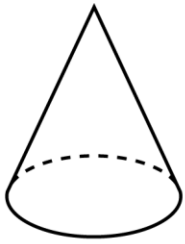
A prism is a solid in which both ends are identical and the cross-section at any point is identical to the two ends. The shape of the two ends gives the prism its name. For example, the diagram below shows a triangular prism.

**Example**

For the above triangular prism, the area of the cross-section is the area of the triangle, which is $\frac{3 \times 4}{2} = 6\text{ cm}^2$. Therefore the volume of the prism is $6 \times 15 = 90\text{ cm}^3$.

Exercise 5

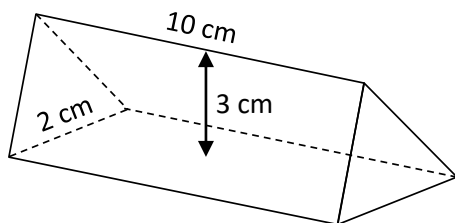
Circle the solids below that are prisms.

**Skill****A**

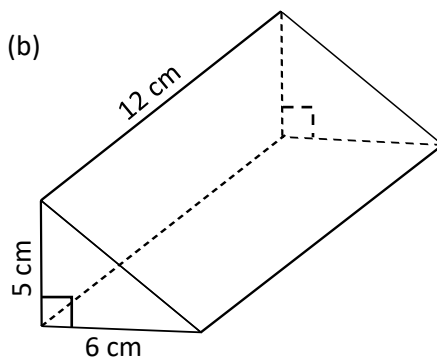
Exercise 6

Calculate the volume of the following prisms.

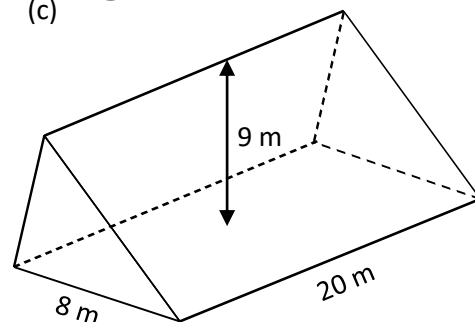
(a)



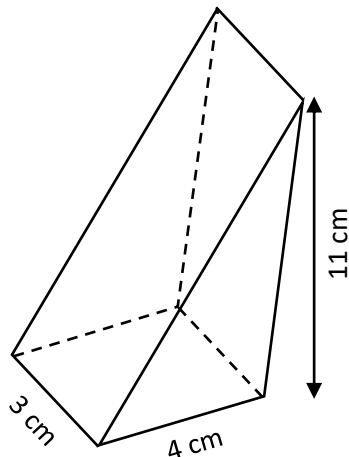
(b)



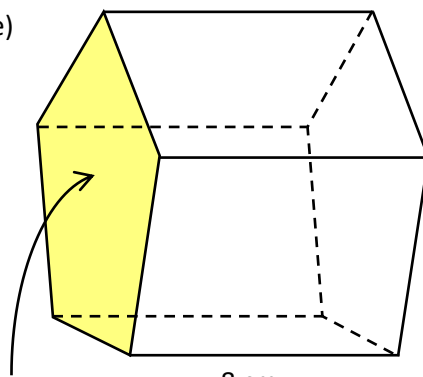
(c)



(d)

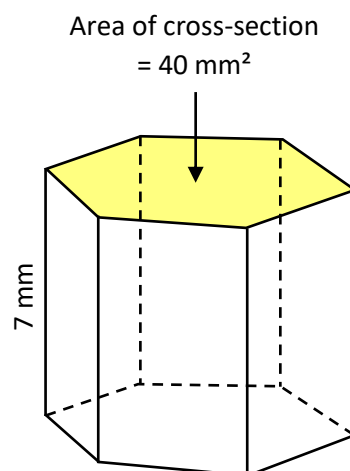


(e)



Area of cross-section
= 50 cm^2

(f)



Area of cross-section
= 40 mm^2

Challenge!

Calculate the surface area of the prism in question (b) above. Give your answer to 1 decimal place.

Exercise 7**Applying**

The picture on the right shows a water fountain.

The depth of the water in the lower part of the fountain is 40 cm.

The area of the cross-section of the water is $38,000 \text{ cm}^2$.

(a) What is the volume of the water in the fountain, in cm^3 ?

(b) What is the volume of the water in the fountain, in ml?

(c) What is the volume of the water in the fountain, to the nearest litre?

(d) Buddug wants to empty the water fountain so that it can be cleaned. The water pump that Buddug uses to empty the fountain works at a rate of 100 litres per minute. To the nearest minute, how long will it take for the pump to empty the fountain?

**Exercise 8**

What is the name of the prism that has the following shapes forming the cross-section?

(a) Rectangle

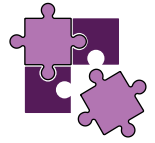
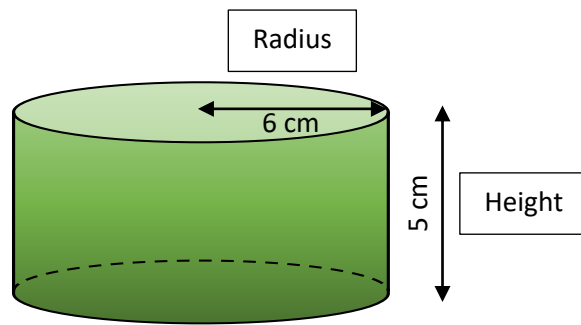
(b) Square

(c) Circle

Skill**A**

Cylinder

A cylinder is a special type of prism where the cross-sectional shape is a circle.



Volume of a Cylinder = Area of the circle $\times$ Height or width of the cylinder

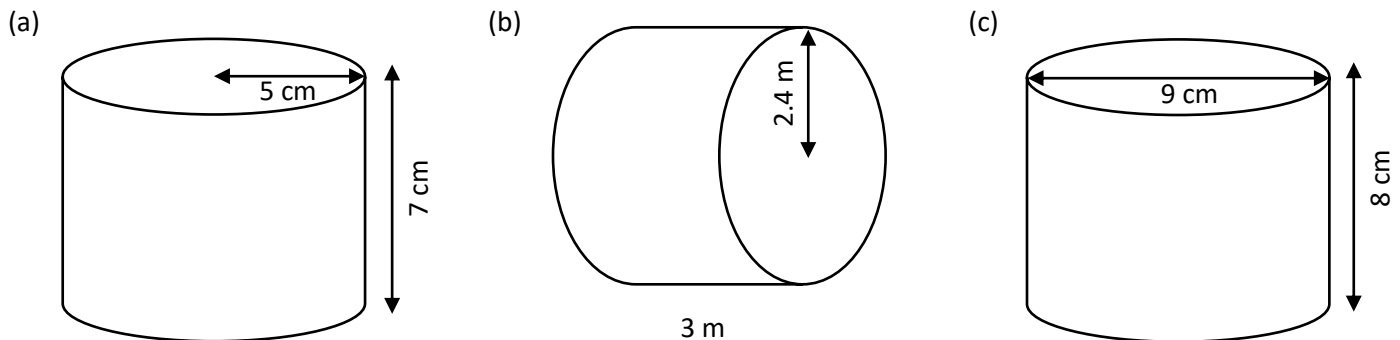
$$\text{Volume of a Cylinder} = \pi \times \text{Radius}^2 \times \text{Height}$$

Example

The volume of the above cylinder is $\pi \times 6^2 \times 5 = 565.49 \text{ cm}^3$, correct to two decimal places.

Exercise 9

Calculate the volume of the following cylinders.

**Exercise 10**

The picture on the right shows an apple cake that has been baked in a baking tin.

- (a) Given that the radius of the tin is 10 cm and its height is 6 cm, calculate the volume of the cake in the tin.
- (b) The cake weighs 1.2 kg. Gwenda wants to cut the cake into equal pieces so that each piece weighs 150 g. How many equal pieces will Gwenda need to cut?
- (c) What is the volume of each of the pieces of cake from part (b)?

**Exercise 11**

A hot water tank is in the shape of a cylinder. The height of the tank is 90 cm, and its diameter is 45 cm.

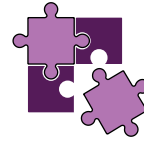
The manufacturer estimates that the tank holds 140 litres of water. Has the manufacturer provided an overestimate or an underestimate?



Composite Solids



A **composite solid** is a solid we can split into simpler solids, like the solids considered in the first chapter of this workbook.



Example

It is possible to split the composite solid on the right into a cuboid (on the left) and a triangular prism (on the right).

Cuboid

$$\begin{aligned}\text{Volume} &= \text{Length} \times \text{Width} \times \text{Height} \\ &= 4 \times 6 \times 8 \\ &= 192 \text{ cm}^3\end{aligned}$$

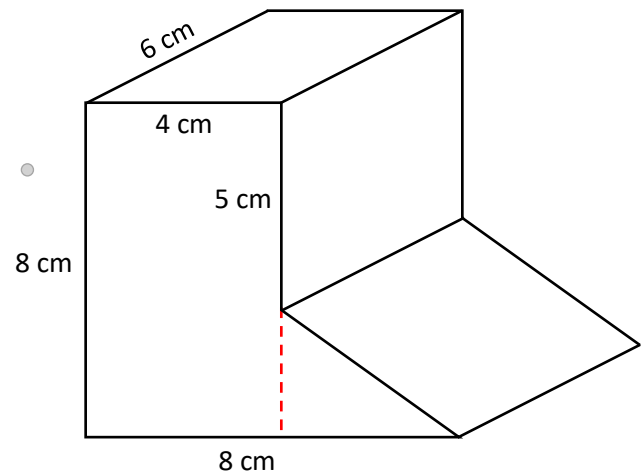
Triangular Prism

$$\begin{aligned}\text{Volume} &= \text{Area of Cross-section} \times \text{Length} \\ &= \left(\frac{4 \times 3}{2}\right) \times 6 \\ &= 36 \text{ cm}^3\end{aligned}$$

Composite Solid

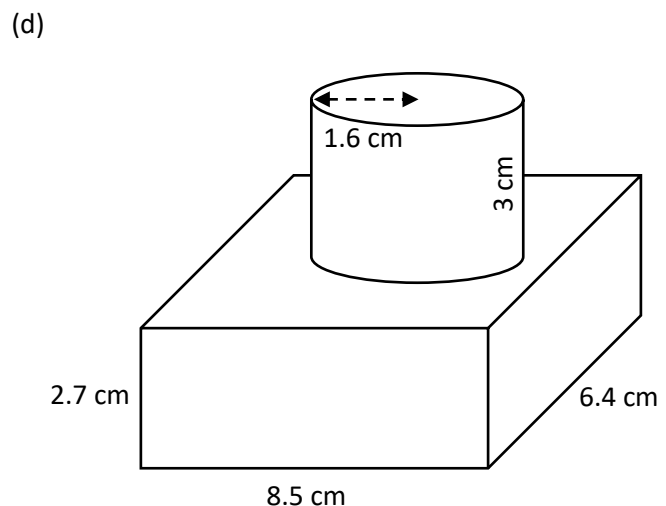
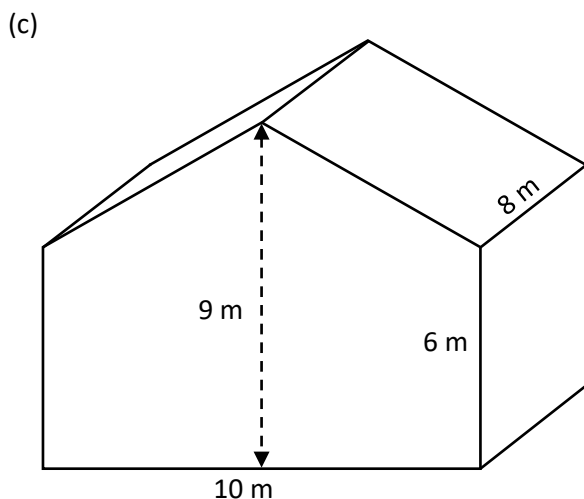
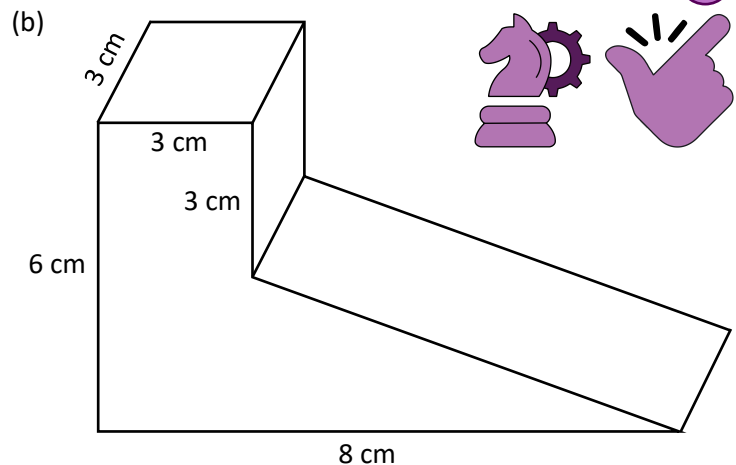
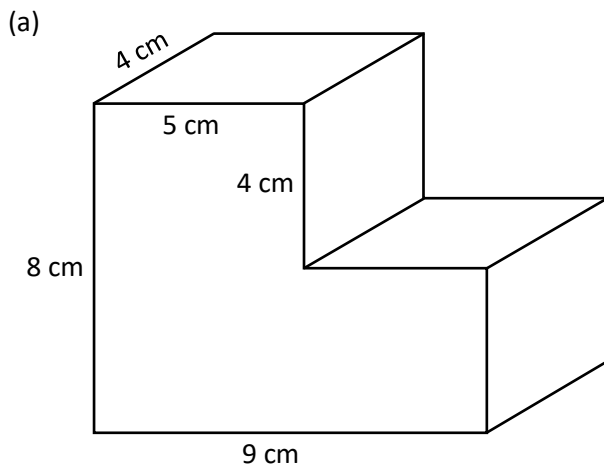
$$\begin{aligned}\text{Volume} &= 192 + 36 \\ &= 228 \text{ cm}^3\end{aligned}$$

It is also possible to treat this solid as one large prism.



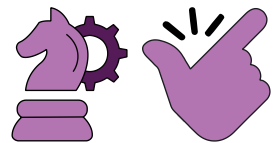
Exercise 12

Calculate the volume of the following composite solids.



Skill

13



Exercise 13

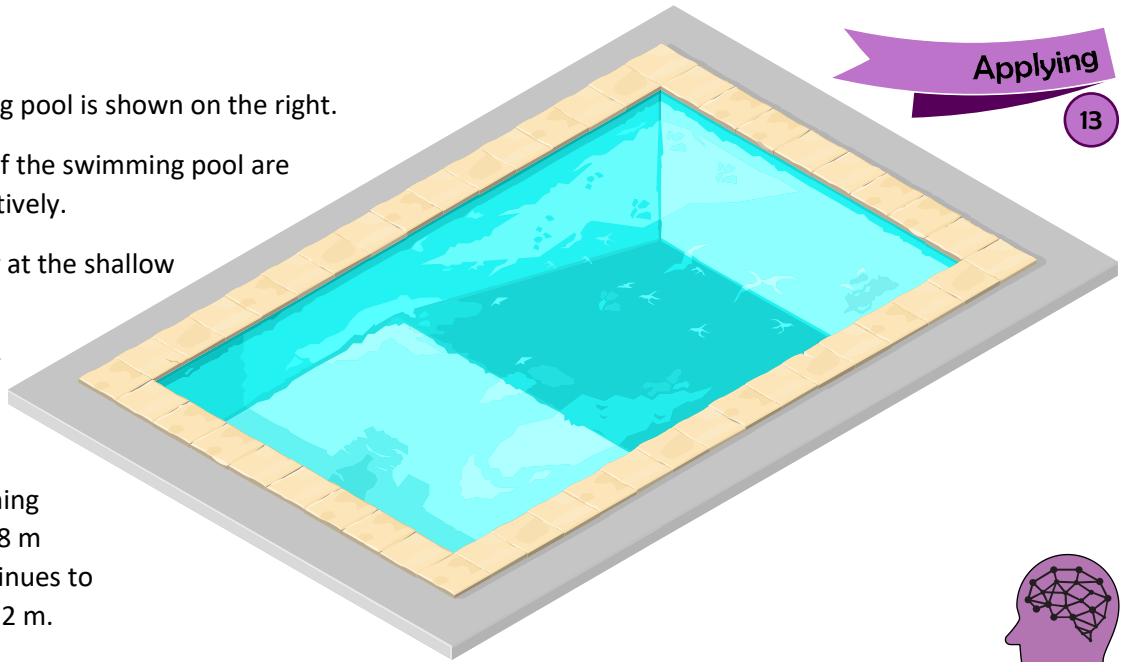
A picture of a swimming pool is shown on the right.

The length and width of the swimming pool are 10 m and 20 m, respectively.

The depth of the water at the shallow end is 1 m.

The depth of the water at the deep end is 2.5 m.

The floor of the swimming pool starts to descend 8 m into the pool, and continues to descend for a total of 12 m.



Applying

13



(a) Calculate the cross-sectional area of the swimming pool.

(b) Calculate the volume of the swimming pool, in m^3 .

(c) Given that there is 1,000 litres of water for every 1 m^3 of volume, calculate how much water is in the pool.

(d) An ordinary hose pipe supplies water at a rate of 10 litres a minute. How long would it take for one hose pipe to fill the swimming pool? Give your answer correct to the nearest day.

Exercise 14

The picture on the right shows some Polo Mint sweets.

- The diameter of each Polo Mint is 1.9 cm.
- The diameter of the hole in the middle is 0.8 cm.
- The thickness of each Polo Mint is 0.4 cm.

(a) Calculate the cross-sectional area of one Polo Mint.

(b) Calculate the volume of one Polo Mint.

(c) Usually, there are 23 Polo Mints in a packet.

(i) What is the volume of sweets in a whole packet?

(ii) What is the volume of the hole in the middle of a packet?

(d) What is the minimum amount of paper needed to make the label around the packet?

(e) What is the minimum amount of foil needed for each packet?



Similar Shapes

Imagine using a photocopier to enlarge a diagram drawn on a piece of paper. If the original paper is A4 sized and the paper that comes out of the photocopier is A3 sized, then you have created a copy of the diagram twice the size.

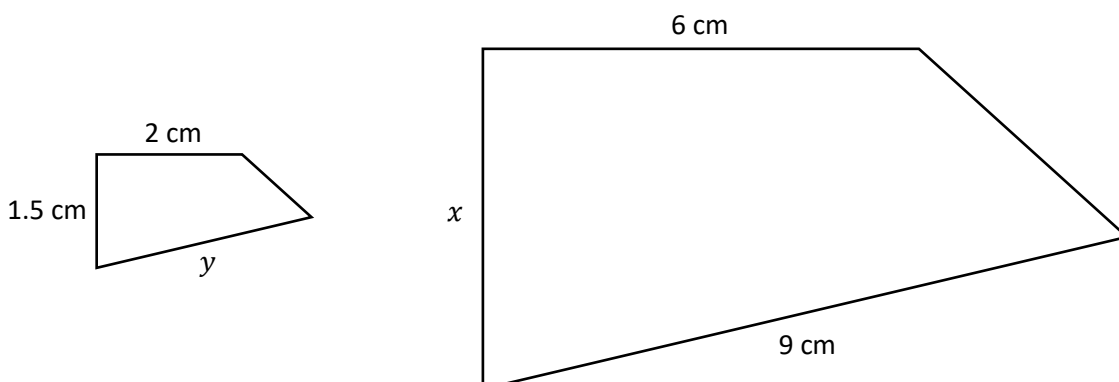
In mathematics, we say that the new diagram is **similar** to the original diagram. This means that the new diagram has the same **shape**, but the **size** has changed. Since the new diagram is twice as big, we can say that the **scale factor** is 2.

Similar shapes are the exact same shape, but of different sizes.

Given two shapes that are similar, we can use the measurements on the shapes either to find the scale factor or to find some missing lengths on the shapes.

Example

The two shapes shown below are similar shapes. Use the measurements on the shapes to find the lengths x and y .



Answer: The first step is to find the **scale factor**. To do this, we need to consider how much bigger the large shape is compared to the small shape. We see that the horizontal lengths at the top of both shapes are given. We can use these two lengths to find the scale factor, by calculating $6 \div 2 = 3$. Therefore, the large shape is three times bigger than the small shape.

Having found the scale factor of 3, we can now use it to find the lengths x and y .

The edge that corresponds to the x edge in the small shape is 1.5 cm. We must **multiply** 1.5 cm by the scale factor to find the length of x , since we are going from the small shape to the large shape. Therefore

$$\begin{aligned} x &= 1.5 \times 3 \\ x &= 4.5 \text{ cm} \end{aligned}$$

The edge that corresponds to the y edge in the large shape is 9 cm. We must **divide** 9 cm by the scale factor to find the length of y , since we are going from the large shape to the small shape. Therefore

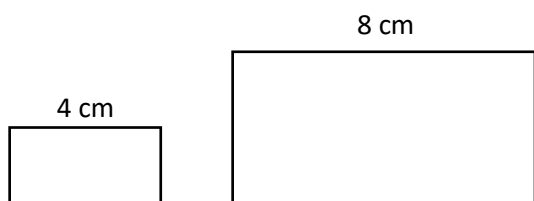
$$\begin{aligned} y &= 9 \div 3 \\ y &= 3 \text{ cm} \end{aligned}$$



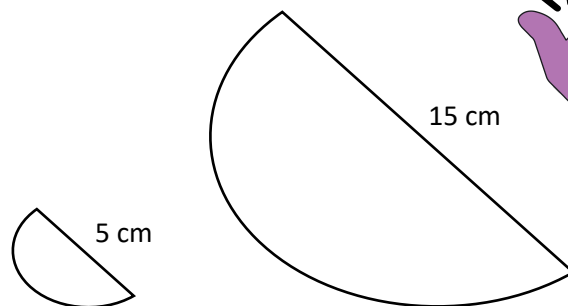
Exercise 15

The following pairs of shapes are similar shapes. Use the measurements on the shapes to find the scale factor.

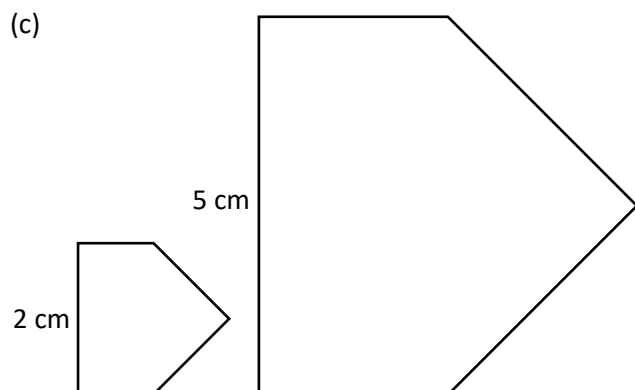
(a)



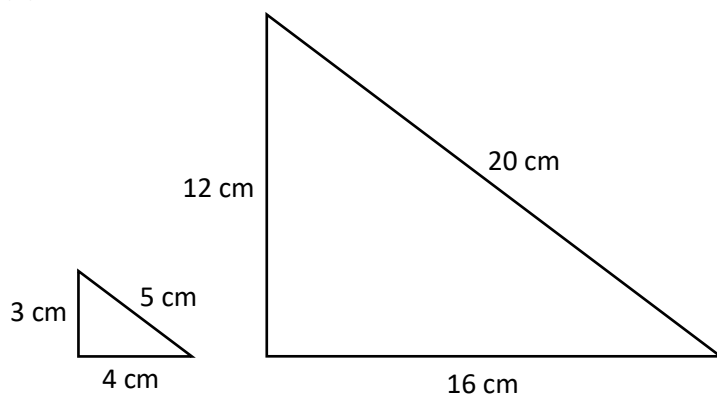
(b)



(c)

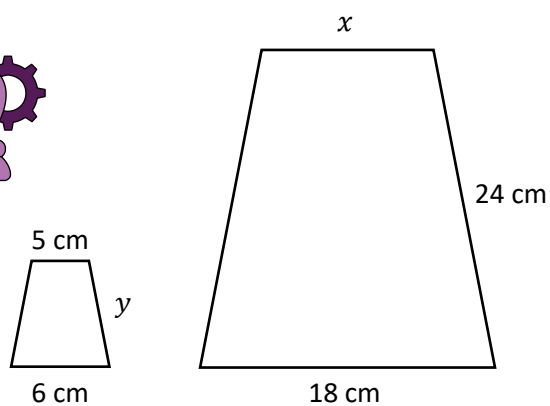


(d)

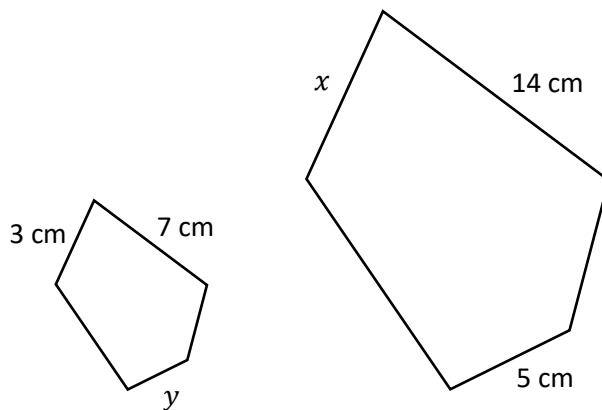
**Exercise 16**

The following pairs of shapes are similar shapes. Use the measurements on the shapes to find the lengths x and y .

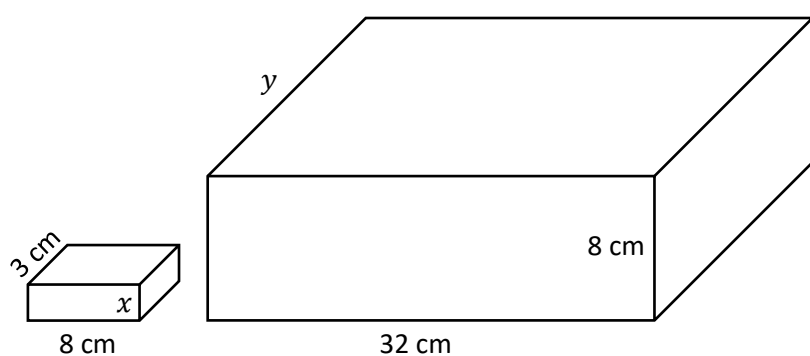
(a)



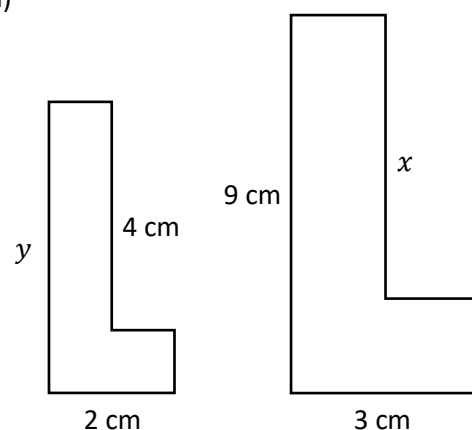
(b)



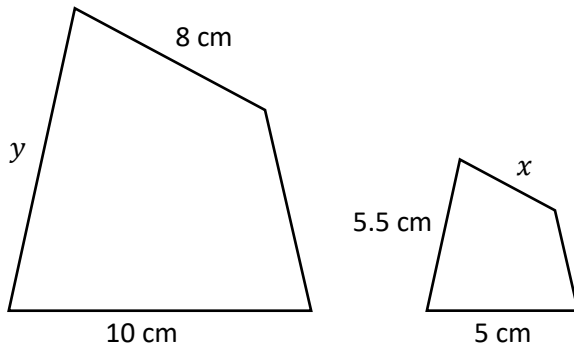
(c)



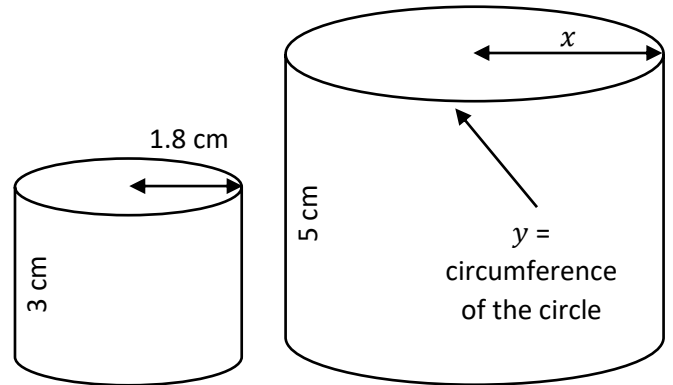
(d)



(e)

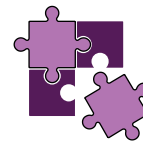


(f)

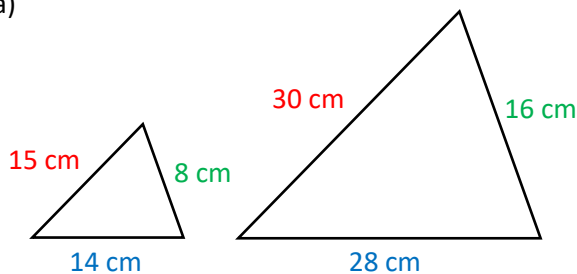
**Similar or not?**

If two shapes are similar, then the **corresponding edges are in the same ratio**.

This means that if we divide a pair of corresponding edges, we will always obtain the same answer.

**Example**

(a)



For the two triangles above, the corresponding edges are in the same ratio.

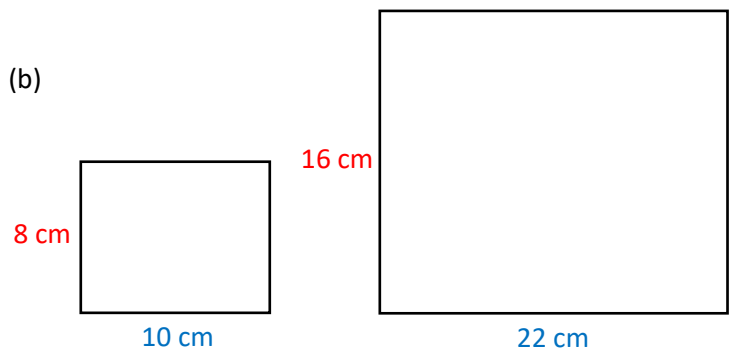
$$30 \div 15 = 2$$

$$28 \div 14 = 2$$

$$16 \div 8 = 2$$

Therefore the two triangles are similar.

(b)



For the two rectangles above, the corresponding edges are not in the same ratio.

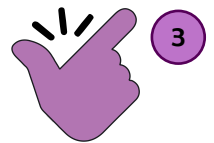
$$16 \div 8 = 2$$

$$22 \div 10 = 2.2$$

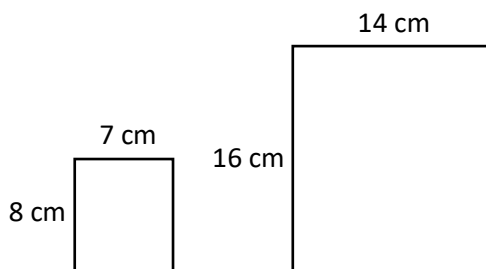
Therefore, the two rectangles are not similar.

Exercise 17

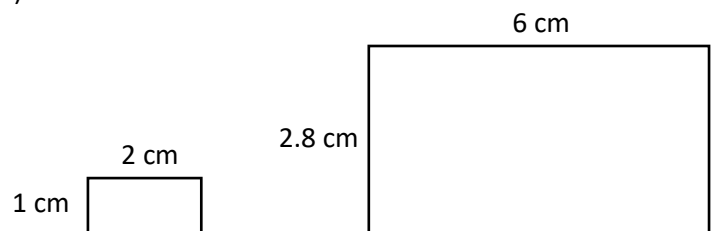
Decide whether the following pairs of shapes are similar or not.



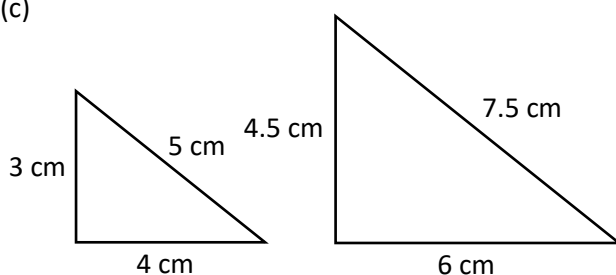
(a)



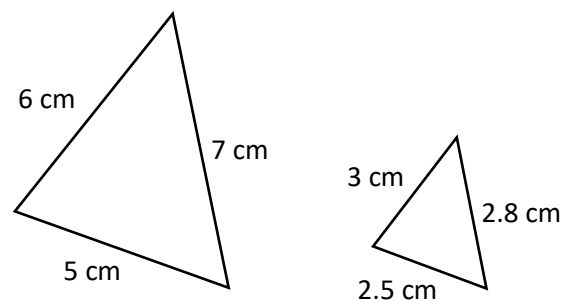
(b)

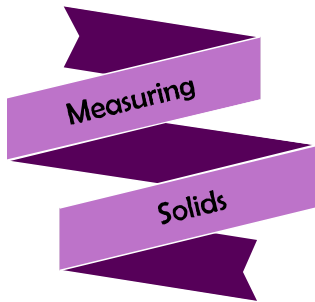


(c)



(d)





Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I know how to calculate the volume and surface area of a cuboid .			1	
I know how to calculate the volume of a prism .			2, 7	
I know how to calculate the volume of a cylinder .			3, 7	
I can calculate the volume of composite solids .			4	
Given two similar shapes, I can calculate the scale factor .			5	
Given two similar shapes, I can calculate missing lengths .			5	
I can recognise whether two triangles are similar .			6	



The Mathematics Department

10

Accuracy of

Measurements

Foundation Tier

Version

3.0

08/2025

Name:

Contents

Chapter	Mathematics	Page Number
Rounding Off: Significant Figures	Rounding a number off to a specified number of significant figures. Labelling the significant figures of a number.	3
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Appropriate Degree of Accuracy	Calculating answers to an appropriate degree of accuracy.	10
Compound Measures	Distance, time and speed. Population, area and population density. Mass, volume and density. Other compound measures.	11



Significant Figures

Sarah sees a television for sale on the internet, for £389.

Delyth, who is Sarah's friend, says that the television costs around £400. Greta, another one of Sarah's friends, says that the television costs around £390.

Sarah's two friends have both given an **estimate** for the price of the television. Delyth has given her estimate to **one significant figure**, whilst Greta has given her estimate to **two significant figures**.



£389

When rounding off to **one significant figure**, we need to find an **estimate** that, **at most**, uses **one non-zero digit**.

When rounding off to **two significant figures**, we need to find an **estimate** that, **at most**, uses **two non-zero digits**.

(And so on for three significant figures; four significant figures; ...)



Example

Price	The price rounded off to one significant figure
£389	£400
£1,799	£2,000
£2.35	£2
£0.41	£0.40

Exercise 1

Round off the following prices to one significant figure.

- | | | | |
|------------|------------|-------------|--------------|
| (a) £799 | (b) £899 | (c) £999 | (d) £1,099 |
| (e) £1,846 | (f) £1,500 | (g) £28,381 | (h) £370,000 |
| (i) £4.19 | (j) £7.93 | (k) £16.70 | (l) £42.99 |
| (m) £0.84 | (n) £0.37 | (o) £0.13 | (p) £0.04 |

Exercise 2

Round off the following numbers to one significant figure.

- | | | | |
|------------|-------------|------------|-----------|
| (a) 329 | (b) 698.2 | (c) 450 | (d) 975 |
| (e) 19 | (f) 14 | (g) 7.3 | (h) 2.8 |
| (i) 0.54 | (j) 0.87 | (k) 1.56 | (l) 0.98 |
| (m) 24,528 | (n) 543,590 | (o) 19,829 | (p) 9,502 |



Example

Price	The price rounded off to two significant figures
£389	£390
£1,799	£1,800
£2.35	£2.40
£0.41	£0.41

Exercise 3

A

Round off the following prices to two significant figures.

- (a) £263 (b) £268 (c) £265 (d) £299
- (e) £4,368 (f) £2,809 (g) £54,238 (h) £876,000
- (i) £6.37 (j) £1.62 (k) £14.28 (l) £27.99
- (m) £1.95 (n) £19.99 (o) £0.13 (p) £0.04

**Exercise 4**

Round off the following numbers to two significant figures.

- (a) 836 (b) 278.5 (c) 913 (d) 976
- (e) 76.3 (f) 16.07 (g) 7.92 (h) 5.37
- (i) 0.283 (j) 0.927 (k) 1.8203 (l) 26.07
- (m) 82,573 (n) 903,400 (o) 49,732 (p) 997

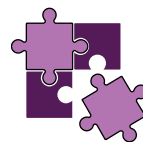
Exercise 5

Complete the following table.

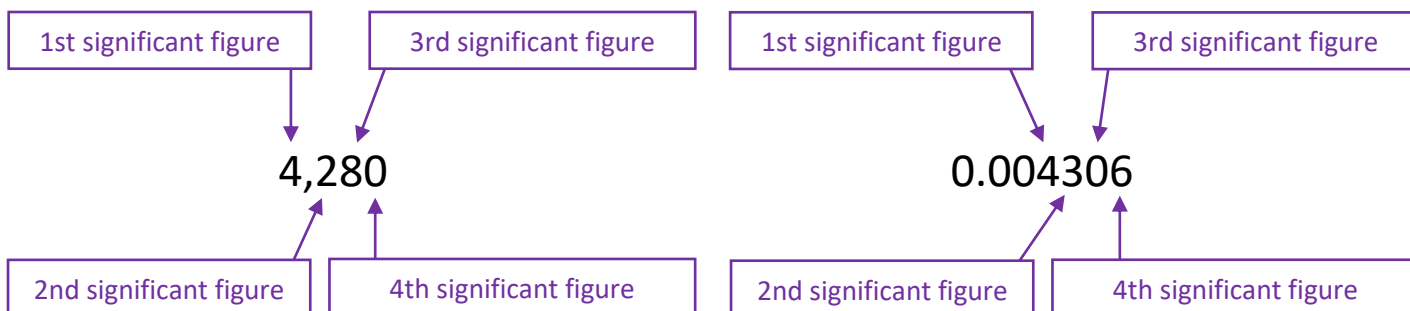
	Original number	Round off to one significant figure	Round off to two significant figures	Round off to three significant figures	Round off to four significant figures
(a)	28,539				
(b)	54.591				
(c)	3.0589				
(d)	1,339				
(e)	106,728				
(f)	4,525,925				
(g)	73,524,000				
(h)	183.078				
(i)	8,406.5				
(j)	0.87325				
(k)	0.018393				
(l)	0.0083736				

Labelling the Significant Figures of a Number

When reading a number from left to right, the **first** significant figure is the first digit encountered that is **not zero**. The other significant figures follow on immediately after the first significant figure.



Example



Exercise 6

For the following numbers, circle the **first** significant figure.

- | | | | |
|------------|-----------|---------------|-----------------|
| (a) 8,205 | (b) 934.2 | (c) 387 | (d) 46 |
| (e) 0.0378 | (f) 0.982 | (g) 0.0400213 | (h) 0.000029002 |

Exercise 7

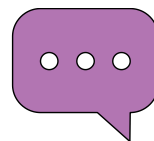
For the following numbers, circle the **second** significant figure.

- | | | | |
|------------|---------------|---------------|--------------------|
| (a) 43,503 | (b) 2,043,293 | (c) 819 | (d) 8.32 |
| (e) 0.0729 | (f) 0.02083 | (g) 0.0004893 | (h) 0.000000072009 |

Sometimes, we need to add zeroes to the end of a number to write it correct to a specified number of significant figures. For example, to write 45 correct to three significant figures, we must write 45.0.

Exercise 8

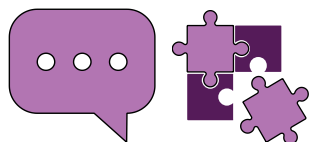
- | | |
|---|--|
| (a) Write 87 correct to three significant figures. | (b) Write 987 correct to four significant figures. |
| (c) Write 3,270 correct to six significant figures. | (d) Write 7 correct to eight significant figures. |



Key words	Corrections	I am happy with...	I need to revise...

Estimation

Dafydd wants to calculate the sum $\frac{482.6 \times 43.5}{2168.12}$, but unfortunately he has lost his calculator. In order to find an approximate value for the correct answer, it would be possible for Dafydd to **estimate** the size of the numbers in the calculation, and perform the calculation with these estimates.



$$\begin{aligned} \frac{482.6 \times 43.5}{2168.12} &\approx \frac{500 \times 40}{2000} \\ &= \frac{20000}{2000} \\ &= 10 \end{aligned}$$



Let us now look at the different parts of the above estimation, explaining each part in turn.

- 1) The first line contains the estimation symbol $\approx$. We say this symbol as 'is around' or 'is approximately', and we need to include the symbol (instead of the equals sign $=$) as the left-hand side is not *equal* to the right-hand side.
- 2) Unless common sense suggests otherwise, the convention for estimating numbers is to round them off to **one significant figure**. So, 482.6 rounds off to 500; 43.5 rounds off to 40; and 2168.12 rounds off to 2000.
- 3) Going from the first line to the second line, we perform the calculation $500 \times 40 = 20000$. To perform this sum quickly, note that $5 \times 4 = 20$, and that the **three zeroes** on the left-hand side lead to **three similar zeroes** on the right-hand side.
- 4) Going from the second line to the third line, we perform the calculation $\frac{20000}{2000} = 10$. To perform this sum quickly, note that three of the zeroes **cancel** on the top and bottom: $\frac{20000}{2000} = \frac{20}{2} = 10$. (The process of 'cancelling' zeroes comes from the fact that we can simplify the fraction by dividing the numerator and denominator by 10 each time.)

Exercise 9

Find an estimate for each of the following calculations.

- | | | |
|-----------------------------|------------------------|----------------------------|
| (a) 423×297 | (b) $19 \times 43,926$ | (c) $876 \times 6,875$ |
| (d) $97 \times 1,024$ | (e) 6.4×2.9 | (f) $0.98 \times 60,536.1$ |
| (g) $75,249 \times 342,192$ | (h) $198 \times 5,034$ | (i) $78 \times 452,138$ |

Exercise 10

Find an estimate for each of the following calculations.

- | | | |
|----------------------------|--------------------------------|------------------------------|
| (a) $\frac{7,924}{403}$ | (b) $\frac{324}{18.4}$ | (c) $\frac{879,054}{12,456}$ |
| (d) $\frac{5,783}{1,985}$ | (e) $\frac{98,160}{502}$ | (f) $\frac{87,204}{2,642}$ |
| (g) $\frac{82,457}{23.46}$ | (h) $\frac{7,304,539}{67,045}$ | (i) $\frac{614}{28}$ |

Exercise 11

Find an estimate for each of the following calculations.

- | | | |
|-------------------------------------|-------------------------------------|---|
| (a) $\frac{798 \times 403}{1,980}$ | (b) $\frac{1,975 \times 435}{787}$ | (c) $\frac{61 \times 90,128}{2,975}$ |
| (d) $\frac{3.24 \times 648}{895}$ | (e) $\frac{78,420}{204 \times 4.8}$ | (f) $\frac{4.9 \times 26.4 \times 7.4}{2.4601}$ |
| (g) $\frac{94.2 \times 398}{552.4}$ | (h) $\frac{0.13 \times 783}{19}$ | (i) $\frac{4.15 \times 5.63}{0.53}$ |

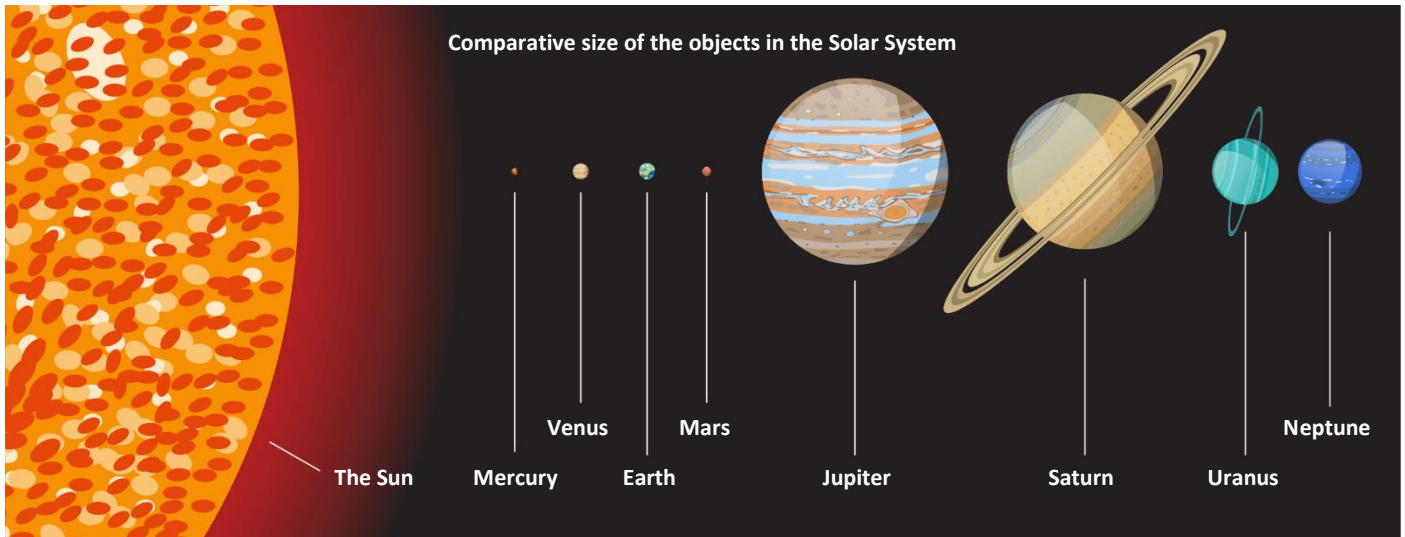


Exercise 12**Applying**

2



- (a) A t-shirt costs £5.99 and a skirt costs £12.99. Sophie wants to buy 2 t-shirts and 5 skirts. She has £50. Use an approximation to decide whether Sophie has enough money.
- (b) David saves £3.80 each week. Approximately how much does he save in a year?
- (c) Eleri has a mobile phone contract costing £29.99 a month. Approximately how much does Eleri pay over a year?



- (d) The diameter of the Sun is around 1,391,400 km, and the diameter of Earth is around 12,742 km. Approximately how many times bigger is the diameter of the Sun compared to the diameter of Earth?

- (e) John works in an office and earns £19.75 per hour. He works 39 hours a week. Find an estimate for John's weekly wage.

- (f) Someone has asked the pupils in year 9 to estimate the individual cost of a trip to watch a musical. There are 98 pupils in year 9.

A packed lunch for each pupil costs £3.95.

The total cost of the tickets to the musical is £915.

The transport costs £290.

Estimate the individual cost for each pupil to watch the musical, correct to the nearest pound. Show all of your method.

**Evaluation**

Key words	Corrections	I am happy with...	I need to revise...



In order to understand how to find **upper and lower bounds**, it's a good idea to first revise different techniques of **rounding off** a number.

Exercise 13

Complete the following table.



Revision

1

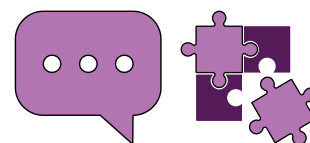
	Number	Round off to the nearest 10	Round off to 1 decimal place	Round off to 1 significant figure
E.g.	825.94	830	825.9	800
(a)	523.86			
(b)	49.15			
(c)	5,284.792			
(d)	3.67			
(e)	284.99			
(f)	43,704.75			
(g)	726			

Given a number rounded off in a particular way, we can consider what the original number was, before rounding occurred. For example, consider the number 470, which has been rounded off to the nearest 10. What could the original number have been? All the numbers between 465 and 474.9999... round off (to the nearest 10) to be 470. If x represents the original number, then we can say that

$$465 \leq x < 475.$$

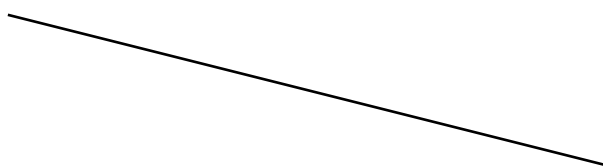
We can also write

$$x = 470 \pm 5$$



but we must remember that x cannot be exactly 475, since this number would round off to give 480. We say that the **lower bound** is 465, and the **upper bound** is 475.

One place where upper and lower bounds are used are when taking measurements, since **every measurement is an approximation**. For example, measure the line below with a ruler.



You should measure the line to be 8.2 cm, but is the line exactly 8.2 cm? How do you know that the line isn't truly 8.19 cm, or 8.21 cm? We would need a more accurate measuring instrument than a ruler to check this, therefore the best we can say (by using a ruler and our eyes) is that the line is 8.2 cm, correct to the nearest 0.1 cm (or mm).

The accuracy limit of a measurement, approximation or calculation uses upper and lower bounds. To measure the above line with a ruler, the lower bound is 8.15 cm and the upper bound is 8.25 cm. The true measurement lies between these two limits. If l denotes the true length of the line in cm, then we can say that

$$8.15 \leq l < 8.25$$

or

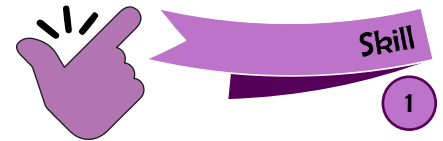
$$l = 8.2 \pm 0.05.$$

Example

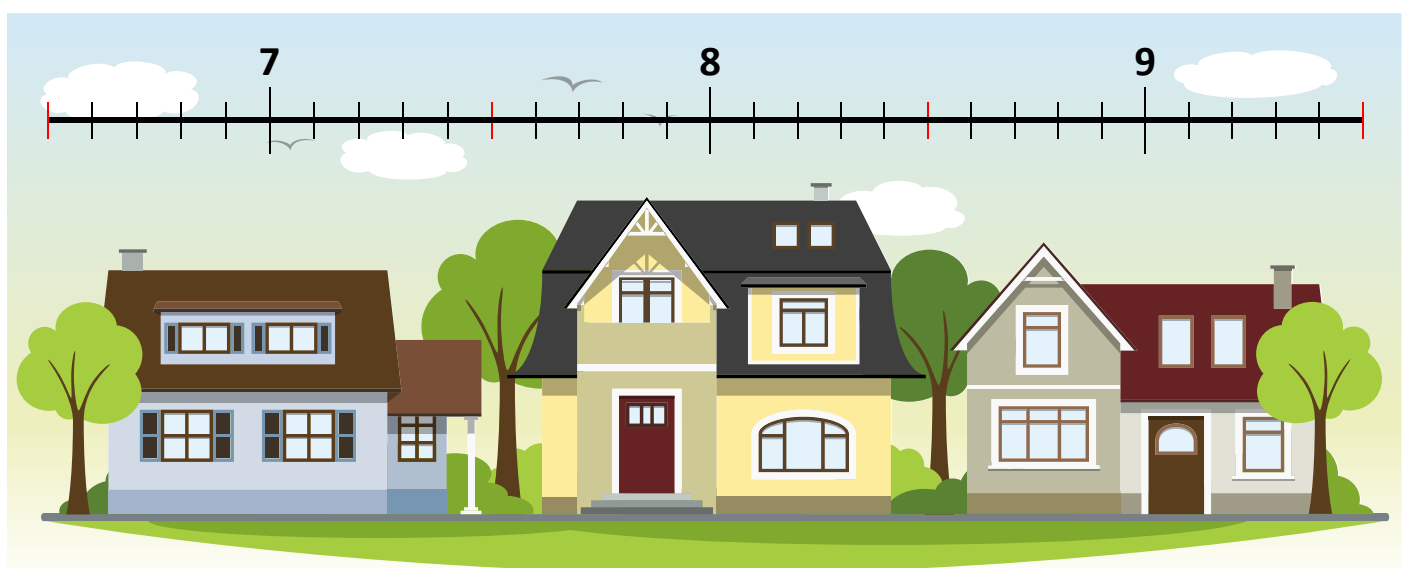
Measurement	Lower Bound	Upper Bound
42 cm (to the nearest cm)	41.5 cm	42.5 cm
85 km (to the nearest 5 km)	82.5 km	87.5 km
60 m (to the nearest 10 m)	55 m	65 m

Exercise 14

Complete the following table.



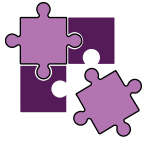
	Measurement	Lower Bound	Upper Bound
(a)	34 cm (to the nearest cm)		
(b)	5 m (to the nearest m)		
(c)	148 kg (to the nearest kg)		
(d)	25 cm (to the nearest 5 cm)		
(e)	420 ml (to the nearest 5 ml)		
(f)	1,825 km (to the nearest 5 km)		
(g)	40 m (to the nearest 10 m)		
(h)	80 cm (to the nearest 10 cm)		
(i)	180 g (to the nearest 10 g)		
(j)	400 cm (to the nearest 100 cm)		
(k)	5,400 m (to the nearest 100 m)		
(l)	27,800 litres (to the nearest 100 litres)		
(m)	8,000 km (to the nearest 1,000 km)		
(n)	45,000 miles (to the nearest 1,000 miles)		
(o)	3,000 tons (to the nearest 1,000 tons)		
(p)	6 cm (to the nearest even number)		
(q)	154 cl (to the nearest even number)		
(r)	4,250 ml (to the nearest even number)		
(s)	80 cm (to the nearest 20 cm)		
(t)	250 ml (to the nearest 50 ml)		
(u)	320 kg (to the nearest 40 kg)		

**Challenge!**

How does the above picture explain how to find upper and lower bounds?

Appropriate Degree of Accuracy

Measurements and calculations should not be too precise for their purpose. For example, advertising a television of size 39.97 inches would be too precise (it would be better to say that the television was 40 inches in size).

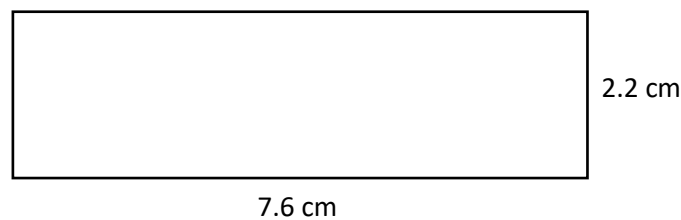


As a general rule, if a question asks you to give an answer to an **appropriate degree of accuracy**,

do not give an answer that is more precise than the values used in the calculation.

Example

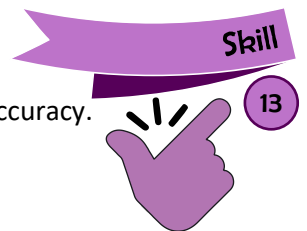
Calculate the area of the following rectangle. Give your answer to an appropriate degree of accuracy.



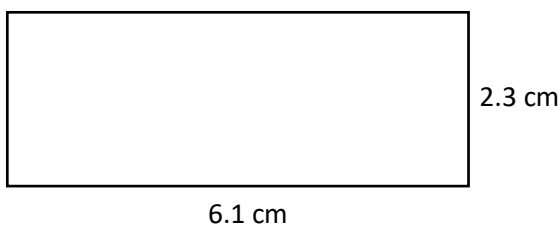
Answer: The area of the rectangle is $7.6 \times 2.2 = 16.72 \text{ cm}^2$. Because the numbers in the question have been rounded off to one decimal place, then our answer should also be rounded off to one decimal place. Therefore, the area of the rectangle (to an appropriate degree of accuracy) is 16.7 cm^2 .

Exercise 15

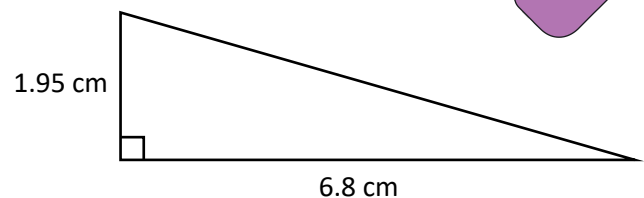
Calculate the area of the following shapes. Give your answers to an appropriate degree of accuracy.



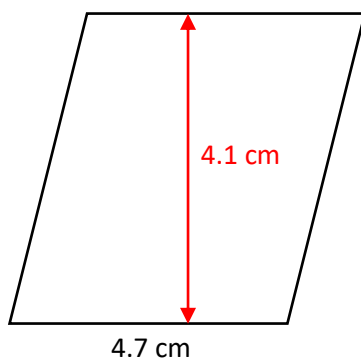
(a)



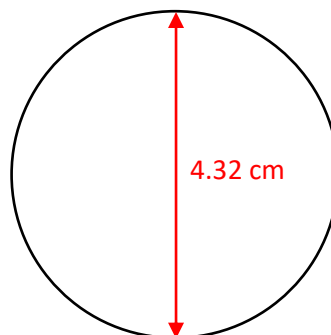
(b)



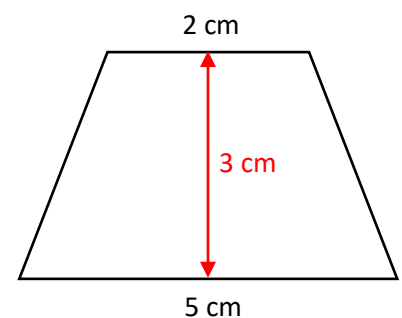
(c)



(d)



(e)



Exercise 16

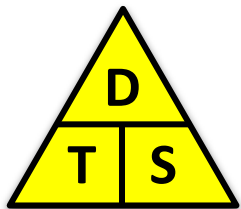
Mabli has a cuboid that measures 5.2 cm by 8.9 cm by 12.8 cm. Calculate, to an appropriate degree of accuracy, the volume of the cuboid.

Compound Measures



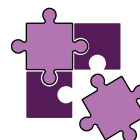
A **compound measure** is a type of measure that combines two different measures.

In the “Movement with Sphero” workbook, we saw an example of a compound measure, namely **speed**.



Distance = Time × Speed
Time = Distance ÷ Speed
Speed = Distance ÷ Time

To use the triangle, *hide* the letter that you want to find.



Example

(a) Iwan cycles a distance of 32 km in $2\frac{1}{2}$ hours.
Calculate Iwan’s average speed in km/hour.

Answer: Speed = Distance ÷ Time
 $= 32 \div 2.5$
 $= 12.8$ km/hour

(b) Moli runs an 800 m race in 2 minutes 48 seconds.
What is Moli’s average speed in metres per second?

Answer: Speed = Distance ÷ Time
 $= 800 \div 168$
 $= 4.8$ metres per second (to one decimal place)

Exercise 17

(a) Hannah cycles a distance of 49 km in $3\frac{1}{2}$ hours.
Calculate Hannah’s average speed in km/hour.

(b) Elin runs a 400 m race in 1 minute 12 seconds.
What is Elin’s average speed in metres per second?

(c) A bus travels the 12 miles from Llandudno to Abergele in 30 minutes.
Find the average speed of the bus in miles per hour.

(d) A train travels at an average speed of 90 km/hour for 2 hours 15 minutes.
How far has the train travelled during this time?

(e) In a 100 m race Dilwyn ran at an average speed of 7.1 metres per second.
How much time did Dilwyn take to finish the race?

(f) An aeroplane flies at an average speed of 885 km/hour. How far did the aeroplane travel between the times 0820 and 0900?

(g) The following chart shows the travelling distance for a car, in miles, between some places in Wales.

	Swansea			
Aberystwyth	70	Aberystwyth		
Bangor	157	86	Bangor	
Cardiff	41	98	180	Cardiff
Wrexham	130	79	70	139

(i) What is the travelling distance for a car between Cardiff and Bangor?

(ii) Dewi travels between Cardiff and Bangor, and then between Bangor and Wrexham. How far has Dewi travelled in total?

(iii) Elis travels between Bangor and Aberystwyth in 2 hours and a half. What is Elis’ average speed, in mph?

(iv) Eryllt travels between Wrexham and Swansea at an average speed of 32 miles per hour.
How much time did Eryllt take to travel between Wrexham and Swansea?



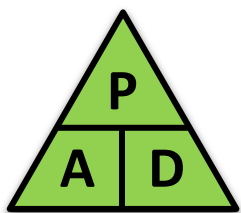
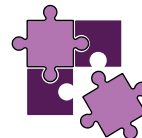
13





Population Density

Population density compares the population of a place to its area.



$$\begin{aligned}\text{Population} &= \text{Area} \times \text{Population Density} \\ \text{Area} &= \text{Population} \div \text{Population Density} \\ \text{Population Density} &= \text{Population} \div \text{Area}\end{aligned}$$

Population density is (usually) measured in people per square kilometre, and is used to compare how developed different areas are.

Example

Paris has a population of 2,265,886 and an area of 105.4 km². What is Paris' population density?

$$\begin{aligned}\text{Answer: Population Density} &= \text{Population} \div \text{Area} \\ &= 2,265,886 \div 105.4 \\ &= 21,498 \text{ people per km}^2 \text{ (correct to the nearest whole number)}\end{aligned}$$



Exercise 18

(a) Mumbai has a population of 12,478,447 and an area of 603 km². What is Mumbai's population density?

(b) San Francisco has a population of 805,816 and an area of 120.93 km². What is San Francisco's population density?

(c) Dublin has a population of 553,165 and an area of 114.99 km². What is Dublin's population density?

(d) The population density of London is 5,584 people per km². If London's population is 8,778,500, what is London's area?

(e) The population density of Chicago is 4,582 people per km². If Chicago's population is 2,695,598, what is Chicago's area?

(f) The population density of Cairo is 19,376 people per km². If Cairo's area is 606 km², what is Cairo's population?

(g) The population density of Miami is 4,324 people per km². If Miami's area is 92.38 km², what is Miami's population?

(h) The following table gives information about some Spanish cities.

City	Madrid	Barcelona	Seville
Population	3,141,991	1,621,537	703,021
Area	604.3 km ²	101.9 km ²	140 km ²

(i) Calculate the population density of the three Spanish cities.

(ii) Which city is the one where people live closest to each other?

Challenge!

Try to calculate the population density of where you live.



Applying

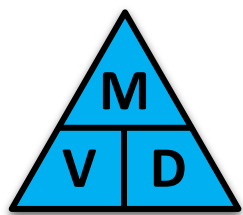
3





Density

Density compares an object's mass to its volume.



$$\begin{aligned}\text{Mass} &= \text{Volume} \times \text{Density} \\ \text{Volume} &= \text{Mass} \div \text{Density} \\ \text{Density} &= \text{Mass} \div \text{Volume}\end{aligned}$$

To use the triangle, *hide* the letter that you want to find.

Density is (usually) measured in g/cm^3 or kg/m^3 , and is used to compare how much mass an object has per unit of volume.

Example

(a) The mass of a 200 cm^3 piece of metal is 1.2 kg . What is the metal's density, in g/cm^3 ?

$$\begin{aligned}\text{Answer: Density} &= \text{Mass} \div \text{Volume} \\ &= 1,200 \div 200 \\ &= 6 \text{ g/cm}^3\end{aligned}$$

(b) The density of a piece of aluminium is 2.7 g/cm^3 . Calculate the mass of a 20 cm^3 piece of aluminium.

$$\begin{aligned}\text{Answer: Mass} &= \text{Volume} \times \text{Density} \\ &= 20 \times 2.7 \\ &= 54 \text{ g}\end{aligned}$$

Exercise 19

- (a) The mass of a stone is 4.4 kg .
 (i) What is the mass of the stone in grams?
 (ii) Find the density of the stone (in g/cm^3) if its volume is $2,000 \text{ cm}^3$.
- (b) A piece of cork weighs 10 kg . Its volume is 0.04 m^3 . Calculate the density of the cork, in kg/m^3 .
- (c) The mass of a $1,200 \text{ ml}$ piece of ice is $1,104 \text{ grams}$. Find its density. (Hint: $1 \text{ ml} = 1 \text{ cm}^3$.)

(d) Gold is expensive and desirable not just for how it looks but also because it does not rust easily. Lewis wishes to build a car made out of gold but is worried about its mass. The density of gold is $19,300 \text{ kg/m}^3$ and the volume of Lewis' car is around 1.5 m^3 .

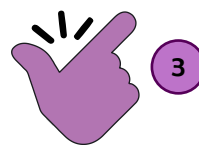
- (i) Calculate the mass of Lewis' gold car.
 (ii) Given that the mass of a normal car is around $1,500 \text{ kg}$, comment on the practicality of Lewis' plan.

(e) Megan is mixing and pouring concrete. She mixes and then pours a total of 2.4 m^3 of concrete. Calculate the mass of this concrete if its density is $2,500 \text{ kg/m}^3$.

(f) The density of water is 1 g/cm^3 . Calculate the mass of 1.5 litres of water.

(g) A clay statue has density 1.4 g/cm^3 . Another statue is carved from wood and has a density of 0.8 g/cm^3 . The two statues are weighed in order to find their mass. The wood statue weighs $4,500 \text{ g}$. The clay statue weighs $7,700 \text{ g}$. Which statue has the greatest volume?

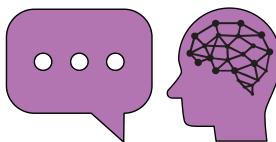
(h) The dimensions of a metal cuboid are 8 cm , 6 cm and 5 cm . The mass of the cuboid is 0.9 kg . Calculate the density of the metal, noting your units clearly.



Other Compound Measures

Exercise 20

13



(a) The fuel consumption of Bethan's car is 50 miles per gallon.

(i) If Bethan travels 200 miles in her car, how many gallons of petrol does she use?

(ii) Today, petrol costs 117.9 pence per litre. Given that a gallon of petrol is around 4.5 litres, calculate the cost of the petrol for Bethan's journey. Give your answer correct to the nearest penny.

(b) John travels 126 miles in his car, using 2.25 gallons of petrol. What is the fuel consumption of John's car, in miles per gallon?

(c) Janet types at a rate of 70 words per minute. How much time would Janet need to type a report containing 3,500 words?

(d) The average person can type at a rate of 40 words per minute. Bob types a 1,200-word report in 40 minutes. Does Bob type at a rate that is slower than the average person, or is quicker than the average person?

(e) A mill produces 20 metric tons/hour of flour. How much flour is produced every 15 minutes?

(f) A paint tin notes that it can cover an area of $13 \text{ m}^2/\text{litre}$. If the paint tin contains 2.5 litres of paint, what total area can the tin cover?

(g) A swimming pool is emptied in order to clean it. The pool (when full) can hold 375,000 litres of water. After it is cleaned, the pool is refilled by using a water pipe that can supply water at a rate of 200 litres per minute. How much time will it take to refill the pool? Give your answer in hours and minutes.

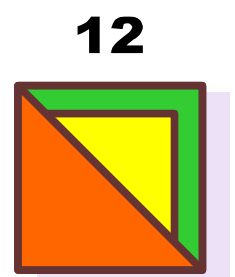
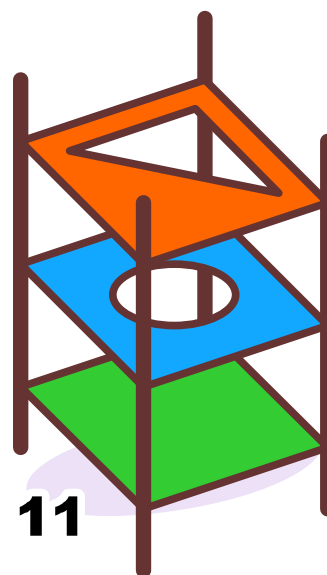
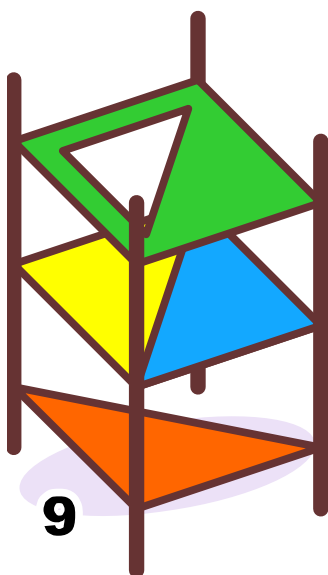
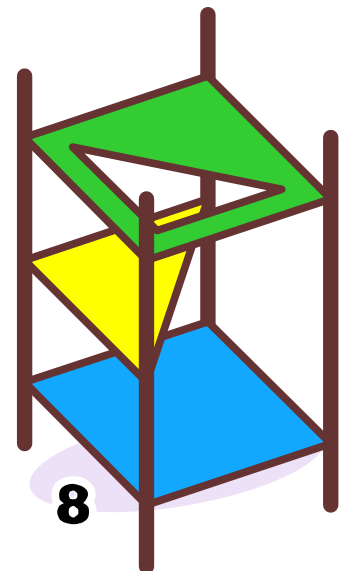
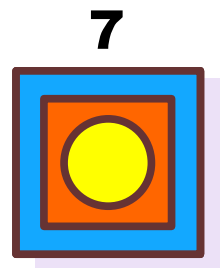
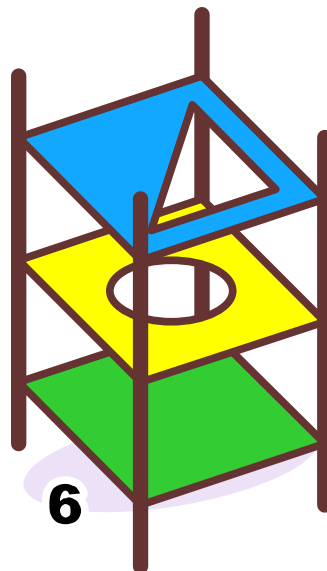
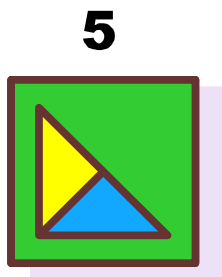
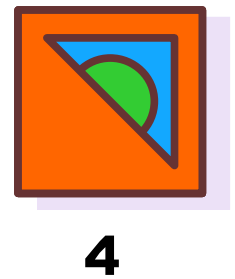
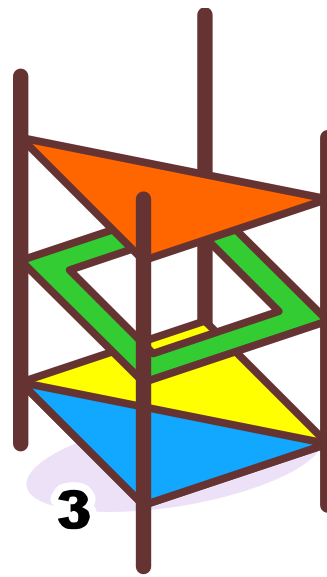
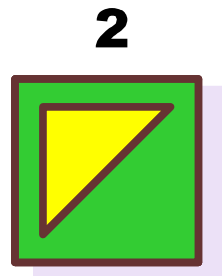
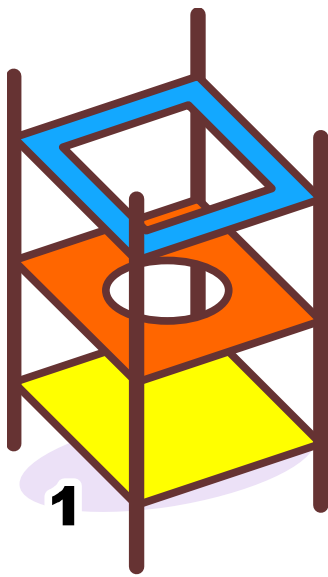
(h) During the first 20 games of a football season, Gareth Bale scored an average of 0.7 goals per game. How many goals is it expected that Gareth Bale scores during the next 6 games?

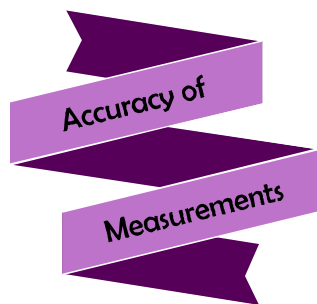


Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Puzzle: Can you find the top view for every stand?



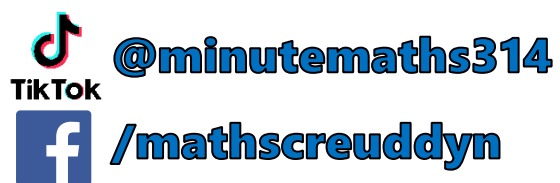


Reflection Sheet

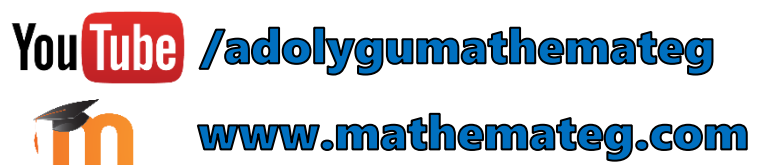
Name:

Percentage in the test:

I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I know how to round off a number to a specified number of significant figures , e.g. to 1 significant figure or to 2 significant figures.		1, 2	
I know how to recognise the significant figures in a given number.		3	
I know how to estimate the answer to a sum, by using rounding off to one significant figure.		4	
I know how to find the upper and lower bounds of a measurement.		5	
I know how to write answers to an appropriate degree of accuracy .		6	
I can use the formulae that are associated with the compound measure speed .		8	
I can use the formulae that are associated with the compound measure population density .		7	
I can use the formulae that are associated with the compound measure density .		9	
I am aware of how to work with other compound measures .		10	



 @minutemaths314



 /adolygumathemateg