Uned 4 Pellach Haf 2022

1) $f(x) = \cosh^3 x - 3\cosh x$ Parth $(-\infty, \infty)$

a) $f(x) = (\cosh x)^3 - 3\cosh x$

$f'(x) = 3(\cosh x)^2 \sinh x - 3\sinh x$

$= 3\sinh x(\cosh^2 x - 1)$

Pwyntiau arhosol $\ni$ $f'(x) = 0$

$3\sinh x(\cosh^2 x - 1) = 0$

Naill ai $3\sinh x = 0$ neu $\cosh^2 x - 1 = 0$

$\sinh x = 0$ $\cosh^2 x = 1$

$x = \sinh^{-1}(0)$ $\cosh x = \pm\sqrt{1}$

$x = 0$ Naill ai $\cosh x = 1$ neu $\cosh x = -1$

$x = \cosh^{-1}(1)$ $x = \cosh^{-1}(-1)$

$x = 0$ Dim datrysiad

Mae'r unig bwynt arhosol yn digwydd pan fo $x = 0$.
Mae $\cosh^3(0) - 3\cosh(0) = -2$ felly'r unig bwynt arhosol yw $(0, -2)$.

b) $f'(x) = 3\sinh x(\cosh^2 x - 1)$

$f''(x) = 3\sinh x(2\cosh x \sinh x) + 3\cosh x(\cosh^2 x - 1)$

$= 6\sinh^2 x \cosh x + 3\cosh^3 x - 3\cosh x$

Os yw $x = 0$, $f''(x) = 6\sinh^2(0)\cosh(0) + 3\cosh^3(0) - 3\cosh(0)$

$= 6(0)(1) + 3(1) - 3(1)$

$= 0$

Nid oes modd dod i gasgliad ar hyn o bryd.

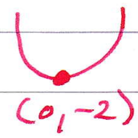
Os yw $x = -0.1$, $f'(x) = 3\sinh(-0.1)(\cosh^2(-0.1) - 1)$

$= -0.003015\ldots$

Os yw $x = 0.1$, $f'(x) = 3\sinh(0.1)(\cosh^2(0.1) - 1)$

$= 0.003015\ldots$

Mae'r graddiant yn negatif cyn $x = 0$ ac yn bositif ar ôl $x = 0$ felly mae $(0, -2)$ yn bwynt minimum.



c) Amrediad $y = \cosh(x)$ yw $[1, \infty)$

Gwerth lleiaf posib $f(x)$ yw $f(x) = 1^3 - 3\times 1$

$= -2$

Felly amrediad mwyaf posib $f(x)$ yw $[-2, \infty)$

2) $z = 9 - 3\sqrt{3}\,i$

Ffurf Trigonomebreg:

$r = \sqrt{9^2 + (-3\sqrt{3})^2}$ $\quad\quad \theta = \tan^{-1}\left(\frac{-3\sqrt{3}}{9}\right)$

$r = \sqrt{108}$ $\quad\quad \theta = -\frac{\pi}{6}$

$z = \sqrt{108}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

Nawr gadewch i $z = \sqrt[4]{9 - 3\sqrt{3}\,i}$

$$z^4 = 9 - 3\sqrt{3}\,i$$
$$z^4 = \sqrt{108}\left(\cos\left(-\tfrac{\pi}{6}\right) + i\sin\left(-\tfrac{\pi}{6}\right)\right)$$
$$z^4 = 108^{\frac{1}{2}}\left(\cos\left(-\tfrac{\pi}{6}\right) + i\sin\left(-\tfrac{\pi}{6}\right)\right)$$
$$z = 108^{\frac{1}{8}}\left(\cos\left(-\tfrac{\pi}{6}\right) + i\sin\left(-\tfrac{\pi}{6}\right)\right)^{\frac{1}{4}}$$

$$(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \cos\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right] + i\sin\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right]$$

$$\left(\cos\left(-\tfrac{\pi}{6}\right) + i\sin\left(-\tfrac{\pi}{6}\right)\right)^{\frac{1}{4}} = \cos\left[\frac{-\frac{\pi}{6}}{4} + \frac{2(k-1)\pi}{4}\right] + i\sin\left[\frac{-\frac{\pi}{6}}{4} + \frac{2(k-1)\pi}{4}\right]$$

$$= \cos\left[\frac{-\pi}{24} + \frac{(k-1)\pi}{2}\right] + i\sin\left[\frac{-\pi}{24} + \frac{(k-1)\pi}{2}\right]$$

$\underline{k=1}$ $\quad z = 108^{\frac{1}{8}}\left(\cos\left(\frac{-\pi}{24}\right) + i\sin\left(\frac{-\pi}{24}\right)\right)$

$z = 1.7801 - 0.2344i$ i 4 lle degol

Bydd y pedwar gwreiddyn yn gorwedd ar gylch efo radiws

$|z| = |1.7801 - 0.2344i|$

$= \sqrt{1.7801^2 + (-0.2344)^2}$

$= 1.80$ i 2 le degol.

Felly hafaliad y cylch yw $x^2 + y^2 = 1.80^2$

$x^2 + y^2 = 3.22$ i 2 le degol

3) a) $4\sin\theta + 5\cos\theta = 3$

$t = \tan\left(\frac{\theta}{2}\right)$ felly $\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$

Felly $4\left(\frac{2t}{1+t^2}\right) + 5\left(\frac{1-t^2}{1+t^2}\right) = 3$

$$4(2t) + 5(1-t^2) = 3(1+t^2)$$
$$8t + 5 - 5t^2 = 3 + 3t^2$$
$$0 = 3 + 3t^2 + 5t^2 - 8t - 5$$
$$0 = 8t^2 - 8t - 2$$
$$0 = 4t^2 - 4t - 1$$
$$4t^2 - 4t - 1 = 0$$

✓

b) $4t^2 - 4t - 1 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$
$$t = \frac{4 \pm \sqrt{32}}{8}$$
$$t = \frac{4 \pm 4\sqrt{2}}{8}$$
$$t = \frac{1 \pm \sqrt{2}}{2}$$

Naill ai $t = \frac{1+\sqrt{2}}{2}$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1+\sqrt{2}}{2}$$
$$\frac{\theta}{2} = \tan^{-1}\left(\frac{1+\sqrt{2}}{2}\right)$$
$$\frac{\theta}{2} = 0.8789605132 + n\pi$$
$$\theta = 1.757921026 + 2n\pi$$

$\theta = 1.758 + 2n\pi$ ar gyfer cyfanrif n (3 lle degol)

neu $t = \frac{1-\sqrt{2}}{2}$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\sqrt{2}}{2}$$
$$\frac{\theta}{2} = \tan^{-1}\left(\frac{1-\sqrt{2}}{2}\right)$$
$$\frac{\theta}{2} = -0.2042195709 + n\pi$$
$$\theta = -0.4084391419 + 2n\pi$$

$\theta = -0.408 + 2n\pi$ ar gyfer cyfanrif n (3 lle degol)

4) $x = \sin y$

Cylchdroi o amgylch yr echelin $\underline{y}$ felly

$$\text{Cynhwysedd} = \pi \int_1^3 x^2\,dy$$

$$= \pi \int_1^3 \sin^2 y\,dy$$

$$\cos(2y) = 1 - 2\sin^2 y$$
$$2\sin^2 y = 1 - \cos(2y)$$
$$\sin^2 y = \frac{1-\cos(2y)}{2}$$

$$= \pi \int_1^3 \frac{1-\cos(2y)}{2}\,dy$$

$$= \frac{\pi}{2} \int_1^3 1 - \cos(2y)\,dy$$

$$= \frac{\pi}{2}\left[y - \frac{\sin(2y)}{2}\right]_1^3$$

$$= \frac{\pi}{2}\left[\left(3 - \frac{\sin(6)}{2}\right) - \left(1 - \frac{\sin(2)}{2}\right)\right]$$

$$= \frac{\pi}{2}\left[3.139707749 - 0.5453512866\right]$$

$$= 4.075205602$$

$= \underline{\underline{4.08}}$ uned ciwb i 2 le degol

PULL 1

5) a)

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 2 & -5 & 3 & 8 \\ 0 & 6 & -2 & 0 \end{array}\right) \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -9 & 3 & 2 \\ 0 & 6 & -2 & 0 \end{array}\right) \begin{array}{l} R_1 \\ R_2 - 2R_1 =: R_4 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -9 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{array}\right) \begin{array}{l} R_1 \\ R_4 \\ 3R_3 + 2R_4 =: R_5 \end{array}$$

Nid yw R_5 yn hafaliad dilys $(0x + 0y + 0z = 4)$ felly nid oes gan y system o hafaliadau unrhyw ddatrysiadau.

b) Nid yw'r llinellau $x + 2y = 3$, $2x - 5y + 3z = 8$, $6y - 2z = 0$ yn croestorri wrth eu plotio mewn gofod 3-D/ ddim yn cyfarfod mewn un pwynt.

5) $x + 2y = 3$ — 1

$2x - 5y + 3z = 8$ — 2

$6y - 2z = 0$ — 3

DULL 2

a) 1 $\Rightarrow$ $x = 3 - 2y$ — 4

3 $\Rightarrow$ $6y = 2z$

$3y = z$ — 5

Yn amnewid am x a z o 4 a 5 i 2:

$2(3-2y) - 5y + 3(3y) = 8$

$6 - 4y - 5y + 9y = 8$

$6 = 8$

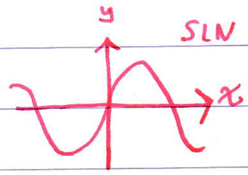
Nid yw'r hafaliad ymain gywir felly nid oes gan y system o hafaliadau unrhyw ddatrysiadau.

b) Nid yw'r llinellau $x + 2y = 3$, $2x - 5y + 3z = 8$, $6y - 2z = 0$ yn croestorri wrth eu plotio mewn gofod 3-D/ ddim yn cyfarfod mewn un pwynt.

6) $\cos 2\theta - \cos 4\theta = \sin 3\theta$ rhung 0 a π

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= -2\sin\left(\frac{2\theta+4\theta}{2}\right)\sin\left(\frac{2\theta-4\theta}{2}\right)$$



$$= -2\sin(3\theta)\sin(-\theta)$$

$$= -2\sin(3\theta)(-\sin\theta)$$

$$= 2\sin(3\theta)\sin(\theta)$$

Felly $\cos 2\theta - \cos 4\theta = \sin 3\theta$

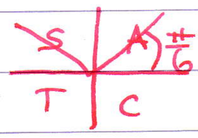
$$2\sin(3\theta)\sin\theta = \sin 3\theta$$

$$2\sin(3\theta)\sin\theta - \sin 3\theta = 0$$

$$\sin(3\theta)(2\sin\theta - 1) = 0$$

Naill ai $\sin 3\theta = 0$ neu $2\sin\theta - 1 = 0$

$3\theta = \sin^{-1}(0)$ $\quad\quad$ $\sin\theta = \frac{1}{2}$



$3\theta = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$ $\quad$ $\theta = \sin^{-1}(\frac{1}{2})$

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \left(\frac{4\pi}{3} \ldots\right)$ $\quad$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$

Rhwng 0 a π y datrysiadau yw $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$

7) a) $4x^2+10x-24 = 4(x^2+\frac{10}{4}x)-24$

$$= 4(x^2+\frac{5}{2}x)-24$$
$$= 4((x+\frac{5}{4})^2-(\frac{5}{4})^2)-24$$
$$= 4((x+\frac{5}{4})^2-\frac{25}{16})-24$$
$$= 4(x+\frac{5}{4})^2-4\times\frac{25}{16}-24$$
$$= 4(x+\frac{5}{4})^2-\frac{121}{4}$$

Felly $a=4$, $b=\frac{5}{4}$, $c=-\frac{121}{4}$

b) $\int_3^5 \frac{6}{\sqrt{4x^2+10x-24}}\,dx$

$$= \int_3^5 \frac{6}{\sqrt{4(x+\frac{5}{4})^2-\frac{121}{4}}}\,dx$$
$$= \int_3^5 \frac{6}{\sqrt{4[(x+\frac{5}{4})^2-\frac{121}{16}]}}\,dx$$
$$= \int_3^5 \frac{6}{\sqrt{4}\times\sqrt{(x+\frac{5}{4})^2-\frac{121}{16}}}\,dx$$
$$= \int_3^5 \frac{6}{2\sqrt{(x+\frac{5}{4})^2-\frac{121}{16}}}\,dx$$
$$= 3\int_3^5 \frac{1}{\sqrt{(x+\frac{5}{4})^2-\frac{121}{16}}}\,dx$$
$$= 3\left[\cosh^{-1}\left(\frac{x+\frac{5}{4}}{\sqrt{\frac{121}{16}}}\right)\right]_3^5$$
$$= 3\left[\cosh^{-1}\left(\frac{4(x+\frac{5}{4})}{11}\right)\right]_3^5$$
$$= 3\left(\cosh^{-1}\left(\frac{4(5+\frac{5}{4})}{11}\right)-\cosh^{-1}\left(\frac{4(3+\frac{5}{4})}{11}\right)\right)$$
$$= 3\times 0.4597627484$$
$$= \underline{1.379} \text{ i 3 lle degol}$$

8) Gadewch i $x = \sinh y$

$$x = \frac{1}{2}(e^{y} - e^{-y}) \quad \text{trwy ddiffiniad } \sinh y$$

Felly $2x = e^{y} - e^{-y}$

$$2x\,e^{y} = (e^{y})(e^{y}) - 1$$

$$2x\,e^{y} = e^{2y} - 1$$

$$0 = e^{2y} - 2x e^{y} - 1$$

Fformiwla Gwadratig $e^{y} = \dfrac{2x \pm \sqrt{(-2x)^2 - 4(1)(-1)}}{2 \times 1}$

$$e^{y} = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$e^{y} = \frac{2x \pm \sqrt{4(x^2+1)}}{2}$$

$$e^{y} = \frac{2x \pm 2\sqrt{x^2+1}}{2}$$

$$e^{y} = x \pm \sqrt{x^2+1}$$

$$\ln(e^{y}) = \ln(x \pm \sqrt{x^2+1})$$

$$y = \ln(x \pm \sqrt{x^2+1})$$

Mae $\sqrt{x^2+1} > x$ felly rhaid dewis
$y = \ln(x + \sqrt{x^2+1})$ [ac nid $y = \ln(x - \sqrt{x^2+1})$]
gan nad yw logarithm o rif negatif yn rhoi rhif real.

Felly $y = \ln(x + \sqrt{x^2+1})$
ac felly $\sinh^{-1}(x) = \ln(x + \sqrt{x^2+1})$ ✓

9) a) i) Ehangiad Binomaidd:

$$\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right)^3$$

$$= \cos^3\frac{\theta}{3} + 3\cos^2\frac{\theta}{3}\left(i\sin\frac{\theta}{3}\right) + 3\cos\frac{\theta}{3}\left(i\sin\frac{\theta}{3}\right)^2 + \left(i\sin\frac{\theta}{3}\right)^3$$

$$= \cos^3\frac{\theta}{3} + 3\cos^2\frac{\theta}{3}\sin\frac{\theta}{3}\,i + 3\cos\frac{\theta}{3}\sin^2\frac{\theta}{3}(-1) + (-1)i\sin^3\frac{\theta}{3}$$

$$= \cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}\sin^2\frac{\theta}{3} + \left(3\cos^2\frac{\theta}{3}\sin\frac{\theta}{3} - \sin^3\frac{\theta}{3}\right)i$$

ii) Theorem de Moivre: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$$\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right)^n = \cos\left(n\left(\frac{\theta}{3}\right)\right) + i\sin\left(n\left(\frac{\theta}{3}\right)\right)$$

$$\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right)^3 = \cos\theta + i\sin\theta$$

$$\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}\sin^2\frac{\theta}{3} + \left(3\cos^2\frac{\theta}{3}\sin\frac{\theta}{3} - \sin^3\frac{\theta}{3}\right)i = \cos\theta + i\sin\theta$$

Yn cymharu darnau real:

$$\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}\sin^2\frac{\theta}{3} = \cos\theta$$

$$\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}\left(1 - \cos^2\frac{\theta}{3}\right) = \cos\theta$$

$$\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3} + 3\cos^3\frac{\theta}{3} = \cos\theta$$

$$4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3} = \cos\theta$$

$$\cos\theta = 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3} \quad ✓$$

b) $$\frac{\cos\theta}{\cos\frac{\theta}{3}} = 1$$

$$\cos\theta = \cos\frac{\theta}{3}$$

$$4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3} = \cos\frac{\theta}{3}$$

$$4\cos^3\frac{\theta}{3} - 4\cos\frac{\theta}{3} = 0$$

$$4\cos\frac{\theta}{3}\left(\cos^2\frac{\theta}{3} - 1\right) = 0$$

Naill ai $4\cos\frac{\theta}{3} = 0$

$$\cos\frac{\theta}{3} = 0$$

Dim datrysiadau (mae $\cos\frac{\theta}{3}$ yn ymddangos yn enwadur yr hafaliad gwreiddiol)

neu $\cos^2\frac{\theta}{3} - 1 = 0$

$$\cos^2\frac{\theta}{3} = 1$$

$\cos\frac{\theta}{3} = 1$ neu $\cos\frac{\theta}{3} = -1$

$\frac{\theta}{3} = \cos^{-1}(1)$ $\qquad$ $\frac{\theta}{3} = \cos^{-1}(-1)$

$\frac{\theta}{3} = 2n\pi$ $\qquad$ $\frac{\theta}{3} = 2n\pi \pm \pi$

$\underline{\theta = 6n\pi}$ $\qquad$ $\underline{\theta = 6n\pi + 3\pi}$

neu $\underline{\theta = 6n\pi - 3\pi}$

ar gyfer cyfanrif n

n	-3	-2	-1	0	1	2	3
$6n\pi$	-18π	-12π	-6π	0	6π	12π	18π
$6n\pi+3\pi$	-15π	-9π	-3π	3π	9π	15π	21π
$6n\pi-3\pi$	-21π	-15π	-9π	-3π	3π	6π	15π

Felly y datrysiad cyffredinol ar gyfer yr hafaliad yw

$$\underline{\underline{\theta = 3n\pi}}$$

10) $A = \begin{pmatrix} 4 & 8 & 0 \\ 0 & \lambda & -2 \\ 4 & 0 & \lambda \end{pmatrix}$

a) $\det(A) = 4 \begin{vmatrix} \lambda & -2 \\ 0 & \lambda \end{vmatrix} - 8 \begin{vmatrix} 0 & -2 \\ 4 & \lambda \end{vmatrix} + 0 \begin{vmatrix} 0 & \lambda \\ 4 & 0 \end{vmatrix}$

$= 4(\lambda^2 + 0) - 8(0+8) + 0(0 - 4\lambda)$

$= 4\lambda^2 - 64$

Mae'r matrics yn hynod os yw $\det(A) = 0$

$4\lambda^2 - 64 = 0$

$\lambda^2 - 16 = 0$

$\lambda^2 = 16$

$\lambda = \pm\sqrt{16}$

$\lambda = 4$ neu $\lambda = -4$ ✓

b) $\lambda = 3$ felly $A = \begin{pmatrix} 4 & 8 & 0 \\ 0 & 3 & -2 \\ 4 & 0 & 3 \end{pmatrix}$

i) $\det(A) = 4(3^2) - 64$ (o ran (a))

$= -28$

$A^T = \begin{pmatrix} 4 & 0 & 4 \\ 8 & 3 & 0 \\ 0 & -2 & 3 \end{pmatrix}$

Minorau $= \begin{pmatrix} 9 & 24 & -16 \\ 8 & 12 & -8 \\ -12 & -32 & 12 \end{pmatrix}$

Atgydiol (cawyddion) $= \begin{pmatrix} 9 & -24 & -16 \\ -8 & 12 & 8 \\ -12 & 32 & 12 \end{pmatrix}$

ii) $A^{-1} = \dfrac{1}{-28} \begin{pmatrix} 9 & -24 & -16 \\ -8 & 12 & 8 \\ -12 & 32 & 12 \end{pmatrix}$

11) a) i) $y = e^{3x}\sin^{-1}(x)$

$$\frac{dy}{dx} = e^{3x}\left(\frac{1}{\sqrt{1-x^2}}\right) + 3e^{3x}\sin^{-1}(x)$$

$$\frac{dy}{dx} = e^{3x}\left(\frac{1}{\sqrt{1-x^2}} + 3\sin^{-1}(x)\right)$$

ii) $y = \ln(\cosh^2(2x^2+7x))$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(\cosh^2(2x^2+7x))}{\cosh^2(2x^2+7x)}$$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}((\cosh(2x^2+7x))^2)}{\cosh^2(2x^2+7x)}$$

$$\frac{dy}{dx} = \frac{2\cosh(2x^2+7x)\frac{d}{dx}(\cosh(2x^2+7x))}{\cosh^2(2x^2+7x)}$$

$$\frac{dy}{dx} = \frac{2\cosh(2x^2+7x)[(4x+7)\sinh(2x^2+7x)]}{\cosh^2(2x^2+7x)}$$

$$\frac{dy}{dx} = \frac{2(4x+7)\sinh(2x^2+7x)}{\cosh(2x^2+7x)}$$

$$\frac{dy}{dx} = 2(4x+7)\tanh(2x^2+7x)$$

b) $x = \sinh^{-1}(y^2)$

Gadewch i $u = y^2$ $\qquad x = \sinh^{-1}(u)$

$\frac{du}{dy} = 2y \qquad \frac{dx}{du} = \frac{1}{\sqrt{1+u^2}}$

$$\frac{dx}{dy} = \frac{dx}{du} \times \frac{du}{dy}$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{1+u^2}} \times 2y$$

$$\frac{dx}{dy} = \frac{2y}{\sqrt{1+y^4}}$$

Os yw $x = 1$ mae $1 = \sinh^{-1}(y^2)$

$$\sinh(1) = y^2$$
$$1.175201194 = y^2$$
$$\pm\sqrt{1.175201194} = y$$
$$y = 1.084066969 \text{ neu } y = -1.084066969$$

Tangiad 1: $x - x_0 = \frac{dx}{dy}(y - y_0)$

$$x - 1 = \left(\frac{2y}{\sqrt{1+y^4}}\right)(y - 1.084066969)$$
$$x - 1 = \left(\frac{2(1.084066969)}{\sqrt{1+1.084066969^4}}\right)(y - 1.084066969)$$
$$x - 1 = 1.405068465(y - 1.084066969)$$
$$x - 1 = 1.405068465y - 1.523188312$$
$$x = 1.405068465y - 0.5231883117$$

Tangiad 2: $x - x_0 = \frac{dx}{dy}(y - y_0)$

$$x - 1 = \left(\frac{2(-1.084066969)}{\sqrt{1+(-1.084066969)^4}}\right)(y - -1.084066969)$$
$$x - 1 = -1.405068465(y + 1.084066969)$$
$$x - 1 = -1.405068465y - 1.523188312$$
$$x = -1.405068465y - 0.5231883117$$

12) $3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 8 + 6x - 2x^2$ —— (1)

Ceisio $x = Ae^{mx}$ fel bod m yn bodloni $am^2 + bm + c = 0$

$a = 3 \quad b = 5 \quad c = -2$

Hafaliad ategol / Auxiliary equation

$3m^2 + 5m - 2 = 0$

$3x - 2 = -6$

$3m^2 + 6m - m - 2 = 0$

$3m(m+2) - 1(m+2) = 0$

$(3m-1)(m+2) = 0$

Naill ai $3m - 1 = 0$ neu $m + 2 = 0$

$m = \frac{1}{3}$ $\qquad m = -2$

Dau ddatrysiad real felly'r ffwythiant cyflenwol (complementary function) yw $y = Ae^{\frac{1}{3}x} + Be^{-2x}$

Ar gyfer yr integryn neilltuol (particular integral) wrth gymharu efo $8 + 6x - 2x^2$ ceisiwn $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = 2a$

Yn amnewid i (1): $3(2a) + 5(2ax + b) - 2(ax^2 + bx + c) = 8 + 6x - 2x^2$

$6a + 10ax + 5b - 2ax^2 - 2bx - 2c = 8 + 6x - 2x^2$

$(6a + 5b - 2c) + (10a - 2b)x - 2ax^2 = 8 + 6x - 2x^2$

Yn cymharu cyfernodau x^2: $-2a = -2$

$a = 1$

Yn cymharu cyfernodau x: $10a - 2b = 6$

$10 - 2b = 6$

$10 - 6 = 2b$

$4 = 2b$

$b = 2$

Yn cymharu cysonion: $6a + 5b - 2c = 8$

$$6\times1 + 5\times2 - 2c = 8$$
$$16 - 2c = 8$$
$$8 = 2c$$
$$c = 4$$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuol

$$y = Ae^{\frac{1}{3}x} + Be^{-2x} + ax^2 + bx + c$$
$$y = Ae^{\frac{1}{3}x} + Be^{-2x} + x^2 + 2x + 4$$

Amnewid $x=0$, $y=6$ i mewn:

$$6 = Ae^{\frac{1}{3}(0)} + Be^{-2(0)} + 0^2 + 2(0) + 4$$
$$6 = A + B + 4$$
$$2 = A + B \quad \text{—— } 2$$

Differu: $\frac{dy}{dx} = \frac{1}{3}Ae^{\frac{1}{3}x} - 2Be^{-2x} + 2x + 2$

Amnewid $x=0$, $\frac{dy}{dx} = 5$ i mewn:

$$5 = \frac{1}{3}Ae^{\frac{1}{3}(0)} - 2Be^{-2(0)} + 2(0) + 2$$
$$5 = \frac{1}{3}A - 2B + 2$$
$$3 = \frac{1}{3}A - 2B$$
$$9 = A - 6B$$
$$9 + 6B = A \quad \text{—— } 3$$

Yn amnewid am A i mewn i 2: $2 = 9 + 6B + B$

$$-7 = 7B$$
$$B = -1$$

Yn ôl yn 3: $9 + 6(-1) = A$

$$A = 3$$

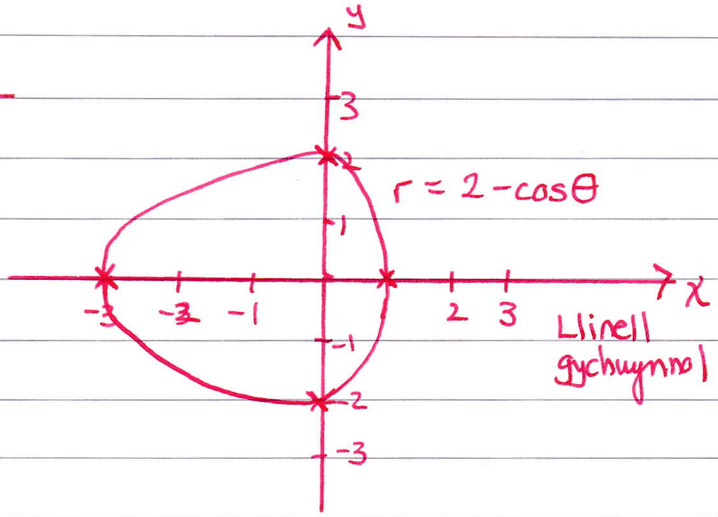
Felly'r datrysiad cyffredinol yw

$$y = 3e^{\frac{1}{3}x} - e^{-2x} + x^2 + 2x + 4$$

13) C: $r = 2 - \cos\theta$ ar gyfer $0 \le \theta \le 2\pi$

a)

θ	$2 - \cos\theta$
0	1
$\frac{\pi}{2}$	2
π	3
$\frac{3\pi}{2}$	2
2π	1



b) $x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x = (2-\cos\theta)\cos\theta$ $\qquad$ $y = (2-\cos\theta)\sin\theta$

$x = 2\cos\theta - \cos^2\theta$ $\qquad$ $y = 2\sin\theta - \cos\theta\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta - 2\cos\theta(-\sin\theta)$ $\qquad$ $\frac{dy}{d\theta} = 2\cos\theta - \cos\theta\cos\theta + \sin\theta\sin\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\cos\theta\sin\theta$ $\qquad$ $\frac{dy}{d\theta} = 2\cos\theta - \cos^2\theta + \sin^2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\frac{dy}{dx} = 2\cos\theta - \cos^2\theta + \sin^2\theta \times \frac{1}{-2\sin\theta + 2\cos\theta\sin\theta}$

$\frac{dy}{dx} = \frac{2\cos\theta - \cos^2\theta + \sin^2\theta}{-2\sin\theta + 2\cos\theta\sin\theta}$

Os yw'r tangiad yn baralel i'r llinell gychwynnol, yna $\frac{dy}{dx} = 0$

$\frac{2\cos\theta - \cos^2\theta + \sin^2\theta}{-2\sin\theta + 2\cos\theta\sin\theta} = 0$

$2\cos\theta - \cos^2\theta + \sin^2\theta = 0$

$2\cos\theta - \cos^2\theta + (1 - \cos^2\theta) = 0$

$2\cos\theta - 2\cos^2\theta + 1 = 0$

$2\cos^2\theta - 2\cos\theta - 1 = 0$ ✓

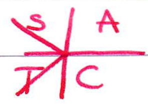
ii) $2\cos^2\theta - 2\cos\theta - 1 = 0$

$$\cos\theta = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2 \times 2}$$

$$\cos\theta = \frac{2 \pm \sqrt{12}}{4}$$

Naill ai $\cos\theta = \frac{2+\sqrt{12}}{4}$ neu $\cos\theta = \frac{2-\sqrt{12}}{4}$

$\theta = \cos^{-1}\left(\frac{2+\sqrt{12}}{4}\right)$ $\theta = \cos^{-1}\left(\frac{2-\sqrt{12}}{4}\right)$



Dim datrysiad

$\theta = 1.94553076$

neu $\theta = 2\pi - 1.94553076$

$\theta = 4.337654548$

$r = 2 - \cos\theta$

$r = 2 - \cos(1.94553076)$

$r = 2.366025404$

I 2 le degol, cyfesurynnau pegynlinol y pwyntiau ar y gromlin lle mae'r tangiad yn baralel i'r llinell gychwynnol yw $(2.37, 1.95)$ a $(2.37, 4.34)$.

14) $\int_2^4 \frac{6x^2+2x+16}{x^3-x^2+3x-3}\,dx$

$= 2\int_2^4 \frac{3x^2+x+8}{x^3-x^2+3x-3}\,dx.$

Ffactorio'r enwadur: Mae $1^3-1^2+3\times 3-3=0$ felly mae $x-1$ yn ffactor.

$$\begin{array}{r l}
 & x^2 \quad +3 \\
x-1 \,\big) & x^3-x^2+3x-3 \\
 & x^3-x^2 \\
 & \qquad\quad 3x-3 \\
 & \qquad\quad 3x-3
\end{array}$$

$2\int_2^4 \frac{3x^2+x+8}{x^3-x^2+3x-3}\,dx$

$= 2\int_2^4 \frac{3x^2+x+8}{(x^2+3)(x-1)}\,dx$

Nawr $\frac{3x^2+x+8}{(x^2+3)(x-1)} \equiv \frac{Ax+B}{x^2+3} + \frac{C}{x-1}$

$3x^2+x+8 \equiv (Ax+B)(x-1) + C(x^2+3)$

Amnewid $x=1$:

$3(1^2)+1+8 \equiv (A(1)+B)(0) + C(1^2+3)$

$12 \equiv 0 + 4C$

$C \equiv 3$

Yn cymharu cyfernodau x^2:

$3 \equiv A + C$

$3 \equiv A + 3$

$A \equiv 0$

Yn cymharu cysonion :

$$8 \equiv -B + 3C$$
$$8 \equiv -B + 9$$
$$-1 \equiv -B$$
$$B \equiv 1$$

Felly $\dfrac{3x^2+x+8}{(x^2+3)(x-1)} = \dfrac{1}{x^2+3} + \dfrac{3}{x-1}$

Felly $2\displaystyle\int_2^4 \frac{3x^2+x+8}{(x^2+3)(x-1)}\,dx$

$$= 2\int_2^4 \frac{1}{x^2+3} + \frac{3}{x-1}\,dx$$
$$= 2\int_2^4 \frac{1}{(\sqrt{3})^2+x^2} + \frac{3}{x-1}\,dx$$
$$= 2\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + 3\ln|x-1|\right]_2^4$$
$$= 2\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{4}{\sqrt{3}}\right) + 3\ln|4-1| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 3\ln|2-1|\right]$$
$$= 2 \times 3.471978653$$
$$= 6.943957305$$
$$= \underline{6.944} \text{ i 3 lle degol}$$