YSGOL CREUDDYN
DAWN DYSG DAIONI

The Mathematics Department

PS 3

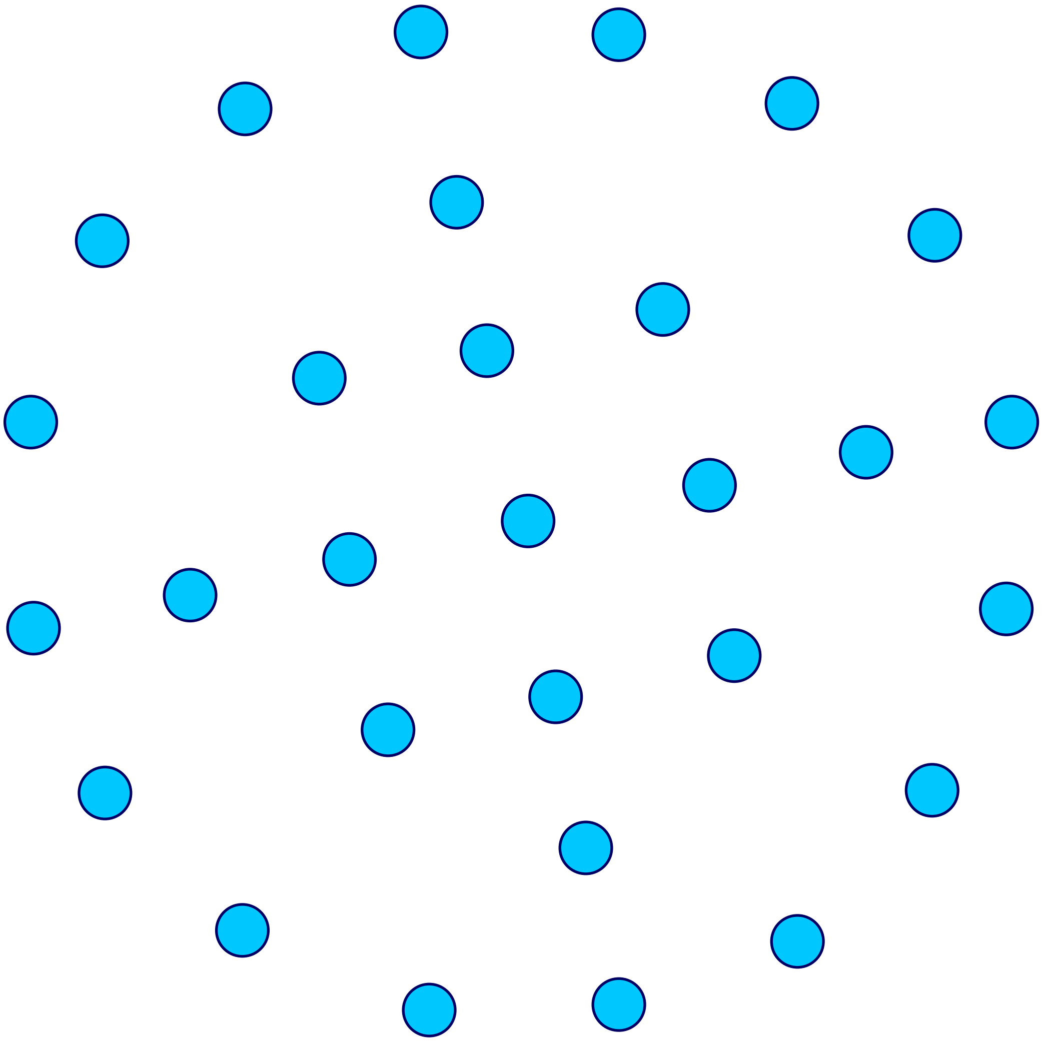
Numbers: Diving Deeper

Name:

Contents

Place Value

How many counters are shown below?

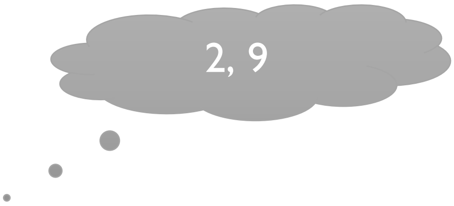


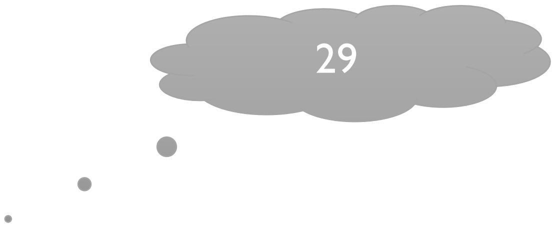
Circle the counters to show how you counted them.

Did other people count the counters in different ways? Discuss.

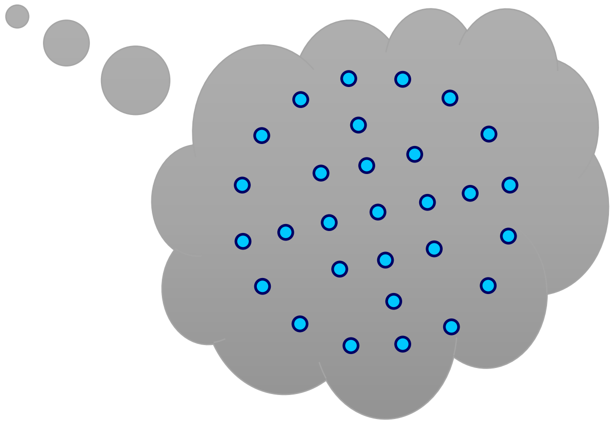
Number, numeral and digit

Hopefully you agree that twenty-nine counters were shown on the previous page. Usually, we write this **number** as the **numeral** 29, with the **digit** 2 showing two tens, and the **digit** 9 showing nine units.





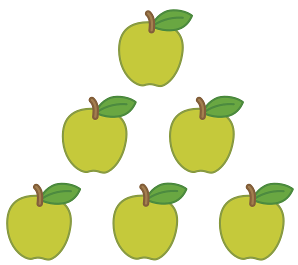
- **Digits** are used to form **numerals**.
- Numerals represent the idea of a **number**.



Exercise

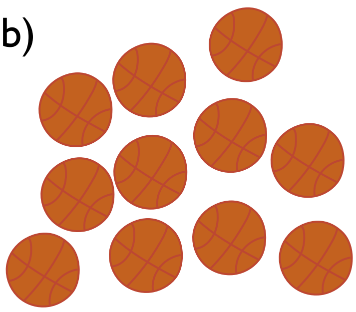
Which numbers are shown in the pictures below? Circle the correct numeral.

(a)



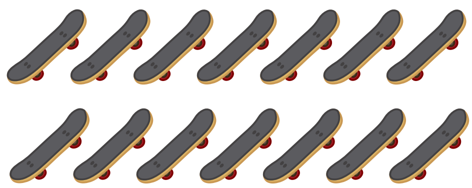
(i) 8 (ii) 7 (iii) 6

(b)



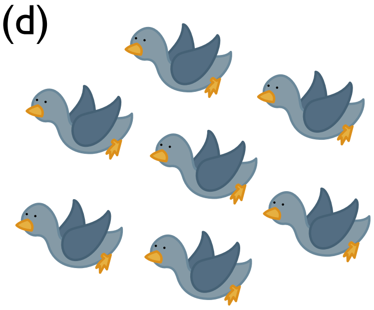
(i) 15 (ii) 13 (iii) 11

(c)



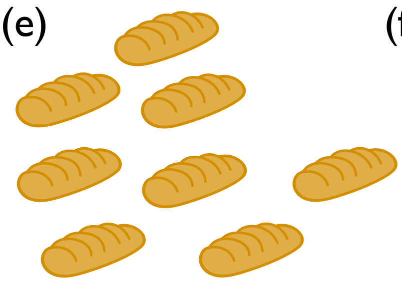
(i) 14 (ii) 16 (iii) 18

(d)



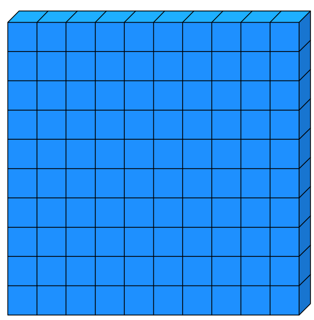
(i) 8 (ii) 7 (iii) 6

(e)



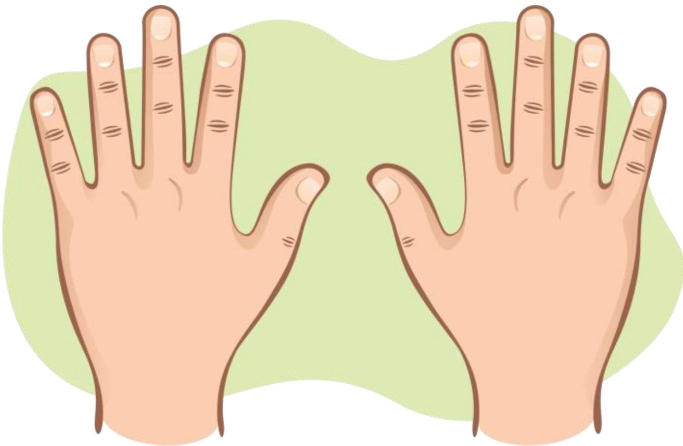
(i) 5 (ii) 7 (iii) 8

(f)



(i) 100 (ii) 10 (iii) 50

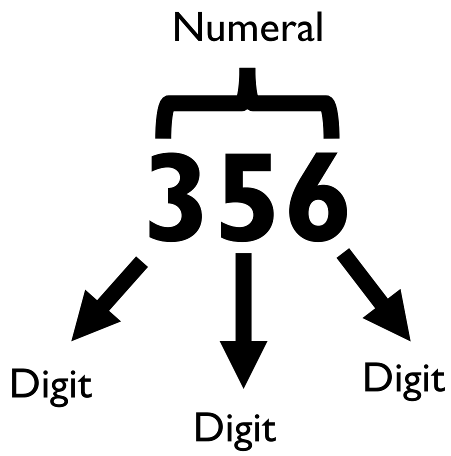
Base ten



Usually, a person has ten fingers and thumbs on their hands, so humans have learnt to write numerals using a **base ten** system.

In this system, we place the **ten digits** 0 to 9 in columns to write different numerals. For example, it is possible to write the number three hundred and fifty-six as follows:

hundreds	tens	units
3	**5**	**6**
$10^2 = 100$	10	1



Exercise

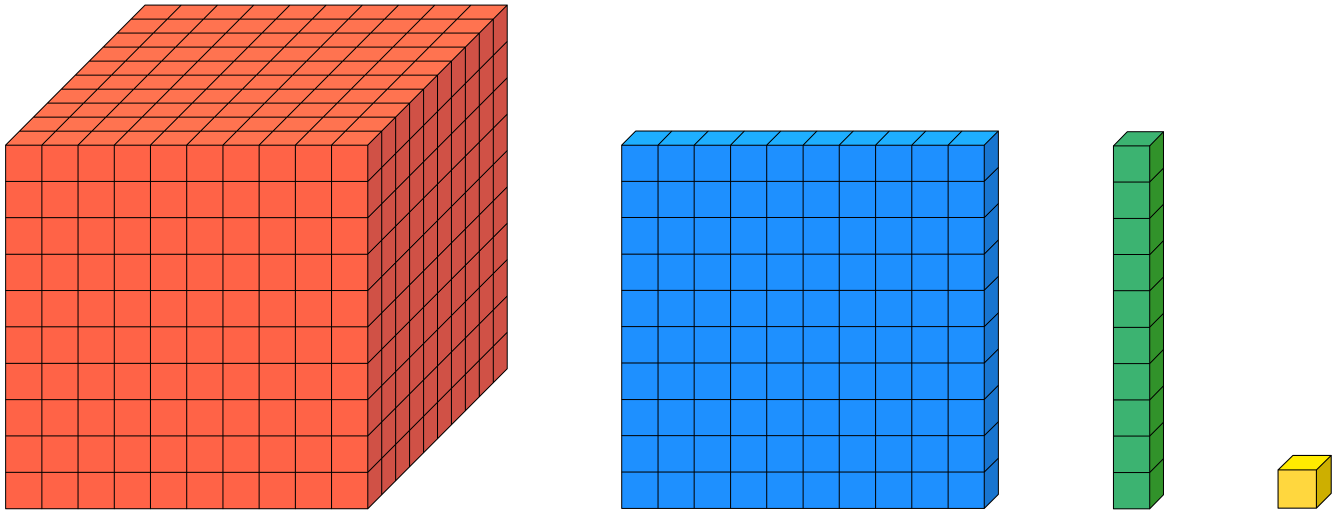
A 3-digit code is used to open the lock shown on the right. The 3-digit code must use all of the digits

2, 5 and 9.

Write down all the different numerals that could represent the code.

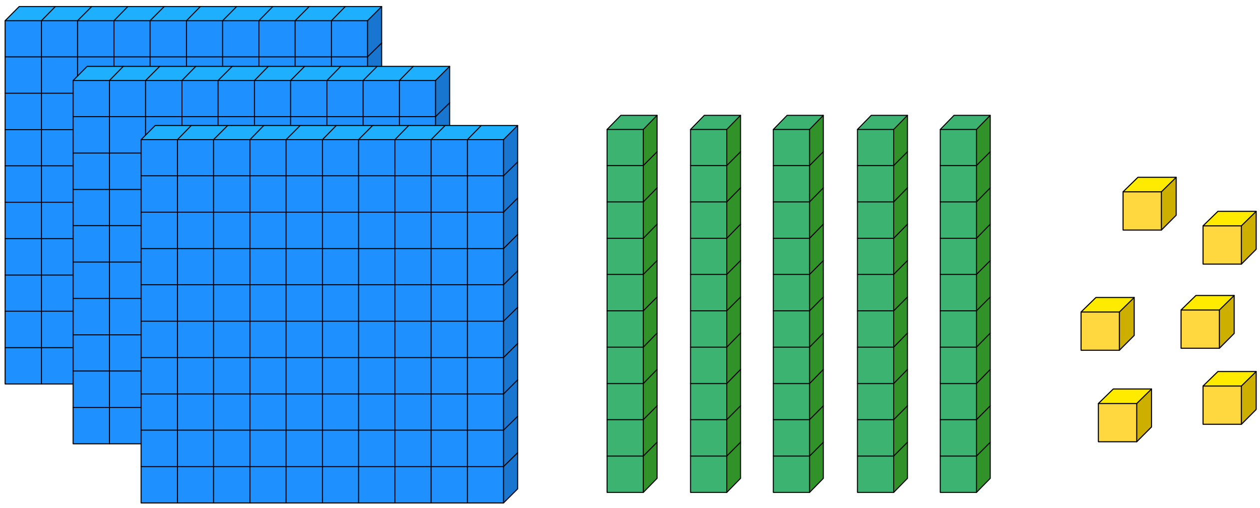
Dienes blocks (base ten)

It is possible to use the following Dienes blocks to show numbers that have been written in base ten.



$10^3 = 1000$, Thousand $10^2 = 100$, Hundred 10, Ten 1, Unit

For example, here is one way of using the blocks to show the numeral 356:



Exercise

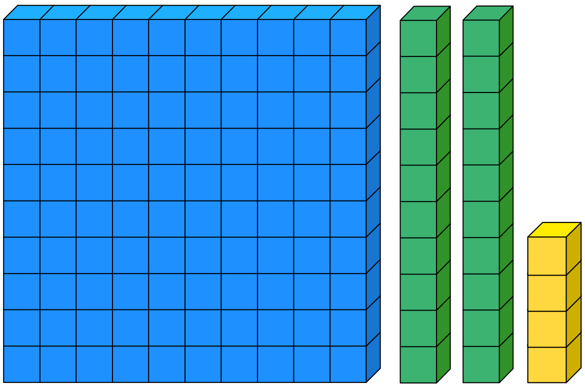
Use the Dienes blocks to show the following numerals.

(a) 3	(b) 30	(c) 300	(d) 3000
(e) 303	(f) 3030	(g) 33	(h) 3330
(i) 2	(j) 24	(k) 243	(l) 2431
(m) 2016	(n) 4007	(o) 280	(p) 5309

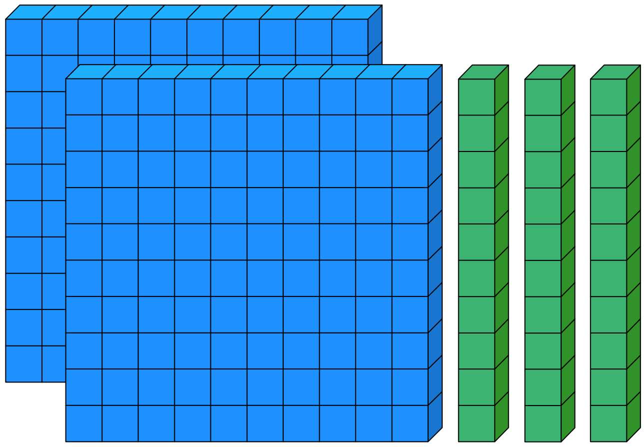
Exercise

Which numbers are shown below?

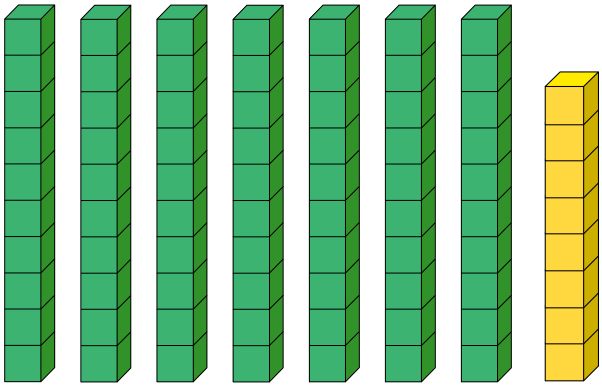
(a)



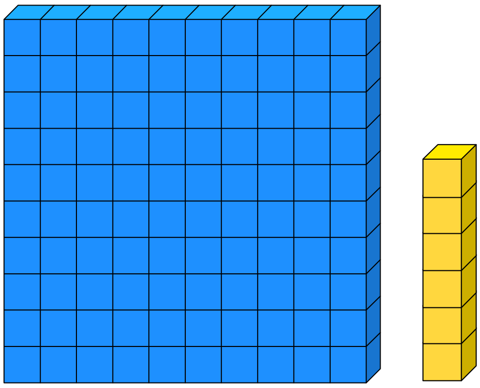
(b)



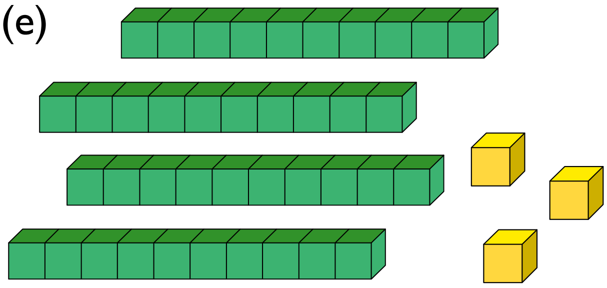
(c)



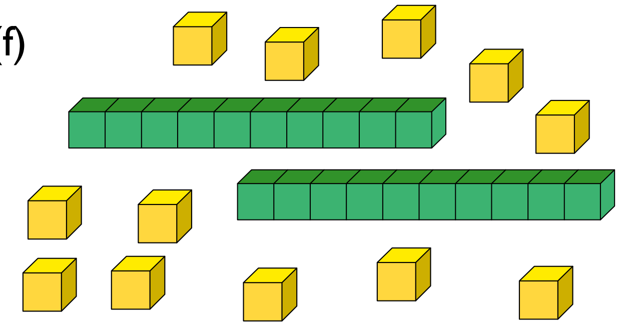
(d)



(e)



(f)



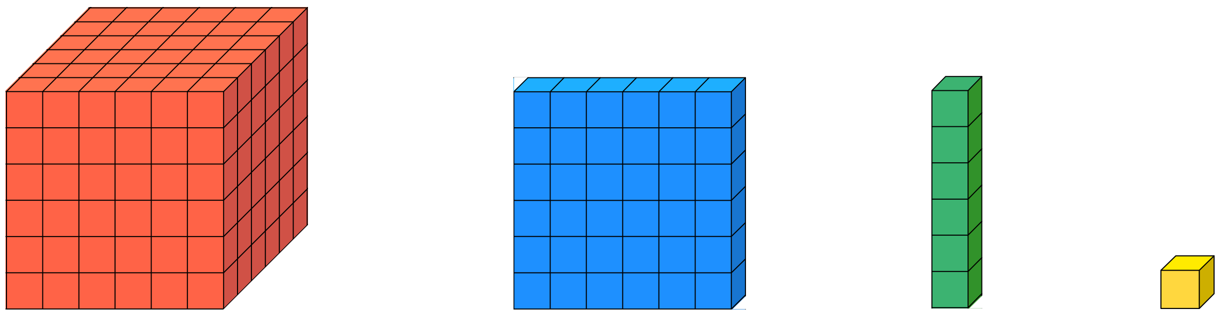
Other Bases

Base ten is not the only way of writing numbers. For example, base six uses the six digits 0, 1, 2, 3, 4, 5 to write numbers that have been grouped into sixes.

Counting in base six

1	One	20	Two sixes
2	Two	21	Two sixes and one
3	Three	22	Two sixes and two
4	Four	23	Two sixes and three
5	Five	24	Two sixes and four
10	Six	25	Two sixes and five
11	One six and one	30	Three sixes
12	One six and two	31	Three sixes and one
13	One six and three	32	Three sixes and two
14	One six and four	33	Three sixes and three
15	One six and five	34	Three sixes and four ....

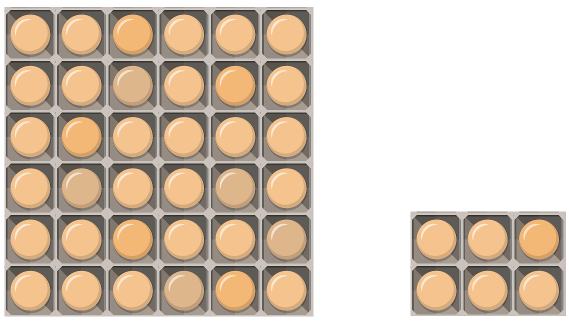
Dienes Blocks (base six)



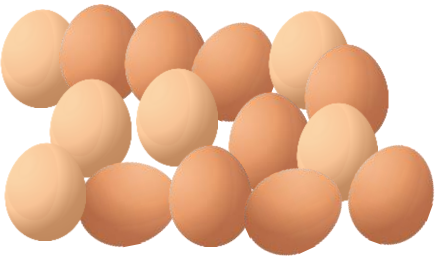
10^3 = 1000, Six cubed 10^2 = 100, Six squared 10, Six 1, Unit

Egg boxes

Base six is useful when packing eggs, as boxes of the following shapes are used.



Egg packing



How many eggs are shown above?

To begin, write down your answer using base ten: ___________

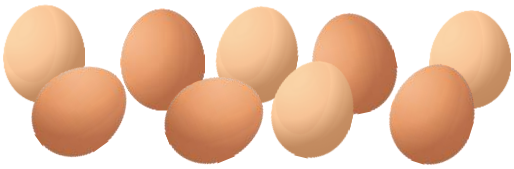
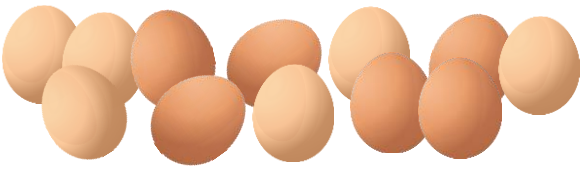
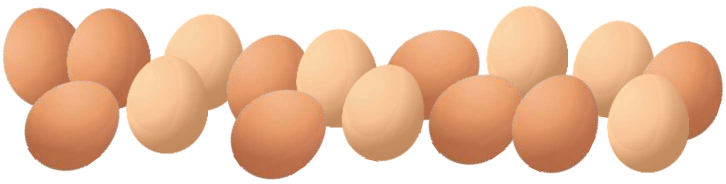
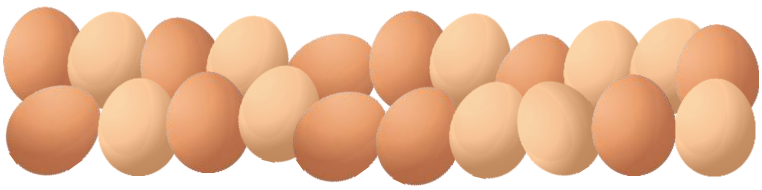
How many **full** boxes of 6 eggs would the above eggs fill? ________

How many eggs would be left over after filling the boxes? _______

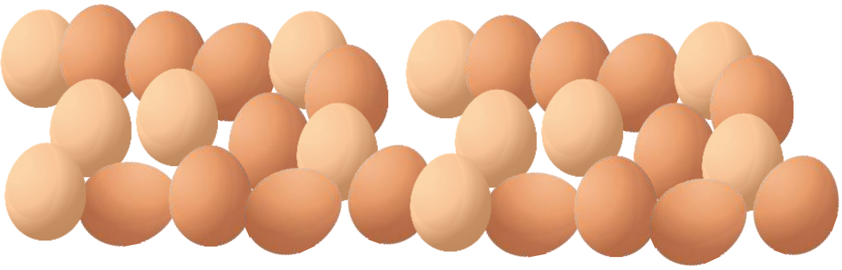
How many eggs are shown above? This time, write down your answer using base six: __________

What is the connection between your answer in base six and the previous two answers? ____

Complete the following table.

Eggs	**Number of eggs in base ten**	**Number of eggs in base six**
		
		
		
		

Egg packing in fours

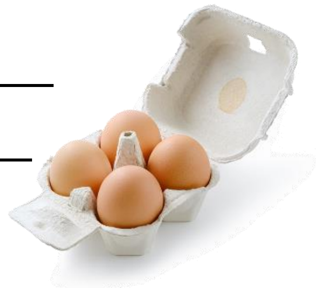


How many eggs are shown above?

To begin, write down your answer using base ten: ____________

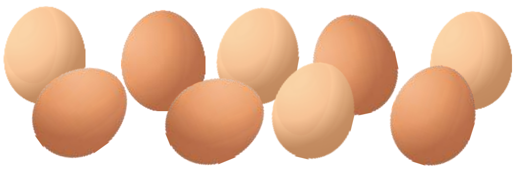
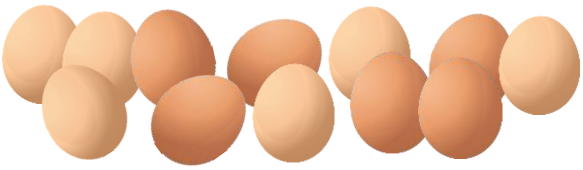
How many **full** boxes of 4 eggs would the above eggs fill? ________

How many eggs would be left over after filling the boxes? _______



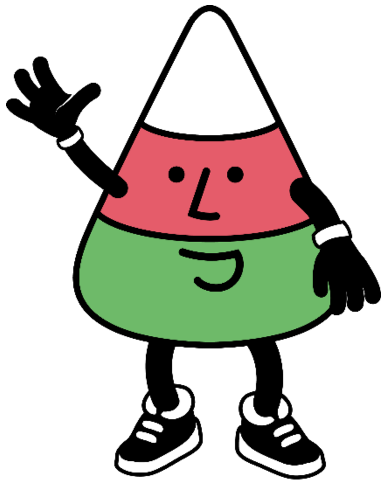
How many eggs are shown above? This time, write down your answer using base four: ____________

Complete the following table.

Eggs	Number of eggs in base ten	Number of eggs in base four
		
		

3 eggs are required to make one cheese and tomato omelette. Rhodri cooks 6 omelettes, one for each member of the family. How many boxes of 4 eggs does Rhodri need to buy?

Base eight

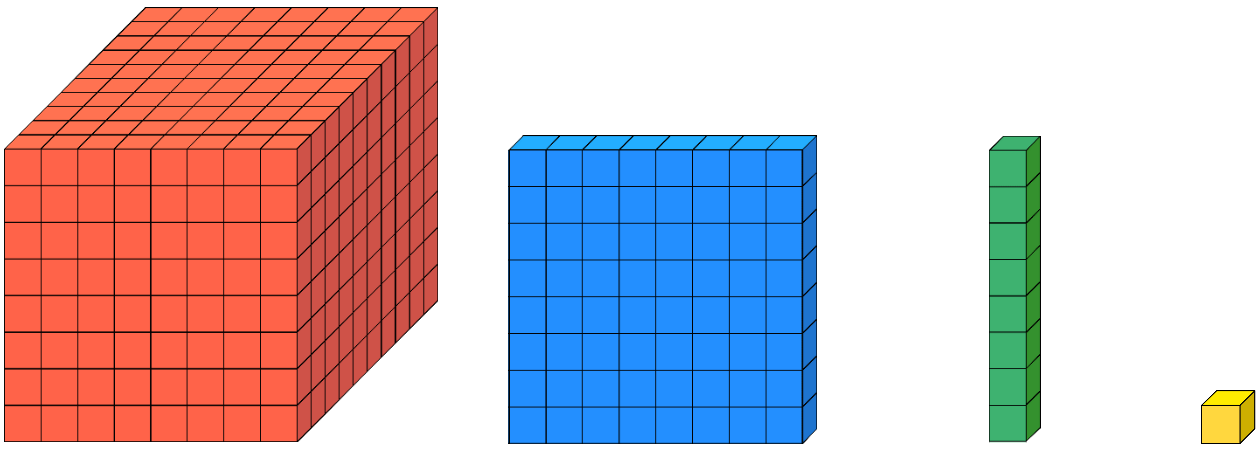


When watching cartoons, have you ever noticed that the characters very often have four fingers and thumbs on each hand? The reason for this is that it is quicker to animate a character with less fingers, but it does mean that Mickey Mouse, Spongebob Squarepants and Mr Urdd need to count using **base eight**.

Counting in base eight

1	One	15	One eight and five
2	Two	16	One eight and six
3	Three	17	One eight and seven
4	Four	20	Two eights
5	Five	21	Two eights and one
6	Six	22	Two eights and two
7	Seven	23	Two eights and three
10	Eight	24	Two eights and four
11	One eight and one	25	Two eights and five
12	One eight and two	26	Two eights and six
13	One eight and three	27	Two eights and seven
14	One eight and four	30	Three eights ....

Dienes blocks (base eight)



$10^3 = 1000$, Eight cubed $10^2 = 100$, Eight squared   10, Eight   1, Unit

Did you know?

The old American language Yuki used base eight to count as the spaces between the fingers were used to count, not the fingers themselves.

Biscuit packing

The company ‘Delicious Biscuits’ packs biscuits in packs of 8 biscuits.

On Monday, the baker bakes 30 delicious biscuits.

How many full packets of biscuits does this give? _______

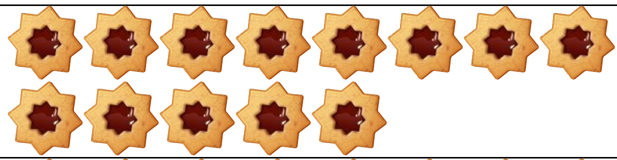
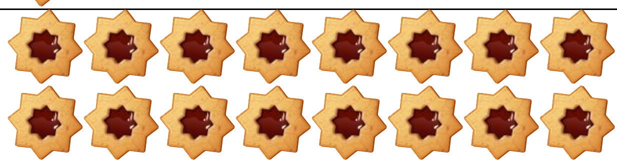
How many biscuits are left over on Monday after filling the boxes? _______

Use the two previous answers to write 30 in base eight. _______

The baker bakes 35 fresh delicious biscuits on Tuesday. He adds the left-over biscuits from Monday to the pile of biscuits. How many full packets of biscuits will be ready to sell on Tuesday? _______

How many biscuits are left over on Tuesday? _______

For the rest of the week, here are the number of fresh biscuits baked each day. Complete the table to show how many fresh biscuits were baked each day.

Day	Fresh biscuits baked today	Number of biscuits in base ten	Number of biscuits in base eight
Wednesday			
Thursday			
Friday			
Saturday			

Addition in base eight

Example

	3	5
+		6
	4	3
	1	

	2	6	3
+	3	4	7
	6	3	2
	1	1	

		5	6
+	3	7
1	1	5
	1	

	4	3
+	1	5
	6	0
	1	

Exercise

	2
+	4

	2
+	5

	2
+	6

	2
+	7

	3	2
+		7

	4	3
+	2	1

	4	3
+	2	6

	4	3
+	3	6

	4	3
+	7	0

	3	4
+	7	6

	2	6	4
+	6	2	3

	2	7	3	4	2	4
+		3	6	5	3	1

	3	0	4	5
+	2	4	7	4

		3	2	5
+	1	7	3	4

		1	2
+	2	7	0

Subtraction in base eight

Example

					3	1			4	1			4	12	1
	6			7	~~4~~	3			~~5~~	2			~~5~~	~~3~~	6
−	2		−		1	5		−	2	6		−	1	4	7
	4			7	2	6			2	4			3	6	7

Exercise

	4	5			4	5			4	5			4	5	
−		4		−		5		−		6		−		7	

	3	4			3	4			3	4			3	4	
−	1	3		−	1	6		−	2	7		−	1	5	

	3	4	2			3	4	2			3	4	2		
−	2	1	6		−	2	5	1		−	1	6	4		

	3	1	5			4	1	3	2	5			4	0	3
−	1	2	6		−	1	3	4	0	6		−	2	3	5

Base two: Binary Numbers

Binary numbers are the numerals that computers understand.
Only two digits are used in this system: 0 and 1. Simply put:

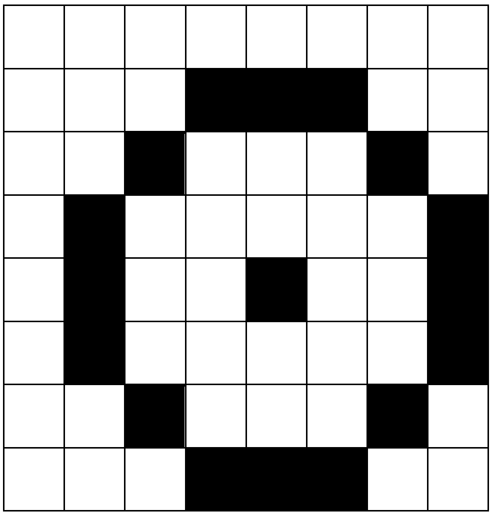
0 (zero) = off (no electrical current);

1 (one) = on (electrical current flowing).

Bitmaps

A bitmap uses binary numbers to represent simple black and white pictures.

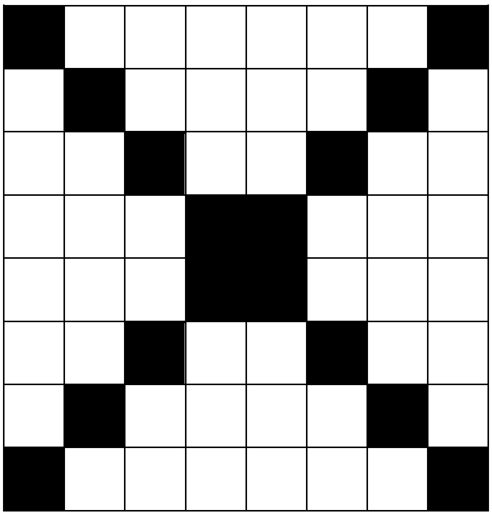
A '0' represents a white square and '1' represents a black square.



0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	0	0	1
0	1	0	0	1	0	0	1
0	1	0	0	0	0	0	1
0	0	1	0	0	0	1	0
0	0	0	1	1	1	0	0

Exercise

Write down the bitmap for the following picture.



Exercise

On the following grids, design your own pictures. Then, write down the bitmap for the picture. Remember to use '0' to represent a white square and '1' to represent a shaded square.

Exercise

Complete the picture for the following bitmap.

0	0	1	0	0	1	0	0
0	1	0	1	1	0	1	0
1	0	0	0	0	0	0	1
0	1	0	0	0	0	1	0
0	0	1	0	0	1	0	0
0	0	0	1	1	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	0	1	0

Changing between base ten and base two

In our usual base ten system, the value of each new column to the left is **ten times** greater than the previous column.

Ten thousand	Thousand	Hundred	Ten	Unit
$10^4 = 10000$	$10^3 = 1000$	$10^2 = 100$	10	1

× 10 × 10 × 10 × 10

In the base two system however, the value of each new column to the left is **two times** greater than the previous column.

Sixteen	Eight	Four	Two	Unit
$10^4 = 10000$ ($2^4 = 16$)	$10^3 = 1000$ ($2^3 = 8$)	$10^2 = 100$ ($2^2 = 4$)	10 (2)	1

× 2 × 2 × 2 × 2

This means that the base two numeral 10110 represents the base ten numeral 16 + 4 + 2 = 22.

Exercise

Change the following base two numerals to be in base ten.

Base two						Base ten	Base two						Base ten
	16	8	4	2	1			16	8	4	2	1	
(a)	0	1	0	1	0		(g)	0	1	1	1	0	
(b)	1	0	1	0	1		(h)	1	0	0	0	1	
(c)	1	1	0	0	1		(i)	1	1	1	0	1	
(d)	0	0	1	0	0		(j)	0	0	1	1	1	
(e)	0	0	1	0	1		(k)	0	0	0	1	1	
(f)	1	0	0	1	0		(l)	1	1	1	1	0	

In order to write 9 in base two, we need to consider which columns can be used to give a total of 9.

16	**8**	**4**	**2**	**1**
0	1	0	0	1

Because 9 = 8 + 1, the base ten numeral 9 is written as 1001 in base two.

Exercise

Complete the following table.

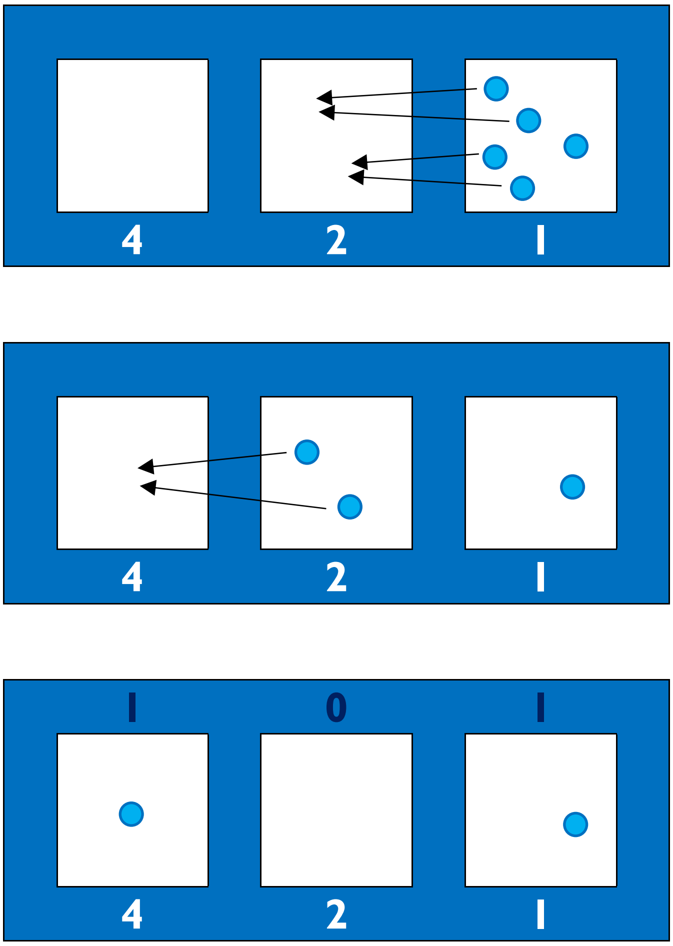
Base ten	Base two					Base ten	Base two				
	16	8	4	2	1		16	8	4	2	1
1						17					
2						18					
3						19					
4						20					
5						21					
6						22					
7						23					
8						24					
9	0	1	0	0	1	25					
10						26					
11						27					
12						28					
13						29					
14						30					
15						31					
16											

Learning more about bases: Exploding dots

Go to the website https://www.explodingdots.org/ and work your way through ISLAND 1 (MECHANIA).

Example

This is what happens in a 2 ➔ 1 machine if 5 dots are placed in the box on the right.



Exercise

Use squared paper to show what happens if the following number of dots are placed in the box on the right.

(a) 1 (b) 2 (c) 3

(d) 4 (e) 6 (f) 7

Extension

Repeat the above exercise, but this time use a 3 ➔ 1 machine.

Alien Invasion

5 aliens are shown below. Each one counts in a different way.

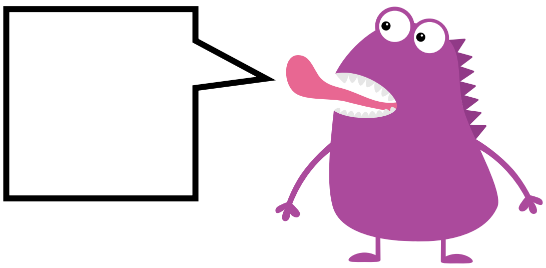
1st: Purple Peter counts in **twos.**

2nd: Blue Billie counts in **threes.**

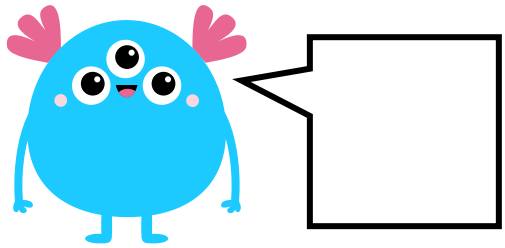
3rd: Orange Ollie counts in **fours.**

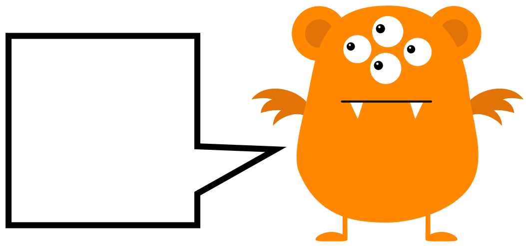
4th: Pink Penny counts in **fives.**

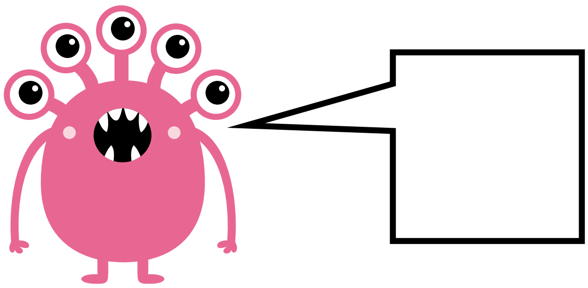
5th: Yellow Yasmin counts in **sixes.**

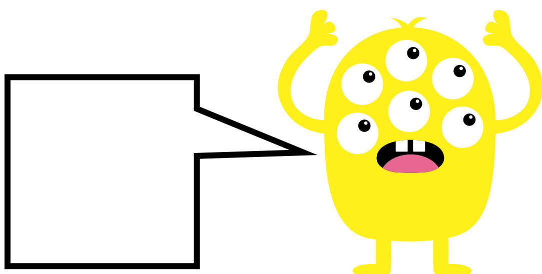


The aliens have all just landed on Earth and want to count the number of fingers and thumbs shown below.









Write down their answers in the speech bubbles.

Bases, bases, bases!

Complete the following table.

Base ten	Base nine	Base eight	Base seven	Base six	Base five	Base four	Base three	Base two
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								

The 1089 Problem

Here is a 3-digit number with decreasing digits:	742
Write the digits in reverse order:	247
Subtract:	495
Write the digits in reverse order:	594
Add:	1,089

What happens if you start with 832?

Here is a 3-digit number with decreasing digits:	832
Write the digits in reverse order:	
Subtract:	
Write the digits in reverse order:	
Add:	

Now start with a 3-digit number of your choice:

Here is a 3-digit number with decreasing digits:	
Write the digits in reverse order:	
Subtract:	
Write the digits in reverse order:	
Add:	

Why is the answer always 1089?

Challenge!

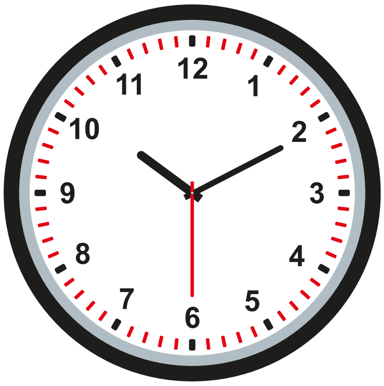
Repeat the above process, but work in base eight, not base ten.

Hint: Be extra careful when 'borrowing' during the subtraction sum.

If the final total is 1089 for base ten, what is the final total for base eight?

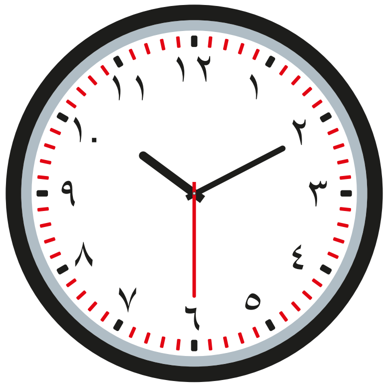
Other Numerals

We are familiar with numerals such as 238 and 873, but other types of numerals exist.



Western Arabic Numerals

Chinese Numerals



Eastern Arabic Numerals

Roman Numerals

The Romans had their own way of writing numbers: they used the following digits.

1	5	10	50	100	500	1,000
I	V	X	L	C	D	M

The above digits can be combined to form the following numerals.

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

Rule 1

If a digit appears **after** a greater (or equal) digit, then the digit is **added**.

For example:

VI = V + I   or   LXX = L + X + X
= 5 + 1   = 50 + 10 + 10
= 6   = 70

Rule 2

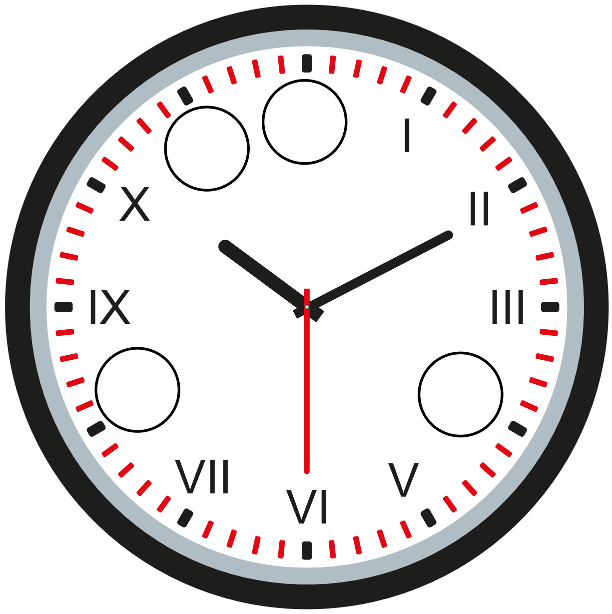
If a digit appears **before** a greater digit, then the digit is subtracted from that digit.

For example:

IV = V – I   or   IX = X – I
= 5 – 1   = 10 – 1
= 4   = 9

Exercise

Write the missing Roman numerals in the clock on the right.



Exercise

Change the following base ten numerals to be Roman numerals.

(a) 31 ______________ (b) 160 ______________

(c) 1,530 ______________ (d) 2,025 ______________

Exercise

Change the following Roman numerals to be base ten numerals.

(a) XXVI ______________ (b) CXVI ______________ (c) CDLII ______________

(d) MMMCLXVII ______________ (e) MMDCCLXXIV ______________ (f) MCLXXVIII ______________

Exercise

Here are two sets of digits from 0 to 9.

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Use the above digits to create the following 4-digit numerals.
Use each digit at most once.

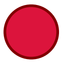
Greatest odd number ______________ Greatest even number ______________

Smallest multiple of 5 ______________ Greatest multiple of 3 ______________

Closest number to 5,000 ______________ Closest multiple of 9 to 2,000 ______________

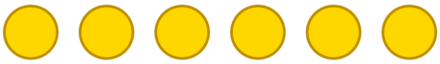
Negative Numbers

Double sided counters

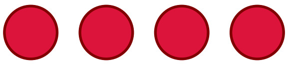
A yellow counter  is worth +1 and a red counter  is worth –1.
We say "one" for +1 and "negative one" for –1.

Example

(a) The diagram below shows 6.



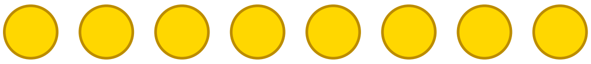
(b) The diagram below shows –4.



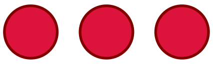
Exercise

Which numbers are shown in the following diagrams?

(a)

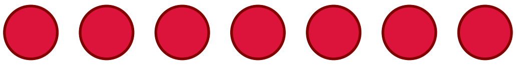


(b)



(c)

(d)



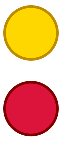
Exercise

Use double sided counters (physical or on-line) to show the following numbers.

(a) 5 (b) –2 (c) 9 (d) –6

Zero Pairs

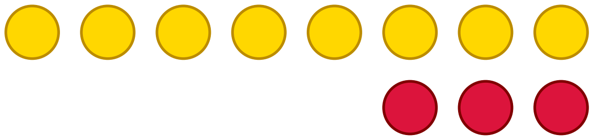
One yellow counter and one red counter, together, represents zero.
(Discuss why this is true.)



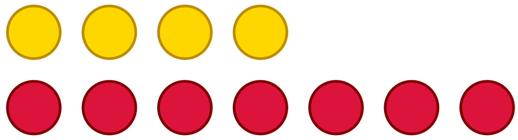
We call a pair of yellow and red counters a "zero pair". It is possible to add a zero pair to a set of counters, or (if possible) remove a zero pair from a set of counters, without affecting the value of the counters.

Example

(a) The diagram below shows 5.



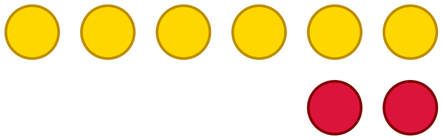
(b) The diagram below shows –3.



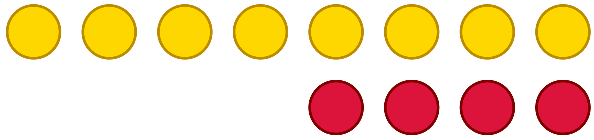
Exercise

Which numbers are shown in the following diagrams?

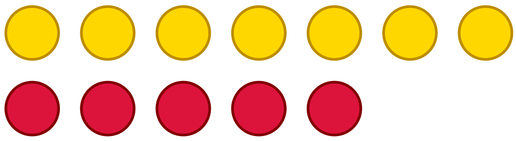
(a)



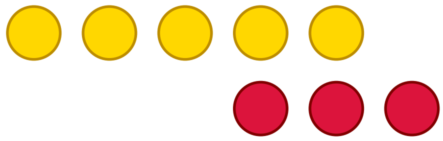
(b)



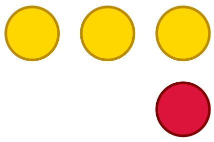
(c)



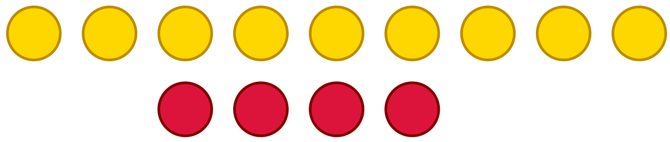
(d)



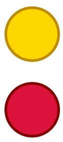
(e)



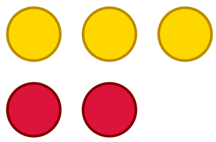
(f)



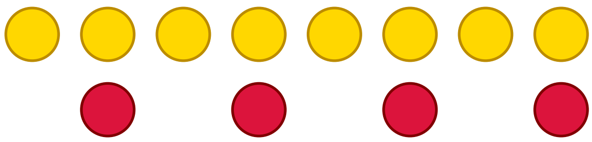
(g)



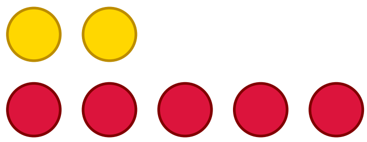
(h)



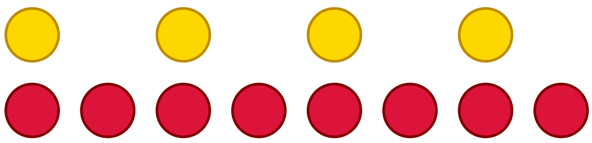
(i)



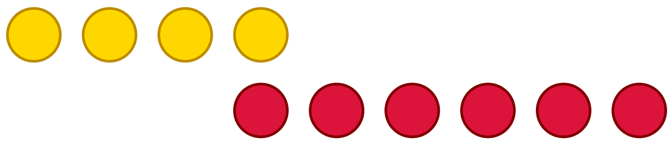
(j)



(k)



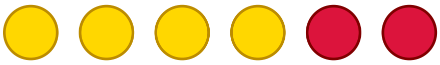
(l)



What's the sum?

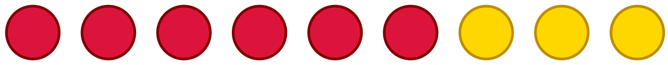
Below we show three ways of representing the same addition sum.
Fill in the blanks.

Example



4 + –2

Four add negative two



–6 + 3

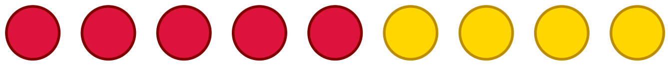
Negative six add three

Exercise

(a)

Five add negative two

(b)

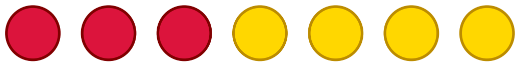


Negative five add four

(c)

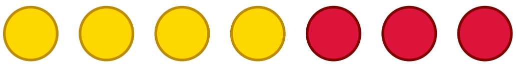
6 + –1

(d)



–3 + 4

(e)



(f)

–2 + 5

(g)

Three add negative four

(h)

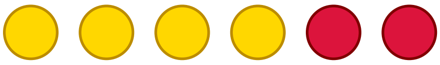
Negative one add seven

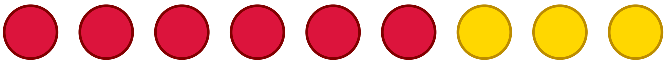
Addition with double sided counters

Example

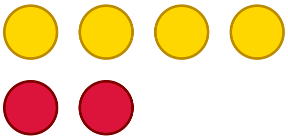
4 + –2 –6 + 3

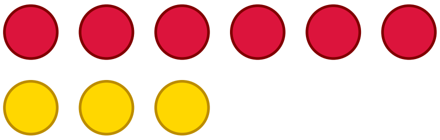
Step 1: Arrange the counters in a row to show the sum.



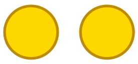


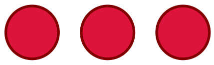
Step 2: Re-arrange the counters to identify any zero pairs.





Step 3: Remove any zero pairs and count what is left.





Answer: 2 Answer: –3

Exercise

Use double sided counters to find the answer to the following sums.

(a) 5 + –3 (b) 5 + –4 (c) 5 + –5

(d) 5 + –6 (e) 5 + –7 (f) 5 + –10

(g) –5 + 2 (h) –5 + 3 (i) –5 + 4

(j) –5 + 5 (k) –5 + 6 (l) –5 + 7

(m) 4 + –7 (n) –4 + –7 (o) –4 + 7

(p) 4 + 7 (q) 4 + –4 (r) –4 + 4

(s) –4 + –4 (t) –4 + 6 (u) –6 + 4

(v) –2 + –3 (w) 9 + –4 (x) –8 + –1

(y) 7 + –8 (z) –7 + 8 (α) –7 + –8

Challenge!

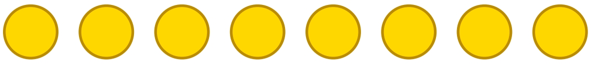
(a) 6 + –2 + 3 (b) –4 + –5 + 7 (c) –2 + 8 + –6

(d) –1 + –5 + 6 (e) 8 + –2 + –4 (f) –9 + 5 + –2 + 4

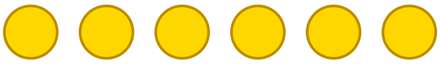
Subtraction with double sided counters

Example

To find the answer to the sum 8 – 2, we start with eight yellow counters...



...and subtract (remove) two of them to leave six yellow counters.



Therefore, 8 – 2 = 6.

Exercise

Use double sided counters to find the answers to the following sums.

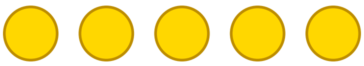
(a) 7 – 2 (b) 7 – 4 (c) 7 – 6

(d) 6 – 1 (e) 5 – 5 (f) 12 – 7

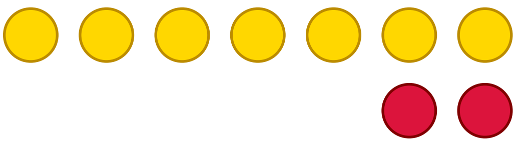
Removing what isn't there

How do you find the answer to the sum 5 – 7 using double sided counters?

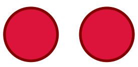
Let us start with five yellow counters.



At the moment, we cannot subtract seven yellow counters (as we only have five of them), so we need to introduce two zero pairs.



Now it is possible to subtract seven yellow counters, and this leaves two red counters. Therefore, 5 – 7 = –2.



Exercise

Use double sided counters to find the answer to the following sums.

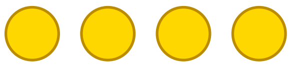
(a) 5 – 8 (b) 3 – 6 (c) 2 – 7

(d) 4 – 8 (e) 5 – 9 (f) 9 – 13

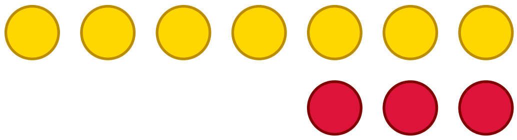
Subtracting a negative number

What is the meaning of the sum 4 – –3, ‘four subtract negative three’?

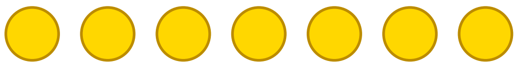
Let us start with four yellow counters.



We can add three zero pairs without affecting the value of the counters (four).



We can now subtract negative three by removing three red counters.



We see that seven yellow counters remain, so that 4 – –3 = 7.

Exercise

Use double sided counters to find the answer to the following sums.

(a) 4 – –2
(b) 4 – –1
(c) 4 – –4
(d) 5 – –2
(e) 5 – –1
(f) 5 – –4
(g) 6 – –3
(h) 6 – –2
(i) 6 – –5
(j) 3 – –2
(k) 3 – –4
(l) 3 – –6
(m) 8 – –3
(n) 8 – –1
(o) 8 – –5
(p) 7 – –4
(q) 7 – –2
(r) 7 – – 3

Starting with a negative number

Discuss how you would use double sided counters to find the answer to the sum –4 – –2.

Exercise

(a) –4 – –1
(b) –4 – –3
(c) –4 – –4
(d) –5 – –1
(e) –5 – –3
(f) –5 – –4
(g) –7 – –1
(h) –7 – –3
(i) –7 – –4

Directed Numbers Grids

How does the grid below work? What is the missing number?

4	5	9	14	23
6	7	13	20	33
10	12	22	34	56
16	19	35	54	89
26	31	57	88	?

Exercise

Fill in the numbers in the following grids.

1	1			
1	2			

1	–1			
0	1			

	3	3		
	3	1		

			5	8
			6	13

		1	5	
		3	5	

2	–5			
–4	7			

–4	–3			
–2	–1			

		1	2	
		3	4	

3	3			
–7	1			

Negative numbers in context

Golf

In a round of golf, players hit a ball into each of 18 different holes. Players need to do this using the least number of strokes possible; the person with the least score wins.

Here are the details of the famous golf course at the Celtic Manor hotel, located near Newport. This course was used during the Ryder Cup competition in 2010.

Hole	1	2	3	4	5	6	7	8	9
Length (yards)	461	605	189	458	447	436	212	437	614
Par	4	5	3	4	4	4	3	4	5

Hole	10	11	12	13	14	15	16	17	18
Length (yards)	211	560	454	187	480	365	502	209	608
Par	3	5	4	3	4	4	4	3	5

What do you notice? What do you wonder?

Exercise

(a) Which hole is the longest?

(b) Which hole is the shortest?

(c) How many 'par 3' holes are there?

(d) How many 'par 4' holes are there?

(e) How many 'par 5' holes are there?

(f) What is the total par of the course?

Challenge! ⚠ What is the total length of the course?

Scoring

A player receives a score for each completed hole of the golf course.

- If Peter uses 6 strokes to get the ball into hole 1, then his score for hole 1 is +2, as 6 is two **more** than the par for the hole (4).
- If Elen uses 2 strokes to get the ball into hole 7, then her score for hole 7 is –1, as 2 is one **less** than the par for the hole (3).

Here is Peter's score card for a round of golf on the course.

Hole	1	2	3	4	5	6	7	8	9
Peter's strokes	6	4	3	5	4	3	2	4	3
Par	4	5	3	4	4	4	3	4	5
Score on the hole	+2	+1	0	–1	0	–1	–1	0	–2
Score	+2	+3	+3	+2	+2	+1	0	0	–2

Hole	10	11	12	13	14	15	16	17	18
Peter's strokes	3	6	4	5	4	3	5	2	4
Par	3	5	4	3	4	4	4	3	5
Score on the hole	0	+1	0	+2	0	–1	+1	–1	–1
Score	–2	–1	–1	+1	+1	0	1	0	–1

Exercise

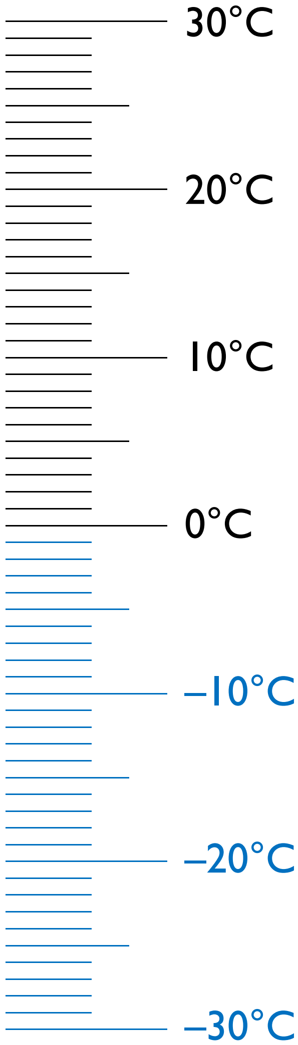
Complete Elen's score card for a round of golf on the course.

Hole	1	2	3	4	5	6	7	8	9
Elen's strokes	3	5	3	5	3	3	2	4	6
Par	4	5	3	4	4	4	3	4	5
Score on the hole									
Score									

Hole	10	11	12	13	14	15	16	17	18
Elen's strokes	3	7	4	3	5	4	3	3	3
Par	3	5	4	3	4	4	4	3	5
Score on the hole									
Score									

Who had the better round of golf on the course: Peter or Elen?

Temperature



Here is the temperature in 10 cities at 18:00 and 06:00 during January.

Paris (France): 10°C and 7°C.

New York (USA): 5°C and –2°C.

Tokyo (Japan): 10°C and 3°C.

Cardiff (Wales): 8°C and –1°C.

Buenos Aires (Argentina): 29°C and 21°C.

Oslo (Norway): –2°C and –7°C.

Sydney (Australia): 23°C and 18°C.

Delhi (India): 17°C and 14°C.

Moscow (Russia): –4°C and –10°C.

Berlin (Germany): 3°C and –3°C.

Exercise

The change in temperature for Paris was
10°C – 7°C = 3°C.
Write down the change in temperature for the other nine cities in a similar way.

Exercise

The temperature falls 7°C in the following cities. Fill in the blanks.

Los Angeles (USA): 18°C to ________.

Edinburgh (Scotland): 7°C to ________.

Rio de Janeiro (Brazil): ________ to 24°C.

Reykjavik (Iceland): ________ to –4°C.

Beijing (China): ________ to –3°C.

Rome (Italy): ________ to 2°C.

Madrid (Spain): 9°C to ________.

Seoul (South Korea): 2°C to _____.

Exercise

Explain what is happening to the temperature in the following sums.

(a) –5°C + 9°C

(b) –8°C – 4°C

(c) 3°C – 4°C

(d) –7°C + 3°C

Money

Many people use a **current account** in a bank to receive and pay money. Often the account allows an **overdraft**, which means that the account **balance** can be negative. (The account is said to be "in the red" when this happens.)

Here is an example of a bank **statement**, showing the payments and deposits for a particular month.

Mrs Meinir Jones Current account statement Sort code: 12-34-56 Account number: 21436587			**Banc Cymru** **1 January – 31 January** Money in: £506.60 Money out: £476.22 Overdraft: £500	
Date	**Description**	**Money out**	**Money in**	**Balance**
01 Jan	Start balance			305.00
04 Jan	Direct Debit: Electricity Bill	103.00		202.00
07 Jan	Card Payment to Tesco	86.25		115.75
10 Jan	Card Payment to Amazon	49.99		65.76
14 Jan	Card Payment to Paypal	45.20		20.56
18 Jan	Card Payment to Lidl	74.35		–53.79
20 Jan	Direct Debit: Water Bill	55.00		–108.79
24 Jan	Received from your employer		506.60	397.81
30 Jan	Card Payment to Shell	62.43		335.38
31 Jan	End balance			335.38

What do you notice? What do you wonder?

Exercise

Fill in the blanks in the following bank statement.

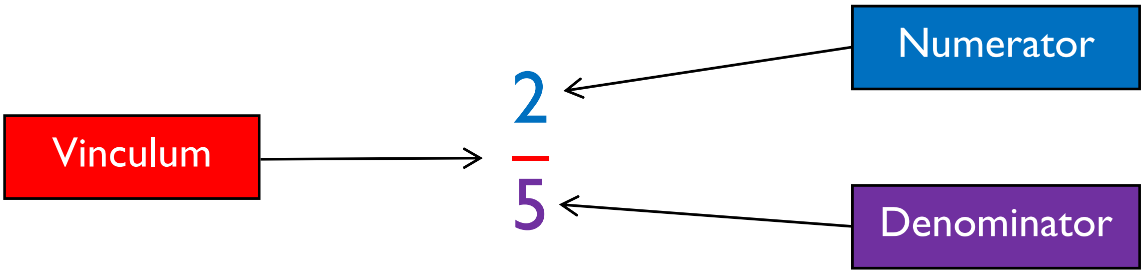
Mrs Meinir Jones Current account statement Sort code: 12-34-56 Account number: 21436587	**Banc Cymru** **1 February – 28 February** Money in: ________ Money out: ________ Overdraft: £500

Date	Description	Money out	Money in	Balance
01 Feb	Start balance			335.38
04 Feb	Direct Debit: Electricity Bill	103.00		______
05 Feb	Card Payment to Aldi	72.48		159.90
07 Feb	Card Payment to Posh Restaurant	48.52		______
10 Feb	Card Payment to Paypal	34.50		76.88
12 Feb	Card Payment to Amazon	______		52.48
17 Feb	Card Payment to Cinema	12.50		______
20 Feb	Direct Debit: Water Bill	55.00		______
21 Feb	Card Payment to McDonalds	5.40		–20.42
21 Feb	Card Payment to Spar	12.38		______
22 Feb	Card Payment to Primark	39.96		–72.76
24 Feb	Received from your employer		541.36	______
25 Feb	Card Payment to Asda	64.20		404.40
25 Feb	Card Payment to Boots	8.42		______
27 Feb	Card Payment to Garej Gethin	______		358.98
28 Feb	End balance			______

Fractions

Terminology

Here are the names for the different parts of any fraction.



- The **vinculum** is the horizontal line in the middle of the fraction.
- The **numerator** is the integer at the top of the fraction.
- The **denominator** is the integer at the bottom of the fraction.

Exercise

(a) Circle the fractions below with a numerator of 3.

$\frac{3}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{3}{7}$ $\frac{2}{3}$ $\frac{33}{34}$ $\frac{3}{7}$

(b) Circle the fractions below with a denominator of 4.

$\frac{4}{5}$ $\frac{1}{4}$ $\frac{4}{7}$ $\frac{3}{4}$ $\frac{1}{44}$ $\frac{5}{4}$ $\frac{4}{9}$ $\frac{4}{11}$

(c) Write down four fractions with a denominator of 7.

(d) Write down three fractions with a numerator of 5.

(e) Write down five fractions where the numerator is 2 less than the denominator.

(f) Write down four fractions where the denominator is double the numerator.

(g) Write down three fractions where the numerator is 1.

(h) Write down four fractions where the sum of the numerator and the denominator is seven.

(i) Write down three fractions where the difference between the numerator and the denominator is four.

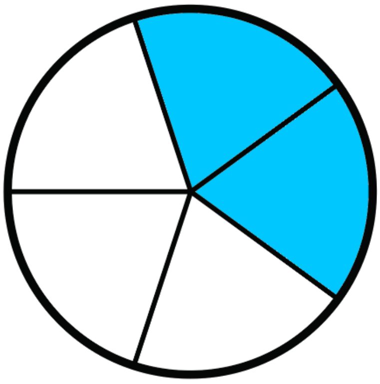
(j) Write down two fractions where the denominator is four times the numerator.

Fraction of a Shape

To shade a fraction of a shape,

- Divide the shape into equal parts according to the number shown in the denominator;
- Shade the number of equal parts shown in the numerator.

For example, to shade $\frac{2}{5}$ of a circle,



- Divide the circle into 5 equal parts;
- Shade 2 of the equal parts.

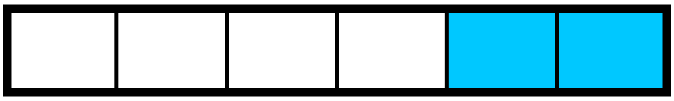
Notice that the parts must be **equal parts**.

Exercise

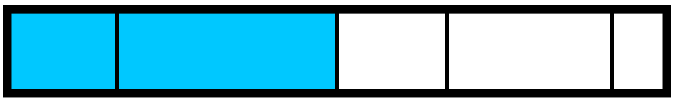
Which pictures below show $\frac{2}{5}$ of a shape being shaded?

(a)

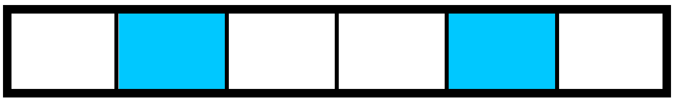
(b)



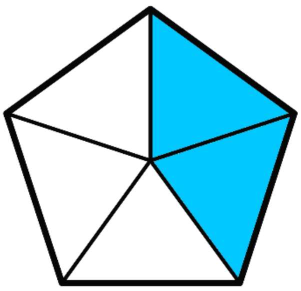
(c)



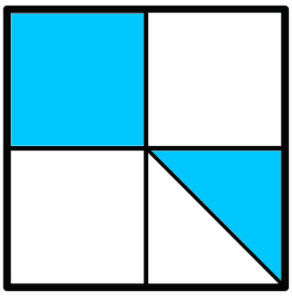
(d)



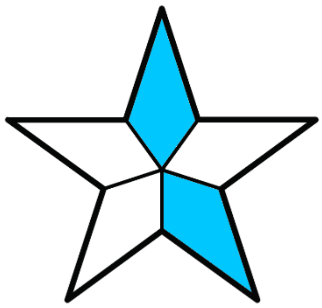
(e)



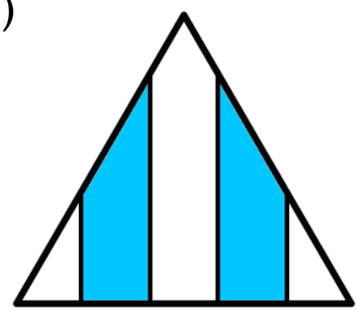
(f)



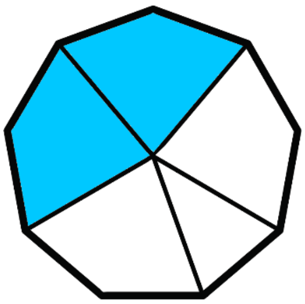
(g)



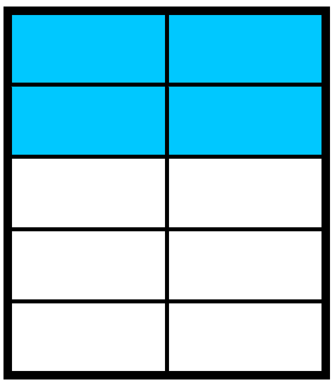
(h)



(i)

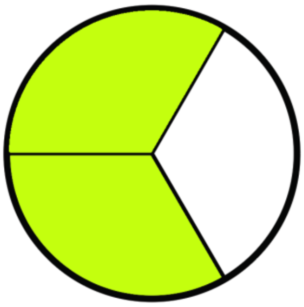


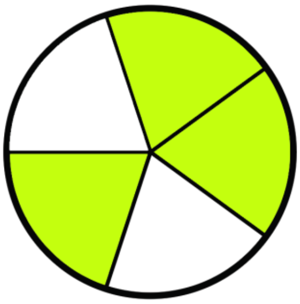
(j)

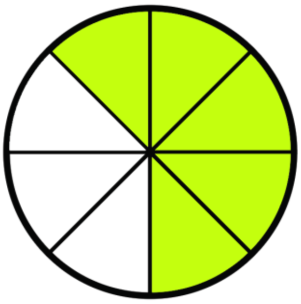


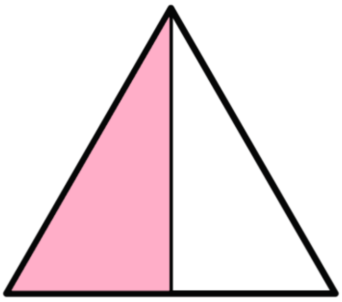
Recognising a Fraction of a Shape

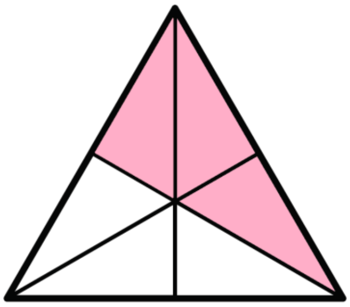
Which fractions of the following shapes are shaded?

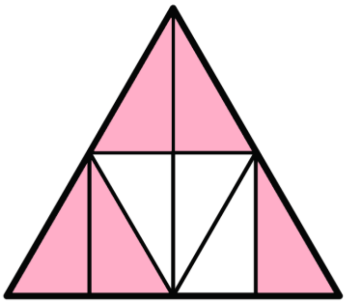

(a)

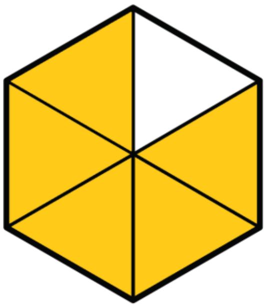

(b)

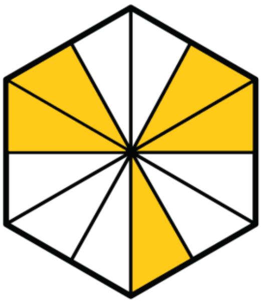

(c)

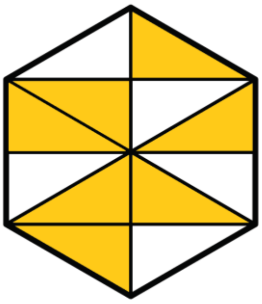

(d)

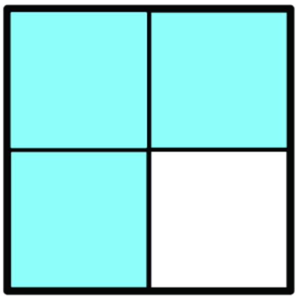

(e)

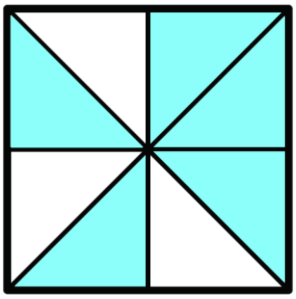

(f)

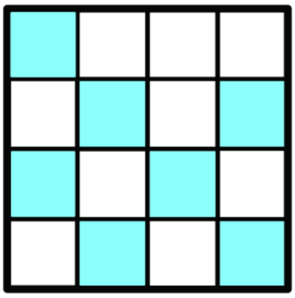

(g)

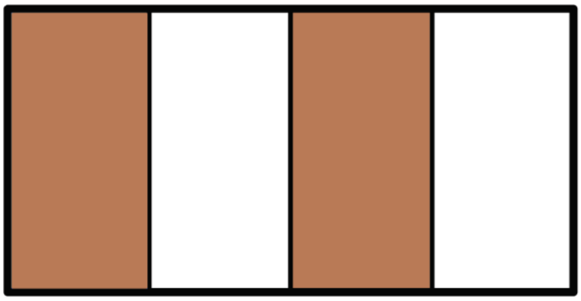

(h)

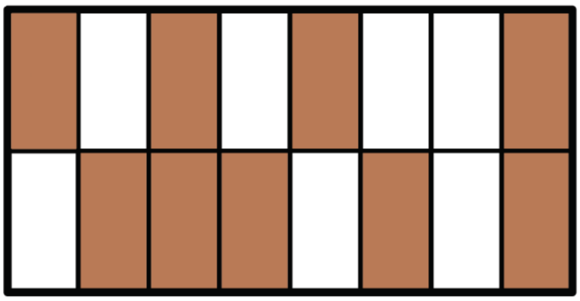

(i)


(j)


(k)


(l)

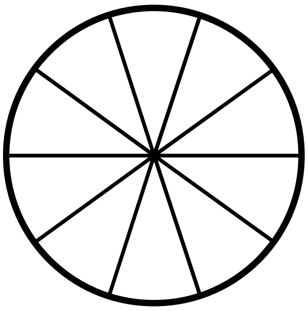

(m)


(n)

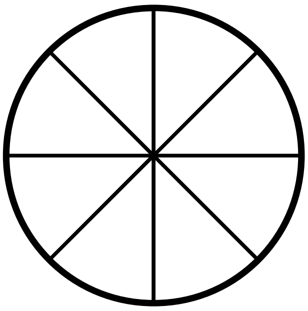
Exercise

Shade the fraction shown of each shape.

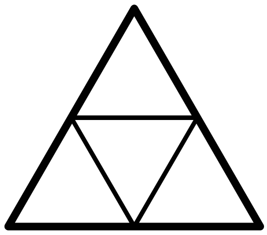
(a) $\frac{3}{10}$



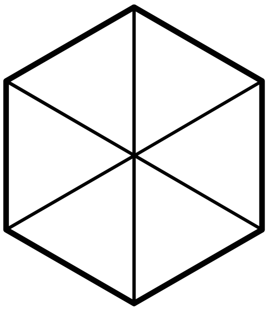
(b) $\frac{5}{8}$



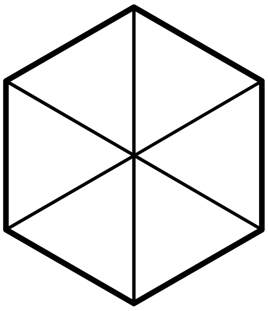
(c) $\frac{1}{4}$



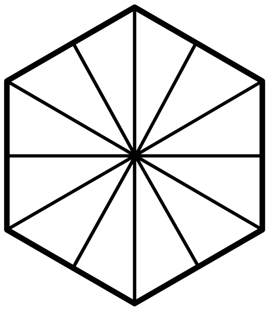
(d) $\frac{2}{6}$



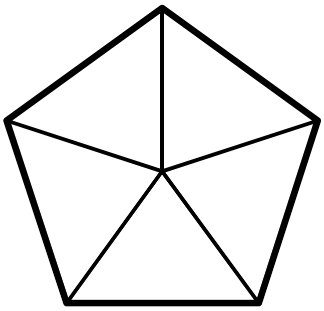
(e) $\frac{6}{6}$



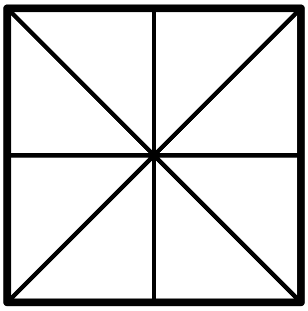
(f) $\frac{7}{12}$



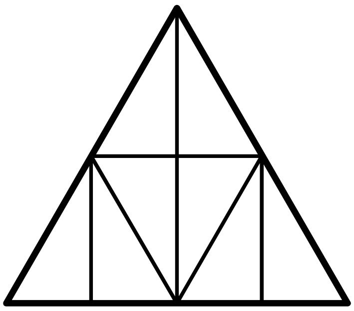
(g) $\frac{3}{5}$



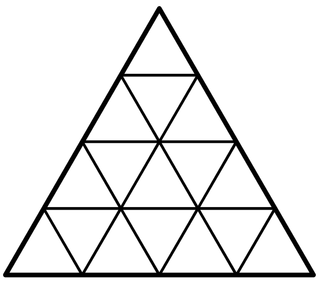
(h) $\frac{7}{8}$



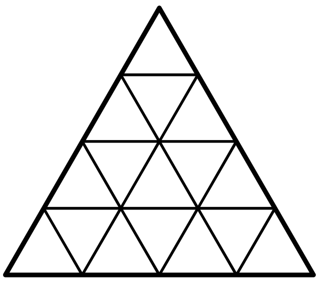
(i) $\frac{3}{8}$



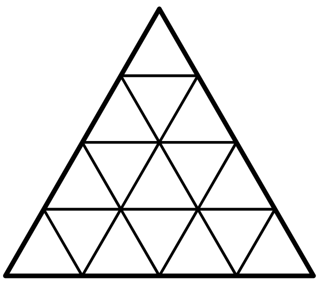
(j) $\frac{9}{16}$



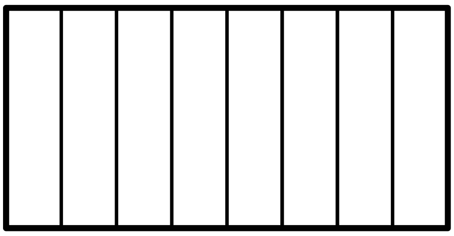
(k) $\frac{1}{2}$



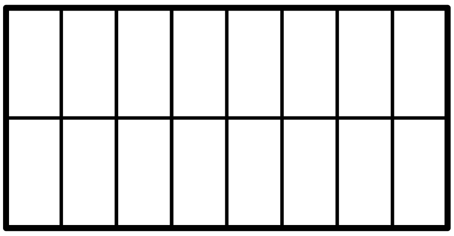
(l) $\frac{1}{4}$



(m) $\frac{5}{8}$

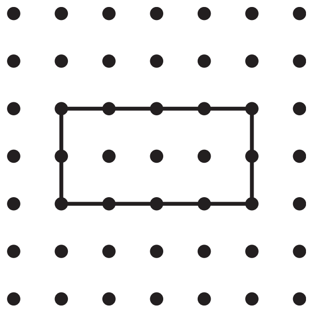


(n) $\frac{11}{16}$

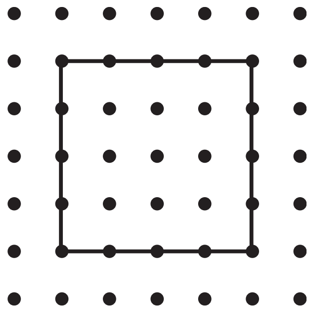


Exercise

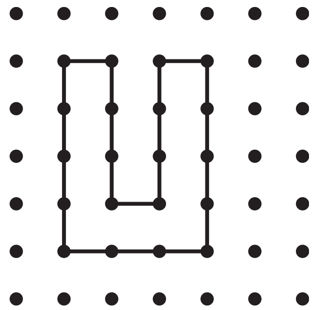
By dividing each shape into parts of equal area, shade the fraction that is shown beside each shape.



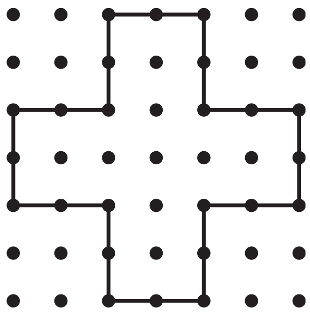
(a) $\frac{1}{2}$



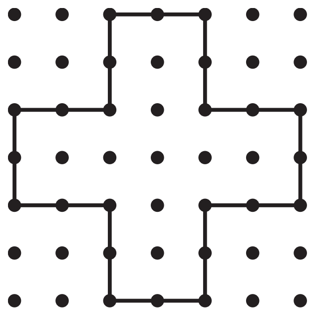
(b) $\frac{1}{4}$



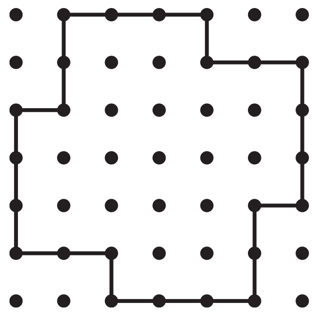
(c) $\frac{2}{3}$



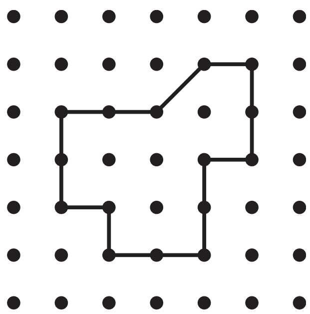
(d) $\frac{2}{5}$



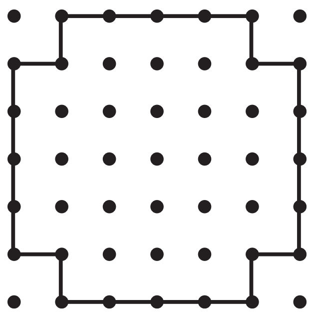
(e) $\frac{3}{4}$



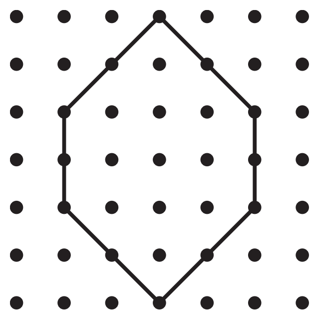
(f) $\frac{3}{4}$



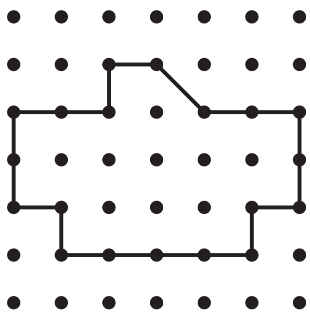
(g) $\frac{2}{3}$



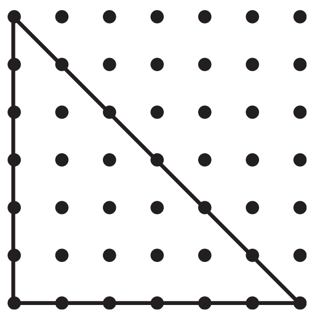
(h) $\frac{5}{8}$



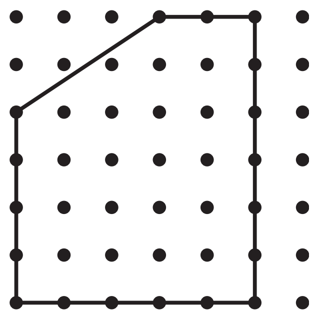
(i) $\frac{3}{8}$



(j) $\frac{3}{5}$



(k) $\frac{5}{9}$



(l) $\frac{4}{9}$

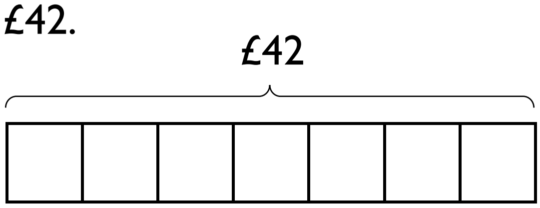
Are there different ways of answering the above questions? Discuss.

Calculating a Fraction of a Number (fraction as an operator)

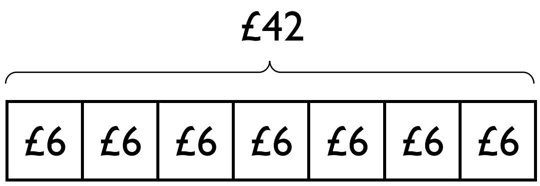
Example

Calculate $\frac{3}{7}$ of £42.

Because the denominator of the fraction is 7, we draw a bar model containing 7 blocks, to represent the £42.



Next, we calculate the value of one block, by dividing the £42 equally between the 7 blocks: £42 ÷ 7 = £6.



The question asks for $\frac{3}{7}$ of £42, so we need to find the value of three of the blocks: 3 × £6 = £18. This is the answer to the question.

Exercise

Draw a suitable bar model to answer the following questions.

(a) $\frac{2}{3}$ of £12

(b) $\frac{3}{5}$ of £15

(c) $\frac{1}{4}$ of £12

(d) $\frac{5}{6}$ of £36

(e) $\frac{2}{7}$ of £21

(f) $\frac{7}{8}$ of £40

Exercise

Connect the following cards in groups of three.

$\frac{1}{2}$ of 30	7 | 7 | 7 | 7 | 7 | 7 | 7	16
$\frac{2}{3}$ of 24	5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5	15
$\frac{7}{9}$ of 45	5 | 5 | 5 | 5 | 5 | 5	28
$\frac{4}{7}$ of 49	15 | 15	35
$\frac{5}{6}$ of 30	8 | 8 | 8	25

Exercise

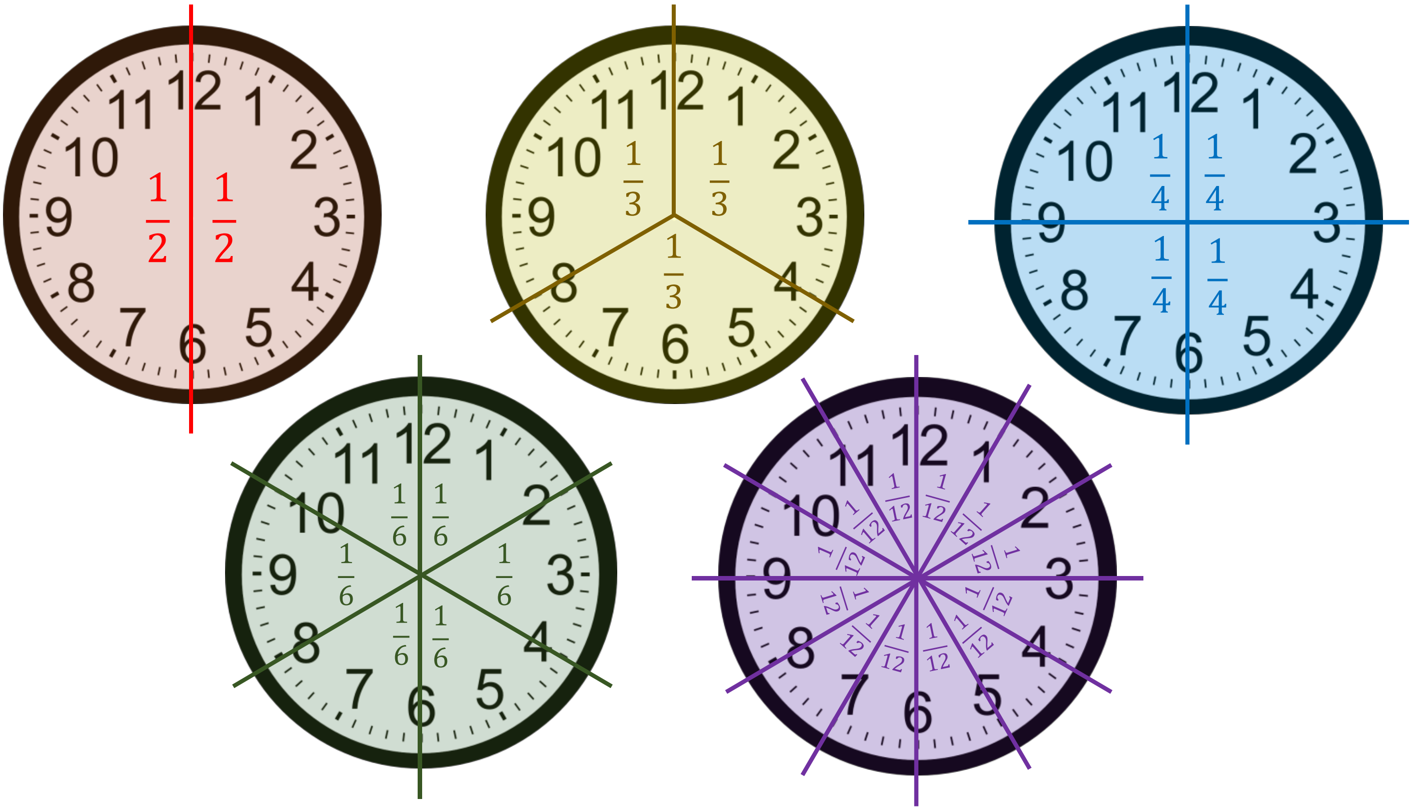
(a) My dog has had 15 puppies, with $\frac{2}{3}$ of the puppies being sold. How many puppies have been sold, and how many puppies are left?

(b) John has 54 stickers. He gives $\frac{4}{9}$ of the stickers to Tom. How many stickers has he given to Tom, and how many stickers are left?

(c) Sam had £5 of pocket money. He spent $\frac{7}{10}$ of the money on a book. How much money did Sam spend, and how much money does he have left?

(d) In a game of cricket, Sali managed to hit $\frac{4}{5}$ of the balls being bowled towards her. If 40 balls in total were bowled towards Sali, how many balls did she manage to hit, and how many balls did she miss?

Fractions and Time



Exercise

(a) In _____ minutes, the minute hand of the clock goes $\frac{1}{4}$ of the way around the clock.

(b) In _____ minutes, the minute hand of the clock goes $\frac{1}{6}$ of the way around the clock.

(c) In 30 minutes, the minute hand of the clock goes ______ of the way around the clock.

(d) In 5 minutes, the minute hand of the clock goes ______ of the way around the clock.

(e) In _____ minutes, the minute hand of the clock goes $\frac{1}{3}$ of the way around the clock.

(f) In _____ minutes, the minute hand of the clock goes $\frac{2}{3}$ of the way around the clock.

(g) In _____ minutes, the minute hand of the clock goes $\frac{3}{4}$ of the way around the clock.

(h) In 50 minutes, the minute hand of the clock goes ______ of the way around the clock.

(i) In 35 minutes, the minute hand of the clock goes ______ of the way around the clock.

(j) In 1 minute, the minute hand of the clock goes ______ of the way around the clock.

(k) In 60 minutes, the **hour** hand of the clock goes ______ of the way around the clock.

(l) In 180 minutes, the **hour** hand of the clock goes ______ of the way around the clock.

Fraction as a Division Sum (fraction as a number)

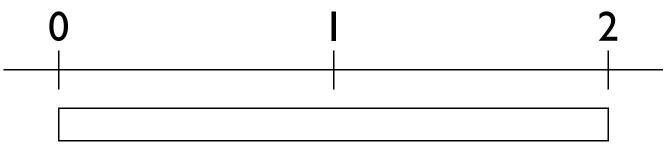
It is possible to write the fraction $\frac{2}{5}$ as the division sum 2 ÷ 5.

The name of the symbol ÷ is the *obelus*, a symbol that looks like a fraction, with dots representing the numerator and the denominator.

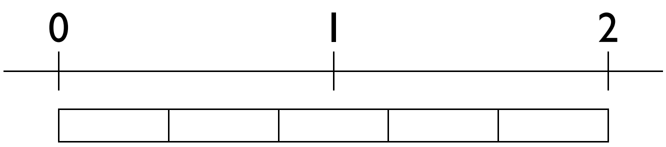
Each fraction has a specific location on the number line. Here are two methods for finding the location of $\frac{2}{5}$ on the number line.

Method 1
(the division method, 2 ÷ 5)

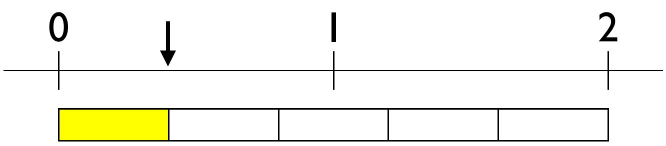
Start with a bar model showing 2 on a number line.



Divide the bar into 5 equal parts.

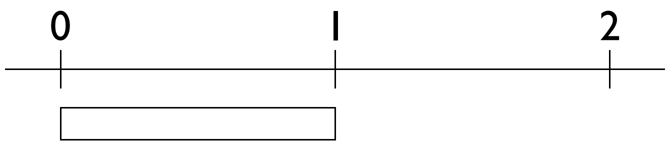


The location of $\frac{2}{5}$ on the number line is where the first part of the bar finishes.

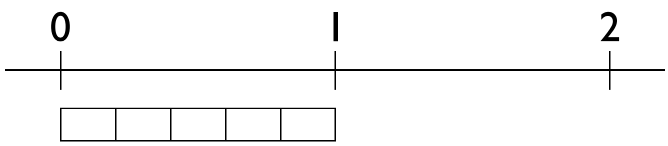


Method 2
(the unit method, finding $\frac{2}{5}$ of 1)

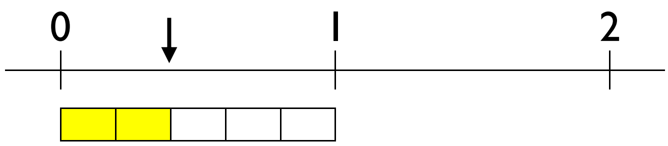
Start with a bar model showing 1 on a number line.



Divide the bar into 5 equal parts.

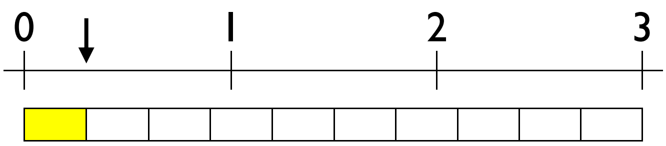


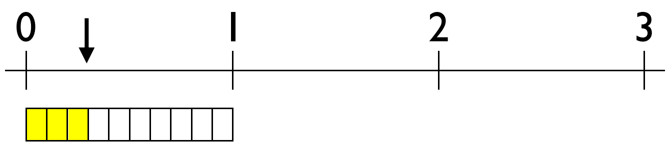
The location of $\frac{2}{5}$ on the number line is where the second part of the bar finishes.



Exercise

Discuss how the following two diagrams show the location of the fraction $\frac{3}{10}$ on the number line.

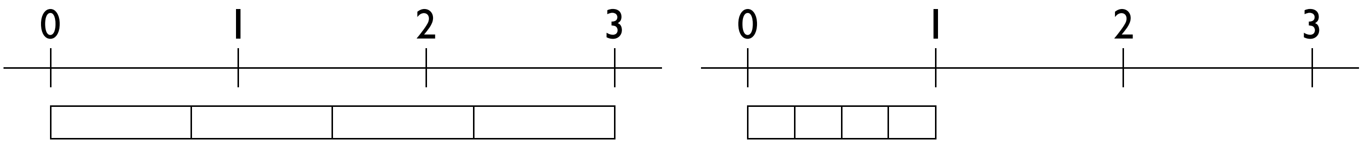




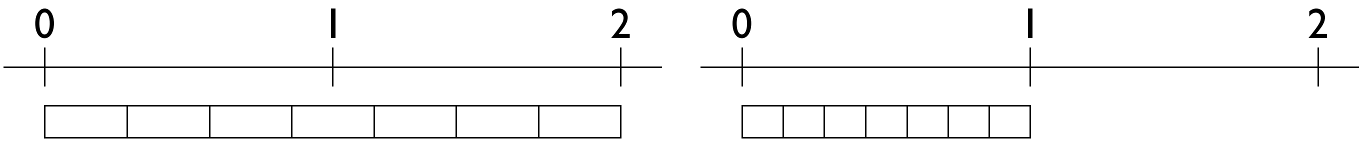
Exercise

Add arrows on the following number lines to show the location of the fractions.

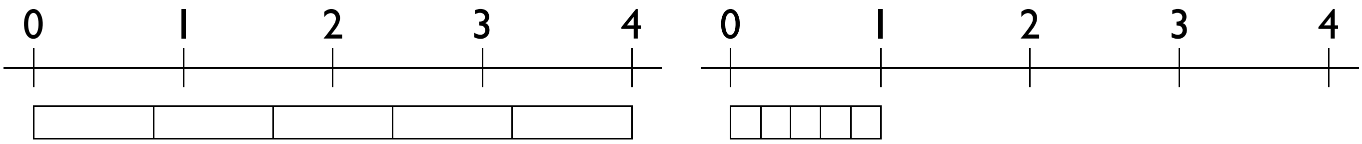
(a) $\frac{3}{4}$



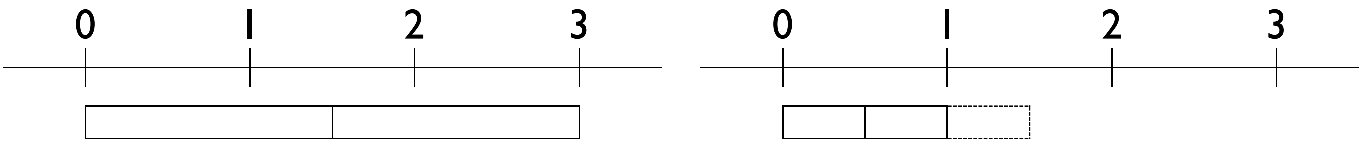
(b) $\frac{2}{7}$



(c) $\frac{4}{5}$



(d) $\frac{3}{2}$



Exercise

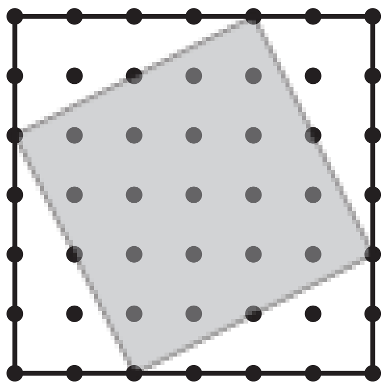
On a piece of squared paper, draw suitable diagrams to show the location of the following fractions on a number line.

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$

(d) $\frac{5}{2}$ (e) $\frac{3}{3}$ (f) $\frac{3}{7}$

Challenge!

What fraction of the square on the right is shaded?



Comparing the Size of Fractions

Which fraction is the greatest: $\frac{2}{5}$ or $\frac{3}{7}$? Using the unit method, we can locate both fractions on a number line:

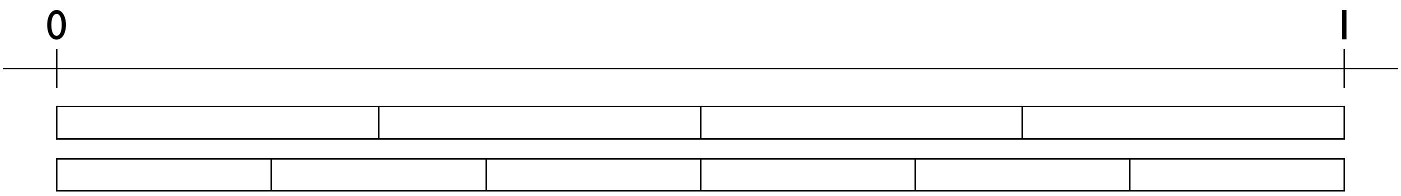
We can see from the diagram that $\frac{2}{5}$ is less than $\frac{3}{7}$, so we write $\frac{2}{5} < \frac{3}{7}$.

< 'less than' > 'greater than' = 'equal to'

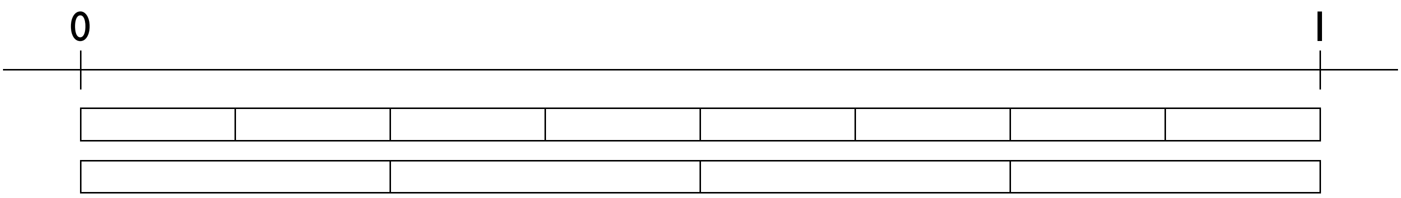
Exercise

Write the symbol <, > or = between the following pairs of fractions.

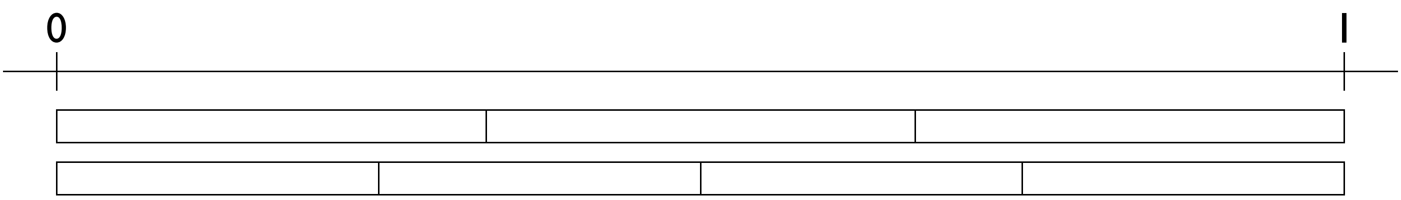
(a) $\frac{3}{4}$ $\frac{5}{6}$



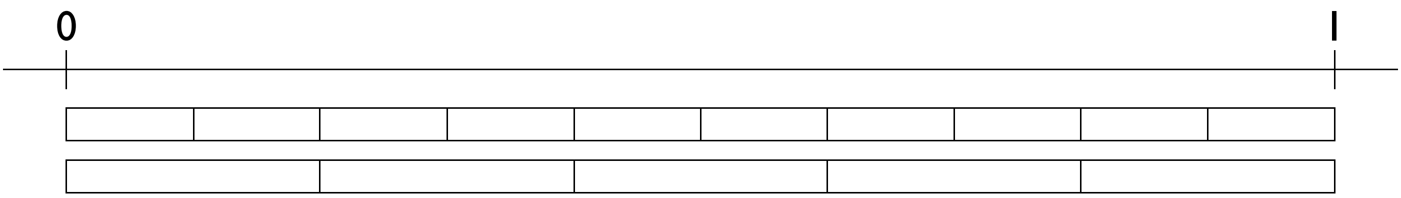
(b) $\frac{2}{8}$ $\frac{1}{4}$



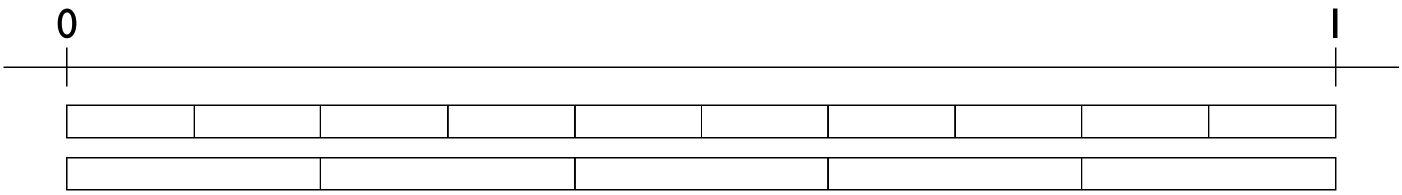
(c) $\frac{2}{3}$ $\frac{1}{4}$



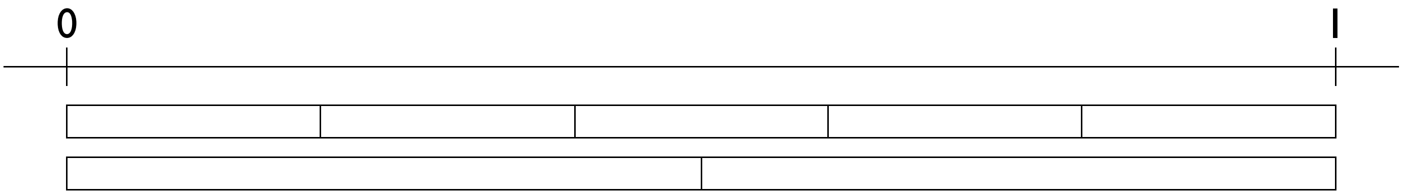
(d) $\frac{7}{10}$ $\frac{4}{5}$



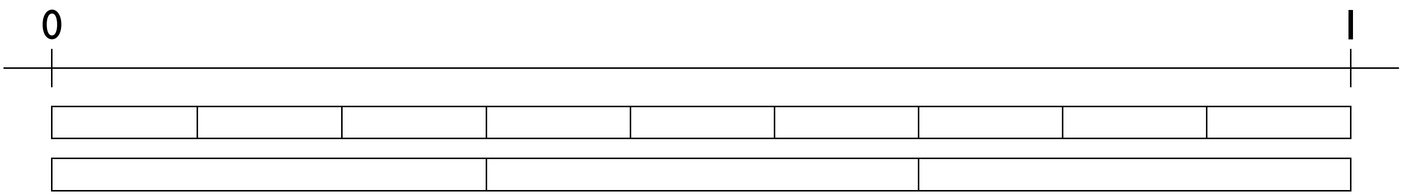
(e) $\frac{4}{10}$ $\frac{2}{5}$



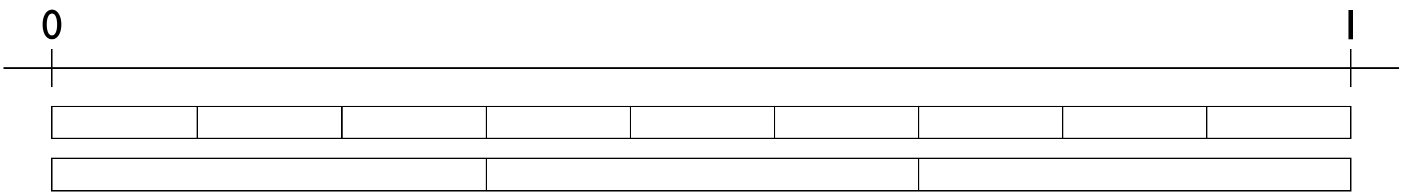
(f) $\frac{2}{5}$ $\frac{1}{2}$



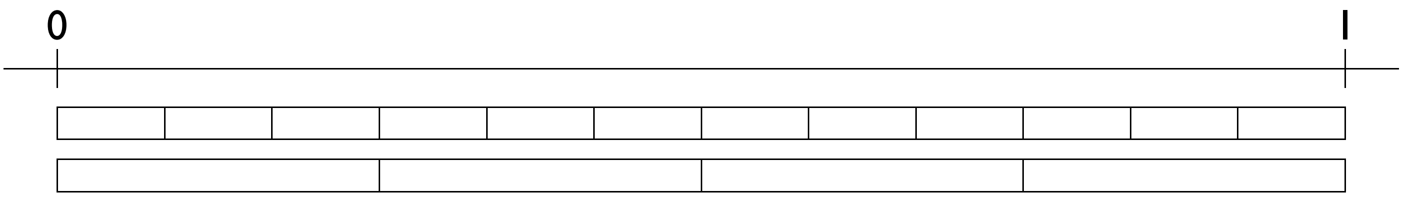
(g) $\frac{5}{9}$ $\frac{2}{3}$



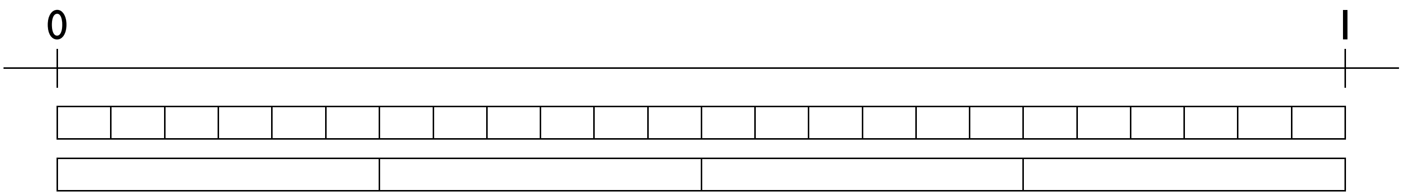
(h) $\frac{6}{9}$ $\frac{2}{3}$



(i) $\frac{7}{12}$ $\frac{3}{4}$



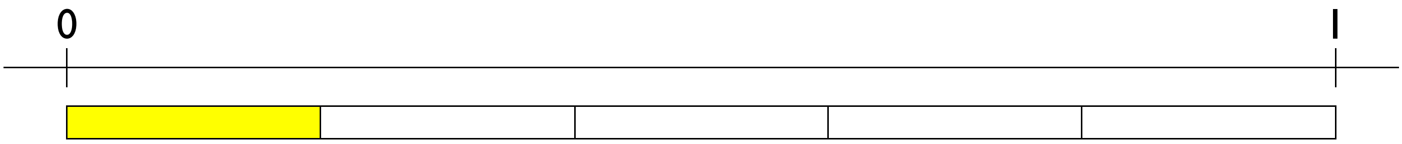
(j) $\frac{18}{24}$ $\frac{3}{4}$



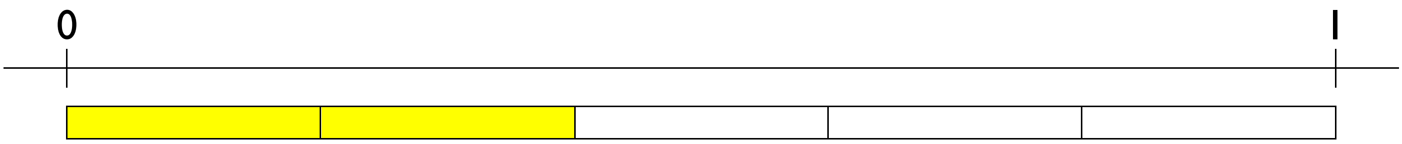
Changing the Numerator

What happens to the size of a fraction when the denominator is kept the same and the numerator is increased?

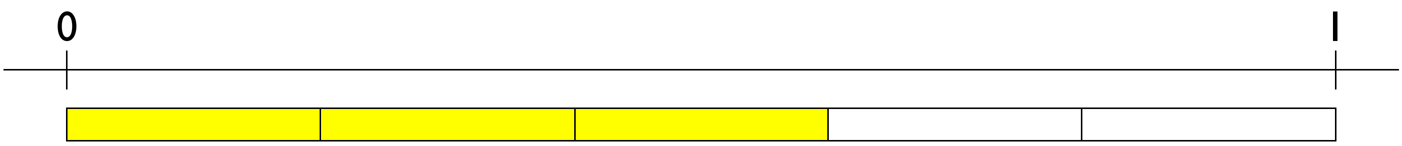
$\frac{1}{5}$



$\frac{2}{5}$



$\frac{3}{5}$



We see from the above diagrams that the size of a fraction **increases** when the denominator is kept the same and the numerator increases.

Exercise

Circle the **greatest** fraction in each of the following pairs of fractions.

(a) $\frac{2}{5}$ and $\frac{3}{5}$ (b) $\frac{3}{5}$ and $\frac{4}{5}$ (c) $\frac{2}{7}$ and $\frac{5}{7}$

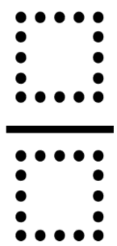
(d) $\frac{6}{7}$ and $\frac{3}{7}$ (e) $\frac{3}{4}$ and $\frac{1}{4}$ (f) $\frac{7}{8}$ and $\frac{3}{8}$

(g) $\frac{4}{9}$ and $\frac{7}{9}$ (h) $\frac{4}{11}$ and $\frac{5}{11}$ (i) $\frac{10}{11}$ and $\frac{8}{11}$

(j) $\frac{3}{2}$ and $\frac{5}{2}$ (k) $\frac{5}{5}$ and $\frac{6}{5}$ (l) $\frac{76}{123}$ and $\frac{107}{123}$

Challenge!

Write a single digit between 1 and 9 in each of the boxes on the right to create the greatest fraction that is less than a half.

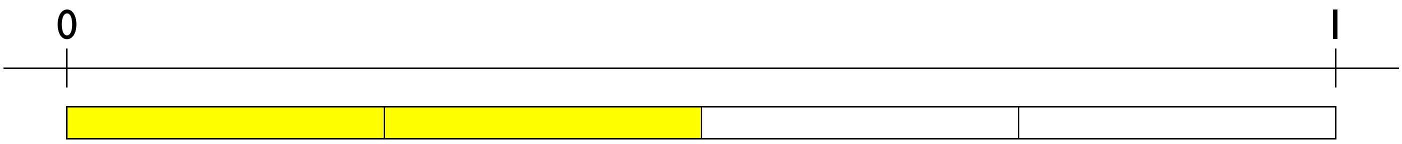


Changing the Denominator

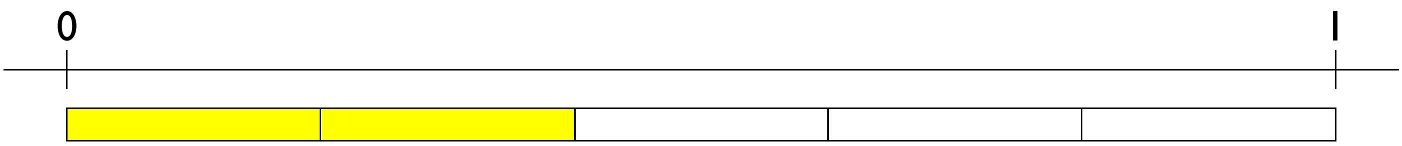
What happens to the size of a fraction when the numerator is kept the same and the denominator is increased?

$\frac{2}{3}$

$\frac{2}{4}$



$\frac{2}{5}$



We can see from the above diagrams that the size of a fraction **decreases** when the numerator is kept the same and the denominator increases.

Exercise

Circle the **greatest** fraction in each of the following pairs of fractions.

(a) $\frac{2}{4}$ and $\frac{2}{5}$ (b) $\frac{3}{5}$ and $\frac{3}{4}$ (c) $\frac{2}{7}$ and $\frac{2}{5}$

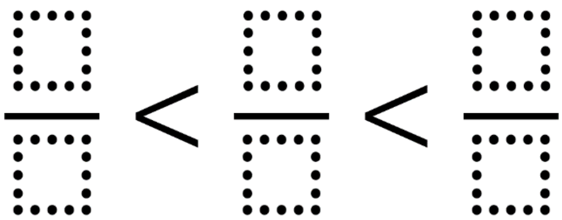
(d) $\frac{6}{7}$ and $\frac{6}{11}$ (e) $\frac{3}{4}$ and $\frac{3}{2}$ (f) $\frac{7}{8}$ and $\frac{7}{9}$

(g) $\frac{2}{9}$ and $\frac{2}{7}$ (h) $\frac{4}{11}$ and $\frac{4}{13}$ (i) $\frac{10}{11}$ and $\frac{10}{7}$

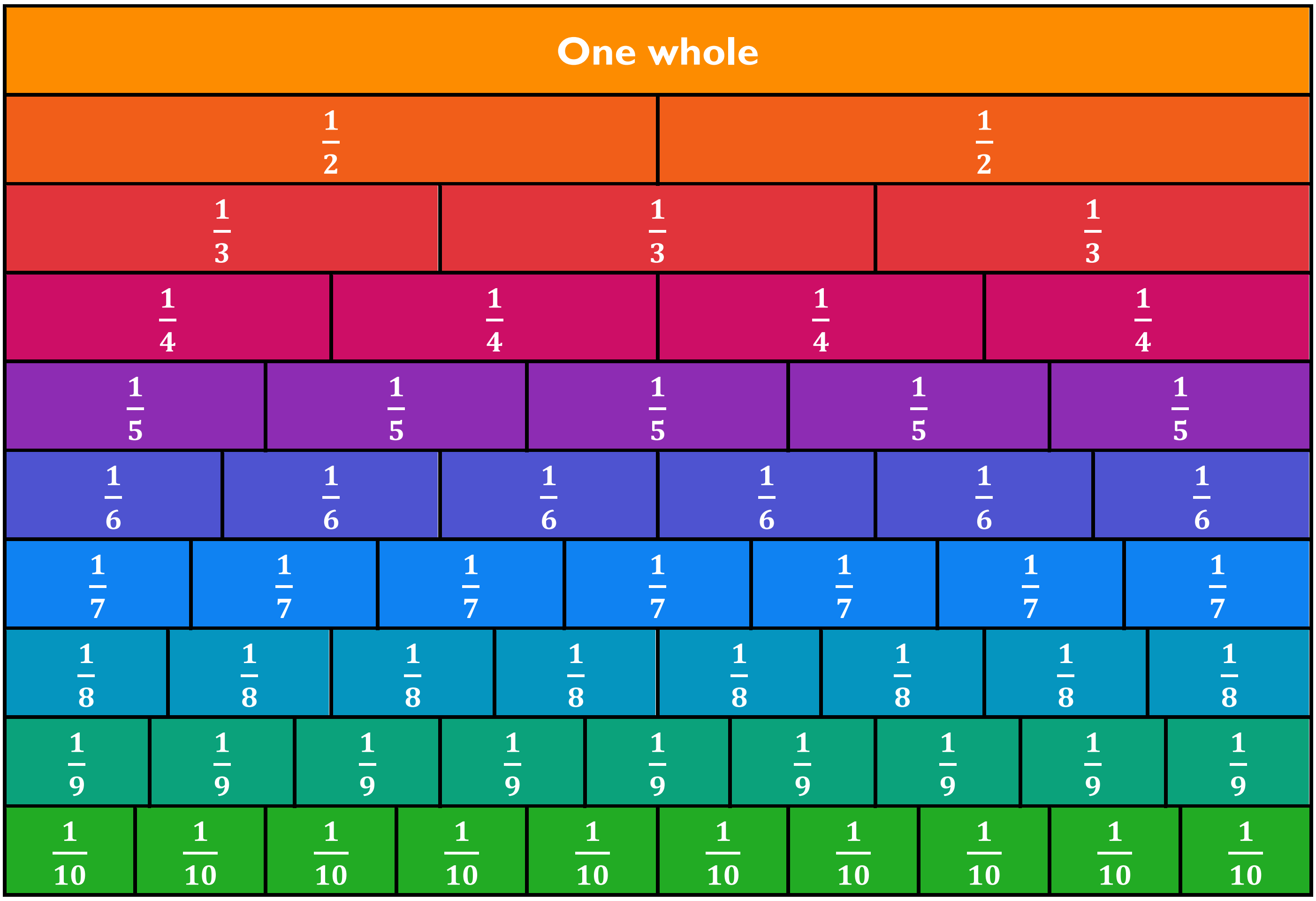
(j) $\frac{4}{3}$ and $\frac{4}{5}$ (k) $\frac{5}{5}$ and $\frac{5}{8}$ (l) $\frac{76}{123}$ and $\frac{76}{122}$

Challenge!

Write a different single digit between 1 and 9 in each of the boxes on the right to form a true statement.



The Fraction Wall



What do you notice? What do you wonder?

Exercise

(a) Which two fractions add together to give $\frac{1}{2}$?

(b) Find two different ways of writing $\frac{1}{3}$.

(c) Find a different way of writing $\frac{3}{4}$.

(d) Which fraction is the greatest: $\frac{3}{8}$ or $\frac{4}{10}$?

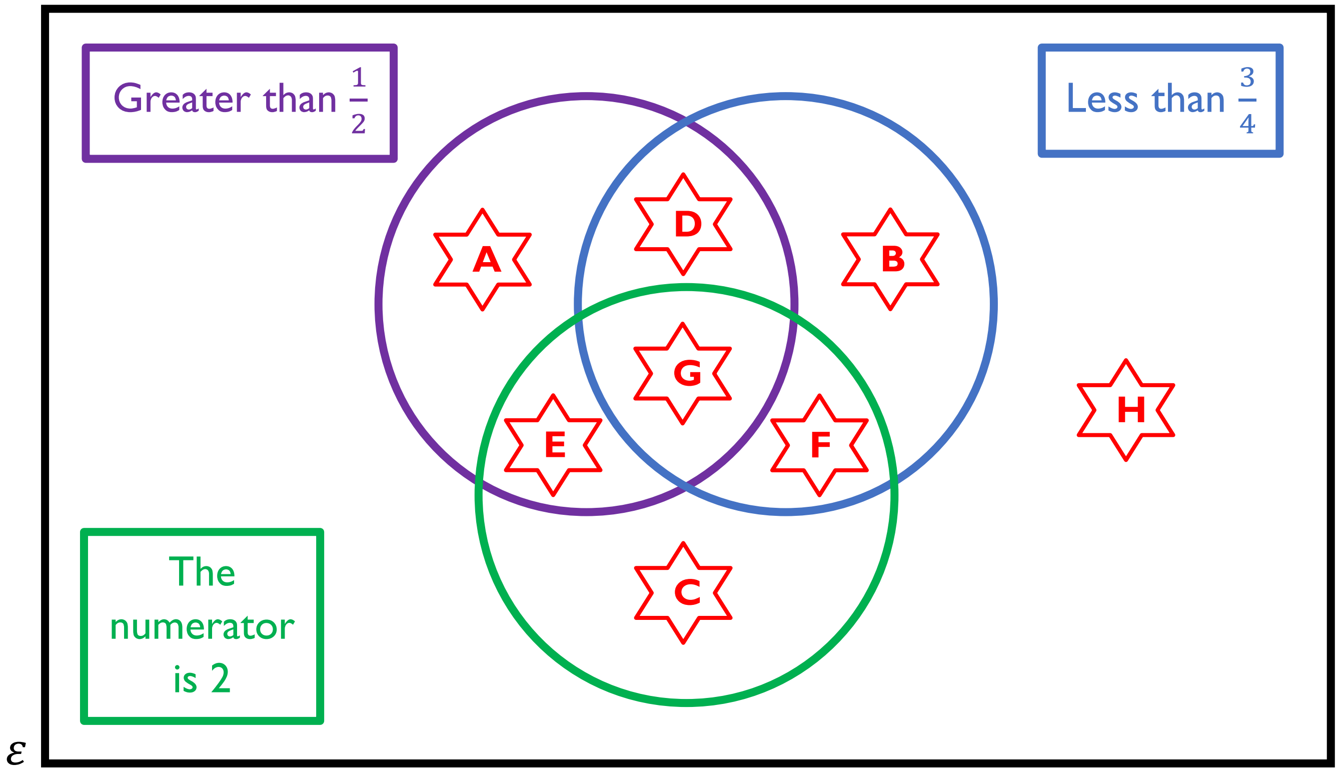
(e) Which fraction is the least: $\frac{3}{5}$ or $\frac{5}{8}$?

(f) How many tenths should be added to $\frac{2}{5}$ to make one whole?

(g) Look at the following pattern of fractions: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

(i) What is the next fraction in the pattern?
(ii) What happens to the value of the fractions as the pattern continues?
(iii) Will a fraction with a value of more than one appear in the pattern?

Venn Diagram Challenge

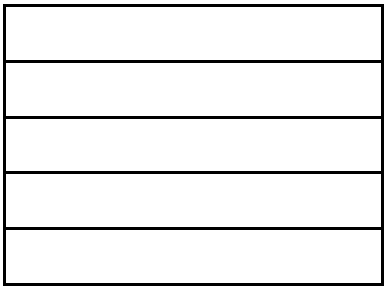


Think of a fraction that could fit into each region.
If you think a region is impossible to fill, explain why!

A		E	
B		F	
C		G	
D		H	

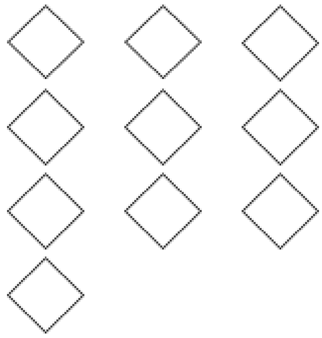
The Story of $\frac{2}{5}$

Shade $\frac{2}{5}$ of the shape.



Shade $\frac{2}{5}$ of the shape.

Shade $\frac{2}{5}$ of the shapes.



Write $\frac{2}{5}$ in words.

What is the numerator of $\frac{2}{5}$?

What is the denominator of $\frac{2}{5}$?

Locate $\frac{2}{5}$ on the number line.

Calculate $\frac{2}{5}$ of 35.

Calculate $\frac{2}{5} \times 15$.

Which division sum does $\frac{2}{5}$ represent?

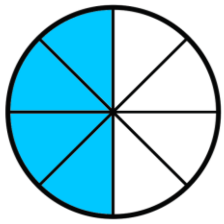
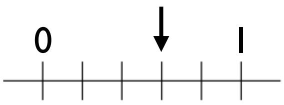
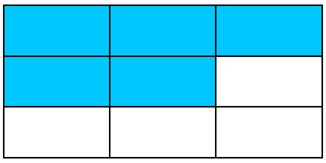
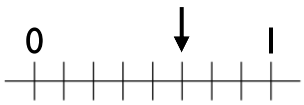
Write down a fraction that is equivalent to (the same as) the fraction $\frac{2}{5}$.

Write down two fractions that add to give $\frac{2}{5}$.

Catrin has 2 apples and 5 bananas. Fill in the missing fractions.

The number of apples is of the total fruit.
The number of bananas is of the total fruit.
The number of apples is of the bananas.

True or False?

 $\frac{3}{8}$ of the shape is shaded.	$\frac{6}{12}$ The denominator is double the numerator.	48 children eat in the school's refectory. If $\frac{1}{4}$ of the children eat sandwiches, 12 children each sandwiches.	$\frac{3}{4} < \frac{3}{5}$
In 10 minutes, the minute hand of the clock goes $\frac{1}{6}$ of the way around the clock.	$\frac{2}{3}$ of £24 is £16.	$\frac{1}{3} < \frac{1}{10}$	$\frac{8}{15}$ The difference between the numerator and the denominator is seven.
There are 49 books in the library. If Nia has read $\frac{3}{7}$ of the books, she has read 14 of the books.	 The arrow points towards the fraction $\frac{3}{4}$.	 $\frac{5}{9}$ of the shape is shaded.	In 40 minutes, the minute hand of the clock goes $\frac{3}{4}$ of the way around the clock.
$\frac{4}{5}$ The numerator of the fraction is 5.	$\frac{4}{5} = \frac{12}{15}$	 The arrow points towards the fraction $\frac{5}{8}$.	$\frac{3}{8}$ of 32 is 7.
 $\frac{2}{5}$ of the stars have been shaded.	$\frac{4}{6}$, when written in words, is 'four sixths'.	$\frac{6}{7} > \frac{5}{7}$	$\frac{4}{7}$ The denominator of the fraction is 7.