Surname	Atebion	Centre Number	Candidate Number
First name(s)			2

wjec cbac

GCE AS/A LEVEL

2305U20-1

S24-2305U20-1

FRIDAY, 17 MAY 2024 – AFTERNOON

FURTHER MATHEMATICS – AS unit 2
FURTHER STATISTICS A

1 hour 30 minutes

2305U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**14**	
2	**13**	
3	**12**	
4	**12**	
5	**12**	
6	**7**	
Total	**70**	

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

Exami only

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. Dave and Llinos like to go fishing. When they go fishing, on average, Dave catches 4·3 fish per day and Llinos catches 3·8 fish per day. A day of fishing is assumed to be 8 hours.

(a) (i) Calculate the probability that they will catch fewer than 2 fish in total on a randomly selected half-day of fishing. [4]

(ii) Justify any distribution you have used in answering (a)(i). [1]

(a) (i) X = nifer o bysgod mae Dave yn dal pob $\frac{1}{2}$ diwrnod

$X \sim Po(2.15)$

Y = nifer o bysgod mae Llinos yn dal pob $\frac{1}{2}$ diwrnod

$Y \sim Po(1.9)$

Z = cyfansum pysgod Dave a Llinos pob $\frac{1}{2}$ diwrnod

$Z \sim Po(2.15+1.9)$

$Z \sim Po(4.05)$

$P(Z < 2) = P(Z \leq 1)$

$= 0.087982999$

$= 0.0880$ i 4 lle degol

(ii) Rwyf yn tybio bod y pysgod yn cael eu dal yn unigol, a bod Dave a Llinos yn dal pysgod yn annibynnol o'i gilydd.

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(b) On a randomly selected day, Dave starts fishing at 7 am. Given that Dave has not caught a fish by 11 am,

(i) find the expected time he catches his first fish,

(ii) calculate the probability that he will not catch a fish by 3 pm. [4]

(b) (i) D = nifer o bysgod mae Dave yn dal pob diwrnod

$D \sim Po(4.3)$

$D \sim Po(8 \times 0.5375)$

A = amser aros nes dal y pysgodyn nesaf

$A \sim Exp(0.5375)$

$Exp(A) = \frac{1}{0.5375}$

$= 1.860465116.$

Felly'r amser disgwyliedig i ddal y pysgodyn nesaf yw 11 am + 1.86... awr = 0.8604... awr ar ôl hanner dydd

Mae $0.860465116 \times 60 = 51.62790698$

felly, i'r funud agosaf, yr amser yw 12:52 pm

$F(x) = 1 - e^{-\lambda x}$

(ii) Angen $P(A > 4)$

$= 1 - P(A \leq 4)$

$= 1 - (1 - e^{-0.5375 \times 4})$

$= 0.1165$ i 4 lle degol

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(c) On average, only 2% of the fish that Llinos catches are trout. Over the course of a year, she catches 950 fish. Calculate the probability that at least 30 of these fish are trout. [3]

B ~ nifer o bysgod Brithyll (trout) mae Llinos yn ei ddal yn y flwyddyn.

$B \sim Bin(950, 0.02)$

$P(B \geq 30) = 1 - P(B \leq 29)$

$= 1 - 0.98891094$

$= 0.01108906$

$= 0.0111$ i 4 lle degol

(d) State, with a reason, a distribution, including any parameters, that could approximate the distribution used in part (c). [2]

Mae n yn fawr a p yn fach felly

$B \approx Po(np)$

$B \approx Po(950 \times 0.02)$

$B \approx Po(19)$

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2. Emlyn aims to produce podcast episodes that are a standard length of time, which he calls the 'target time'. The time, X minutes, above or below the target time, which he calls the 'allowed time', can be modelled by the following cumulative distribution function.

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$$F(x)=\begin{cases}0 & x<-2\\ \frac{x+2}{5} & -2\leqslant x<1\\ \frac{x^2-x+3}{5} & 1\leqslant x\leqslant 2\\ 1 & x>2\end{cases}$$

(a) Calculate the upper quartile for the 'allowed time'. [5]

Chwartel uchaf $\Rightarrow$ $F(x)=0.75$

Os yw $x=1$ mae $F(x)=\frac{1^2-1+3}{5}$

$=0.6$ felly mae'r chwartel uchaf ble mae $F(x)=\frac{x^2-x+3}{5}$.

$F(x)=0.75$

$\frac{x^2-x+3}{5}=0.75$

$x^2-x+3=3.75$

$x^2-x-0.75=0$

$(x+0.5)(x-1.5)=0$

Naill ai $x+0.5=0$ neu $x-1.5=0$

$x=-0.5$ $x=1.5$ munud

D dim yn yr amrediad

$1\leqslant x\leqslant 2$

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(b) Find $f(x)$, the probability density function, for all values of x. [4]

$\frac{d}{dx}\left(\frac{x+2}{5}\right) = \frac{1}{5}$

$\frac{d}{dx}\left(\frac{x^2-x+3}{5}\right) = \frac{2x}{5} - \frac{1}{5}$

$f(x) = \frac{1}{5}$ ar gyfer $-2 \leq x < 1$

$= \frac{2}{5}x - \frac{1}{5}$ ar gyfer $1 \leq x \leq 2$

$= 0$ fel arall

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(c) (i) Calculate the mean 'allowed time'. [3]

(ii) Interpret your answer in context. [1]

(i) $E(X) = \int_{-\infty}^{\infty} f(x)\, x\, dx$

$= \int_{-2}^{1} \frac{1}{5}(x)\, dx + \int_{1}^{2} \left(\frac{2}{5}x - \frac{1}{5}\right) x\, dx$

$= \int_{-2}^{1} \frac{1}{5}x\, dx + \int_{1}^{2} \frac{2}{5}x^2 - \frac{1}{5}x\, dx$

$= \left[\frac{1}{5}\frac{x^2}{2}\right]_{-2}^{1} + \left[\frac{2}{5}\frac{x^3}{3} - \frac{1}{5}\frac{x^2}{2}\right]_{1}^{2}$

$= \left[\frac{x^2}{10}\right]_{-2}^{1} + \left[\frac{2x^3}{15} - \frac{x^2}{10}\right]_{1}^{2}$

$= \left(\frac{1^2}{10} - \frac{(-2)^2}{10}\right) + \left(\frac{2\times 2^3}{15} - \frac{2^2}{10}\right) - \left(\frac{2\times 1^3}{15} - \frac{1^2}{10}\right)$

$= \left(\frac{1}{10} - \frac{4}{10}\right) + \left(\frac{16}{15} - \frac{4}{10}\right) - \left(\frac{2}{15} - \frac{1}{10}\right)$

$= \frac{3}{30} - \frac{12}{30} + \frac{32}{30} - \frac{12}{30} - \frac{4}{30} + \frac{3}{30}$

$= \frac{10}{30}$

$= \frac{1}{3}$ munud

(ii) Ar gyfartaledd, mae bob pennod o'r podcast yn rhedeg 20 eiliad yn hirach na'r amser darged.

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3. A company makes bags. The table below shows the number of bags sold on a random sample of 50 days. A manager believes that the number of bags sold per day can be modelled by the Poisson distribution with mean 2·2.

Number of bags sold	0	1	2	3	4	5 or more
Frequency	7	10	11	9	6	7

(a) Carry out a chi-squared goodness of fit test, using a 10% significance level. [11]

H_0: Gall y data gael ei fodelu gan y dosraniad Poisson Po(2.2)

H_1: Ni all y data gael ei fodelu gan y dosraniad Poisson Po(2.2) yn erbyn

Gadewch i X gynrychioli nifer o fagiau sy'n cael eu werthu ar hapsampl o 50 diwrnod.

x	P(X=x)	P(X=x) × 50
0	0.11080315	5.5401575
1	0.24376694	12.188347
2	0.26814364	13.407182
3	0.1966386 7	9.8319335
4	0.10815126	5.407563
5 neu fwy	1 − 0.92750369 = 0.0724963 1	3.6248155

Rhaid i'r gwerth disgwyliedig fod o leiaf 5 bob tro felly rhaid cyfuno'r 2 res olaf.

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x	Arsylwad	Amlder Disgwyliedig	$\frac{(A-D)^2}{D}$
0	7	5.5401575	0.3846713969
1	10	12.188347	0.3929050094
2	11	13.407182	0.4321956084
3	9	9.8319335	0.0703944293 8
4neu fwy	13	9.032375	1.742843302

$X = \sum \frac{(A-D)^2}{D}$, yr ystadegyn prawf

$X = 3.023009746$

J = nifer o gategoriau

$J = 5 - 1$

$J = 4$

Felly rydym yn derbyn bod y data'n gallu cael ei fodelu gan y dosraniad Poisson Po(2.2).

Y gwerth critigol ($J = 4$, lefel arwyddocâd 10%) yw 7.779. Mae $3.023009746 < 7.779$ felly nid oes tystiolaeth digonol ar gyfer gwrthod Ho.

(b) A chi-squared goodness of fit test for the Poisson distribution with mean 2·5 is conducted. This uses the same number of degrees of freedom as part (a) and gives a test statistic of 1·53. State, with a reason, which of these two Poisson models is a better fit for the data. [1]

Y model efo cymedr 2.5 gan fod yr ystadegyn prawf yn llai. Golygai hyn fod y model efo cymedr 2.5 yn fwy agos at y data sydd wedi'i arsylwi.

4. An author poses the following question:

Does using cash for transactions affect people's financial behaviour?

She collects data on 'Cash transactions as a % of all transactions' and 'Household debt as a % of net disposable income' from a random sample of 25 countries. The table below shows the data she collected. There are missing values, p and q, for Malta and Denmark respectively.

Country	Cash transactions as a % of all transactions x	Household debt as a % of net disposable income y	Country	Cash transactions as a % of all transactions x	Household debt as a % of net disposable income y
Malta	92	p	**France**	68	120
Mexico	90	–14	**Luxembourg**	64	177
Greece	88	107	**Belgium**	63	113
Spain	87	110	**Finland**	54	137
Italy	86	87	**Estonia**	48	82
Austria	85	91	**The Netherlands**	45	247
Portugal	81	131	**UK**	42	147
Slovenia	80	56	**Australia**	37	214
Germany	80	95	**USA**	32	109
Ireland	79	154	**Sweden**	20	187
Slovakia	78	74	**South Korea**	14	182
Lithuania	75	46	**Denmark**	q	261
Latvia	71	43			

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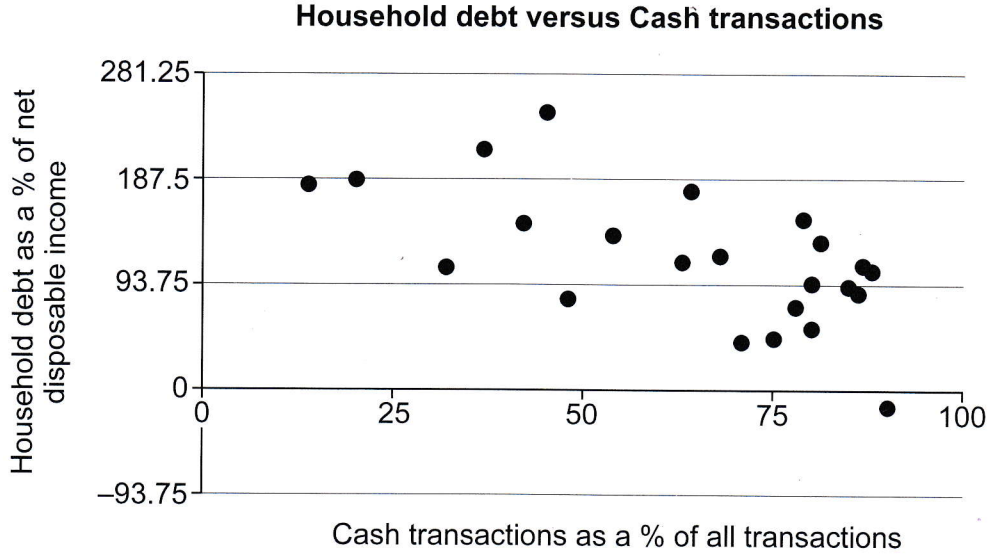
The summary statistics and scatter diagram below are for the other 23 countries.



$\sum x = 1467$ $\sum y = 2695$ $\sum x^2 = 105073$ $S_{xx} = 11503 \cdot 91304$ $S_{yy} = 78669 \cdot 30435$

$\sum y^2 = 394453$ $\sum xy = 152999$ $S_{xy} = -18895 \cdot 13043$

(a) Using the summary statistics for the 23 countries, calculate and interpret Pearson's product moment correlation coefficient. [3]

$r = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$= \dfrac{-18895.13043}{\sqrt{11503.91304 \times 78669.30435}}$

$= -0.628093707099$

Ho: $\rho = 0$ yn erbyn
H_1: $\rho \neq 0$ ble mae
ρ yn cynrychioli
cyfernod cydberthyniad
y boblogaeth.

Gydag n = 23, a phrawf dwy-ochrog, y gwerth critigol ôr tablau ystadegol (ar lefel arwyddocâd 5%) yw 0.4132. Mae maint r, 0.6281, yn fwy nâr gwerth critigol, felly mae tystiolaeth gref ar gyfer gwrthod Ho a ffafrio H_1. Felly mae tystiolaeth i gefnogi'r ddamcaniaeth bod defnyddio arian yn dylanwadu ymddygiad ariannol.

Y mwyafo arian sy'n cael ei ddefnyddio, lleiaf yw'r ddyled fel % o incwm gros net

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(b) Calculate the equation of the least squares regression line of Household debt as a % of net disposable income (y) on Cash transactions as a % of all transactions (x). [5]

Itafaliad y llinell atchwel sgwariau lleiaf:

$y = a + bx$

$b = \frac{S_{xy}}{S_{xx}}$ $\quad \bar{x} = \frac{\Sigma x}{n}$ $\quad \bar{y} = \frac{\Sigma y}{n}$

$b = \frac{-18895.13043}{11503.91304}$ $\quad \bar{x} = \frac{1467}{23}$ $\quad \bar{y} = \frac{2695}{23}$

$b = -1.642495937$ $\quad \bar{x} = 63.7826087$ $\quad \bar{y} = 117.173913$

$a = \bar{y} - b\bar{x}$

$a = 117.173913 - -1.642495937 \times 63.7826087$

$a = 221.9365886$

Felly $y = 221.9365886 - 1.642495937x$

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The regression line x on y is given below.

$$x = -0 \cdot 24y + 91 \cdot 92$$

(c) By selecting the appropriate regression line in each case, estimate the values of p and q in the table. [2]

(p) $y = 221.9365886 - 1.642495937x$

$y = 221.9365886 - 1.642495937 \times 92$

$p = 70.8269624$

$p = 71\%$ i'r ganran agosaf

(q) $x = -0.24y + 91.92$

$x = -0.24 \times 261 + 91.92$

$x = 29.28$

$q = 29\%$ i'r ganran agosaf

(d) Comment on the reliability of your answers in part (c). [1]

Mae angen cymryd gofal efo dibynadwyedd yr atebion gan fod 92% (Malta) a 261% (Denmark) tu allan yr amrediad o werthoedd eraill sydd yn y tabl. Golygai hyn ein bod yn defnyddio allosodiad (extrapolation), nid mewnosodiad (interpolation), sydd ddim mor ddibynadwy.

(e) Interpret the negative value of y for Mexico. [1]

Mae'r dyled negatif yn golygu bod cynilion gan bob cartref, ar gyfartaledd.

5. Lily is interested in the relationship between the way in which students learned Welsh and their attitude towards the Welsh language.

Students were categorised as having learned Welsh in one of three ways:

- from one Welsh-speaking parent/carer at home,
- from two Welsh-speaking parents/carers at home,
- at school only, for those with no Welsh-speaking parents/carers at home.

The students were asked to rate their attitude towards the Welsh language from 'Very negative' to 'Very positive'.

The following data for a random sample of 253 students were collected as part of a project.

	Learned Welsh			
Attitude	**From two parents/carers**	**From one parent/carer**	**At school only**	**Total**
Very negative	2	14	30	**46**
Slightly negative	4	20	21	**45**
Neutral	12	17	8	**37**
Slightly positive	21	19	11	**51**
Very positive	25	21	28	**74**
Total	**64**	**91**	**98**	**253**

Lily intends to carry out a chi-squared test for independence at the 5% level. She produces the following tables which are incomplete.

Expected Frequencies	**Learned Welsh**		
Attitude	**From two parents/carers**	**From one parent/carer**	**At school only**
Very negative	11·64	16·55	17·82
Slightly negative	11·38	16·19	17·43
Neutral	9·36	13·31	14·33
Slightly positive	12·90	18·34	19·75
Very positive	F	26·62	28·66

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Chi-Squared Contributions	Learned Welsh		
Attitude	**From two parents/carers**	**From one parent/carer**	**At school only**
Very negative	7·98	0·39	8·33
Slightly negative	4·79	0·90	0·73
Neutral	0·74	1·02	G
Slightly positive	5·08	0·02	3·88
Very positive	2·11	1·19	0·02
Total	20·70	3·52	H

(a) Calculate the values of F, G and H. [4]

$F = \frac{74 \times 64}{253}$

$F = 18.719367 59$

$F = 18.72$ i 2 le degol

$G = \frac{(8 - 14.33)^2}{14.33}$

$G = 2.79615492$

$G = 2.80$ i 2 le degol

$H = 8.33 + 0.73 + \overbrace{2.80}^{G} + 3.88 + 0.02$

$H = 15.76$

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(b) Carry out Lily's chi-squared test for independence at the 5% level. [6]

H_0: Nid oes cysylltiad rhwng dull i ddysgu Cymraeg ag agwedd tuag at y Gymraeg yn erbyn

H_1: Mae cysylltiad rhwng dull i ddysgu Cymraeg ag agwedd tuag at y Gymraeg.

$$X = \sum \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}$$

$= 20.70 + 3.52 + 15.76$

$= 39.98$

Graddau rhyddid $= (5-1)(3-1)$

$= 4 \times 2$

$= 8$

o'r tablau ystadegol, y gwerth critigol (ar lefel arwyddocâd 5%) yw $X_c = 15.507$

Gan fod $39.98 > 15.507$, mae tystiolaeth gref ar gyfer gwrthod H_0 o blaid H_1. Felly rydym yn derbyn bod cysylltiad rhwng dull i ddysgu Cymraeg ag agwedd tuag at y Gymraeg.

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(c) By referring to the figures in the tables on pages 16 and 17, give two comments on the relationship between the way students learned Welsh and their attitude towards the Welsh language. [2]

Ar gyfer y data 'dau riant' / 'negyddol iawn', mae llai nag 1 disgybl o ddysgwyr yn fan hyn (2 o'i gymharu ag amlder disgwyliedig 11.64).

Ar gyfer y data 'ysgol gynnig' / 'negyddol iawn', mae mwy nag 1 disgybl o ddysgwyr yn fan hyn (30 o'i gymharu ag amlder disgwyliedig 17.82).

Arholwr yn unig

6. Mae Penelope yn gwneud 8 teisen yr wythnos. Mae pob teisen yn costio £20 i'w gwneud ac yn gwerthu am £60. Mae hi bob amser yn gwerthu o leiaf 5 teisen yr wythnos. Mae unrhyw deisennau sydd ar ôl ar ddiwedd yr wythnos yn cael eu rhoi i fanc bwyd. Y tebygolrwydd o werthu 5 teisen mewn wythnos yw 0·3. Mae hi ddwywaith mor debygol o werthu 6 teisen mewn wythnos na gwerthu 7 teisen mewn wythnos. Yr elw disgwyliedig bob wythnos yw £206.

Lluniwch ddosraniad tebygolrwydd ar gyfer yr elw wythnosol. [7]

Gadewch i X gynrychioli sawl cacen sy'n cael ei werthu bob wythnos.

x	5	6	7	8
$P(X=x)$	0.3	$2p$	p	q

$0.3 + 2p + p + q = 1$

$3p + q = 0.7$

$q = 0.7 - 3p$ —— (1)

Gadewch i E gynrychioli'r elw sy'n cael ei uned.

x	5	6	7	8
Banc bwyd	3	2	1	0
e	$40\times5-3\times20$ $=£140$	$40\times6-2\times20$ $=£200$	$40\times7-20$ $=£260$	40×8 $=£320$
$P(E=e)$	0.3	$2p$	p	q

$E(P) = 206$

$140 \times 0.3 + 200 \times 2p + 260 \times p + 320 \times q = 206$

$42 + 400p + 260p + 320q = 206$

$42 + 660p + 320q = 206$

Arholwr yn unig

Yn amnewid am q o (1):

$42 + 660p + 320(0.7 - 3p) = 206$

$42 + 660p + 224 - 960p = 206$

$266 - 300p = 206$

$266 - 206 = 300p$

$60 = 300p$

$p = 0.2$

Felly $q = 0.7 - 3 \times 0.2$

$q = 0.1$

Dosraniad tebygolrwydd

x	5	6	7	8
$P(X=x)$	0.3	0.4	0.2	0.1

DIWEDD Y PAPUR

Trowch y dudalen.

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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