Surname Atebion	Centre Number	Candidate Number
First name(s)		2

GCE A LEVEL

1305U40-1

S24-1305U40-1

MONDAY, 3 JUNE 2024 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	11	
2	13	
3	9	
4	21	
5	14	
6	8	
7	12	
8	11	
9	9	
10	12	
Total	**120**	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

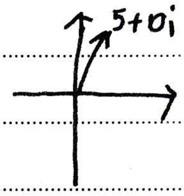
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. (a) Express the three cube roots of $5+10\mathrm{i}$ in the form $re^{\mathrm{i}\theta}$, where $0 \leqslant \theta < 2\pi$. [6]

$Z = 5 + 10i$



Ffurf Trigonometreg:

$r = \sqrt{5^2 + 10^2}$ $\qquad \theta = \tan^{-1}\left(\frac{10}{5}\right)$

$r = \sqrt{125}$ $\qquad \theta = 1.107148718$

$r = 5\sqrt{5}$

Yn defnyddio $z = r(\cos\theta + i\sin\theta)$

$z = re^{i\theta}$

$z = 5\sqrt{5}\, e^{i(1.107148718 + 2n\pi)}$

Gadewch i $w = \sqrt[3]{z}$

$w^3 = z$

$w^3 = 5\sqrt{5}\, e^{i(1.107148718 + 2n\pi)}$

$w = \left(5\sqrt{5}\, e^{i(1.107148718 + 2n\pi)}\right)^{\frac{1}{3}}$

$w = \left(\sqrt{125}\right)^{\frac{1}{3}} e^{i(0.3690495726 + \frac{2}{3}n\pi)}$

$w = \left((125)^{\frac{1}{2}}\right)^{\frac{1}{3}} e^{i(0.3690495726 + \frac{2}{3}n\pi)}$

$w = \left((125)^{\frac{1}{3}}\right)^{\frac{1}{2}} e^{i(0.3690495726 + \frac{2}{3}n\pi)}$

$w = \sqrt{5}\, e^{i(0.3690495726 + \frac{2}{3}n\pi)}$

$$\left.\begin{array}{l} n=0 \Rightarrow w_1 = \sqrt{5}\, e^{0.369i} \\ n=1 \Rightarrow w_2 = \sqrt{5}\, e^{2.463i} \\ n=2 \Rightarrow w_3 = \sqrt{5}\, e^{4.558i} \end{array}\right\} \text{ i 3 lle degol}$$

Examiner only

(b) The three cube roots of $5+10\mathrm{i}$ are plotted in an Argand diagram. The points are joined by straight lines to form a triangle. Find the area of this triangle, giving your answer correct to two significant figures. [5]

$w = \sqrt[3]{5+10i}$

$w^3 = 5+10i$

$w^3 = 5\sqrt{5}(\cos 1.107 + i\sin 1.107)$

$w = \sqrt{5}(\cos 1.107 + i\sin 1.107)^{\frac{1}{3}}$

$(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \cos\left[\frac{\theta}{n} + \frac{2(K-1)\pi}{n}\right] + i\sin\left[\frac{\theta}{n} + \frac{2(K-1)\pi}{n}\right]$

$(\cos 1.107 + i\sin 1.107)^{\frac{1}{3}} = \cos\left[\frac{1.107}{3} + \frac{2(K-1)\pi}{3}\right]$

$+ i\sin\left[\frac{1.107}{3} + \frac{2(K-1)\pi}{3}\right]$

$K=1$: $w = \sqrt{5}\left(\cos\left(\frac{1.107}{3}\right) + i\sin\left(\frac{1.107}{3}\right)\right)$

$w = 2.086 + 0.807i$ i 3 lle degol (A)

$K=2$: $w = \sqrt{5}\left(\cos\left(\frac{1.107}{3} + \frac{2\pi}{3}\right) + i\sin\left(\frac{1.107}{3} + \frac{2\pi}{3}\right)\right)$

$w = -1.741 + 1.403i$ i 3 lle degol (B)

$K=3$: $w = \sqrt{5}\left(\cos\left(\frac{1.107}{3} + \frac{4\pi}{3}\right) + i\sin\left(\frac{1.107}{3} + \frac{4\pi}{3}\right)\right)$

$w = -0.344 - 2.209i$ i 3 lle degol (C)

Mae'r tri phwynt uchod yn ffurfio triongl hafalochrog.

Hyd y triongl = Pellter AB

$= \sqrt{(-1.741 - 2.086)^2 + (1.403 - 0.807)^2}$

$= 3.873131162$

Arwynebedd y triongl = $\frac{1}{2}ab\sin C$

$= 0.5 \times 3.873\cdots \times 3.873\cdots \times \sin 60^\circ$

$= \underline{\underline{6.5}}$ uned sgwâr i 2ff. yst.

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2. The function f is defined by $f(x) = \cosh\left(\frac{x}{2}\right)$.

(a) State the Maclaurin series expansion for $\cosh\left(\frac{x}{2}\right)$ up to and including the term in x^4. [2]

Cyfres Maclaurin: $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f''''(0) + \dots$

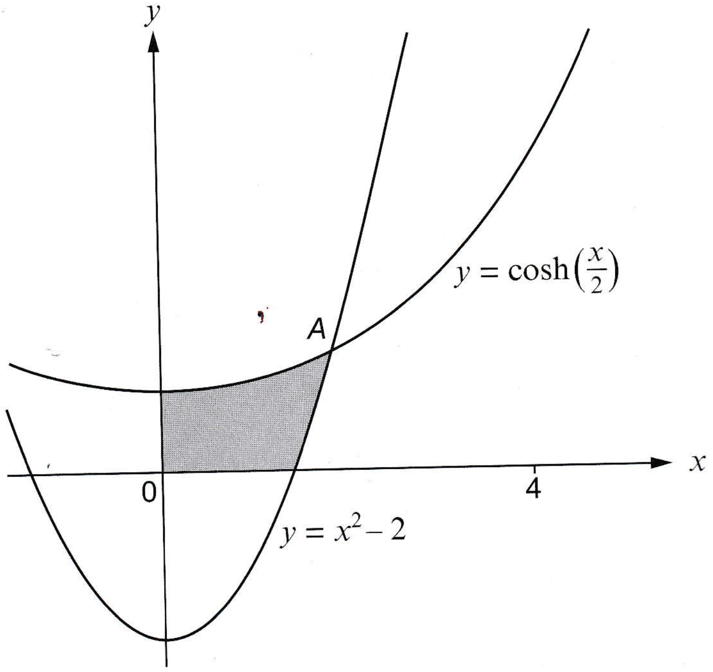
$f(x) = \cosh\left(\frac{0}{2}\right) + x\left(\frac{1}{2}\sinh\left(\frac{0}{2}\right)\right) + \frac{x^2}{2!}\left(\frac{1}{4}\cosh\left(\frac{0}{2}\right)\right)$

$+ \frac{x^3}{3!}\left(\frac{1}{8}\sinh\left(\frac{0}{2}\right)\right) + \frac{x^4}{4!}\left(\frac{1}{16}\cosh\left(\frac{0}{2}\right)\right) + \dots$

$f(x) = 1 + x\left(\frac{1}{2} \times 0\right) + \frac{x^2}{2}\left(\frac{1}{4} \times 1\right) + \frac{x^3}{6}\left(\frac{1}{8} \times 0\right) + \frac{x^4}{24}\left(\frac{1}{16} \times 1\right) + \dots$

$f(x) = 1 + \frac{x^2}{8} + \frac{x^4}{384} + \dots$

Another function g is defined by $g(x) = x^2 - 2$. The diagram below shows parts of the graphs of $y = f(x)$ and $y = g(x)$.



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(b) The two graphs intersect at the point A, as shown in the diagram. Use your answer from part (a) to find an approximation for the x-coordinate of A, giving your answer correct to two decimal places. [5]

Mae angen datrys $\cosh\left(\frac{x}{2}\right) = x^2 - 2$

$$1 + \frac{x^2}{8} + \frac{x^4}{384} \approx x^2 - 2$$

$$\frac{x^4}{384} + \frac{x^2}{8} - x^2 + 1 + 2 \approx 0$$

$$\frac{1}{384}x^4 - \frac{7}{8}x^2 + 3 \approx 0$$

$$x^4 - 336x^2 + 1152 \approx 0$$

Gadewch i $u = x^2$.

$$u^2 - 336u + 1152 \approx 0$$

Datrys efo'r fformiwla gwadratig:

$$u = \frac{336 \pm \sqrt{(-336)^2 - 4(1)(1152)}}{2 \times 1}$$

$$u = \frac{336 \pm \sqrt{108288}}{2}$$

Naill ai $u = \frac{336 + \sqrt{108288}}{2}$ neu $u = \frac{336 - \sqrt{108288}}{2}$

$u = 332.5357104$ $\quad$ $u = 3.46428959$

$x^2 = 332.5357104$ $\quad$ $x^2 = 3.46428959$

$x = \pm\sqrt{332.5357104}$ $\quad$ $x = \pm\sqrt{3.46428959}$

$x = \pm 18.2355617$ $\quad$ $x = \pm 1.861260216$

Trwy ystyried y diagram ar dudalen 4, rhaid bod $\underline{x = 1.86}$ i 2 le degol.

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(c) Using your answer to part (b), find an approximation for the area of the shaded region enclosed by the two graphs, the x-axis and the y-axis. [6]

$$\int_0^{1.86} \cosh\left(\frac{x}{2}\right) dx = \left[2\sinh\left(\frac{x}{2}\right)\right]_0^{1.86}$$

$$= 2\sinh\left(\frac{1.86}{2}\right) - 2\sinh\left(\frac{0}{2}\right)$$

$$= 2.139955467 - 0$$

$$= 2.139955467$$

Ble mae $y = x^2 - 2$ yn torri echelin-x ?

$$0 = x^2 - 2$$

$$2 = x^2$$

$$x = \pm\sqrt{2}.$$

$$\int_{\sqrt{2}}^{2.15} x^2 - 2\,dx = \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$$

$$= \left(\frac{1.86^3}{3} - 2 \times 1.86\right) - \left(\frac{(\sqrt{2})^3}{3} - 2\sqrt{2}\right)$$

$$= -1.575048 - -\frac{4\sqrt{2}}{3}$$

$$= 0.3105700832$$

Arwynebedd wedi'i dywyllu $= 2.139955467 - 0.3105700832$

$$= 1.829385384$$

$= \underline{1.83}$ uned sgwâr i 2le degol

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3. Given the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y\sin x = 4\cos^3 x \sin x + 5$$

and $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$, find an equation for y in terms of x. [9]

$\cos x \frac{dy}{dx} + y \sin x = 4\cos^3 x \sin x + 5$

$\frac{dy}{dx} + y \tan x = 4\cos^2 x \sin x + 5\sec x$

Ar gyfer $\frac{dy}{dx} + Fy = G$,

ble mae F a G yn ffwythiannau mewn x ynunig,

y ffactor integru yw $I = e^{\int F dx}$

$I = e^{\int \tan x \, dx}$

$I = e^{\ln|\sec x|}$

$I = \sec x$

Felly $\frac{dy}{dx} + y\tan x = 4\cos^2 x \sin x + 5\sec x$

Lluosi bob ochr yr hafaliad â'r ffactor integru:

$\sec x \frac{dy}{dx} + y \tan x \sec x = 4\cos^2 x \sin x \sec x + 5\sec^2 x$

$\sec x \frac{dy}{dx} + y \tan x \sec x = 4\cos x \sin x + 5\sec^2 x$

$\int \sec x \frac{dy}{dx} + y\tan x \sec x \, dx = \int 4\cos x \sin x + 5\sec^2 x \, dx$

$\sec x \, y = 2\sin^2 x + 5\tan x + c$

Amnewid $y = 3\sqrt{2}$, $x = \frac{\pi}{4}$:

$\sec\left(\frac{\pi}{4}\right) 3\sqrt{2} = 2\sin^2\left(\frac{\pi}{4}\right) + 5\tan\left(\frac{\pi}{4}\right) + c$

$\frac{1}{\frac{1}{\sqrt{2}}}(3\sqrt{2}) = 2\left(\frac{1}{\sqrt{2}}\right)^2 + 5(1) + c$

$\sqrt{2}(3\sqrt{2}) = 1 + 5 + c$

$6 = 6 + c$

$c = 0$

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Felly $\sec x \, y = 2\sin^2 x + 5\tan x + 0$

$$y = \frac{2\sin^2 x}{\sec x} + \frac{5\tan x}{\sec x}$$

$$y = 2\sin^2 x\cos x + 5\left(\frac{\sin x}{\cancel{\cos x}}\right)\cancel{\cos x}$$

$$y = 2\sin^2 x\cos x + 5\sin x$$

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Exa

4. (a) Given that $z^n + \frac{1}{z^n} = 2\cos n\theta$, where $z = \cos\theta + \mathrm{i}\sin\theta$, express $16\cos^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$, where a, b, c are integers whose values are to be determined. [5]

Ehangiad Binomaidd:

$$\left(z+\frac{1}{z}\right)^4 = z^4 + 4z^3\left(\frac{1}{z}\right) + 6z^2\left(\frac{1}{z}\right)^2 + 4z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$$

$$= z^4 + 4z^2 + 6 + \frac{4}{z^2} + \frac{1}{z^4}$$

$$= \left(z^4 + \frac{1}{z^4}\right) + 4\left(z^2 + \frac{1}{z^2}\right) + 6$$

$$\left(z+\frac{1}{z}\right)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$$

$$(2\cos\theta)^4 = 2\cos 4\theta + 8\cdot\cos 2\theta + 6$$

$$2^4\cos^4\theta = 2\cos 4\theta + 8\cdot\cos 2\theta + 6$$

$$16\cos^4\theta = 2\cos 4\theta + 8\cdot\cos 2\theta + 6$$

Felly $a=2$, $b=8$, $c=6$.

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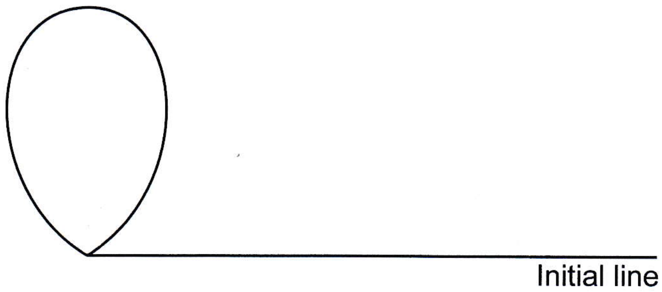
The diagram below shows a sketch of the curve C with polar equation

$$r = 3 - 4\cos^2\theta, \qquad \text{where } \frac{\pi}{6} \leqslant \theta \leqslant \frac{5\pi}{6}.$$



(b) Calculate the area of the region enclosed by the curve C. [8]

$$\text{Arwynebedd} = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3-4\cos^2\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} 9 - 24\cos^2\theta + 16\cos^4\theta \, d\theta$$

$$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} 9 - 24\cos^2\theta + 2\cos 4\theta + 8\cos 2\theta + 6 \, d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 9 - 24\left(\frac{\cos 2\theta + 1}{2}\right) + 2\cos 4\theta + 8\cos 2\theta + 6 \, d\theta$$

$$= \frac{1}{2}\left[9\theta - 24\left(\frac{1}{4}\sin 2\theta + \frac{\theta}{2}\right) + \frac{1}{2}\sin 4\theta + 4\sin 2\theta + 6\theta\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{1}{2}\left[15\theta - 6\sin 2\theta - 12\theta + \frac{1}{2}\sin 4\theta + 4\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{1}{2}\left[3\theta - 2\sin 2\theta + \frac{1}{2}\sin 4\theta\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{1}{2}\left[\left(3\left(\frac{5\pi}{6}\right) - 2\sin\left(\frac{10\pi}{6}\right) + \frac{1}{2}\sin\left(\frac{20\pi}{6}\right)\right) - \left(3\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{2\pi}{6}\right) + \frac{1}{2}\sin\left(\frac{4\pi}{6}\right)\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{15\pi}{6} - 2\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)\right) - \left(\frac{\pi}{2} - 2\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$$

$$= \frac{1}{2}\left[2\pi + \sqrt{3} - \frac{\sqrt{3}}{4} + \sqrt{3} - \frac{\sqrt{3}}{4}\right]$$

$$= \frac{1}{2}\left[2\pi + \frac{3}{2}\sqrt{3}\right]$$

$= \pi + \frac{3}{4}\sqrt{3}$ uned sgwâr ($\simeq$ 4.44 i 2 le degol).

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(c) Find the exact polar coordinates of the points on C at which the tangent is perpendicular to the initial line. [8]

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x = (3-4\cos^2\theta)\cos\theta$ $\qquad$ $y = (3-4\cos^2\theta)\sin\theta$

$x = 3\cos\theta - 4\cos^3\theta$ $\qquad$ $y = 3\sin\theta - 4\cos^2\theta\sin\theta$

$\frac{dx}{d\theta} = -3\sin\theta - 4(3\cos^2\theta(-\sin\theta))$ $\qquad$ $\frac{dy}{d\theta} = 3\cos\theta - 4\cos^2\theta(\cos\theta) - 4(2\cos\theta(-\sin\theta))\sin\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta$ $\qquad$ $\frac{dy}{d\theta} = 3\cos\theta - 4\cos^3\theta + 8\cos\theta\sin^2\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \frac{3\cos\theta - 4\cos^3\theta + 8\cos\theta\sin^2\theta}{-3\sin\theta + 12\cos^2\theta\sin\theta}$$

I fod yn berpendicwlar i'r llinell gychwynnol, rhaid i graddiant y tangiad fod yn ∞. Felly, rhaid cael $\frac{dx}{d\theta} = 0$

$$-3\sin\theta + 12\cos^2\theta\sin\theta = 0$$

$$\sin\theta(-3 + 12\cos^2\theta) = 0$$

Naill ai $\sin\theta = 0$ $\quad$ neu $-3 + 12\cos^2\theta = 0$

S | A, T | C $\quad$ $\theta = \sin^{-1}(0)$ $\qquad$ $\cos^2\theta = \frac{1}{4}$

$\theta = 0, \pi, 2\pi, \ldots$ $\qquad$ $\cos\theta = \pm\sqrt{\frac{1}{4}}$

Dim ongl yn yr amrediad $\quad$ Naill ai $\cos\theta = \frac{1}{2}$ neu $\cos\theta = -\frac{1}{2}$

S | A, T | C $\quad$ $\theta = \cos^{-1}(\frac{1}{2})$ $\qquad$ $\theta = \cos^{-1}(-\frac{1}{2})$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$r = 3 - 4\cos^2(\frac{\pi}{3})$ $\qquad$ $r = 3 - 4\cos^2(\frac{2\pi}{3})$

$r = 2$ $\qquad$ $r = 2$

Cyfesurynnau (r, θ) fel atebion:

$(2, \frac{\pi}{3})$, $(2, \frac{2\pi}{3})$.

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5. Find each of the following integrals.

(a) $\displaystyle\int \frac{3-x}{x\left(x^2+1\right)}\,\mathrm{d}x$ [8]

$$\frac{3-x}{x(x^2+1)} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\frac{3-x}{x(x^2+1)} \equiv \frac{A(x^2+1)}{x(x^2+1)} + \frac{(Bx+C)x}{x(x^2+1)}$$

$$3-x \equiv A(x^2+1) + x(Bx+C)$$

Amnewid $x=0$:

$$3-0 \equiv A(0^2+1) + 0(B(0)+C)$$

$$3 \equiv A$$

Yn cymharu cyfernodau x^2:

$$0 \equiv A + B$$

$$0 \equiv 3 + B$$

$$-3 \equiv B$$

Yn cymharu cyfernodau x:

$$-1 \equiv C$$

Felly $\displaystyle\int \frac{3-x}{x(x^2+1)}\,dx = \int \frac{3}{x} + \frac{-3x-1}{x^2+1}\,dx$

$$= \int \frac{3}{x} - \frac{3x}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$$= 3\ln|x| - \frac{3}{2}\ln|x^2+1| - \tan^{-1}(x) + C$$

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(b) $\displaystyle\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,dx$ [6]

$$\cosh^4 x - 9\cosh^2 x$$
$$= \left(\cosh^2 x - \frac{9}{2}\right)^2 - \frac{81}{4}$$

Felly $\displaystyle\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,dx = \int \frac{\sinh 2x}{\sqrt{(\cosh^2 x - \frac{9}{2})^2 - \frac{81}{4}}}\,dx$

Gadewch i $u = \cosh^2 x - \frac{9}{2}$

$$\frac{du}{dx} = 2\cosh x \sinh x$$

$$\frac{du}{2\cosh x \sinh x} = dx$$

Felly $\displaystyle\int \frac{\sinh 2x}{\sqrt{(\cosh^2 x - \frac{9}{2})^2 - \frac{81}{4}}}\,dx$

$$= \int \frac{\sinh 2x}{\sqrt{u^2 - \frac{81}{4}}} \frac{du}{2\cosh x \sinh x}$$

$$= \int \frac{2\sinh \cancel{x \cosh x}}{\sqrt{u^2 - \frac{81}{4}}} \frac{du}{\cancel{2\cosh x \sinh x}}$$

$$= \int \frac{1}{\sqrt{u^2 - \frac{81}{4}}}\,du$$

$$= \cosh^{-1}\left(\frac{u}{\frac{9}{2}}\right) + c$$

$$= \cosh^{-1}\left(\frac{\cosh^2 x - \frac{9}{2}}{\frac{9}{2}}\right) + c$$

$$= \cosh^{-1}\left(\frac{2}{9}\cosh^2 x - \frac{9}{2} \times \frac{2}{9}\right) + c$$

$$= \cosh^{-1}\left(\frac{2}{9}\cosh^2 x - 1\right) + c$$

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6. The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{pmatrix} 12 & 30 & 8 \\ 18 & 25 & 20 \\ 19 & 50 & 16 \end{pmatrix}.$$

(a) Given that $\det \mathbf{M} = -1040$, give a geometrical interpretation of the solution to the following equation. [2]

$$\begin{pmatrix} 12 & 30 & 8 \\ 18 & 25 & 20 \\ 19 & 50 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2668 \\ 3402 \\ 4581 \end{pmatrix}$$

Gan fod det $M \neq 0$ mae datrysiad unigryw i'r hafaliad, sef pwynt (x, y, z) mewn gofod tri-dimensiwn.

(b) Three hotels A, B, C each have different types of room available to book: single, double and family rooms. For each type of room, the price per night is the same in each of the three hotels.

The table below gives, for each hotel, details of the number of each type of room and the total revenue per night when the hotel is full.

	Types of room			
Hotel	**Single**	**Double**	**Family**	**Total revenue**
A	12	30	8	£2,668
B	18	25	20	£3,402
C	19	50	16	£4,581

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Find the price per night of each type of room. [6]

det M = −1040

$$M^T = \begin{pmatrix} 12 & 18 & 19 \\ 30 & 25 & 50 \\ 8 & 20 & 16 \end{pmatrix}$$

$$\text{Minorau} = \begin{pmatrix} -600 & 80 & 400 \\ -92 & 40 & 96 \\ 425 & 30 & -240 \end{pmatrix}$$

$$\text{Atgydiol (Cyfwyddion)} = \begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}$$

$$= \frac{1}{-1040} \begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}$$

$$\text{Felly } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-1040} \begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix} \begin{pmatrix} 2668 \\ 3402 \\ 4581 \end{pmatrix}$$

$$= \begin{pmatrix} 39 \\ 56 \\ 65 \end{pmatrix}$$

Felly cost ystafell sengl yw £39; cost ystafell ddwbl yw £56; a chost ystafell deulu yw £65.

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7. (a) A curve C is defined by the equation $y = \dfrac{1}{\sqrt{16-6x-x^2}}$ for $-3 \leqslant x \leqslant 1$.

(i) Find the mean value of $y = \dfrac{1}{\sqrt{16-6x-x^2}}$ between $x=-3$ and $x=1$. [4]

$$\text{Gwerth cymedrig} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{1--3}\int_{-3}^{1} \frac{1}{\sqrt{16-6x-x^2}}\,dx$$

$$= \frac{1}{4}\int_{-3}^{1} \frac{1}{\sqrt{16-(x^2+6x)}}\,dx$$

$$= \frac{1}{4}\int_{-3}^{1} \frac{1}{\sqrt{16-((x+3)^2-3^2)}}\,dx$$

$$= \frac{1}{4}\int_{-3}^{1} \frac{1}{\sqrt{16-((x+3)^2-9)}}\,dx$$

$$= \frac{1}{4}\int_{-3}^{1} \frac{1}{\sqrt{25-(x+3)^2}}\,dx$$

Gadewch i $u = x+3$ $\quad$ $[-3]$ $u = -3+3$

$\frac{du}{dx} = 1$ $\quad$ $u = 0$

$du = dx$ $\quad$ $[1]$ $u = 1+3$

$u = 4.$

$$\text{Gwerth cymedrig} = \frac{1}{4}\int_0^4 \frac{1}{\sqrt{25-u^2}}\,du$$

$$= \frac{1}{4}\left[\sin^{-1}\left(\frac{u}{5}\right)\right]_0^4$$

$$= \tfrac{1}{4}\left[\sin^{-1}\left(\tfrac{4}{5}\right) - \sin^{-1}(0)\right]$$

$$= \tfrac{1}{4}\sin^{-1}(0.8)$$

$(\approx 0.23$ i 2 ledegol$)$

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(ii) The region R is bounded by the curve C, the x-axis and the lines $x = -3$ and $x = 1$. Find the volume of the solid generated when R is rotated through four right-angles about the x-axis. [5]

$$\int_{-3}^{1} \pi r^2 \, dr = \pi \int_{-3}^{1} \left(\frac{1}{\sqrt{16-6x-x^2}} \right)^2 dr$$

$$= \pi \int_{-3}^{1} \frac{1}{16-6x-x^2} \, dr$$

$$= \pi \int_{-3}^{1} \frac{1}{25-(x+3)^2} \, dx$$

Gadewch i $u = x + 3$ $\qquad$ $[-3]$ $u = -3 + 3$

$\frac{du}{dx} = 1$ $\qquad$ $u = 0$

$du = dx$ $\qquad$ $[0]$ $u = 1 + 3$

$u = 4$

Ateb $= \pi \int_{0}^{4} \frac{1}{25-u^2} \, du$

$$= \pi \left[\frac{1}{5} \tanh^{-1}\left(\frac{u}{5} \right) \right]_0^4$$

$$= \frac{\pi}{5} \left[\tanh^{-1}\left(\frac{4}{5} \right) - \tanh^{-1}\left(\frac{0}{5} \right) \right]$$

$$= \frac{\pi}{5} \tanh^{-1}(0.8)$$

(≈ 0.69 uned ciwb i 2 le degol)

 (1305U40-1) **Turn over.**

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(b) Evaluate the improper integral

$$\int_1^{\infty} \frac{-8e^{-2x}}{4e^{-2x}-5}\,dx,$$

giving your answer correct to three decimal places. [3]

$\int_1^{\infty} \frac{-8e^{-2x}}{4e^{-2x}-5}\,dx$

$= \left[\ln|4e^{-2x}-5|\right]_1^{\infty}$

$= \ln|4e^{-2(\infty)}-5| - \ln|4e^{-2(1)}-5|$

Nawr fel mae $x \to \infty$ mae $e^{-2x} \to 0$

felly'r ateb yw $\ln|4(0)-5| - \ln|4e^{-2}-5|$

$= \ln|-5| - \ln|4e^{-2}-5|$

$= \ln(5) - \ln|4e^{-2}-5|$

$= \ln(5) - \ln|-4.458658867|$

$= \ln(5) - \ln(4.458658867)$

$= 0.1145898941$

$= \underline{\underline{0.115}}$ i 3 lle degol

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8. (a) By writing $y=\sinh^{-1}(4x+3)$ as $\sinh y = 4x+3$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{4}{\sqrt{16x^2+24x+10}}$. [5]

$$y=\sinh^{-1}(4x+3)$$

$$\sinh(y) = 4x+3$$

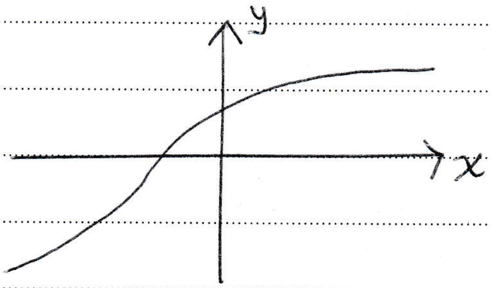
$$\cosh(y)\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{\cosh y}$$

Nawr $\cosh^2 y - \sinh^2 y = 1$

$$\cosh^2 y = 1+\sinh^2 y$$

$$\cosh y = \pm\sqrt{1+\sinh^2 y}$$



Trwy edrych ar graff $y=\sinh^{-1}(4x+3)$, neu ystyried gwerthoedd wrth amnewid i mewn iddo (efo'r TABLE MODE), mae graddiant y o hyd yn bositif. Felly dewiswn $\cosh y = +\sqrt{1+\sinh^2 y}$

Felly $\dfrac{dy}{dx} = \dfrac{4}{\sqrt{1+\sinh^2 y}}$

$$\frac{dy}{dx} = \frac{4}{\sqrt{1+(4x+3)^2}}$$

$$\frac{dy}{dx} = \frac{4}{\sqrt{1+16x^2+24x+9}} \rightarrow \frac{dy}{dx} = \frac{4}{\sqrt{16x^2+24x+10}}$$

✓

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(b) Show that the graph of $e^{-3x}y = \sinh 2x$ has only one stationary point. [6]

$y = \dfrac{\sinh 2x}{e^{-3x}}$

$y = e^{3x}\sinh 2x$

$\dfrac{dy}{dx} = e^{3x}(2\cosh 2x) + 3e^{3x}\sinh 2x$

Pwyntiau arhosol $\Rightarrow \dfrac{dy}{dx} = 0$

$2e^{3x}\cosh 2x + 3e^{3x}\sinh 2x = 0$

Mae e^{3x} o hyd yn bositif felly gallwn rannu bob ochr ag ef.

$2\cosh 2x + 3\sinh 2x = 0$

$3\sinh 2x = -2\cosh 2x$

$\tanh 2x = -\frac{2}{3}$

$2x = \tanh^{-1}(-\frac{2}{3})$

$2x = -0.8047189562$

$x = -0.4023594781$

Dim ond un datrysiad sydd ar gyfer x felly dim ond un pwynt arhosol sydd. ✓

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9. Find the general solution of the equation

$$\sin 6\theta + 2\cos^2\theta = 3\cos 2\theta - \sin 2\theta + 1.$$

[9]

$\sin 6\theta + 2\cos^2\theta = 3\cos 2\theta - \sin 2\theta + 1$

$\sin 6\theta + \sin 2\theta = 3\cos 2\theta - 2\cos^2\theta + 1$

Nawr $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin 6\theta + \sin 2\theta = 2\sin\left(\frac{6\theta+2\theta}{2}\right)\cos\left(\frac{6\theta-2\theta}{2}\right)$

$= 2\sin 4\theta \cos 2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$
$\cos 2\theta + 1 = 2\cos^2\theta$

$3\cos 2\theta - 2\cos^2\theta + 1$
$= 3\cos 2\theta - (\cos 2\theta + 1) + 1$
$= 3\cos 2\theta - \cos 2\theta - 1 + 1$
$= 2\cos 2\theta$

Felly'r hafaliad yw
$2\sin 4\theta \cos 2\theta = 2\cos 2\theta$
$2\sin 4\theta \cos 2\theta - 2\cos 2\theta = 0$
$\cos 2\theta(\sin 4\theta - 1) = 0$

Naill ai $\cos 2\theta = 0$
$2\theta = \cos^{-1}(0)$
$2\theta = \frac{\pi}{2} \pm n\pi$
$\theta = \frac{\pi}{4} \pm \frac{n\pi}{2}$

neu $\sin 4\theta - 1 = 0$
$\sin 4\theta = 1$
$4\theta = \sin^{-1}(1)$
$4\theta = \frac{\pi}{2} \pm 2n\pi$
$\theta = \frac{\pi}{8} \pm \frac{1}{2}n\pi$

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Ar gyfer cyfanrif positif n,

Naill ai $\theta = \frac{\pi}{4} + \frac{n}{2}\pi$

neu $\theta = \frac{\pi}{4} - \frac{n}{2}\pi$

neu $\theta = \frac{\pi}{8} + \frac{n}{2}\pi$

neu $\theta = \frac{\pi}{8} - \frac{n}{2}\pi$

Examiner only

10. The following simultaneous equations are to be solved.

$$\frac{dx}{dt} = 4x + 2y + 6e^{3t} \quad \text{— (1)}$$

$$\frac{dy}{dt} = 6x + 8y + 15e^{3t} \quad \text{— (2)}$$

(a) Show that $\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$. [5]

$$\frac{dx}{dt} = 4x + 2y + 6e^{3t}$$

$$\frac{1}{2}\frac{dx}{dt} = 2x + y + 3e^{3t}$$

$$y = \frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t} \quad \text{— (3)}$$

$$\frac{dy}{dt} = \frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t}$$

Amnewid i (2):

$$\frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t} = 6x + 8\left(\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}\right) + 15e^{3t}$$

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 18e^{3t} = 12x + 16\left(\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}\right) + 30e^{3t}$$

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 18e^{3t} = 12x + 8\frac{dx}{dt} - 32x - 48e^{3t} + 30e^{3t}$$

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} \cancel{-18e^{3t}} = -20x \cancel{-18e^{3t}}$$

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$$

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(b) Given that $\frac{\mathrm{d}x}{\mathrm{d}t} = 9$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 10$ when $t = 0$, find the particular solution for x in terms of t. [7]

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$$

Ceisio $x = Ae^{mt}$ fel bod m yn bodloni $am^2 + bm + c = 0$

$a = 1,\ b = -12,\ c = 20$

Hafaliad ategol / Auxiliary equation

$m^2 - 12m + 20 = 0$

$(m-2)(m-10) = 0$

Naill ai $m = 2$ neu $m = 10$.

Dau ddatrysiad real felly'r ffwythiant cyflenwol (complementary function) yw $x = Ae^{2t} + Be^{10t}$

Does dim angen integryn neilltuol yma gan fod ochr dde yr hafaliad gwreiddiol yn sero.

Os yw $t = 0$ mae $\frac{dx}{dt} = 9$, $\frac{d^2x}{dt^2} = 10$.

Differu: $\frac{dx}{dt} = 2Ae^{2t} + 10Be^{10t}$

Amnewid: $9 = 2Ae^{2(0)} + 10Be^{10(0)}$

$9 = 2A + 10B$ — (1)

Differu: $\frac{d^2x}{dt^2} = 4Ae^{2t} + 100Be^{10t}$

Amnewid: $10 = 4Ae^{2(0)} + 100Be^{10(0)}$

$10 = 4A + 100B$

$5 = 2A + 50B$ — (2)

(1) − (2) $\Rightarrow$ $4 = -40B$

$B = -0.1$

→ Wele tudalen 28

END OF PAPER

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.
10b	Amnewid yn ôl yn ①: $9 = 2A + 10(-0.1)$
	$9 = 2A - 1$
	$10 = 2A$
	$A = 5$
	Felly'r datrysiad cyffredinol yw
	$x = 5e^{2t} - 0.1e^{10t}$.

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(1305U40-1)

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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