



Mathematics

Year 7



Workbook	Mathematics	Length	Page Number
01 Welcome to Ysgol y Creuddyn	(a) The mathematics department. (b) Multiples. (c) Polygons; special lines. (d) Odd numbers and even numbers; square numbers; cube numbers. (e) The clock; the calendar; timetables. (f) Symmetry. (g) Rotational symmetry; translation using a column vector.	8–9 weeks	5
02 Introducing Angles	(a) Types of angles. (b) Measuring angles. (c) Drawing angles. (d) Calculating angles. (e) Angles in triangles.	6–7 weeks	37
03 Data Handling and Statistics 1	(a) The sequential data handling cycle. (b) The mean. (c) The range. (d) Bar charts. (e) Mean of grouped data. (f) Pie charts. (g) Investigation.	8–9 weeks	63
04 Introducing Percentages	(a) Preparation. (b) Calculating percentages. (c) Using percentages.	7–8 weeks	95
05 Co-ordinates in the Four Quadrants	(a) The first quadrant. (b) The four quadrants. (c) Mid-point of a line. (d) Using co-ordinates.	4 weeks	115



The Mathematics Department

7

Welcome to

Ysgol y Creuddyn

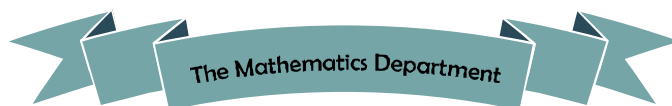
Version
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Contents

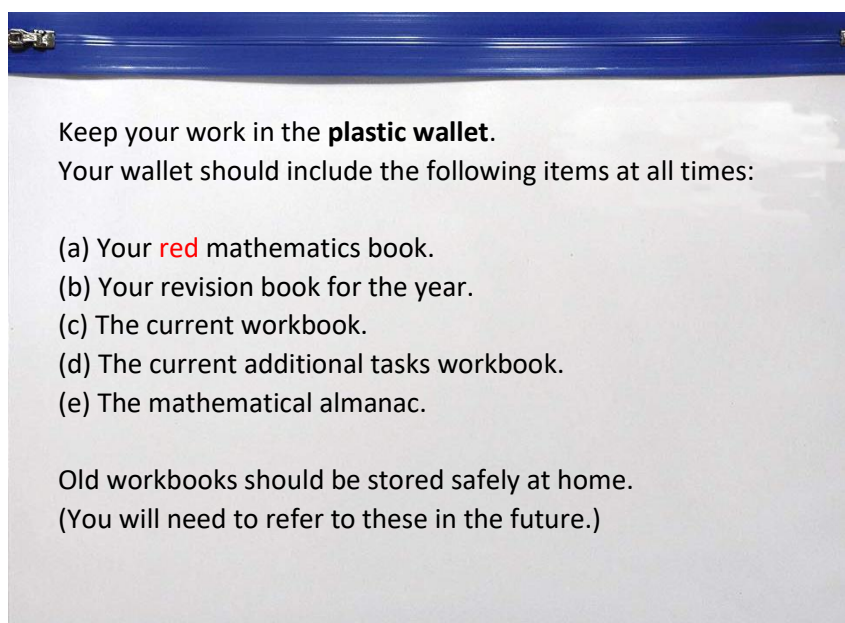
Chapter	Mathematics	Page Number
The Mathematics Department	Looking after your work. Content of the workbooks. Mathematics learning journey.	3
The Library	Multiples.	6
The Hall	Polygons. Special Lines.	10
The Sports Hall	Odd and Even Numbers. Square Numbers. Cube Numbers.	14
The School Timetable	The Clock. The Calendar. Timetables.	18
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Welcome to the mathematics department! Here are some guidelines for looking after your work.

- At the start of every lesson, write “Classwork”, the date and a suitable title for the work. Each of these should be underlined.
- No blank pages should be left in the red mathematics book.
- Draw diagrams using a pencil and, where appropriate, using the correct equipment, e.g. ruler, protractor, compass.
- Show your method fully.
- Remember to include relevant units in your answers, e.g. cm, £, ml.



Equipment

- Black, **red** and **blue** biros.
- HB Pencil.
- Ruler (a 30 cm one is better).
- Eraser.
- Protractor.
- Compass.
- Scientific calculator (e.g. Casio FX-83GTCW).
- Highlighter.

The Revision Book

This will form the basis for your GCSE revision.

- Complete at least 4 pages in your revision book for each unit of work.
- You should include material you will need in the future for remembering the work quickly. This can include notes on the work; examples; important facts; and revision posters.

Workbooks

You will receive 1 copy of the workbook and 1 copy of the additional tasks workbook at the start of each new unit of work. (If you lose the workbook, a new one will cost 50p.)

A Welsh copy of the workbook, and additional supporting materials, can be found on the department's website, www.mathemateg.com



Content of the workbooks

When you see a QR code (like the one on the left), scan it using your mobile device in order to reach a Welsh YouTube video hosted on the following channel.

www.youtube.com/adolygumathemateg

The numbers in circles, for example **3**, show which GCSE unit the work appears in.

Circle	1	2	3	12	13	23	A
Units	1	2	3	1 and 2	1 and 3	2 and 3	1, 2 and 3

All the workbooks contain a variety of exercises, labelled as follows.



Exercises on new topics.



Answering a question in context, or solving a problem.



A more difficult question.



Revision of material from previous workbooks.



There are evaluation boxes at the end of each chapter to revise the completed work.

Key words	Corrections	I am happy with...	I need to revise...
Write down the new or important mathematical terms from the chapter.	What do you need to remember when completing this type of work in the future?	Write down the topics you had success with.	Write down the topics you need to look at again.

Curriculum for Wales Proficiencies



Conceptual understanding



Communication using symbols



Strategic competence



Logical reasoning



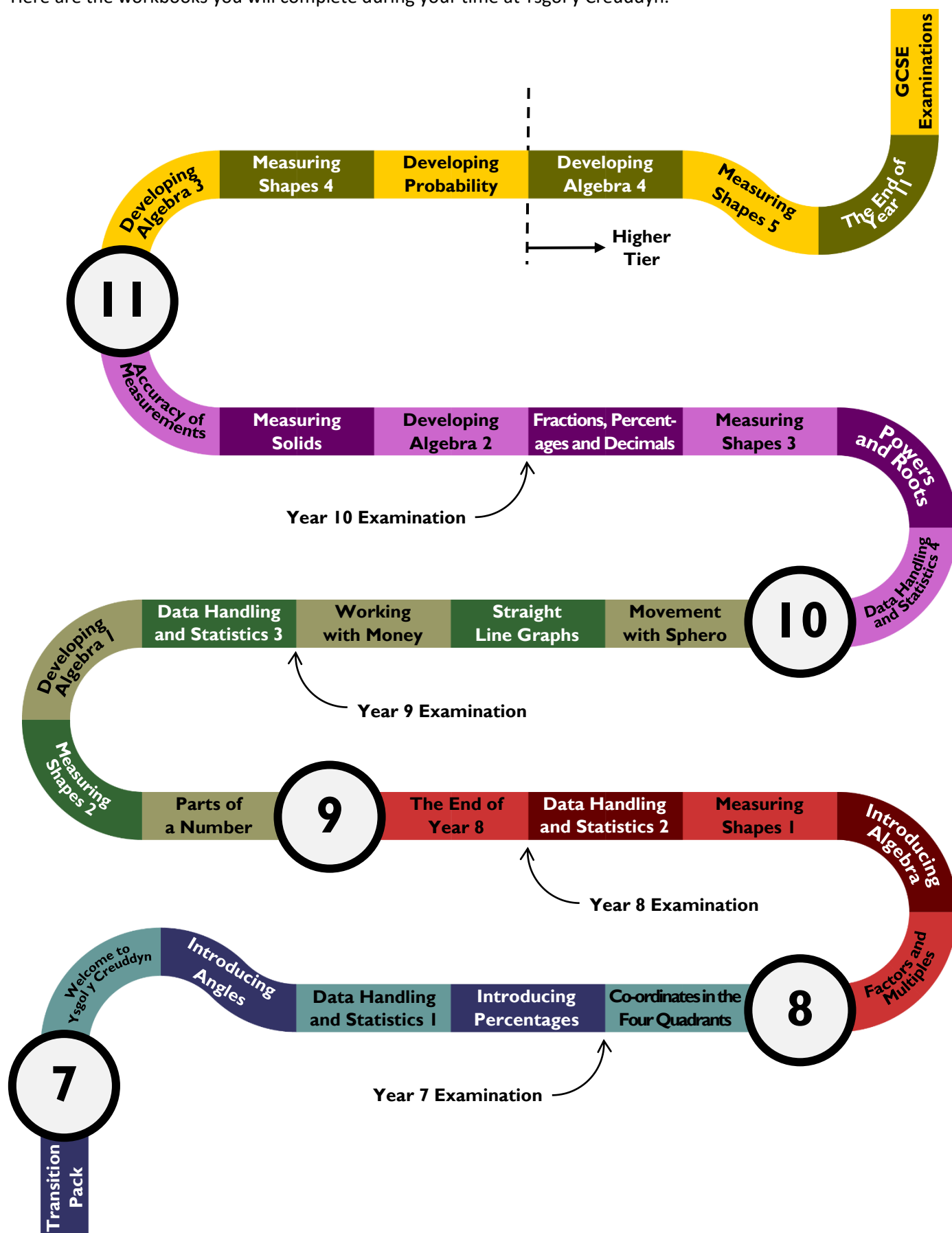
Fluency

Supporting Materials:

- Diagnostic Questions
 - A quiz for each workbook on the website www.diagnosticquestions.com.
- Reflection Sheet
 - An opportunity to assess your understanding of a workbook, and to see the test question order.
- Old WJEC examination questions; worksheets; investigations; puzzles.
 - Available for some topics.

Mathematics Learning Journey for Ysgol y Creuddyn

Here are the workbooks you will complete during your time at Ysgol y Creuddyn.



The Library



The library is located above the English block, in block A. The sixth form study in the library.

Each book has a special number, an **ISBN**, which is an acronym for *International Standard Book Number*. Since 2007, ISBN numbers have 13 digits. The final digit is a **check digit** to help bar code readers read an ISBN for a particular book.

For example, the picture on the left shows the bar code for the book *Gwyddoniadur Cymru*.

To ensure that the ISBN is correct, we must alternatively multiply the digits by 1 and 3, and then find the total of all the multiplication sums.

Write down the ISBN digits in the first column.

Write one number or symbol in each square of the 1 cm squared paper.

Carry the 6 tens into the tens column.

9	×	1	=			9
7	×	3	=		2	1
8	×	1	=			8
0	×	3	=			0
7	×	1	=			7
0	×	3	=			0
8	×	1	=			8
3	×	3	=			9
1	×	1	=			1
9	×	3	=		2	7
5	×	1	=			5
4	×	3	=		1	2
3	×	1	=			3
				1	1	0
					6	

Place the units under the units, and the tens under the tens.

If you find adding the units in one sum difficult, split the sum into smaller pieces.

Because 110 is a **multiple of 10**, this ISBN is correct. If the total had not been a multiple of 10, then there would have been a mistake somewhere.

Exercise 1**A**

Which of the following are valid ISBN numbers?

- (a) 978-1-8452-7516-7 (b) 978-1-8485-1997-9 (c) 978-1-8468-8423-7
 (d) 978-1-7847-5263-7 (e) 978-1-4088-5565-2 (f) 978-1-4088-5868-3
 (g) 978-1-3545-5102-4 (h) 978-0-0074-8831-5 (i) 978-0-2342-2359-2

Applying**Challenge!**

For the valid ISBN numbers above, which books give these numbers? (Investigate using the Internet.)

Extension**Multiples**

With ISBN numbers, it was easy to verify whether a total was a multiple of 10, as all multiples of 10 end with a zero. But what about other multiples? Here are some techniques to help.

**2**

A number is a multiple of 2 if it is an even number, so ends with either a 2, 4, 6, 8 or 0.

3

A number is a multiple of 3 if the sum of its digits is a multiple of 3. For example, 47268 is a multiple of 3, as $4 + 7 + 2 + 6 + 8 = 27$ is a multiple of 3.

4

A number is a multiple of 4 if it is a multiple of 2 **and** half of its last two digits is a multiple of 2. For example, 486 is not a multiple of 4, as half of 86, 43, is not a multiple of 2.

5

A number is a multiple of 5 if it ends with either a 5 or a 0.

**Exercise 2****Skill****A**

(a) Which of the following numbers are multiples of 2?

27 82 192 2828 1293 28 95 8629 2380 29 0 238 92

(b) Which of the following numbers are multiples of 3?

39 512 1569 6495 84652 24155 543 918 64578 9 254 26 54374

(c) Which of the following numbers are multiples of 4?

48 66 75 100 128 150 240 364 601 2710 5864 12912 531052

(d) Which of the following numbers are multiples of 5?

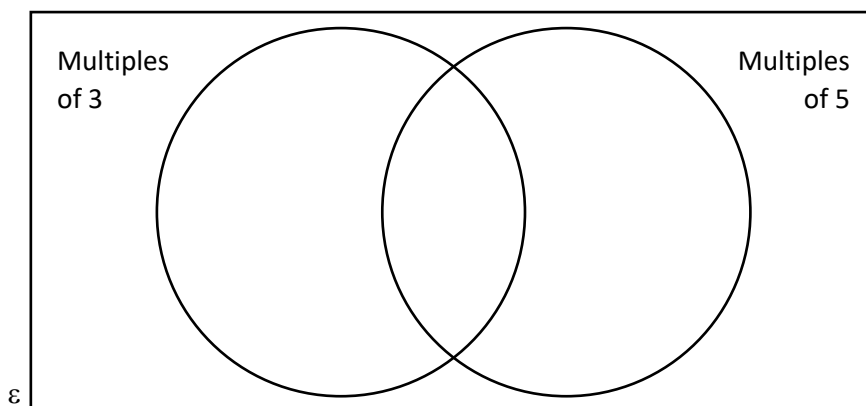
51 165 70 502 3215 5400 214 15 39552 5001 6020 687 654650

Exercise 3

Complete the following Venn diagram using the numbers below.

A

25 18 26 30 95 36 60 14 31 45 75 72 100 105



Challenge!

- (a) Which numbers between 1 and 100 are multiples of 3 and 5?
 (b) Which numbers between 1 and 100 are multiples of 4 and 5?
 (c) Which numbers between 1 and 100 are multiples of 2, 3, 4 and 5?

Extension**Mwy o Luosrifau****6**

A number is a multiple of 6 if it is a multiple of 2 **and** a multiple of 3.

7

A number is a multiple of 7 if multiplying the final digit by 5 and then adding the rest of the number gives a multiple of 7. For example, considering 147:
 $7 \times 5 = 35$, $35 + 14 = 49$.
 49 is a multiple of 7
 so 147 is a multiple of 7.

8

A number is a multiple of 8 if it is a multiple of 2 **and** half of the number is a multiple of 4.

9

A number is a multiple of 9 if the sum of its digits is a multiple of 9.

10

A number is a multiple of 10 if it ends with a 0.

Exercise 4

- (a) Which of the following numbers are multiples of 6?

18 23 32 42 78 120 130 250 300 301 340 500 600

- (b) Which of the following numbers are multiples of 7?

15 28 35 44 63 76 84 105 119 132 140 217 279

- (c) Which of the following numbers are multiples of 8?

56 88 100 105 140 160 200 250 256 328 360 420 1600

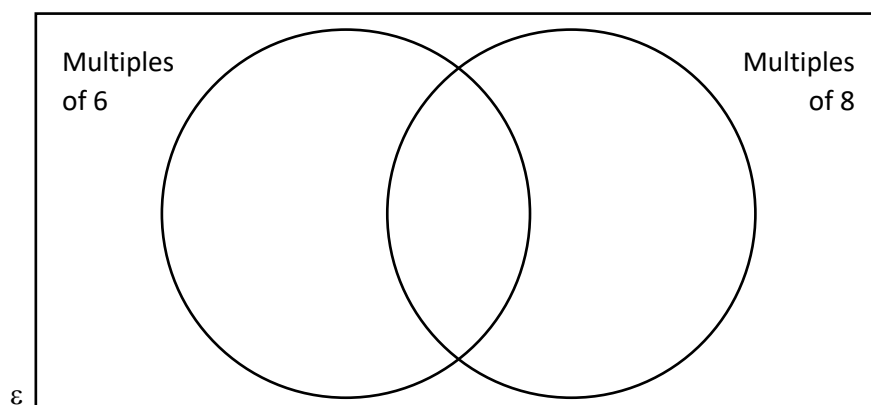
- (d) Which of the following numbers are multiples of 9?

45 65 99 124 351 486 845 909 1542 1978 5229 9571 26946

Exercise 5

Complete the following Venn diagram using the numbers below.

3 12 16 20 24 30 38 40 44 48 54 56 58 96

**Exercise 6**

True or False?

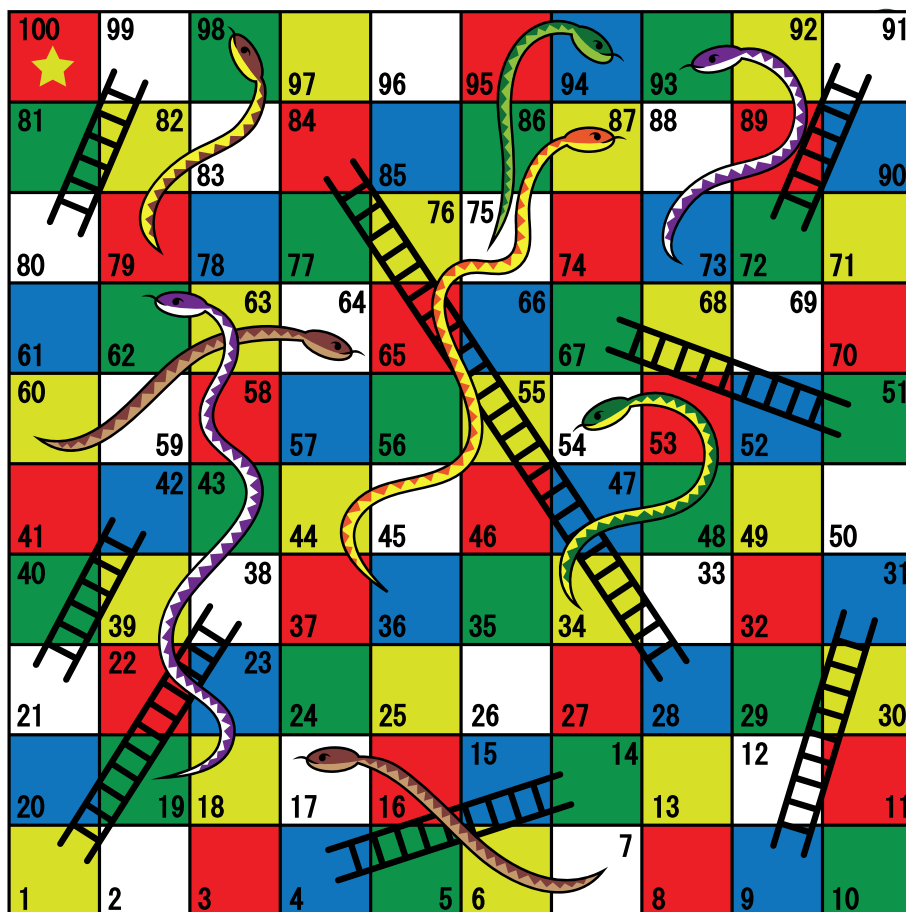
- (a) 432 is a multiple of 2. (b) 792 is a multiple of 3. (c) 2632 is a multiple of 4.
 (d) 8273 is a multiple of 5. (e) 824 is a multiple of 6. (f) 432 is a multiple of 7.
 (g) 1620 is a multiple of 8. (h) 72513 is a multiple of 9. (i) 70281 is a multiple of 10.

Exercise 7

A

The following questions refer to the Snakes & Ladders game on the right.

- (a) The first ladder goes from 1 to 38. Is 38 a multiple of 2?
- (b) The final snake goes from 98 to 79. Is 98 a multiple of 3?
- (c) The longest ladder goes from 28 to 84. Is 84 a multiple of 7?
- (d) (i) What is the total number of snakes and ladders on the board?
(ii) Is your answer to part (i) a multiple of 8?
- (e) Find the ladder that doubles your number. Is the number you end on a multiple of 6?
- (f) The top row of the board (91 to 100) contains 3 snakes, but how many numbers in the top row are multiples of 3?



- (g) The goal of the game is to finish on 100. Is 100 a multiple of 20?

Exercise 8

A

- (a) Which is the first multiple of 7 over 50? (b) Which is the first multiple of 9 over 60?
- (c) Which is the first multiple of 4 over 130? (d) Which is the first multiple of 6 over 90?
- (e) Which is the first multiple of 11 over 100? (f) Which is the first multiple of 3 over 700?

Exercise 9 (Revision)

A

- (a) When is a number a multiple of 3? (b) When is a number a multiple of 8?
- (c) Write down the multiples of 6 between 50 and 80. (d) Write down the multiples of 9 between 30 and 60.

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

The Hall

The hall is located in the centre of the school, by reception. Morning assembly; drama lessons; performances of musicals and much more take place here!

Can you see from the picture the **shape** of the **floor of the hall**?

Exercise 10

Name the shapes with the following number of sides.

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) 8
- (g) 9
- (h) 10



Skill

A

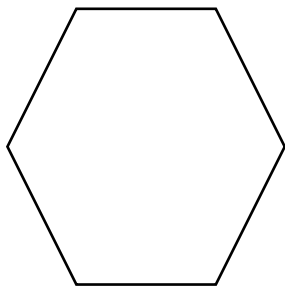


A

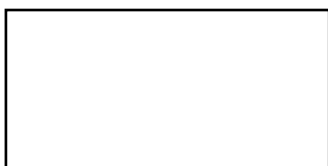
Exercise 11

What is the name of the following shapes?

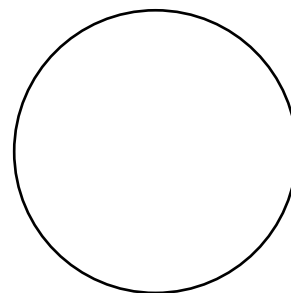
(a)



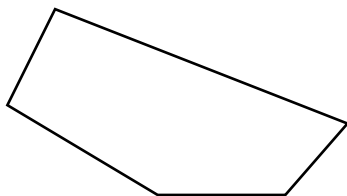
(b)



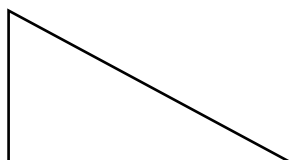
(c)



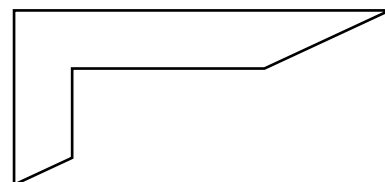
(d)



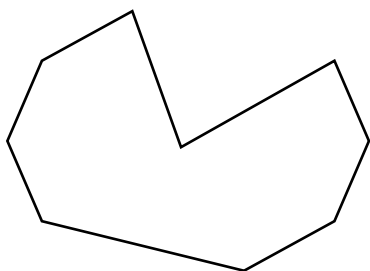
(e)



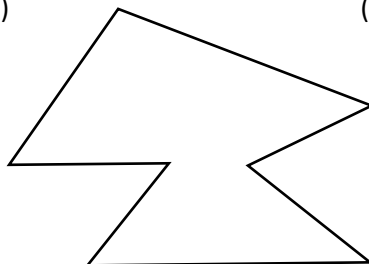
(f)



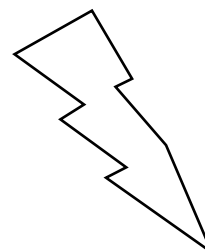
(g)



(h)



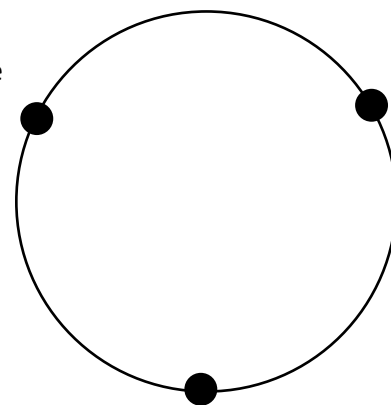
(i)



Investigation

Using a compass, draw a circle in your book (use a radius of around 4cm). Add three dots to your circle, as shown in the diagram on the right. Using straight lines, connect the dots in every possible way. Verify that this divides the circle into four regions. Repeat with a circle that has four dots, five dots, and so on. How many regions are formed each time? Is there a pattern?

Dots	2	3	4	5	6	7
Regions	2	4				



Regular and Irregular Polygons

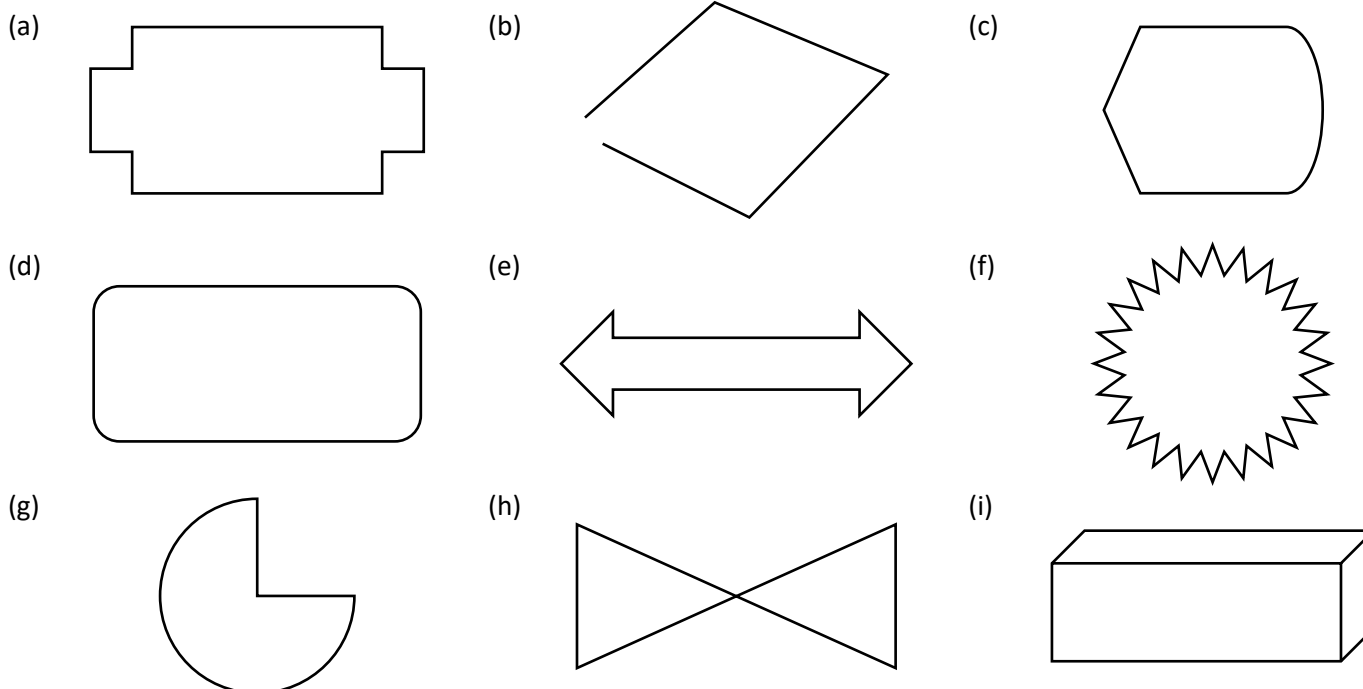
A closed shape that uses straight lines only is called a **polygon**.



A

Exercise 12

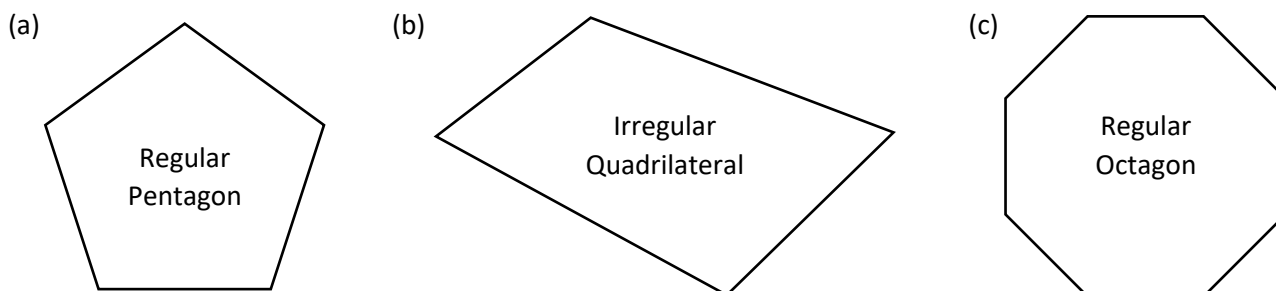
Explain which of the following shapes are polygons, and which are not polygons.



A polygon is **regular** if its sides all have the same length and its angles are also all equal. If the polygon is not regular, then it is an **irregular** polygon.



Example



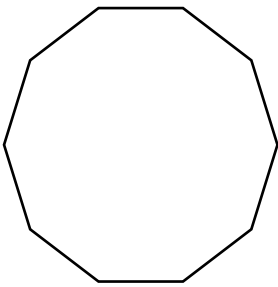
Exercise 13

Name the following polygons.

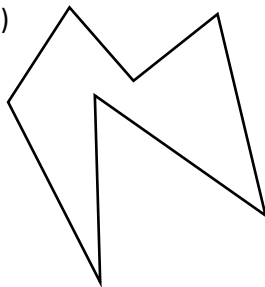
Skill

A

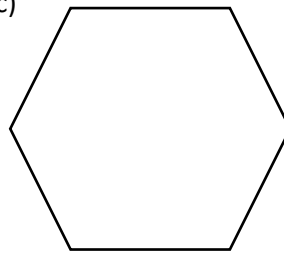
(a)



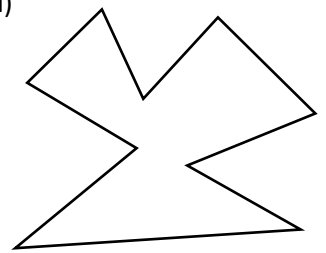
(b)



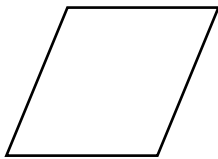
(c)



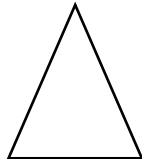
(d)

(e) Explain why the following shapes are **not** regular polygons.

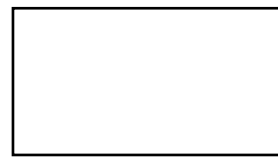
(i) Rhombus



(ii) Isosceles Triangle



(iii) Rectangle

**Challenge!** 

Try to say the names of the following polygons...

Number of sides	Name	Number of sides	Name
13	Triskaidecagon	39	Triacontakaienneagon
21	Icosikaihenagon	45	Tetracontakai pentagon
25	Icosikaipentagon	100000	Megagon
31	Triacontakaihenagon	10^{100}	Googolgonalmost

Special Lines

Consider the picture shown of the musical "Herspre" (December 2014).

Can you see any **horizontal** lines?Can you see any **vertical** lines?Can you see any **parallel** lines?Can you see any **perpendicular** lines?**Exercise 14**

A

In your book, explain what is meant by a *horizontal* line; a *vertical* line; *parallel* lines; and *perpendicular* lines.



Exercise 15

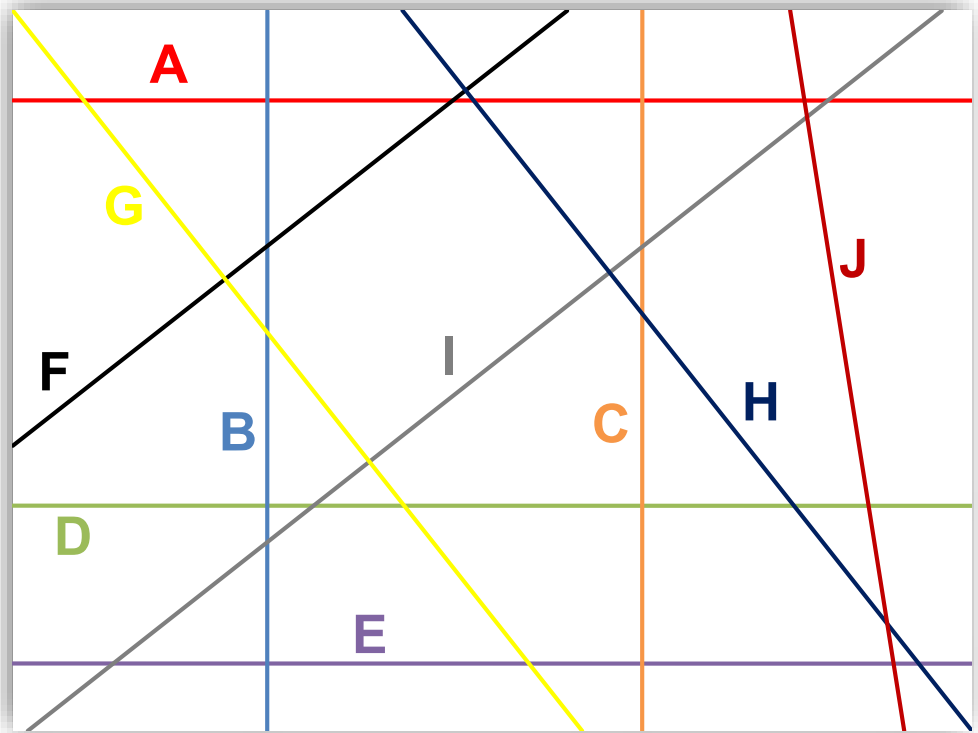
A

Use the diagram on the right to list:

- (a) the horizontal lines;
- (b) the vertical lines;
- (c) the pairs of parallel lines;
- (d) the pairs of perpendicular lines.

Did you know?

The word "parallel" comes from the Latin word "parallēlus" meaning side-by-side.

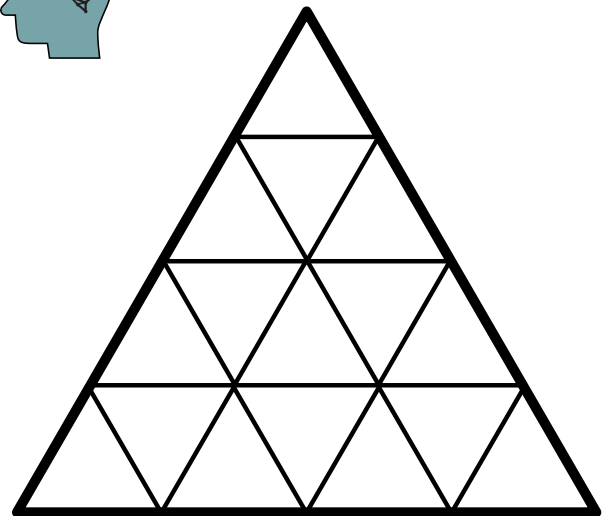
**Puzzle**

How many triangles can be seen in the diagram on the right?

**Exercise 16 (Revision)**

A

- (a) How many sides does a heptagon have?
- (b) Draw a regular quadrilateral.
What is the usual name for this shape?
- (c) Draw a pair of perpendicular lines.
- (d) I have 4 octagons, 3 hexagons and 2 nonagons.
How many sides do I have?

**Evaluation**

Key Words	Corrections	I am happy with...	I need to revise...

The Sports Hall

The sports hall was built during 2005-06, with the official opening taking place in the summer term of 2006. The school uses the hall during the day, and the local community at night.

It is possible to modify the hall for the use of many different sports.

Exercise 17

How many players are needed to form a **team** in these sports?

- (a) Netball
- (b) Football
- (c) Basketball
- (d) Cricket
- (e) Rugby league
- (f) Rugby union.

What do you notice about all your answers to Exercise 17?

Exercise 18

- (a) Which of the following numbers are even numbers?

23 86 99 104 158 2603 3547 6712 95420 451245874

- (b) Which of the following numbers are odd numbers?

8 41 59 84 107 245 659 1054 5249 3154287

- (c) List the 5 even numbers after 95.

- (d) List the 5 odd numbers after 36.

- (e) Explain what the difference is between an odd number and an even number.

Exercise 19

- (a) What is the largest odd number that can be made using 7, 1, 6, 4, 8?
- (b) What is the smallest even number that can be made using 6, 5, 2, 1, 9?
- (c) Answer “always”, “sometimes” or “never” to the following questions.

- (i) Adding two even numbers gives an even number.
- (ii) Adding two odd numbers gives an odd number.
- (iii) Multiplying two even numbers gives an even number.
- (iv) Multiplying two odd numbers gives an odd number.
- (v) Multiplying an odd number by an even number gives an odd number.



Challenge!

Think of a sport which requires an even number of players to form a team.



Skill

A



Sports Day

Square Numbers

Just as the even numbers form a number pattern, the **square numbers** also form a number pattern. To illustrate this pattern, consider the following sequence of squares.



1

4

9

16

....

What will be the next number in the sequence? And the one after that?

It is possible to write the numbers in this sequence as multiplication sums:

 1×1 2×2 3×3 4×4

....

To get 16, which is the fourth number in the sequence, we multiply four by itself. This is written mathematically as 4^2 , and we say that we are calculating “**four squared**”. Therefore, $4^2 = 4 \times 4 = 16$.

Exercise 20

Evaluate the following square numbers.

(a) 7^2

(b) 5^2

(c) 9^2

(d) 10^2

(e) 8^2

(f) 6^2

Skill

A

Exercise 21

A

Calculate the following sums.

(a) $1^2 + 3^2$

(b) $4^2 + 5^2$

(c) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

(d) $10^2 - 6^2$

(e) $12^2 - 2^2$

(f) $5^2 - 4^2 - 3^2$

(g) $2^2 - 1^2$

(h) $3^2 \times 4^2$

(i) $1^2 \times 13^2$

(j) $2^2 \times 9^2 \times 1^2$

(k) $5^2 \times 2^2 \times 1^2$

(l) $10^2 \div 5^2$

(m) $8^2 \div 4^2$

(n) $5^2 \div 1^2$

(o) $5^2 \div 2^2$

(p) $5^2 \times 2^2 - 1^2$

(q) $4^2 \div 1^2 + 3^2$

(r) $8^2 \times 1^2 \div 2^2$

(s) $7^2 + 4^2 + 10^2 - 9^2$

Challenge!

A

(a) A number is palindromic if it reads the same backwards as it does forwards. For example, 373 is a palindromic number. Find the first three square numbers that are palindromic. (Clue: The answers are greater than 100, but less than 1000.)

(b) Find two square numbers that add to give another square number.



(c) Find three square numbers that add to give another square number.

Extension



Investigation: The Golden Number

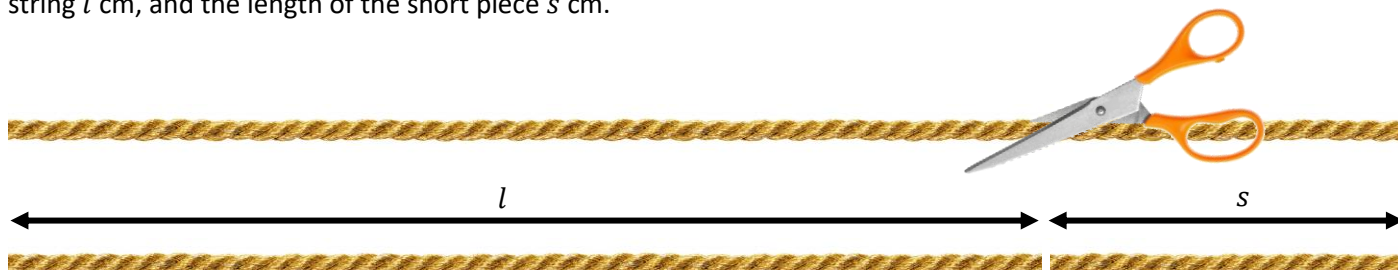
Applying

Problem 1

When we square any number more than one, we obtain an answer that is greater¹. For example, when squaring 3, we obtain 9; it follows that 3 is six less than its square. Question: Which number is **exactly one less** than its square? Investigate for a few minutes using your calculator...

Problem 2

Consider a piece of string with a length of 1m, or 100cm. You are allowed to cut the string in any place, but not in the middle. This leaves two pieces of string – a long piece and a short piece. Let us call the length of the long piece of string l cm, and the length of the short piece s cm.



Question: Where should the string be cut so that $100 \div l$ is the same as $l \div s$? Investigate for a few minutes using your calculator...

The Answer

It turns out that both problems have a connection to a special number known as the golden number. The symbol for the golden number is Φ ("Phi"). One way of finding Φ is to consider the **Fibonacci sequence**. In this sequence, we obtain the next number by adding the two previous numbers. Starting with 1 and 1, the sequence continues like this:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Exercise 22

A

Find the next five numbers in the Fibonacci sequence.

Consider dividing the number in the Fibonacci sequence like this: $1 \div 1$, $2 \div 1$, $3 \div 2$, $5 \div 3$, $8 \div 5$, ... (can you see the pattern?) As Fibonacci's sequence continues, these division sums give an answer that is closer and closer to the golden number Φ .

Exercise 23

A

Using your answer to Exercise 22, write down your best estimate for Φ . What is the link between Φ and problems 1 and 2 above?

Extension

The Golden Number and the Body

Calculate the following ratios. What do you notice?

- Distance from the floor to your belly button $\div$ Distance from the floor to your knee.
- Distance from your wrist to your elbow $\div$ Length of your hand.

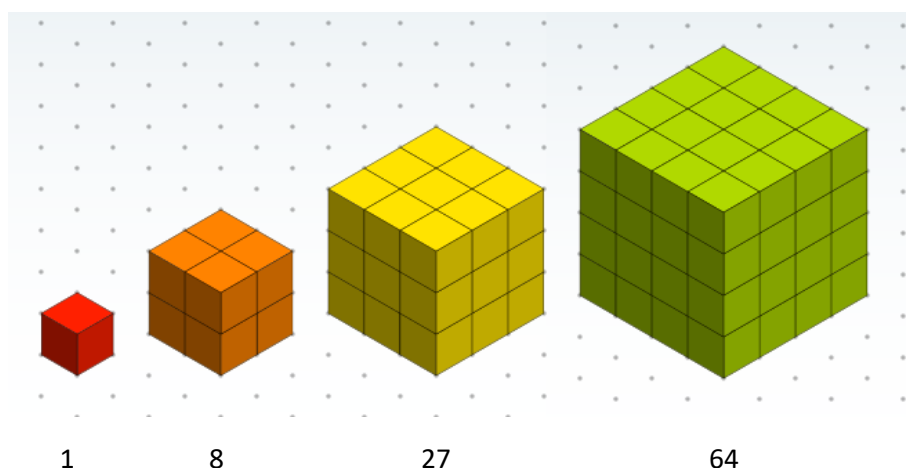


Sports Day

¹ What happens if you square a number between 0 and 1?

Cube Numbers

The **cube numbers** come from the following set of diagrams of cubes.



What will be the next number in the sequence? And the one after that?

It is possible to write the numbers in this sequence as multiplication sums:

$$1 \times 1 \times 1 \quad 2 \times 2 \times 2 \quad 3 \times 3 \times 3 \quad 4 \times 4 \times 4 \quad \dots$$

To get 64, which is the fourth number in the sequence, we multiply four by itself three times. This is written mathematically as 4^3 , and we say that we are calculating "**four cubed**". Therefore, $4^3 = 4 \times 4 \times 4 = 64$.

Exercise 24

Evaluate the following cube numbers.

- (a) 5^3 (b) 6^3 (c) 7^3 (d) 8^3 (e) 9^3 (f) 10^3

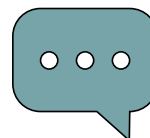
Skill

A

Exercise 25

Calculate the following sums.

- (a) $2^3 + 3^3$ (b) $3^3 + 4^3$ (c) $7^3 - 4^3$ (d) $10^3 \times 1^3$
 (e) $4^3 + 5^3$ (f) $6^3 + 1^3 + 2^3$ (g) $2^3 \times 5^3$ (h) $1^3 \times 5^3$
 (i) $8^3 + 9^3$ (j) $4^3 \div 1^3$ (k) $10^3 \div 2^3$ (l) $10^3 - 2^3$



A

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...



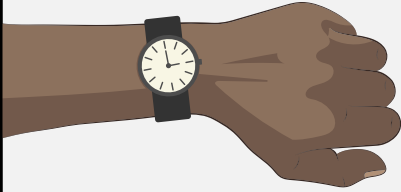
The School Timetable

Exercise 26

Skill

13

Fill in the following blanks.



1 minute = ____ seconds

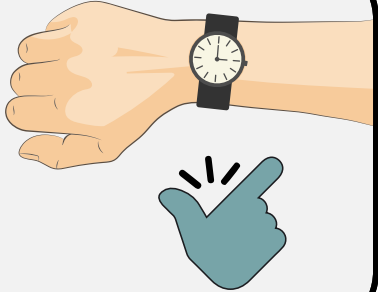
1 hour = ____ minutes

1 day = ____ hours

1 week = ____ days

1 year = ____ or ____ days

1 century = ____ years



Exercise 27

13

Fill in the following blanks.

- | | |
|------------------------------|--|
| (a) 3 minutes = ____ seconds | (b) Half a minute = ____ seconds |
| (c) 2 hours = ____ minutes | (d) An hour and a quarter = ____ minutes |
| (e) Fortnight = ____ days | (f) 1 week = ____ hours |
| (g) 1 year = ____ weeks | (h) 52 weeks = ____ days |
- (i) What do you notice about your answers to (g) and (h) above?

Exercise 28

13

Fill in the following blanks for dates in 2024.

Numbers	Words
25/12/24	
23/2/24	
10/7/24	
6/9/24	
	August 31st
	March 1st
17/2/24	
	May 20th
	November 5th

Numbers	Words
4/3/24	
12/8/24	
	February 27th
	April 19th
30/1/24	
13/5/24	
	June 12th
11/4/24	
	October 3rd

Exercise 29

A

How many minutes are there between the following times?

- | | | |
|-------------------------|------------------------|-----------------------|
| (a) 3:00pm and 3:30pm | (b) 2:15pm and 2:47pm | (c) 6:47am and 7:02am |
| (d) 10:23pm and 11:10pm | (e) 9:24am and 11:00am | (f) 1:57pm and 4:28pm |

The Timetable

Here is the school timetable for Ysgol y Creuddyn.

08:55 – 09:15	Registration Period / Assembly
09:15 – 10:05	Lesson 1
10:05 – 10:55	Lesson 2
10:55 – 11:10	Break
11:10 – 12:00	Lesson 3
12:00 – 12:50	Lesson 4
12:50 – 13:45	Dinner Time and Activities
13:45 – 13:50	Warning Bell
13:50 – 14:40	Lesson 5
14:40 – 15:30	Lesson 6



Exercise 30

- What is the length of the registration period, in minutes?
- How many minutes are there between the end of lesson 4 and the end of lesson 6?
- What is the length of the school day, in hours and minutes?
- You are exactly half way into lesson 2. What is the time?
- The headmaster grants an extension of twenty five minutes to the usual break. When will lesson 3 start?
- What is the length of break time in seconds?

Applying

A

The 12 hour clock and the 24 hour clock

There are two methods for telling the time.

- The **12 hour** clock, e.g. 2:45 a.m., 11:40 p.m., 7:45 a.m.
- The **24 hour** clock, e.g. 02:45, 16:42, 12:32.



The 24 hour clock runs from 00:00 to 23:59. The times from 00:00 to 11:59 are a.m. times, and the times from 12:00 to 23:59 are p.m. times.

The letters a.m. stand for Ante Meridiem (from the Latin, meaning “before midday”). The letters p.m. stand for Post Meridiem (again from the Latin, meaning “after midday”).

Exercise 31

Complete the following table.

12 hour clock	24 hour clock
2:34 p.m.	
4:56 a.m.	
9:20 p.m.	
12:00 a.m.	
2:45 a.m.	
11:38 p.m.	
3:30 p.m.	
7:34 a.m.	
8:30 p.m.	
12:00 p.m.	

12 hour clock	24 hour clock
	11:15
	18:31
	00:45
	06:30
	16:54
	22:13
	08:55
	13:04
	01:25
	21:59

Skill

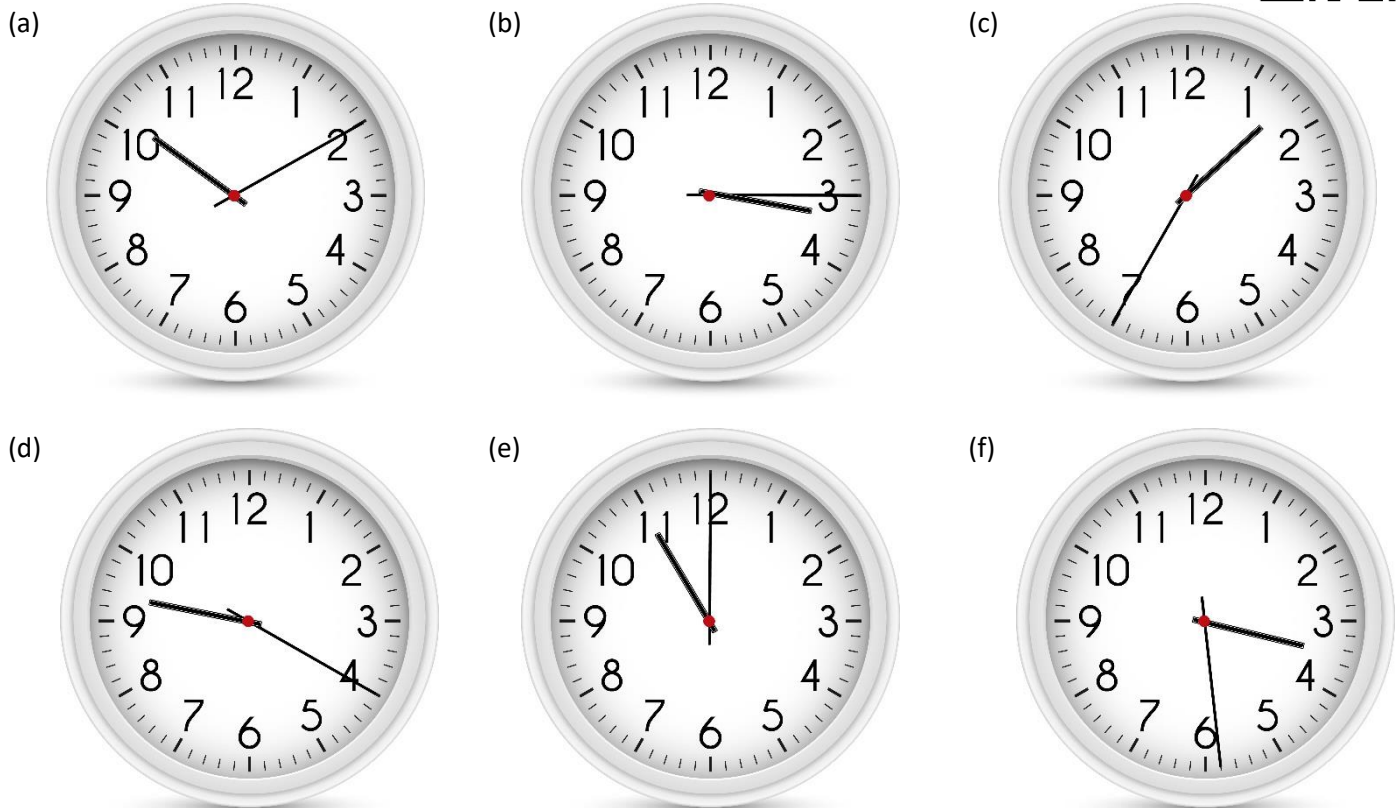
A

Exercise 32

A



The following clocks show the time at different stages during the school day. For each one, write down
(i) The time using the 12 hour clock; (ii) The time using the 24 hour clock; (iii) The time in words.

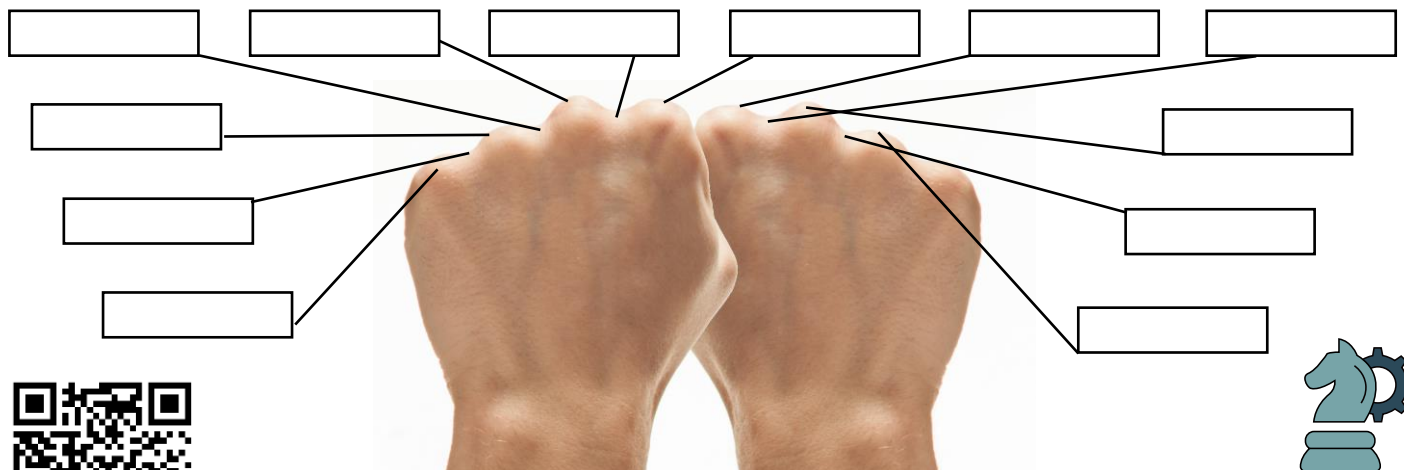
**The Calendar**

There are 12 months in a year, but how many days are there in each month? Try to complete the following table.

Name of the month	Number of days
January	

Name of the month	Number of days
July	

How can the following picture help you remember how many days make up each month?



Exercise 33

Here is a calendar for the year 2025.

Applying

A

January							February							March							April						
Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
		1	2	3	4	5						1	2						1	2		1	2	3	4	5	6
6	7	8	9	10	11	12	3	4	5	6	7	8	9	3	4	5	6	7	8	9	7	8	9	10	11	12	13
13	14	15	16	17	18	19	10	11	12	13	14	15	16	10	11	12	13	14	15	16	14	15	16	17	18	19	20
20	21	22	23	24	25	26	17	18	19	20	21	22	23	17	18	19	20	21	22	23	21	22	23	24	25	26	27
27	28	29	30	31			24	25	26	27	28			24	25	26	27	28	29	30	28	29	30				
														31													

May							June							July							August						
Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
			1	2	3	4							1		1	2	3	4	5	6					1	2	3
5	6	7	8	9	10	11	2	3	4	5	6	7	8	7	8	9	10	11	12	13	4	5	6	7	8	9	10
12	13	14	15	16	17	18	9	10	11	12	13	14	15	14	15	16	17	18	19	20	11	12	13	14	15	16	17
19	20	21	22	23	24	25	16	17	18	19	20	21	22	21	22	23	24	25	26	27	18	19	20	21	22	23	24
26	27	28	29	30	31		23	24	25	26	27	28	29	28	29	30	31				25	26	27	28	29	30	31
							30																				

September							October							November							December						
Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
1	2	3	4	5	6	7			1	2	3	4	5						1	2	1	2	3	4	5	6	7
8	9	10	11	12	13	14	6	7	8	9	10	11	12	3	4	5	6	7	8	9	8	9	10	11	12	13	14
15	16	17	18	19	20	21	13	14	15	16	17	18	19	10	11	12	13	14	15	16	15	16	17	18	19	20	21
22	23	24	25	26	27	28	20	21	22	23	24	25	26	17	18	19	20	21	22	23	22	23	24	25	26	27	28
29	30						27	28	29	30	31			24	25	26	27	28	29	30	29	30	31				

- Find 1st January 2025. Which day of the week is this?
- How many months in 2025 begin on Saturday?
- Is 2025 a leap year? How do you know?
- How many Saturdays are in September?
- Which month is the first in the year with more Fridays than Thursdays?
- On which day will Fireworks Night be in 2025?
- How many Mondays are there in the year?
- Which day will be the final day of January **2026**?

Challenge!

Are there more days in the first six months of a year or the final six months of a year?

School History

Ysgol y Creuddyn was opened on Wednesday, September 2nd, 1981. There were 218 pupils to begin with, including exactly 100 in year seven. The pupils were welcomed by the headmaster Mr. Roland Jones (Year 3); Mrs. Ellen Kent (Year 1); and Mr. Dafydd Whittall (Year 2).

The official opening took place on Wednesday, October 17th, 1984.

Question: Was 1981 a leap year? What about 1984? How do we go about deciding?



Leap Years

- There is a leap year every four years.
- There is one extra day (February 29th) during a leap year.
- There are 366 days in a leap year.
- Each leap year is a multiple of 4. From the first chapter, this means that half of the final two digits needs to be a multiple of 2.

For 1956, half of 56, 28, is a multiple of 2, so 1956 was a leap year.

For 1862, half of 62, 31, is not a multiple of 2, so 1862 was not a leap year.



Exercise 34



A

Skill

Decide whether the following years were leap years.

- (a) 1936 (b) 1832 (c) 1873 (d) 1990 (e) 1906 (f) 1740 (g) 1808

Decide whether the following years will be leap years.

- (h) 2056 (i) 2140 (j) 2562 (k) 3483 (l) 4800 (m) 5136 (n) 13276

Timetables

Exercise 35

A

Applying

(a) What is the length of a television programme that starts at 17:55 and ends at 18:40?

(b) The film started at 21:35. If the film lasted 1 hour 40 minutes, when did it finish?

(c) When cooking a turkey, it requires 20 minutes cooking for each pound of weight and then an extra 20 minutes. You have a 12 pound turkey and need it to be ready by 19:30. When should you start cooking the turkey?

(d) Film A starts at 13:52 and finishes at 16:35. Film B starts at 14:33 and finishes at 16:48. Which film is the longest, and by how much?

(e) A bus leaves Llandudno at 11:35 and reaches Bangor at 12:50. How long was the journey?

(f) Look at the following timetable and answer the questions that follow.



Holyhead	0551	0628	0715	0805	0923	1040	1127
Bangor	0618	0706	0802	0902	1002	1107	1200
Llandudno Junction	0635	0722	0823	0923	1023	1123	1222
Colwyn Bay	0642	0731	0831	0931	1031	1131	1229
Rhyl	0653	0741	0841	0941	1041	1141	1240
Prestatyn	0658	0747	0847	0947	1047	1147	1245
Flint	0712	0800	0900	1000	1100	1200	1259
Chester	0726	0815	0914	1015	1115	1216	1313

- (i) What time does the first train reach Llandudno Junction?
- (ii) How many minutes does the 0841 from Rhyl take to reach Chester?
- (iii) Siwan lives in Colwyn Bay. She works in Prestatyn and starts work at 09:00. Which train should Siwan catch in order to reach work on time?
- (iv) Geraint reaches Bangor train station at 10:23. How many minutes does he have to wait until the next train arrives?
- (v) Of the seven trains on the timetable, which is the **slowest** in travelling from Holyhead to Chester?



Challenge!

Which is the largest: the number of seconds in a week, or the number of minutes in a year?



A

Exercise 36 (Revision)

- (a) How many minutes are there in 4 hours?
- (b) How many years and months make up 35 months?
- (c) Change 10:34 p.m. to be in the 24 hour clock.
- (d) The clock on the right is three quarters of an hour early. What is the correct time?
- (e) How many days are there in April and October combined?
- (f) Was 1846 a leap year?
- (g) The concert finished at ten minutes to ten at night. The concert's length was 2 hours 35 minutes. When did the concert start?
- (h) If December 1st was a Monday, what day was December 16th?
- (i) Write 'twenty-five minutes to three in the morning' in figures, using the 24 hour clock.



a.m.

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

The Refectory

Each dinner time, the refectory serves hot food; salad; sandwiches and different puddings. An extension to the refectory was built during 2014.

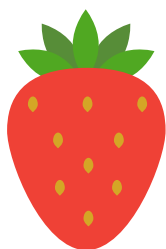
Exercise 37

2

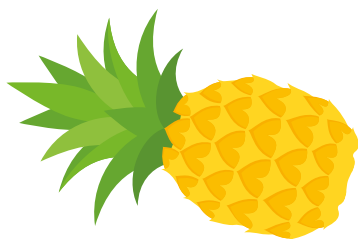
Skill

Add symmetry lines to the following fruits.

(a)



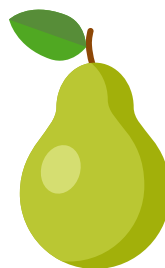
(b)



(c)



(d)

**Challenge!**

Can you identify the names of each of the fruits above?

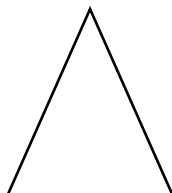
**Exercise 38**

Add symmetry lines to the following diagrams.

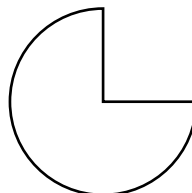
(a)



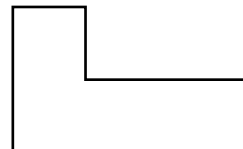
(b)



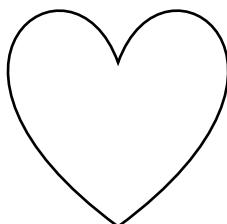
(c)



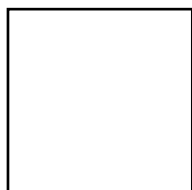
(d)



(e)



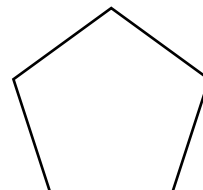
(f)



(g)



(h)



Exercise 39

2

Applying

Classify the following flags according to whether they have (a) vertical symmetry only; (b) horizontal symmetry only; (c) both horizontal and vertical symmetry; (d) no symmetry.

**Challenge!**

Which countries appear on the above flags?

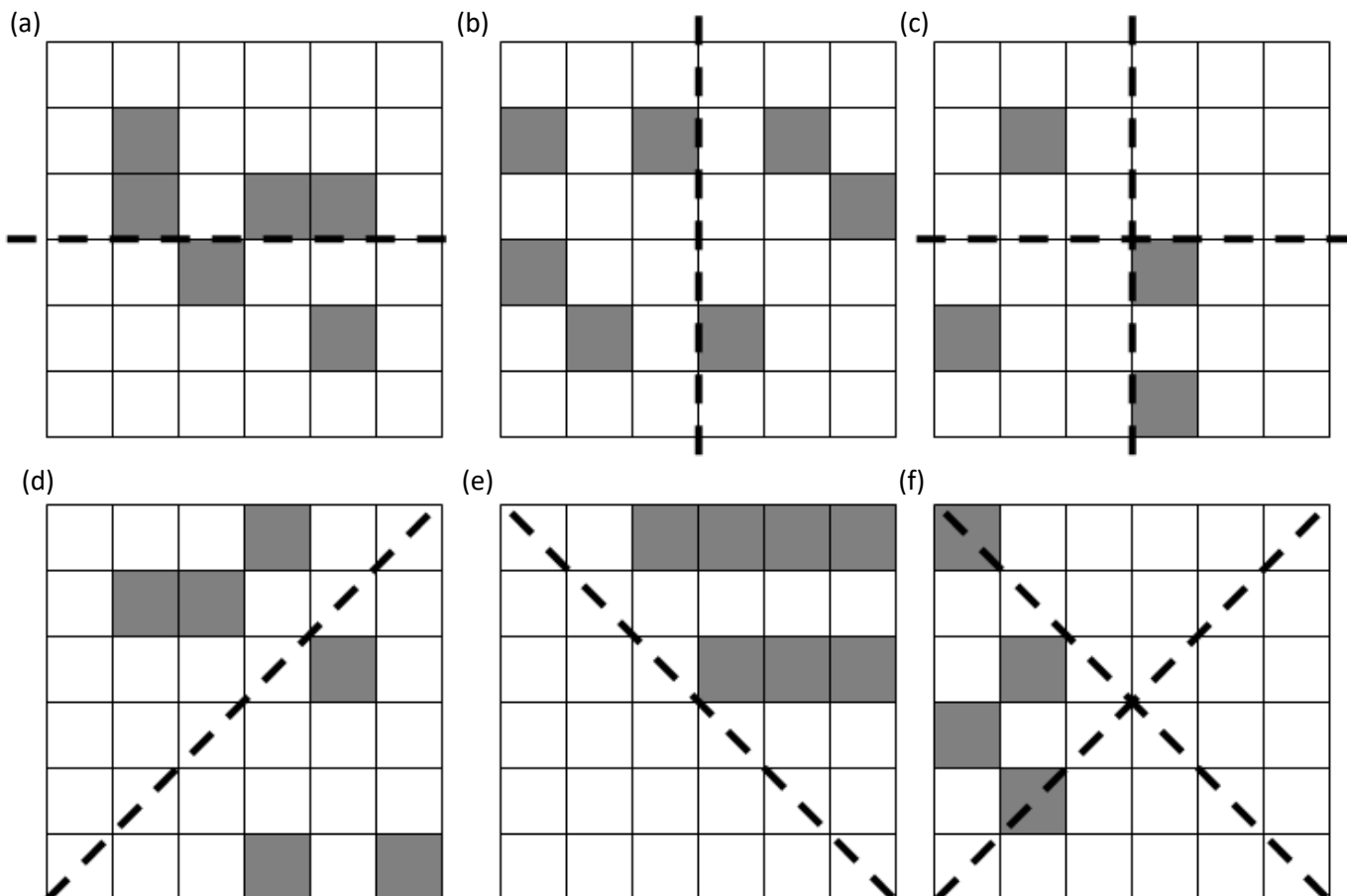
Exercise 40

Skill

2



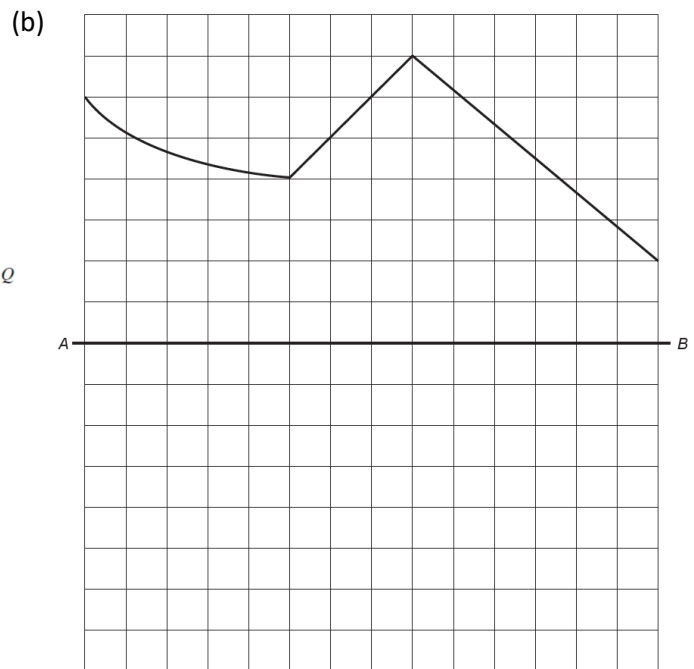
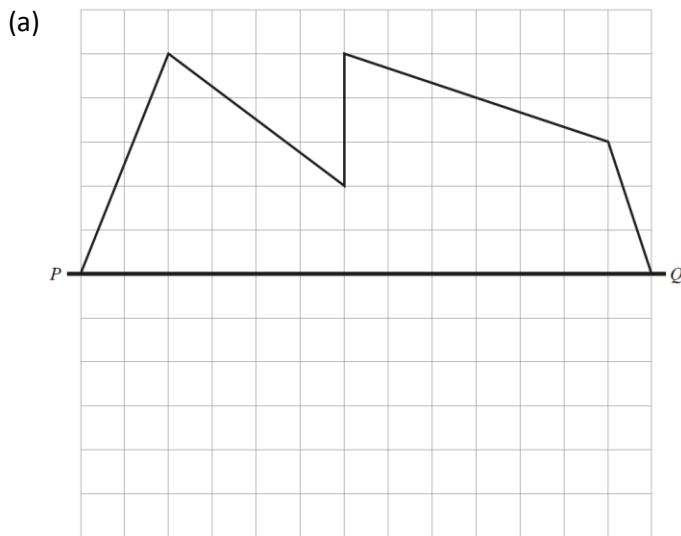
Shade the least number of squares so that the dotted lines are symmetry lines.



Exercise 41

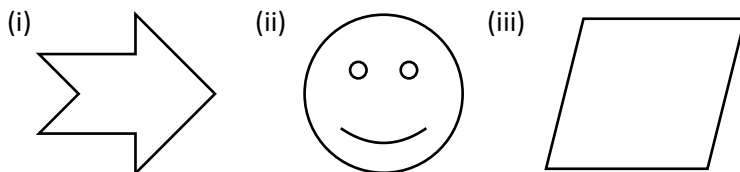
2

Complete the following shapes so that the horizontal line is a symmetry line.

**Exercise 42 (Revision)**

2

(a) Add symmetry lines to the following diagrams.



(b) Explain whether France's flag has any symmetry lines.

(c) How many symmetry lines does a regular hexagon have?

(d) How many symmetry lines does a circle have?

**Evaluation**

Key Words	Corrections	I am happy with...	I need to revise...

The Eisteddfod

The Eisteddfod is one of the highlights of the school year. The houses **Bodysgallen**, **Penrhyn** and **Gloddaeth** battle against each other in competitions such as the Solo Instrumental; Speaking for a Minute; Solo from a Musical; and Gargling the National Anthem.

Many of the competitions are musical in nature, so it is advantageous to be able to read musical notation. Here are some of the symbols used in musical notation.



Treble clef



Semibreve (4 beats)



Rest



Minim (2 beats)



Flat



Crotchet (1 beat)



Sharp

Imagine rotating these symbols. Which symbols will look exactly the same before you finish a complete rotation?

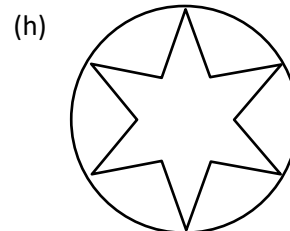
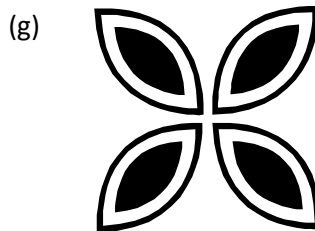
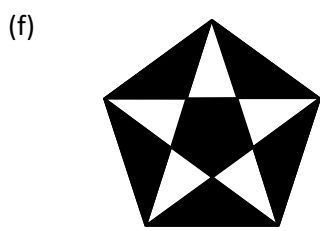
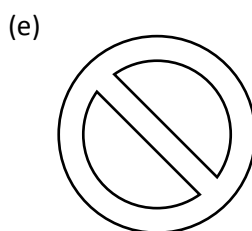
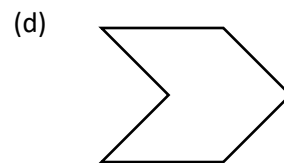
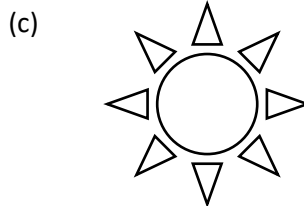
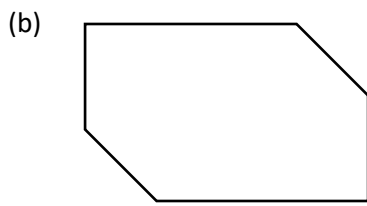
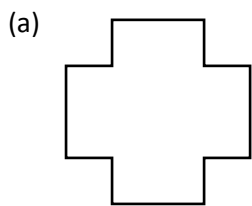
Rotational Symmetry

A shape has **rotational symmetry** if it comes to fit onto itself more than once during a complete rotation. The **order of rotational symmetry** is the number of times the shape comes to fit onto itself during the complete rotation. A shape does not have rotational symmetry if it fits onto itself only once.



Exercise 43

What is the order of rotational symmetry of the following shapes?



(i) Regular Decagon

(j) Square

(k) Rectangle

(l) Circle



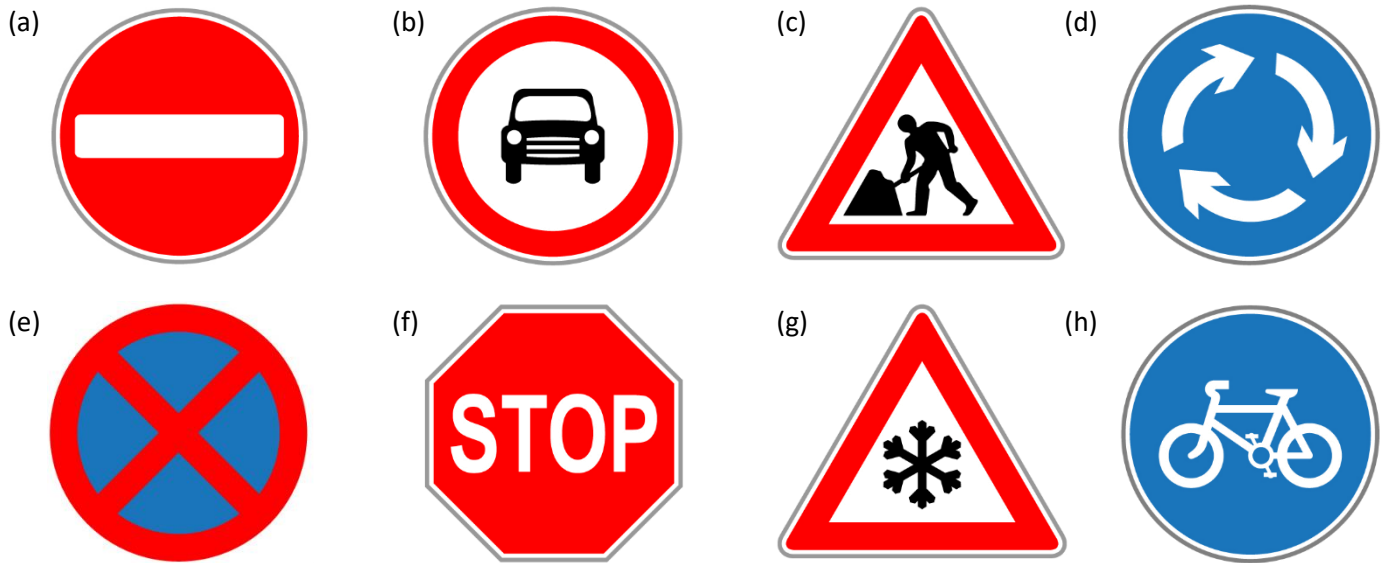


Exercise 44

What is the order of rotational symmetry of the following road signs?

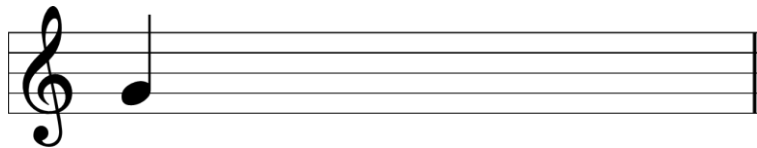
2

Applying

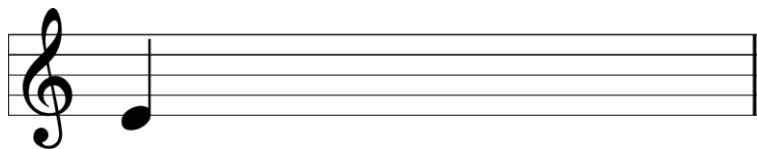


Writing Music

It is possible to write music on a **five line musical stave**. The location of a symbol like a crotchet tells the reader which note to play. For example, the following stave shows a G note.



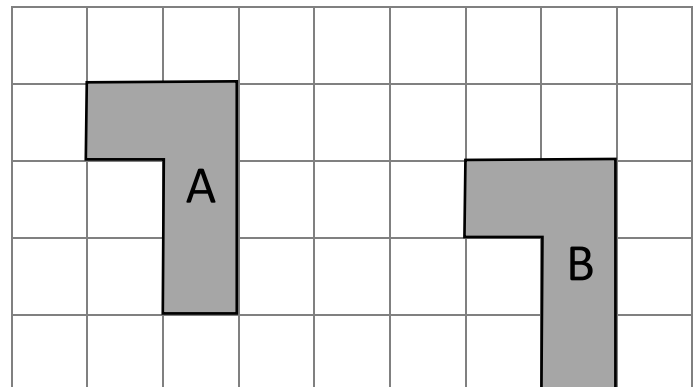
By moving the crotchet up or down, a different note can be played. For example, moving the crotchet down one line changes the note to be an E.



The mathematical name for moving a symbol is **translation**.

Translations

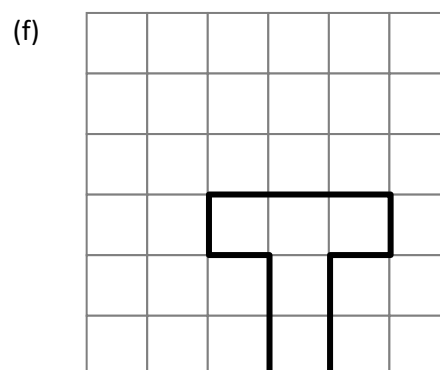
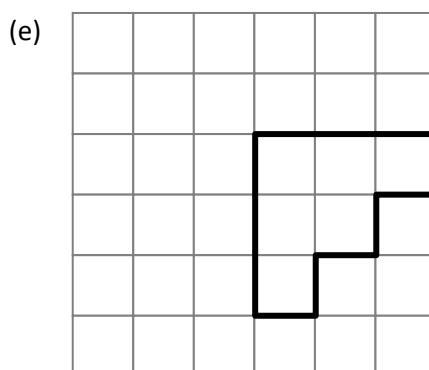
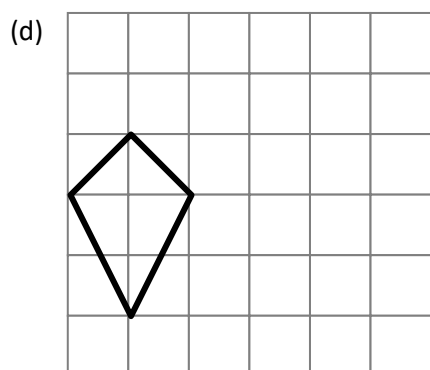
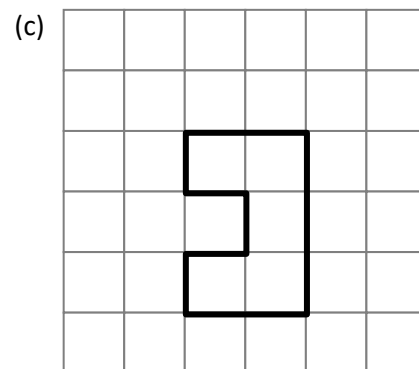
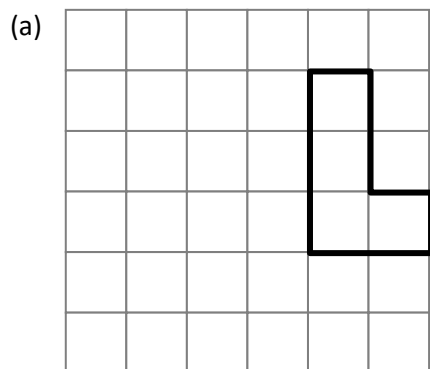
In the diagram on the right, the shape A has been moved 5 squares to the right and 1 square down in order to form the shape B.



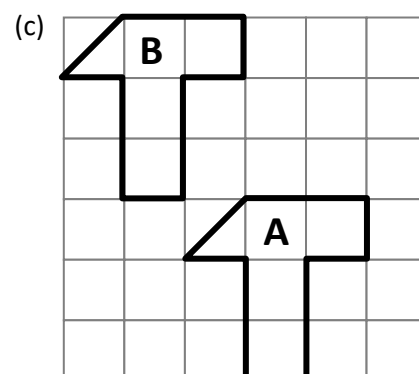
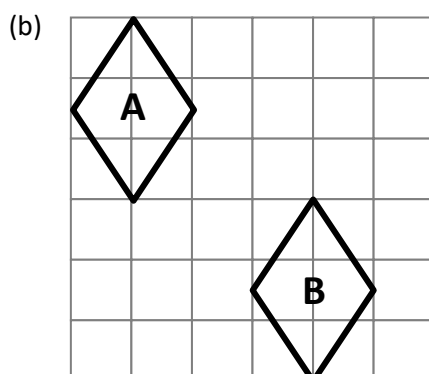
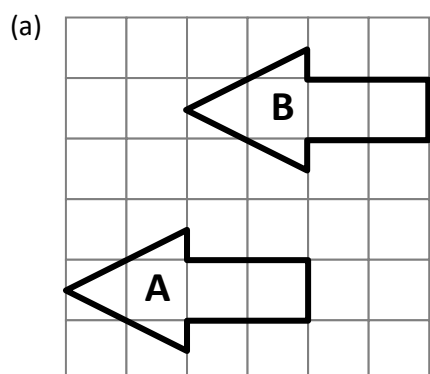
Exercise 45**Skill****2**

Translate the following shapes using the instructions below.

- (a) Translate the shape 4 units to the right and 3 units up.
- (b) Translate the shape 3 units to the left and 7 units up.
- (c) Translate the shape 8 units to the right and 2 units down.
- (d) Translate the shape 5 units up.
- (e) Translate the shape 2 units down and 4 units to the left.

**Exercise 46****2**

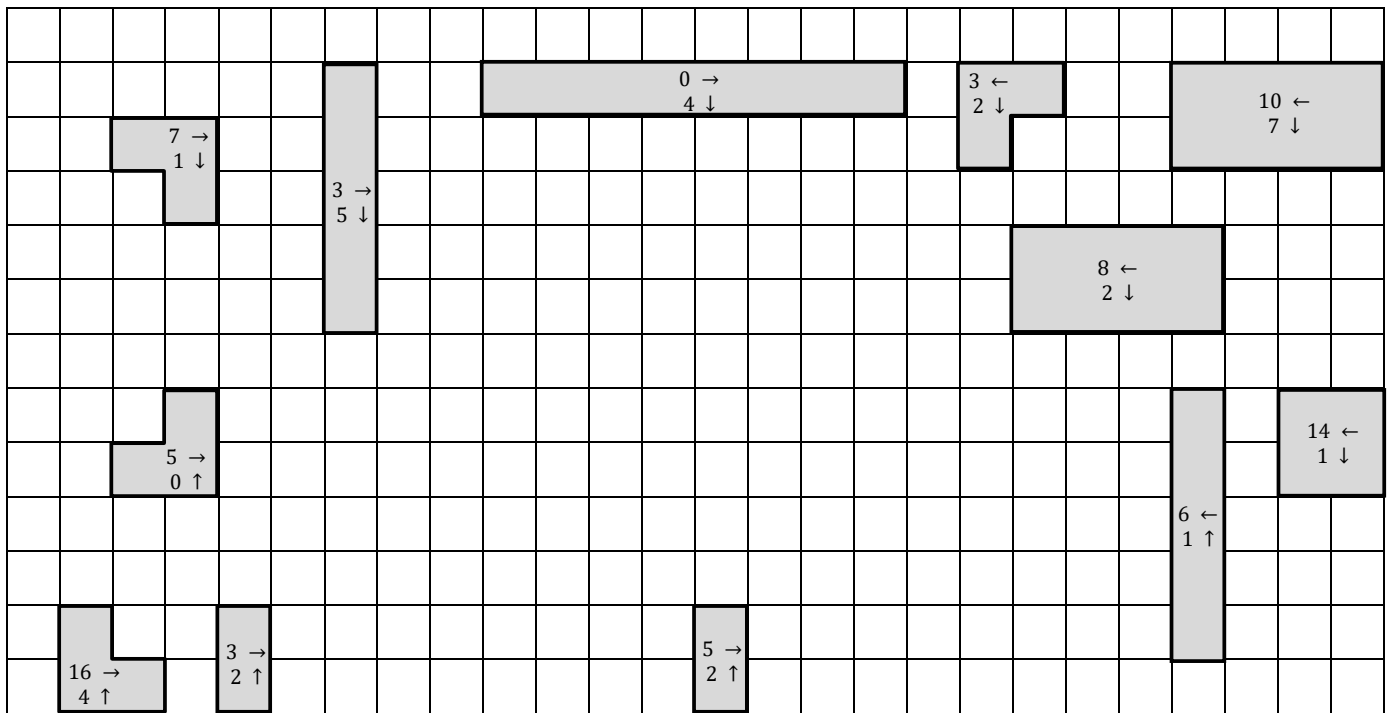
Write down the translation that translates the shape **A** to the shape **B** in the following diagrams.



Exercise 47

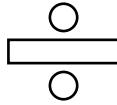
2

Translate the following shapes to form one big shape...

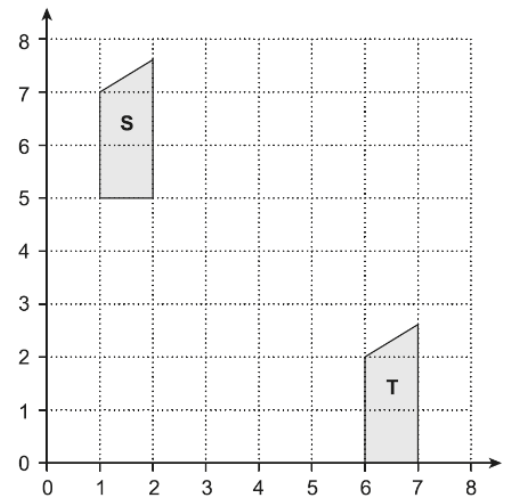


Exercise 48 (Revision) 2

(a) What is the order of rotational symmetry of the division sign shown on the right?



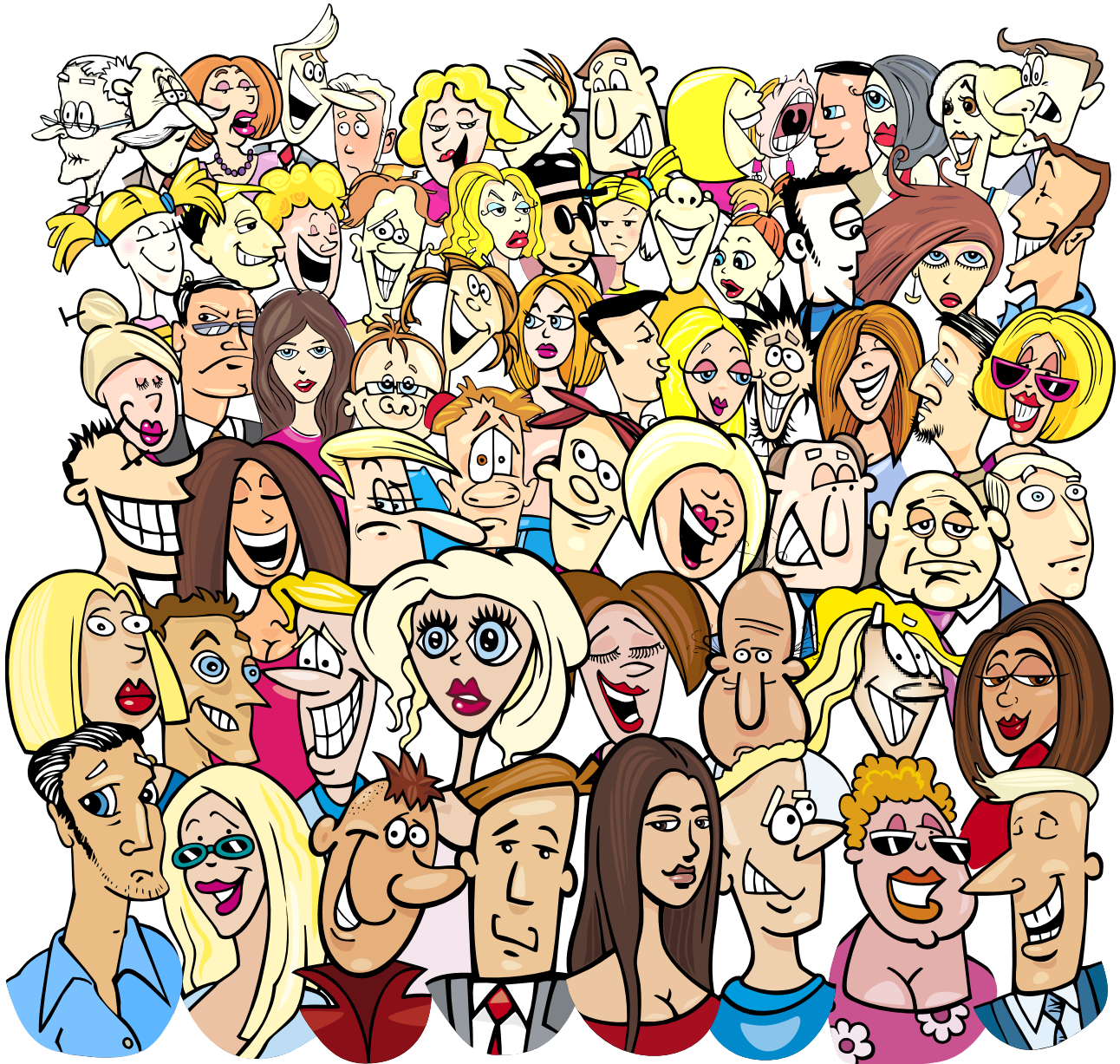
(b) Which translation translates the shape **S** to the shape **T** in the diagram on the right?



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Puzzle: How many people can be seen in the picture?





Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
I know the meaning of the word ' multiple '.			5, 9	
I know how to decide if a number is a multiple of 2 .			1	
I know how to decide if a number is a multiple of 3 .			5	
I know how to decide if a number is a multiple of 5 .			9	
I know how to decide if a number is a multiple of 7 .			9	
I know how to decide if a number is a multiple of 9 .			12	
I know how to complete a Venn diagram .			12	
I know the names of the shapes with between 3 and 10 sides.			2	
I know the difference between a regular polygon and an irregular polygon .			14	
I know the difference between a vertical line and a horizontal line .			7	
I know the difference between parallel lines and perpendicular lines .			7	
I know the difference between an even number and an odd number .			1, 3	
I can calculate a square number such as 7^2 .			9, 11	
I can calculate a cube number such as 6^3 .			11	
I know the difference between times in the 12-hour clock and times in the 24-hour clock .			4, 5	
I can read the time shown on an analogue clock .			4	
I know the number of days in each month of the year.			5	
I know how to decide whether a year is a leap year .			13	
I can answer questions that use a timetable .			17	
I can add symmetry lines to different shapes.			8	
I can complete a shape to ensure it is symmetrical .			15	
I know how to decide what order of symmetry a shape has.			10	
I know how to translate shapes.			16	



The Mathematics Department

7

Introducing

Angles

Version

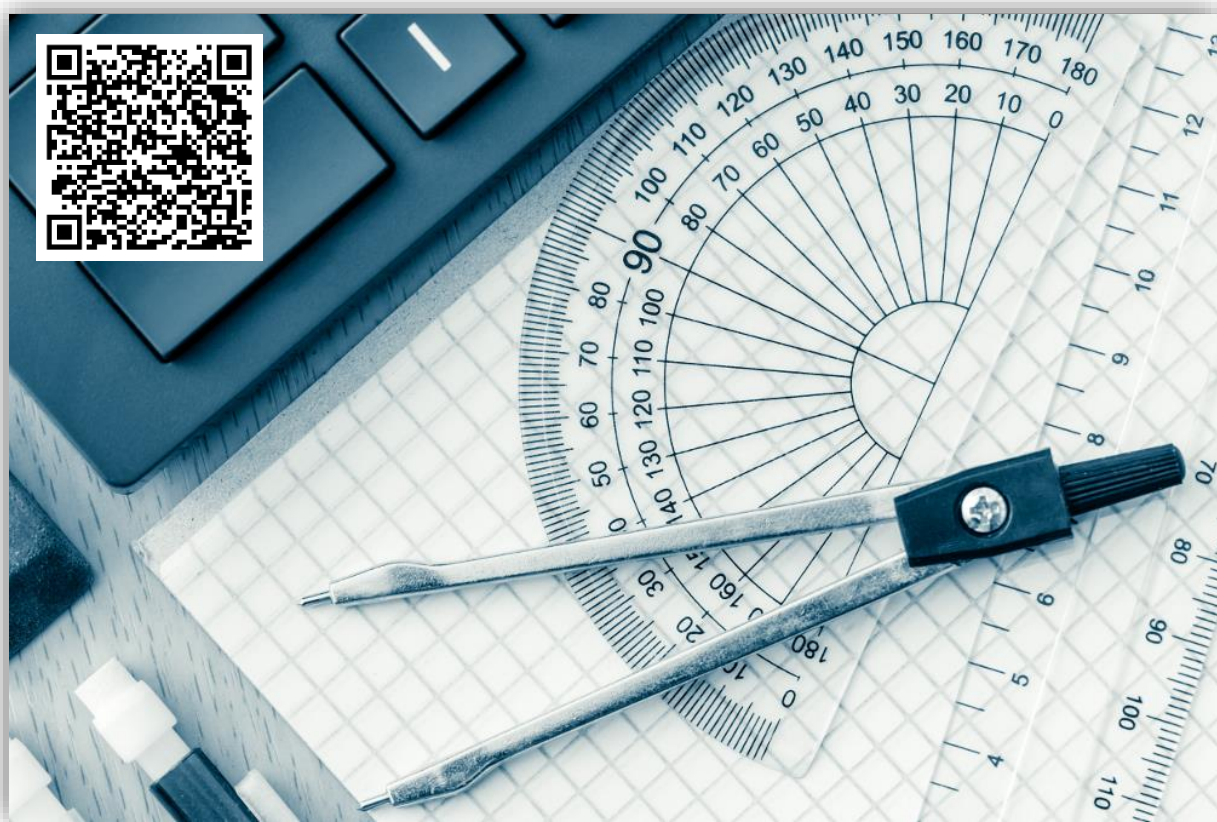
3.0

07/2025

Name:

Contents

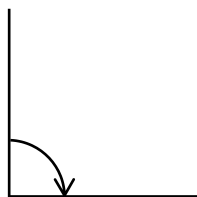
Chapter	Mathematics	Page Number
Types of Angles	Turns. Acute, obtuse and reflex angles. Naming angles.	3
Measuring Angles	Estimating angles. Using a protractor to measure angles.	5
Drawing Angles	Using a protractor to draw angles.	13
Calculating Angles	Angles around a point. Angles on a straight line. Vertically opposite angles.	14
Angles in Triangles	Angles in a triangle. Types of triangles. The exterior angle of a triangle. Constructing triangles.	19



Types of Angles



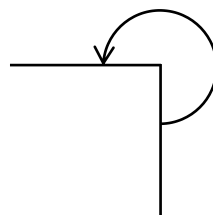
An angle measures **turn**. Here are some examples of different turns.



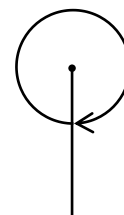
A quarter of a turn clockwise



Half a turn anticlockwise



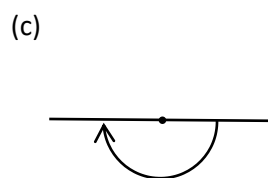
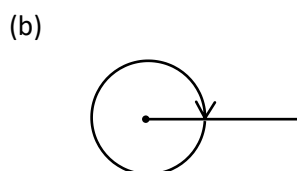
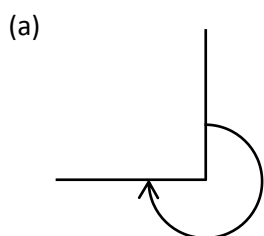
Three quarters of a turn anticlockwise



A full turn clockwise

Exercise 1

What **type** of turns do you see below?



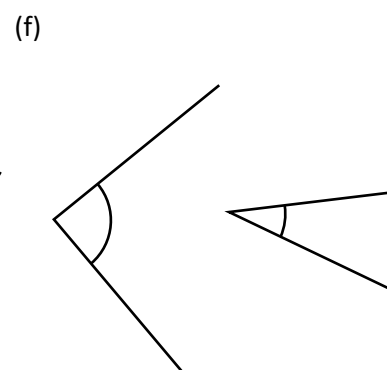
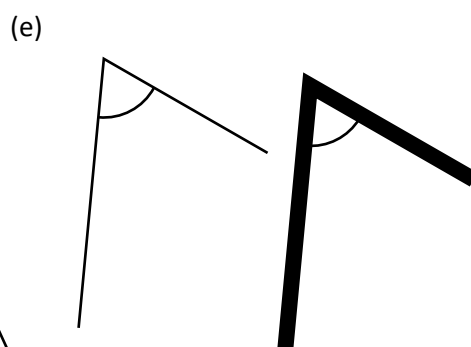
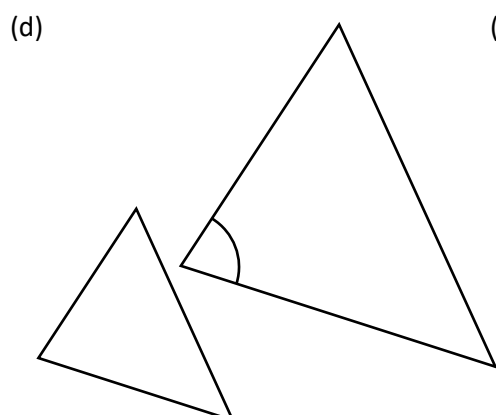
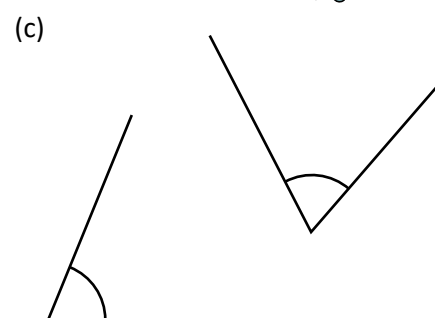
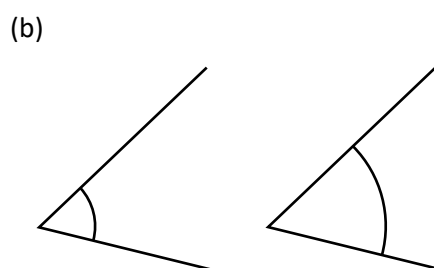
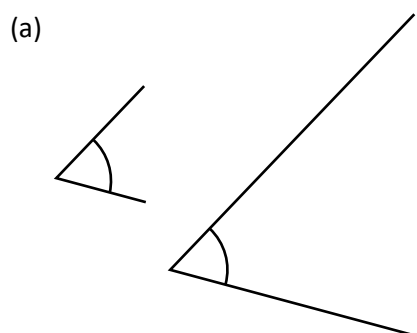
Exercise 2

Experiment with turning in class in different ways.



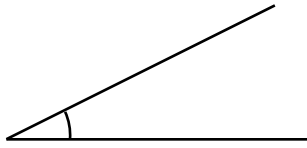
Exercise 3

Explain which angle is the **largest** in each pair of angles below.



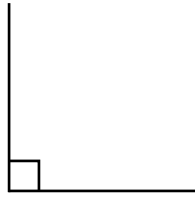
It is possible to measure an angle in degrees. There are **360°** in a full turn, so there are 180° in half a turn; 90° in a quarter of a turn, and so on.

We use the following names for the different types of angles.



Acute Angle

Any angle between 0° and 90°



Right Angle

An angle that is exactly 90°



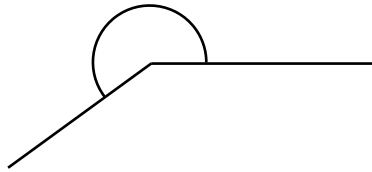
Obtuse Angle

Any angle between 90° and 180°



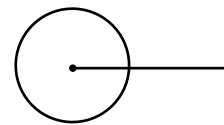
Straight Line

An angle that is exactly 180°



Reflex Angle

Any angle between 180° and 360°



Full Turn

An angle that is exactly 360°

Challenge!

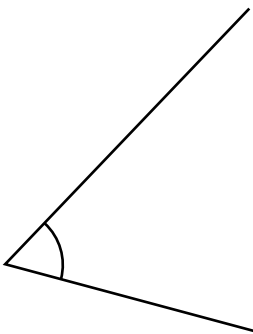
Why do we use 360° for the size of a full turn, and not some other number?



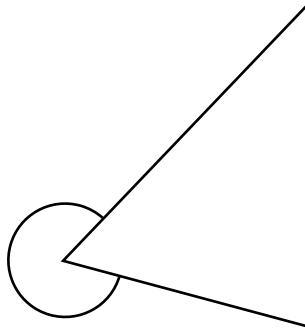
Exercise 4

What **type** of angles are the following angles?

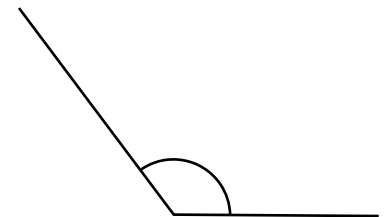
(a)



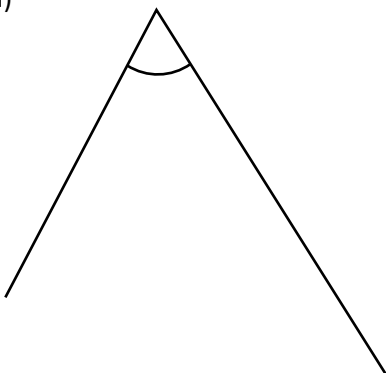
(b)



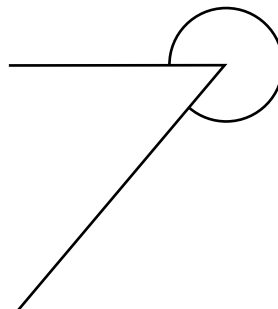
(c)



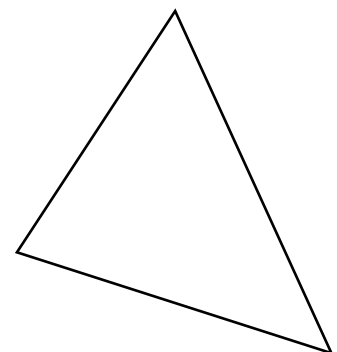
(d)



(e)



(f)



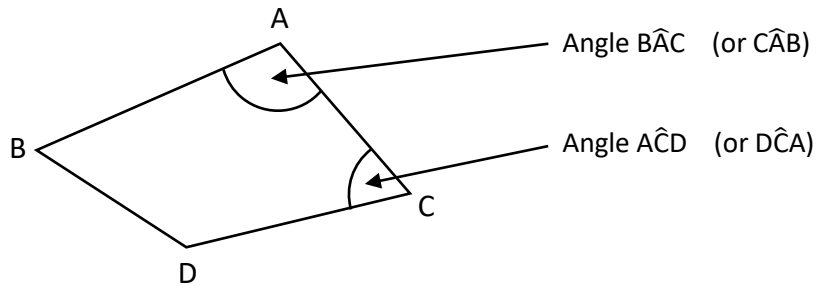
Exercise 5

What **type** of angles are the following?

- (a) 140° (b) 14° (c) 280° (d) 104° (e) 325° (f) 94° (g) 180°
 (h) 54° (i) 205° (j) 90° (k) 270° (l) 2° (m) 360° (n) 103°

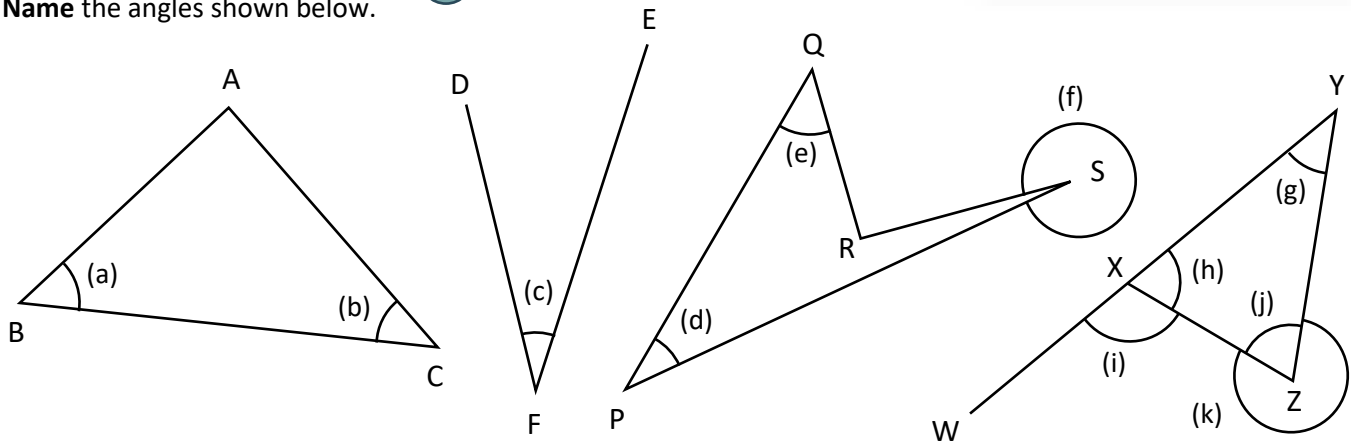
In order to **name** an angle, we label the vertices in a diagram with capital letters, before using **three letters** to identify a specific angle. (We need to add a **circumflex accent** to the letter in the middle.)

Example



Exercise 6

Name the angles shown below.



Exercise 7

What **type** of angles are the ones shown in Exercise 6?

A



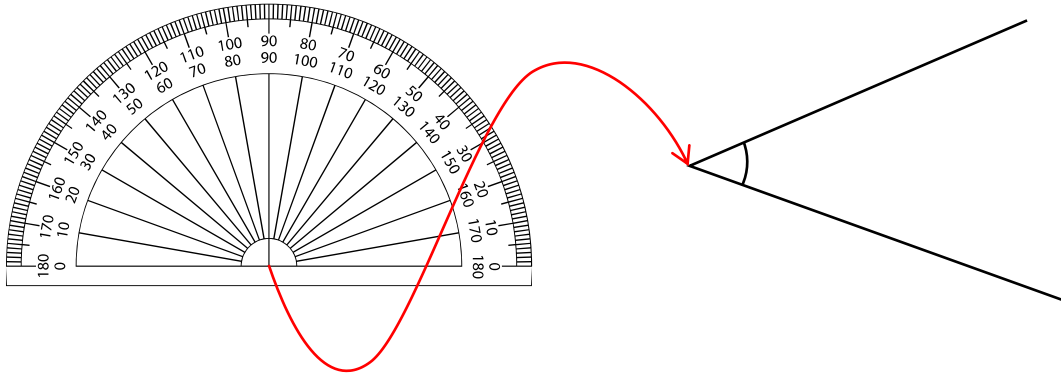
Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

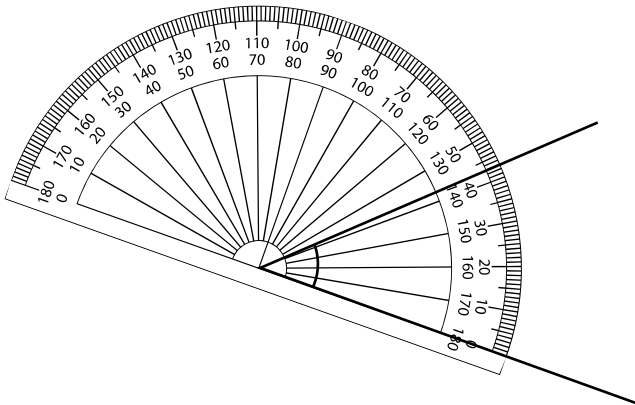
Measuring Angles

We use a **protractor** to measure angles. It is possible to obtain a protractor in the shape of half a circle (measures up to 180°) or in the shape of a full circle (measures up to 360°). To measure an angle:

(i) Put the **centre of the protractor** on the **vertex** (corner) of the angle.



(ii) **Turn** the protractor so that the angle 0° lies on one of the **lines** of the angle.

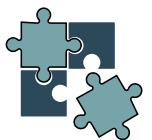
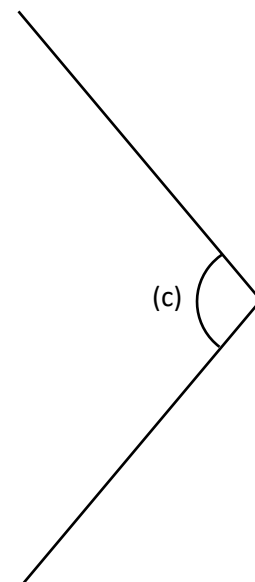
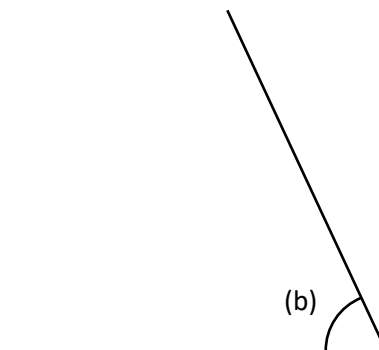
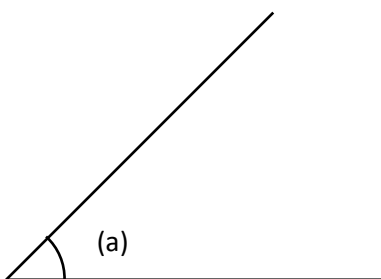


(iii) **Count up from 0°** in order to measure the angle. (It is important to do this as there are two sets of numbers on a protractor.) What angle is shown by the protractor above?

Exercise 8

13

Estimate the size of each of the following angles in degrees. Write your estimate down before **measuring** the angle using a protractor. How accurate were you in estimating the size of the angles?

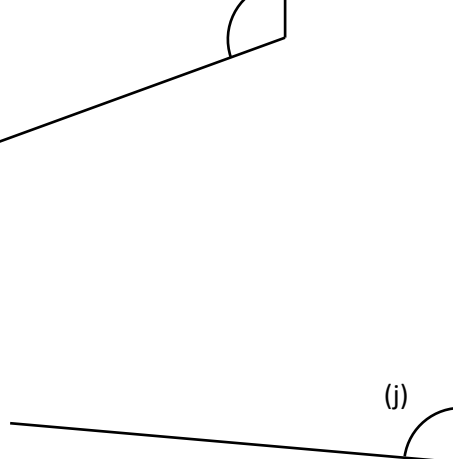
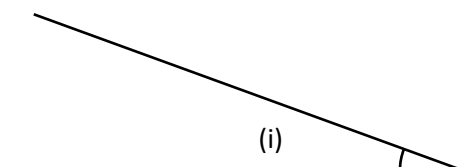
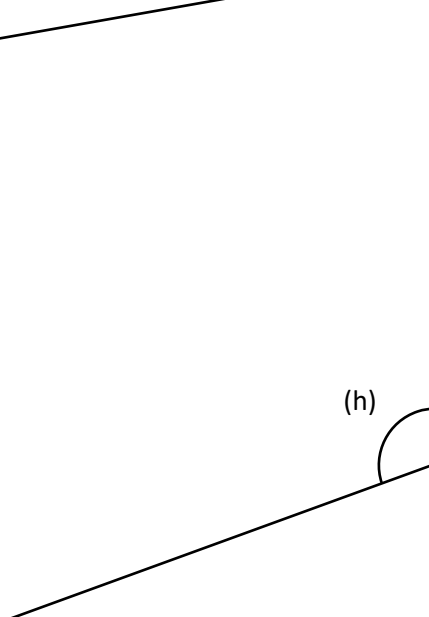
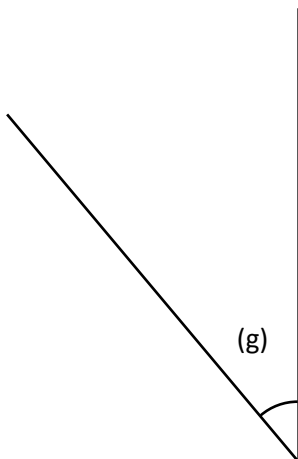
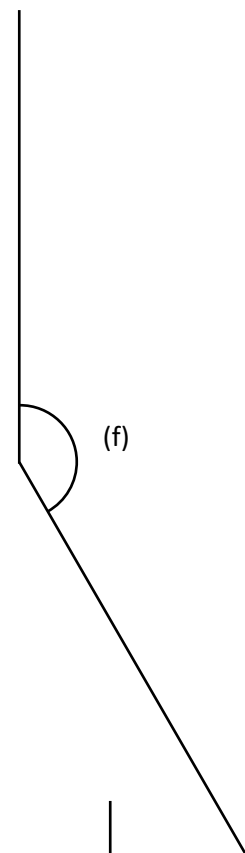
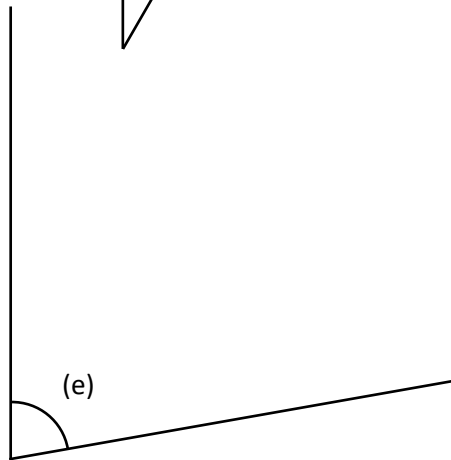
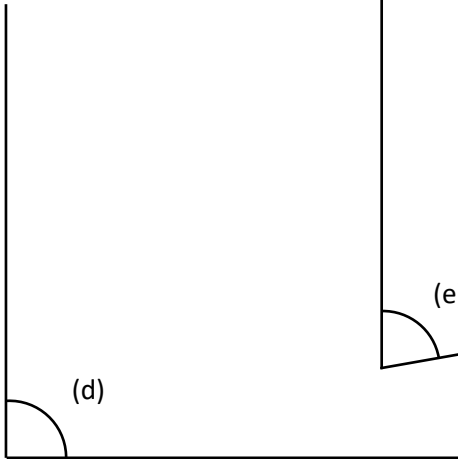
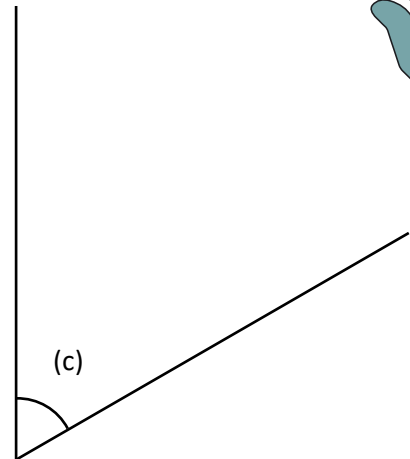
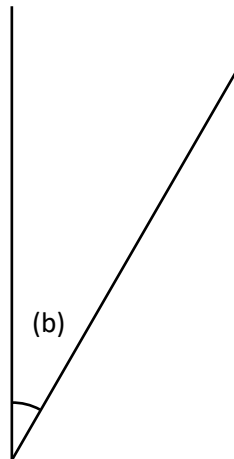
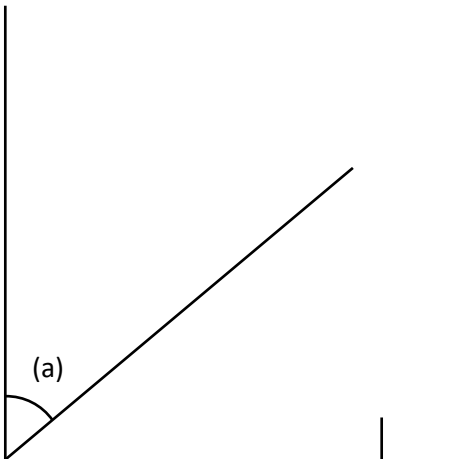


Exercise 9

13

Measure the following angles.

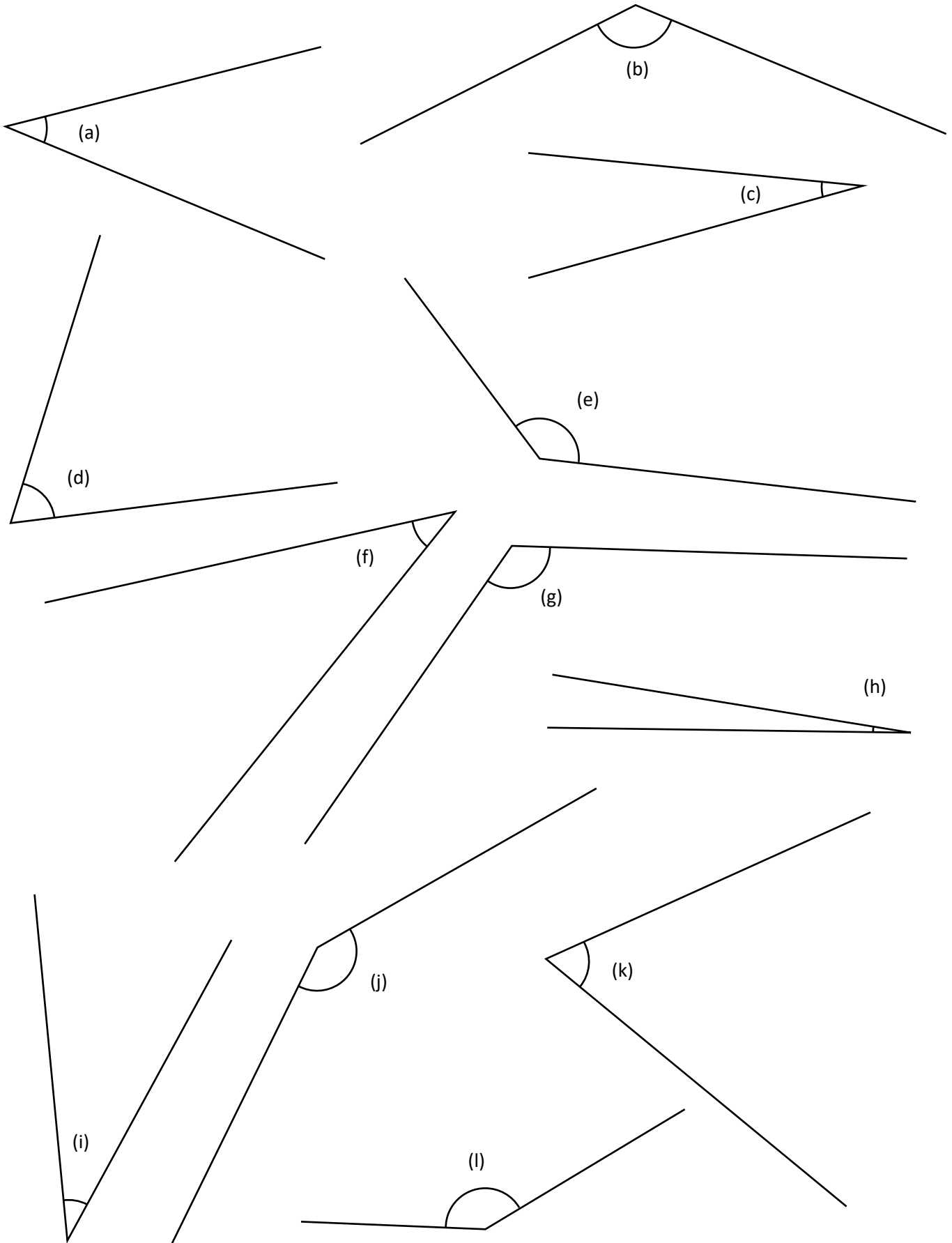
Skill



Exercise 10

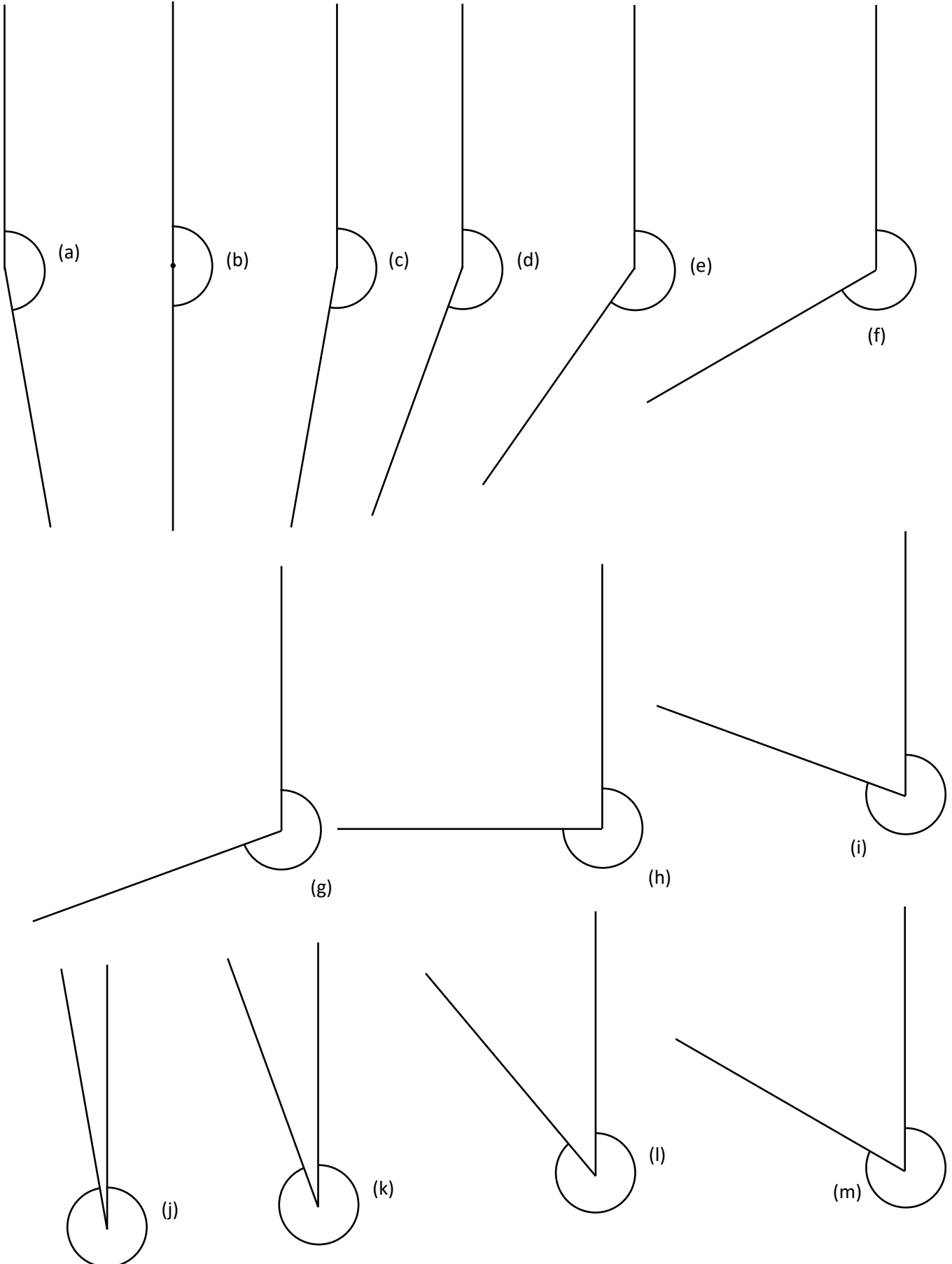
13

Measure the following angles and state what **type** of angles they are.



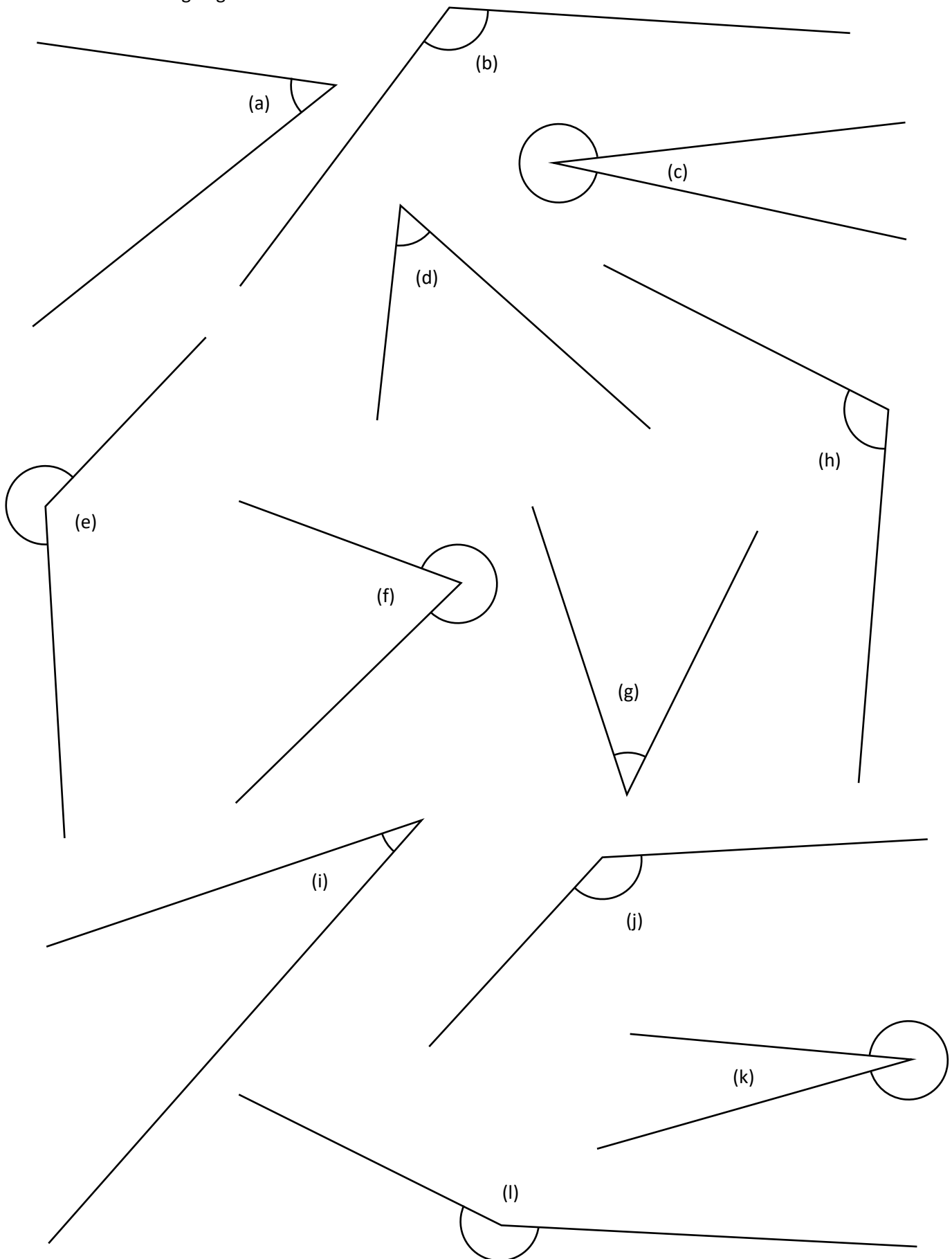
Exercise 11**Measure** the following angles.

13



Exercise 12

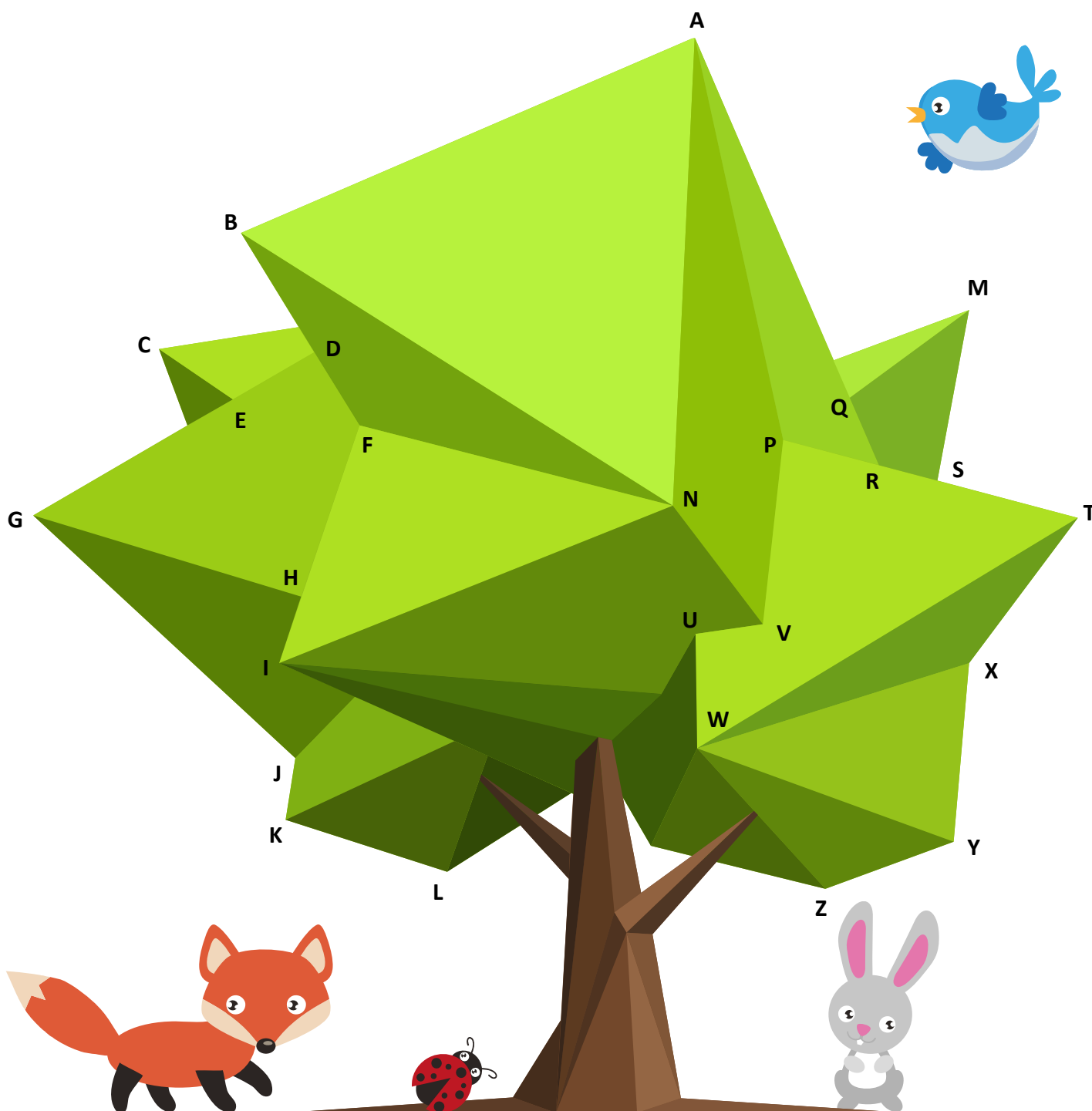
13

Measure the following angles.

Exercise 13**Measure** the following angles.

13

Applying

(a) $\widehat{BAN}$ (b) $\widehat{ABN}$ (c) $\widehat{ANB}$ (d) $\widehat{BFH}$ (e) $\widehat{HGE}$ (f) $\widehat{NIF}$ (g) $\widehat{PTW}$ (h) Reflex angle $\widehat{PTW}$ (i) $\widehat{QMS}$ (j) $\widehat{TXW}$ (k) $\widehat{WUV}$ (l) $\widehat{VNA}$ (m) Reflex angle $\widehat{HGJ}$ (n) $\widehat{LKJ}$ (o) Reflex angle $\widehat{ZYX}$ 

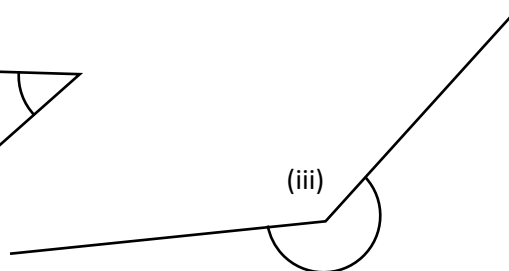
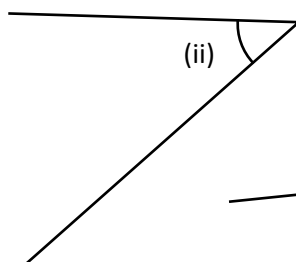
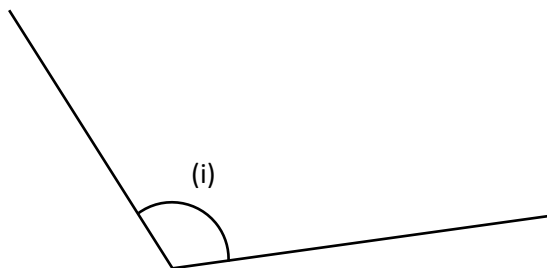
Exercise 14 (Revision)

(a) **Estimate** the size of each of the following angles in degrees.

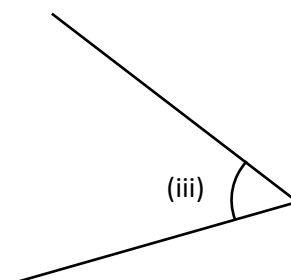
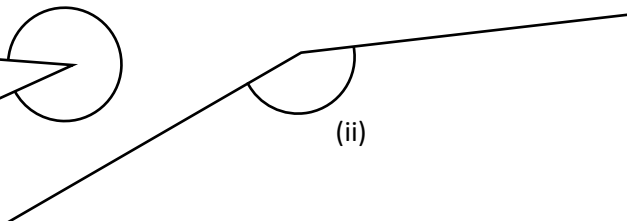
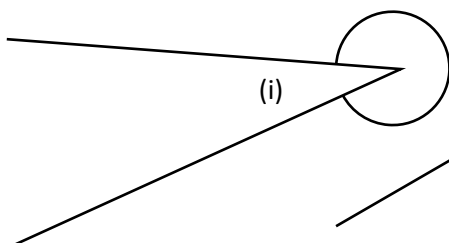
Write down your estimate before **measuring** the size of the angle using a protractor.

13

Skill



(b) **Measure** the following angles and state what **type** of angles they are.

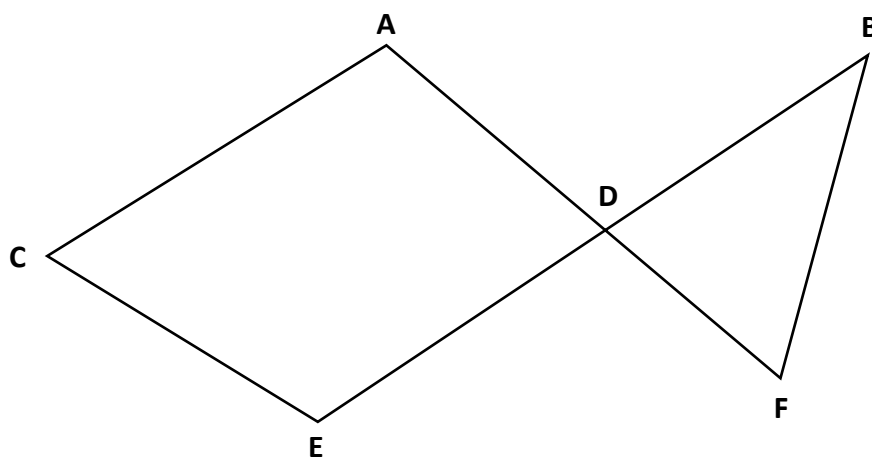


(c) **Measure** the following angles.

(i) $\widehat{ADE}$

(ii) $\widehat{DBF}$

(iii) Reflex angle $\widehat{ACE}$



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

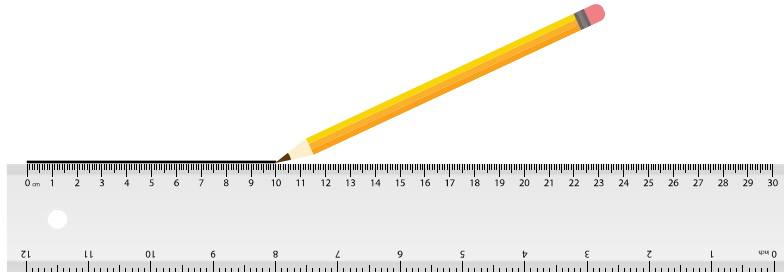
Drawing Angles



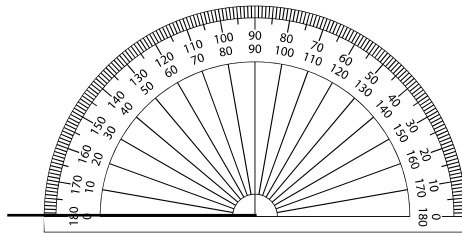
It is possible to draw any angle using a protractor.

Drawing Angles Using a Protractor

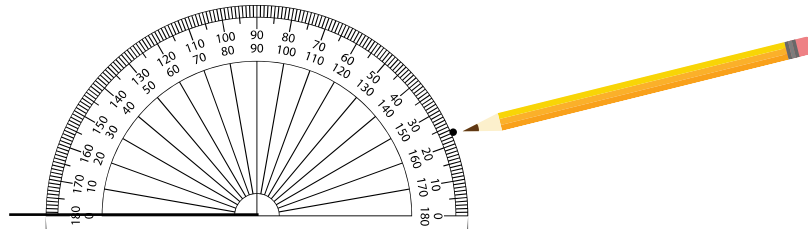
- (1) Draw a straight line using a ruler. (Allow plenty of space around the line in order to draw the angle.)



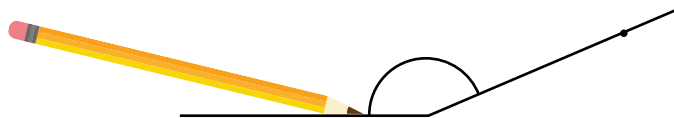
- (2) Place the centre of the protractor on one end of the line. Turn the protractor so that the angle 0° lies on the line.



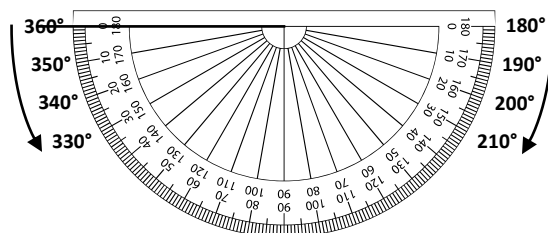
- (3) Count up from 0° to the angle you are attempting to draw. Mark the angle (e.g. 157°) using a pencil.



- (4) Draw a straight line through the dot, starting from where the centre of your protractor was. Draw the arc of the angle.



Note: If you are drawing a **reflex** angle, hold your protractor as shown below during step (2). Then, during step (3), either count up from 180° on the right side of the protractor, or count down from 360° on the left side of the protractor.



Exercise 15

Draw the following angles using a protractor.

- | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| (a) 13° | (b) 40° | (c) 78° | (d) 93° | (e) 107° | (f) 138° | (g) 164° |
| (h) 193° | (i) 216° | (j) 254° | (k) 265° | (l) 284° | (m) 303° | (n) 341° |

13

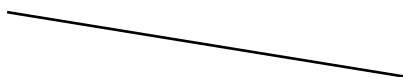
Skill

Exercise 16

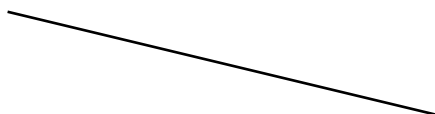
13

(a) Draw an angle of 53° on the left side of the line.(b) Draw an angle of 147° on the right side of the line.(c) Draw an angle of 237° on the left side of the line.

(d) Draw a right angle on the right side of the line.



(e) Draw an obtuse angle on the left side of the line.

(f) Draw an angle of 357° on the right side of the line.**Exercise 17 (Revision)**

Draw the following angles using a protractor.

(a) 25° (b) 74° (c) 172° (d) 208° (e) 270° (f) 317° 

Key Words	Corrections	I am happy with...	I need to revise...

Calculating Angles

With enough information, it is possible to find the size of some angles without using a protractor, by **calculation**.

Angles Around a Point

From the information on page 3, we know that a complete turn is 360° . It follows that the **angles around any point** must sum to 360° .

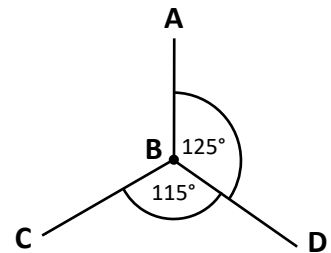
Example

In the diagram on the right, it is possible to find the size of the angle $\hat{A}BC$ without using a protractor, as the sum of $\hat{A}BC$, $\hat{C}BD$ and $\hat{A}BD$ must be 360° .

$$125^\circ + 115^\circ = 240^\circ$$

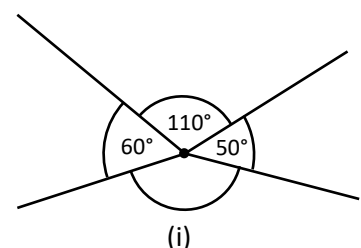
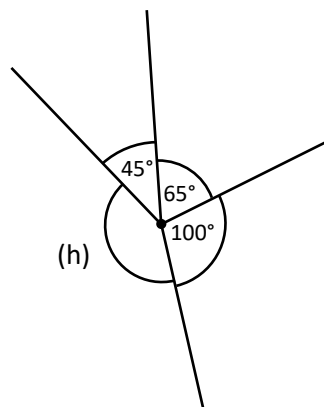
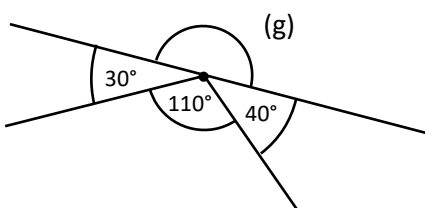
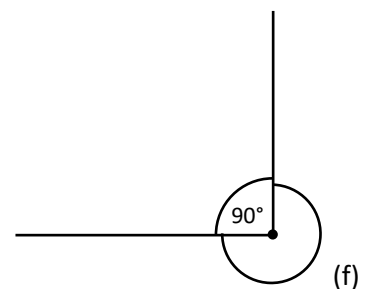
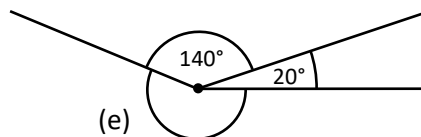
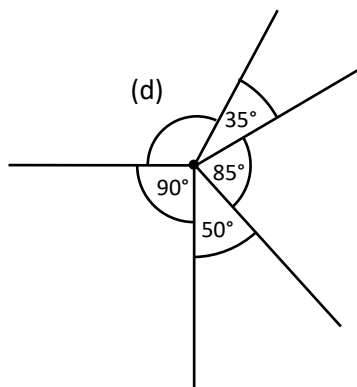
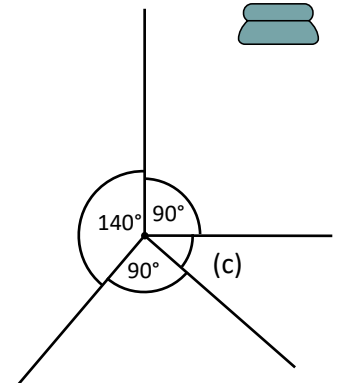
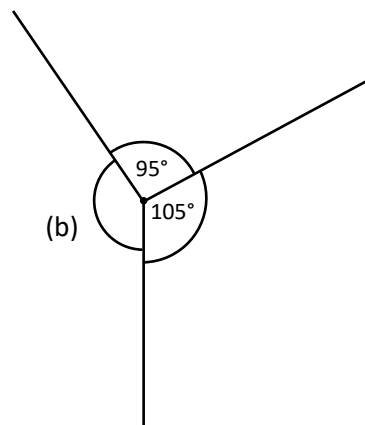
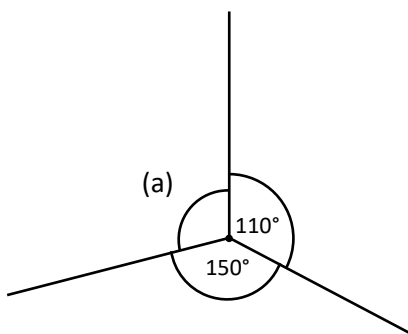
$$\hat{A}BC = 360^\circ - 240^\circ$$

$$\hat{A}BC = 120^\circ.$$



Exercise 18

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Angles on a Straight Line

The angle for half a turn is 180° , so the angles on any **straight line** must sum to 180° .

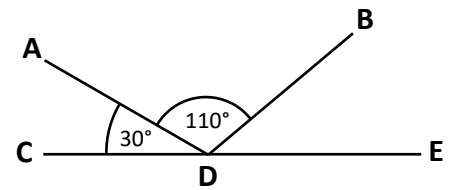
Example

In the diagram on the right, it is possible to find the size of the angle $\widehat{BDE}$ without using a protractor, as the sum of $\widehat{ADC}$, $\widehat{ADB}$ and $\widehat{BDE}$ must be 180° .

$$30^\circ + 110^\circ = 140^\circ$$

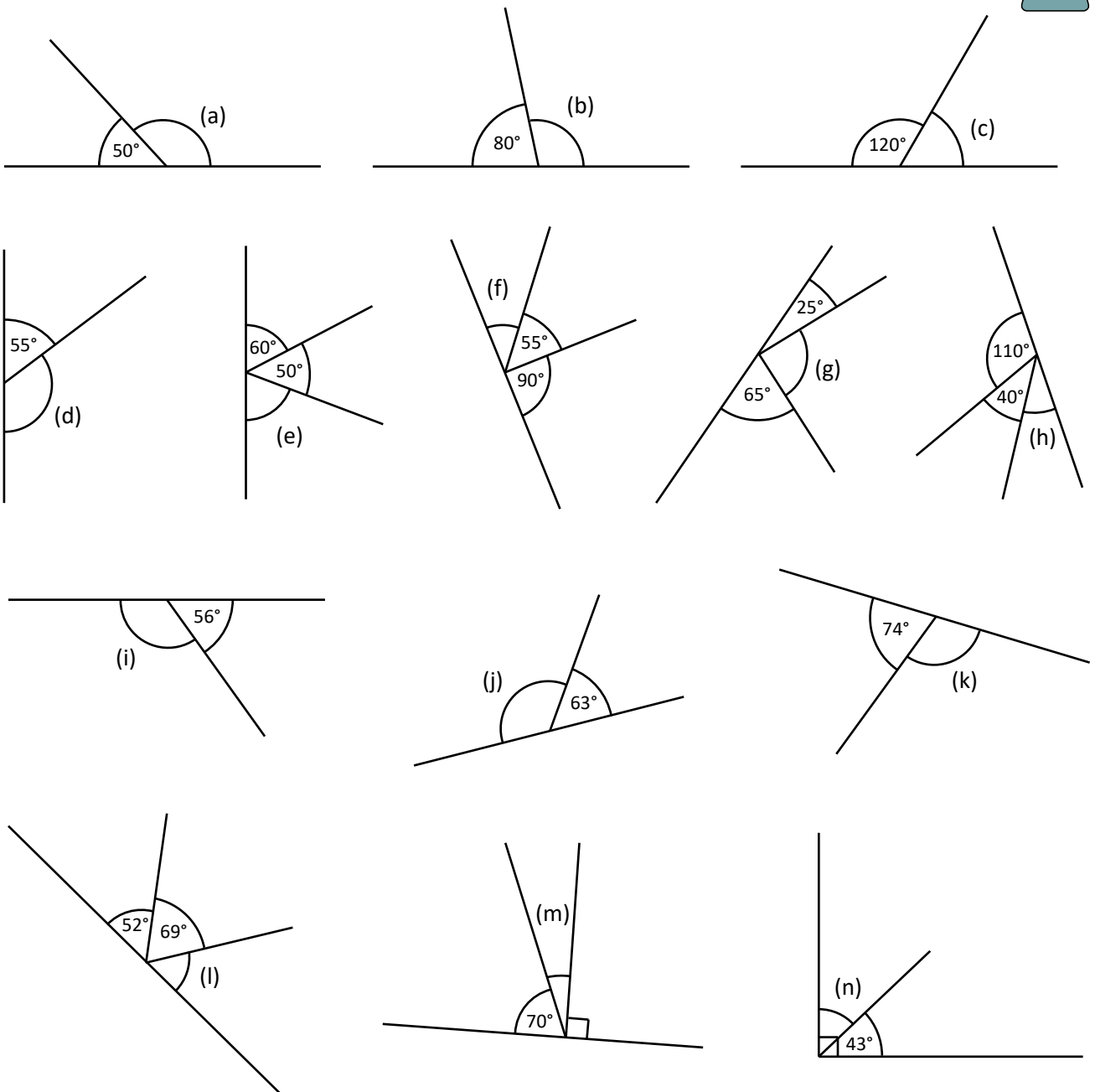
$$\widehat{BDE} = 180^\circ - 140^\circ$$

$$\widehat{BDE} = 40^\circ.$$



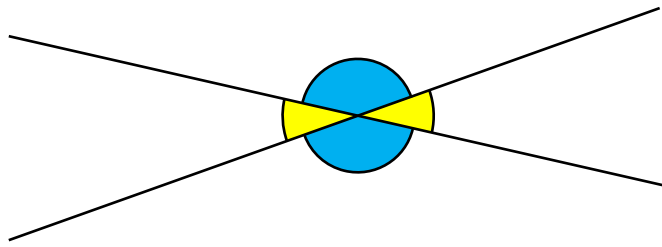
Exercise 19

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



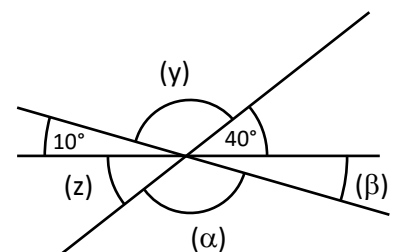
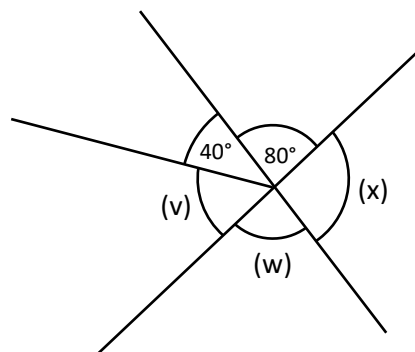
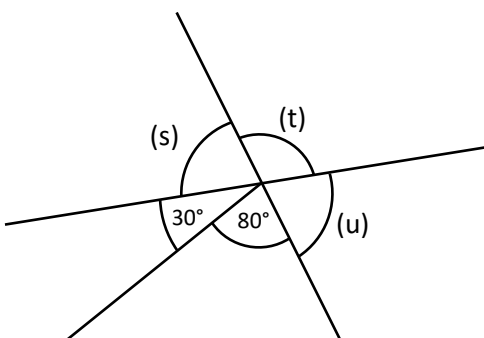
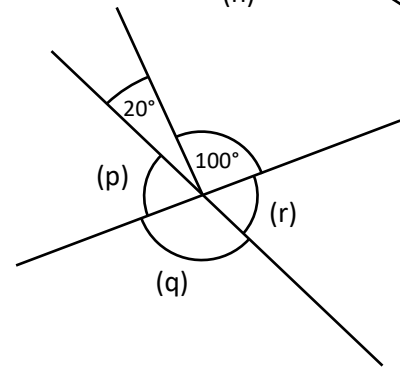
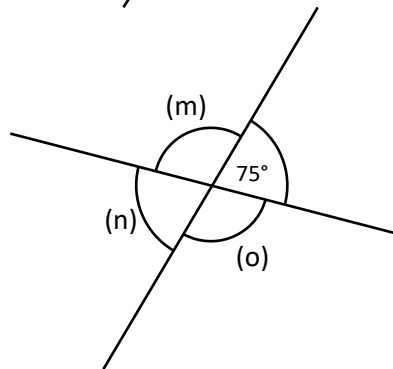
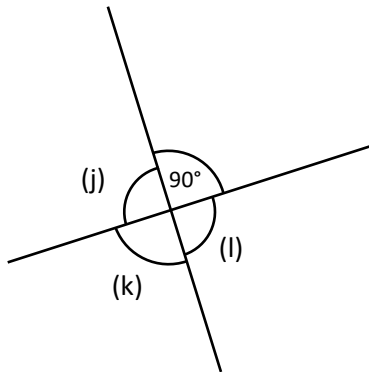
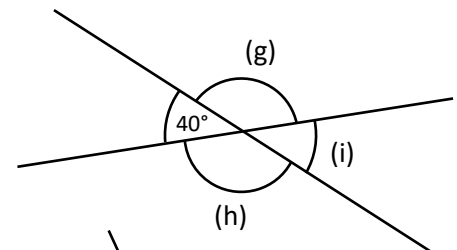
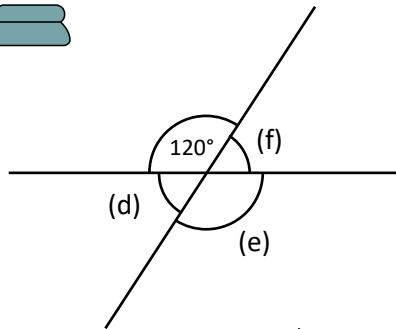
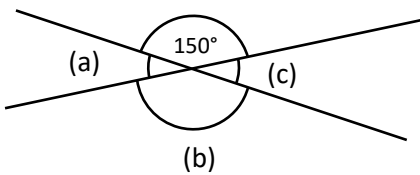
Vertically Opposite Angles

Every time that two lines cross, four angles are formed around the vertex. The angles that are opposite each other (not next to each other) are **equal**. We call these angles **vertically opposite angles**.



Exercise 20

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



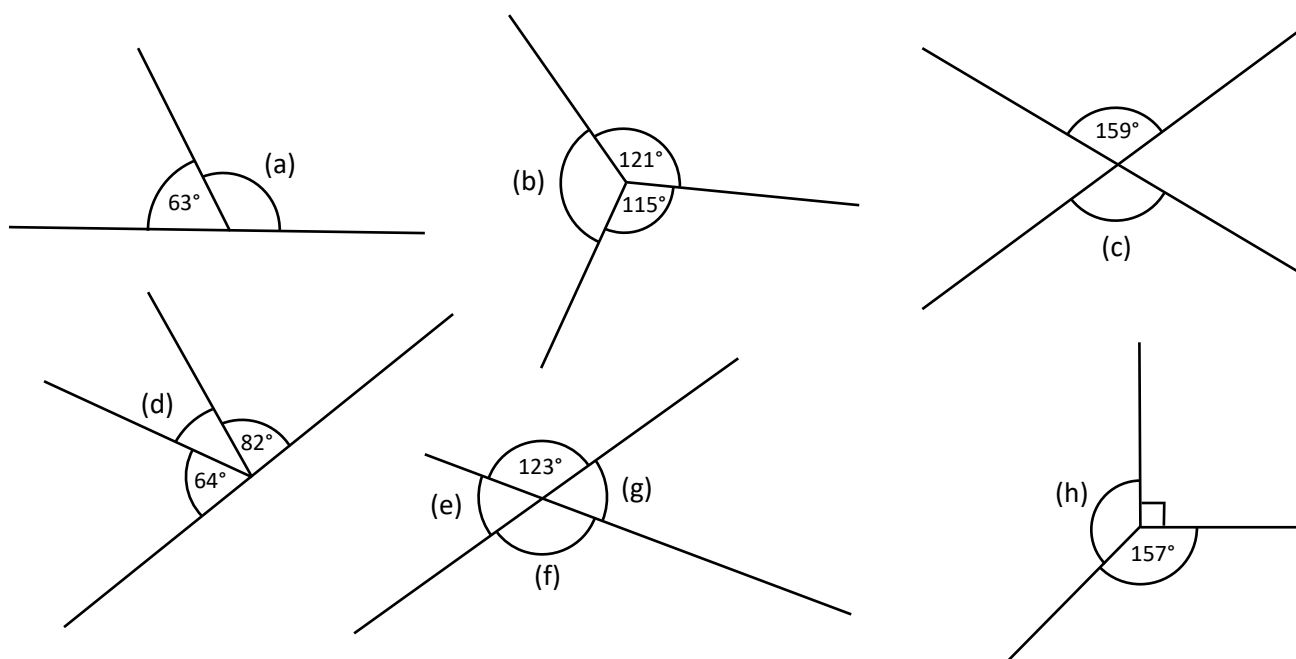
Challenge!

The width of a football goal is 8 yards. The shortest distance from the goal to the penalty spot is 12 yards. Construct a scale drawing in order to find the angle shown in the picture, which is the angle to score a penalty.

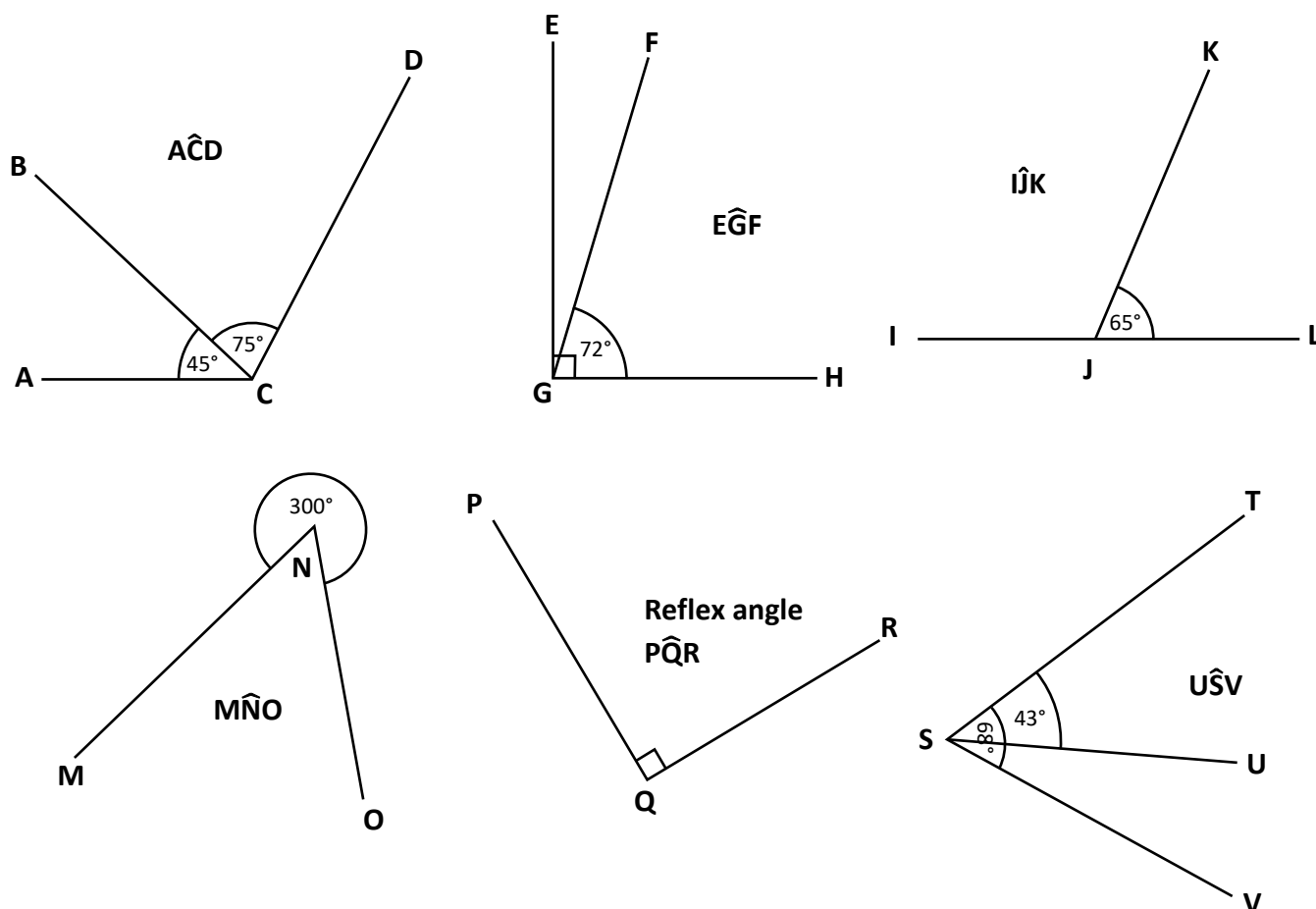


Exercise 21

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

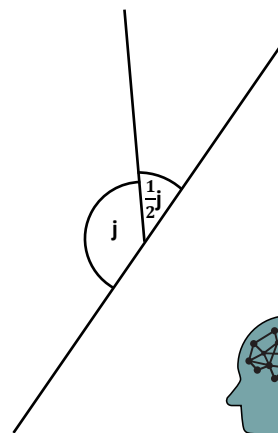
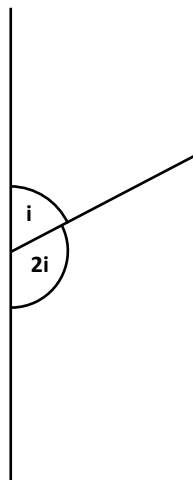
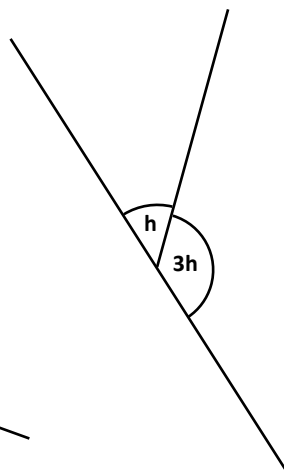
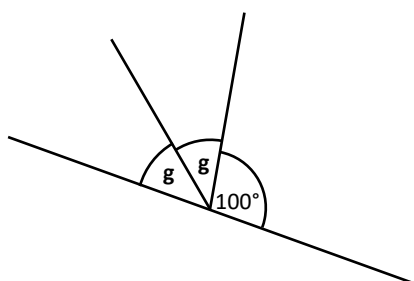
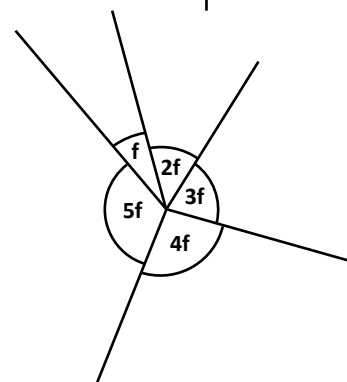
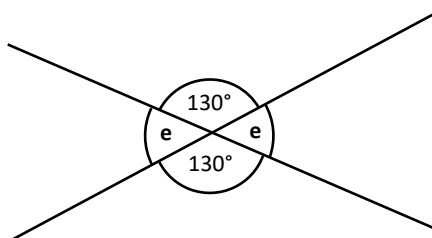
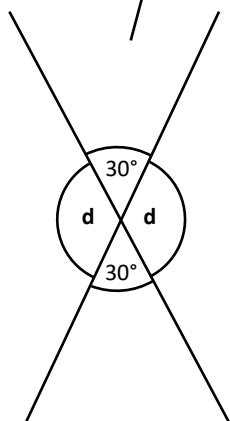
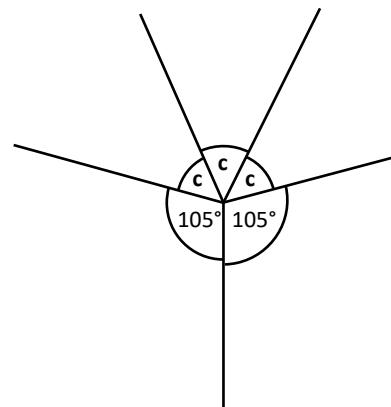
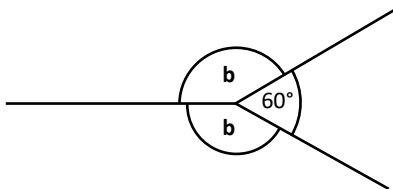
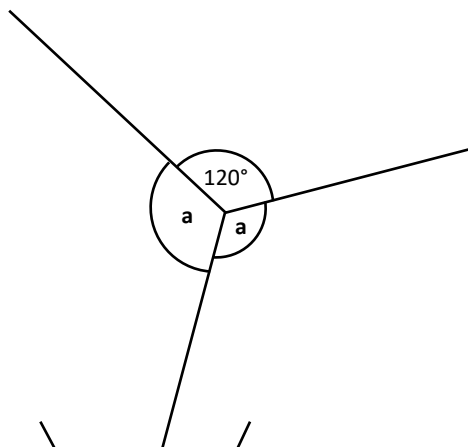
**Exercise 22**

Calculate the **size** of each of the named angles. (The diagrams are not drawn to scale.)



Challenge!

Can you find the following angles? (The diagrams are not drawn to scale.)

**A****Extension****Evaluation**

Key Words	Corrections	I am happy with...	I need to revise...

Angles in a Triangle

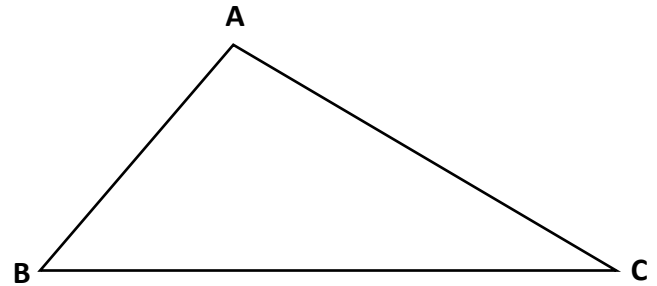
The angles inside a shape are called **interior angles**.

Use your protractor to measure the three interior angles of the triangle shown on the right.

$$\hat{A}BC = \underline{\hspace{2cm}}^\circ$$

$$\hat{B}AC = \underline{\hspace{2cm}}^\circ$$

$$\hat{A}CB = \underline{\hspace{2cm}}^\circ$$



What is the total of your answers?

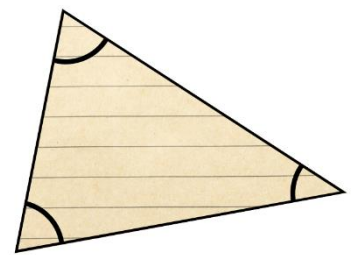
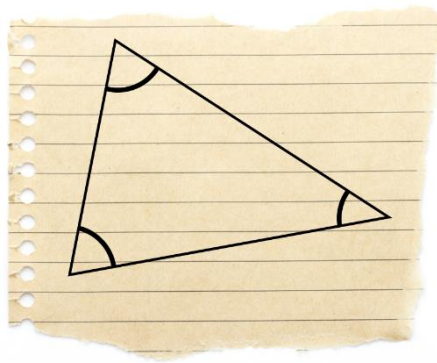
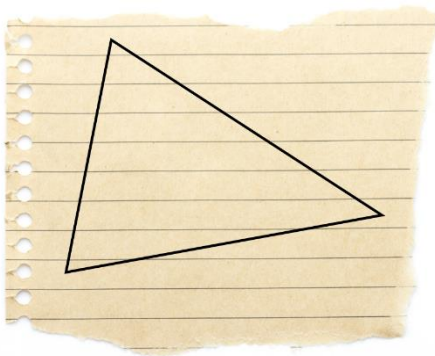
$$\text{Total} = \underline{\hspace{2cm}}^\circ$$

Proving the Sum of the Interior Angles of a Triangle

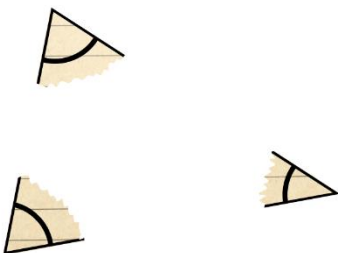


Follow the steps below to show that the sum of the interior angles of a triangle is always the same.

1. Draw any triangle on a piece of scrap paper. The triangle does **not** need to look like the triangle below.
2. Mark the interior angles of your triangle with thick arcs.
3. Cut your triangle out using a pair of scissors.



4. Tear off the corners of your triangle.



5. Arrange the corners to form a straight line.



6. Glue your corners (in a straight line) in the box below.



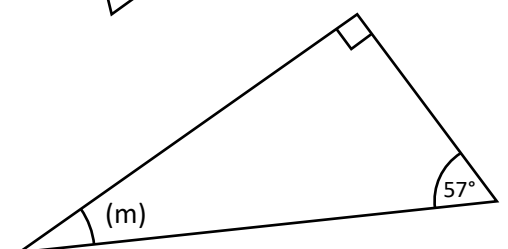
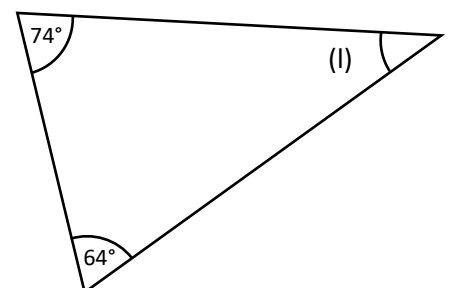
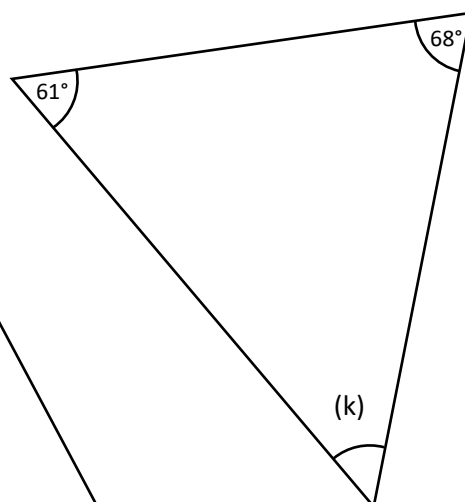
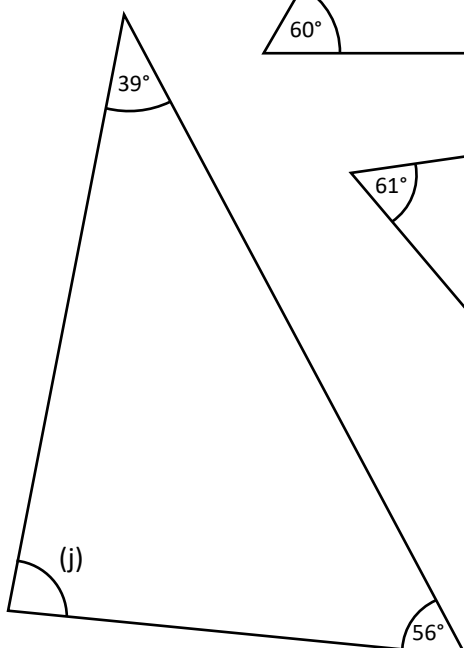
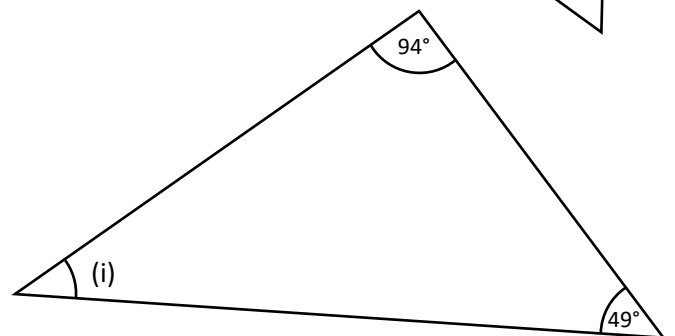
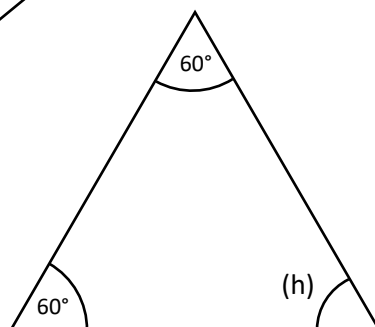
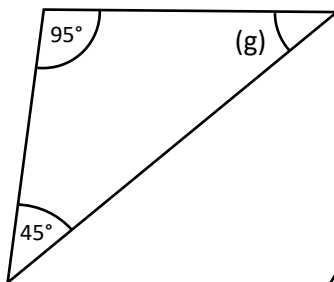
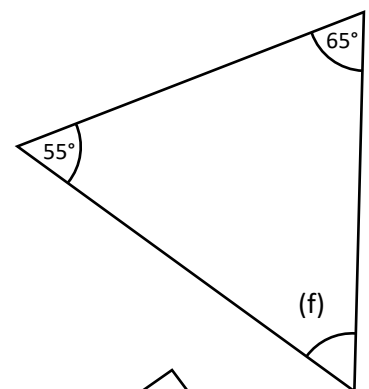
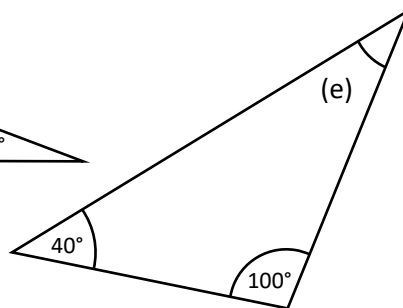
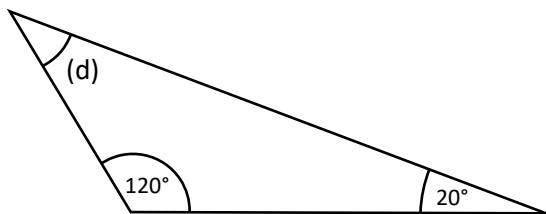
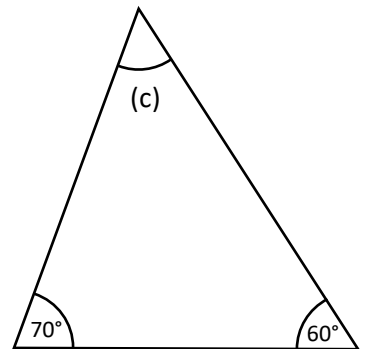
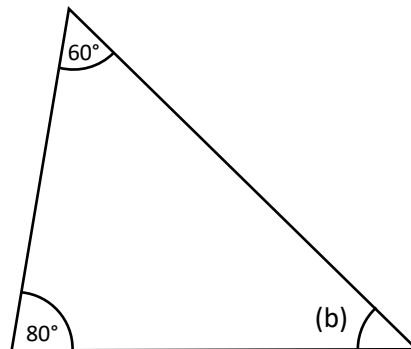
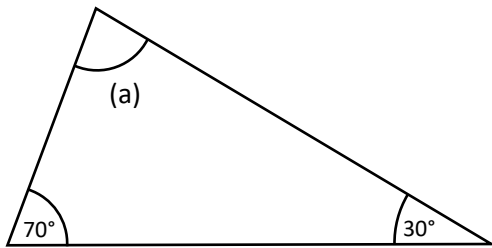
7. Complete the following sentence:

The sum of the interior angles of a triangle is always $\underline{\hspace{2cm}}^\circ$



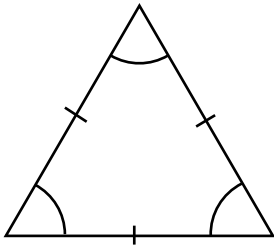
Exercise 23

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

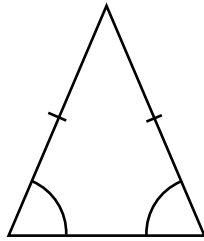
**Skill**

Types of Triangle

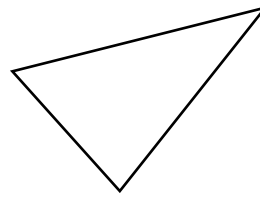
An **equilateral** triangle has three equal sides and three equal angles.



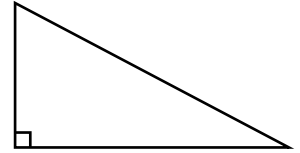
An **isosceles** triangle has two equal sides and two equal angles.



A **scalene** triangle does not have any equal sides or any equal angles.



A **right-angled** triangle has one angle that is a right angle, or 90° .



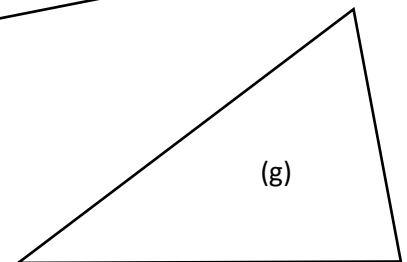
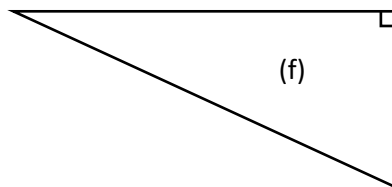
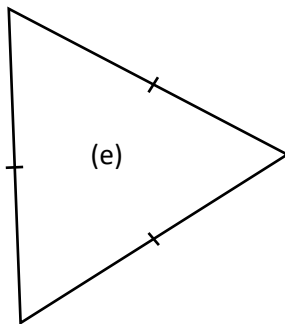
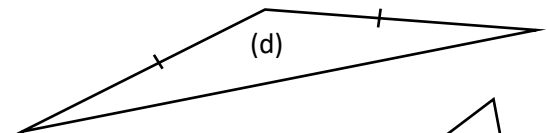
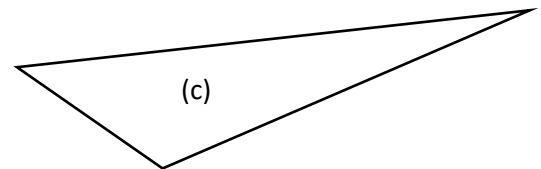
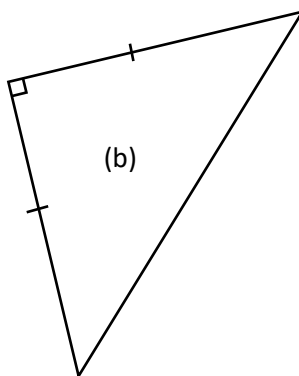
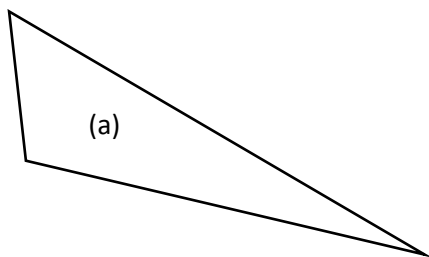
Exercise 24

What **type** of triangles are shown below?



A

Skill

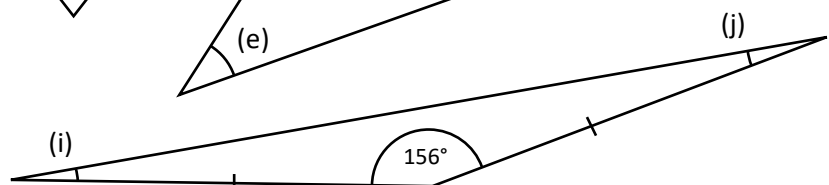
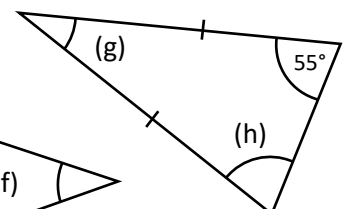
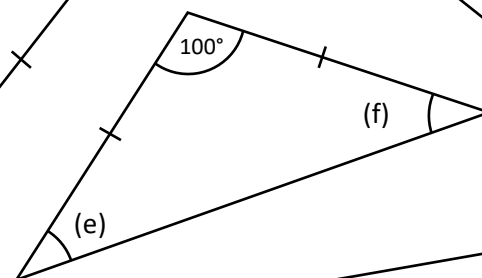
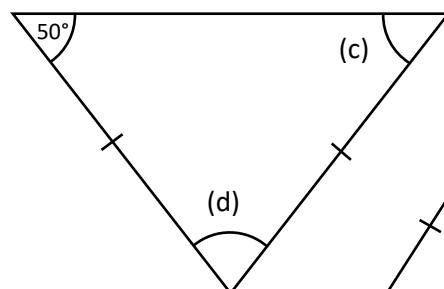
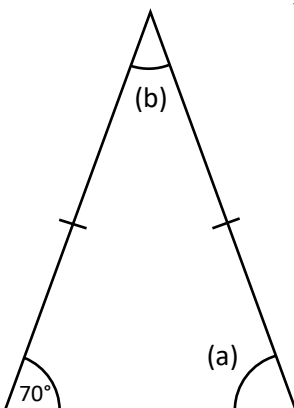


Exercise 25

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



23



23

**Exercise 26**

(a) Dewi draws a triangle with angles 50° , 80° and 50° .

What type of triangle is this?

(b) Explain whether or not it is possible to draw a triangle with two obtuse angles.

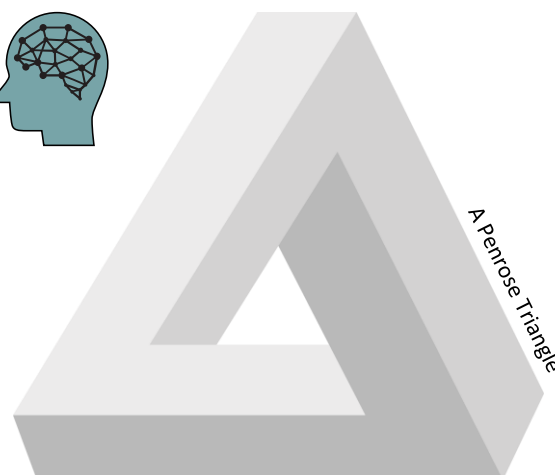
(c) Llinos draws an isosceles triangle that is also a right-angled triangle.

What are the size of the angles in Llinos' triangle?

(d) Explain whether or not it is possible to draw an equilateral triangle that is also a right-angled triangle.

(e) Tomos draws a triangle and measures its angles as 95° , 95° and 170° .

Erin measures the angles of the same triangle and gets 85° , 85° and 10° . Explain who has measured the angles correctly. What mistake has the other person made?

**Investigation**

Draw triangles on the dotted paper below (each corner of a triangle must be placed on a dot).

Identify the type of triangle underneath the dots. How many different triangles are possible?



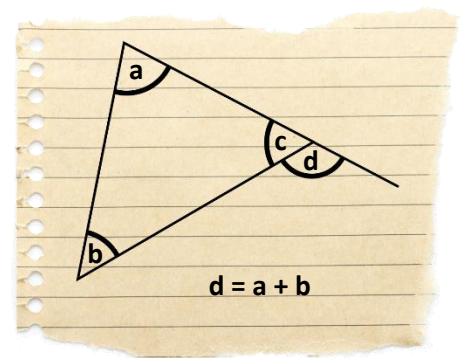
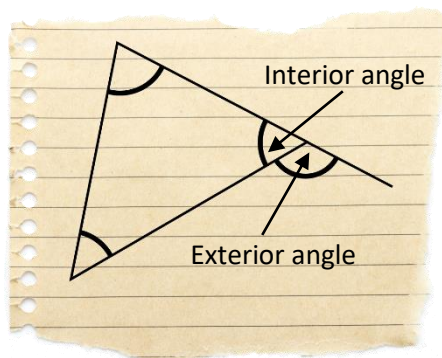
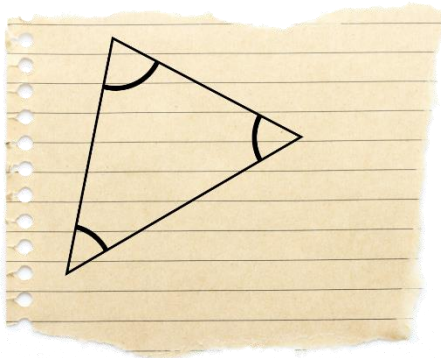
Isosceles; Right-Angled			

The Exterior Angle of a Triangle

Consider any triangle.

By extending one of the sides of the triangle, we create an **exterior angle**.

The size of the exterior angle is the sum of the **two opposite interior angles**.

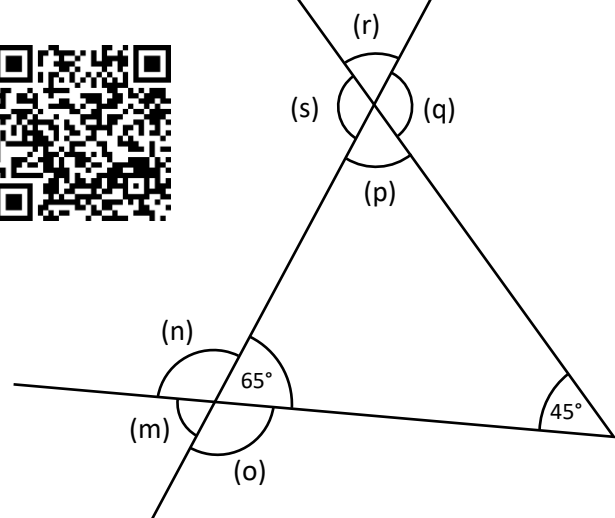
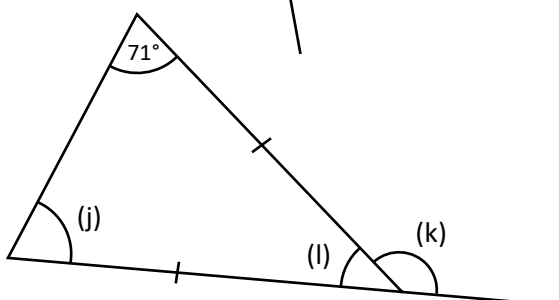
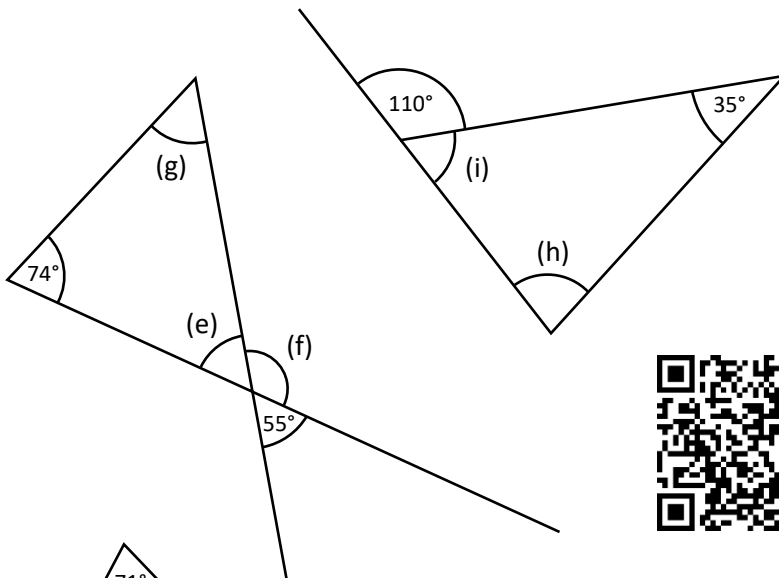
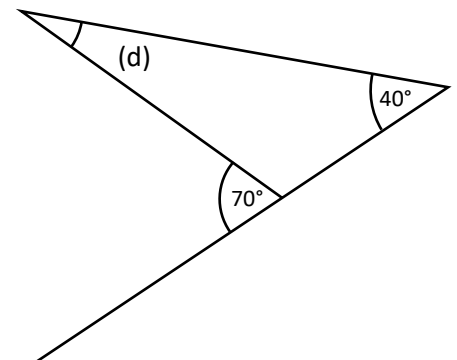
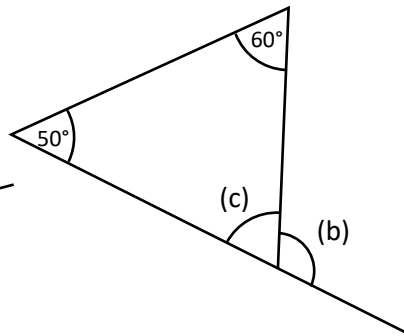
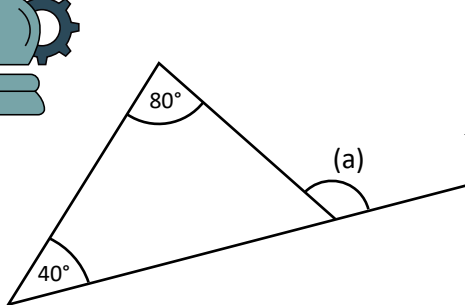


Exercise 27

Calculate the **size** of the marked angles. (The diagrams are not drawn to scale.)

23

Skill



Constructing Triangles

Using a ruler, a protractor and a compass, it is possible to draw any triangle if we know one of the following.

- The length of **one side** of the triangle and the size of **two of the angles**.
- The length of **two sides** of the triangle and the size of the **angle joining** these two sides.
- The length of **all three sides** of the triangle.



3



Exercise 28

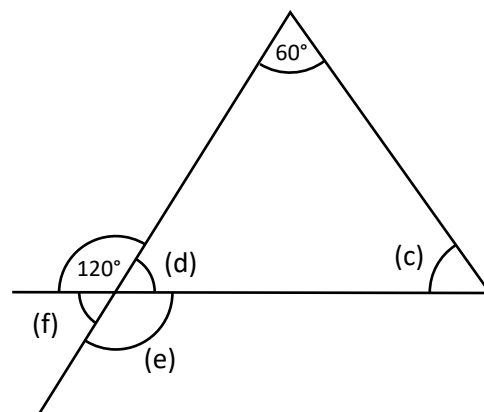
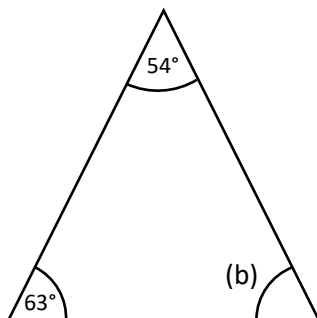
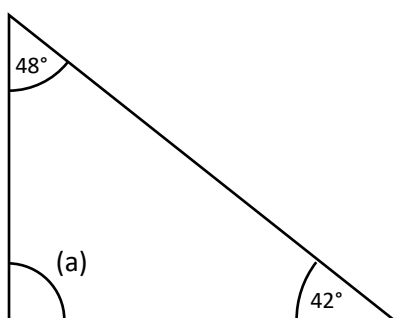
Use a ruler, a protractor and a compass to draw a triangle ABC of the following forms.

- | | |
|---|---|
| (a) $AB = 5\text{cm}$; $\widehat{BAC} = 40^\circ$; $\widehat{ABC} = 60^\circ$. | (b) $AB = 5.5\text{cm}$; $\widehat{BAC} = 30^\circ$; $\widehat{ABC} = 80^\circ$. |
| (c) $AB = 6.2\text{cm}$; $\widehat{BAC} = 45^\circ$; $\widehat{ABC} = 36^\circ$. | (d) $AB = 4.8\text{cm}$; $\widehat{BAC} = 27^\circ$; $\widehat{ACB} = 79^\circ$. |
| (e) $AB = 6\text{cm}$; $BC = 8\text{cm}$; $\widehat{ABC} = 64^\circ$. | (f) $AB = 5\text{cm}$; $BC = 6.2\text{cm}$; $\widehat{ABC} = 105^\circ$. |
| (g) $AC = 7\text{cm}$; $BC = 6\text{cm}$; $\widehat{ACB} = 90^\circ$. | (h) $AB = 4.7\text{cm}$; $BC = 4.7\text{cm}$; $AC = 4.7\text{cm}$. |
| (i) $AB = 6.7\text{cm}$; $BC = 7.2\text{cm}$; $\widehat{ABC} = 38^\circ$. | (j) $AB = 4.5\text{cm}$; $BC = 7.3\text{cm}$; $AC = 7.3\text{cm}$. |
| (k) $AB = 5.7\text{cm}$; $BC = 8.1\text{cm}$; $AC = 9.2\text{cm}$. | (l) $AB = 3.5\text{cm}$; $BC = 8\text{cm}$; $AC = 6\text{cm}$. |

Exercise 29 (Revision)

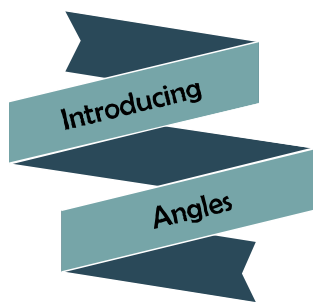
Find the **size** of the marked angles. What **type** of triangles are shown?
(The diagrams are not drawn to scale.)

23



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...



Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
I know the names for the different types of angle, e.g. acute angle.			1, 5, 10	
I know how to name an angle, e.g. $\hat{ABC}$.			4, 5	
I know how to estimate an angle.				
I know how to measure an angle with a protractor.			3, 5	
I know how to draw an angle using a protractor and ruler .			2	
I know what the angles around a point total to.			6, 12	
I know what the angles on a straight line total to.			6, 12	
I know how to use vertically opposite angles to find the size of some angles.			6, 12	
I know the total of the angles in any triangle .			6, 9, 12	
I know what the different types of triangle are, e.g. equilateral triangle.			7, 8	
I know how to use external angles to find the size of some angles.			6	
I know how to construct triangles using a ruler, compass and protractor.			11	



The Mathematics Department

7

Data Handling

and Statistics 1

Version

3.0

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Name:

Contents

Chapter	Mathematics	Page Number
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The Sequential Data Handling Cycle

- Designing questionnaires
- Criticising questions
- Hypotheses



- **Types of data**

- Simple random sampling
- Systematic sampling
- Stratified sampling



1
Specify the problem and plan

2
Collect data from a variety of sources

3
Process and represent the data

4
Interpret and discuss the data

5
Evaluate the process



- **The mean**
- **The range**

- The mode
- The median
- The interquartile range
- Box and whisker diagrams
- Choosing the most appropriate average
- Comparing averages

- **Bar charts**
- **Frequency diagrams**
- **Pie charts**
- Pictograms
- Vertical line diagrams
- Line graphs
- Scatter diagrams
- Cumulative frequency diagrams
- Frequency polygons



The Mean

In the qualifying games for Euro 2016, Wales' football team scored the following number of goals.

2, 0, 2, 0, 3,
1, 1, 0, 0, 2.

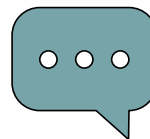
Which number would best describe the number of goals scored by Wales per game? When we are looking for a number of this type, we are looking for an **average** for the data. An average is a number which attempts to represent the typical or middle value of the data. The **mean** is an example of an average.



Calculating the Mean

In order to calculate the mean of a set of data:

- (1) Find the **total** of all the data items.
- (2) **Divide** by the number of data items.



For the data about Wales' goals, the total of all the data items would be

$$2 + 0 + 2 + 0 + 3 + 1 + 1 + 0 + 0 + 2 = 11.$$

It follows that the mean is $11 \div 10 = 1.1$. This means that, on average, Wales scored around 1 goal per game.

Exercise 1

Find the mean number of goals scored by the following football teams during the first games of the new football season.

(a) Arsenal	5,	0,	2,	0,	4.
(b) Chelsea	0,	2,	1,	2,	3.
(c) Everton	2,	1,	1,	2,	0.
(d) Liverpool	2,	3,	0,	2,	1.
(e) Manchester United	3,	2,	3,	4,	0.
(f) Tottenham Hotspur	2,	0,	0,	4,	1.



Exercise 2

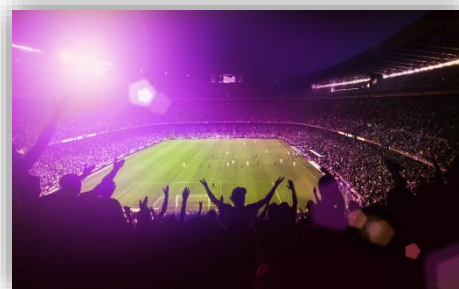
In Exercise 1, which team has the **largest** mean number of goals?

Exercise 3

Sara asked her friends how much pocket money they received. She wrote down their responses:

£1, £3, £2, £5, £2, £10, £8, £7.

What is the **mean amount** of pocket money?

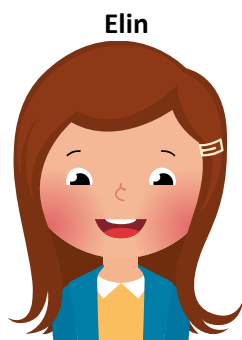


Exercise 4

Here is some information about five children.

3

Applying



Elin
Height: 132cm
Weight: 45kg
Shoe size: 3



Hari
Height: 148cm
Weight: 50kg
Shoe size: 5



Catrin
Height: 136cm
Weight: 39kg
Shoe size: 4



Siôn
Height: 150cm
Weight: 49kg
Shoe size: 5



Eysyllt
Height: 150cm
Weight: 52kg
Shoe size: 6

- What is the mean height of these children?
- Which children are taller than the mean height?
- What is the mean weight of these children?
- Which children are lighter than the mean weight?
- What is the mean shoe size of these children?
- Can you buy shoes of mean shoe size?



Exercise 5

Here is some information about four babies.

**Carwyn**

Weight: 8kg

Height: 69cm

Bethan

Weight: 5kg

Height: 70cm

Eryl

Weight: 7kg

Height: 75cm

Osian

Weight: 9kg

Height: 77cm

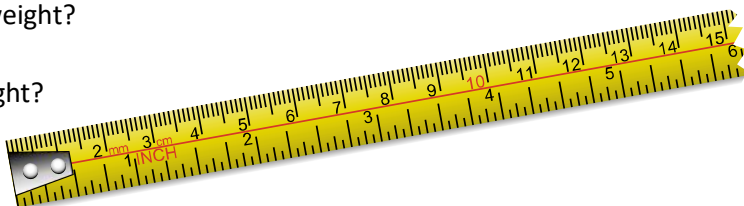
- What is the mean weight of these 4 babies?
- Which of the babies are heavier than the mean weight?
- What is the mean height of these 4 babies?
- Which of the babies are taller than the mean height?

Exercise 6

Here is the weekly shopping cost of 15 families.

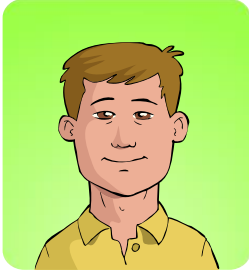
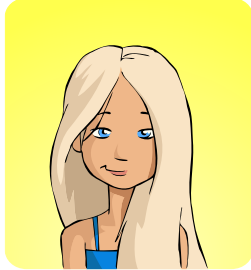
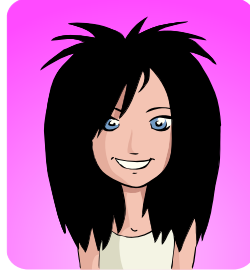
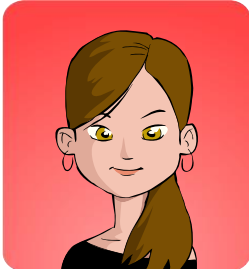
£75 £86 £83 £62 £86 £78 £64 £94 £101 £79 £91 £71 £59 £91 £95

Find the **mean amount** spent, per week, by these families.



Exercise 7

Here is some information about ten adults.

Sophie  Age: 32 Height: 154cm Number of pets: 2	David  Age: 38 Height: 163cm Number of pets: 1	Jessica  Age: 18 Height: 145cm Number of pets: 3	Ben  Age: 24 Height: 165cm Number of pets: 0	Ffion  Age: 26 Height: 150cm Number of pets: 1
Jack  Age: 74 Height: 149cm Number of pets: 1	Jane  Age: 34 Height: 165cm Number of pets: 0	Iwan  Age: 30 Height: 155cm Number of pets: 2	Sara  Age: 44 Height: 139cm Number of pets: 1	Jeff  Age: 48 Height: 153cm Number of pets: 0

- What is the mean age of these 10 people?
- Who is older than the mean age?
- What is the mean height of these 10 people?
- Who is shorter than the mean height?
- What is the mean number of pets of these 10 people?
- How many people have less pets than the mean amount?
- What is the mean age of the 5 males?
- What is the mean age of the 5 females?
- On average, who is older: the males or the females?
- On average, who is taller: the males or the females?

**Exercise 8**

Dafydd asked his friends how much money they had in their pockets. He wrote down their responses:

99p, 50p, 25p, 12p, 52p, 32p, 90p, 80p.

What is the **mean** of this data?

**Exercise 9**

Here are the times, in seconds, that the females in 7C took to run a 100m race.

14.1, 15.3, 15.5, 15.6, 15.0, 14.7, 13.9, 16.1, 15.8, 14.5, 15.7, 14.6, 16.3, 15.7.

- Find the **mean time taken** to complete the race.
- How many females were **faster** than the mean time?

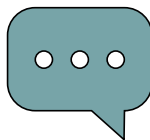
Calculating the mean from a table

Sometimes, when we have a lot of data, it is easier (and quicker) to use a table to present the data.

Example

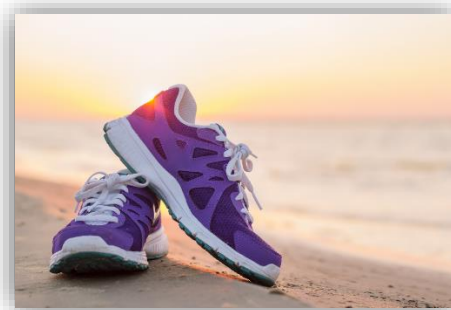
The following table shows the shoe size of the learners in 7R.

Shoe size	Number of children
3	4
4	5
5	10
6	9
7	2



In order to calculate the mean shoe size for 7R, we first need to find the **total of all the data values**. To do this, we must consider each row in the table in turn. The first row tells us that four pupils in 7R have a shoe size of 3. The total of the shoe sizes of these four pupils would be $3 + 3 + 3 + 3$, or 3×4 . The other rows of the table would also lead to multiplication sums, as shown below.

Shoe size	Number of children	Multiplication
3	4	$3 \times 4 = 12$
4	5	$4 \times 5 = 20$
5	10	$5 \times 10 = 50$
6	9	$6 \times 9 = 54$
7	2	$7 \times 2 = 14$
Total	30	150



The total of all the data values would be the total of the final column, namely $12 + 20 + 50 + 54 + 14 = 150$.

To find the mean, we must divide this total by the **number of data items**. In this example, the number of data items is the number of pupils in 7R, which is the total of the second column of the table.

$$4 + 5 + 10 + 9 + 2 = 30.$$

It follows that the mean shoe size for 7R is $150 \div 30 = 5$.

Exercise 10

100 packets of Smarties are chosen randomly. The following table shows the number of Smarties in each tube.

Number of Smarties in a tube	Number of tubes
35	14
36	24
37	26
38	20
39	16



Find the mean number of Smarties in a tube.

Challenge!

Today, the first 30 customers visiting a shop spent a mean total of £4 each. After the next 50 customers had visited the shop, the mean had risen to £6. What was the mean total spent by these 50 customers?



Exercise 11

The following table shows the number of M&Ms in several big packets of M&Ms.

Number of M&Ms in a big packet	Number of big packets
96	2
97	13
98	8
99	17

- (a) Calculate the mean number of M&Ms in a big packet.
 (b) How many packets contained more than the mean value?
 (c) How many packets contained less than the mean value?

**Exercise 12**

A dice is thrown 100 times.

The scores are listed in the following table.

Score	Number of times the dice was thrown
1	14
2	17
3	16
4	18
5	17
6	18



Calculate the mean score.

Exercise 13

On a golf course, the hole shown on the right is called a Par 3. This means that the majority of the players are expected to complete the hole in three shots.

The table below shows the number of shots taken by some players in completing the hole.

Number of shots	Number of people
1	1
2	18
3	30
4	26
5	25

- (a) Calculate the mean number of shots needed to complete the hole.
 (b) In your opinion, should this hole be called a Par 3? Explain your answer.

**Challenge!**

The mean age of a pop group with 3 members is 21. What will be the mean age of the group if a new, 25-year-old member joins the group?



Comparing Two Distributions

Exercise 14

3

Applying

(a) Two mathematics tests were given to ten students. Here are their results:

Test 1:	42	55	47	61	59	71	49	53	65	61
Test 2:	54	49	63	63	58	48	68	61	48	58



Use the mean to compare the **performance** of the students in these tests.

(b) In 8 games, the netball team of Ysgol y Creuddyn scored the following number of goals:

34 41 33 48 42 33 36 38

In 6 games the netball team of Ysgol John Bright scored the following number of goals:

48 29 52 34 42 28

Use the mean to decide, in your opinion, which team is the **best team**.

(c) The following data shows the golf scores of two players.

Morris:	81	76	79	85	89	78	74	83	74	87
Parry:	79	88	87	76	91	73	74	76		

Use the mean to decide, in your opinion, who is the **best golfer**.

(d) The teachers in the physical education department measure, in seconds, the times taken by the football and rugby teams to complete a 100m race.

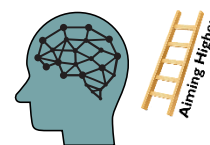
The football team: 13, 14, 15, 11, 14, 12, 12, 13, 11, 13, 14.

The rugby team: 12, 13, 14, 11, 12, 14, 15, 13, 15, 14, 11, 16, 14, 15, 13.

Use the mean to decide, in your opinion, which team is **quickest** over 100 metres.

Challenge! 

The mean of 20 numbers is 20. If the mean of the first 4 numbers is 4, what is the mean of the final 16 numbers?



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

The Range



Arwyn and Megan travel to school on different school buses. Over a period of a fortnight, Arwyn and Megan wrote down the times the buses arrived to take them to school.

Arwyn's Bus

08:13 08:16 08:14 08:15 08:14
08:15 08:14 08:17 08:14 08:15

Megan's Bus

08:29 08:27 08:33 08:28 08:28
08:33 08:26 08:31 08:34 08:30



Looking at the above times, can you tell which bus is **most reliable**?

Measures of Spread

In order to discuss which bus is most reliable, we need to measure the **spread** of the data. A data set has **low data spread** if the data values are all close to each other. We could depend on a bus with low data spread to reach the bus stop at the same time each day, or close to this time. A bus with **high data spread** wouldn't be as reliable.

The Range

The **range** is one measure of data spread. To calculate the range for a specific data set, find the **difference** between the greatest data item and the least data item. Usually, this means **taking away the least number from the greatest number**.

Arwyn and Megan

For Arwyn's bus, the latest time is 08:17, and the earliest time is 08:13. The difference between these two times is 4 minutes, so the range for Arwyn's bus is 4 minutes.

For Megan's bus, the latest time is 08:34, and the earliest time is 08:26. The difference between these two times is 8 minutes, so the range for Megan's bus is 8 minutes.

Because the range of Megan's bus is more than the range of Arwyn's bus, we can say that the **data spread** of Megan's bus is higher, and so Megan's bus is **less reliable** than Arwyn's bus.

Exercise 15

Find the range of the following data sets.

- (a) 4 8 9 10 11 15 16
- (b) 1 2 4 8 16 17 19 22
- (c) 10,800 15,000 15,500 18,300 21,300
- (d) 2.8 3.1 4.9 8.6 14.3 21.6
- (e) 7.2 1.3 8.4 2.1 3.6 4.7 9.3 8.4
- (f) 5 4 0 2 4
- (g) 150 150 141 137 146
- (h) 70 69 69 51 80

3

Skill



Exercise 16

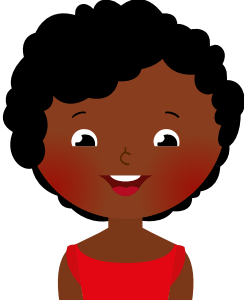
- (a) The range of a data set is 16. If the largest data value is 26, what is the **smallest** data value?
- (b) Peter collected data from 10 of his friends concerning the weekly pocket money they received. He forgot to write down the final value in his list.

£1 £1.20 £1.15 £1.25 £1 £1.40 £1.90 70c 90c

He remembered that the range of values was £1.40. What could be the **final** data value?

**Exercise 17**

Here is some information about five children.

Huw	Erin	William	Lili	Iestyn
				
Height: 150cm Weight: 52kg Shoe size: 4	Height: 158cm Weight: 52kg Shoe size: 6	Height: 150cm Weight: 50kg Shoe size: 5	Height: 149cm Weight: 46kg Shoe size: 4	Height: 146cm Weight: 45kg Shoe size: 3

- (a) Who is the tallest pupil?
- (b) Who is the shortest pupil?
- (c) What is the range of the heights of the pupils?
- (d) What is the range of the weights of the pupils?
- (e) What is the range of the shoe sizes of the pupils?

Exercise 18

Alys and Hywel travel to school on different school buses. Over a period of a fortnight, Alys and Hywel wrote down the times the buses arrived to take them to school.

Alys' Bus

08:25 08:27 08:24 08:29 08:31
08:30 08:25 08:32 08:26 08:26

Hywel's Bus

08:09 08:14 08:11 08:12 08:10
08:10 08:09 08:12 08:13 08:10

- (a) What is the range of Alys' times?
- (b) What is the range of Hywel's times?
- (c) In your opinion, which bus is most reliable? Explain your answer.



Exercise 19

Two groups of 6 people took part in a quiz.

(a) The 6 members of group A had the following scores.

52 29 78 56 24 37

What was the range of group A's scores?

(b) The scores for the 6 members of group B are shown in the following table.

Score	Number of people
22	2
25	2
26	1
28	1



Which group had the biggest range for their scores? You must give a reason for your answer.

Exercise 20 (Revision)

3

Here are the ages (in years) of 8 members of an evening class:

36 28 45 24 31 34 27 47

- Find the range of the ages of the members of the evening class.
- What was the range of the ages exactly one year ago? Give a reason for your answer.
- Find the mean age of the members of the evening class.
- What was the mean age of the ages exactly one year ago? Give a reason for your answer.
- A new, 40-year-old member joins the evening class. Without doing any further calculations, state
 - Will the range of the evening class increase, decrease or stay the same?
 - Will the mean of the evening class increase, decrease or stay the same?



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Bar Charts

We use bar charts to display data. The type of bar chart depends on the type of data that has been collected.

Types of Data

It is possible to place data into two categories: **qualitative** and **quantitative**.



Qualitative Data	Quantitative Data
Deals with descriptions . Data that can be observed but not measured. <i>Examples:</i> Method of travelling to school; eye colour; favourite pet; mobile phone manufacturer.	Deals with numbers . Data that can be measured. <i>Examples:</i> The height of a person; temperature; number of brothers; time to complete a race.

Exercise 21

State whether the following data is **qualitative** or **quantitative**.

- (a) Favourite board game (e.g. Monopoly; Scrabble).
- (b) The weight of a cake.
- (c) The name of your dentist.
- (d) The number of learners in your registration class.
- (e) The daily rainfall in Llandudno.
- (f) The day of the week (e.g. Monday) that you were born on.



13

Skill

Quantitative Data

It is possible to place quantitative data into two categories: discrete and continuous.

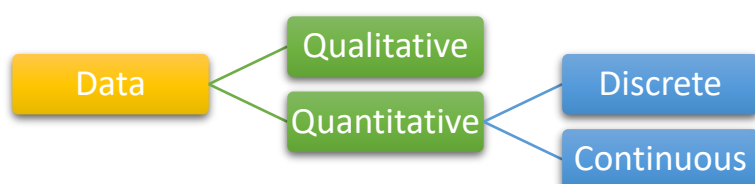
Discrete Data	Continuous Data
Only takes specific values . <i>Examples:</i> Shoe size; number of songs on an album; the salary of a nurse; number of sisters.	Takes any values within a specific range. <i>Examples:</i> Length of your foot; the temperature at midday; the time taken to cycle from Rhyl to Abergel.

Exercise 22

State whether the following quantitative data is **discrete** or **continuous**.

- (a) Population of the United Kingdom.
- (b) The length of your thumb.
- (c) The age of women when they marry.
- (d) The amount of money earnt for babysitting one night.
- (e) The number of cars on sale at a showroom.
- (f) The weight of the letters in a postman's sack.

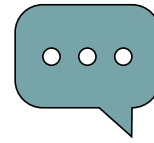
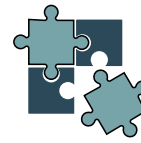
Summary



Drawing a Bar Chart

We can draw a **bar chart** for the following types of data.

- Qualitative data.
- Discrete quantitative data that is not grouped.



We need to include a **gap** between each bar in a bar chart.

Example

Dylan asked each member of 7Dd how they got to school one morning. Here are the results:

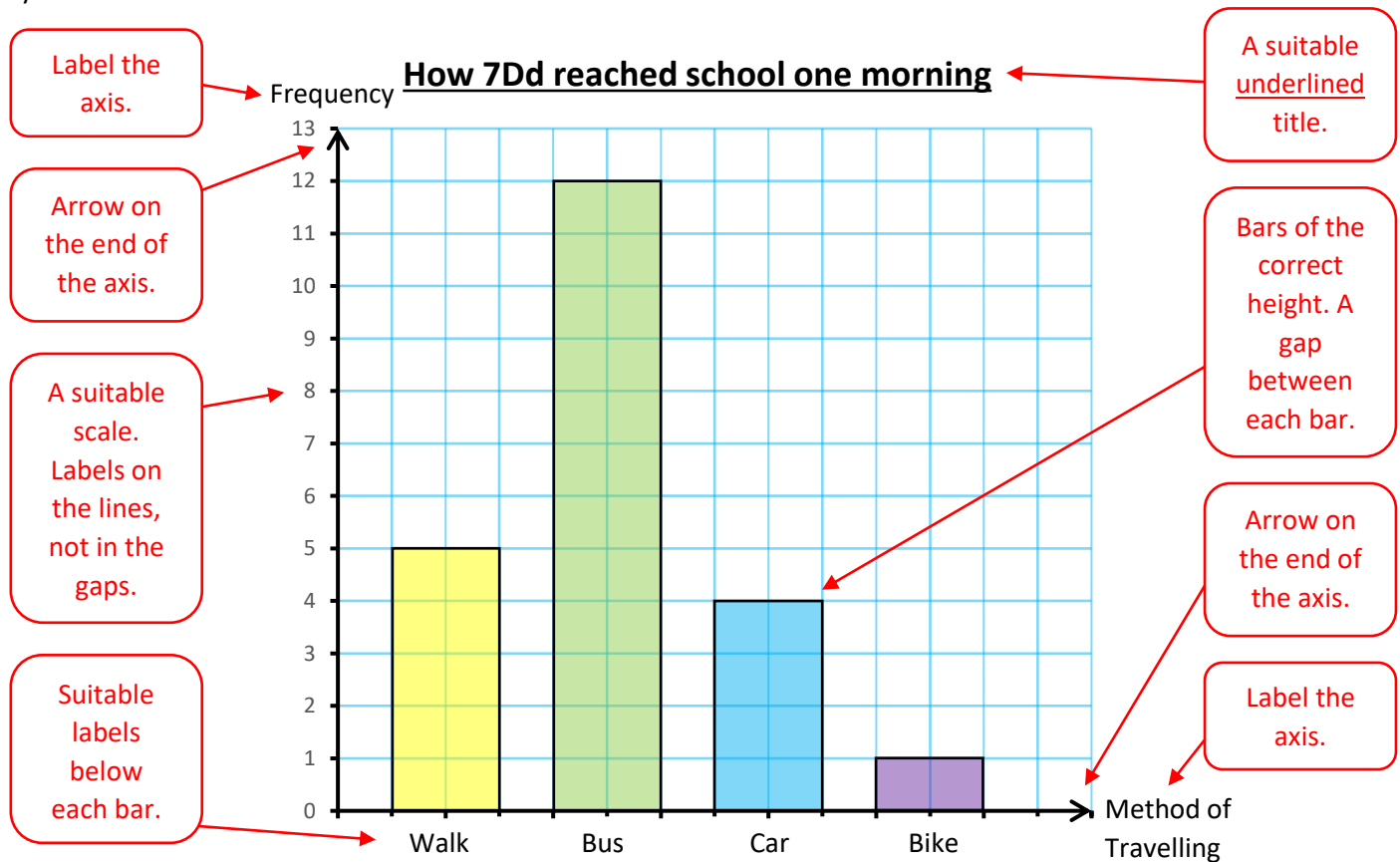
Walk 5

Bus 12

Car 4

Bike 1

Dylan drew a bar chart for the data.



Exercise 23

On a piece of squared paper, draw a **bar chart** for the following sets of data.



Skill

13

(a) Number of hours of sunshine in Colwyn Bay during one week.

Monday: 3 hours Tuesday: 4 hours Wednesday: 4 hours Thursday: 3 hours
Friday: 8 hours Saturday: 7 hours Sunday: 6 hours

(b) 7C's favourite pet.

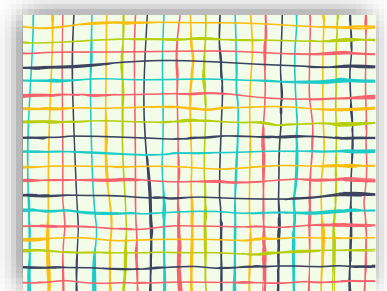
Dog: 8 Cat: 7 Rabbit: 4 Guinea Pig: 2 Snake: 1

(c) 7R's eye colour.

Blue: 6 Brown: 15 Green: 2

(d) The number of pets of each member of 7E.

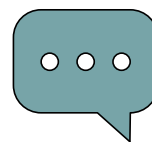
No pets: 5 One pet: 10 Two pets: 7 Three pets: 2



Drawing a Frequency Diagram

We can draw a frequency diagram for the following types of data.

- Discrete quantitative data that has been grouped.
- Continuous quantitative data.



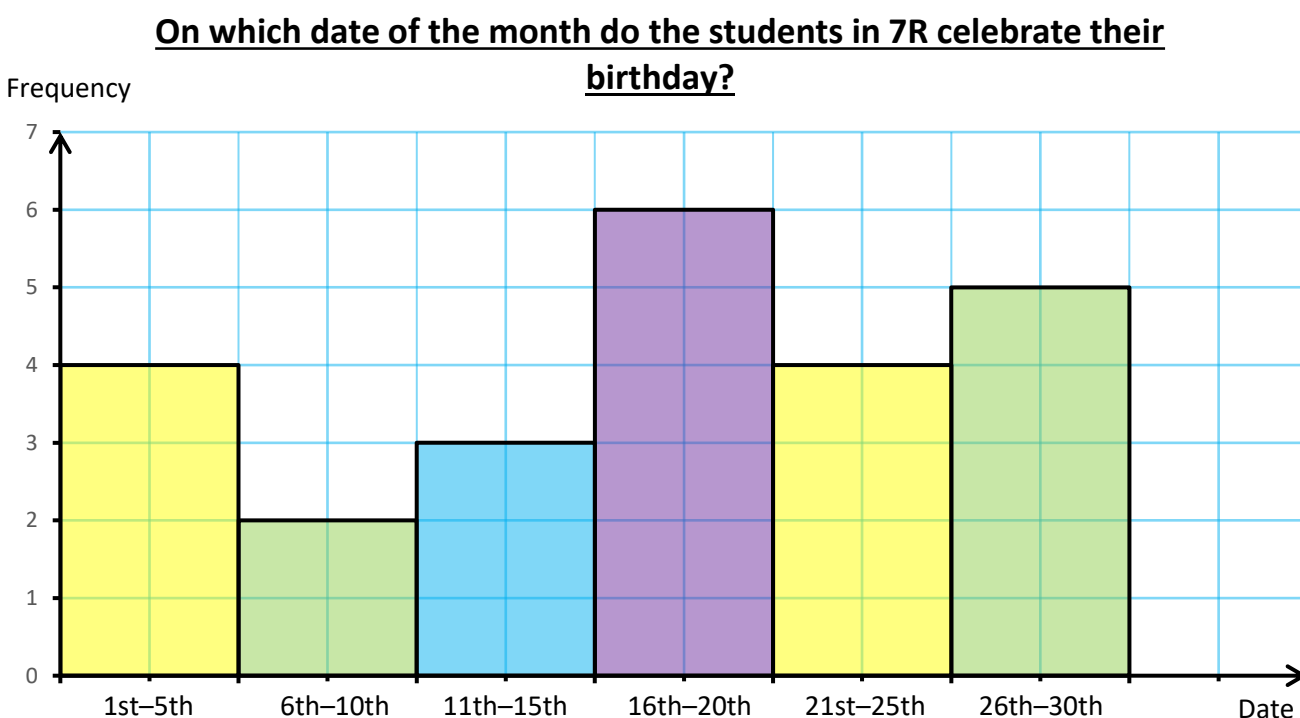
There are **no gaps** between the bars in a frequency diagram, but the **width** of each bar must be **equal**.

Example

Siwan asked each member of 7R which date of the month their birthday fell on. She received the following responses.

1st–5th: 4 6th–10th: 2 11th–15th: 3 16th–20th: 6 21st–25th: 4 26th–30th: 5

Siwan drew the following frequency diagram to show the data.



Exercise 24

On a piece of squared paper, draw a **frequency diagram** for the following sets of data.

(a) The number of books the pupils in 7E have bought during the past year.

Zero to four books: 12 Five to nine books: 5 Ten to fourteen books: 6 Fifteen to nineteen books: 1

(b) The number of minutes the dentist spends with each patient.

1–5 minutes: 2 6–10 minutes: 4 11–15 minutes: 9 16–20 minutes: 5
 21–25 minutes: 3 26–30 minutes: 3 31–35 minutes: 0 36–40 minutes: 1

(c) The number of days the pupils in 7C were absent from school last term.

0–4 days: 11 5–9 days: 8 10–14 days: 6 15–19 days: 0 20–24 days: 5

(d) Number of words in each sentence during the first 50 sentences of a book.

1–10 words: 2 11–20 words: 9 21–30 words: 14 31–40 words: 7
 41–50 words: 4 51–60 words: 8 61–70 words: 6



Grouping Discrete Data

In Exercise 24, we saw that we must use grouped data in order to draw a frequency diagram. We must use a **frequency table** to group data that is not yet grouped.



Example

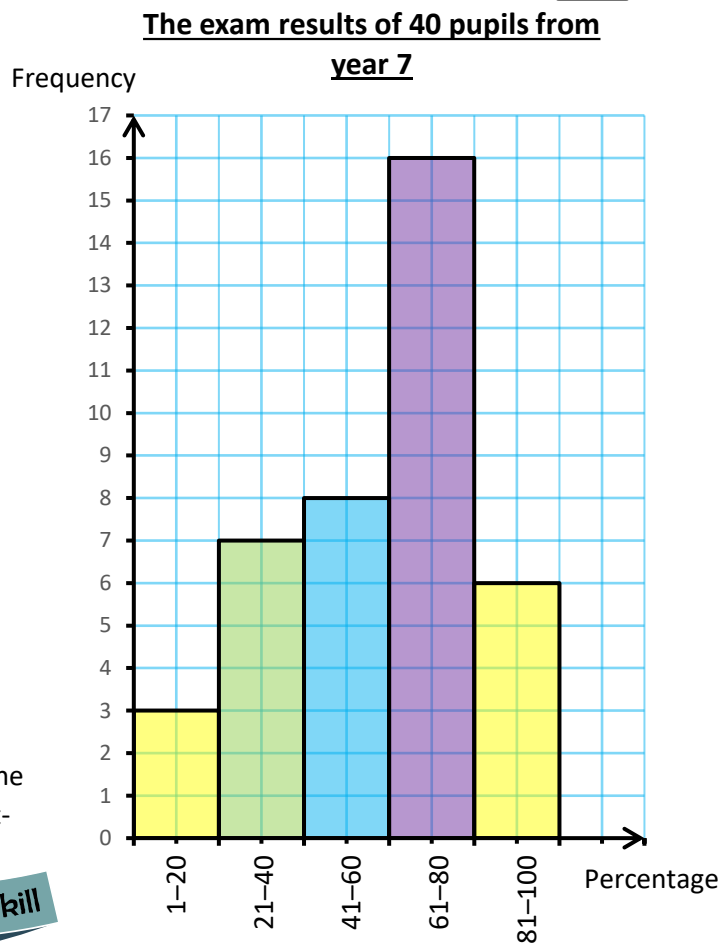
Here are the exam percentages of 40 pupils in year 7.

56	14	61	67	90	38	47	67
72	75	80	41	21	54	92	44
68	85	43	71	69	19	39	64
83	80	65	42	20	35	27	81
70	75	39	36	71	88	50	79

Let us group this data using the groups
1–20, 21–40, 41–60, 61–80, 81–100.

Group	Tally Marks	Frequency
1–20		3
21–40		7
41–60		8
61–80		16
81–100		6
Total		40

After forming the above frequency table, we can now draw the frequency diagram for the data. This can be seen on the right-hand side of this page.



Exercise 25

3

Skill

(a) Here are the times taken by the pupils in 7E to come to school today, correct to the nearest minute.

15	5	8	22	12	10	30	25	27	22	35	8	18	17	22
15	44	35	45	55	12	28	35	48	48	30	40	19	25	50

Draw a **frequency table** for the data, using the groups 1–10, 11–20, 21–30 up to 51–60.

Draw a **frequency diagram** to show the data.

(b) A class measured the arm span of each pupil. Here are the results, to the nearest cm.

132	129	147	127	128	124	134	139	138	136	144
143	133	141	139	130	142	143	140	137	148	136
146	151	123	131	135	135	130	140	138	133	



Draw a **frequency table** for the data, using the groups 120–124, 125–129, 130–134 up to 150–154.

Draw a **frequency diagram** to show the data.

(c) 7C decided to see how good they were at estimating the length of a minute. Here are the results, correct to the nearest second.

72	70	52	48	52	48	63	71	63	70	61	62	59	62	50
64	46	65	69	65	58	64	61	48	63	68	67	73	55	51

Draw a **frequency table** for the data, using the groups 46–50, 51–55, 56–60 up to 71–75.

Draw a **frequency diagram** to show the data.

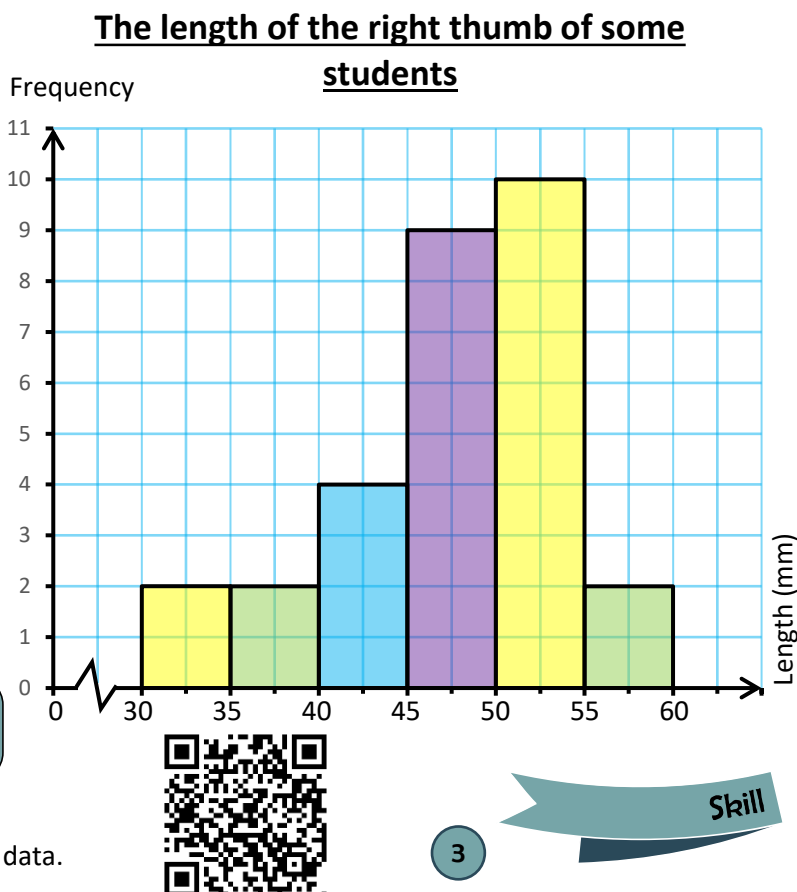
Grouped Continuous Data

The following frequency table shows the length of the right thumb of the students of a class.

Class interval (l mm)	Frequency
$30 \leq l < 35$	2
$35 \leq l < 40$	2
$40 \leq l < 45$	4
$45 \leq l < 50$	9
$50 \leq l < 55$	10
$55 \leq l < 60$	2

We read $30 \leq l < 35$ as " l is at least 30 but is less than 35" or " l lies between 30 and 35, but can be equal to 30".

The above information is shown in the frequency diagram on the right. Notice the jump in the graph between 0 and 30.

**Exercise 26**

Draw **frequency diagrams** for the following sets of data.

- (a) How much weight the members of a weight loss club lost in 6 months. (b) The height of 60 pupils. (c) The noise of 60 items of electrical equipment.

Weight (w kg)	Frequency
$0 \leq w < 6$	4
$6 \leq w < 12$	11
$12 \leq w < 18$	12
$18 \leq w < 24$	7
$24 \leq w < 30$	3

Height (h cm)	Frequency
$168 \leq h < 172$	2
$172 \leq h < 176$	6
$176 \leq h < 180$	17
$180 \leq h < 184$	22
$184 \leq h < 188$	10
$188 \leq h < 192$	3

Noise (n db)	Frequency
$15 \leq n < 20$	4
$20 \leq n < 25$	12
$25 \leq n < 30$	15
$30 \leq n < 35$	6
$35 \leq n < 40$	8
$40 \leq n < 45$	3
$45 \leq n < 50$	12

Exercise 27

The following figures give the time, correct to the nearest minute, taken by 50 cross-country runners to complete a race.

30	37	43	55	52	47	49	36	44	40
41	49	52	53	39	41	46	42	50	49
39	53	54	57	43	59	34	38	40	42
48	53	50	52	37	36	45	53	48	42
52	39	41	46	50	52	38	58	57	46

Time (t minutes)	Tally Marks	Frequency
$30 \leq t < 35$		
$35 \leq t < 40$		
$40 \leq t < 45$		
$45 \leq t < 50$		
$50 \leq t < 55$		
$55 \leq t < 60$		



- (a) Complete the frequency table for the data. (Remember that 35 minutes would appear in the class $35 \leq t < 40$, not $30 \leq t < 35$.)
- (b) Draw a **frequency diagram** for the data.

Exercise 28

The following data gives the weights of books on a shelf in a library, in grams.

742 241 332 478 1245 832 746 291 160 635 1042 368 760 590
 748 800 564 1340 824 683 174 960 236 746 328 387 823 894

(a) Complete a **frequency table** for the data. (Use the following classes for the weight, w :
 $0 \leq w < 200$, $200 \leq w < 400$, $400 \leq w < 600$, ..., $1200 \leq w < 1400$.)

(b) Draw a **frequency diagram** for the data.

Exercise 29

The following data shows the hand width of several 13-year-old girls, in mm.

171 182 202 164 198 211 164 173 177 173 192 205 232 223
 189 186 191 194 197 205 231 207 169 173 184 209 176 181

Draw a **frequency diagram** for the data. (You will need to complete a suitable **frequency table** first.)

Challenge!

What is wrong with the following frequency table, which shows the length of leaves that fell from a tree one morning? What would you have to do to create a frequency diagram for the data?

Length (l cm)	Tally Marks	Frequency
$0 \leq l < 5$		7
$5 \leq l < 10$		11
$10 \leq l < 20$		9
$20 \leq l < 25$		3
$25 \leq l < 30$		8

**Interpreting Charts****Exercise 30**

(a) How many pupils chose **red**?

(b) How many pupils chose **yellow**?

(c) Which colour was the **most popular**?

(d) How many pupils are in class 8E?

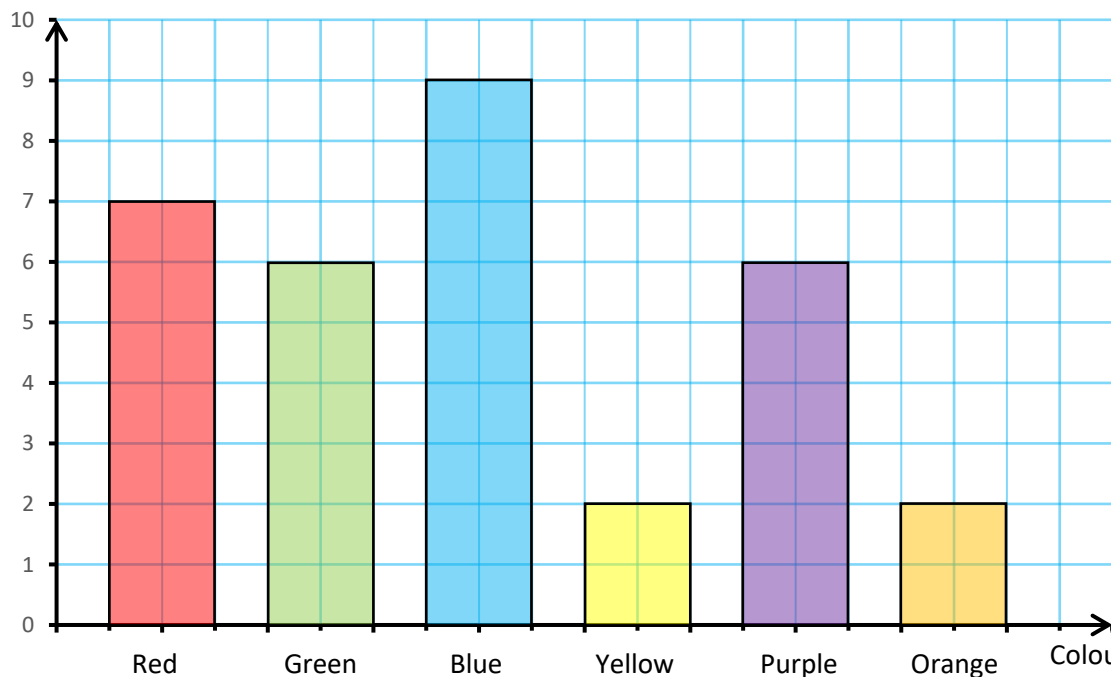
(e) How many pupils in 8E did **not** like purple?

Frequency

8E's favourite colour

13

Applying



Exercise 31

3

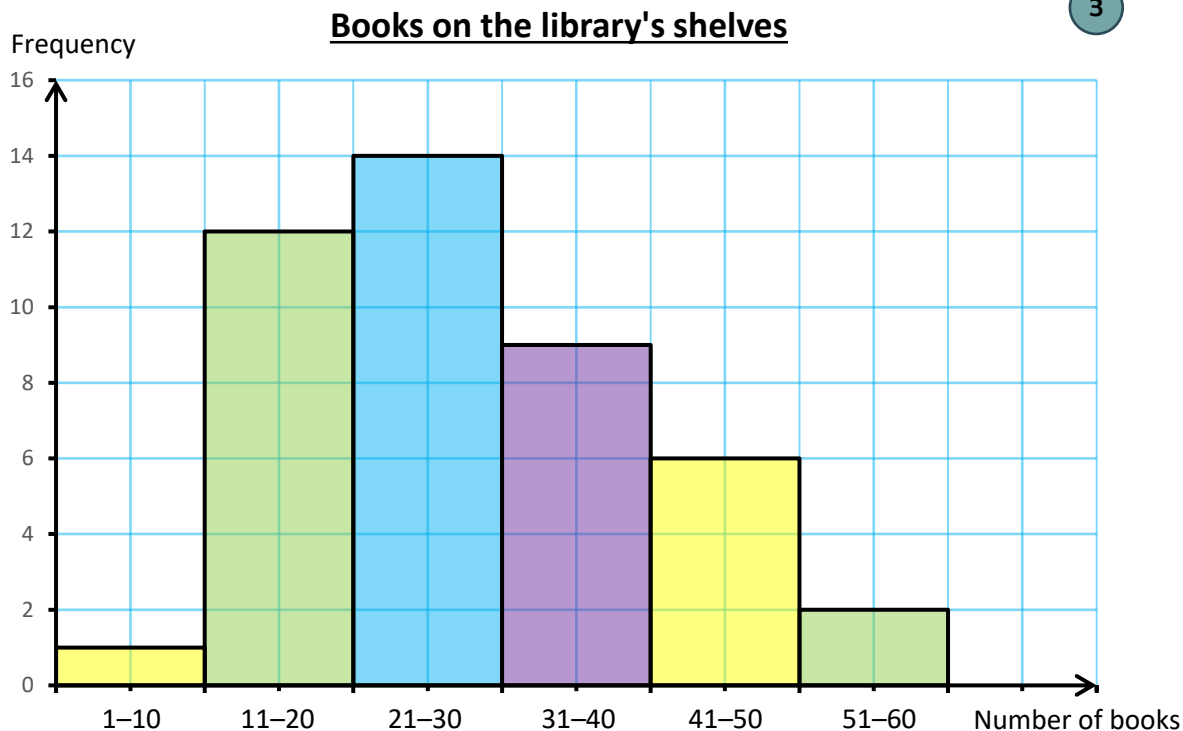
(a) How many shelves contain between **11 and 20** books?

(b) How many shelves contain between **31 and 40** books?

(c) How many shelves contain **more than 30** books?

(d) **How many shelves** are there in total in the library?

(e) Based on this information, can you state how many shelves in the library contain **exactly 20 books**? Explain your answer.

**Exercise 32**

13

(a) How many pupils got **7 marks**?

(b) How many pupils got **less than 5 marks**?

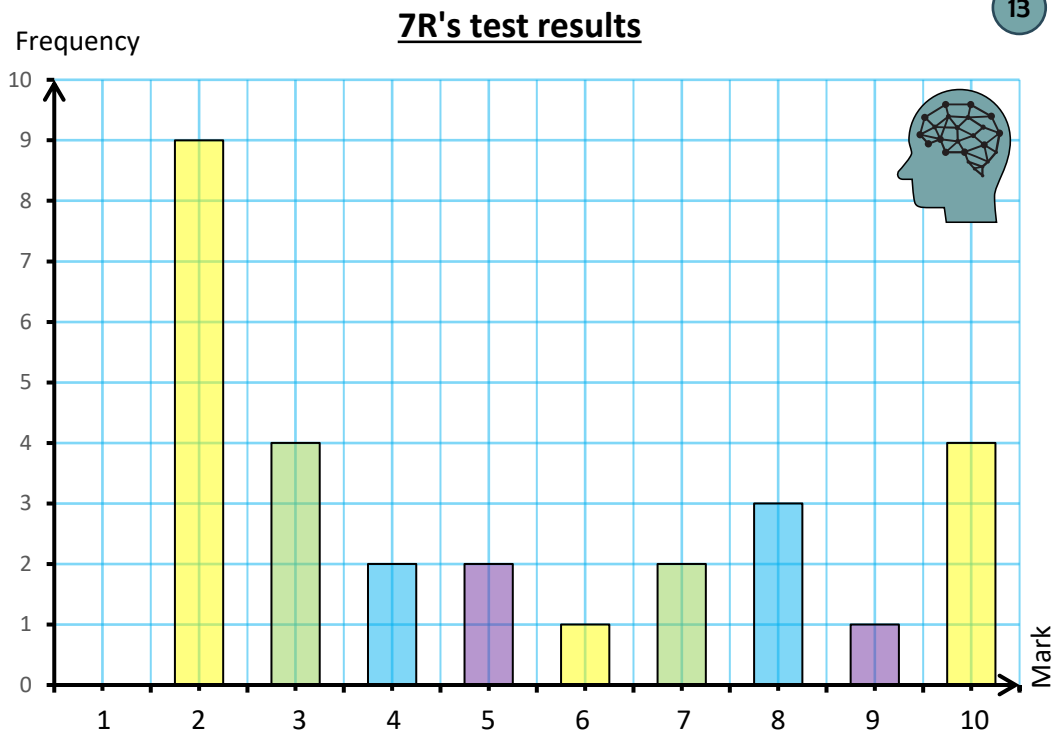
(c) How many pupils got **more than 6 marks**?

(d) **How many pupils** are there in the class?

(e) What was the **total** of the class' marks?

(f) What was the **mean** of the class' marks?

(g) What was the mark **3 of the pupils** got?

**Exercise 33**

3

(a) What **type of data** is shown in Exercises 30, 31 and 32?

(b) Were the graphs in Exercises 30–32 **bar charts** or **frequency diagrams**?

Exercise 34

3

The length of the phone calls in an office one afternoon

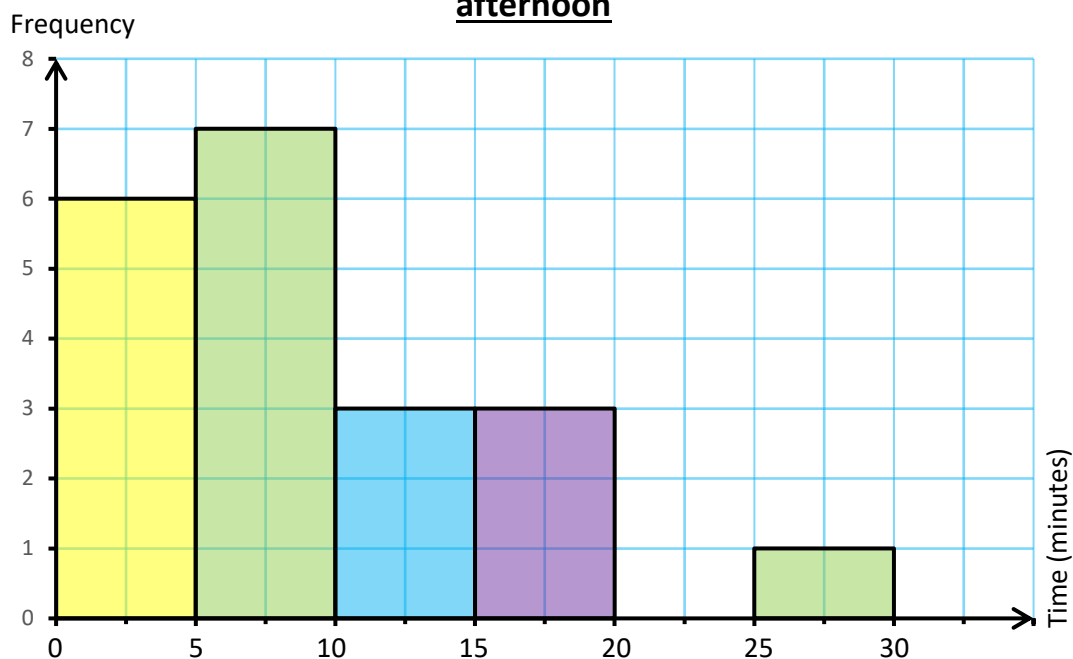
(a) How many phone calls lasted between **10 and 15** minutes?

(b) How many phone calls lasted **at least 15 minutes**?

(c) How many phone calls were there **in total** during the afternoon?

(d) What **type** of data is shown here?

(e) Alwen phoned the office during the afternoon. The length of her phone call was 590 seconds. Which bar in the frequency diagram shows this phone call?

**Exercise 35 (Revision)**

The following list shows the **number of complete games** a squad of football players played during one season.

16 10 9 3 21 30 4 12 0 36 20 14 12 5 16
 0 5 6 14 23 0 0 18 1 38 15 18 3 12 16

(a) What **type** of data is this?

(b) Draw a **frequency table** for the data, using the groups 0–4, 5–9, 10–14 up to 35–39.

(c) Draw a **frequency diagram** for the data.

(d) How many players played more than 25 complete games during the season?

(e) How many players played less than 10 complete games during the season?

Did you know?

The first person to draw a bar chart was William Playfair, in 1786, to show Scottish exports.

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

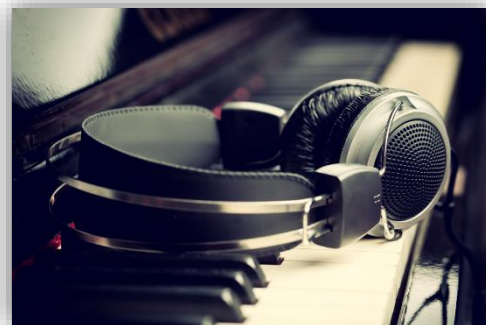
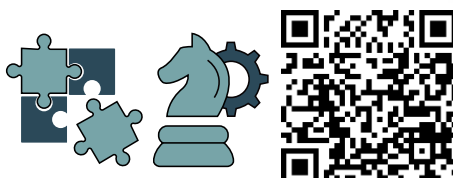
Mean of Grouped Data

In the previous chapter, we used a **frequency table** to group data into equal classes. Given a table of this sort, is it possible to calculate the **mean** of the original data?

Example

The following table shows the number of music albums the pupils of 10R have bought during November.

Number of albums	Frequency
0–4	11
5–9	8
10–14	5
15–19	1



What is the **mean number** of music albums the pupils of 10R have bought during November? Without access to the original data (*before it is grouped*), it is **impossible to state** what is the exact mean, because the frequency table does not give us complete data. For example, the table notes that one person has bought between 15 and 19 albums, but how many exactly? It could be 15, but it could also be 16, 17, 18 or 19. Without this information, we cannot calculate the mean exactly. We can, however, **estimate** the mean using the frequency table, by using the following method.

(1) Find the **mid-point** of each class by adding the two numbers that form the class and halving. For example, the numbers 5 and 9 form the class 5–9. By adding these numbers, we obtain 14. Half of 14 is 7, so the mid-point of the class 5–9 is 7.

Number of albums	Mid-point	Frequency
0–4	2	11
5–9	7	8
10–14	12	5
15–19	17	1

(2) Because the mid-point of each class gives a single value that **represents** the class, we can now use the method from page **Error! Bookmark not defined.** to estimate the mean from the table.

To begin, **multiply** each mid-point by the frequency of the class.

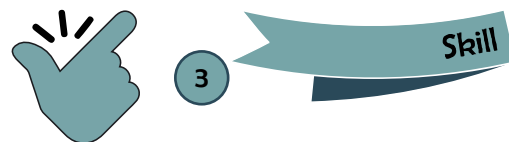
Number of albums	Mid-point	Frequency	Multiplication
0–4	2	11	$2 \times 11 = 22$
5–9	7	8	$7 \times 8 = 56$
10–14	12	5	$12 \times 5 = 60$
15–19	17	1	$17 \times 1 = 17$

(3) Find the **total** of the final two columns.

Number of albums	Mid-point	Frequency	Multiplication
0–4	2	11	$2 \times 11 = 22$
5–9	7	8	$7 \times 8 = 56$
10–14	12	5	$12 \times 5 = 60$
15–19	17	1	$17 \times 1 = 17$
Total		25	155

(4) Divide the total of the 'multiplication' column by the total of the 'frequency' column:
 $155 \div 25 = 6.2$. This is the estimate for the mean of this set of data.



Exercise 36Find an **estimate** for the mean of the following data sets.

(a)

Number of cars sold in a day	Number of days
0–4	18
5–9	11
10–14	6
15–19	4
20–24	1

(b)

Number of flowers per m ²	Number of square metres
0–14	43
15–29	21
30–44	13
45–59	2
60–74	1

(c)

Height of a horse chestnut tree, in metres	Frequency
0–3	13
4–7	17
8–11	9
12–15	4
16–19	5
20–23	0
24–27	2

(d)

Number of books sold in a week	Frequency
100–124	1
125–149	9
150–174	8
175–199	5
200–224	2

(e)

Number of customers in a café	Frequency
70–74	3
75–79	21
80–84	30
85–89	28
90–94	14
95–99	4

(f)

Number of sweets in a jar	Frequency
300–324	4
325–349	15
350–374	27
375–399	63
400–424	74
425–449	32
450–474	11
475–499	2



The Range

Let us consider again the data about music from page 21.

Number of albums	Mid-point	Frequency
0–4	2	11
5–9	7	8
10–14	12	5
15–19	17	1



Is it possible to find the **range** of the data set using the above table? As with the mean, it is not possible to calculate the range exactly, as the table does not tell us the maximum and minimum data values. The best we can do is to state that the least value is *between 0 and 4*; and that the greatest value is *between 15 and 19*. We must use the **mid-point** of the classes to find an **estimate** for the range, by taking away the lower mid-point from the higher mid-point. For this data, our estimate for the range is $17 - 2 = 15$.



3

Exercise 37

Find an **estimate for the range** of the data sets from Exercise 36.

Continuous Grouped Data

The methods of estimating the mean and range also work with continuous data that is grouped.

Example

A golfer hits 40 golf balls using one of his clubs. He records the distance that each ball has travelled. The following frequency table shows his results.

Distance travelled, d , in yards	Frequency
$75 \leq d < 80$	4
$80 \leq d < 85$	13
$85 \leq d < 90$	17
$90 \leq d < 95$	6

We can find an estimate for the **mean** of this data by finding the class mid-points; multiplying by the frequency; and finding the total of the final two columns.

Distance travelled, d , in yards	Mid-point	Frequency	Multiplication
$75 \leq d < 80$	77.5	4	$77.5 \times 4 = 310$
$80 \leq d < 85$	82.5	13	$82.5 \times 13 = 1072.5$
$85 \leq d < 90$	87.5	17	$87.5 \times 17 = 1487.5$
$90 \leq d < 95$	92.5	6	$92.5 \times 6 = 555$
Total		40	3425

The estimate for the **mean** is $3425 \div 40 = 85.625$ yards.

The estimate for the **range** is $92.5 - 77.5 = 15$ yards.



Exercise 38

3

Find an estimate for the **mean** and **range** of each of the following data sets.

(a)

The length of the cars, l , that are parked in a supermarket car park, in metres	Frequency
$3 \leq l < 4$	34
$4 \leq l < 5$	41
$5 \leq l < 6$	5

(b)

The time, t , taken to complete a race, in minutes	Frequency
$20 \leq t < 25$	5
$25 \leq t < 30$	11
$30 \leq t < 35$	19
$35 \leq t < 40$	12
$40 \leq t < 45$	3

(c)

The head circumference, c , of people in a factory	Frequency
$50 \leq c < 54$	10
$54 \leq c < 58$	31
$58 \leq c < 62$	16
$62 \leq c < 66$	3

(d)

The thickness, t , of books in a library, in mm	Frequency
$0 \leq t < 10$	5
$10 \leq t < 20$	16
$20 \leq t < 30$	41
$30 \leq t < 40$	32
$40 \leq t < 50$	6

(e)

The length of phone calls, l , in minutes	Frequency
$0 \leq l < 5$	32
$5 \leq l < 10$	42
$10 \leq l < 15$	24
$15 \leq l < 20$	12
$20 \leq l < 25$	7
$25 \leq l < 30$	2



Exercise 39

Over a period of 50 days, Bethan and Lowri recorded how many minutes per day they spent on their mobile phones. The data is given in the following frequency tables.

Bethan's time, t , in minutes	Frequency
$0 \leq t < 30$	14
$30 \leq t < 60$	17
$60 \leq t < 90$	10
$90 \leq t < 120$	9
$120 \leq t < 150$	0
$150 \leq t < 180$	0

Lowri's time, t , in minutes	Frequency
$0 \leq t < 30$	23
$30 \leq t < 60$	14
$60 \leq t < 90$	5
$90 \leq t < 120$	3
$120 \leq t < 150$	2
$150 \leq t < 180$	3

Applying

3

- (a) Calculate an estimate for **Bethan's mean time** and **Lowri's mean time**.
 (b) Calculate an estimate for **Bethan's range** and **Lowri's range**.
 (c) Who would you say uses their mobile phone **the most**? Explain your answer.
 (d) Who would you say is **most consistent** in using their mobile phone? Explain your answer.

Exercise 40 (Revision)

The height of 25 Year 7 learners was measured. The results are given below, correct to the nearest inch.

57	53	59	62	61
66	51	58	59	62
60	62	54	64	59
58	62	64	53	67
61	63	57	48	60

- (a) Calculate the **mean height** of the learners.
 (b) Calculate the **range** of the heights.
 (c) What **type** of data is this?
 (d) Complete the following **frequency table** in order to group the data.

Height, h , in inches	Tally Marks	Frequency
$45 \leq h < 50$		
$50 \leq h < 55$		
$55 \leq h < 60$		
$60 \leq h < 65$		
$65 \leq h < 70$		

- (e) Draw a **frequency diagram** for the data.
 (f) Use the frequency table to find an **estimate for the mean and range**.
 (g) How close were your estimates to the true values?

Challenge! 

Repeat the above exercise using the heights of the learners in your class.



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Pie Charts

It is possible to display data by dividing a circle into **sectors** of appropriate sizes. We form a **pie chart** by doing this.

Example

Here is the weather for the 30 days in June.

<i>Rainy days</i>	4
<i>Cloudy days</i>	8
<i>Sunny days</i>	18



In order to draw a pie chart for this data, we need to start by dividing 360° (which is the angle around a point in the centre of a circle) by 30 (which is the number of data items).

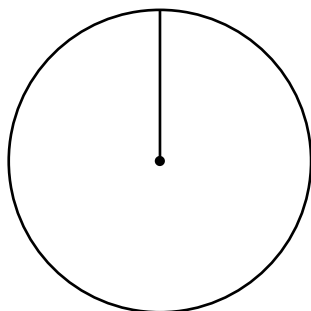
$$360^\circ \div 30 = 12^\circ$$

Next, we multiply the 12° (which is the angle for one day) by the number of days for a particular type of weather.

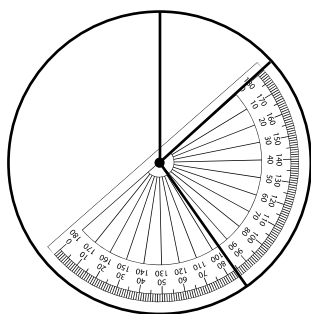
<i>Rainy days</i>	$4 \times 12^\circ = 48^\circ$
<i>Cloudy days</i>	$8 \times 12^\circ = 96^\circ$
<i>Sunny days</i>	$18 \times 12^\circ = 216^\circ$

To finish, we construct the pie chart itself.

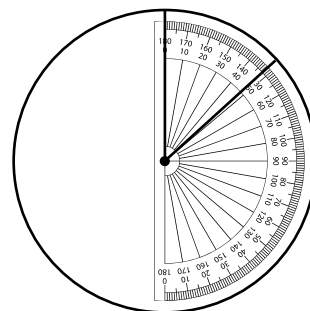
- (a) Draw a **circle** using a compass. Mark the centre of the circle with a **dot**. Draw a **vertical line** from the centre of the circle to the top of the circle.



- (c) Draw the next angle (96°) using a protractor.

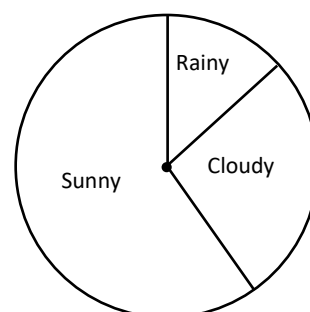


- (b) Measure the first angle (48°) using a protractor. Mark the angle using a pencil before drawing a straight line **from the centre of the circle to your mark**.



- (d) Label the sectors and add a suitable title to the pie chart.

June's weather



Exercise 41

Draw a **pie chart** for the following sets of data.

(a) 8C's favourite fruit.

Apple: 10 Banana: 8
Orange: 4 Strawberry: 2

(b) Year 9's holiday destinations, for last summer.

Europe: 40 Britain: 70
U.S.A.: 10

(c) Tomos' typical day (24 hours).

Sleeping: 8 hours Eating: 1 hour
Working: 8 hours Leisure: 6 hours
Travelling: 1 hour

(d) The foods that were eaten in a hotel.

Beef: 7 Lamb: 26
Chicken: 18 Vegetarian: 5
Fish: 14 Duck: 2

(e) Methods of heating a house.

Coal: 7 Electricity: 9
Gas: 11 Oil: 3



13

Skill



(f) How do the pupils of Ysgol y Creuddyn reach school?

Bus: 500 Car: 150
Bike: 50 Walk: 100

(g) The votes in a recent election.

Conservatives: 13,000 Labour: 9,000
Plaid Cymru: 7,000 UKIP: 4,000
Liberal Democrats: 3,000

(h) The colour of the cars in a car park.

Red: 8 Blue: 4
Green: 5 Yellow: 7

(i) 8E's favourite sport.

Football: 13 Rugby: 5
Golf: 2 Other: 3

(j) How many pets do the learners of 9E have?

None: 7 One: 12
Two: 4 Three: 3

**Exercise 42**

What **type** of data is given in the questions of Exercise 41?

13

Interpreting Pie Charts

Exercise 43

13

Applying

(a) The pie chart shows how 7C come to school.

(i) What is the **most popular method** of coming to school?

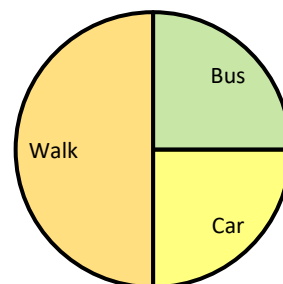
8 of the pupils come by car.

(ii) How many pupils come by **bus**?

(iii) How many pupils **walk** to school?



How do 7C come to school?



(b) Paul has drawn a pie chart. It shows when pupils like to complete their homework.

(i) Which time is the **most popular**?

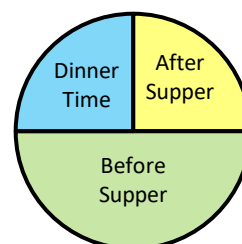
(ii) What **fraction** of the pupils like to complete their homework after supper?

There were 20 pupils in the survey.

(iii) How many pupils like to complete their homework **before** supper?

(iv) How many pupils chose **dinner time**?

Favourite time for doing homework



(c) Siân asked her class, 9C, what type of **bike** they had. There are 32 pupils in Siân's class.

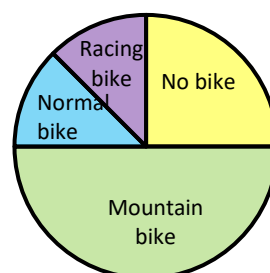
(i) What is the **most popular** bike?

(ii) How many pupils **do not** have a bike?

(iii) Complete the following table.

Bike	Mountain	Racing	Normal	No Bike
Number of pupils				

The type of bike that the pupils of Siân's class have



(d) This pie chart shows the **favourite colour** of 40 people.

(i) Which **fraction** liked **red**?

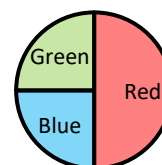
(ii) How many people liked **red**?

(iii) Which fraction liked **blue**?

(iv) How many people liked **blue**?

(v) How many people did **not** choose green?

40 people's favourite colour



(e) Siôn spends 4 hours on different activities at night.

How much time is spent on

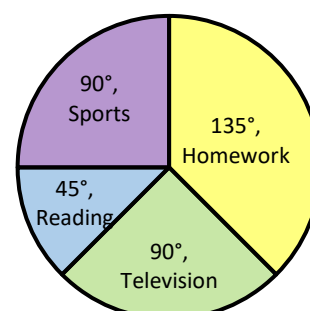
(i) **sports**;

(ii) watching **television**;

(iii) **reading**;

(iv) **homework**?

Siôn's Activities



Exercise 44

13

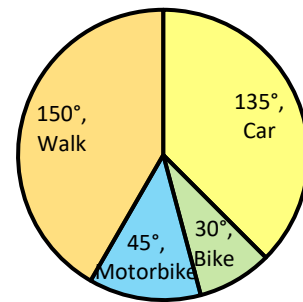
(a) This pie chart shows how 24 people travelled to town.

Find how many people

- (i) **walked** to town;
- (ii) came in a **car**;
- (iii) came on a **motorbike**;
- (iv) came on a **bike**.



How did 24 people travel to town?



(b) Llinos surveyed 210 pupils in years 7 and 8.

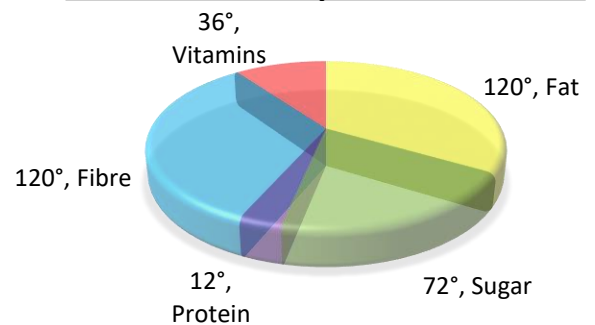
She asked the pupils which of the following, in their opinion, was most important in their diet.

Fat; Sugar; Protein; Fibre; Vitamins.

The pie chart shows her results.

How many pupils are in each category?

What is most important in a diet?

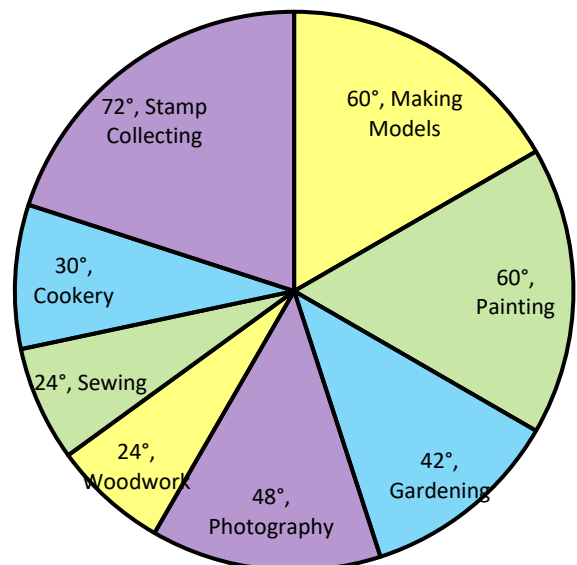


(c) This pie chart shows what several people do in their leisure time.

12 people collect stamps. How many people:

- (i) were surveyed **in total**?
- (ii) like **gardening**?
- (iii) enjoy **photography**?
- (iv) Which two hobbies had the same number of people partaking in them?

Leisure time activities

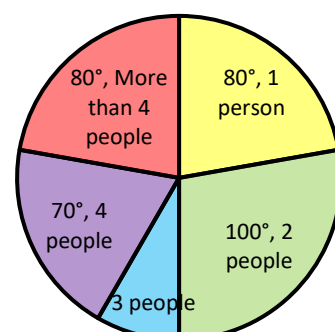


(d) This pie chart shows how many people were in a car when surveyed by Siwan.

Siwan looked at a total of 180 cars.

- (i) How many cars had **2 people** in them?
- (ii) How many cars had **3 people** in them?
- (iii) In how many cars were there **4 people**?

How many people were in the car?



(e) This pie chart shows the nutritional value of a packet of crisps weighing 36g.

- (i) How much of the crisps is made up of **carbohydrate**?
- (ii) How much of the crisps is made up of **fat**?
(Use your protractor to help.)
- (iii) How much of the crisps is **protein**?

Exercise 45 (Revision)

13

(a) Draw a pie chart for the following data showing the **manufacturer** of 20 cars belonging to teachers.

Ford 5	Vauxhall 4	Toyota 3
Audi 2	Volkswagen 5	Fiat 1

(b) Jessica drew the pie chart shown on the right which shows where a group of people had been on holiday last year.

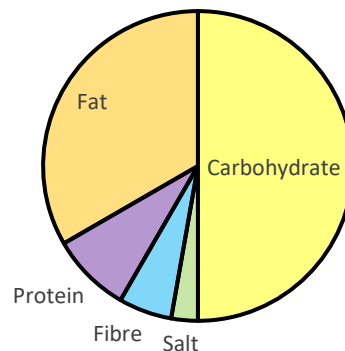
16 people had been in Scotland.

- (i) Daniel looked at the chart and said “*Most of the people had spent their holiday in England*”. Jessica said that this was not true. Explain why Daniel is **wrong**.
- (ii) Dafydd looked at the chart and said “*Because 16 people have been in Scotland, it must be that 1% represents 4 people*”. Is Dafydd **correct**?
- (iii) How many people had been to **Spain** on their holiday?

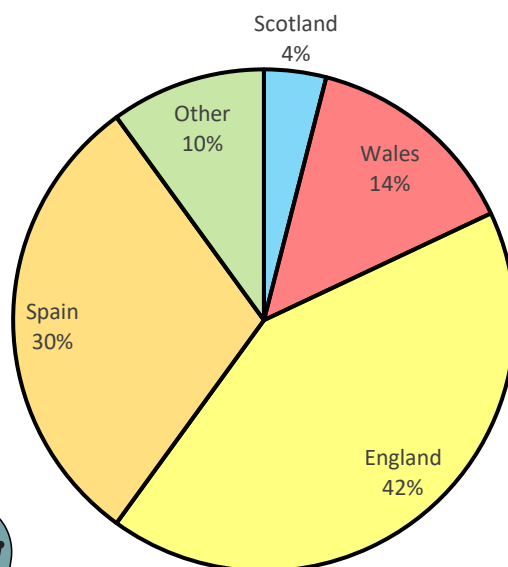
Dafydd added that Jessica had surveyed **400** people. Jessica gave two reasons why Dafydd was **wrong** in saying this. What could these reasons be?



Nutritional value of a packet of 36g crisps



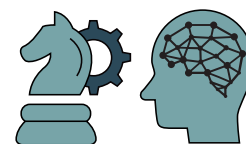
Jessica's Holiday Survey



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Investigation



In this chapter, you will complete a mini project on the theme 'keeping healthy'.

STEP 1: Use the box on the right to write down which data you intend to collect.

Here are some ideas:

- Data from a football game.
- Data about the food tins in your house.
- Data from an electronic device, e.g. number of steps taken each day, number of hours slept each night.

STEP 2: Use the box on the right to write down what you intend to do with your data. **Rule:** you must do at least 4 different things with your data.

Here are some ideas:

- Draw a bar chart to show how many shots each player in a football game has taken.
- Draw a pie chart to show the possession for each team in a football game.
- Draw a bar chart to show the different types of food tins in your house.
- Calculate the mean number of calories for each food tin.
- Calculate the range of steps taken for each day during the past week.

STEP 3: Write a report in your book on this project.

The report should contain:

- A short introduction about the project;
- The data you have collected;
- At least two different bar charts / frequency diagrams / pie charts;
- Calculations for at least two different means / ranges;
- A conclusion to explain what you have discovered about your data.



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...



Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
I know how to calculate the mean of a set of data.			3	
I know how to calculate the mean of a set of data given in a table .			5	
I know how to calculate the range of a set of data.			3	
I know how to compare two data sets .			3	
I know the difference between qualitative data and quantitative data, and the difference between discrete quantitative data and continuous quantitative data.			2	
I know how to draw a bar chart for qualitative data or for discrete quantitative data that is ungrouped.			1	
I know how to draw a frequency diagram for discrete quantitative data that is grouped or for continuous quantitative data.			7	
I know how to estimate the mean for grouped data.			9	
I know how to estimate the range for grouped data.			9	
I know how to draw a pie chart for qualitative data.			6	
I know how to interpret bar charts, frequency diagrams and pie charts.			4, 8	



The Mathematics Department

7

Introducing

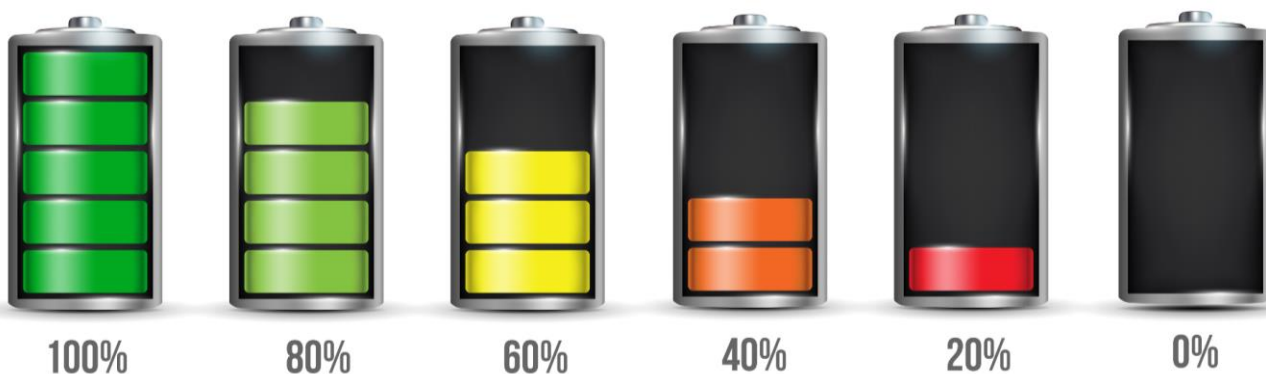
Percentages

Version
3.0
07/2025

Name:

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Chapter	Mathematics	Page Number
Preparation	Place value. Writing numbers in words and in figures. Multiplying and dividing by a power of 10. Multiplying a decimal by a whole number less than ten. Adding and subtracting decimals.	3
Calculating Percentages	Percentages of shapes. Simple percentages. Calculating a percentage without a calculator. Calculating a percentage with a calculator.	10
Using Percentages	Profit and loss. Simple interest. Salaries. Percentage in a test.	14



Preparation

Place Value

When we write any number, we use the **base 10 decimal system**. The place value of each digit is **ten times more** than the place value of the digit that is immediately to the right.



It is possible to write any number on a **place value chart**. Here is the chart for the number 7289134:

Millions			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
		7	2	8	9	1	3	4

We read the number 7289134 as “seven million, two hundred and eighty-nine thousand, one hundred and thirty-four. It is possible to use commas to make reading the number easier; the number 7289134 is written as 7,289,134. (We group the digits into groups of three.)

Here is the place value chart for the number 457.208:

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
4	5	7	.	2	0	8

We read the number 457.208 as “four hundred and fifty-seven point two zero eight”, or “four hundred and fifty-seven and two hundred and eight thousandths”.

Exercise 1

Write the following numbers on the place value chart shown below.



- (a) 47289 (b) 40.25 (c) 8344903 (d) 91837.234 (e) 0.208
 (f) 139847431986 (g) 30003.001 (h) 300 (i) 31298129381398 (j) 200904.435

	Trillions			Billions			Millions			Thousands						
	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point
(a)																
(b)																
(c)																
(d)																
(e)																
(f)																
(g)																
(h)																
(i)																
(j)																

Exercise 2

Write the numbers from Exercise 1 in words.

Challenge!

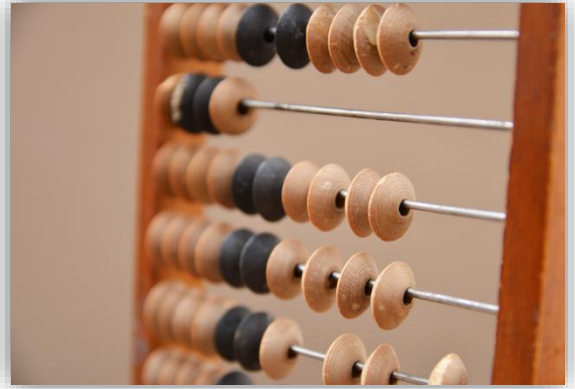
Use the internet to investigate names for numbers more than a trillion, or less than a thousandth.



Exercise 3

Write the following numbers in **words**.
(Hint: use commas to help you.)

A



- | | | |
|---------------|-------------|---------------|
| (a) 3008 | (b) 7521 | (c) 43978 |
| (d) 12045 | (e) 662907 | (f) 250018 |
| (g) 12650 | (h) 34800 | (i) 1590307 |
| (j) 25482134 | (k) 5062011 | (l) 923010520 |
| (m) 744300042 | (n) 280403 | (o) 2900302 |

Exercise 4

Write the following numbers in **figures**.

- | | |
|---|--|
| (a) ten thousand, nine hundred and sixty-four | (b) one hundred and forty-one |
| (c) fifty-two thousand and sixty-six | (d) four million |
| (e) twenty thousand and six hundred | (f) ninety thousand and forty-five |
| (g) nine hundred and eighty thousand, four hundred and five | (h) three billion and seventy-nine |
| (i) forty and four tenths | (j) nine hundredths |
| (k) eight thousand, four hundred and fifty-three | (l) nine thousand and three & fourteen thousandths |

Example

In the number 675, the 7 is in the tens column. The place value of the 7 is therefore seven **tens**. In the number 870031, the 7 is in the tens of thousands column. The place value of the 7 is therefore seven **tens of thousands**.

Exercise 5

What is the **place value** of the 8 in the following numbers?

- | | | | | |
|-------------|-------------|---------------|-------------|--------------|
| (a) 34859 | (b) 584392 | (c) 12346.248 | (d) 4348213 | (e) 38050712 |
| (f) 1800904 | (g) 7342814 | (h) 81234620 | (i) 5263087 | (j) 234.08 |

**Egyptian Numbers**

In the past, the following system was used in Egypt for writing numbers.

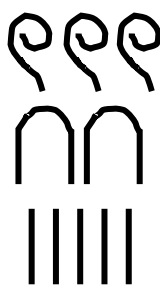
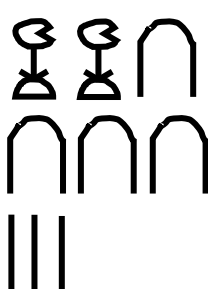
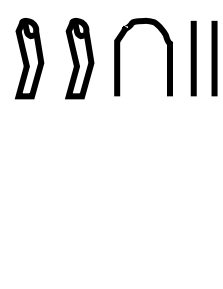
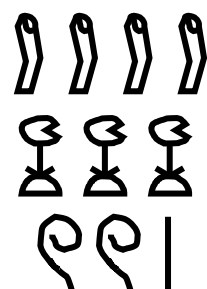
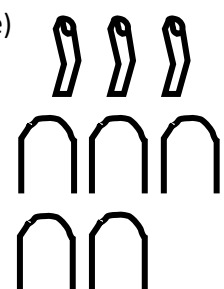


Number	1	10	100	1,000	10,000
Symbol		∩	🌀	👤	👤

**Exercise 6**

Write the following numbers in **words**.



- | | | | | |
|---|---|---|---|---|
| (a)  | (b)  | (c)  | (d)  | (e)  |
|---|---|---|---|---|

Multiplying and Dividing by a Power of 10

Look at the following place value chart for the numbers 5, 23.65, and 284,340,630.2.

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
														5				
													2	3	.	6	5	
						2	8	4	3	4	0	6	3	0	.	2		

By multiplying each of these numbers by 10, we obtain the numbers 50, 236.5, and 2,843,406,302.

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
													5	0				
												2	3	6	.	5		
					2	8	4	3	4	0	6	3	0	2				

Discuss the effect that multiplying by 10 has had on the place value chart. What would be the effect of multiplying by 100 or 1,000?

Exercise 7

- (a) 34×10 (b) 23.56×10 (c) 2837×10 (d) 2.3×10
 (e) 450×10 (f) 0.24×10 (g) 0.00323×10 (h) 7.8×10
 (i) 328×100 (j) 280×100 (k) 234.232×100 (l) 2.31×100
 (m) 0.24×100 (n) 0.0345×100 (o) 0.00302×100 (p) 9383.2134×100
 (q) 82×1000 (r) 928.26332×1000 (s) 2.3×1000 (t) 0.0983×1000
 (u) 0.0003729×1000 (v) 2.88×1000 (w) 0.0006929×1000 (x) 3.1×1000
 (y) 7293.1×10 (z) 0.0048×10 (α) 2.829×100 (β) 0.204×100



By dividing the numbers 5, 23.65, and 284,340,630.2 by 10, we obtain the numbers 0.5, 2.365 and 28,434,063.02.

Trillions			Billions			Millions			Thousands										
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	
														0	.	5			
														2	.	3	6	5	
							2	8	4	3	4	0	6	3	.	0	2		

Discuss the effect that dividing by 10 has had on the place value chart. What would be the effect of dividing by 100 or 1,000?

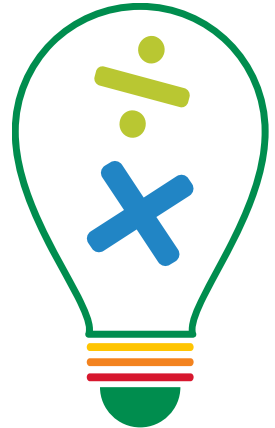
Exercise 8

- | | | | |
|---------------------------|----------------------------|---------------------------|----------------------------|
| (a) $520 \div 10$ | (b) $45 \div 10$ | (c) $2 \div 10$ | (d) $2.3 \div 10$ |
| (e) $4534 \div 10$ | (f) $0.83 \div 10$ | (g) $0.00267 \div 10$ | (h) $124654 \div 10$ |
| (i) $745 \div 100$ | (j) $48000 \div 100$ | (k) $32 \div 100$ | (l) $0.3 \div 100$ |
| (m) $0.86 \div 100$ | (n) $0.0325 \div 100$ | (o) $0.0005 \div 100$ | (p) $346356.4554 \div 100$ |
| (q) $460000 \div 1000$ | (r) $5657.43232 \div 1000$ | (s) $2.3 \div 1000$ | (t) $6 \div 1000$ |
| (u) $0.0003475 \div 1000$ | (v) $5.82 \div 1000$ | (w) $0.0008821 \div 1000$ | (x) $8.5 \div 1000$ |
| (y) $345 \div 10$ | (z) $0.0024 \div 10$ | (α) $7.433 \div 100$ | (β) $0.8 \div 100$ |



Exercise 9

- | | | | |
|--------------------------|------------------------|---------------------------|-----------------------|
| (a) 436×10 | (b) $256 \div 100$ | (c) 2.43×1000 | (d) $0.002 \div 10$ |
| (e) 0.0026×100 | (f) $1829 \div 1000$ | (g) 928×100 | (h) $0.239 \div 100$ |
| (i) 0.23×10 | (j) $9 \div 100$ | (k) 0.00238×1000 | (l) $23 \div 10$ |
| (m) 0.000202×10 | (n) $2839 \div 10$ | (o) 2.04×10 | (p) $29 \div 1000$ |
| (q) 900×100 | (r) $1.92 \div 100$ | (s) 8×1000 | (t) $0.0342 \div 100$ |
| (u) 0.08×100 | (v) 78×100 | (w) $0.023 \div 1000$ | (x) $7 \div 100$ |
| (y) 13.45×10 | (z) $19293.2 \div 100$ | (α) 2×10 | (β) $0.004 \div 10$ |



Multiplying a Decimal by a Whole Number less than 10



To calculate a sum like 75.82×6 , follow the steps shown below.

1. Set out the sum so that the 6 is below the 2.

2. Working from right to left, start by working out $2 \times 6 = 12$.

3. Next, we calculate $8 \times 6 = 48$, and add the 1 to obtain 49.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 2 \\ 1 \end{array}$$

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 92 \\ 4 \quad 1 \end{array}$$

4. We must ensure that the decimal points are lined up.

5. The next sum is $5 \times 6 = 30$, and add 4 to obtain 34.

6. To finish we do $7 \times 6 = 42$, and add the 3 to obtain 45.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline .92 \\ 4 \quad 1 \end{array}$$

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 4.92 \\ 3 \quad 4 \quad 1 \end{array}$$

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 454.92 \\ 3 \quad 4 \quad 1 \end{array}$$

Puzzle

Awel, Bob and Catrin are counting their money. Awel and Bob have a total of £4.60. Bob and Catrin have a total of £4.97. Awel and Catrin have a total of £6.63. How much money do they have, all together?



Exercise 10

- | | | | |
|------------------------|-------------------------|------------------------|------------------------|
| (a) 43.61×3 | (b) 2.53×4 | (c) 9.46×5 | (d) 2.3×7 |
| (e) 93.68×9 | (f) 102.4×6 | (g) 29.437×2 | (h) 20.31×5 |
| (i) 9.3×8 | (j) 128.432×7 | (k) 26.43×8 | (l) 65.43×6 |
| (m) 23.392×7 | (n) 245.3×4 | (o) 925.32×6 | (p) 902.1×8 |
| (q) 7256.42×3 | (r) 294.04×5 | (s) 92.239×7 | (t) 1002.3×6 |
| (u) 1039.24×8 | (v) 293.9×4 | (w) 23.4×9 | (x) 2849.23×5 |
| (y) 3289.2×6 | (z) 3409.223×4 | (α) 239.44×10 | (β) 2839×6 |



Adding Decimals

To calculate a sum like $24.52 + 387.963$, follow the steps shown below.

1. Set out the sum so that the decimal points line up.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline \end{array}$$

2. Working from right to left, we start by calculating $0 + 3 = 3$.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 3 \end{array}$$

3. Next, we do $2 + 6 = 8$ and then $5 + 9 = 14$.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 483 \end{array}$$

4. Remember to insert the decimal point in the correct position.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline .483 \\ 1 \end{array}$$

5. We now continue with the addition sums: $4 + 7 = 11$, and add one to obtain 12.

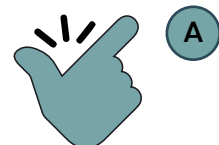
$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 2.483 \\ 1 \quad 1 \end{array}$$

6. We complete the final two additions to arrive at the final solution.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 412.483 \\ 1 \quad 1 \quad 1 \end{array}$$

Exercise 11

- | | | | |
|----------------------|----------------------|-------------------------|---------------------------|
| (a) $24.67 + 83.49$ | (b) $8.28 + 3.66$ | (c) $23.384 + 98.324$ | (d) $9.3987 + 3.2893$ |
| (e) $4.87 + 23.23$ | (f) $432.2 + 2.7$ | (g) $54.23 + 293.234$ | (h) $85.23 + 829.385$ |
| (i) $839.28 + 2.394$ | (j) $23.2 + 284.284$ | (k) $48.3 + 2.95$ | (l) $87.23 + 7.49$ |
| (m) $9.2839 + 54.23$ | (n) $2.34 + 21.492$ | (o) $204.93 + 48.943$ | (p) $92.3 + 24.503$ |
| (q) $452.95 + 21.2$ | (r) $2004.32 + 3.87$ | (s) $45.828 + 58834.2$ | (t) $2883.23 + 2948.3939$ |
| (u) $0.234 + 2.3$ | (v) $0.0283 + 5.43$ | (w) $256.37 + 0.092$ | (x) $25 + 0.32$ |
| (y) $38.5 + 924.85$ | (z) $0.43 + 0.068$ | (α) $2982.323 + 399.21$ | (β) $399.2 + 299.3567$ |



Puzzle

One of the following numbers has been **typed incorrectly**.

9, 7, 4, 6, 3, 5, 2, 4, 1, 3

Which number is incorrect, and why?



Subtracting Decimals

To calculate a sum like $53.8 - 1.46$, follow the steps shown below.

1. Set out the sum so that the decimal points line up.

$$\begin{array}{r} 53.8 \\ - 1.46 \\ \hline \end{array}$$

2. If required, add zeroes after the decimal point so that an equal number of digits appear after the decimal point.

$$\begin{array}{r} 53.80 \\ - 1.46 \\ \hline \end{array}$$

3. To avoid negative numbers in the answer, we borrow from the next column to the left: 8 changes to 7 and 0 changes to 10.

$$\begin{array}{r} 7 \quad 1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline \end{array}$$

4. Subtracting, we calculate $10 - 6 = 4$; and $7 - 4 = 3$.

$$\begin{array}{r} 7 \quad 1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline 34 \end{array}$$

5. Remember to insert the decimal point in the correct position.

$$\begin{array}{r} 7 \quad 1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline .34 \end{array}$$

6. We complete the final two subtractions to arrive at the final solution.

$$\begin{array}{r} 7 \quad 1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline 52.34 \end{array}$$

Exercise 12

- | | | | |
|----------------------|---------------------|----------------------|--------------------------|
| (a) $25.78 - 13.24$ | (b) $7.8 - 6.4$ | (c) $128.76 - 45.5$ | (d) $95.9 - 58.6$ |
| (e) $8.3 - 5$ | (f) $19.8 - 12.9$ | (g) $23.9 - 4.67$ | (h) $48.4 - 7.84$ |
| (i) $256.8 - 135.78$ | (j) $14 - 7.2$ | (k) $23 - 9.7$ | (l) $34 - 4.8$ |
| (m) $689 - 23.6$ | (n) $45.1 - 9.83$ | (o) $208.5 - 123.88$ | (p) $700.34 - 216.74$ |
| (q) $45.67 - 23.895$ | (r) $128.2 - 2.32$ | (s) $27.7 - 23.53$ | (t) $2983 - 2837.3$ |
| (u) $23.4 - 14.528$ | (v) $56.7 - 43.238$ | (w) $87.5 - 23.282$ | (x) $74.2 - 17.854$ |
| (y) $25 - 12.447$ | (z) $90 - 43.245$ | (α) $100 - 23.4329$ | (β) $101010 - 94738.282$ |

**Exercise 13**

- (a) Steffan cuts a piece of string into three pieces. The length of the pieces are 4.24cm; 7.58cm; and 3.5cm. What was the length of the string **before** Steffan made the cuts?
- (b) Emma cycles to work, a journey of 7.8 miles. One morning, on the way to work, Emma stops to buy a newspaper from a shop. If the shop is 2.9 miles from Emma's home, how far is the shop from Emma's place of work?
- (c) In a gymnastics competition, Lisa scores 12.6 on vault; 11.9 on bars; 12.7 on beam; and 12.9 on floor. What was **Lisa's total score**?
- (d) Dewi's height is 1.45m. Dewi has grown 0.12m in 18 months. What was Dewi's height **18 months ago**?



Applying

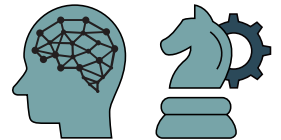
Investigation

Imagine that you have a pair of fair dice, as shown in the picture. When rolled, we can combine the numbers on the dice to make **two decimals that are less than 10**.

For example, if the two numbers on the dice are 6 and 2, then the two decimals are 6.2 and 2.6.

By rounding off the two decimals, we obtain two whole numbers. In the example, the whole numbers are 6 (rounding off 6.2) and 3 (rounding off 2.6).

Investigate to see if it is possible to obtain two **equal** whole numbers after rounding the decimals. If this is possible, list the different ways of obtaining two equal whole numbers.



Exercise 14 (Revision)

A

(a) Write the number 2439245.2 in **words**.

(b) Write the number three million, seventy thousand and sixty in **figures**.

(c) Calculate the answer to the following.

- | | |
|---------------------------|---------------------------|
| (i) 45.7×100 | (ii) $2.56 \div 10$ |
| (iii) 0.03245×10 | (iv) $24 \div 1000$ |
| (v) 245.6×9 | (vi) 0.234×4 |
| (vii) $123.54 + 29.829$ | (viii) $23.29 + 9283.928$ |
| (ix) $245.32 - 25.17$ | (x) $45 - 27.54$ |

(d) Alun bought six gas canisters for a camping holiday. If each of the canisters weighed 0.56kg, what was the **total weight** of the canisters?



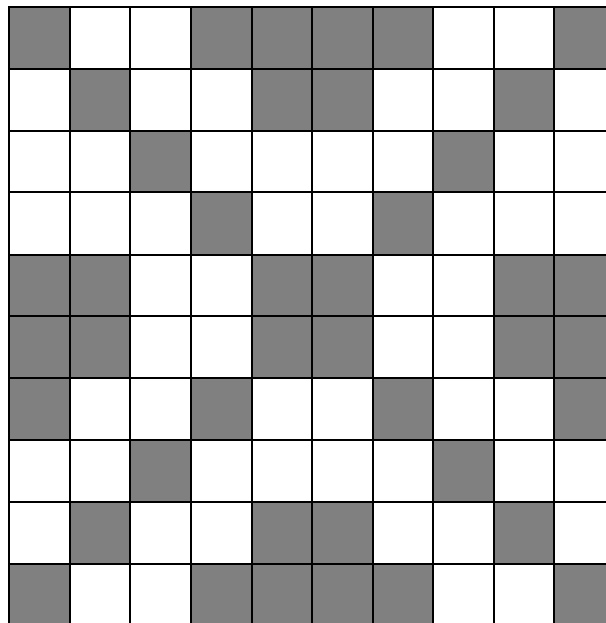
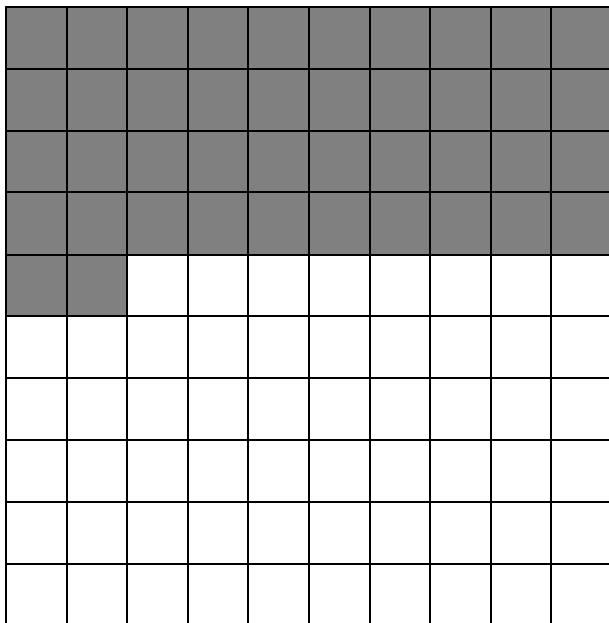
Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Calculating Percentages



A percentage represents a fraction out of 100. For example, 42% is **equivalent** to $\frac{42}{100}$, or 42 out of 100. We read 42% as “forty-two percent”. In each of the following diagrams, 42% of the diagram has been shaded.

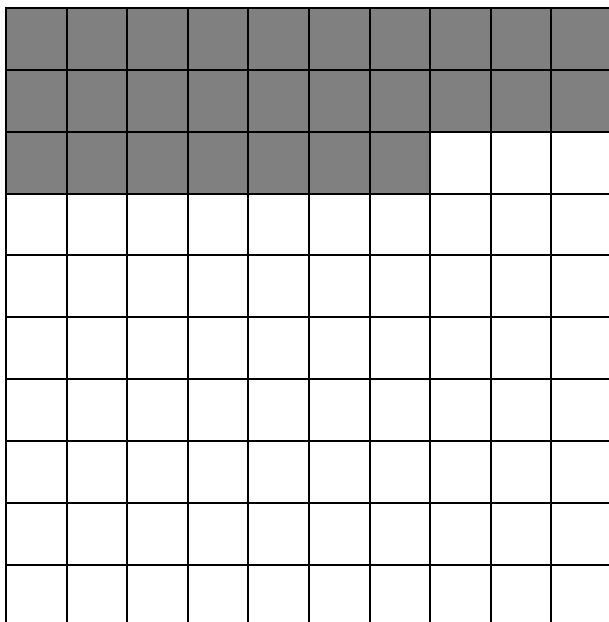


Exercise 15

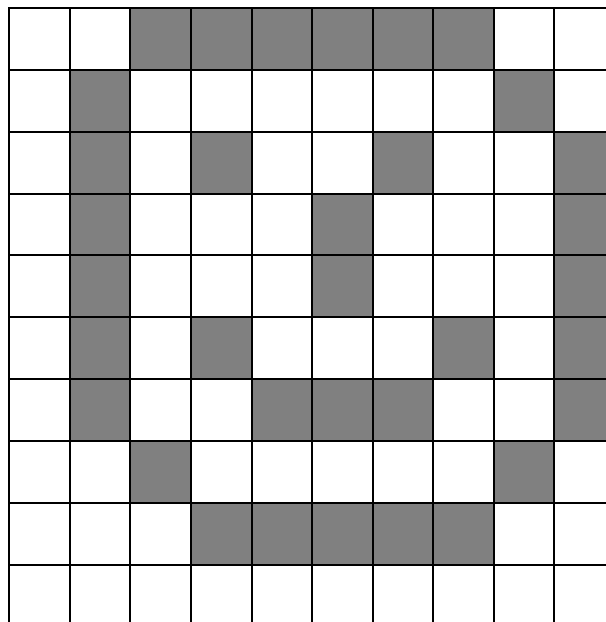


(a) What percentage of the following shapes have been shaded?

(i)



(ii)



(b) What percentage of the above shapes have **not** been shaded?

Simple Percentages

Some percentages are easier to calculate than others. Try to complete the following table which explains how to calculate some simple percentages.

Percentage	Fraction	To calculate this percentage:
50%	$\frac{1}{2}$, because 50% fits into 100% twice.	Divide by 2, or halve.
25%		
10%		
20%		
5%		
1%		



Exercise 16

A

Shade in the shown percentage.

(a) 50%

(b) 25%

(c) 20%

(d) 10%

(e) 50%

(f) 5%

(g) 60%

(h) 25%

(i) 40%

--	--	--	--	--

Exercise 17

Calculate the shown percentage.

(a) 50% of 80

(b) 25% of 24

(c) 10% of 70

(d) 20% of 30

(e) 50% of 12

(f) 25% of 36

(g) 10% of 160

(h) 5% of 80

(i) 1% of 600

(j) 20% of 35

(k) 50% of 7

(l) 25% of 30

(m) 20% of 65

(n) 10% of 85

(o) 50% of 19

(p) 50% of £48

(q) 25% of £180

(r) 10% of £2500

(s) 20% of £50

(t) 1% of £190

(u) 5% of £120

(v) 25% of £34

(w) 20% of £45

(x) 10% of £19

(y) 40% of £200



A

Calculating a Percentage without a Calculator

To calculate 10% of a number, we divide the number by 10. For example, 10% of £38 is $£38 \div 10 = £3.80$.

Exercise 18

A

Calculate **10%** of the following values.

- | | | | | |
|----------|-----------|----------|------------|-----------|
| (a) 80 | (b) 170 | (c) 600 | (d) 18 | (e) 49 |
| (f) £28 | (g) £187 | (h) £16 | (i) £286 | (j) £182 |
| (k) \$24 | (l) \$920 | (m) 25.4 | (n) £78.50 | (o) £0.70 |

To calculate 1% of a number, we divide the number by 100. For example, 1% of £38 is $£38 \div 100 = £0.38$.

Another way of calculating 1% would be to calculate 10% first, and then divide this answer by 10. For example, 10% of £38 is $£38 \div 10 = £3.80$, so 1% of £380 is $£3.80 \div 10 = £0.38$.

Exercise 19

Calculate **1%** of the following values.

- | | | | | |
|----------|-----------|----------|----------|-----------|
| (a) 200 | (b) 260 | (c) 400 | (d) 17 | (e) 53 |
| (f) £58 | (g) £276 | (h) £12 | (i) £253 | (j) £176 |
| (k) \$26 | (l) \$298 | (m) 45.8 | (n) 64.3 | (o) 97.25 |

Knowing 10% and 1% of a number, it is possible to calculate **any whole number percentage** between 1% and 99%.

Example

What is 37% of £85?



The first step is to calculate **10%** and **1%** of the number.

Next, we calculate **30%** of the number by multiplying the 10% by 3; and calculate **7%** of the number by multiplying the 1% by 7.

Finally, we **add** the 30% to the 7% to obtain the final solution.

10%
 $£85 \div 10 = £8.50$
1%
 $£8.50 \div 10 = £0.85$

$\begin{array}{r} 8.50 \\ \times 3 \\ \hline 25.50 \\ 1 \end{array}$	$\begin{array}{r} 0.85 \\ \times 7 \\ \hline 5.95 \\ 5 \quad 3 \end{array}$
--	---

$$\begin{array}{r} 25.50 \\ + 5.95 \\ \hline £31.45 \\ 1 \quad 1 \end{array}$$

Exercise 20

Calculate the following percentages.

- | | | | |
|-----------------|-----------------|-------------------|-------------------|
| (a) 48% of £75 | (b) 73% of £94 | (c) 42% of £36 | (d) 93% of £71 |
| (e) 16% of £82 | (f) 24% of £68 | (g) 66% of £84 | (h) 79% of £52 |
| (i) 46% of £168 | (j) 32% of £262 | (k) 74% of £375 | (l) 95% of £186 |
| (m) 22% of 450 | (n) 47% of 820 | (o) 81% of 1042 | (p) 15% of 2,400 |
| (q) 6% of £28 | (r) 99% of £80 | (s) 28% of £5,400 | (t) 12% of £8,400 |
| (u) 2% of \$240 | (v) 41% of \$95 | (w) 88% of \$88 | (x) 90% of \$740 |
| (y) 38% of £94 | (z) 46% of £143 | (α) 33% of £400 | (β) 87% of £2,145 |



Calculating a Percentage using a Calculator

Every scientific calculator has a percentage button (%). Sometimes the **CATALOG** or **SHIFT** button must be pressed to access the percentage button.

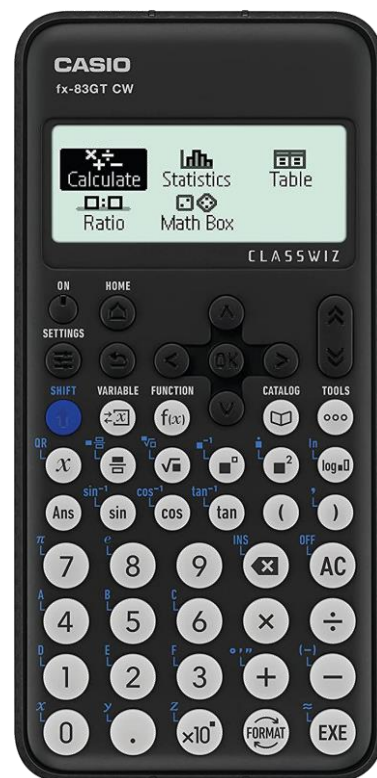
To calculate 37% of £85 using the calculator shown on the right, the following buttons must be pressed.

8	5	×	3	7	CATALOG	↓	OK	OK	EXE
---	---	---	---	---	---------	---	----	----	-----

The screen of the calculator should show 31.45.

(Press the **FORMAT**, **↓**, **DECIMAL** buttons if you see $\frac{629}{20}$.)

Maybe your calculator is different. In the box below, write down the buttons you must press to calculate 43% of £97.



Exercise 21

13

Use your calculator to calculate the following percentages.

- | | | | |
|------------------|-----------------|-------------------|-------------------|
| (a) 74% of £24 | (b) 28% of £57 | (c) 35% of £96 | (d) 4% of £125 |
| (e) 18% of £74 | (f) 27% of £57 | (g) 68% of £15 | (h) 73% of £4 |
| (i) 42% of 285 | (j) 35% of 685 | (k) 77% of 457 | (l) 98% of 35.2 |
| (m) 23% of \$875 | (n) 2% of \$241 | (o) 74% of €1,095 | (p) 35% of €7,800 |



Exercise 22 (Revision)

A

- (a) Draw a circle with a compass. Colour in 25% of the circle.
- (b) Calculate the following **without** using a calculator.
- (i) 50% of 38 (ii) 25% of 500 (iii) 10% of £12 (iv) 20% of \$58 (v) 48% of £76 (vi) 87% of €146
- (c) Calculate the following using a calculator.
- (i) 24% of 57 (ii) 73% of £25 (iii) 83% of \$1,950

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Using Percentages

5% COMMISSION!

COTTON 30%

POLYESTER 70%



Percentages can be seen in many situations, as shown above. Can you think of more examples where a percentage is used?

Profit and Loss

Profit and loss are important in business. In making a profit we are receiving **more** money, and in making a loss we are **losing** money.



Example

A car dealer buys a car for £7,800. If the dealer wants to make a profit of at least 12% in selling the car, what price should the dealer place on the car?

Answer: 10% of £7,800 is $£7,800 \div 10 = £780$,
and 1% of £7,800 is $£780 \div 10 = £78$.

It follows that 2% of £7,800 is $£78 \times 2 = £156$,
and 12% of £7,800 is $£780 + £156 = £936$.

So the dealer should sell the car for at least
 $£7,800 + £936 = £8,736$. (It is probable that the dealer will place a sticker like £8,750 or £9,000 on the car.)



Exercise 23

(a) A car dealer buys a car for £9,200. If the dealer wishes to make a profit of **at least 15%** on selling the car, what price should the dealer place on the car?

(b) A furniture company holds a '**40% off**' sale. Complete the following table showing the prices in the sale.

Item	Original price	Discount	Price in the sale
Sofa	£800		
Table	£250		
Cushion	£15		
Wardrobe	£550		
4 Chairs	£160		

Applying

1



(c) Deiniol went into a clothes shop and saw a pair of trousers with a marked price of £40. If there was a '20% off' sale in the shop, how much did Deiniol **pay** for the trousers at the till?

(d) An estate agent received 3% commission on selling a house for £185,000. **How much commission** did the estate agent receive?

(e) After advertising on social media, a company saw a **16% increase** in their profit. If the company's profit was £2,400 a week before advertising started, what was the company's profit (per week) after advertising started?

(f) A jeweller bought a ring for £1,200, hoping to sell it for **20% profit**. Unfortunately, there was no interest in the ring, so the jeweller had to sell it in a sale, making a **loss of 14%**.

(i) At what price was the jeweller hoping to sell the ring?

(ii) At what price did the jeweller sell the ring in the end?

(g) Megan changed £400 into euros. The bank claimed **2% commission**. How much was the commission?

(h) A shop holds a '**70% off**' closing down sale. Siwan buys a pair of shoes in the sale, usually costing £60. How much did Siwan pay for the shoes?



Simple Interest

When borrowing money from a bank, the bank charges you **interest**. You must pay back this interest on top of the money you have borrowed. On the other hand, if you **invest** money into a bank – which is to place money in a bank for safe keeping – the bank pays you interest. The **interest rate** is usually given as a percentage. There are several types of interest; in year 7 we will consider one type, namely **simple interest**.



Example

Rheinallt borrows **£12,000** from Barclays Bank at a simple interest rate of **3% a year**. Rheinallt wants to pay back all the money after **four years**. How much money will Rheinallt have to pay back after four years?

Answer: With a simple interest rate of 3% a year, the bank will charge 3% of £12,000 for each year that Rheinallt borrows the money.

1) Calculate 3% of £12,000.

2) Multiply the £360 by the number of years that Rheinallt wants to borrow the money for, namely 4 years.

3) Add the simple interest (£1,440) to the amount borrowed (£12,000) to obtain the final solution.

$$\begin{aligned} &\underline{1\%} \\ &£12,000 \div 100 = £120 \\ &\underline{3\%} \\ &£120 \times 3 = £360 \end{aligned}$$

$$\begin{array}{r} 360 \\ \times 4 \\ \hline £1440 \\ 2 \end{array}$$

$$\begin{array}{r} 12000 \\ + 1440 \\ \hline £13440 \end{array}$$

Exercise 24

1

The following table shows the simple interest rate for different sums to borrow over a specific period of time. In each case, calculate the amount of money that must be **paid back** to the bank at the end of the period of borrowing.

	Sum to borrow	Simple interest rate, per year	Period to borrow
(a)	£15,000	3%	4 years
(b)	£18,000	4%	3 years
(c)	£9,500	2%	3 years
(d)	£13,500	5%	6 years
(e)	£2,500	7%	2 years
(f)	£26,000	6%	5 years
(g)	£14,750	3%	2 years
(h)	£50,000	4%	3 years



Example

Siriol wants to invest **£8,000** in HSBC Bank at a simple interest rate of **4% a year**. Siriol wants to withdraw all the money from the bank after **five years**. How much money will Siriol be able to withdraw after five years?

Answer: With a simple interest rate of 4% a year, the bank will add 4% of £8,000 for each year that Siriol keeps the money in the bank.

1) Calculate 4% of £8,000.

2) Multiply the £320 by the number of years Siriol wants to keep the money in the bank, namely 5 years.

3) Add the simple interest (£1,600) to the original sum (£8,000) to obtain the final solution.

1%
 $£8,000 \div 100 = £80$
4%
 $£80 \times 4 = £320$

$$\begin{array}{r} 320 \\ \times 5 \\ \hline £1600 \\ 1 \end{array}$$

$$\begin{array}{r} 8000 \\ + 1600 \\ \hline £9600 \end{array}$$

Exercise 25

1

The following table shows the simple interest rate for different sums to invest over a specific period of time. In each case, calculate how much money is available to **withdraw** from the bank at the end of the period of investment.

	Sum to invest	Simple interest rate, per year	Period to invest
(a)	£5,000	4%	3 years
(b)	£8,500	6%	2 years
(c)	£12,000	3%	4 years
(d)	£1,500	4%	5 years
(e)	£2,800	2%	3 years
(f)	£100,000	7%	2 years
(g)	£24,600	1%	5 years
(h)	£6,540	3%	8 years



Personal Finance

Percentages often appear when considering how much money comes into and out of a home.



Salaries

When receiving a salary, you are not allowed to keep every penny of what your employer gives you. Everyone starts off with a **gross salary**, out of which items such as *income tax*; *national insurance*; and *pension* are taken. We call the amount that is left a **net salary**, and this is what reaches your bank account.

Example

Elin receives a gross salary of £450 a week. Elin must **deduct** 26% of her salary as income tax; national insurance and pension. What is Elin's net salary, per week?

Answer: Using a calculator, 26% of £450 is

$$450 \times 26\% = £117.$$

Elin's net salary is therefore $£450 - £117 = £333$ per week.



Exercise 26

1

Use a calculator to complete the following table. (The above example forms the first row of the table.)

	Gross salary (per week)	Percentage of income tax; national insurance and pension	Contributions (or Deductions)	Net salary (per week)
	£450	26%	£117	£333
(a)	£350	22%		
(b)	£720	27%		
(c)	£290	19%		
(d)	£210	15%		
(e)	£596	26%		
(f)	£740	31%		
(g)	£185	12%		
(h)	£245	16%		
(i)	£539	29%		
(j)	£1,050	38%		

Challenge!

Mr. Jones' gross salary is £26,624 per year.

Can you work out Mr. Jones' **net salary** using the following information?



1

Extension

1. Income Tax Basic Rate: 20% on your yearly earnings between £10,600 and £31,785 Higher Rate: 40% on your yearly earnings between £31,786 and £150,000.	2. National Insurance Contributions Mr. Jones pays Class 1 National Insurance Contributions. The rates are: 12% on weekly earnings between £155 and £815 2% on weekly earnings over £815	3. Pension The rates for pension contributions are: 7.5% on your yearly earnings between £0 and £25,999.99 8.5% on your yearly earnings between £26,000 and £34,999.99.
---	---	--

Bills

When you pay for goods, some of the money goes to the government as a **tax** on what you have bought. The name of this tax is **Value Added Tax (VAT)**. At the current time, the rate of VAT is 20%, but some goods use a different rate.

- The rate for *energy bills; child car seats; and mobility aids for the elderly* is 5%.
- The rate for *children's clothes and shoes; books; magazines; and bike helmets* is 0%.



Example

The price of a television before adding VAT is £250. What is the price of the television after adding VAT?

Answer: The VAT rate for a television is 20%. Using a calculator, 20% of £250 is $£250 \times 20\% = £50$. It follows that the price of a television after adding VAT is $£250 + £50 = £300$.

Exercise 27

Complete the following table showing the prices of items **after** adding VAT.
(The above example forms the first row of the table.)



	Item	Price before adding VAT	VAT Rate	Amount of VAT	Price after adding VAT
	Television	£250	20%	£50	£300
(a)	Mobile Phone	£275	20%		
(b)	Kettle	£30	20%		
(c)	Electricity Bill	£1,400	5%		
(d)	Sofa	£600	20%		
(e)	Chocolate Bar	£1.25	20%		
(f)	Gas Bill	£740	5%		
(g)	Magazine	£3.60	0%		
(h)	Washing Machine	£187.50	20%		
(i)	Children's Shoes	£7.50	0%		
(j)	Toy	£35	20%		

Special Offers

Exercise 28



(a) Two shops sell the same mobile phone. The first shop sells the mobile phone for £120. The second shop sells the mobile phone for £150, but today there is a special offer available: "25% off". Which shop sells the mobile phone for the **cheapest** price today?

(b) Two shops sell the same television. The first shop sells the television for £425. The second shop sells the television for £500, but today there is a special offer available: "20% off". Which shop sells the television for the **cheapest** price today?

(c) Two shops sell the same hairdryer. The first shop sells the hairdryer for £25. The second shop sells the hairdryer for £35, but today there is a special offer available: "30% off". Which shop sells the hairdryer for the **cheapest** price today?

(d) Two shops sell the same vacuum cleaner. The first shop sells the vacuum cleaner for £199. The second shop sells the vacuum cleaner for £350, but today there is a special offer available: "40% off". Which shop sells the vacuum cleaner for the **cheapest** price today?

Percentage in a Test

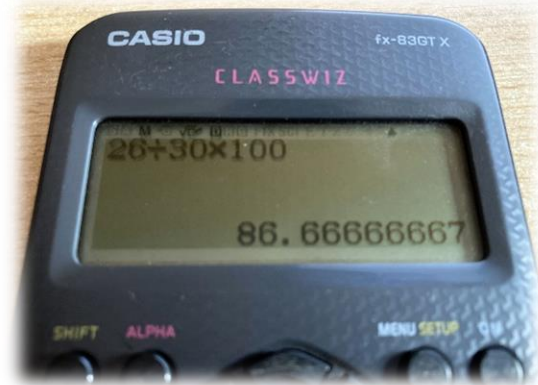
If Hari has received 26 out of 30 in a test, what **percentage** has Hari received in the test?

Answer: To change Hari's mark to a percentage, we need to multiply the fraction $\frac{26}{30}$ by 100%. On a calculator, the buttons that need to be pressed to do this are

2	6	÷	3	0	×	1	0	0	EXE
---	---	---	---	---	---	---	---	---	-----

The number 86.66666667 should appear on the screen of the calculator. (Perhaps you must press **FORMAT** to see the number.)

This is Hari's percentage in the test. Rounded off to the nearest whole number, Hari's percentage in the test is **87%**.

**Exercise 29**

13

Change the following test results to **percentages**. Give your answers to the nearest percentage.

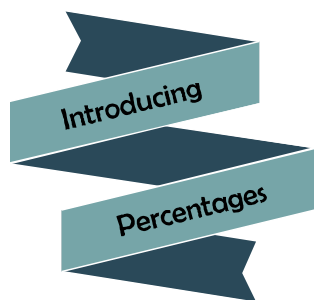
- | | | | |
|------------------|--------------------|--------------------|-------------------|
| (a) 18 out of 25 | (b) 11 out of 20 | (c) 23 out of 28 | (d) 5 out of 18 |
| (e) 45 out of 63 | (f) 123 out of 150 | (g) 4 out of 7 | (h) 32 out of 70 |
| (i) 25 out of 80 | (j) 14 out of 23 | (k) 241 out of 300 | (l) 47 out of 100 |

**Exercise 30 (Revision)**

- (a) John changes £500 to dollars. The post office take **3% commission**. How much is the commission?
- (b) Imogen wants to invest **£6,000** into Lloyds Bank at a simple interest rate of **3% a year**. Imogen wants to take all the money out of the bank after **two years**. How much money will Imogen be able to withdraw after two years?
- (c) Sean receives a **gross salary** of £450 per week. Sean must deduct 24% of his salary as income tax; national insurance and pension. What is Sean's **net salary**, per week?
- (d) The price of a computer **before** adding VAT (at 20%) is £400. What is the price of the computer **after** adding VAT?
- (e) Emma scores 28 out of 43 in an English test. What **percentage** did Emma score in the test?
- (f) Out of all 650 pupils in Ysgol y Creuddyn, 381 come from Colwyn Bay. What **percentage** of the school's pupils do **not** come from Colwyn Bay?


Evaluation

Key Words	Corrections	I am happy with...	I need to revise...



Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test: Without a calculator With a calculator	Correct in the test?
I know how to write numbers in words and in figures .			1, 2	
I know the place value of different digits in numbers, e.g. the place value of the 8 in the number 28,105.			3	
I know how to multiply and divide numbers by 10 , 100 or 1,000 .			7	
I know how to multiply a decimal by a whole number less than 10 , e.g. 83.548×7 .			8	
I know how to add decimals .			9	
I know how to subtract decimals , using "borrowing" if required.			9	
I know how to decide which percentage of a shape is shaded .			4, 5	
I know how to calculate simple percentages quickly, e.g. 25% of £80.			6	
I know how to calculate any percentage without using a calculator , e.g. 73% of £170.			10	
I know how to calculate any percentage using a calculator .			1	
I know the meaning of profit and loss .			11, 2	
I know how to use simple interest when borrowing from or investing into a bank.			3	
I know how to use percentages in questions involving salaries , bills or special offers .			4, 5, 6, 8	
I know how to use a calculator to calculate the percentage obtained in a test .			7	



The Mathematics Department

7

Co-ordinates

in the Four

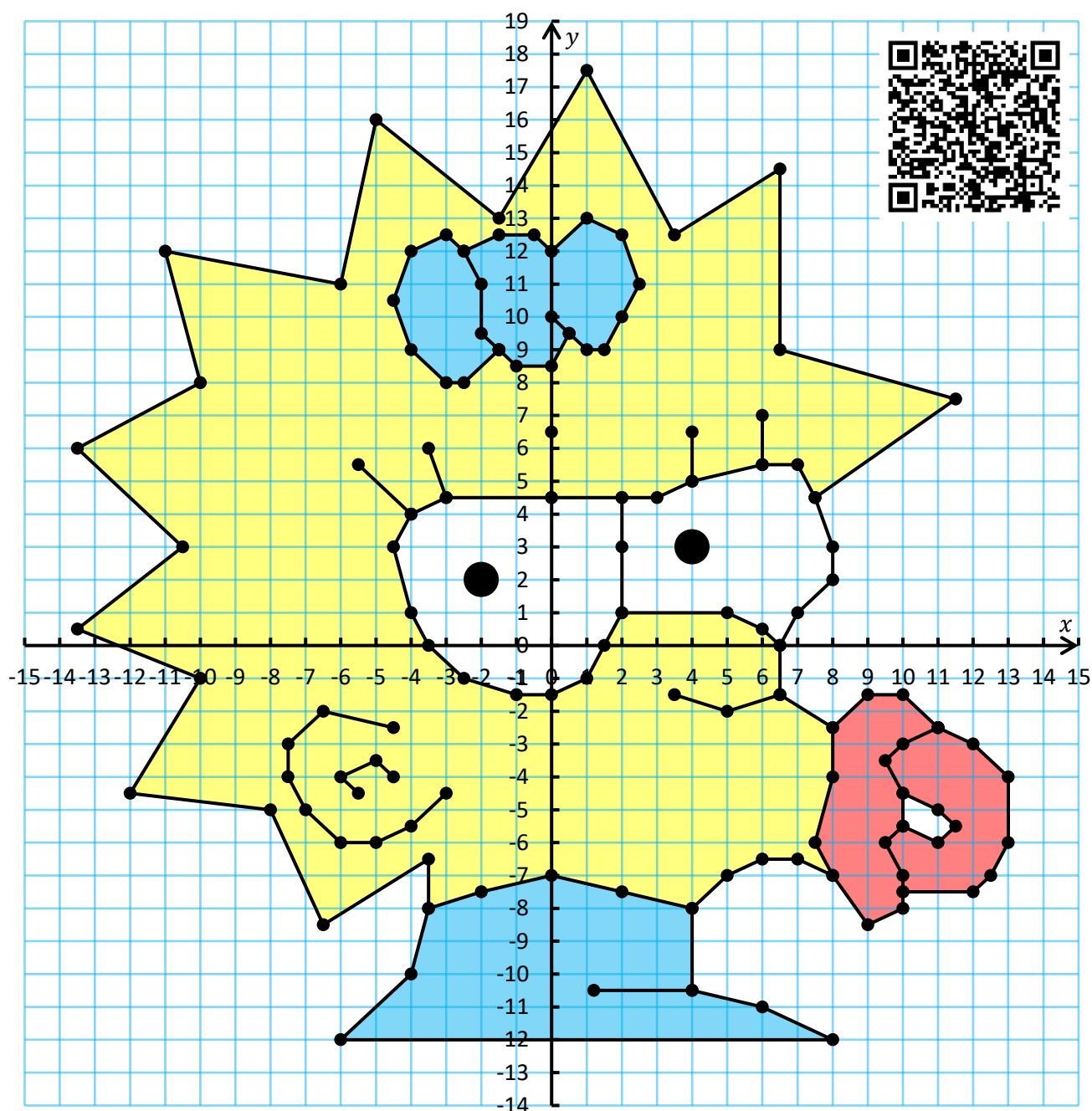
Quadrants

Version
3.0
07/2025

Name:

Contents

Chapter	Mathematics	Page Number
The First Quadrant	Plotting co-ordinates in the first quadrant.	3
The Four Quadrants	The four quadrants. Plotting co-ordinates in the four quadrants.	7
Mid-point of a Line	The first quadrant. Adding with negative numbers. The four quadrants.	11
Using Co-ordinates	Missing co-ordinates. Translations.	14



The First Quadrant

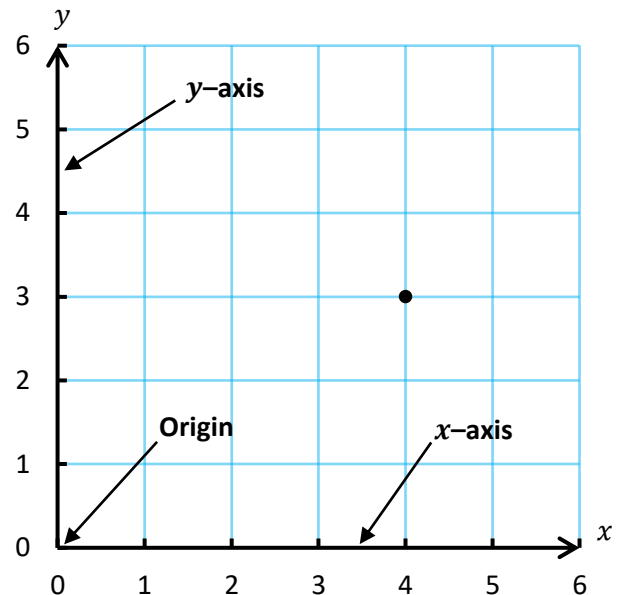
One way of locating a point is to use a pair of **numbered, perpendicular lines**. These lines are called **axes**.

- The **horizontal** axis is the **x -axis**.
- The **vertical** axis is the **y -axis**.
- The **origin** is the point where the two axes **meet**.
- To **plot a point** on the axes:
 - Start at the origin.
 - Move **across** to begin with.
 - Then move **up**.



We use **co-ordinates** to represent points on a set of axes. The co-ordinate (4, 3) is shown on the axes to the right. For this particular co-ordinate,

- The **x -coordinate** is 4. This is the distance we move **across**.
- The **y -coordinate** is 3. This is the distance we move **up**.



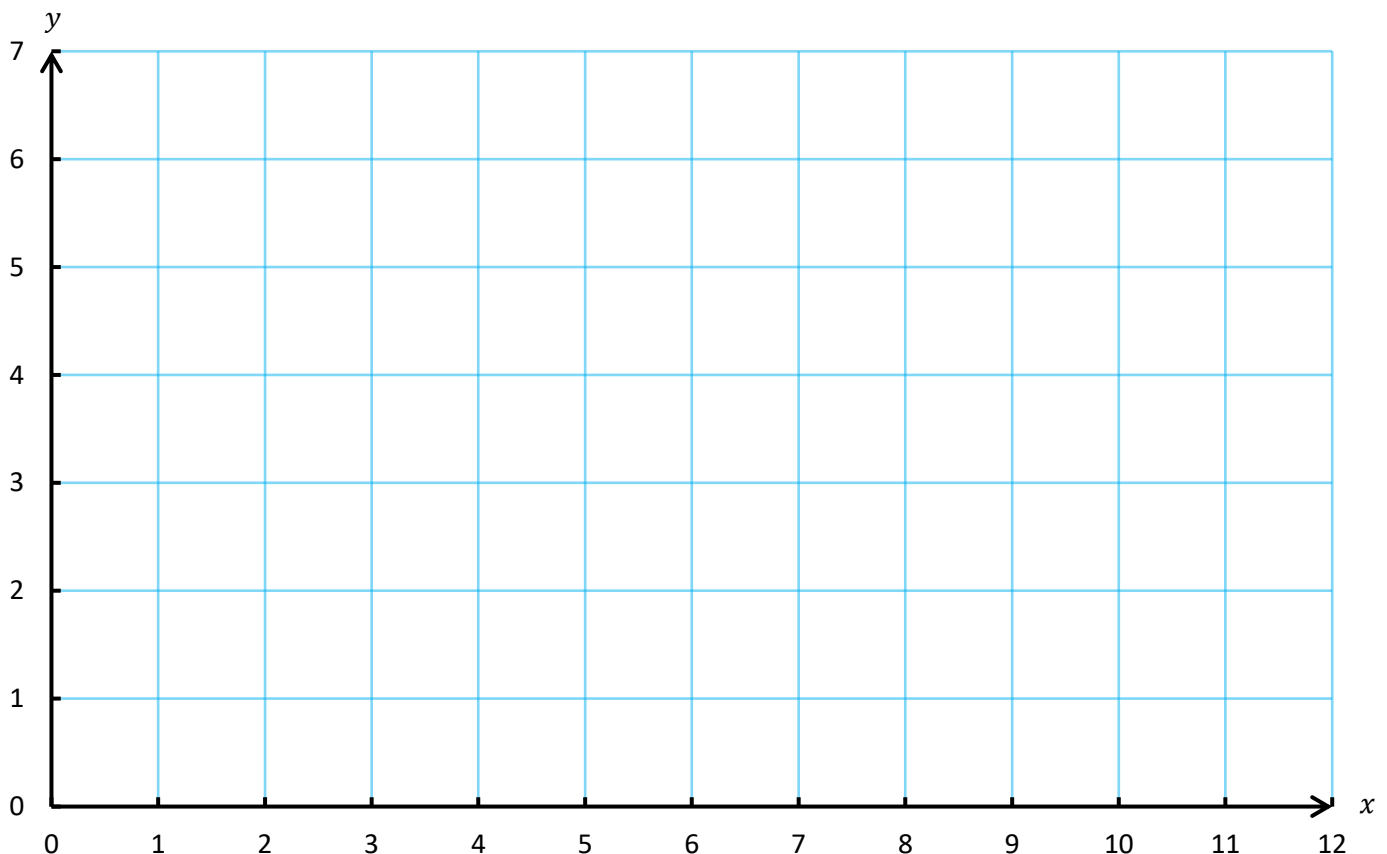
2

Skill

Exercise 1

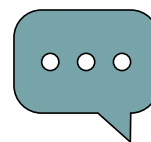
Plot the following points on the axes shown below.

- | | | | | |
|------------|------------|-------------|--------------|----------------|
| (a) (4, 5) | (b) (8, 1) | (c) (11, 4) | (d) (1, 5) | (e) (6, 2) |
| (f) (9, 2) | (g) (0, 4) | (h) (6, 0) | (i) (2.5, 3) | (j) (6.5, 4.5) |

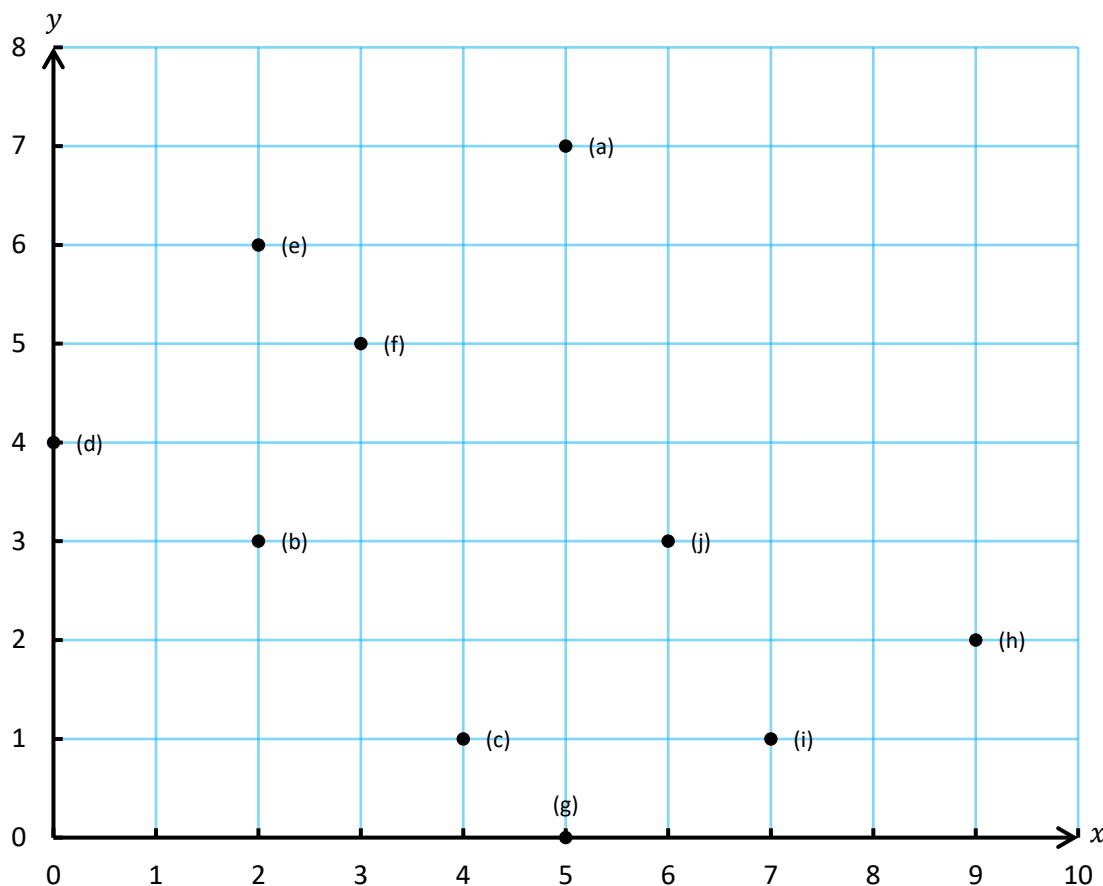


Exercise 2

Write down the co-ordinates of each of the points (a), (b), ..., (j) on the axes shown below.



2



(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

Exercise 3

Plot the following co-ordinates on the next page to draw the **face of a famous person**...

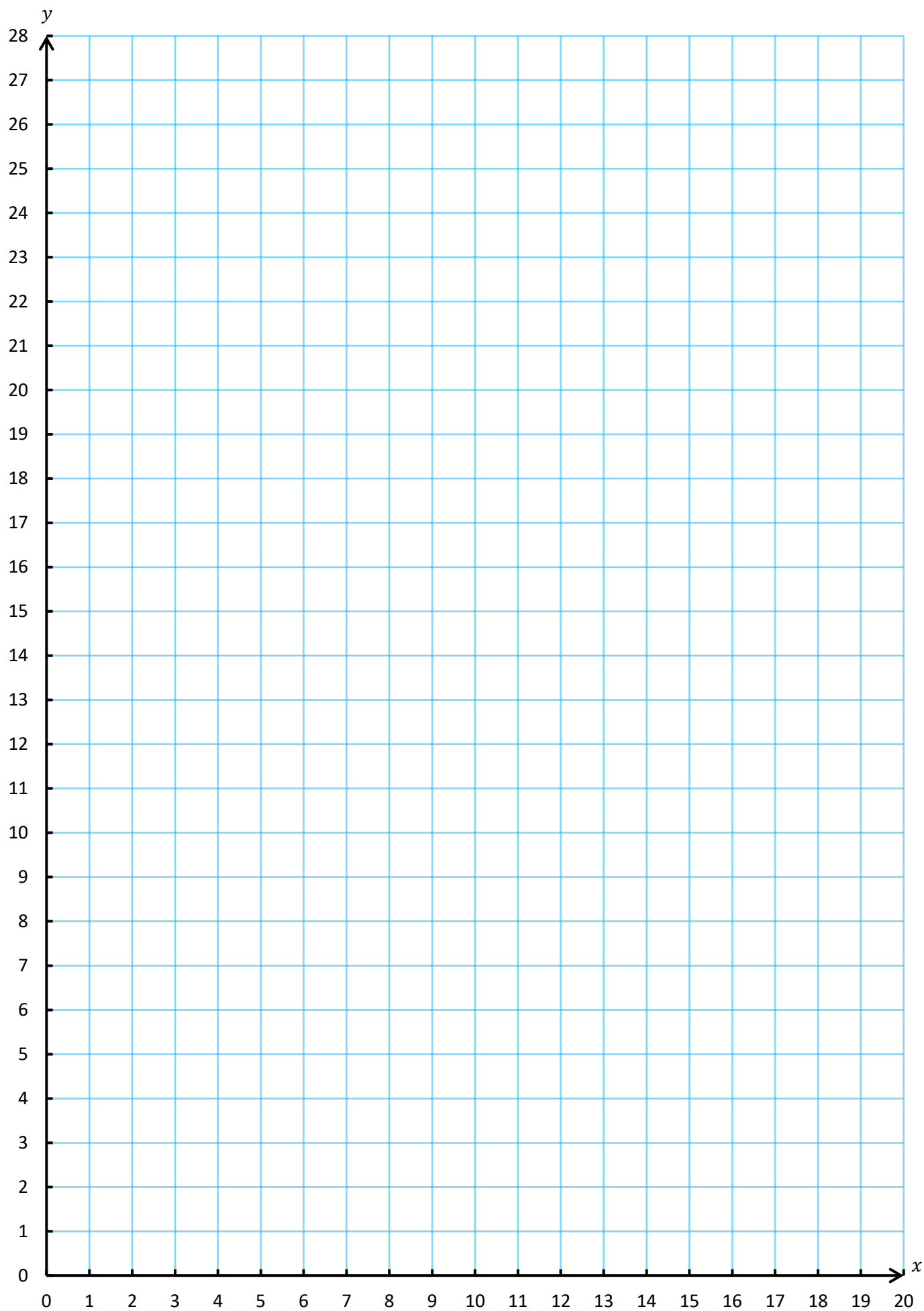
Connect the co-ordinates as you go on, starting a new line for each new bullet point.

For the **red** co-ordinates, you must **double** the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (5, 8), then you must plot the point (10, 16).

For the **blue** co-ordinates, you must **subtract three** from the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (7, 11), then you must plot the point (4, 8).



- **Face:** (10, 0), (13, 6), (8, 3), (6, 5), (2, 5), (1, 6), (2, 6), (2, 11), (1, 12), (1, 16), (2, 17), (2, 26), (3, 25), (2, 13), (5, 25), (6, 26), (10, 28), (8, 26), (9, 25), (10, 26), (11, 25), (12, 26), (13, 25), (7, 13), (15, 25), (16, 26), (17, 25), (18, 26), (18, 15), (17, 4), (16, 3), (19, 3)
- **Right Eye:** (8, 11), (6, 13), (6, 16), (7, 18), (12, 22), (11, 19), (13, 18), (7, 8), (13, 12), (10, 11), (9, 11)
- **Nose:** (6, 9), (5, 8), (4, 8), (3, 9), (3, 10), (7, 14), (5, 11), (3, 6), (7, 12)
- **Eyebrow:** (6, 17), (5, 18), (2, 9), (2, 17)
- **Left Eye:** (3, 11), (5, 14)
- **Mouth:** (7, 5), (13, 8), (11, 6), (13, 6), (14, 5)
- **Mouth:** (13, 7), (13, 6)
- **Ear:** (17, 11), (9, 6), (19, 11), (19, 9), (9, 4), (17, 8), (16, 9)
- **Place a Big Dot at (4, 15) and at (14, 18)**



Exercise 4

2

Write down the letter that is next to each of the following points. After you finish, you should have a Welsh sentence. What is the sentence?

(4, 9) (8, 1)

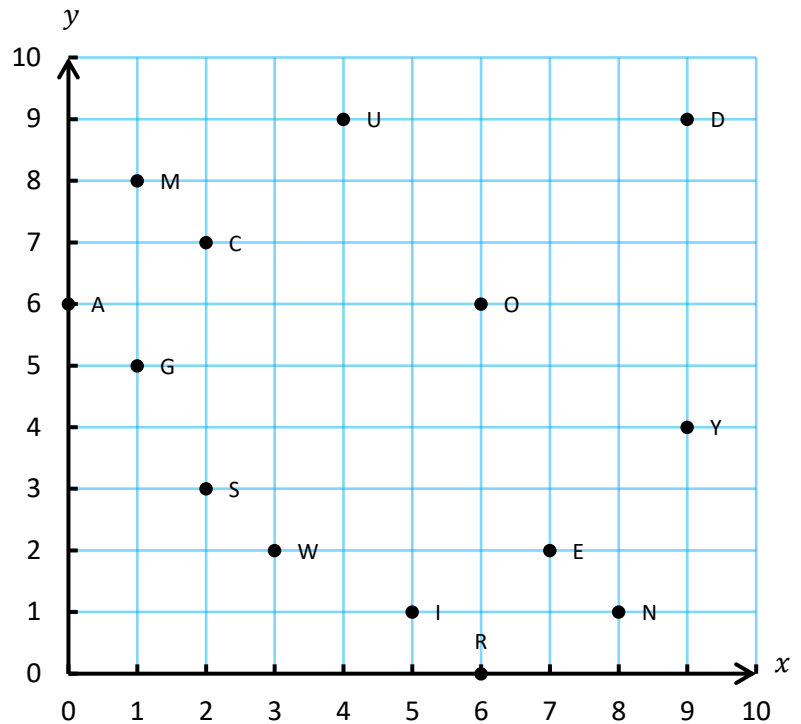
(2, 7) (0, 6) (1, 8) (1, 5) (9, 4) (1, 8)

(7, 2) (6, 0) (5, 1) (0, 6) (9, 9)

(3, 2) (8, 1) (0, 6)

(8, 1) (7, 2) (1, 5) (7, 2) (2, 3)

(3, 2) (5, 1) (6, 0) (5, 1) (6, 6) (8, 1)

**Exercise 5 (Revision)**

Draw x and y axes on a piece of squared paper. Number the axes from 0 to 18.

Plot the following points, connecting them with straight lines as you go on. (Work row-by-row.)

(4, 8),	(6, 7),	(8, 7),	(9, 6),	(8, 5),	(6, 5),	(5, 4),
(5, 3),	(6, 3),	(6, 4),	(9, 4),	(10, 5),	(13, 4),	(14, 2),
(13, 1),	(13, 0),	(14, 0),	(14, 1),	(15, 2),	(14, 5),	(13, 5),
(13, 7),	(12, 9),	(17, 8),	(14, 9),	(17, 9),	(14, 10),	(17, 11),
(15, 11),	(17, 12),	(13, 11),	(12, 9),	(10, 9),	(8, 11),	(8, 12),
(5, 15),	(4, 15),	(3, 16),	(3, 15),	(1, 13),	(1, 12),	(2, 12),
(4, 13),	(5, 11),	(4, 10),	(2, 11),	(0, 9),	(0, 8),	(1, 8),
(1, 9),	(2, 10),	(3, 9),	(2, 7),	(4, 5),	(5, 5),	(5, 6),
(4, 6),	(3, 7),	(4, 8).				

Evaluation

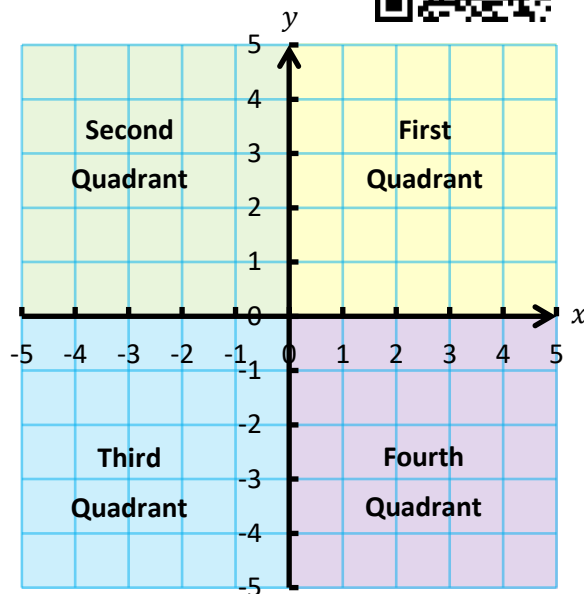
Key Words	Corrections	I am happy with...	I need to revise...



The Four Quadrants



In the previous chapter, only co-ordinates with **positive** numbers were used. To plot co-ordinates that use **negative** numbers, we must extend the x -axis to the left and the y -axis downwards. By doing this, the x and y axes divide the **plane** into **four quadrants**. These quadrants are numbered in an anti-clockwise direction, as shown in the diagram on the right.



Exercise 6

2

Skill

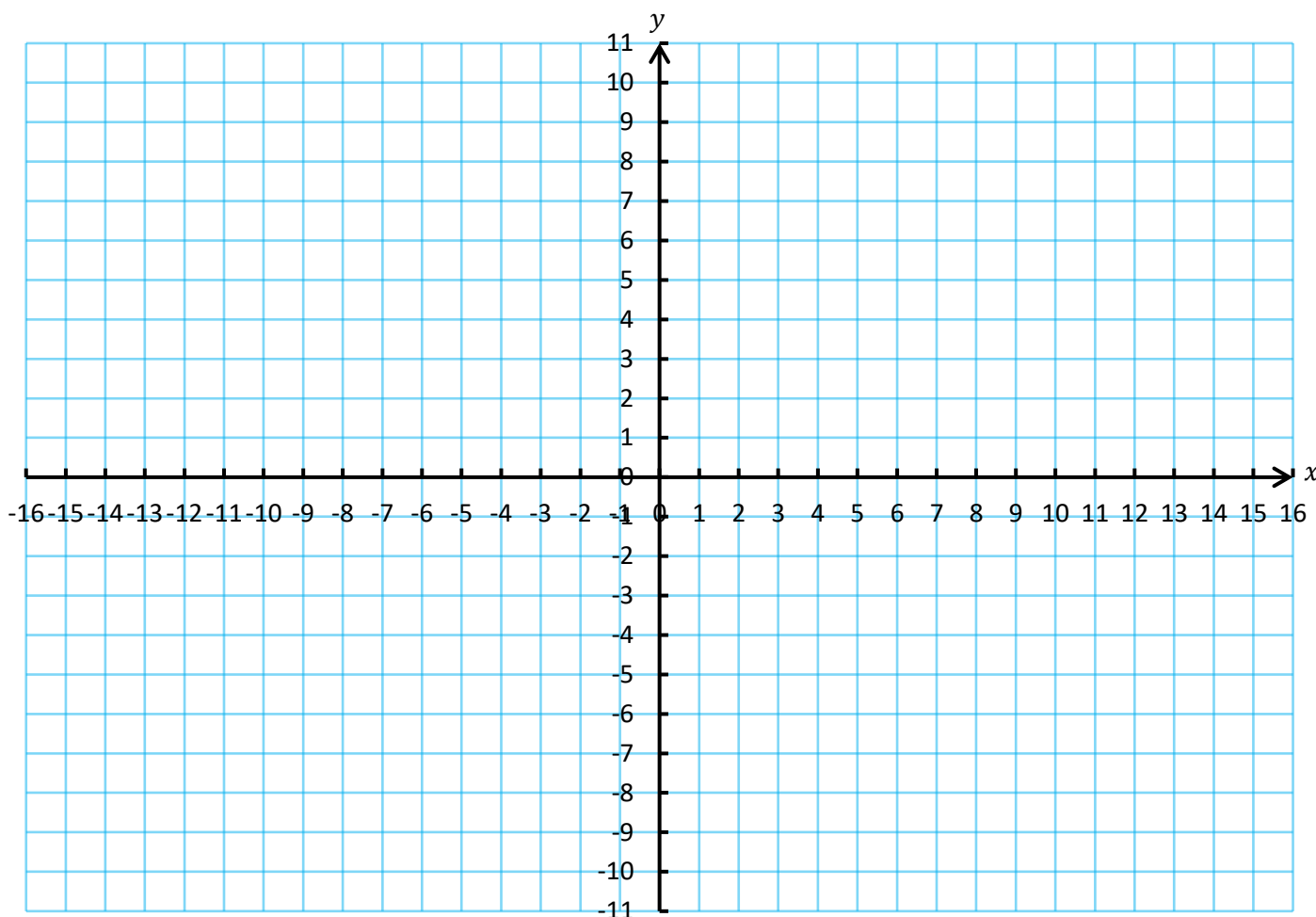
In which quadrant do the following points appear?

- | | | | |
|--------------|------------------|-------------|------------------|
| (a) (2, 5) | (b) (-2, 3) | (c) (4, -2) | (d) (-4, 1) |
| (e) (-4, -3) | (f) (2, 8) | (g) (1, -6) | (h) (-2, 4.5) |
| (i) (5, 0) | (j) (-9, -2) | (k) (-4, 3) | (l) (5, -1) |
| (m) (0, -2) | (n) (-3.5, -1.5) | (o) (0, 0) | (p) (-0.5, 3.25) |

Exercise 7

Plot the following points on the axes shown below.

- | | | | | | | |
|--------------|--------------|---------------|----------------|---------------|-------------------|----------------|
| (a) (13, 7) | (b) (-3, 5) | (c) (8, -2) | (d) (-10, -4) | (e) (-14, 7) | (f) (6, -3) | (g) (0, -6) |
| (h) (13, 10) | (i) (4.5, 8) | (j) (-5.5, 3) | (k) (4.5, -10) | (l) (-7.5, 0) | (m) (-11.5, -6.5) | (n) (14.5, -3) |



Exercise 8

2

(a) Write down the letter that is next to each of the following points. After you finish, you should have a Welsh sentence. What is the sentence?

(7, 3), (6, 5), (1, 1), (6, 5), (-4, -6), (2, 2),

(4, 5), (7, -1)

(6, 5)

(-4, -6), (6, 5), (3, 3), (-2, -2), (0, -4), (-4, 0),

(-4, 0), (7, 5), (1, 5)

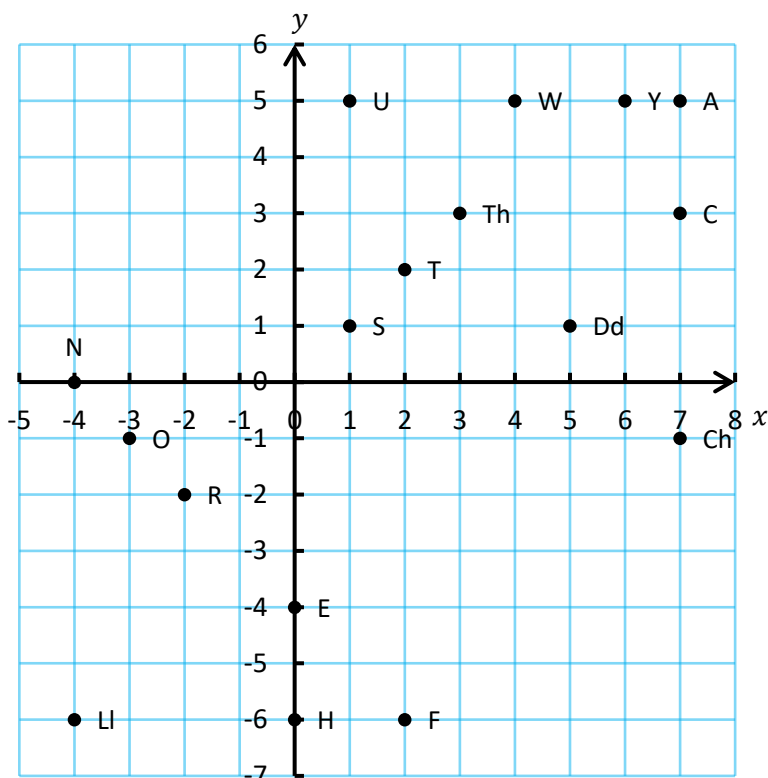
(6, 5), (-4, 0)

(-4, 0), (0, -6), (-2, -2), (0, -4), (2, -6), (-4, 0)

(6, 5), (-2, -2)

(4, 5), (6, 5), (5, 1), (-3, -1), (-2, -2)

(b) Do what is asked of you in part (a). What is the result?



Exercise 9

Plot the following co-ordinates on the next page to draw the **face of a famous person...**

Connect the co-ordinates as you go on, starting a new line for each new bullet point.

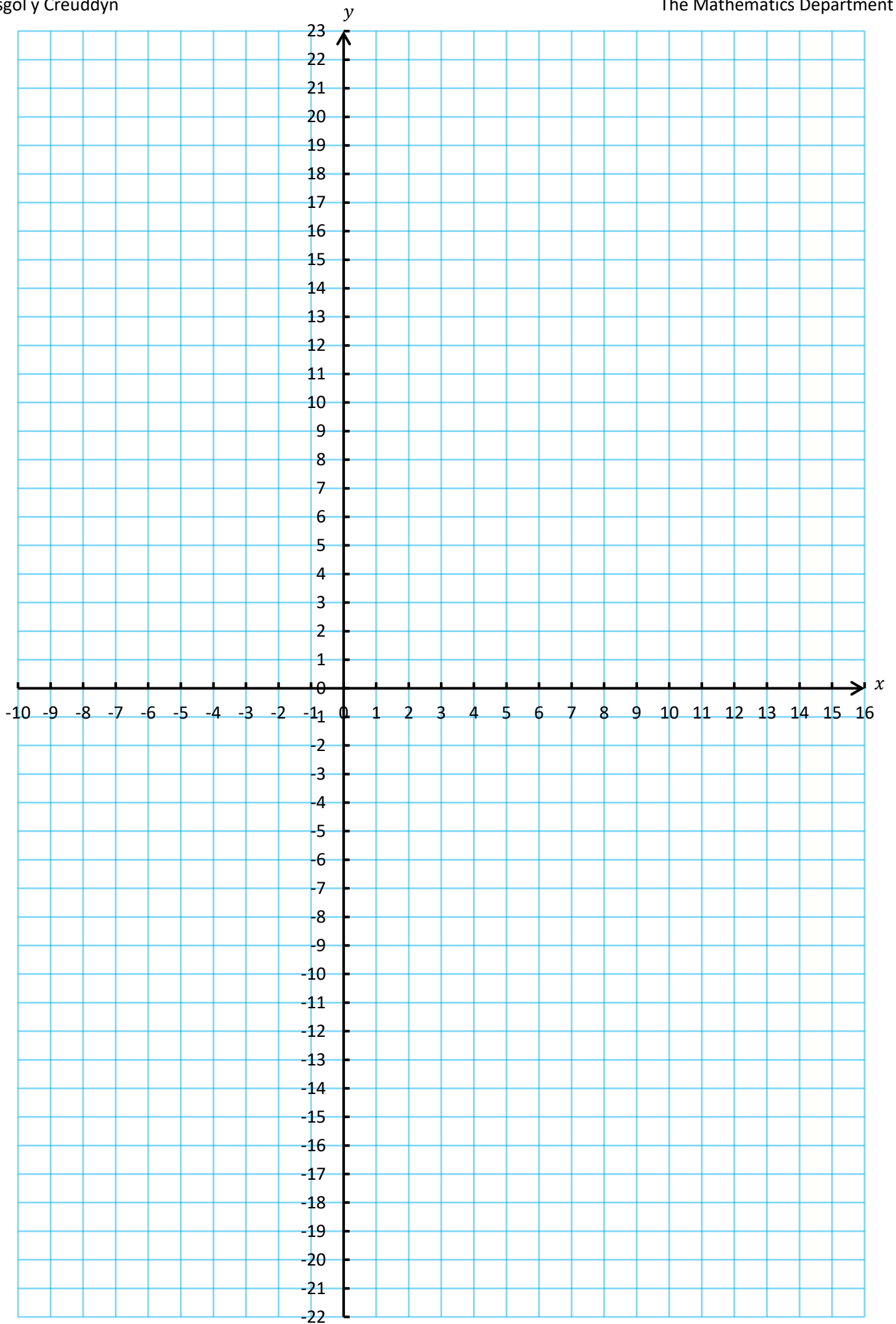
For the **red** co-ordinates, you must **halve** the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (8, 10), then you must plot the point (4, 5).

For the **blue** co-ordinates, you must **add four** to the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (2, 5), then you must plot the point (6, 9).



- **Mouth:** (11, -11), (11, -12), (9, -14), (5, -16), (3, -16), (0, -15), (-2, -11), (-2, -9), (-1, -7), (2, -4), (4, -3), (12, -3), (14, -4), (15, -6), (15, -9), (11, -11), (8, -12), (5, -11), (2, -10)
- **Lips:** (0, -11), (4, -9)
- **Neck:** (-9, -20), (-7, -12), (-5, -7), (-7, -7), (-9, -5), (-9, -4), (-8, -3), (-6, -2), (-5, -3)
- **Ear:** (-7, -5), (-7, -4), (-6, -4)
- **Neck:** (8, -21), (7, -19), (7, -17), (8, -15), (9, -14)
- **Hair:** (-9, -3), (-9, 3), (-7, -2), (-6, 3), (-4, -2)
- **Left Eye:** (12, -3), (13, -2), (14, 0), (12, 1), (6, 1), (4, 0), (3, -1), (1, -1), (-1, 0), (-2, 1), (-3, 4), (-1, 7), (4, 16), (4, 8), (2, 2), (7, 4), (6, 1)
- **Right Eye:** (12, 1), (13, 2), (10, 0), (13, 6), (24, 14), (9, 8), (7, 7), (6, 6)
- **Head:** (-7, 0), (-9, 6), (-9, 12), (-8, 15), (-6, 17), (-3, 19), (0, 20), (2, 20), (1, 15), (7, 17), (9, 15), (20, 26), (11, 11), (11, 9), (9, 5), (12, 7)
- **Hair:** (-8, 12), (-9, 15), (-8, 18), (-6, 20), (-4, 20), (-3, 19)
- **Hair:** (-6, 14), (-7, 17), (-6, 20), (-4, 22), (-1, 22), (1, 21), (2, 20)
- **Place a Big Dot at (0, 3) and at (20, 6).**





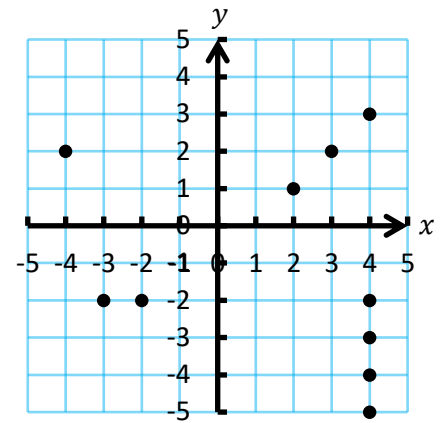
Battleships (a game for two players)

Each player will need the worksheet for the game.

Place the following ships on your graph without the other player seeing:

1 submarine (1 dot); **1 destroyer** (2 dots); **1 cruiser** (3 dots); **1 battleship** (4 dots). An example is shown to the right.

Each player names a co-ordinate in turn. The other player announces “*hit*” or “*miss*”. When each of the dots representing a ship have been hit, the opponent announces, “*ship destroyed*”. The first player to lose **all** of his/her ships is the loser.

**Exercise 10 (Revision)**

2

Draw x and y axes on a piece of squared paper. Number the x -axis from -18 to 18 , and the y -axis from -26 to 16 . Plot the following points, connecting them with straight lines as you go on. (Work row-by-row.)

(-13, -25)	(-11, -22)	(-8, -18)	(-6, -15)	(-5, -12)	(-5, -11)	(-6, -10)
(-7, -10)	(-8, -9)	(-7, -7)	(-6, -7)	(-5, -8)	STOP	(-7, -9)
(-6, -8)	STOP	(-6.5, -8.5)	(-5, -10)	STOP	(-7, -25)	(-6, -23)
(-4, -21)	STOP	(-5, -22)	(-3, -23)	(1, -23)	(4, -21)	STOP
(6, -25)	(4, -21)	(4, -17)	(5, -14)	(7, -13)	(10, -12)	(13, -10)
(13, -9)	(11, -6)	STOP	(-2, -11)	(2, -13)	(7, -13)	STOP
(-3, -12)	(-1, -10)	STOP	(6, -6)	(10, -6)	(11, -7)	(11, -8)
(10, -9)	(8, -9)	STOP	(6, -2)	(7, -1)	(9, 0)	(11, 0)
(13, -2)	(13, -4)	(12, -6)	(11, -6)	STOP	(4, -8)	(0, -8)
(-2, -6)	(-2, -4)	(-1, -2)	(2, 0)	(4, 0)	(6, -2)	(7, -4)
(6, -6)	(4, -8)	STOP	(-2, -5)	(-4, -4)	STOP	(-1, -2)
(-3, 0)	STOP	(2, 0)	(1, 2)	STOP	(4, 0)	(5, 2)
STOP	(9, 0)	(8, 2)	STOP	(11, 0)	(12, 2)	STOP
(12, -1)	(14, 0)	STOP	(13, -3)	(15, -2)	STOP	(-5, -12)
(-9, -13)	(-10, -9)	(-14, -8)	(-11, -4)	(-15, -1)	(-10, 2)	(-12, 7)
(-6, 7)	(-3, 13)	(1, 9)	(6, 14)	(8, 9)	(14, 10)	(12, 4)
(17, 2)	(13, -2)	A big dot at (1, -6) and at (9, -5).				

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...



Mid-point of a Line

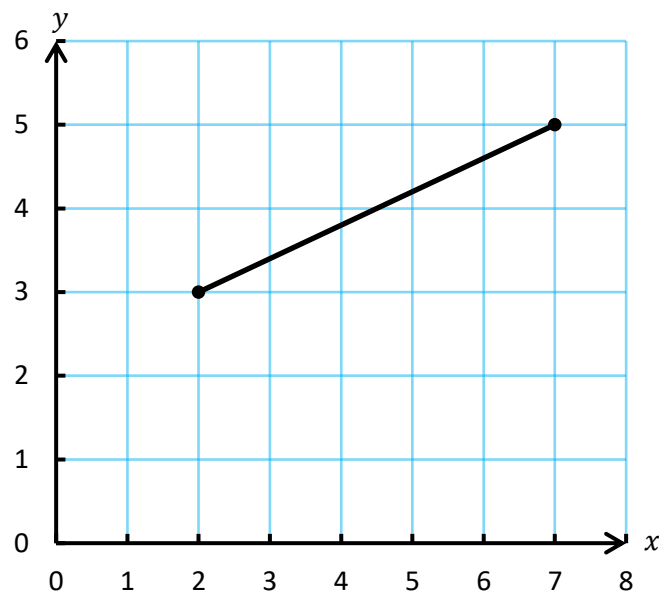
It is possible to use a straight line to connect any two different co-ordinates. What is the **mid-point** of a straight line of this type?

Example

Let us consider the two coordinates (2, 3) and (7, 5), as shown in the graph on the right.

We can see from the graph that the mid-point of the line joining (2, 3) to (7, 5) is (4.5, 4), but it is possible to calculate this *without* using the graph. To do this, we use the **mean** of the *x*-coordinates and the mean of the *y*-coordinates.

- The *x*-coordinates are 2 and 7. The total of 2 and 7 is 9, so the **mean of the *x*-coordinates** is $9 \div 2 = 4.5$.
- The *y*-coordinates are 3 and 5. The total of 3 and 5 is 8, so the **mean of the *y*-coordinates** is $8 \div 2 = 4$.
- It follows that the mid-point of the line joining (2, 3) to (7, 5) is (4.5, 4).



Exercise 11

Find the mid-point of the straight line joining the following pairs of co-ordinates.

- | | | | |
|-------------------------|---------------------------|---------------------------|---------------------------|
| (a) (2, 6) and (8, 2) | (b) (4, 5) and (2, 3) | (c) (9, 6) and (3, 10) | (d) (11, 7) and (9, 1) |
| (e) (2, 4) and (3, 8) | (f) (4, 1) and (10, 6) | (g) (1, 2) and (2, 1) | (h) (10, 4) and (5, 12) |
| (i) (1, 6) and (12, 12) | (j) (24, 10) and (14, 18) | (k) (34, 2) and (26, 5) | (l) (0, 4) and (6, 4) |
| (m) (9, 9) and (8, 8) | (n) (18, 4) and (20, 0) | (o) (13, 17) and (14, 16) | (p) (45, 67) and (83, 24) |

2

Skill



Adding Negative Numbers

In Exercise 11, the co-ordinates all came from the first quadrant. To find the mid-point of lines joining co-ordinates in other quadrants, we must first learn how to work with addition sums that contain negative numbers.

Two Sided Counters

Two sided counters have a **yellow** face and a **red** face. The **yellow** face represents +1 (positive one) and the **red** face represents -1 (negative one).



Example

+3



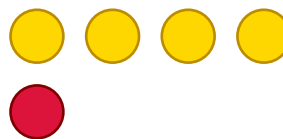
-4



0



+3



One yellow counter and one red counter gives a **zero pair**. It is possible to remove or add zero pairs to any set of counters without affecting the value of the counters.



Exercise 12

A

Which numbers are shown below?

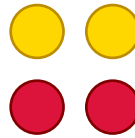
(a)



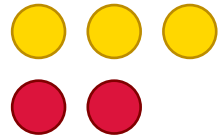
(b)



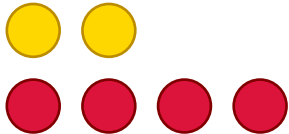
(c)



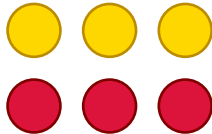
(d)



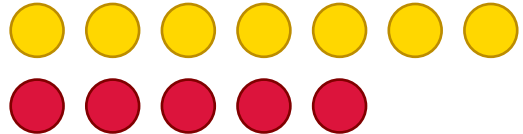
(e)



(f)



(g)



It is possible to use two sided counters to find the answer to addition sums that contain negative numbers.

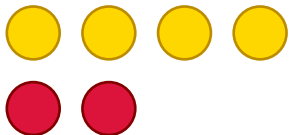
Example

$4 + -2$

Start with 4 yellow counters



Add 2 red counters



Remove any zero pairs



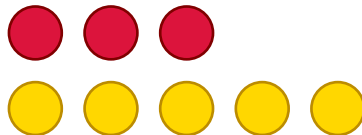
Count what is left: 2

$-3 + 5$

Start with 3 red counters



Add 5 yellow counters



Remove any zero pairs



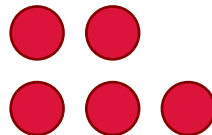
Count what is left: 2

$-2 + -3$

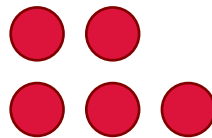
Start with 2 red counters



Add 3 red counters



Remove any zero pairs



Count what is left: -5

**Exercise 13**

A

Use two sided counters to calculate the answer to the following sums.

(a) $5 + 2$

(b) $5 + -2$

(c) $-5 + 2$

(d) $-5 + -2$

(e) $6 + -1$

(f) $-1 + 6$

(g) $-6 + 1$

(h) $1 + -6$

(i) $7 + -3$

(j) $3 + -7$

(k) $-3 + -7$

(l) $-7 + 3$

(m) $3 + -6$

(n) $-6 + 3$

(o) $-3 + -6$

(p) $6 + -3$

(q) $9 + -2$

(r) $7 + -6$

(s) $-2 + -6$

(t) $-3 + -4$

(u) $-3 + 7$

(v) $-4 + 2$

(w) $4 + -8$

(x) $2 + -2$

(y) $3 + -10$

(z) $-4 + 6$

(\alpha) $-2 + -3$

(\beta) $12 + -3$



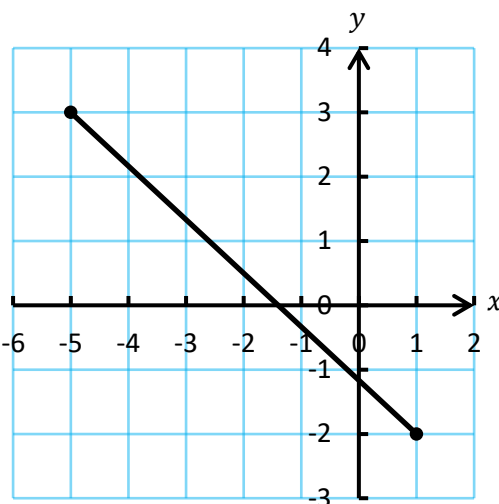
Mid-point of Lines in the Four Quadrants

Let us consider the two co-ordinates $(1, -2)$ and $(-5, 3)$, as shown in the graph on the right.

We can see from the graph that the mid-point of the line joining $(1, -2)$ to $(-5, 3)$ is $(-2, 0.5)$, but it is possible to calculate this *without* using the graph. To do this, we use (as before) the **mean** of the x -coordinates and the mean of the y -coordinates.



- The x -coordinates are 1 and -5 .
The total of 1 and -5 is $1 + -5 = -4$,
so the **mean of the x -coordinates** is $-4 \div 2 = -2$.
- The y -coordinates are -2 and 3 . The total of -2 and 3 is $-2 + 3 = 1$, so the **mean of the y -coordinates** is $1 \div 2 = 0.5$.
- It follows that the mid-point of the line joining $(1, -2)$ to $(-5, 3)$ is $(-2, 0.5)$.



Exercise 14

Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) $(4, -2)$ and $(2, 8)$ (b) $(8, 4)$ and $(-2, 6)$ (c) $(9, -4)$ and $(5, 2)$ (d) $(-4, -6)$ and $(10, 12)$
 (e) $(4, 2)$ and $(-10, -6)$ (f) $(-8, 5)$ and $(1, 3)$ (g) $(-6, -2)$ and $(11, 15)$ (h) $(34, -5)$ and $(-4, 10)$
 (i) $(-8, 14)$ and $(2, -6)$ (j) $(-4, -6)$ and $(-2, -4)$ (k) $(-5, 4)$ and $(-7, -12)$ (l) $(-14, 0)$ and $(-6, -8)$
 (m) $(15, 13)$ and $(11, -1)$ (n) $(-25, -20)$ and $(7, -4)$ (o) $(-1, 0)$ and $(0, -1)$ (p) $(-12.5, -3.5)$ and $(0.5, -1.5)$



Exercise 15

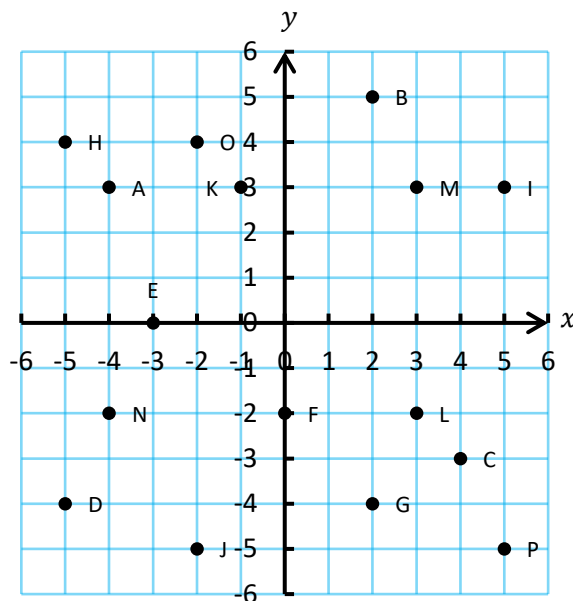
Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) A and B (b) C and D
 (c) E and F (d) G and H
 (e) I and J (f) K and L
 (g) M and N (h) O and P

Exercise 16 (Revision)

Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) $(9, 3)$ and $(1, 9)$ (b) $(9, -3)$ and $(1, 9)$
 (c) $(-9, 3)$ and $(1, 9)$ (d) $(-9, -3)$ and $(-1, -9)$



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Using Co-ordinates

Applying

2

Missing Co-ordinates



Exercise 17

In the diagram on the right, the length of the sides of the square **ABCD** is 6 units. Find the co-ordinates of the points **B**, **C** and **D**.

Exercise 18

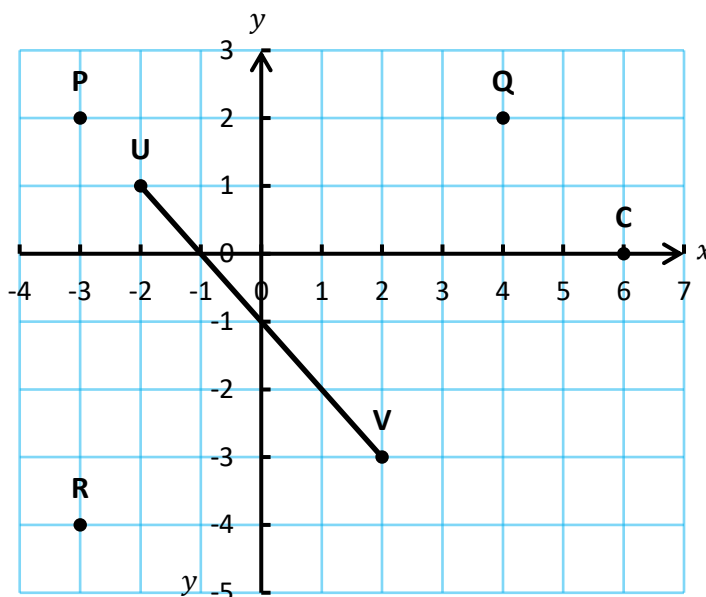
Hayley plots a square on graph paper. The co-ordinates of three of the corners are $(-4, 4)$, $(-1, 4)$ and $(-4, 1)$. What is the co-ordinate of the **fourth corner** of the square?

Exercise 19

Guto plots a rectangle on graph paper. The co-ordinates of three of the corners are $(2, -1)$, $(5, -1)$ and $(5, -5)$. What is the co-ordinate of the **fourth corner** of the rectangle?

Exercise 20

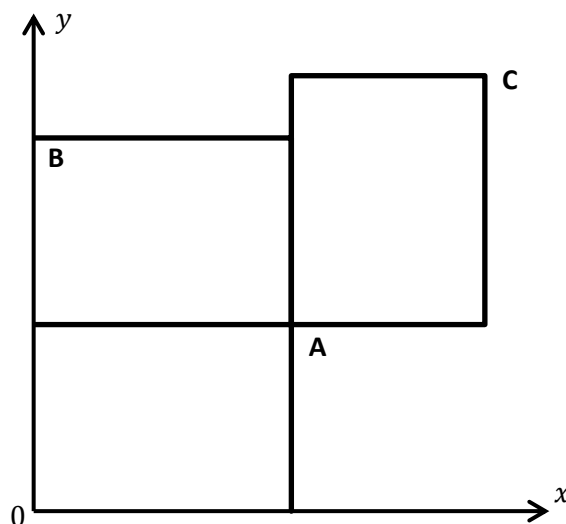
- Write down the co-ordinates of the point **P**.
- The point **W** lies on the line **UV**. The y -coordinate of **W** is -2 . What is **W**'s x -coordinate?
- What are the co-ordinates of the mid-point of the line **UV**?
- The co-ordinates of the point $(1, 3)$ add to 4. What number do you get if you add the co-ordinates of any point on the line **UV**?
- Write down the co-ordinates of the point **S** so that **PQRS** forms a rectangle.
- CD** is a vertical line. What would be the co-ordinates of the point **D** if **CD** was 4 units in length and in the fourth quadrant?



Exercise 21

The diagram on the right shows 3 rectangles, each of them 4 units by 3 units.

- Find the co-ordinates of the points **A**, **B** and **C**.
- What would be the co-ordinates of the mid-point of the line joining **A** to **B**?
- What would be the co-ordinates of the mid-point of the line joining **A** to **C**?

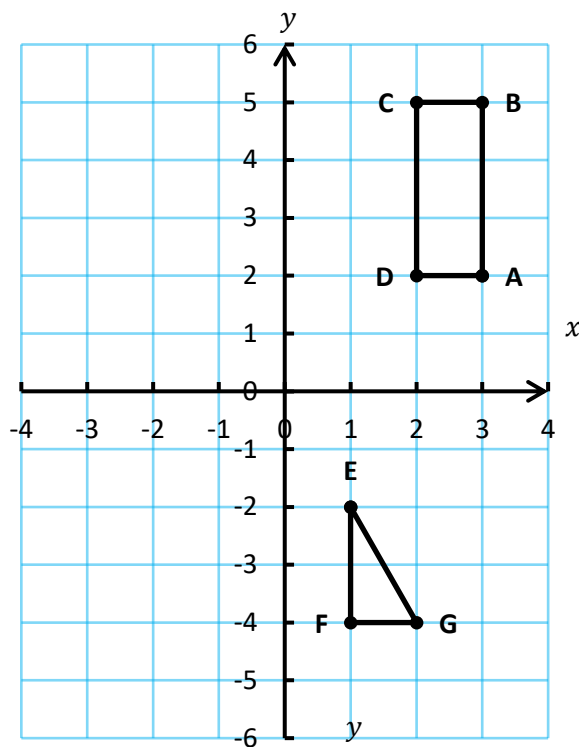


Translation

2

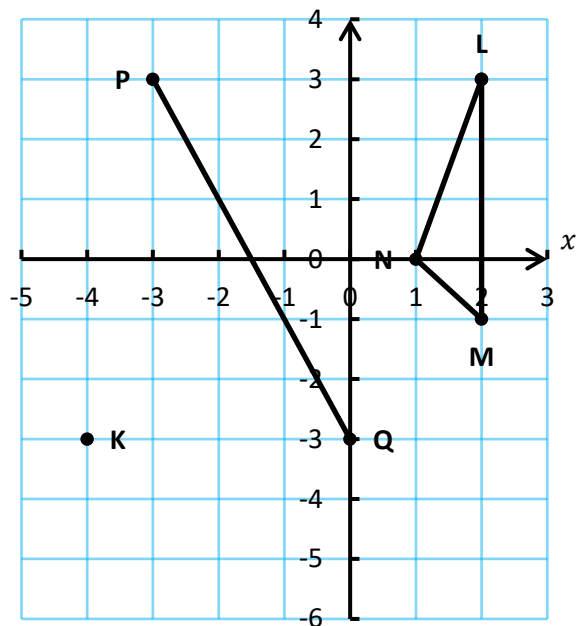
Exercise 22

- (a) The rectangle **ABCD** is translated 4 units to the left. Write down the co-ordinates of the new vertices.
- (b) The rectangle **ABCD** is translated 1 unit to the right and 7 units down. Write down the co-ordinates of the new vertices.
- (c) The rectangle **ABCD** is translated so that its vertices move to the co-ordinates (0, 1), (1, 1), (1, 4), (0, 4). What vector column represents this translation?
- (d) The triangle **EFG** is translated 3 units to the left and 2 units up. Write down the co-ordinates of the new vertices.



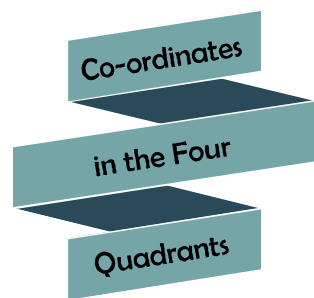
Exercise 23 (Revision)

- (a) Write down the co-ordinates of the point **K**.
- (b) The point **R** lies on the line **PQ**. The y -coordinate of **R** is 1. What is the x -coordinate of **R**?
- (c) What are the co-ordinates of the mid-point of the line **PQ**?
- (d) The shape **LMN** is translated 4 units to the left and 5 units down. Write down the co-ordinates of the new vertices.
- (e) The line **PQ** forms one side of a square. What could be the co-ordinates of the other two corners?



Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

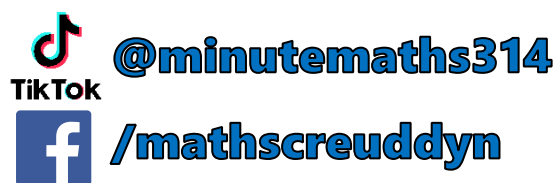


Reflection Sheet

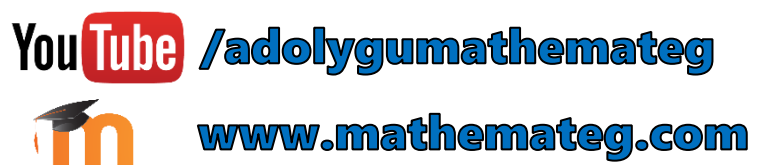
Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
I know how to plot co-ordinates in the first quadrant , e.g. (5, 2).			2	
I know how to recognise co-ordinates that have been plotted in the first quadrant .			1	
I know what the co-ordinate of the origin is.			5	
Given a co-ordinate such as (7, 3), I know what the x-coordinate is and what the y-coordinate is.			3, 4	
I know how to plot co-ordinates in all four quadrants , e.g. (-7, 3).			7	
I know how to recognise co-ordinates that have been plotted in all four quadrants .			11	
I know how to recognise in which quadrant a given co-ordinate appears.			8	
I know how to add a negative number , e.g. $5 + -2$.			6	
I can find the mid-point of a straight line that connects a pair of co-ordinates.			9, 11	
I can find missing co-ordinates in diagrams.			11	
I can translate co-ordinates.			10	



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 /adolygumathemateg