GCE AS/A Level – LEGACY

0984/01

S19-0984-01

MATHEMATICS – S2
**Statistics**

WEDNESDAY, 12 JUNE 2019 – MORNING

1 hour 30 minutes

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent, continuous random variables $X$ and $Y$ are uniformly distributed on the interval [0, 4].

   (a) Show that

   $$E(X^n) = \frac{4^n}{n+1},$$

   where $n$ is a positive integer. [3]

   (b) Determine the mean and the variance of

   (i) $X^2$,
   (ii) $XY$. [9]

2. The weights of potatoes in a shop are normally distributed with mean 60 grams and standard deviation 8 grams.

   (a) Determine the lower quartile of the weights of these potatoes. [2]

   (b) The shopkeeper places a random selection of 16 of these potatoes on his weighing scales. Calculate the probability that the total weight of these 16 potatoes exceeds 1000 grams. [6]

   (c) Anne randomly selects 10 of these potatoes and Huw randomly selects 9 of these potatoes. Calculate the probability that the total weight of Huw's potatoes is greater than the total weight of Anne's potatoes. [8]

3. Sue has two ducks of different breeds. During a particular fortnight, the Breed 1 duck lays 7 eggs and the Breed 2 duck lays 6 eggs. Their weights, in grams, were as follows.

| Breed 1 duck | 80·2 | 82·3 | 79·5 | 82·2 | 81·6 | 78·4 | 80·7 |
|---|---|---|---|---|---|---|---|
| Breed 2 duck | 77·1 | 79·8 | 78·1 | 77·4 | 75·8 | 76·2 | |

You may assume that these are two random samples from normal distributions with means $\mu_1$ grams and $\mu_2$ grams respectively and common standard deviation 1·5 grams. Determine a 95% confidence interval for $\mu_1 - \mu_2$. [7]

4. An electrician knows that the number of calls that she receives each day follows a Poisson distribution with mean $\mu = 2.8$.

   (a) Calculate the probability that, in a 5-day period, she receives

      (i) exactly 12 calls,

      (ii) fewer than 16 calls. [5]

   (b) The electrician wants more work so she places an advert in the local Post Office.

      (i) State suitable hypotheses for investigating whether or not this results in an increase in the value of $\mu$.

      (ii) In the first 5-day period after placing the advert, she receives 19 calls. Calculate and interpret the *p*-value of this result.

      (iii) In the next 80-day period, she receives 255 calls. Calculate an approximate *p*-value for this result and interpret it in context. [12]

5. Alun has a coin which he thinks might be biased so that, when tossed, the probability $p$ of obtaining heads may not be 0·5. He therefore defines the following hypotheses.

$$H_0 : p = 0.5; \quad H_1 : p \neq 0.5$$

   (a) He decides to toss the coin 20 times and to accept $H_0$ if the number of heads obtained lies in the interval $[10 - k, 10 + k]$, where $k$ is a positive integer.

      (i) Assuming $H_0$ to be true, determine the probability that the number of heads obtained lies in the interval $[10 - k, 10 + k]$ for $k = 2, 3, 4$.

      (ii) Hence state the value of $k$ which gives a significance level closest to 10% and state this significance level. [7]
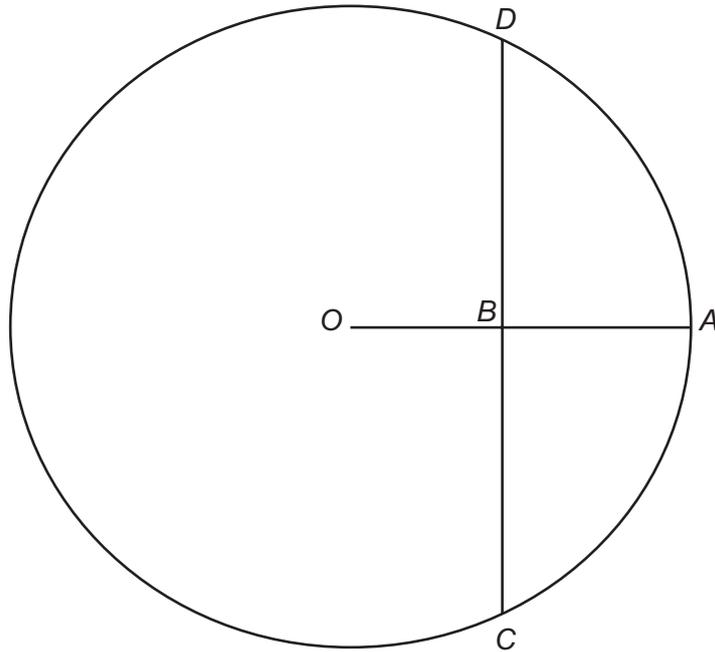
   (b) He now decides to toss the coin 120 times and he obtains 52 heads.

      (i) Calculate an approximate *p*-value for this result.

      (ii) What does your *p*-value tell you about the bias of the coin? [8]

# TURN OVER

**6.**



The diagram shows a fixed point *A* lying on a circle with centre *O* and fixed radius $r$. The random point *B* lies on *OA* such that the length *OB* is uniformly distributed on the interval [0, $r$]. The chord of the circle passing through *B*, perpendicular to *OA*, meets the circle at the points *C* and *D*. The length *OB* is denoted by $X$ and the length *CD* is denoted by $Y$.

*(a)* Show that

$$Y^2 = 4(r^2 - X^2).$$ 

[2]

*(b)* Determine the value of $P(Y \geqslant r)$. 

[6]

**END OF PAPER**