

Uned 6 Pelloch Haf 2022

1)  $v = \frac{24}{4x+9}$

a)  $v = 24(4x+9)^{-1}$

$$a = \frac{dv}{dt}$$

$$a = \frac{dx}{dt} \times \frac{dv}{dx} \quad [\text{rheol y gadwyn}]$$

$$a = v \times \frac{dv}{dx}$$

$$a = \left( \frac{24}{4x+9} \right) \times 24(-1)(4x+9)^{-2}(4)$$

$$a = \frac{-2304}{(4x+9)^3}$$

b) i) Os yw  $t = T$ , mae  $a = -\frac{4}{3} \text{ ms}^{-2}$ .

Yn yr achos hwn, mae  $-\frac{4}{3} = \frac{-2304}{(4x+9)^3}$

$$-\frac{4}{3}(4x+9)^3 = -2304$$

$$(4x+9)^3 = \frac{-2304 \times 3}{-4}$$

$$(4x+9)^3 = 1728$$

$$4x+9 = 12$$

$$4x = 3$$

$$x = \frac{3}{4} \text{ m}$$

$$\text{ii) } v = \frac{24}{4x+9}$$

$$\frac{dx}{dt} = \frac{24}{4x+9}$$

$$(4x+9)dx = 24dt$$

$$\int (4x+9) dx = \int 24 dt$$

$$2x^2 + 9x = 24t + K$$

Os yw  $x = -2$  pan fo  $t = 0$ , yna

$$2(-2)^2 + 9(-2) = 24(0) + K$$

$$-10 = K$$

$$\text{Felly } 2x^2 + 9x = 24t - 10$$

Pan fo  $t = T$ , mae  $x = \frac{3}{4}$ , felly

$$2\left(\frac{3}{4}\right)^2 + 9\left(\frac{3}{4}\right) = 24T - 10$$

$$7.875 = 24T - 10$$

$$17.875 = 24T$$

$$T = \frac{143}{192}$$

$$192$$

$$T = 0.745 \text{ i 2 le degol}$$

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2)  $x = \sin(\pi t) + \sqrt{3} \cos(\pi t)$ .

a) (i)  $x = \sin(\pi t) + \sqrt{3} \cos(\pi t)$

$$\frac{dx}{dt} = \pi \cos(\pi t) - \pi\sqrt{3} \sin(\pi t)$$

$$\frac{d^2x}{dt^2} = -\pi^2 \sin(\pi t) - \pi^2\sqrt{3} \cos(\pi t)$$

$$\frac{d^2x}{dt^2} = -\pi^2 (\sin(\pi t) + \sqrt{3} \cos(\pi t))$$

$$\frac{d^2x}{dt^2} = -\pi^2 x$$

Mae hwn yn hafaliad ar gyfer Mudiant Harmonig Symbl, gan fod  $\frac{d^2x}{dt^2}$  o'r ffurf  $-w^2x$ .

Mae'r mudiant yn osgiliadu rhwng y gwerthoedd ar gyfer yr osgled  $a$ ,  $-a$ ; gwerth  $x$  ar ganol y mudiant yw  $x = 0$ .

(ii) Cyfnod y mudiant =  $\frac{2\pi}{w}$

$$= \frac{2\pi}{\pi}$$

$$= 2s \quad \checkmark$$

Beth yw'r osgled  $a$ ?

Hwn yw'r dadlediad pan fôr gronyn yn llonydd,

h.y.  $v = 0$

$$\frac{dx}{dt} = 0$$

$$\pi \cos(\pi t) - \pi \sqrt{3} \sin(\pi t) = 0$$

$$\cos(\pi t) - \sqrt{3} \sin(\pi t) = 0$$

$$\cos(\pi t) = \sqrt{3} \sin(\pi t)$$

$$\frac{1}{\sqrt{3}} = \frac{\sin(\pi t)}{\cos(\pi t)}$$

$$\frac{1}{\sqrt{3}} = \tan(\pi t)$$

$$\pi t = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$t = \frac{1}{\pi} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$t = \frac{1}{\pi} \left(\frac{\pi}{6}\right)$$

$$t = \frac{1}{6} \text{ s}$$

Y dudleoliad ar gyfer yr amser yma yw

$$x = \sin(\pi t) + \sqrt{3} \cos(\pi t)$$

$$x = \sin\left(\frac{\pi}{6}\right) + \sqrt{3} \cos\left(\frac{\pi}{6}\right)$$

$$x = \frac{1}{2} + \sqrt{3} \left(\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{1}{2} + \frac{3}{2}$$

$$\underline{x = 2 \text{ m}}$$

Fellyr osgled yw 2m

(b) cyfnod  $Q = 2s$  fel bod  $\omega = \pi$   
 Pan fo  $t = 0$ ,  $x = 0m$   
 Pan fo  $x = 2\sqrt{3}m$ ,  $v = 2\pi ms^{-1}$

Gan ddefnyddio  $v^2 = \omega^2(a^2 - x^2)$   
 $(2\pi)^2 = \pi^2(a^2 - (2\sqrt{3})^2)$   
 $4\pi^2 = \pi^2(a^2 - 12)$   
 $4 = a^2 - 12$   
 $16 = a^2$   
 $a = \pm\sqrt{16}$   
 $a = \pm 4m$

Felly'r osgled yw 4m

(c) Yn gyffredinol, mae mudiant  $Q$  o'r ffurf  $x = a \sin(\omega t + \epsilon)$ .  
 Gwyddom fod  $a = 4$ ,  $\omega = \pi$ , a  $\epsilon = 0$  gan fod  $x = 0$  pan fo  $t = 0$ . Felly  $x = 4 \sin(\pi t)$ .

Pryd mae  $P$  a  $Q$  yn cyfarfod am y tro cyntaf?

(P)  $x = \sin(\pi t) + \sqrt{3} \cos(\pi t)$       (Q)  $x = 4 \sin(\pi t)$

Rydym angen  $\sin(\pi t) + \sqrt{3} \cos(\pi t) = 4 \sin(\pi t)$   
 $\sqrt{3} \cos(\pi t) = 3 \sin(\pi t)$   
 $\frac{\sqrt{3}}{3} = \tan(\pi t)$

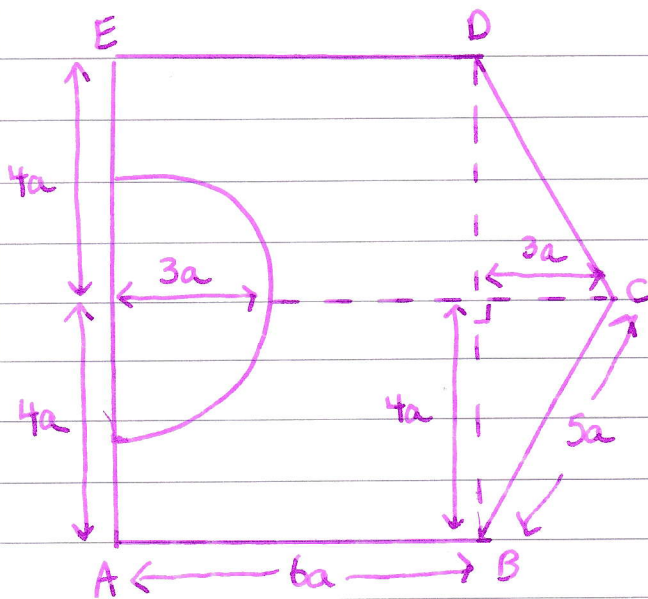
$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \pi t$

Dewis yr  
 ongl leiaf  
 posib

$\frac{\pi}{6} = \pi t$   
 $t = \frac{1}{6} s$

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3)



(a) Trwy gymesuredd, pellter craidd mäs y lamina o AB yw 4a.

(b) Siâp	Amlynebedd	Pellter craidd mäs o AE
Peblyd ABDE	$6a \times 8a = 48a^2$	3a
Triangl BDC	$\frac{1}{2} \times 8a \times 3a = 12a^2$	$6a + a = 7a$
ffanner cylch	$\frac{\pi r^2}{2} = \frac{\pi(3a)^2}{2} = 4.5\pi a^2$	$\frac{\frac{4r}{3\pi} = \frac{4(3a)}{3\pi} = \frac{4a}{\pi}}$
Lamina	$48a^2 + 12a^2 - 4.5\pi a^2 = a^2(60 - 4.5\pi)$	$\bar{x}$

Yn cymryd momentau o amgylch AE:

$$a^2(60 - 4.5\pi) \bar{x} = 48a^2 \times 3a + 12a^2 \times 7a - 4.5\pi a^2 \left(\frac{4a}{\pi}\right)$$

$$(60 - 4.5\pi) \bar{x} = 144a + 84a - 18a$$

$$(60 - 4.5\pi) \bar{x} = 210a$$

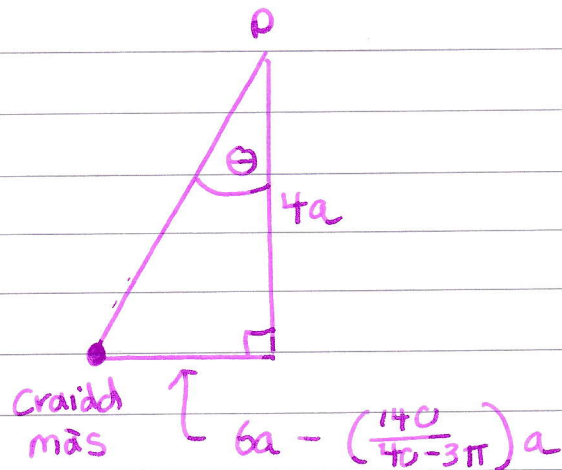
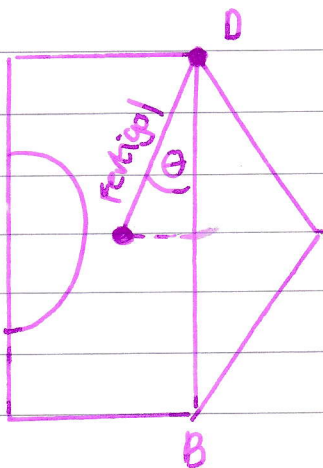
$$\bar{x} = \frac{210a}{60 - 4.5\pi}$$

$$\bar{x} = \frac{420a}{120 - 9\pi}$$

$$\bar{x} = \frac{140a}{40 - 3\pi}$$

$$\bar{x} = \frac{140}{40 - 3\pi} a \quad \checkmark$$

(c) i) Mae  $\frac{140}{40-3\pi} \approx 4.58$ .



$$\tan \theta = \frac{\text{cyferbyrn}}{\text{agos}}$$

$$\tan \theta = \frac{6a - \left(\frac{140}{40-3\pi}\right)a}{4a}$$

$$\tan \theta = \frac{6 - \frac{140}{40-3\pi}}{4}$$

$$\theta = \tan^{-1} \left( \frac{6 - \frac{140}{40-3\pi}}{4} \right)$$

$$\theta = \underline{19.56^\circ} \text{ i 2 ledogol}$$

ii) Os yw AB i fod yn llobrweddol mae angen i BD fod yn fertigol.

Gwrthrych	Mäs	Pellter craidd mäs o AE
Lamina	M	$\frac{140}{40-3\pi} a$
Gronyn	KM	$9a$
System	$M+KM = M(1+K)$	$\bar{x}$

Yn cymryd momentau o amgylch AE:

$$M(1+K)\bar{x} = M\left(\frac{140}{40-3\pi}\right)a + KM(9a)$$

$$(1+K)\bar{x} = \frac{140}{40-3\pi}a + 9Ka.$$

Rydym angen  $\bar{x} = 6a$ .

$$(1+K)6a = \frac{140}{40-3\pi}a + 9Ka$$

$$6(1+K) = \frac{140}{40-3\pi} + 9K$$

$$6 + 6K = \frac{140}{40-3\pi} + 9K$$

$$6 - \frac{140}{40-3\pi} = 3K$$

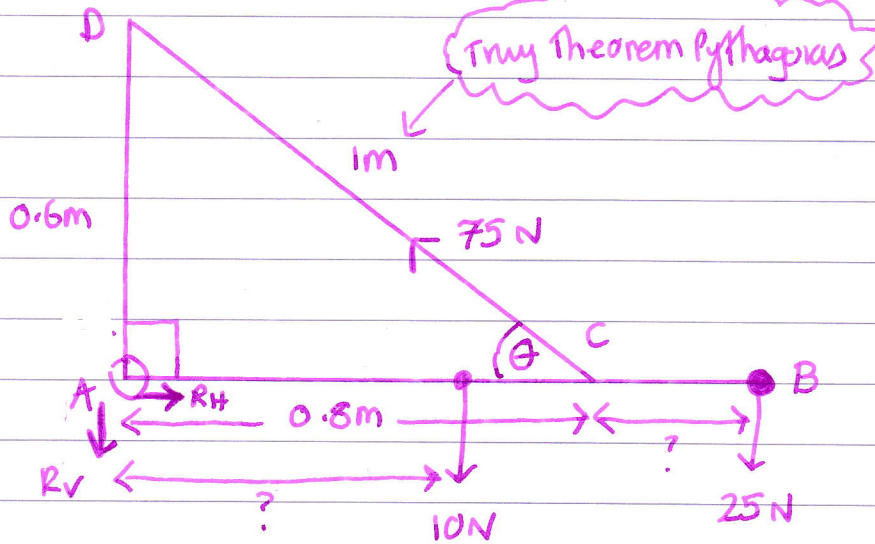
$$2 - \frac{140}{3(40-3\pi)} = K$$

$$K = 2 - \frac{140}{120-9\pi}$$

$$\underline{K = 0.4737} \text{ i 4 lle degol}$$

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4)



(a) Gadewch i hyd y rhodlen AB fod yn  $x$ .

Yn cymryd momentau o amgylch A:

Momentau cloccwedd = momentau gwrthglodwedd

$$\frac{x \times 10 + x \times 25}{2} = 0.8 \times 75 \sin \theta$$

$$5x + 25x = 0.8 \times 75 \times \left(\frac{0.6}{1}\right)$$

$$30x = 36$$

$$x = \frac{36}{30}$$

$$= 1.2$$

$$x = \underline{\underline{1.2m}}$$

(b) Ar gyfer rhodlen AB, yn defnyddio grymoedd i fyny = grymoedd i lawr:

$$75 \times \sin \theta = R_v + 10 + 25$$

$$75 \times \left(\frac{0.6}{1}\right) = R_v + 35$$

$$45 = R_v + 35$$

$$R_v = \underline{\underline{10N}}$$

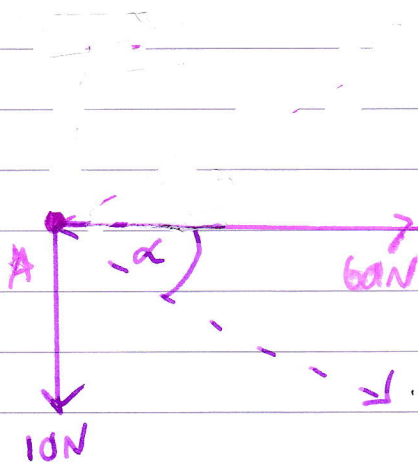
Ar gyfer rhodlen AB, yn defnyddio grymoedd i'r dde = grymoedd i'r chwith:

$$R_H = 75 \times \cos \theta$$

$$R_H = 75 \times \left(\frac{0.8}{1}\right)$$

$$R_H = 60 \text{ N (i'r dde)}$$

Adwaith yn A:



Maint yr adwaith

$$= \sqrt{10^2 + 60^2}$$

$$= 10\sqrt{37} \text{ N}$$

$$= \underline{60.83 \text{ N}} \text{ i 2 le degol}$$

Cyfeiriad yr adwaith

$$\tan \alpha = \frac{\text{cyferbyrn}}{\text{agos}}$$

$$\tan \alpha = \frac{10}{60}$$

$$\alpha = \tan^{-1}\left(\frac{1}{6}\right)$$

$$\alpha = \underline{9.46^\circ} \text{ i 2 le degol,}$$

o dan y llorweddol.

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5) a) Eguyddor Cadwraeth Momentum:

Momentum A cyn y gwrthdrawiad + Momentum B cyn y gwrthdrawiad  
= Momentum A ar ôl y gwrthdrawiad + Momentum B ar ôl y gwrthdrawiad

$$4(c\mathbf{i} + b\mathbf{j}) + 2(c\mathbf{i} + d\mathbf{j}) = 4(-2\mathbf{i} - 5\mathbf{j}) + 2(\mathbf{i} + 3\mathbf{j})$$

$$4a\mathbf{i} + 4b\mathbf{j} + 2c\mathbf{i} + 2d\mathbf{j} = -8\mathbf{i} - 20\mathbf{j} + 2\mathbf{i} + 6\mathbf{j}$$

$$(4a + 2c)\mathbf{i} + (4b + 2d)\mathbf{j} = -6\mathbf{i} - 14\mathbf{j} \quad \text{--- (1)}$$

Mae llinell y canolau yn baralel i  $\mathbf{j}$  felly gallun ddefnyddio deddf adferiad Newton efor cyfernodau  $\mathbf{i}$ .

$$v_B - v_A = -e(u_B - u_A)$$

$$1 - -2 = -\frac{2}{5}(c - a)$$

$$3 = -\frac{2}{5}(c - a)$$

$$15 = -2(c - a)$$

$$-7.5 = c - a$$

$$a - 7.5 = c$$

Yn amnewid  $c$  mewn i (1), ar gyfer y cyfernodau  $\mathbf{j}$ :

$$4a + 2c = -6$$

$$4a + 2(a - 7.5) = -6$$

$$4a + 2a - 15 = -6$$

$$6a = 9$$

$$\underline{a = 1.5}$$

$$\text{Felly } a - 7.5 = c$$

$$1.5 - 7.5 = c$$

$$\underline{c = -6}$$

Nid gwir cyfernodau  $\mathbf{j}$  yn newid felly  $b = -5$ ,  $d = 3$

Cyflymder A cyn y gwrthdrawiad yw  $1.5\mathbf{i} - 5\mathbf{j}$

Cyflymder B cyn y gwrthdrawiad yw  $-6\mathbf{i} + 3\mathbf{j}$ .

b) Ma'r wal yn baralel i'r factor  $\underline{i}$  gannad oes cydran  $\underline{i}$  gan yr ergyd.

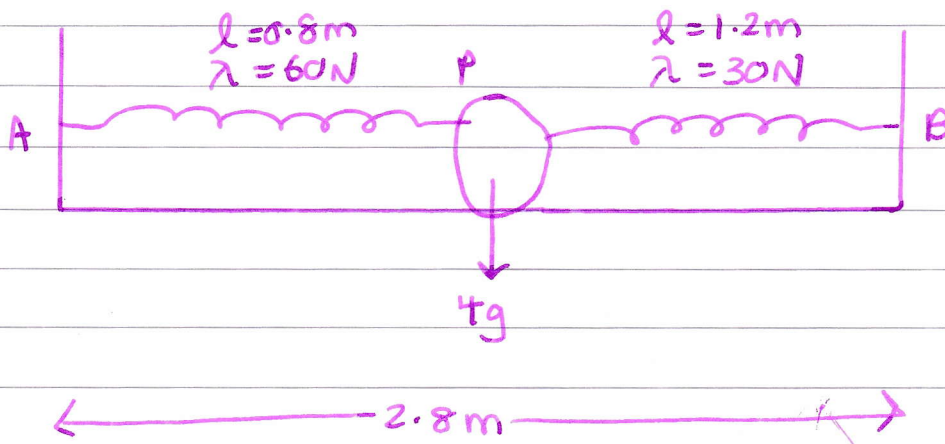
c) Ergyd a gunaeth y wal ei roi ar  $A =$  newid momentum  $A$   
 $0\underline{i} + 32\underline{j} = 4(\underline{e}_i + \underline{f}_j) - 4(-2\underline{i} - 5\underline{j})$   
 $8\underline{j} = \underline{e}_i + \underline{f}_j + 2\underline{i} + 5\underline{j}$   
 $-2\underline{i} + 3\underline{j} = \underline{e}_i + \underline{f}_j$

Felly cyflymder  $A$  ar ôl y gwrthdrawiad â'r wal yw  $-2\underline{i} + 3\underline{j}$ .  
Buanedd  $A$  ar ôl y gwrthdrawiad â'r wal =  $\sqrt{(-2)^2 + 3^2}$   
 $= \sqrt{13} \text{ ms}^{-1}$ .

ch) Colled meun egni cinetig = EC  $A$  gynt - EC  $A$  wedyn  
 $= \frac{1}{2} m u^2 - \frac{1}{2} m v^2$   
 $= \frac{1}{2} \times 4 \times (\sqrt{(-2)^2 + (-5)^2})^2 - \frac{1}{2} \times 4 \times (\sqrt{13})^2$   
 $= 2 \times (\sqrt{29})^2 - 2 \times (\sqrt{13})^2$   
 $= 58 - 26$   
 $= \underline{\underline{32 \text{ J}}}$

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b)



a) Yn y pwynt ecwilibriwm, tensiwn yn AP = tensiwn yn PB

$$\frac{\lambda_A x_A}{l_A} = \frac{\lambda_B x_B}{l_B}$$

$$\frac{60 x_A}{0.8} = \frac{30 x_B}{1.2}$$

$$75 x_A = 25 x_B \quad \text{--- (1)}$$

Ona hefyd rhaid bod  $x_A + x_B = 2.8 - l_A - l_B$

$$x_A + x_B = 2.8 - 0.8 - 1.2$$

$$x_A + x_B = 0.8$$

$$x_A = 0.8 - x_B.$$

Yn amnewid i mewn i (1):  $75(0.8 - x_B) = 25x_B$

$$60 - 75x_B = 25x_B$$

$$60 = 100x_B$$

$$x_B = 0.6 \text{ m}$$

$$\text{Felly } x_A = 0.8 - 0.6$$

$$x_A = 0.2 \text{ m}$$

Y pellter AC yw  $l_A + x_A$

$$= 0.8 + 0.2$$

$$= 1 \text{ m} \quad \checkmark$$

b) i) Ystyriwn estyniad cyffredinol  $x$  o'r safle cydbuysedd ( $\rightarrow = +$ if).

$F = ma$  ar P, yn llonreddol,  $\rightarrow = +$ if.

$$-\frac{\lambda_A x_A}{l_A} + \frac{\lambda_B x_B}{l_B} = 4a$$

$$-\frac{60x_A}{0.8} + \frac{30x_B}{1.2} = 4a$$

$$-75x_A + 25x_B = 4a$$

$$-75x + 25(-x) = 4a$$

$$-100x = 4a$$

$$-25x = a$$

$$a = -25x$$

$$\frac{d^2x}{dt^2} = -25x$$

Yn cymharu efo'r hafaliad  $\frac{d^2x}{dt^2} = -\omega^2 x$  ar gyfer mudiant

harmonig syml, gwelwn bod  $\omega^2 = 25$ .

$$\begin{aligned} \text{Cyfnod y mudiant yw } \frac{2\pi}{\omega} &= \frac{2\pi}{\sqrt{25}} \\ &= \frac{2\pi}{5} \text{ s.} \end{aligned}$$

ii) Osgled  $a$  y mudiant yw  $1.4 - 1 = 0.4 \text{ m}$

Yn defnyddio  $x = a \sin(\omega t + \epsilon)$ , gan fesur o'r safle cydbuysedd fel bod  $\epsilon = 0$ , rydym angen darganfod yr amser pan fo  $x = -0.2 \text{ m}$  (dim tyniant yn AP) ac  $x = 0.4 \text{ m}$  (safle hydychau).

$$x = -0.2\text{m}$$

$$-0.2 = 0.4 \sin(5t)$$

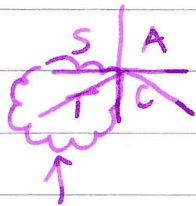
$$-0.5 = \sin(5t)$$

$$\sin^{-1}(-0.5) = 5t$$

$$\frac{7\pi}{6} = 5t$$

$$t = \frac{7\pi}{30}$$

$$t = 0.7330382858\text{s.}$$



Amser  
cynharaf

$$x = 0.4\text{m}$$

$$0.4 = 0.4 \sin(5t)$$

$$1 = \sin(5t)$$

$$\sin^{-1}(1) = 5t$$

$$\frac{\pi}{2} = 5t$$

$$t = \frac{\pi}{10}$$

$$t = 0.3141592654$$

Felly'r amser lleiaf maen ei gymryd i P gyrraedd safle lle  
does dim tyniant yn y sbring AP yw

$$0.7330382858 - 0.3141592654$$

$$= \underline{0.4189\text{s}} \text{ i 4 lle degol.}$$