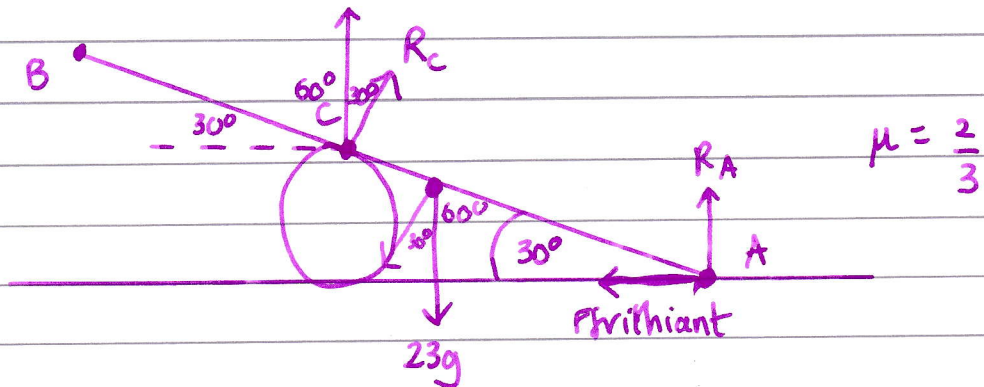


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1)



Yn cydrannu'n fertigol:

$$R_C \times \cos 30^\circ + R_A = 23g$$

$$\frac{\sqrt{3}}{2} R_C + R_A = 23g \quad \text{--- (1)}$$

Yn cydrannu'n llorweddol:

$$R_C \times \sin 30^\circ = F_{\text{frithiant}}$$

$$\frac{1}{2} R_C = F_{\text{frithiant}}$$

$$\text{Ond } F_{\text{frithiant}} = \mu R_A$$

$$= \frac{2}{3} R_A$$

$$\text{Felly } \frac{1}{2} R_C = \frac{2}{3} R_A$$

$$R_C = \frac{4}{3} R_A$$

$$\text{Amnewid i mewn i (1): } \frac{\sqrt{3}}{2} \left(\frac{4}{3} R_A \right) + R_A = 23g$$

$$\frac{3 + 2\sqrt{3}}{3} R_A = 23g$$

$$R_A = 104.6085041 \text{ N}$$

$$\begin{aligned} \text{Felly } R_c &= \frac{4}{3} R_A \\ &= \frac{4}{3} \times 104.608504 \\ &= 139.4780054 \text{ N} \end{aligned}$$

$$\text{I 2ie degol, } R_A = \underline{104.61 \text{ N}}, \quad R_c = \underline{139.48 \text{ N}}$$

b) Yn cymryd momentau o amgylch A:

Momentau clocwedd = Momentau gwrthglocwedd

$$\text{Pellter AC} \times R_c = 23g \times \cos 30^\circ \times 4$$

$$\text{Pellter AC} \times 139.4780054 = 780.8085041$$

$$\text{Pellter AC} = 5.598076211 \text{ m}$$

$$\text{Pellter AC} = \underline{5.60 \text{ m}} \quad \text{i 2ie degol}$$

c) Bydd yr atebion yn aros yr un peth ohermydd ni ddefnyddiwyd y pellter o A i'r craidd mäs yn rhan (a).

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Cyfaint x Duysedd

2) Siâp	Mâs	Pellter craidd mäs ôr sail
Côn Mawr	$\frac{1}{3} \pi (3x)^2 6y \rho$ $= \frac{1}{3} \pi \times 9x^2 \times 6y \rho$ $= 18 \pi x^2 y \rho$	$\frac{1}{4} \times 6y$ $= \frac{3}{2} y$
Côn Bach	$\frac{1}{3} \pi (x)^2 \left(\frac{1}{3} \times 6y\right) \rho$ $= \frac{1}{3} \pi x^2 (2y) \rho$ $= \frac{2}{3} \pi x^2 y \rho$	$\frac{1}{4} \times \left(\frac{1}{3} \times 6y\right) + \text{uchder y ffrwrthum}$ $= \frac{1}{4} \times 2y + (6y - 2y)$ $= \frac{1}{2} y + 4y$ $= \frac{9}{2} y$
Ffrwrthum	$18 \pi x^2 y \rho - \frac{2}{3} \pi x^2 y \rho$ $= \frac{52}{3} x^2 y \rho$	\bar{y}

Yn cymryd momentau o amgylch y sail:

$$\frac{52}{3} x^2 y \rho \times \bar{y} = 18 \pi x^2 y \rho \times \frac{3}{2} y - \frac{2}{3} \pi x^2 y \rho \times \frac{9}{2} y$$

$$\frac{52}{3} \bar{y} = 18 \times \frac{3}{2} y - \frac{2}{3} \times \frac{9}{2} y$$

$$\frac{52}{3} \bar{y} = 24 y$$

$$\bar{y} = \frac{18}{13} y \quad \checkmark$$

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3) a) Cyfnod = 12 awr
Osgled = $\frac{1}{2}(10-2)$
= 4 m

b) $\omega = \frac{2\pi}{\text{cyfnod}}$
 $\omega = \frac{2\pi}{12}$
 $\omega = \frac{1}{6}\pi$

Yn defnyddio $x = \pm a \cos(\omega t)$

Cychwyn ar llanwr isel ($x = -4\text{m}$) felly

$$x = -4 \cos(\omega t)$$

$$x = -4 \cos\left(\frac{1}{6}\pi t\right)$$

c) Mae angen $-2 = -4 \cos\left(\frac{1}{6}\pi t\right)$

$$\frac{1}{2} = \cos\left(\frac{1}{6}\pi t\right)$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{6}\pi t$$

$$\frac{1}{3}\pi = \frac{1}{6}\pi t$$

$$\frac{6}{3} = t$$

$$t = 2 \text{ awr}$$

Felly'r amser cynharaf yw 7 a.m. a'r amser dychwelyd hwyraf posib yw 3 p.m.

$$ch) \quad v = \frac{dx}{dt}$$

$$v = \frac{d}{dt} \left(-4 \cos \left(\frac{1}{6} \pi t \right) \right)$$

$$v = \frac{1}{6} \pi (-4) \left(-\sin \left(\frac{1}{6} \pi t \right) \right)$$

$$v = \frac{2}{3} \pi \sin \left(\frac{1}{6} \pi t \right)$$

Am 2 p.m., mae $t = 9$ aur.

$$v = \frac{2}{3} \pi \sin \left(\frac{1}{6} \pi (9) \right)$$

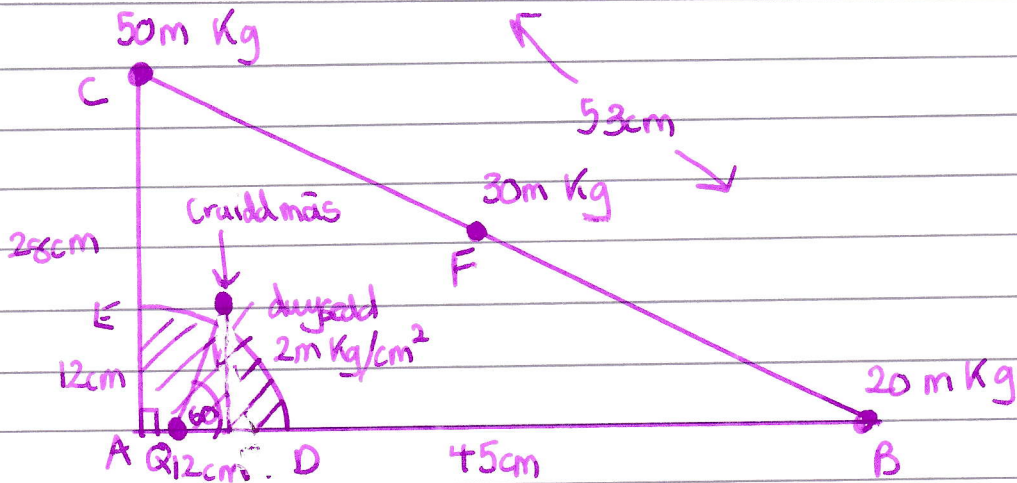
$$v = \frac{2}{3} \pi (-1)$$

$$v = -\frac{2}{3} \pi$$

Felly'r gyfradd mae lfeil y dŵr yn gostung am 2 p.m.
yw $\frac{2}{3} \pi$ m yr aur

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4)



$$\sqrt{28^2 + 45^2} = 53$$

	Mäs (Kg)	Pellter craidd mäs o AC	Pellter craidd mäs o AB
Gwrthrych			
Chwrâcher	$\pi \times 12^2 \times 2m$	$\frac{4 \times 12}{3\pi} = \frac{16}{\pi}$ cm	$\frac{16}{\pi}$ cm
Cylch	$4 = 72m\pi$		
C	50m	0cm	28cm
F	30m	22.5cm	14cm
B	20m	45cm	0cm
Lamina	$72m\pi + 100m$	\bar{x}	\bar{y}

Yn cymryd momentau o amgylch AC:

$$72m\pi \times \frac{16}{\pi} + 50m \times 0 + 30m \times 22.5 + 20m \times 45$$

$$= (72m\pi + 100m) \bar{x}$$

$$1152m + 675m + 900m = (72m\pi + 100m) \bar{x}$$

$$2727 = (72\pi + 100) \bar{x}$$

$$\bar{x} = \frac{2727}{72\pi + 100}$$

$$\bar{x} = \underline{\underline{8.36\text{cm}}} \quad ; \quad 2 \text{ le degol}$$

Yn cymryd momentau o amgylch AB:

$$72\text{m}\pi \times \frac{16}{\pi} + 50\text{m} \times 28 + 30\text{m} \times 14 + 20\text{m} \times 0$$

$$= (72\text{m}\pi + 100\text{m}) \bar{y}$$

$$1152\pi + 1400\pi + 420\pi = (72\text{m}\pi + 100\text{m}) \bar{y}$$

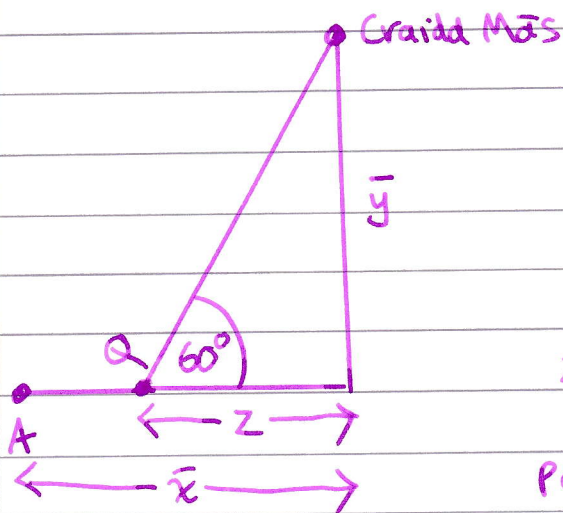
$$2972 = (72\pi + 100) \bar{y}$$

$$\bar{y} = \frac{2972}{72\pi + 100}$$

$$\bar{y} = \underline{\underline{9.11\text{cm}}} \text{ i 2 le degol}$$

b) Y pellter yw \bar{y} , sef 9.11cm i 2 le degol

c)



$$\tan 60^\circ = \frac{\bar{y}}{z}$$

$$z = \frac{\bar{y}}{\tan 60^\circ}$$

$$z = 5.260309724\text{cm}$$

$$\text{Pellter AQ} = \bar{x} - z$$

$$= 8.360038474$$

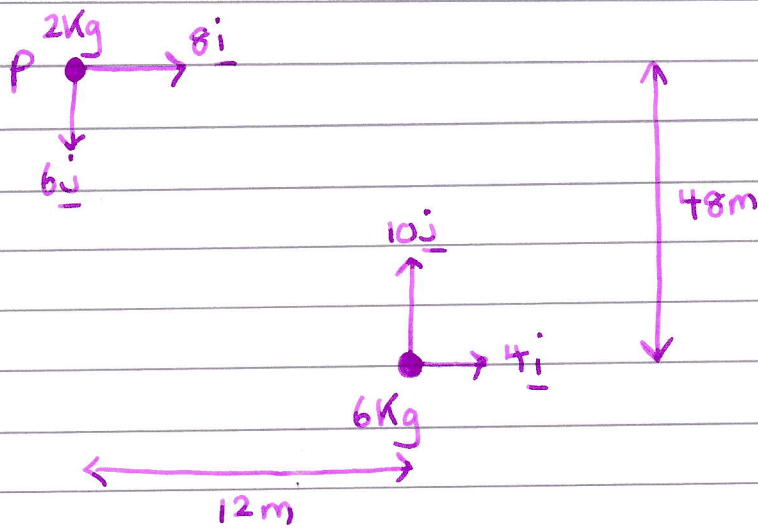
$$- 5.260309724$$

$$= 3.09972875\text{cm}$$

$$= \underline{\underline{3.10\text{cm}}} \text{ i 2 le degol}$$

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5)



Gadewch i safle gwreiddiol P gynrychiolir tarddbwynt.

a) $\underline{r}_P = \int \underline{v}_P dt$
 $= \int (8\underline{i} - 6\underline{j}) dt$
 $= (8\underline{i} - 6\underline{j})t + \underline{c}$

Os yw $t=0$, $\underline{r}_P = 0$, felly $0 = (8\underline{i} - 6\underline{j})(0) + \underline{c}$
 $0 = \underline{c}$

Felly $\underline{r}_P = (8\underline{i} - 6\underline{j})t$

$$\underline{r}_Q = \int \underline{v}_Q dt$$
$$= \int (4\underline{i} + 10\underline{j}) dt$$
$$= (4\underline{i} + 10\underline{j})t + \underline{c}$$

Os yw $t=0$, $\underline{r}_Q = 12\underline{i} - 48\underline{j}$, felly $12\underline{i} - 48\underline{j} = (4\underline{i} + 10\underline{j})(0) + \underline{c}$
 $12\underline{i} - 48\underline{j} = \underline{c}$

Felly $\underline{r}_Q = (4\underline{i} + 10\underline{j})t + 12\underline{i} - 48\underline{j}$

Os yw P a Q yn gwlthdaro, yna $\underline{r}_P = \underline{r}_Q$ ar gyfer gwerth arbennig o t .

Yn gymharu cyfernodau i :

$$8t = 4t + 12$$

$$4t = 12$$

$$t = \underline{\underline{3s}}$$

Yn gymharu cyfernodau j :

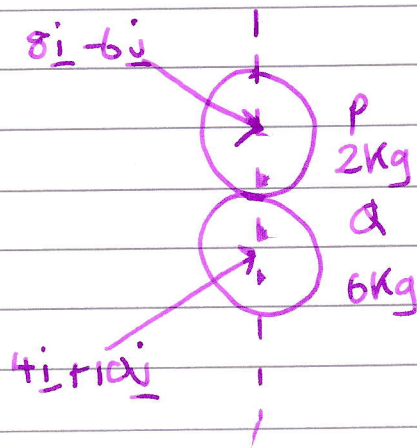
$$-6t = 10t - 48$$

$$-16t = -48$$

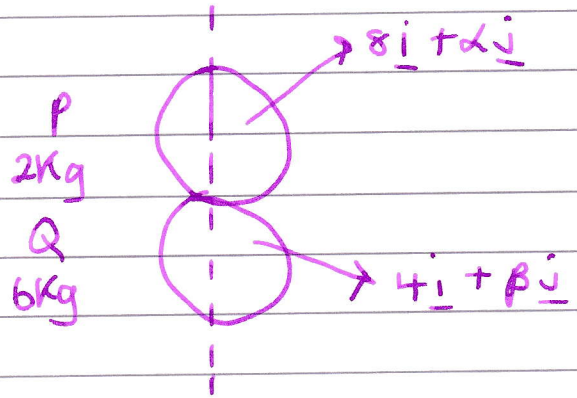
$$t = \underline{\underline{3s}}$$

Mae'r ddau werth t yn hafal felly bydd P a Q yn gwrthdaro pan fydd $t = \underline{\underline{3s}}$.

b)



C/N



AR O/L

Buanedd Q wedyn = 5 ms^{-1} .

$$\sqrt{4^2 + \beta^2} = 5$$

$$16 + \beta^2 = 25$$

$$\beta^2 = 9$$

$$\beta = \pm 3 \text{ ms}^{-1}$$

Egwyddor Cadwraeth Momentum (ar hyd llinell y candau)

$$2 \times -6 + 6 \times 10 = 2\alpha + 6\beta$$

$$48 = 2\alpha + 6\beta$$

$$24 = \alpha + 3\beta$$

$$\text{os yw } \beta = 3$$

$$24 = \alpha + 3\beta$$

$$24 = \alpha + 9$$

$$\alpha = 15$$

$$\text{os yw } \beta = -3$$

$$24 = \alpha + 3\beta$$

$$24 = \alpha - 9$$

$$\alpha = 33$$

Deddf Adferiad Newton (ar hyd llinell y canolau)

$$V_a - V_p = -e(u_a - u_p)$$

$$\beta - \alpha = -e(10 - -6)$$

$$\beta - \alpha = -e(16)$$

Naill a_i

$$3 - 15 = -16e$$

$$-12 = -16e$$

$$e = \frac{3}{4}$$

neu

$$-3 - 33 = -16e$$

$$-36 = -16e$$

$$e = \frac{9}{4}$$

Yma mae $e > 1$ felly nid yw hwn yn ddabrysiad dilys.

Felly $\alpha = 15$, $\beta = 3$, $e = \frac{3}{4}$.

Cyflymder P ar ôl y gwrthdrawiad yw $8\mathbf{i} + 15\mathbf{j}$

$$\begin{aligned} \text{c) Ergyd} &= \text{Newid mewn momentum} \\ &= \text{mas (cyflymder wedyn, llynged - cyflymder gynt)} \\ &= 2 (0\mathbf{i} + 0\mathbf{j} - (8\mathbf{i} + 15\mathbf{j})) \\ &= -16\mathbf{i} - 30\mathbf{j} \end{aligned}$$

$$\begin{aligned} \text{Maint yr ergyd} &= \sqrt{(-16)^2 + (-30)^2} \\ &= \underline{\underline{34 \text{ Ns}}} \end{aligned}$$

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$$\begin{aligned} \text{b) a) Tensiwn yn y sbring} &= \frac{\lambda x}{l} \\ &= \frac{312500 x}{1} \\ &= 312500 x \end{aligned}$$

Cyfeiriad mudiant y trêen = cyfeiriad positif.

$F = ma$ ar y system, yn llonweddol, ← = tîf

- Tensiwn yn y sbring - Gwrthiant y chwanegol = ma

$$-312500 x - 250000 v = 50 \times 1000 a$$

$$-312500 x - 250000 v = 50000 a$$

$$-312500 x - 250000 \frac{dx}{dt} = 50000 \frac{d^2 x}{dt^2}$$

$$-3125 x - 2500 \frac{dx}{dt} = 500 \frac{d^2 x}{dt^2} \quad (\div 100)$$

$$-125 x - 100 \frac{dx}{dt} = 20 \frac{d^2 x}{dt^2} \quad (\div 25)$$

$$-25 x - 20 \frac{dx}{dt} = 4 \frac{d^2 x}{dt^2} \quad (\div 5)$$

$$0 = 4 \frac{d^2 x}{dt^2} + 20 \frac{dx}{dt} + 25 x$$

$$4 \frac{d^2 x}{dt^2} + 20 \frac{dx}{dt} + 25 x = 0 \quad \checkmark$$

b) $4 \frac{d^2 x}{dt^2} + 20 \frac{dx}{dt} + 25 x = 0.$

Ceisis $x = Ae^{mt}$ fel bod m yn bodloni $am^2 + bm + c = 0$
 $a = 4, b = 20, c = 25$

Hafaliad ategol / Auxiliary equation

$$4m^2 + 20m + 25 = 0$$

Hafaliad cwadrateg: $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-20 \pm \sqrt{20^2 - 4(4)(25)}}{2 \times 4}$$

$$m = \frac{-20 \pm \sqrt{0}}{8}$$

$$m = -2.5 \quad (\text{dwywaith})$$

Un gwreiddyn wedi ailadrodd felly'r ffynhiant cyflenwol (Complementary function) yw $x = e^{-2.5t}(A + Bt)$

Ar gyfer yr integryn neilltoll (particular integral) mae'r ochr dde yn sero felly does dim angen integryn neilltoll.

Oo yw $t=0$ mae $x=0, v=0$

Yn amnewid: $x = e^{-2.5t}(A + Bt)$

$$0 = e^{-2.5(0)}(A + B(0))$$

$$0 = 1(A + 0)$$

$$0 = A$$

Difffrenn: $x = e^{-2.5t}(A + Bt)$

$$\frac{dx}{dt} = -2.5 e^{-2.5t}(A + Bt) + B e^{-2.5t}$$

Amnewid: $u = -2.5 e^{-2.5(0)}(0 + B(0)) + B e^{-2.5(0)}$

$$u = 0 + B$$

$$u = B$$

Felly'r datbysiad cyffredinol yw $x = e^{-2.5t}(A + Bt)$

$$x = e^{-2.5t}(0 + ut)$$

$$x = u e^{-2.5t} t$$

c) Os yw $v = 0 \text{ ms}^{-1}$, gallun amnewid hwn i $\frac{dx}{dt}$ uchod.

$$\frac{dx}{dt} = -2.5 e^{-2.5t}(A + Bt) + B e^{-2.5t}$$

$$\frac{dx}{dt}$$

$$\frac{dx}{dt} = -2.5 e^{-2.5t}(0 + ut) + u e^{-2.5t}$$

$$\frac{dx}{dt}$$

$$\frac{dx}{dt} = e^{-2.5t}(-2.5ut + u)$$

$$\frac{dx}{dt}$$

$$0 = e^{-2.5t}(-2.5ut + u)$$

$$0 = -2.5ut + u$$

(rhannu efo $e^{-2.5t}$)

$$0 = -2.5t + 1$$

(rhannu efo u)

$$2.5t = 1$$

$$t = \underline{0.4 \text{ s}}$$

Amnewid $x = 0.3$, $t = 0.4$ i meun i $x = u e^{-2.5t} t$

$$0.3 = u e^{-2.5(0.4)}(0.4)$$

$$\frac{3}{4} = u e^{-1}$$

$$\frac{3}{4} e = u$$

$$u = \frac{3}{4} e$$

$$u = \underline{2.04 \text{ ms}^{-1}} \text{ i 2 l.d.}$$

ch) Gwanychu critigol (critical damping) ohenydd y gwreiddyn sy'n ailadrodd yn rhan (b).