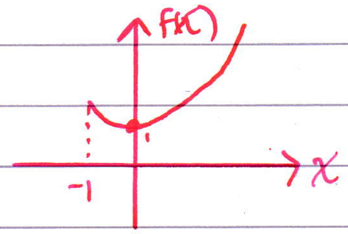
Uned 4 Pellach Haf 2023

1) $f(x) = \cosh(x)$ $\qquad$ $g(x) = x^2 - 1$

Parth $(-1, \infty)$ $\qquad$ Parth $(0, \infty)$

a) Amrediad $f(x) = (1, \infty)$



Amrediad $g(x) = (-1, \infty)$

Gellid defnyddio unrhyw rif yn yr amrediad yma ym mharth $f(x)$.

Parth $f(g(x)) = (0, \infty)$

Amrediad $f(g(x)) = [1, \infty)$.

b) $fg(x) = 3$

$f(g(x)) = 3$

$f(x^2 - 1) = 3$

$\cosh(x^2 - 1) = 3$

$$x^2 - 1 = \cosh^{-1}(3)$$
$$x^2 - 1 = 1.762747174$$
$$x^2 = 2.762747174$$
$$x = \pm 1.662151369$$

Dewis y gwerth positif gan nad yw'r gwerth negatif ym mharth $fg(x)$.

Felly $x = \underline{1.662}$ i 3 lle degol.

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2) $A = \begin{pmatrix} \lambda & 1 & 14 \\ -1 & 2 & 8 \\ -3 & 2 & \lambda \end{pmatrix}$

a) $\det(A) = \lambda \begin{vmatrix} 2 & 8 \\ 2 & \lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 8 \\ -3 & \lambda \end{vmatrix} + 14 \begin{vmatrix} -1 & 2 \\ -3 & 2 \end{vmatrix}$

$= \lambda(2\lambda - 16) - 1(-\lambda + 24) + 14(-2+6)$

$= 2\lambda^2 - 16\lambda + \lambda - 24 + 56$

$= 2\lambda^2 - 15\lambda + 32$ (felly $a = 2$, $b = -15$, $c = 80$)

b) Mae'r matrics yn hynod os yw $\det(A) = 0$

$2\lambda^2 - 15\lambda + 32 = 0$

Y gwahanolyn yw $b^2 - 4ac = (-15)^2 - 4(2)(32)$

$= -31$

Gan fod y gwahanolyn yn negatif nid oes datrysiadau real i'r hafaliad $2\lambda^2 - 15\lambda + 80 = 0$. Felly, mae'r matrics A yn anhynod ar gyfer pob gwerth o λ.

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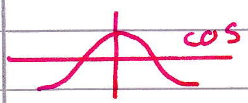
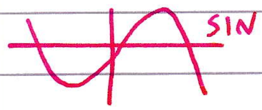
3) $z = \cos\theta + i\sin\theta$

a) Theorem de Moivre: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$$z^n + \frac{1}{z^n} = z^n + z^{-n}$$





$$= (\cos\theta + i\sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$$
$$= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$$
$$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$$
$$= 2\cos n\theta \checkmark$$

b)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Ehangiad Binomaidd:

$$\left(z + \frac{1}{z}\right)^6 = z^6 + 6z^5\left(\frac{1}{z}\right) + 15z^4\left(\frac{1}{z}\right)^2 + 20z^3\left(\frac{1}{z}\right)^3 + 15z^2\left(\frac{1}{z}\right)^4 + 6z\left(\frac{1}{z}\right)^5 + \left(\frac{1}{z}\right)^6$$
$$= z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$$
$$= \left(z^6 + \frac{1}{z^6}\right) + 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) + 20$$
$$= 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

Felly

$$\left(z+\frac{1}{z}\right)^6 = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

$$(2\cos\theta)^6 = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

$$2^6\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

$$64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

(Felly $a=1$, $b=6$, $c=15$, $d=10$)

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4) $4x - 2y + 3z = 8$

$2x - 3y + 8z = -1$

$2x + 4y - z = 0$

$$\left(\begin{array}{ccc|c} 4 & -2 & 3 & 8 \\ 2 & -3 & 8 & -1 \\ 2 & 4 & -1 & 0 \end{array}\right) \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 4 & -2 & 3 & 8 \\ 0 & -4 & 13 & -10 \\ 0 & 10 & -5 & -8 \end{array}\right) \begin{array}{l} \\ 2R_2 - R_1 =: R_4 \\ 2R_3 - R_1 =: R_5 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 4 & -2 & 3 & 8 \\ 0 & -4 & 13 & -10 \\ 0 & 0 & 55 & -66 \end{array}\right) \begin{array}{l} \\ \\ 2R_5 + 5R_4 =: R_6 \end{array}$$

$R_6 \Rightarrow 55z = -66$

$z = -\frac{66}{55}$

$\underline{z = -1.2}$

$R_4 \Rightarrow -4y + 13z = -10$

$-4y + 13(-1.2) = -10$

$-4y - 15.6 = -10$

$-4y = 5.6$

$\underline{y = -1.4}$

$R_1 \Rightarrow 4x - 2y + 3z = 8$

$4x - 2(-1.4) + 3(-1.2) = 8$

$4x - 0.8 = 8$

$4x = 8.8$

$\underline{x = 2.2}$

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5) (a) $f(x) = \sin(2x).$

$f(0) = \sin(2 \times 0)$
$f(0) = 0$

$f'(x) = 2\cos(2x)$
$f'(0) = 2\cos(0)$
$f'(0) = 2$

$f''(x) = -4\sin(2x)$
$f''(0) = -4\sin(2 \times 0)$
$f''(0) = 0$

$f'''(x) = -8\cos(2x)$
$f'''(x) = -8\cos(0)$
$f'''(x) = -8$

$f''''(x) = 16\sin(2x)$
$f''''(0) = 16\sin(0)$
$f''''(0) = 0$

$f'''''(x) = 32\cos(2x)$
$f'''''(0) = 32\cos(0)$
$f'''''(0) = 32$

Cyfres Maclaurin:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f''''(0) + \frac{x^5}{5!}f'''''(0) + \dots$$

$$= 0 + x(2) + \frac{x^2}{2}(0) + \frac{x^3}{6}(-8) + \frac{x^4}{24}(0) + \frac{x^5}{120}(32) + \dots$$

$$= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \dots$$

b) $\sin(2x) = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \dots$

$\downarrow$ DIFFERU

$2\cos(2x) = 2 - 4x^2 + \frac{4}{3}x^4 + \dots$

Nawr $\cos(2x) = 2\cos^2(x) - 1$

Felly
$$2(2\cos^2(x) - 1) = 2 - 4x^2 + \frac{4}{3}x^4 + \dots$$
$$4\cos^2(x) - 2 = 2 - 4x^2 + \frac{4}{3}x^4 + \dots$$
$$4\cos^2(x) = 4 - 4x^2 + \frac{4}{3}x^4 + \dots$$
$$\cos^2(x) = 1 - x^2 + \frac{4}{12}x^4 + \dots$$

Hyd at y term yn x^4,
$$\cos^2(x) = 1 - x^2 + \frac{1}{3}x^4.$$

Uned 4 Pellach Haf 2023

6) a) Os yw $t = \tan\left(\frac{x}{2}\right)$ mae $\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$.

Felly $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$$

$$= \frac{2t}{\cancel{1+t^2}} \times \frac{\cancel{1+t^2}}{1-t^2}$$

$$\tan\theta = \frac{2t}{1-t^2} \quad \checkmark$$

b) $r = \cos\left(\frac{\theta}{2}\right)$, $\quad -\pi < \theta \leq \pi$.

$x = r\cos\theta \qquad\qquad y = r\sin\theta$

$x = \cos\left(\frac{\theta}{2}\right)\cos\theta \qquad\qquad y = \cos\left(\frac{\theta}{2}\right)\sin\theta$

$\frac{dx}{d\theta} = -\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cos\theta - \sin\theta\cos\left(\frac{\theta}{2}\right) \qquad \frac{dy}{d\theta} = -\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\sin\theta + \cos\theta\cos\left(\frac{\theta}{2}\right)$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= -\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\sin\theta + \cos\theta\cos\left(\frac{\theta}{2}\right) \times \frac{1}{-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cos\theta - \sin\theta\cos\left(\frac{\theta}{2}\right)}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\sin\theta + \cos\theta\cos\left(\frac{\theta}{2}\right)}{-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cos\theta - \sin\theta\cos\left(\frac{\theta}{2}\right)}$$

Os yw'r tangiad yn berpendicwlar i'r llinell gychwynnol yna mae enwadur $\frac{dy}{dx}$ yn sero. (Neu $\frac{dx}{d\theta} = 0$.) Fel yma mae $\frac{dy}{dx} = \infty$.

Felly $-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cos\theta - \sin\theta\cos\left(\frac{\theta}{2}\right) = 0$

$$-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cos\theta = \sin\theta\cos\left(\frac{\theta}{2}\right)$$

$$-\frac{1}{2}\tan\left(\frac{\theta}{2}\right) = \tan\theta$$ [Rhannu efo $\cos\theta$ a $\cos\left(\frac{\theta}{2}\right)$]

$$\tan\theta = -\frac{1}{2}\tan\left(\frac{\theta}{2}\right) \checkmark$$

c) $\tan\theta = -\frac{1}{2}\tan\left(\frac{\theta}{2}\right)$

$$\frac{2t}{1-t^2} = -\frac{1}{2}(t)$$

$$4t = -t(1-t^2)$$

$$4t = -t + t^3$$

$$0 = t^3 - 5t$$

$$0 = t(t^2-5)$$

Naill ai $t=0$ neu $t^2-5=0$

$$t^2 = 5$$

$$t = \pm\sqrt{5}$$

$t = 0$

$t = \tan\left(\frac{\theta}{2}\right)$

$0 = \tan\left(\frac{\theta}{2}\right)$

$\tan^{-1}(0) = \frac{\theta}{2}$

$0 = \frac{\theta}{2}$

$\theta = 0$

$r = \cos\left(\frac{\theta}{2}\right)$

$r = \cos\left(\frac{0}{2}\right)$

$r = 1$

$\underline{(1, 0)}$

$t = \sqrt{5}$

$t = \tan\left(\frac{\theta}{2}\right)$

$\sqrt{5} = \tan\left(\frac{\theta}{2}\right)$

$\tan^{-1}(\sqrt{5}) = \frac{\theta}{2}$

$1.150261992 = \frac{\theta}{2}$

$\theta = 2.300523983$

$r = \cos\left(\frac{\theta}{2}\right)$

$r = \cos\left(\frac{2.300523983}{2}\right)$

$r = 0.4082482905$

$\underline{(0.41, 2.30)}$ (2 l.d.)

$t = -\sqrt{5}$

$t = \tan\left(\frac{\theta}{2}\right)$

$-\sqrt{5} = \tan\left(\frac{\theta}{2}\right)$

$\tan^{-1}(-\sqrt{5}) = \frac{\theta}{2}$

$-1.150261992 = \frac{\theta}{2}$

$\theta = -2.300523983$

$r = \cos\left(\frac{\theta}{2}\right)$

$r = \cos\left(\frac{-2.300523983}{2}\right)$

$r = 0.4082482905$

$\underline{(0.41, -2.30)}$ (2 l.d.)

ch) Arwynebedd y rhanbarth $= \frac{1}{2}\int_0^{\pi} r^2 \, d\theta$

$$= \frac{1}{2}\int_0^{\pi} \left(\cos\left(\frac{\theta}{2}\right)\right)^2 d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \cos^2\left(\frac{\theta}{2}\right) d\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\frac{\cos\theta + 1}{2} = \cos^2\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{2}\int_0^{\pi} \frac{\cos\theta + 1}{2} \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi} \cos\theta + 1 \, d\theta$$

$$= \frac{1}{4}\left[\sin\theta + \theta\right]_0^{\pi}$$

$$= \frac{1}{4}\left[(\sin(\pi) + \pi) - (\sin(0) + 0)\right]$$

$$= \frac{1}{4}\left[0 + \pi - (0 + 0)\right]$$

$$= \frac{\pi}{4} \text{ uned sgwâr}$$

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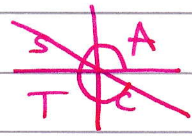
7) $z = 11 - 2i$

$|z| = \sqrt{11^2 + (-2)^2}$
$= \sqrt{121 + 4}$
$= \sqrt{125}$
$= 5\sqrt{5}$

$\operatorname{Arg}(z) = \tan^{-1}\left(\frac{-2}{11}\right)$
$= -0.17985,$
$2.961739154,$
$\underline{\underline{6.103331807}}$



Gadewch i $w = \sqrt[3]{11 - 2i}$

$w^3 = 11 - 2i$

$w^3 = 5\sqrt{5}\,(\cos(6.10\ldots) + i\sin(6.10\ldots))$

$w = [5\sqrt{5}\,(\cos(6.10\ldots) + i\sin(6.10\ldots))]^{\frac{1}{3}}$

$w = 2.236067977\,[\cos(6.10\ldots) + i\sin(6.10\ldots)]^{\frac{1}{3}}$

Gadewch i $\alpha = 6.103331807$, $\beta = 2.236067977$.

Felly $w = \beta[\cos(\alpha) + i\sin(\alpha)]^{\frac{1}{3}}$

$$(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \cos\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right] + i\sin\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right]$$

$$(\cos\alpha + i\sin\alpha)^{\frac{1}{3}} = \cos\left[\frac{\alpha}{3} + \frac{2(k-1)\pi}{3}\right] + i\sin\left[\frac{\alpha}{3} + \frac{2(k-1)\pi}{3}\right]$$

$k=1$ $w = \beta\left(\cos\left(\frac{\alpha}{3}\right) + i\sin\left(\frac{\alpha}{3}\right)\right)$

$w = \beta(-0.4472135954 + 0.894429238\,8i)$

$w = -1 + 2i$

$k=2$ $w = \beta\left(\cos\left(\frac{\alpha}{3} + \frac{2\pi}{3}\right) + i\sin\left(\frac{\alpha}{3} + \frac{2\pi}{3}\right)\right)$

$w = \beta(-0.5509898716 - 0.83451193i)$

$w = -1.232 - 1.866i$ i 3 lle degol

$K=3$ $w = \beta\left(\cos\left(\frac{\alpha}{3}+\frac{4\pi}{3}\right)+i\sin\left(\frac{\alpha}{3}+\frac{4\pi}{3}\right)\right)$

$w = \beta(0.998203467 - 0.05991526101i)$

$w = 2.232 - 0.134i$ i 3 lledegol

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8) $f(x) = \dfrac{1}{\sqrt{x^2+4x+3}}$

a) Gwerth cymedrig $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

$$= \frac{1}{2-0}\int_0^2 \frac{1}{\sqrt{x^2+4x+3}}\,dx$$

$$= \frac{1}{2}\int_0^2 \frac{1}{\sqrt{(x+2)^2-4+3}}\,dx$$

$$= \frac{1}{2}\int_0^2 \frac{1}{\sqrt{(x+2)^2-1}}\,dx$$

Gadewch i $u = x+2$ $\quad [0]\ u = 0+2$

$\dfrac{du}{dx} = 1$ $\quad u = 2$

$\quad [2]\ u = 2+2$

$du = dx$ $\quad u = 4$

Gwerth cymedrig $= \dfrac{1}{2}\displaystyle\int_2^4 \frac{1}{\sqrt{u^2-1}}\,du$

$$= \frac{1}{2}\left[\cosh^{-1}\left(\frac{u}{1}\right)\right]_2^4$$

$$= \tfrac{1}{2}\cosh^{-1}(4) - \tfrac{1}{2}\cosh^{-1}(2)$$

$= 0.373$ i 3 lle degol

b) $\int_0^2 \pi r^2 dr = \pi \int_0^2 \left(\frac{1}{\sqrt{x^2+4x+3}}\right)^2 dx$

$= \pi \int_0^2 \frac{1}{x^2+4x+3} dx$

$= \pi \int_0^2 \frac{1}{(x+2)^2-2^2+3} dx$

$= \pi \int_0^2 \frac{1}{(x+2)^2-1} dx$

Gadewch i $u = x+2$ 　　[0] $u = 0+2$

$\frac{du}{dx} = 1$ 　　$u = 2$

$du = dx$ 　　[2] $u = 2+2$

　　$u = 4$

Mteb $= \pi \int_2^4 \frac{1}{u^2-1} du$

$= \pi \left[\frac{1}{2(1)} \ln\left|\frac{u-1}{u+1}\right|\right]_2^4$

$= \pi \left[\frac{1}{2}\ln\left|\frac{4-1}{4+1}\right| - \frac{1}{2}\ln\left|\frac{2-1}{2+1}\right|\right]$

$= \pi \left[\frac{1}{2}\ln\left|\frac{3}{5}\right| - \frac{1}{2}\ln\left|\frac{1}{3}\right|\right]$

$= \frac{\pi}{2}\left(\ln\left(\frac{3}{5}\right) - \ln\left(\frac{1}{3}\right)\right]$

$= \frac{\pi}{2}\ln\left(\frac{\frac{3}{5}}{\frac{1}{3}}\right)$

$= \frac{\pi}{2}\ln\left(\frac{9}{5}\right)$

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9) $(x+1)\frac{dy}{dx} + 5y = (x+1)^2, \quad x > -1$

$$\frac{dy}{dx} + \frac{5y}{x+1} = x+1$$

Ar gyfer $\frac{dy}{dx} + Fy = G$,

ble mae F a G yn ffwythiannau mewn x yn unig,
y ffactor integru yw $I = e^{\int F\,dx}$

$$I = e^{\int \frac{5}{x+1}dx}$$
$$I = e^{5\ln|x+1|}$$
$$I = e^{5\ln|(x+1)^5|}$$
$$I = (x+1)^5$$

Felly $\frac{dy}{dx} + \frac{5y}{x+1} = x+1$

Lluosi bob ochr yr hafaliad â'r ffactor integru:

$$(x+1)^5\frac{dy}{dx} + \frac{5y}{x+1}(x+1)^5 = (x+1)(x+1)^5$$

$$(x+1)^5\frac{dy}{dx} + 5y(x+1)^4 = (x+1)^6$$

$$\int\left((x+1)^5\frac{dy}{dx} + 5y(x+1)^4\right)dx = \int (x+1)^6\,dx$$

$$(x+1)^5 y = \frac{1}{7}(x+1)^7 + K$$

Os yw $x=1$, $y=\frac{1}{4}$:

$$(1+1)^5 \times \frac{1}{4} = \frac{1}{7}(1+1)^7 + K$$
$$8 = \frac{128}{7} + K$$
$$-\frac{72}{7} = K$$

Felly $(x+1)^5 y = \frac{1}{7}(x+1)^7 - \frac{72}{7}$.

Os yw $x=0$, $(0+1)^5 y = \frac{1}{7}(0+1)^7 - \frac{72}{7}$

$$y = \frac{1}{7} - \frac{72}{7}$$
$$y = \frac{-71}{7}$$

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10) $y = \sin^{-1}(2x+5)$

a) $\sin y = \sin(\sin^{-1}(2x+5))$

$\sin y = 2x+5$

$\cos y \left(\frac{dy}{dx}\right) = 2$

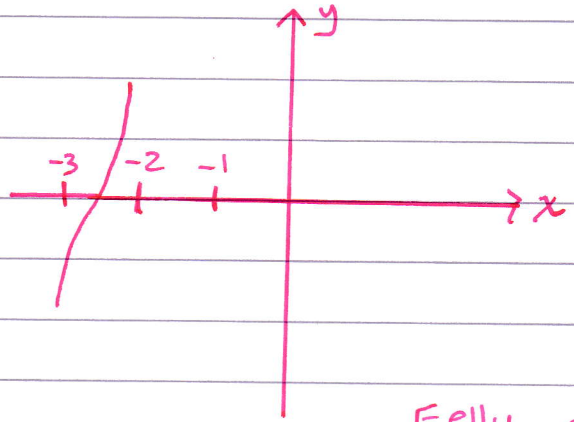
$\frac{dy}{dx} = \frac{2}{\cos y}$

Nawr $\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1-\sin^2 y}$

$\cos y = \pm\sqrt{1-(2x+5)^2}$



Trwy edrych ar graff $y = \sin^{-1}(2x+5)$, neu ystyried gwerthoedd wrth amnewid i mewn iddo (efo'r TABLE MODE), mae graddiant y o hyd yn bositif. Felly dewisun $\cos y = +\sqrt{1-(2x+5)^2}$

Felly $\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x+5)^2}}$ ✓

b) Rhaid bod $\sqrt{1-(2x+5)^2}$ yn bositif.

Felly rhaid cael $(2x+5)^2 < 1$

$2x+5 < \sqrt{1}$ a $2x+5 > -\sqrt{1}$

$2x+5 < 1$ a $2x+5 > -1$

$2x < -4$ $\quad$ $2x > -6$

$x < -2$ $\quad$ $x > -3$

Ateb: $\underline{-3 < x < -2}$

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11) a) $\int_{-2}^{0} e^{2x} \sinh x \, dx$

Nawr $\sinh x = \frac{1}{2}(e^{x} - e^{-x})$

$$\int_{-2}^{0} e^{2x}\left(\frac{1}{2}(e^{x} - e^{-x})\right) dx$$

$$= \int_{-2}^{0} \frac{1}{2} e^{3x} - \frac{1}{2} e^{x} \, dx$$

$$= \left[\frac{e^{3x}}{6} - \frac{1}{2} e^{x}\right]_{-2}^{0}$$

$$= \left(\frac{e^{3\times 0}}{6} - \frac{1}{2} e^{0}\right) - \left(\frac{e^{3\times -2}}{6} - \frac{1}{2} e^{-2}\right)$$

$$= \frac{1}{6} - \frac{1}{2} - \frac{e^{-6}}{6} + \frac{1}{2} e^{-2}$$

$$= -\frac{1}{3} - \frac{e^{-6}}{6} + \frac{1}{2} e^{-2}$$

(≈ -0.266 i 3 lle degol)

b) $\int_{\frac{3}{2}}^{3} \frac{5}{(x-1)(x^2+9)} \, dx$

Nawr $\frac{5}{(x-1)(x^2+9)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$

$$\frac{5}{(x-1)(x^2+9)} \equiv \frac{A(x^2+9)}{(x-1)(x^2+9)} + \frac{(Bx+C)(x-1)}{(x-1)(x^2+9)}$$

$$5 \equiv A(x^2+9) + (Bx+C)(x-1)$$

Os yw $x = 1$, $5 \equiv A(1^2+9) + (B(1)+C)(1-1)$

$$5 \equiv 10A$$

$$A = \frac{1}{2}$$

Yn cymharu cyfernodau x^2:

$$0 \equiv A + B$$
$$0 \equiv \frac{1}{2} + B$$
$$B \equiv -\frac{1}{2}$$

Yn cymharu cysonion:

$$5 \equiv 9A - C$$
$$5 \equiv 9\left(\frac{1}{2}\right) - C$$
$$C \equiv -\frac{1}{2}$$

Felly $\displaystyle\int_{\frac{3}{2}}^{3} \frac{5}{(x-1)(x^2+9)}\,dx = \int_{\frac{3}{2}}^{3} \frac{1}{2(x-1)} + \frac{-\frac{1}{2}x - \frac{1}{2}}{x^2+9}\,dx$

$$= \frac{1}{2}\int_{\frac{3}{2}}^{3} \frac{1}{x-1} + \frac{-1-x}{x^2+9}\,dx$$

$$= \frac{1}{2}\int_{\frac{3}{2}}^{3} \frac{1}{x-1} - \frac{1}{x^2+9} - \frac{x}{x^2+9}\,dx$$

$$= \frac{1}{2}\int_{\frac{3}{2}}^{3} \frac{1}{x-1} - \frac{1}{3^2+x^2} - \frac{x}{x^2+9}\,dx$$

$$= \frac{1}{2}\left[\ln|x-1| \quad \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) - \frac{\ln|x^2+9|}{2}\right]_{\frac{3}{2}}^{3}$$

$$= \frac{1}{2}\left[\left(\ln|3-1| - \frac{1}{3}\tan^{-1}\left(\frac{3}{3}\right) - \frac{\ln|3^2+9|}{2}\right) - \left(\ln\left|\frac{3}{2}-1\right| - \frac{1}{3}\tan^{-1}\left(\frac{\frac{3}{2}}{3}\right) - \frac{\ln\left|\frac{3}{2}^2+9\right|}{2}\right)\right]$$

$$= \frac{1}{2}\left[\left(\ln(2) - \frac{1}{3}\tan^{-1}(1) - \frac{\ln(18)}{2}\right) - \left(\ln\left(\frac{1}{2}\right) - \frac{1}{3}\tan^{-1}\left(\frac{1}{2}\right) - \frac{\ln\left(\frac{45}{4}\right)}{2}\right)\right]$$

$$= \frac{1}{2} \times 1.044042362$$

$$= 0.5220211808$$

$= \underline{\underline{0.522}}$ i 3 lle degol

Uned 4 Pellach Haf 2023

12) $\cos 4\theta + \cos 2\theta = \cos\theta$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos 4\theta + \cos 2\theta = 2\cos\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)$$

$$= 2\cos(3\theta)\cos\theta$$

$2\cos 3\theta \cos\theta = \cos\theta$

$2\cos 3\theta \cos\theta - \cos\theta = 0$

$\cos\theta(2\cos 3\theta - 1) = 0$

Naill ai $\cos\theta = 0$ neu $2\cos 3\theta - 1 = 0$

$\theta = \cos^{-1}(0)$

$\theta = 2n\pi \pm \frac{\pi}{2}$

ar gyfer cyfanrif n

Felly naillai $\underline{\theta = 2n\pi + \frac{\pi}{2}}$

neu $\underline{\theta = 2n\pi - \frac{\pi}{2}}$

$2\cos 3\theta = 1$

$\cos 3\theta = \frac{1}{2}$

$3\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$3\theta = 2n\pi \pm \frac{\pi}{3}$

ar gyfer cyfanrif n

$\theta = \frac{2}{3}n\pi \pm \frac{\pi}{9}$

Felly naill ai $\underline{\theta = \frac{2}{3}n\pi + \frac{\pi}{9}}$

neu $\underline{\theta = \frac{2}{3}n\pi - \frac{\pi}{9}}$

Uned 4 Pellach Haf 2023

13) $\frac{dx}{dt} = 3x + 10y$ — (1) $\qquad \frac{dy}{dt} = x + 6y$ — (2)

a) i) $\frac{dx}{dt} = 3x + 10y$

$$\frac{1}{10}\frac{dx}{dt} = \frac{3}{10}x + y$$

$$y = \frac{1}{10}\frac{dx}{dt} - \frac{3}{10}x \quad — (3)$$

$$\frac{dy}{dt} = \frac{1}{10}\frac{d^2x}{dt^2} - \frac{3}{10}\frac{dx}{dt}$$

Amnewid i (2):

$$\frac{1}{10}\frac{d^2x}{dt^2} - \frac{3}{10}\frac{dx}{dt} = x + 6\left(\frac{1}{10}\frac{dx}{dt} - \frac{3}{10}x\right)$$

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 10x + 6\left(\frac{dx}{dt} - 3x\right)$$

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 10x + 6\frac{dx}{dt} - 18x$$

$$\frac{d^2x}{dt^2} - 9\frac{dx}{dt} + 8x = 0 \quad \checkmark$$

ii) Ceisio $x = Ae^{mt}$ fel bod m yn bodloni $am^2 + bm + c = 0$

$a = 1 \quad b = -9 \quad c = 8$

Hafaliad ategol / Auxiliary equation

$$m^2 - 9m + 8 = 0$$

$$(m-1)(m-8) = 0$$

Naill ai $m - 1 = 0$ neu $m - 8 = 0$

$m = 1 \qquad m = 8$

Dau ddatrysiad real felly'r ffwythiant cyflenwol (complementary function) yw $x = Ae^{t} + Be^{8t}$ — ④

Does dim angen integryn neilltuol yma gan fod ochr dde yr hafaliad gwreiddiol yn sero.

b) Differu: $\frac{dx}{dt} = Ae^{t} + 8Be^{8t}$ — ⑤

Amnewid yn ôl i ③ (tudalen gynt) a hefyd defnyddio ④:

$$y = \frac{1}{10}\left(Ae^{t} + 8Be^{8t}\right) - \frac{3}{10}\left(Ae^{t} + Be^{8t}\right)$$

$$y = -\frac{1}{5}Ae^{t} + \frac{1}{2}Be^{8t} \quad — ⑥$$

c) Os yw $t = 0$, $\frac{dx}{dt} = 5$. Amnewid i ⑤:

$$5 = Ae^{0} + 8Be^{8\times 0}$$
$$5 = A + 8B$$
$$5 - 8B = A \quad — ⑥$$

Amnewid i ①:

$$5 = 3x + 10y$$
$$5 = 3x + 10(4x)$$
$$5 = 43x$$
$$\frac{5}{43} = x$$

Amnewid i ④: $\frac{5}{43} = Ae^{0} + Be^{8\times 0}$

$$\frac{5}{43} = A + B$$
$$\frac{5}{43} - B = A \quad — ⑦$$

⑥, ⑦ $\Rightarrow$ $5 - 8B = \frac{5}{43} - B$

$$5 - \frac{5}{43} = 7B$$
$$B = \frac{30}{43}$$

Yn ôl yn ⑥:

$$A = 5 - 8\left(\frac{30}{43}\right)$$
$$A = -\frac{25}{43}$$

Y datrysiad neilltuol yw

$$x = -\frac{25}{43}e^{t} + \frac{30}{43}e^{8t}$$