YSGOL CREUDDYN

DAWN DYSG DAIONI

The Mathematics Department

8

Factors and Multiples

Name:

Version 3.0

07/2025

Contents

The Mathematics Department

Welcome back to year 8! Here is a reminder of the guidelines for looking after your work.

- At the start of every lesson, write "Classwork", the date and a suitable title for the work.
  Each of these should be underlined.
- No blank pages should be left in the red mathematics book.
- Draw diagrams using a pencil and, where appropriate, using the correct equipment, e.g. ruler, protractor, compass.
- Show your method fully.
- Remember to include relevant units in your answers, e.g. cm, £, ml.

Keep your work in the **plastic wallet**.
Your wallet should include the following items at all times:

(a) Your red mathematics book.
(b) Your revision book for the year.
(c) The current workbook.
(d) The current additional tasks workbook.
(e) The mathematical almanac.

Old workbooks should be stored safely at home.
(You will need to refer to these in the future.)

Equipment

- Black, red and blue biros.
- HB Pencil.
- Ruler (a 30 cm one is better).
- Eraser.
- Protractor.
- Compass.
- Scientific calculator (e.g. Casio FX-83GTCW).
- Highlighter.

The Revision Book

This will form the basis for your GCSE revision.

- Complete at least 4 pages in your revision book for each unit of work.
- You should include material you will need in the future for remembering the work quickly. This can include notes on the work; examples; important facts; and revision posters.

Workbooks

You will receive 1 copy of the workbook and 1 copy of the additional tasks workbook at the start of each new unit of work. (If you lose the workbook, a new one will cost 50p.)

A Welsh copy of the workbook, and additional supporting materials, can be found on the department's website, www.mathemateg.com

Content of the workbooks

When you see a QR code (like the one on the left), scan it using your mobile device in order to reach a Welsh YouTube video hosted on the following channel.

www.youtube.com/adolygumathemateg

The numbers in circles, for example (3), show which GCSE unit the work appears in.

Circle	1	2	3	12	13	23	A
Units	1	2	3	1 and 2	1 and 3	2 and 3	1, 2 and 3

All the workbooks contain a variety of exercises, labelled as follows.

Skill
Exercises on new topics.

Applying
Answering a question in context, or solving a problem.

Extension
A more difficult question.

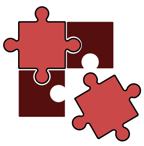
Revision
Revision of material from previous workbooks.

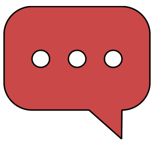
Evaluation

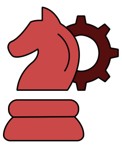
There are evaluation boxes at the end of each chapter to revise the completed work.

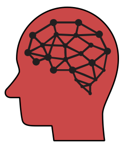
Key words	Corrections	I am happy with...	I need to revise...
Write down the new or important mathematical terms from the chapter.	What do you need to remember when completing this type of work in the future?	Write down the topics you had success with.	Write down the topics you need to look at again.

Curriculum for Wales Proficiencies


Conceptual understanding


Communication using symbols


Strategic competence


Logical reasoning

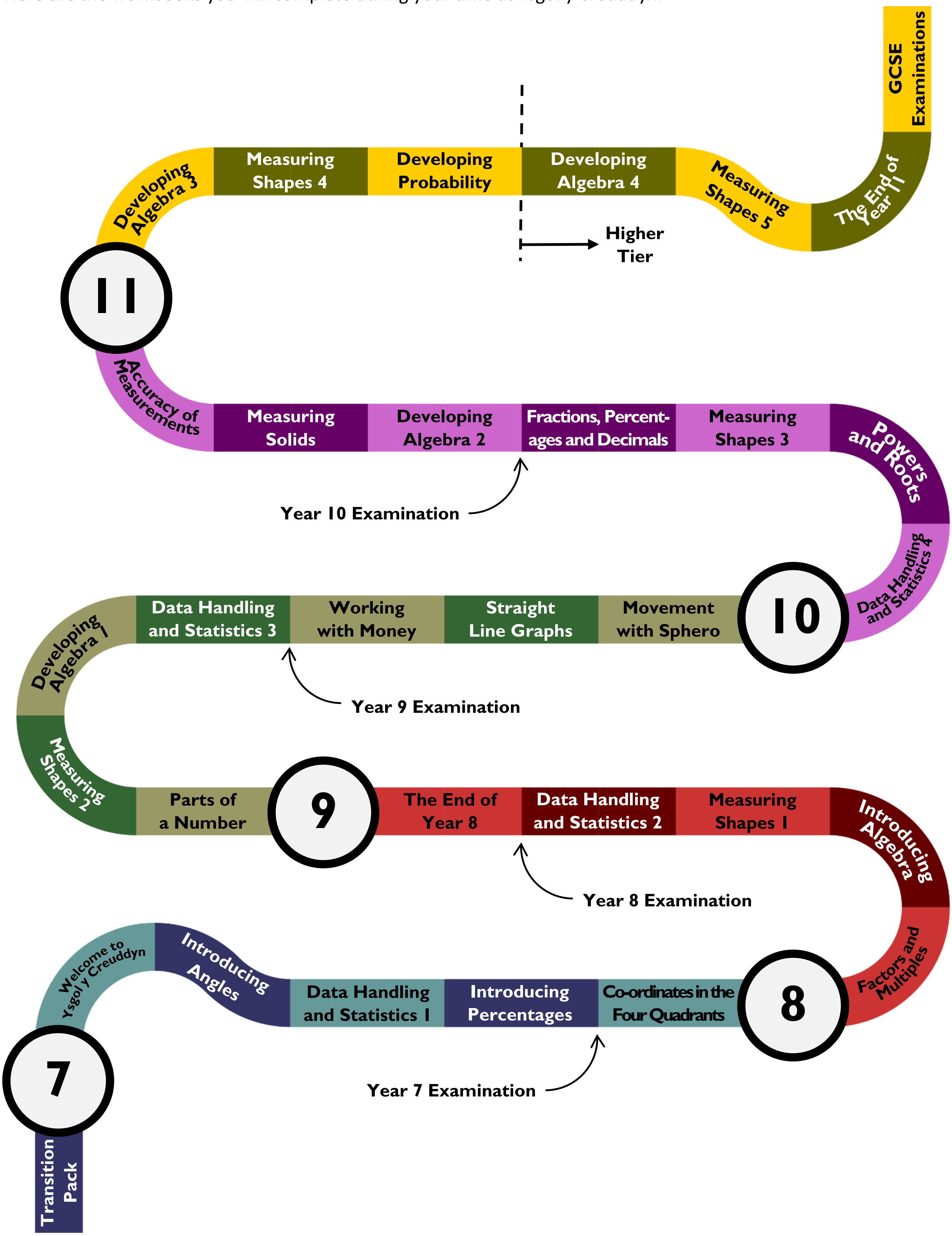

Fluency

Supporting Materials:

- Diagnostic Questions
  - A quiz for each workbook on the website www.diagnosticquestions.com.
- Reflection Sheet
  - An opportunity to assess your understanding of a workbook, and to see the test question order.
- Old WJEC examination questions; worksheets; investigations; puzzles.
  - Available for some topics.

Mathematics Learning Journey for Ysgol y Creuddyn

Here are the workbooks you will complete during your time at Ysgol y Creuddyn.



Square Root and Cube Root

Revision

In year 7, you learnt how to calculate different square numbers and cube numbers.

Example

$5^2 = 5 \times 5$
$= 25$

$9^3 = 9 \times 9 \times 9$
$= 81 \times 9$
$= 729$

Exercise 1

A

(a) 4^2 (b) 7^2 (c) 3^2 (d) 9^2 (e) 8^2
(f) 2^3 (g) 4^3 (h) 3^3 (i) 5^3 (j) 8^3
(k) $6^2 + 2^2$ (l) $4^2 \div 2^3$ (m) $6^3 + 6^2$ (n) $10^3 - 1^2$ (o) $7^3 + 11^2$
(p) Which numbers between 1 and 1,000 are both square numbers and cube numbers?

Square Root

The square root of a number is the number that **squares** to give the original number.
For example, the square root of 25 is 5 because 5^2 is 25.
The symbol for square root is $\sqrt{\ }$, therefore $\sqrt{25} = 5$.

Exercise 2

A

Skill

(a) $\sqrt{9}$ (b) $\sqrt{36}$ (c) $\sqrt{49}$ (d) $\sqrt{100}$ (e) $\sqrt{16}$
(f) $\sqrt{1}$ (g) $\sqrt{64}$ (h) $\sqrt{4}$ (i) $\sqrt{81}$ (j) $\sqrt{144}$

Cube Root

The cube root of a number is the number that **cubes** to give the original number. For example, the cube root of 729 is 9 because $9^3 = 729$. The symbol for cube root is $\sqrt[3]{\ }$, therefore $\sqrt[3]{729} = 9$.

Exercise 3

A

(a) $\sqrt[3]{8}$ (b) $\sqrt[3]{64}$ (c) $\sqrt[3]{1000}$ (d) $\sqrt[3]{125}$ (e) $\sqrt[3]{27}$
(f) $\sqrt[3]{1}$ (g) $\sqrt[3]{216}$ (h) $\sqrt[3]{512}$ (i) $\sqrt[3]{0}$ (j) $\sqrt[3]{343}$

Exercise 4

A

(a) $\sqrt{16} + \sqrt{25}$ (b) $\sqrt[3]{8} \times \sqrt{25}$ (c) $\sqrt{81} + \sqrt[3]{125}$ (d) $\sqrt[3]{512} \div \sqrt{64}$ (e) $\sqrt{1} + \sqrt[3]{1}$
(f) $\sqrt{49} \times \sqrt{64}$ (g) $\sqrt[3]{64} - \sqrt{4}$ (h) $\sqrt{121} + \sqrt[3]{216}$ (i) $\sqrt[3]{343} \times \sqrt{49}$ (j) $\sqrt[3]{27} \times \sqrt{100}$

Exercise 5

A

(a) $5^2 - \sqrt{25}$ (b) $5^3 - \sqrt{1}$ (c) $6^3 - \sqrt[3]{8}$ (d) $5^2 + \sqrt{49}$ (e) $7^2 + \sqrt{169}$
(f) $3^3 \times \sqrt{100}$ (g) $2^2 - \sqrt[3]{1}$ (h) $5^2 \times \sqrt[3]{8}$ (i) $2^3 + 3^2$ (j) $8^2 - \sqrt{49}$
(k) $8^3 \times \sqrt{4}$ (l) $3^3 - 4^2 - \sqrt{1}$ (m) $\sqrt{49} + \sqrt[3]{343}$ (n) $\sqrt[3]{216} \times 1^3$ (o) $2^3 + 5^2$
(p) $10^3 \div \sqrt{100}$ (q) $\sqrt{81} \times \sqrt[3]{1000}$ (r) $5^2 + 5^3$ (s) $5^3 - 5^2 + 5$ (t) $\sqrt{81} + (3^2 \times 2^3)$

Challenge! ⚠

Can you calculate the following sums?

(a) 2^4 (b) $\sqrt[4]{81}$ (c) 3^5 (d) $\sqrt[6]{64}$ (e) 1^{2500}

Exercise 6: Answer these using a calculator. 13

(a) $\sqrt{49}$ (b) 8^2 (c) $\sqrt[3]{8}$ (d) 4^3 (e) $\sqrt{9} + 3^2$

Exercise 7: Answer these using a calculator. 13

(a) 26^2 (b) 33^2 (c) $20^2 - 9^2$ (d) $18^2 - 6^2 - 3^2$ (e) $3^2 + 19^2$
(f) $24^2 - 5^2 + 3^2$ (g) $40^2 - 16^2 - 4^2 - 7^2$ (h) $\sqrt{625} - \sqrt{196}$ (i) $\sqrt{441} - \sqrt{36}$ (j) $\sqrt{361} + \sqrt{121}$
(k) $\sqrt{900} - \sqrt{400}$ (l) $\sqrt{961} + \sqrt{3600}$ (m) $\sqrt{289} + \sqrt{196}$ (n) $\sqrt{1600} - \sqrt{144}$ (o) $7^3 - 5^3$
(p) $1^3 + 2^3 + 3^3$ (q) $9^3 - 4^3$ (r) $6^3 + 2^3 - 3^3$ (s) $8^3 - 3^3 - 1^3$ (t) $10^3 + 5^3$
(u) $12^3 - 6^3 - 4^3$ (v) $\sqrt[3]{27} + \sqrt[3]{216}$ (w) $\sqrt[3]{1728} - \sqrt[3]{1000}$ (x) $\sqrt[3]{2744} - \sqrt[3]{729}$ (y) $\sqrt[3]{3375} + \sqrt[3]{27}$
(z) $\sqrt[3]{5832} - \sqrt[3]{2744}$ (α) $\sqrt[3]{15625} + \sqrt[3]{216}$ (β) $\sqrt[3]{32768} - \sqrt[3]{1000}$ (χ) $\sqrt[3]{46656} + \sqrt[3]{9261} - \sqrt[3]{2197}$

Exercise 8: Answer these using a calculator. 13

(a) $7^2 + 3^3 + \sqrt{49}$ (b) $\sqrt{121} + \sqrt{81} - \sqrt{64}$ (c) $5^3 - \sqrt{100} + \sqrt{36}$ (d) $8^3 + 3^2 - \sqrt{64}$ (e) $12^2 + \sqrt[3]{2197}$
(f) $\sqrt[3]{5832} - \sqrt{169}$ (g) $\sqrt[3]{32768} - \sqrt{441}$ (h) $\sqrt[3]{35937} - 5^2 + 13^2$ (i) $7^2 + \sqrt[3]{19683}$ (j) $\sqrt{276676}$

Exercise 9 13

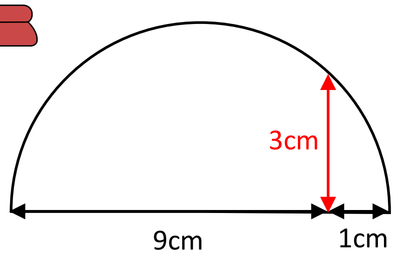
(a) Using a calculator, write the answer to $\sqrt{50}$.
(b) Draw a picture of a 5 cm by 5 cm square.
(c) Measure the length of one of the square's diagonals. What do you notice about your answer?

Descartes Investigation: Finding square roots with a semi-circle

René Descartes was born on the 15th of March 1596, in France. He used graphs to connect algebra with geometry. Graphs that use an x-axis and a y-axis are called Cartesian graphs, after him.

As he lay in bed one morning (he usually didn't get up until after midday!), he thought about the following method for calculating the square root of any number.

- Draw a **straight line** of the number you want to find the square root of e.g. 9 cm.
- Add **one unit extra** to the length.
- Draw a **semi-circle** over the whole line.
- Draw a **vertical line** from the point where the original line finished.
- The length of the vertical line will be the square root of the additional number, in this case 3 cm. This method works for any initial number!



Exercise 10

Applying 13

Use Descartes' method to calculate the square roots of the following numbers. Measure the length of the vertical line as accurately as possible. Compare your answer with the correct answer from the calculator.

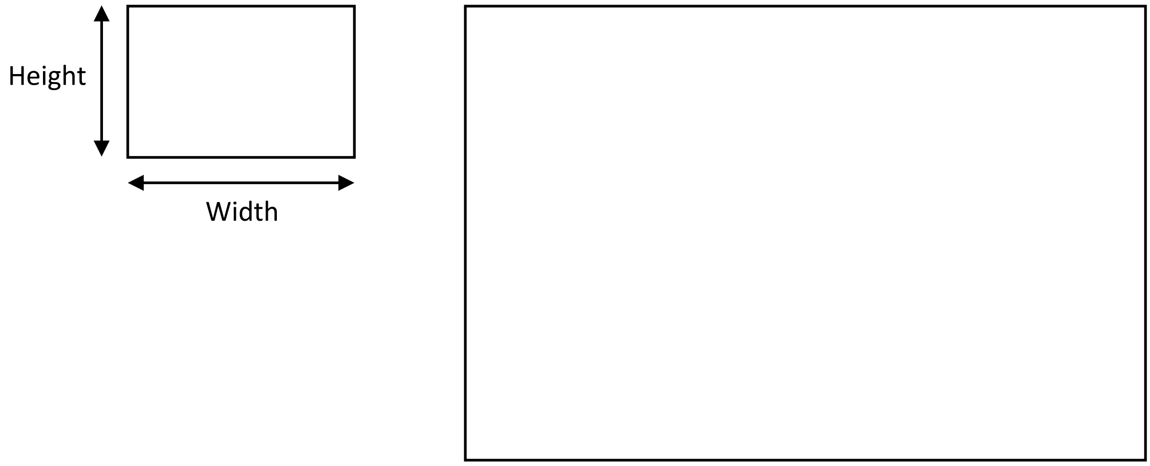
(a) 8 (b) 10 (c) 5.6 (d) 7.8 (e) 6.5

Similar Shapes Investigation

Set 1 only

Two shapes are **similar** if they are the same shape but have different size.

Consider the two similar rectangles below.



Exercise 11

(a) Use a ruler to measure the height and width of the two rectangles.
(b) **How many times larger** is the height of the large rectangle compared to the height of the small rectangle?
(c) Calculate the **area** of the small rectangle and the area of the large rectangle.
(d) **How many times larger** is the area of the large rectangle compared to the area of the small rectangle?
(e) What is the **connection** between your answers to (b) and (d)?
(f) Now consider a new rectangle that has a length and width **four times** the size of the small rectangle.
 (i) What is the height and width of the new rectangle?
 (ii) How many times larger is the area of the new rectangle compared to the area of the small rectangle?
(g) What if another rectangle had a length and width **seven times** the size of the small rectangle? How about **eight** times larger? How about x times larger?

Exercise 12 (Revision)

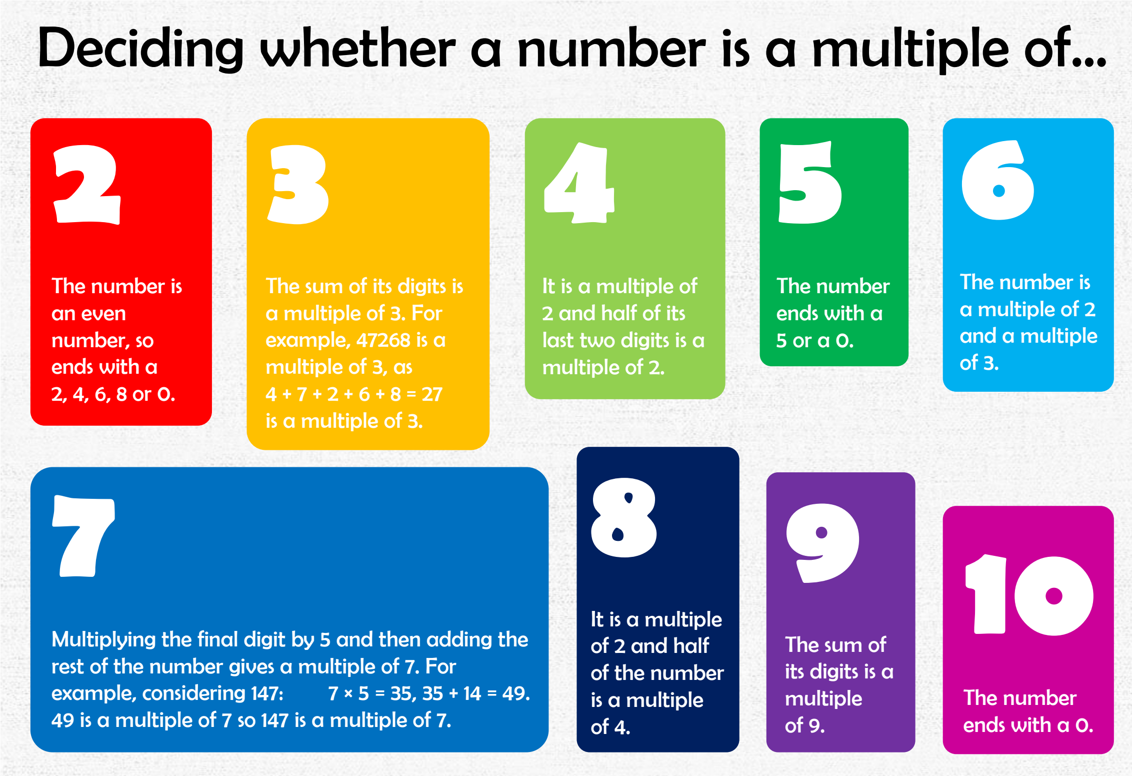
(a) 6^2	(b) 4^3	(c) $\sqrt{64}$	(d) $\sqrt[3]{27}$	(e) $5^2 + \sqrt{16}$
(f) $\sqrt[3]{8} + 8^2$	(g) $8^2 - 2^3$	(h) $\sqrt{81} \times \sqrt{4}$	(i) 15^2	(j) 11^3

Evaluation

Key Words	Corrections	I am happy with…	I need to revise…

Factors and Prime Numbers

Before we discuss how to find the **factors** of a number, it will be useful to revise the year 7 work on **multiples**.



Exercise 13

A Revision

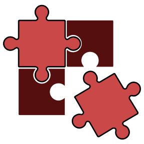
Complete the following table using a tick (✓) or a cross (×).

Number	Multiple of 2?	Multiple of 3?	Multiple of 4?	Multiple of 5?	Multiple of 6?	Multiple of 7?	Multiple of 8?	Multiple of 9?	Multiple of 10?
12									
20									
36									
42									
60									
72									
96									
105									
120									
156									
203									
900									

Factors

A number is a **factor** of another number if it **divides exactly** into that number. For example, 3 is a factor of 12 because 3 divides exactly into 12. (Or, we can say that 12 is a multiple of 3.) 5 is not a factor of 12 as 5 does not divide exactly into 12. (Or, we can say that 12 is not a multiple of 5.)

To list all the factors of a given number, we can think of all the multiplication sums that give that number. For example, to find all the factors of 16, we can list the multiplication sums that give 16.



16 = 1 × 16
16 = 2 × 8
16 = 4 × 4

So, the factors of 16, ordered from smallest to largest, are 1, 2, 4, 8 and 16.

Exercise 14

Fill in the blanks.

15 = 1 × _____
15 = _____ × 5
The factors of 15 are 1, _____, 5, _____

24 = 1 × 24
24 = 2 × _____
24 = _____ × 8
24 = 4 × _____
The factors of 24 are 1, 2, _____, 4, _____, 8, _____, 24

52 = _____ × 52
52 = 2 × _____
52 = _____ × 13
The factors of 52 are _____, 2, _____, 13, _____, 52

77 = 1 × _____
77 = _____ × _____
The factors of 77 are 1, _____, _____, _____

How many sums?

When finding the factors of a number, we must consider whether the number is a multiple of 1, a multiple of 2, a multiple of 3, and so on. The **largest multiple** we must consider is the **square root of the largest square number less than or equal to the number**. For example, considering 67, the largest square number that is less than or equal to 67 is 64. Because $\sqrt{64} = 8$, we must consider if 67 is a multiple of 1 up to a multiple of 8.

Exercise 15

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
The number squared	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Complete the following table.

Number	67	10	18	24	25	32	48	50	64	90	110	132	200
The largest square number that is less than or equal to the number	64												
Square root of the square number	8												
We must consider multiples of	1 to 8												

Exercise 16: List all the factors of the following numbers.

(a) 4	(b) 12	(c) 20	(d) 26	(e) 30	(f) 36	(g) 42
(h) 50	(i) 54	(j) 60	(k) 72	(l) 75	(m) 84	(n) 100
(o) 1	(p) 8	(q) 48	(r) 90	(s) 120	(t) 128	(u) 360

Exercise 17

(a) Which numbers in this list are factors of 24? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
(b) List all the factors of 96.
(c) Which numbers in this list are both factors of 60 and factors of 96? 3, 8, 10, 6, 18, 12, 24, 4.
(d) Which numbers in this list are both factors of 36 and factors of 48? 6, 9, 12, 4, 8, 3, 16, 18.
(e) True or False? 1 is a factor of every whole number.
(f) Which numbers in this list are factors of 45, 75 and 120? 2, 10, 5, 20, 3, 12, 15, 9
(g) Which numbers in this list are factors of 28, 84 and 126? 4, 7, 2, 12, 14, 21, 6, 9
(h) Write the odd numbers that are factors of 36.
(i) Write every even number that is a factor of 40.
(j) Apart from 42, what is the highest factor of 42?

Exercise 18

Here is a list of numbers.

2,	4,	6,	10,	15,	16,	20,	27,	32,	42,	48,	50,	75

Using only numbers from the list above, write

(a) the factors of 100 (b) a square number (c) a cube number (d) multiples of 3
(e) the cube root of 64 (f) the square root of 225 (g) factors of 20 (h) two numbers that add to 63
(i) even numbers (j) factors of 32 (k) multiples of 5 (l) two numbers with a difference of 17

Investigation: Perfect Numbers

- The factors of 6 are 1, 2, 3 and 6.
  On adding the factors 1, 2, 3 (but not the 6) we obtain an answer of 6. So, 6 is a perfect number.
- The factors of 10 are 1, 2, 5 and 10.
  On adding the factors 1, 2, 5 (but not the 10) we obtain an answer of 8. So, 10 is not a perfect number (we did not obtain a total of 10).
- Apart from 6, what is the only other perfect number between 1 and 100?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Challenge!

A person goes shopping and buys a loaf of bread for £1.50; a drink for £1.20 and two other items. Using a calculator, the person multiplies the cost of the items instead of adding. At the till the person says “So, I need to pay £7.11?” and the till operator (that has added the prices correctly) agrees! What was the price of the other two items?

Prime Numbers

A

A number is a **prime number** if it has **exactly two factors:** 1 and the number itself. For example, 5 is a prime number, because the factors of 5 are 1 and 5 (exactly two factors). 6 is not a prime number, because the factors of 6 are 1, 2, 3 and 6 (more than 2 factors).

Exercise 19: Are the following numbers prime numbers?

(a) 3	(b) 4	(c) 7	(d) 9	(e) 11	(f) 12	(g) 15
(h) 16	(i) 19	(j) 23	(k) 27	(l) 29	(m) 39	(n) 41
(o) 45	(p) 49	(q) 59	(r) 65	(s) 67	(t) 71	(u) 77

The Sieve of Eratosthenes

Eratosthenes was a Greek mathematician born in 276 B.C. He created an **algorithm** to find all the prime numbers that exist. Use the algorithm to find all the prime numbers between 1 and 100, using the hundred square on the right.

(1) Put a **cross** through 1 as it is not a prime number (one factor only, not two).
(2) **Circle** the number 2 as it is a prime number (the factors of 2 are 1 and 2).
(3) Cross out any numbers that are **multiples of 2** (4, 6, 8, 10, ...) as these are not prime numbers.
(4) The next **empty** number is 3. Circle this number to show that it is a prime number.
(5) Cross out numbers that are **multiples of 3** (6, 9, 12, ...).
(6) **Repeat** this (circle the next number that is empty; cross out any of the multiples) until every number is either circled or crossed out.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Factors Investigation

Applying

- Give a number to each member of the class.
- Everyone to start with their hands down.
- If 1 is a factor of your number, raise your hand. (What should happen here?)
- If 2 is a factor of your number, put your hand down.
- If 3 is a factor of your number, change the position of your hand. (If your hand is up, put it down. If your hand is down, put it up.)
- Repeat this with the rest of the numbers, changing the position of your hand when needed.

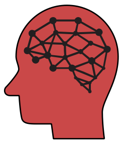
At the end, whose hands are raised?

Have another go at the investigation to see if the answer is the same (watch out for people making mistakes!)

Can you explain the end result?

Exercise 20

(a) List all the prime numbers between 50 and 60.
(b) What is the next prime number after 23?
(c) What is the next prime number after 100?
(d) List all the prime numbers between 110 and 120.
(e) How many prime numbers are there between 1 and 100?



Exercise 21

Which number am I?

(a) I'm greater than 8. I'm less than 15. I'm an odd number. I'm not a prime number.
(b) I'm a factor of 60. I'm a two-digit number. I'm an odd number.
(c) I'm a square number. I'm less than 100. I have 9 factors.
(d) I'm a prime number. I'm less than 30. I'm one greater than a multiple of 7.
(e) I'm a multiple of 5. I'm a three-digit number. I'm less than 120. I'm a multiple of 3.

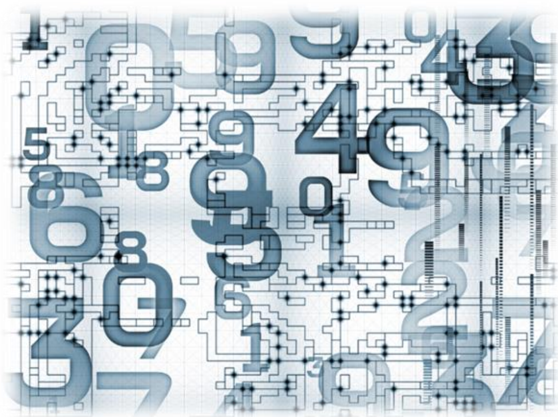
Exercise 22

Use the internet to research how prime numbers keep the internet safe. Write half a page presenting your research.

Challenge!

It is possible to write some square numbers as the sum of two prime numbers. For example, 4 = 2 + 2; 9 = 2 + 7; 16 = 5 + 11. What is the smallest square number (apart from 1) that is impossible to write as the sum of two prime numbers?

Exercise 23 (Revision)



(a) List all the factors of 24.
(b) List all the factors of 80 that are even numbers.
(c) Apart from 91, what is the highest factor of 91?
(d) True or False? Every prime number is odd.
(e) List all the prime numbers between 60 and 70.
(f) List all the prime numbers between 100 and 110.

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

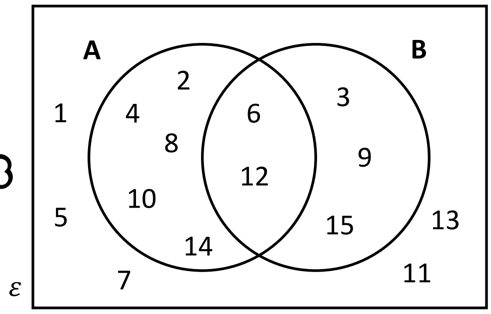
Venn Diagrams

In 1880, John Venn introduced a method for displaying information using overlapping circles.
A Venn diagram always includes:

- A **universal set** ε, a rectangle that contains all the information under inspection.
- One or more **circles**, each with a specific **label**.
- Items from the universal set placed into the correct location.

For example, let the universal set ε be the numbers between 1 and 15. Let the circle **A** represent the even numbers, whilst the circle **B** represents the multiples of 3. It is possible to construct a Venn diagram to represent this situation.

The numbers 6 and 12 in the middle are the items in ε that are even numbers and multiples of 3.



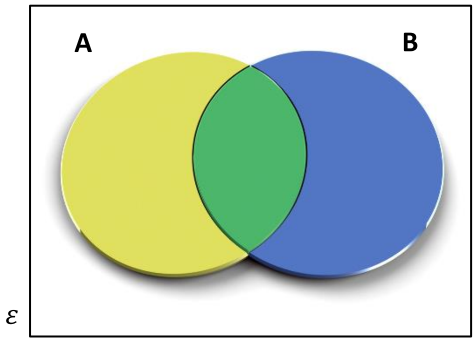
The numbers outside the circles are the items in ε that are neither even nor a multiple of 3.

Exercise 24

Skill 12

Construct a Venn diagram for the following situations.

(a) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = multiples of 5.
(b) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = even numbers; **B** = multiples of 4.
(c) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = even numbers.
(d) $\varepsilon = \{1, 2, 3, \cdots, 15\}$; **A** = factors of 15; **B** = factors of 5.
(e) $\varepsilon = \{1, 2, 3, \cdots, 15\}$; **A** = factors of 20; **B** = even numbers.
(f) $\varepsilon = \{1, 2, 3, \cdots, 15\}$; **A** = prime numbers; **B** = odd numbers.
(g) $\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = factors of 8; **B** = multiples of 8.
(h) $\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = multiples of 3; **B** = factors of 60.
(i) $\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = square numbers; **B** = odd numbers.

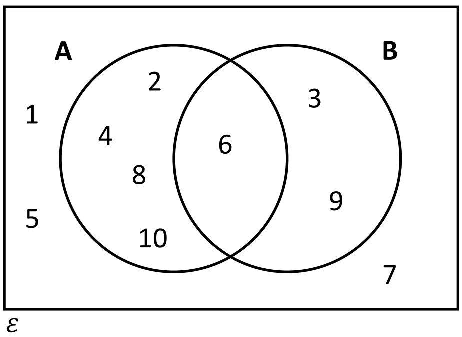


Exercise 25

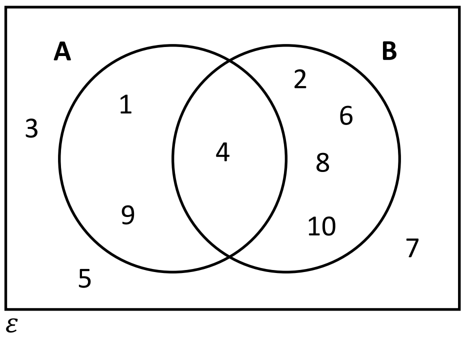
12

In the following Venn diagrams, the universal set ε consists of the numbers between 1 and 10.
What could the sets **A** and **B** represent?

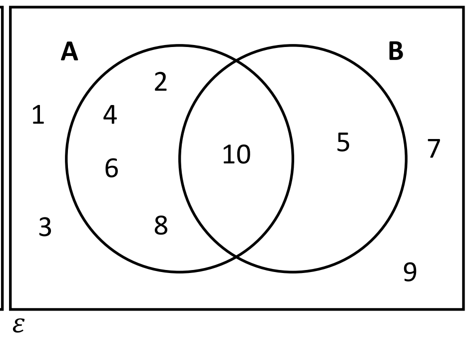
(a)



(b)



(c)

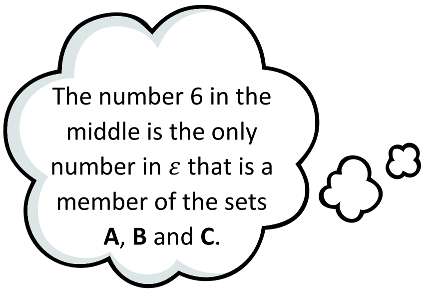


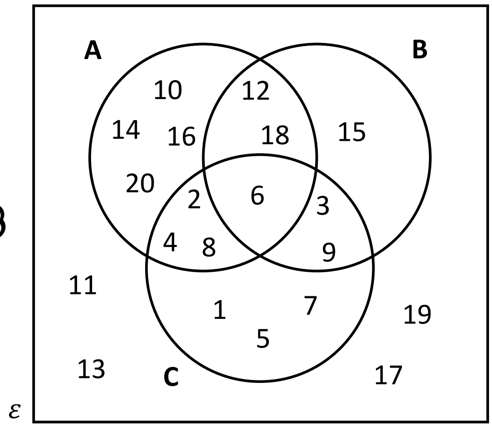
Challenge! Construct a Venn diagram for the following situation.

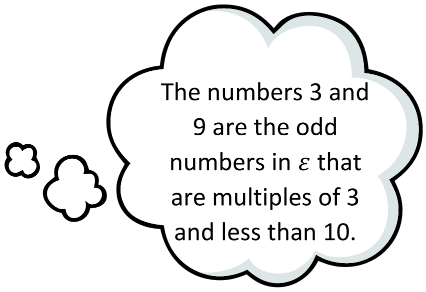
$\varepsilon = \{1, 2, 3, \cdots, 40\}$; **A** = factors of 72; **B** = multiples of 3.

Example

$\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = even numbers; **B** = multiples of 3; **C** = numbers less than 10.







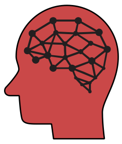
Exercise 26

12

Construct a Venn diagram for the following situations.

(a) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = even numbers; **B** = multiples of 4; **C** = numbers less than 5.
(b) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = prime numbers; **C** = numbers greater than 6.
(c) $\varepsilon = \{1, 2, 3, \cdots, 15\}$; **A** = even numbers; **B** = multiples of 3; **C** = multiples of 5.
(d) $\varepsilon = \{1, 2, 3, \cdots, 15\}$; **A** = square numbers; **B** = factors of 16; **C** = numbers less than 7.
(e) $\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = multiples of 2; **B** = multiples of 3; **C** = multiples of 9.
(f) $\varepsilon = \{1, 2, 3, \cdots, 20\}$; **A** = prime numbers; **B** = factors of 24; **C** = two-digit numbers.

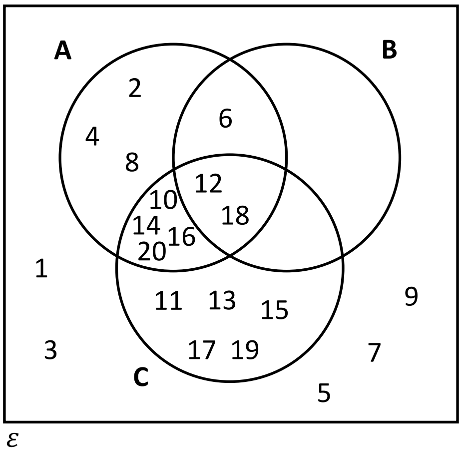
Exercise 27



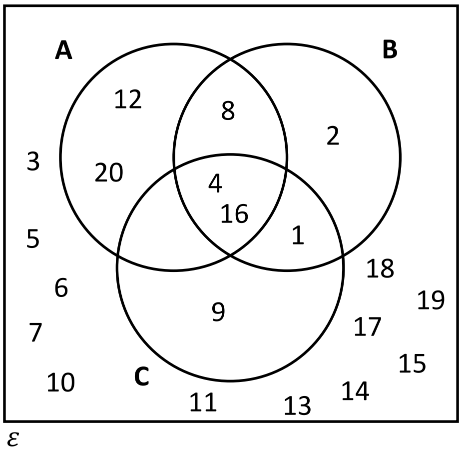
12

In the following Venn diagrams, $\varepsilon = \{1, 2, 3, \cdots, 20\}$. What could the sets **A, B** and **C** represent?

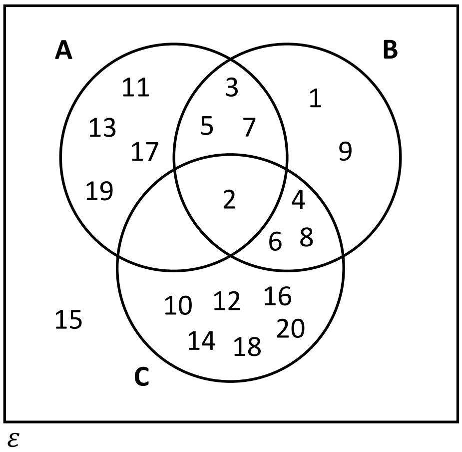
(a)



(b)

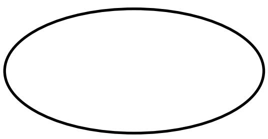


(c)

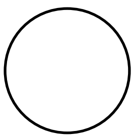


Challenge! ⚠

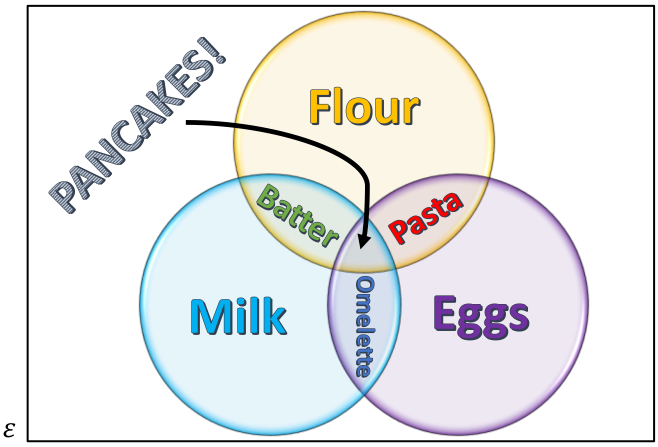
Try to construct a Venn diagram for four sets **A**, **B**, **C** and **D**.
Hint: You will need to use ellipses (not circles) to show each region.



Ellipse

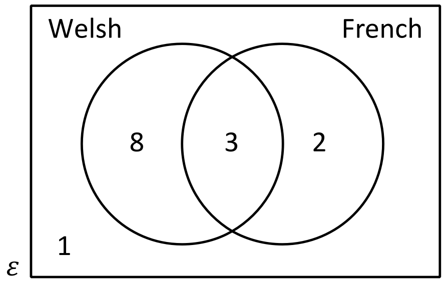


Circle



Exercise 28

Someone asks a group of learners if they can speak Welsh, French, both languages, or neither language. Their responses are shown in the Venn diagram below.
The universal set, ε, contains all the learners in the group.

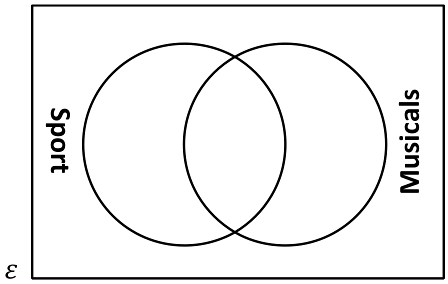


(a) How many of the learners can speak French?
(b) How many of the learners cannot speak French?
(c) How many of the learners can speak both languages?
(d) How many of the learners can speak Welsh or French?
(e) How many learners are there in the group in total?

Exercise 29

(a) In a class of 26 pupils, 14 participate in sport; 12 participate in musicals; and 5 participate in both.

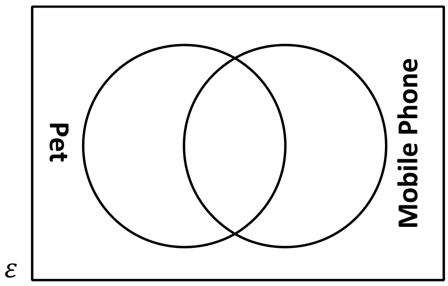
(i) Complete the following Venn diagram to show the above information.



(ii) How many pupils in the class either participate in sport or participate in musicals?

(b) In a primary school of 150 pupils, 110 have a pet; 95 have a mobile phone; and 74 have both.

(i) Complete the following Venn diagram to show the above information.

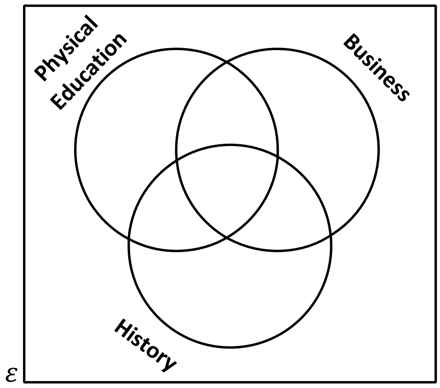


(ii) How many pupils in the school have neither a pet nor a mobile phone?

(c) There are 30 learners in a class. 20 of the learners have a brother, 18 have a sister, and 12 have a brother but no sisters. How many learners have a sister but no brothers?

(d) The head teacher wants to arrange the timetable for year 10 pupils. There are 100 pupils in the year. 28 have chosen P.E. 31 have chosen Business. 42 have chosen History. 9 have chosen P.E. and Business. 10 have chosen P.E. and History. 6 have chosen Business and History. 4 have chosen all three subjects.

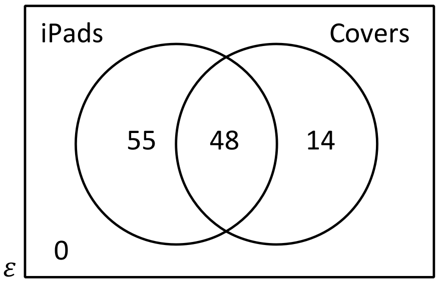
(i) Complete the following Venn diagram to show the above information.



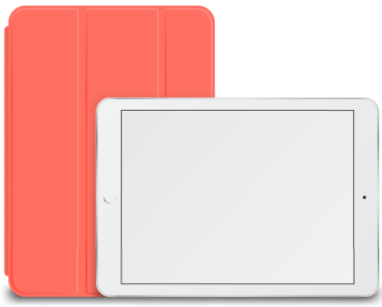
(ii) How many pupils did not choose any of these subjects?

Exercise 30 12

The Apple shop in Cardiff considers how many iPads and how many covers were sold during the first fortnight in September.



(a) How many customers purchased both an iPad and a cover at the same time?
(b) How many customers purchased only an iPad?
(c) How many customers purchased only a cover?
(d) How many customers purchased an iPad or a cover?
(e) If an iPad costs £320 and a cover costs £50, how much money did the shop take during the first fortnight in September?



Evaluation

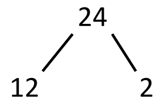
Key Words	Corrections	I am happy with...	I need to revise...

Index Form

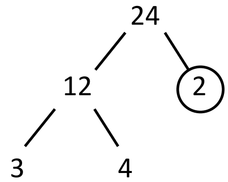
It is possible to write any whole number greater than one as a **product of its prime factors.** This means writing the number as a multiplication sum using only prime numbers. We can use a **factor tree** to help us write a number as a product of its prime factors.

Example

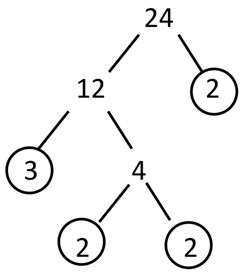
Consider the number 24. Think of a multiplication sum that makes 24, for example 12 × 2. Write 24 and then split 24 using the numbers 12 and 2.



As 2 is a prime number, we circle it; however as 12 is not a prime number, we continue to split the number, using (for example) 12 = 3 × 4.



To complete the factor tree, we continue to circle prime numbers, and split numbers that are not prime numbers, until all numbers have been considered. Notice that 1 will never appear in a factor tree, as 1 is not a prime number.



After completing the factor tree, we can now write 24 as a product of its prime factors. We must consider all the prime numbers that are circled and write a multiplication sum using these numbers. It is good practice to write these numbers in order, from smallest to largest. We can therefore write 24 as a product of its prime factors as

$$24 = 2 \times 2 \times 2 \times 3$$

Exercise 31 2

Verify that it does not matter how you split the numbers in a factor tree, by considering all the possible factor trees for the number 24 (a forest!), showing that each factor tree ends with 24 = 2 × 2 × 2 × 3.

Exercise 32 Skill 2

Write the following numbers as a product of their prime factors.

(a) 6	(b) 15	(c) 20	(d) 25	(e) 28
(f) 36	(g) 42	(h) 48	(i) 55	(j) 56
(k) 60	(l) 64	(m) 70	(n) 72	(o) 75
(p) 78	(q) 80	(r) 84	(s) 92	(t) 96
(u) 100	(v) 115	(w) 120	(x) 152	(y) 180
(z) 256	(α) 630	(β) 1080	(χ) 1225	

Challenge!

Write 415,800 as a product of its prime factors.

Index Form

You will remember from work in Year 7 that 5^2 ("five squared") means 5 × 5, whilst 5^3 ("five cubed") means 5 × 5 × 5. Following this pattern, 5^4 ("five to the power of four") means the sum 5 × 5 × 5 × 5, whilst 5^6 ("five to the power of six") means the sum 5 × 5 × 5 × 5 × 5 × 5. When writing a sum such as 7 × 7 × 7 × 7 in the form 7^4, we say that we have written the sum in **index form**.

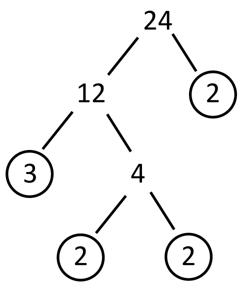
Exercise 33

Write the following sums in index form.

(a) 5 × 5 × 5
(b) 8 × 8
(c) 9 × 9 × 9 × 9
(d) 12 × 12 × 12
(e) 2 × 2 × 2 × 2 × 2
(f) 3 × 3 × 3 × 3 × 3 × 3 × 3
(g) 7 × 7 × 7 × 7 × 7 × 7 × 7 × 7
(h) 20 × 20 × 20 × 20
(i) 4 × 4 × 4 × 4 × 4 × 4 × 4 × 4 × 4 × 4 × 4
(j) 11 × 11 × 11 × 11 × 11 × 11
(k) 5 × 5 × 5 × 5 × 5
(l) 1 × 1 × 1 × 1 × 1 × 1
(m) 6 × 6 × 6 × 6 × 6 × 6 × 6 × 6 × 6
(n) 8 × 8 × 8 × 8
(o) 14

Writing a number in index form

Earlier in the chapter, we wrote the number 24 as a product of its prime factors, using the following factor tree.



$$24 = 2 \times 2 \times 2 \times 3$$

We can now write 24 in index form using our work above.

$$24 = 2^3 \times 3$$

(Notice that it would also be possible to write 24 as $24 = 2^3 \times 3^1$, however as 3 to the power of 1 (3^1) is the same as 3 on its own, we do not usually write the power of 1.)

Exercise 34

The following numbers have been written as a product of their prime factors. Re-write these numbers in index form.

(a) 16 = 2 × 2 × 2 × 2
(b) 45 = 3 × 3 × 5
(c) 90 = 2 × 3 × 3 × 5
(d) 225 = 3 × 3 × 5 × 5
(e) 704 = 2 × 2 × 2 × 2 × 2 × 2 × 11
(f) 26325 = 3 × 3 × 3 × 3 × 5 × 5 × 13

Exercise 35

Use a factor tree to help you write the following numbers in index form.

(a) 8
(b) 12
(c) 18
(d) 27
(e) 30
(f) 32
(g) 40
(h) 52
(i) 63
(j) 88
(k) 98
(l) 125
(m) 128
(n) 160
(o) 212
(p) 288
(q) 360
(r) 500
(s) 648
(t) 1125

Square Numbers in Index Form

When we write a square number in index form, we notice something special about the powers.

Exercise 36

2

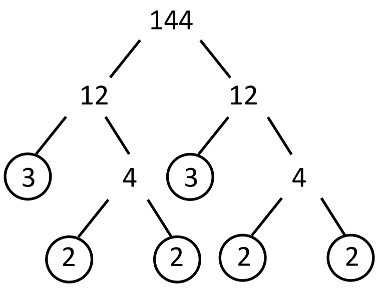
Use a factor tree to write the following square numbers in index form.

(a) 4	(b) 9	(c) 16	(d) 25	(e) 36
(f) 49	(g) 64	(h) 81	(i) 100	(j) 121

If you completed the exercise correctly, you should notice that the powers are always **even numbers**. This is always true for square numbers.

Fact: When writing a square number in index form, all the powers are always **even numbers**.

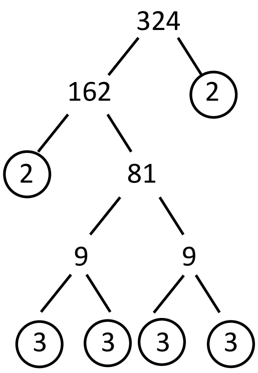
This happens because, if we start the factor tree with the **square root** of the square number, everything **doubles up**.

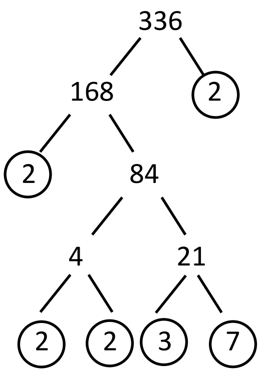


We can use the above fact to decide whether or not a number is a square number.

Example: Are the numbers 324 and 336 square numbers?

To start, let's construct factor trees for 324 and 336.





As a product of its prime factors, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$ and $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$.
In index form, $324 = 2^2 \times 3^4$ and $336 = 2^4 \times 3 \times 7$.

For 324, the powers in the index form are all even numbers, so 324 is a square number.
For 336, the powers in the index form are not all even numbers (remember that 3 on its own is the same as 3^1). Therefore 336 is not a square number.

Exercise 37 2

Write the following numbers in index form. Hence, decide whether or not the numbers are square numbers.

(a) 144	(b) 200	(c) 324	(d) 400	(e) 460
(f) 576	(g) 600	(h) 729	(i) 800	(j) 900

Finding the square root of a square number

It is possible to use the index form to find the square root of any square number.

For example, consider again the number 324 from the previous page. We saw that 324, when written in index form, is $324 = 2^2 \times 3^4$. To find the square root of 324, we **halve the powers** in the index form. In this case, we halve the powers 2 and 4 to obtain 1 and 2 so that

$$\sqrt{324} = 2^1 \times 3^2$$
$$\sqrt{324} = 2 \times 9$$
$$\sqrt{324} = 18$$

(So, $18^2 = 324$.)

Rule: To find the square root of a square number, **halve** the powers in the index form.

Exercise 38 2

Find the square root of the following square numbers, which have been written in index form.

(a) $1444 = 2^2 \times 19^2$	(b) $2304 = 2^8 \times 3^2$	(c) $2500 = 2^2 \times 5^4$
(d) $3136 = 2^6 \times 7^2$	(e) $3600 = 2^4 \times 3^2 \times 5^2$	(f) $4096 = 2^{12}$
(g) $4900 = 2^2 \times 5^2 \times 7^2$	(h) $5184 = 2^6 \times 3^4$	(i) $5625 = 3^2 \times 5^4$

Multiplying a number to make a square number

If a number is **not** a square number, what is the **smallest** whole number that we need to **multiply** our number by to make a square number?

Again, consider the number 336 from the previous page. We saw that 336, when written in index form, is $336 = 2^4 \times 3 \times 7$. As the powers are not all even, 336 is not a square number. We could multiply 336 with itself to make a square number, $336 \times 336 = 112{,}896 = 336^2$. But we can also multiply 336 by 21 to make a square number: $336 \times 21 = 7056 = 84^2$.

Why 21? Again, consider the index form for 336, $336 = 2^4 \times 3 \times 7$. The numbers that have odd powers are 3 and 7. By multiplying 336 by 3 and 7, the index form for the new number is $2^4 \times 3^2 \times 7^2$. This index form **does** represent a square number as all the powers are even. So, by multiplying 336 by $3 \times 7 = 21$, we have made a square number. (There is not a number less than 21 that works as the powers in the index form would not all be even.)

Rule: To multiply a number to make a square number, (a) write the number in index form;
(b) multiply the number by the numbers in the index form that do not have even powers.

Exercise 39 2

What is the smallest whole number that **multiplies** with the following numbers to make a square number?

(a) $60 = 2^2 \times 3 \times 5$	(b) $72 = 2^3 \times 3^2$	(c) $308 = 2^2 \times 7 \times 11$
(d) $405 = 3^4 \times 5$	(e) $540 = 2^2 \times 3^3 \times 5$	(f) $1617 = 3 \times 7^2 \times 11$
(g) 18	(h) 44	(i) 156
(j) 252	(k) 720	(l) 850

Dividing a number to make a square number

If a number is **not** a square number, what is the **smallest** whole number that we need to **divide** our number by to make a square number?

Again, consider the number 336 = $2^4 \times 3 \times 7$. We could divide 336 by itself to make a square number: 336 ÷ 336 = 1, and 1 is a square number (1^2). But we can also divide 336 by 21 to make a square number: 336 ÷ 21 = 16 = 4^2.

Why 21? Again, the numbers in the index form without even powers are important. By dividing by these numbers, the numbers will either disappear from the index form (which happens with 336), or the power becomes one less, and change to be even. In each case, the powers in the index form for the new number will all be even numbers, so we end up with a square number.

Rule: To divide a number to make a square number, (a) write the number in index form; (b) divide the number by the numbers in the index form that do not have even powers.

Exercise 40

What is the smallest whole number that **divides** the following numbers to make a square number?

(a) 68 = $2^2 \times 17$ (b) 84 = $2^2 \times 3 \times 7$ (c) 260 = $2^2 \times 5 \times 13$
(d) 567 = $3^4 \times 7$ (e) 792 = $2^3 \times 3^2 \times 11$ (f) 4114 = $2 \times 11^2 \times 17$
(g) 20 (h) 27 (i) 42
(j) 288 (k) 350 (l) 940

Exercise 41 (Revision)

(a) Write the following numbers as a product of their prime factors.
(i) 68 (ii) 140 (iii) 304
(b) Using a factor tree, write the following numbers in index form.
(i) 92 (ii) 196 (iii) 1200
(c) Are the following numbers square numbers? If they **are** square, find the square root of the square number. If they are **not** square, find the smallest whole number that **multiplies** with the number to make a square number.
(i) 484 (ii) 888 (iii) 1600

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

HCF, LCM

The **Highest Common Factor** (HCF) of two numbers is the largest number that is a factor of both numbers.
The **Lowest Common Multiple** (LCM) of two numbers is the smallest number that is a multiple of both numbers.

Example

What is the Highest Common Factor and Lowest Common Multiple of 24 and 36?

Method 1

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

The **largest** number that appears in **both** lists is 12.
Therefore, 12 is the Highest Common Factor of 24 and 36.

24 = 1 × 24
24 = 2 × 12
24 = 3 × 8
24 = 4 × 6

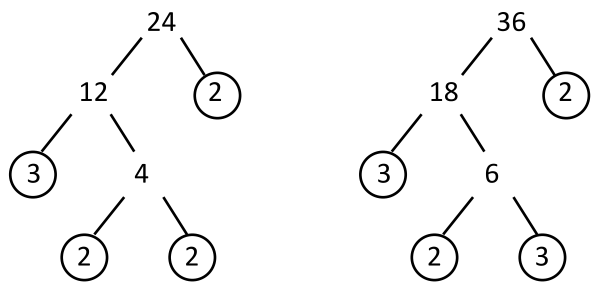
36 = 1 × 36
36 = 2 × 18
36 = 3 × 12
36 = 4 × 9
36 = 6 × 6

The multiples of 24 are 24, 48, 72, 96, 120, ...
The multiples of 36 are 36, 72, 108, 144, 180, ...

The **smallest** number that appears in both lists is 72. Therefore, 72 is the Lowest Common Multiple of 24 and 36.

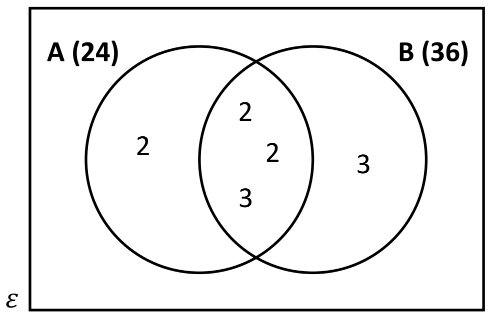
Method 2

Here are some factor trees for 24 and 36.



As a product of their prime factors, 24 = 2 × 2 × 2 × 3 and 36 = 2 × 2 × 3 × 3.

We can construct the following Venn diagram for the prime numbers that form the product of prime factors for 24 and 36. ε = Prime factors of 24 and 36; **A** = Prime factors of 24; **B** = Prime factors of 36.



To find the Highest Common Factor of 24 and 36, we multiply all the numbers that appear in the centre of the Venn diagram (in the **intersection**). Therefore, the Highest Common Factor of 24 and 36 is 2 × 2 × 3 = 12.

To find the Lowest Common Multiple of 24 and 36, we multiply all the numbers that appear in the **union** of the Venn diagram (inside the circles). Therefore, the Lowest Common Multiple of 24 and 36 is 2 × 2 × 2 × 3 × 3 = 72.

Note: In this method, if no numbers appear in the middle of the Venn diagram (in the intersection), then the Highest Common Factor of the two numbers is 1 (remember that 1 is always a factor of any whole number).

Exercise 42

Find the Highest Common Factor and Lowest Common Multiple of the following numbers.
(Suggestion: Use a factor tree and Venn diagram if the numbers are large.)

(a) 6 and 8 (b) 8 and 18 (c) 8 and 9 (d) 36 and 48 (e) 25 and 55 (f) 33 and 55 (g) 54 and 72
(h) 30 and 40 (i) 45 and 63 (j) 24 and 50 (k) 27 and 63 (l) 20 and 49 (m) 48 and 84 (n) 50 and 64
(o) 42 and 49 (p) 84 and 96 (q) 120 and 125 (r) 128 and 160 (s) 180 and 200 (t) 75 and 315 (u) 125 and 144
(v) 240 and 250 (w) 256 and 280 (x) 340 and 380 (y) 350 and 400 (z) 640 and 800 (α) 700 and 750 (β) 1296 and 1536

Challenge! ⚠

(a) What is the Highest Common Factor and Lowest Common Multiple of the numbers 24, 84 and 126?
(b) For any two numbers a and b, what is the connection between $a \times b$ and the product of the highest common factor by the lowest common multiple?

Exercise 43

(a) Dewi and John are cycling around a bicycle track. Dewi takes 50 seconds to complete a lap of the track. John takes 80 seconds to complete a lap of the track. Dewi and John start cycling at the same time from the start line. When is the next time they will they **both** be at the start line?
(b) Buses to Llandudno leave the station every 18 minutes. Buses to Abergele leave the same station every 16 minutes. A bus to Llandudno and a bus to Abergele leave the station at 08:20. When will be the next time that buses leave to the two places at the same time?
(c) Lydia pays £6 for a ticket every time she goes to the cinema. Megan goes to a different cinema, and she pays £8 for a ticket each time. Lydia and Megan meet and realise that they have spent the same amount on cinema tickets this year. Megan has not spent more than £40. How many times have they each been to the cinema this year?
(d) Fiona is organising a party. She would like to buy sausage rolls, small pork pies and cakes for the guests. The table below shows what is available to buy in the local shop.

	Sausage Rolls	Small Pork Pies	Cakes
Amount in each pack	12	6	9
Cost per pack	£2.40	£1.80	£2.10

Fiona wants to be sure that each person in the party has at least one sausage roll, one pork pie and one cake each. Fiona has £30 to spend on food for the party. Does Fiona have enough money to buy the required food?

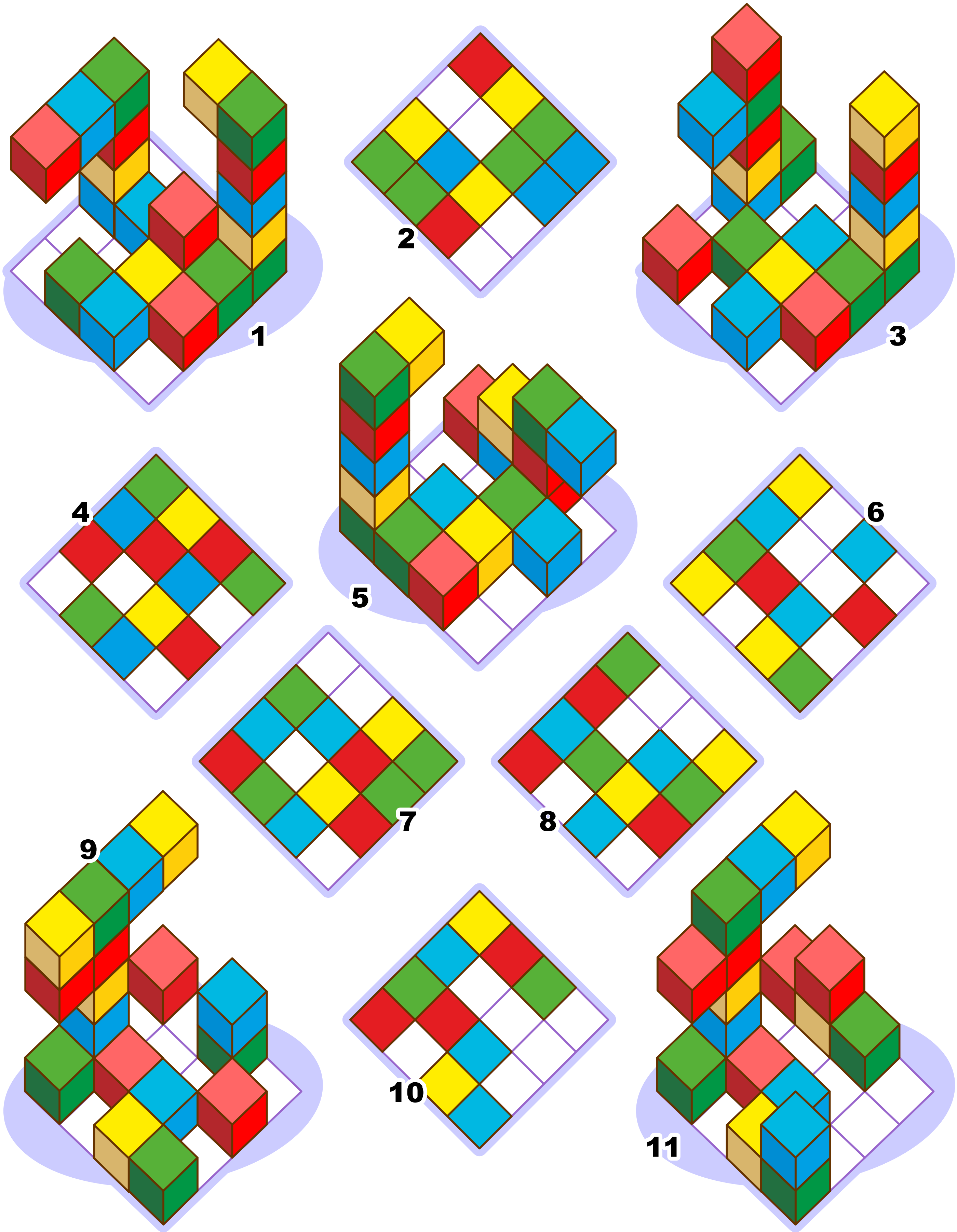
Exercise 44 (Revision)

Find the Highest Common Factor and Least Common Multiple of the following numbers.
(a) 12 and 16 (b) 32 and 40 (c) 35 and 45 (d) 100 and 150 (e) 392 and 400

Evaluation

Key Words	Corrections	I am happy with...	I need to revise...

Puzzle: Pair the solid to its top view.



Factors and Multiples

Reflection Sheet

Name: ....

Percentage in the test: ....

	I know this.	I need to revise this.	Question in the test:	Correct in the test?
I know how to find the **square root** of a square number.			1	
I know how to find the **cube roo**t of a cube number.			1	
I can perform calculations that include **squaring**, **cubing**, **square root** and **cube root**.			1	
I can list the **factors** of any whole number.			2	
I know the meaning of the terms **sum**, **difference**, **multiple** and **even number**.			5	
I know how to decide whether or not a number is a **prime number**.			3	
I can **list all the prime numbers** in a certain range, e.g. between 70 and 80.			4	
I know how to draw a **Venn diagram** that uses **two circles**.			5	
I know how to draw a **Venn diagram** that uses **three circles**.			8	
I can answer **worded problems** using Venn diagrams.			11	
I can write a number as a **multiple of its prime factors**.			7	
I can write numbers in **index form**.			6, 9	
I know what's special about **square numbers** when written in index form.			10	
I can **recognise square numbers** using index form.			10	
I can use index form to find the **square root of a square number**.			10	
I can use index form to decide what to **multiply/divide by** in order to **change a number into a square number**.			10	
I can find the **Highest Common Factor** of two numbers.			7	
I can find the **Lowest Common Multiple** of two numbers.			7	
I can answer **worded problems** using the Highest Common Divisor or Lowest Common Multiple.			12	