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1) a) i) Nid yw'n bosib i'r gwthiant fod yn fwy na 980 kN.

$$(80 + 0.1v^2)\text{ kN} \leq 980\text{ kN}$$
$$80 + 0.1v^2 \leq 980$$
$$0.1v^2 \leq 900$$
$$v^2 \leq 9000$$
$$v \leq \sqrt{9000}$$
$$v \leq 30\sqrt{10}\ \text{ms}^{-1}$$

Buanedd uchaf yr awyren yw $30\sqrt{10}\ \text{ms}^{-1}$

ii) $F=ma$ ar yr awyren, yn defnyddio cyfeiriad teithio'r awyren fel y cyfeiriad positif.

$$980 \times 1000 - (80 + 0.1v^2) \times 1000 = ma$$
$$980{,}000 - 80{,}000 - 100v^2 = (360 \times 1000)a$$
$$900{,}000 - 100v^2 = 360{,}000a$$
$$9000 - v^2 = 3600a$$

Ond mae $a = \dfrac{dv}{dt}$

$$a = \frac{dx}{dt} \times \frac{dv}{dx} \qquad \text{Rheol y gadwyn}$$

$$a = v \times \frac{dv}{dx}$$

Felly $9000 - v^2 = 3600v\dfrac{dv}{dx}$

$$3600v\frac{dv}{dx} = 9000 - v^2$$

QED

b) $$3600v\frac{dv}{dx} = 9000 - v^2$$

$$3600v\,dv = (9000 - v^2)\,dx$$

$$\int \frac{3600v}{9000-v^2}\,dv = \int dx$$

$$-1800\ln|9000 - v^2| = x + K$$

Os yw $t = 0s$, mae $v = 0ms^{-1}$ ag $x = 0m$

felly [ cychwyn y medfa

$$-1800\ln|9000 - 0^2| = 0 + K$$

$$-1800\ln|9000| = K$$

Felly $$-1800\ln|9000 - v^2| = x - 1800\ln(9000)$$

$$1800\ln(9000) - 1800\ln|9000 - v^2| = x$$

$$1800\ln\left|\frac{9000}{9000-v^2}\right| = x$$

$$\ln\left|\frac{9000}{9000-v^2}\right| = \frac{x}{1800}$$

$$\frac{9000}{9000-v^2} = e^{\frac{x}{1800}}$$

$$9000 = (9000 - v^2)e^{\frac{x}{1800}}$$

$$\frac{9000}{e^{\frac{x}{1800}}} = 9000 - v^2$$

$$v^2 = 9000 - \frac{9000}{e^{\frac{x}{1800}}}$$

$$\underline{v^2 = 9000(1 - e^{-\frac{x}{1800}})}$$

c) os yw $v = 85\,ms^{-1}$, yna

$$85^2 = 9000\left(1 - e^{-\frac{x}{1800}}\right)$$

$$\frac{7225}{9000} = 1 - e^{-\frac{x}{1800}}$$

$$e^{-\frac{x}{1800}} = 1 - \frac{7225}{9000}$$

$$-\frac{x}{1800} = \ln\left(\frac{71}{360}\right)$$

$$-\frac{x}{1800} = -1.623424154$$

$$x = 1800 \times 1.623424154$$

$$x = 2922.163478 \ldots m$$

Hyd lleiaf y rhedfa yw 2922m (i'r metr agosaf).

ch) Mae'r model wedi'i selio ar gychwyn ar y rhedfa.
Mae'n debygol bydd y gwrthiant ar uchder fawr ddim
yn dilyn yr un patrwm â'r gwrthiant ar y rhedfa.

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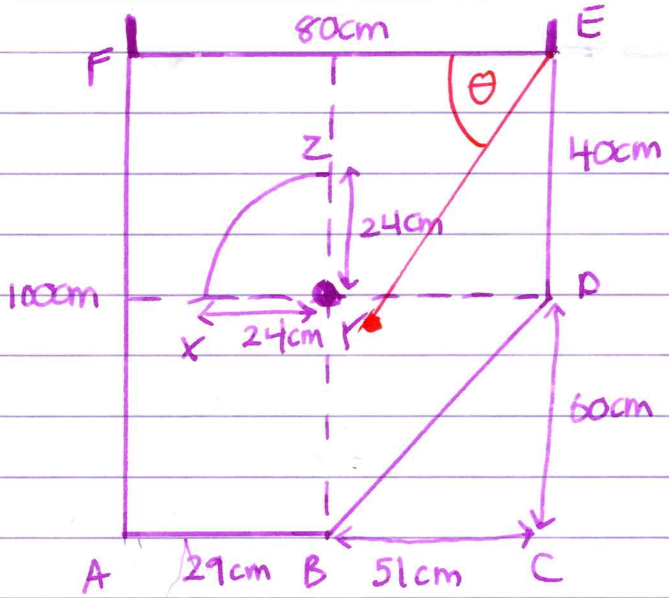
2)




Arwynebedd y triongl coll:

$$\frac{51 \times 60}{2} = 1530\text{cm}^2$$

a)

| Siâp | Arwynebedd | Pellter craidd màs o AF | Pellter craidd màs o AC |
|---|---|---|---|
| Petryal FECA | $8000\text{cm}^2$ | 40cm | 50cm |
| Triongl BCD | $1530\text{cm}^2$ | $80 - \frac{51}{3}$ cm | 20 cm |
| $\frac{1}{4}$ cylch XYZ | $\frac{1}{4}\pi(24)^2\text{cm}^2$ | $29 - \frac{4(24)}{3\pi}$ cm | $60 + \frac{4(24)}{3\pi}$ cm |
| Gronyn Y | Trinfel $765\text{cm}^2$ | 29cm | 60 cm |
| Lamina | ↑ | $\bar{x}$ | $\bar{y}$ |

$$8000 - 1530 + \tfrac{1}{4}\pi(24)^2 + 765$$
$$= 7235 + 144\pi$$

i) Yn cymryd momentau o amgylch AF:

$$(7235 + 144\pi)\bar{x} = 8000 \times 40 - 1530\left(80 - \frac{51}{3}\right) + \frac{1}{4}\pi(24)^2\left(29 - \frac{4(24)}{3\pi}\right) + 765 \times 29$$

$$(7235 + 144\pi)\bar{x} = 320{,}000 - 96{,}390 + 144\pi\left(29 - \frac{32}{\pi}\right) + 22185$$

$$(7235 + 144\pi)\bar{x} = 245795 + 4176\pi - 4608$$

$$(7235 + 144\pi)\bar{x} = 241187 + 4176\pi$$

$$\bar{x} = \frac{241187 + 4176\pi}{7235 + 144\pi}$$

$\bar{x} = 33.08\text{cm}$ i 2 le degol

ii) Yn cymryd momentau o amgylch AC:

$$(7235 + 144\pi)\bar{y} = 8000 \times 50 - 1530 \times 20 + \tfrac{1}{4}(\pi)(24)^2\left(60 + \frac{4(24)}{3\pi}\right) + 765 \times 60$$

$$(7235 + 144\pi)\bar{y} = 400{,}000 - 30{,}600 + 144\pi\left(60 + \frac{32}{\pi}\right) + 45900$$

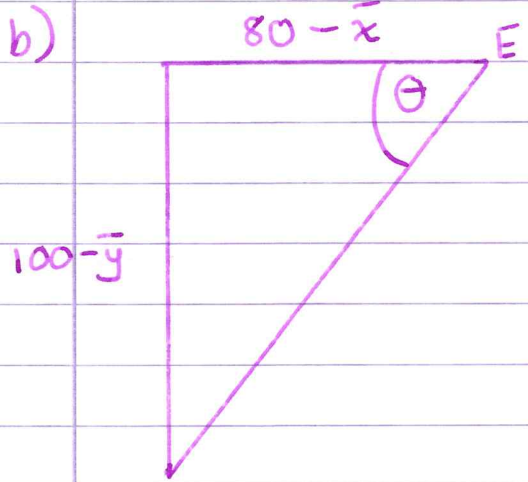
$$(7235 + 144\pi)\bar{y} = 415{,}300 + 8640\pi + 4320$$

$$(7235 + 144\pi)\bar{y} = 419620 + 8640\pi$$

$$\bar{y} = \frac{419620 + 8640\pi}{7235 + 144\pi}$$

$\bar{y} = 58.12$ cm i 2 le degol

b)




$$\tan\theta = \frac{\text{cyferbyn}}{\text{agos}}$$

$$\tan\theta = \frac{100 - \bar{y}}{80 - \bar{x}}$$

$$\tan\theta = \frac{100 - 58.11639565}{80 - 33.08096931}$$

$$\theta = \tan^{-1}(0.8926783809)$$

$$\theta = 41.75460014^\circ$$

$\theta = 41.75^\circ$ i 2 le degol

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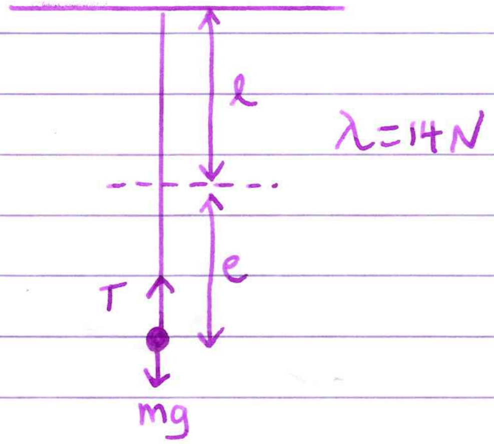
3)




a) Pan fo'r gronyn mewn cydbwysedd, mae'r pwysau'n hafal i'r tensiwn yn yr elastig.

$$mg = T$$
$$mg = \frac{\lambda e}{l}$$
$$mgl = 14e$$
$$e = \frac{mgl}{14}$$

b) i) Ar bwynt cyffredinol,

$$\text{Tensiwn} = \frac{\lambda \times \text{estyniad}}{l}$$
$$T = \frac{\lambda(e+x)}{l}$$
$$T = \frac{14(e+x)}{l}$$
$$T = \frac{14\left(\frac{mgl}{14}+x\right)}{l}$$
$$T = \frac{(mgl+14x)}{l}$$
$$T = mg + \frac{14x}{l}$$

ii) Yn defnyddio $F = ma$ ar y gronyn, yn fertigol, $\downarrow = +$if

$$mg - T = ma$$
$$mg - \left(mg + \frac{14x}{l}\right) = ma$$

$$mg - mg - \frac{14x}{l} = ma$$

$$-\frac{14x}{ml} = a$$

$$a = -\frac{14}{ml}x$$

$$\frac{d^2x}{dt^2} = -\frac{14}{ml}x$$

Mae hwn yn hafaliad ar gyfer mudiant harmonig syml o'r ffurf $\frac{d^2x}{dt^2} = -\omega^2 x$, fel bod $\omega^2 = \frac{14}{ml}$

iii) Y pellter mwyaf posib yw $l + e$.
Os yw'r pellter yn fwy na hyn bydd y gronyn yn taro'n erbyn y pwynt ble mae'r llinyn wedi'i osod yn y top, yn sefydlog.

c) $m = 0.5\,Kg$, $l = 0.7\,m$, $x = 0.2\,m$

i) osgled y mudiant yw $0.2m$

Mae'r buanedd uchaf ar y pwynt cydbwysedd ac mae'n hafal i $a\omega = 0.2 \times \sqrt{\frac{14}{ml}}$

$$= 0.2 \times \sqrt{\frac{14}{0.5 \times 0.7}}$$

$$= 0.2 \times \sqrt{40}$$

$$= \frac{2\sqrt{10}}{5}$$

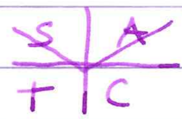
$$\approx 1.26\,ms^{-1} \text{ i 2l.d.}$$

ii) Yn defnyddio $x = a\sin(\omega t + \varepsilon)$
gwelwn fod $\varepsilon = \frac{\pi}{2}$ gan fod y mudiant yn cychwyn o'r dadleoliad mwyaf, nid o'r canol.

$$x = a\sin\left(\omega t + \frac{\pi}{2}\right)$$
$$x = 0.2\sin\left(\sqrt{\frac{14}{0.5\times 0.7}}\, t + \frac{\pi}{2}\right)$$
$$x = 0.2\sin\left(\sqrt{40}\, t + \frac{\pi}{2}\right).$$

Os yw $x = 0.15\text{m}$,

$$0.15 = 0.2\sin\left(\sqrt{40}\, t + \frac{\pi}{2}\right)$$
$$0.75 = \sin\left(\sqrt{40}\, t + \frac{\pi}{2}\right)$$
$$\sqrt{40}\, t + \frac{\pi}{2} = \sin^{-1}(0.75)$$




$$\sqrt{40}\, t + \frac{\pi}{2} = \ldots,\ 0.848062079,$$
$$2.293530575,\ \ldots$$
$$\sqrt{40}\, t = \ldots,\ -0.7227942478,\ 0.7227342482,\ \ldots$$
$$t = \ldots,\ -0.1142743184,\ 0.1142743184,\ \ldots$$

Mae'r gronyn yn cyrraedd $x = 0.15\text{m}$ am y tro cyntaf ar amser $\underline{t = 0.1143\text{s}}$ (i 4 lle degol).

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4) a) Egwyddor Cadwraeth Momentum:

Momentum A cyn y gwrthdrawiad + Momentum B cyn y gwrthdrawiad = Momentum A ar ôl y gwrthdrawiad + Momentum B ar ôl y gwrthdrawiad

$$\cancel{m}(-\underline{i}+8\underline{j}) + \cancel{m}(0\underline{i}+0\underline{j}) = \cancel{m}(2\underline{i}+\underline{j}) + \cancel{m}(a\underline{i}+b\underline{j})$$

$$-\underline{i}+8\underline{j} = 2\underline{i}+\underline{j}+a\underline{i}+b\underline{j}$$

$$-3\underline{i}+7\underline{j} = a\underline{i}+b\underline{j}$$

Felly cyflymder B ar ôl y gwrthdrawiad yw $-3\underline{i}+7\underline{j}$ ✓

b) $e = \frac{5}{7}$

cydran $\underline{i}$: $v = eu$

$v = \frac{5}{7} \times -3$

$v = -\frac{15}{7}\ ms^{-1}$

cydran $\underline{j}$: $v = -eu$

$v = -\frac{5}{7} \times 7$

$v = -5\ ms^{-1}$

Ateb: $(-\frac{15}{7}\underline{i} - 5\underline{j})\ ms^{-1}$.

c) Ergyd a gunaeth C ei roi ar B

$= $ Newid momentum B

$= m(-\frac{15}{7}\underline{i} - 5\underline{j}) - m(-3\underline{i}+7\underline{j})$

$= m(\frac{6}{7}\underline{i} - 12\underline{j})$ Ns

Maint yr ergyd $= \sqrt{(\frac{6}{7}m)^2 + (-12m)^2}$

$= \sqrt{\frac{36}{49}m^2 + 144m^2}$

$= \sqrt{\frac{7092\,m^2}{49}}$

$= \frac{6\sqrt{197}\,m}{7}$ Ns

ch) Cyflymder B ar ôl y gwrthdrawiad

$= \left(-\frac{15}{7}\underline{i} - 5\underline{j}\right) ms^{-1}$

Yn fertigol, faint o amser mae'r bêl yn ei gymryd i deithio 1.75m?

$$\text{Amser} = \frac{\text{Pellter}}{\text{Buanedd}}$$

$$\text{Amser} = \frac{1.75}{5}$$

$$\text{Amser} = \underline{0.35 \text{ eiliad}}$$

ii) Mewn 0.35 eiliad, faint mae'r bêl yn ei deithio'n llorweddol?

$$\text{Pellter} = \text{Amser} \times \text{Buanedd}$$

$$= 0.35 \times \frac{15}{7}$$

$$= 0.75m$$

Felly $x = 1.75 + 0.75$

$\underline{x = 2.5m}$

d) Gallai'r model gael ei wella trwy ystyried y ffrithiant rhwng y peli a'r bwrdd snwcer. Bydd hyn yn golygu y bydd yr amser yn rhan (ch)(i) ychydig yn fwy.

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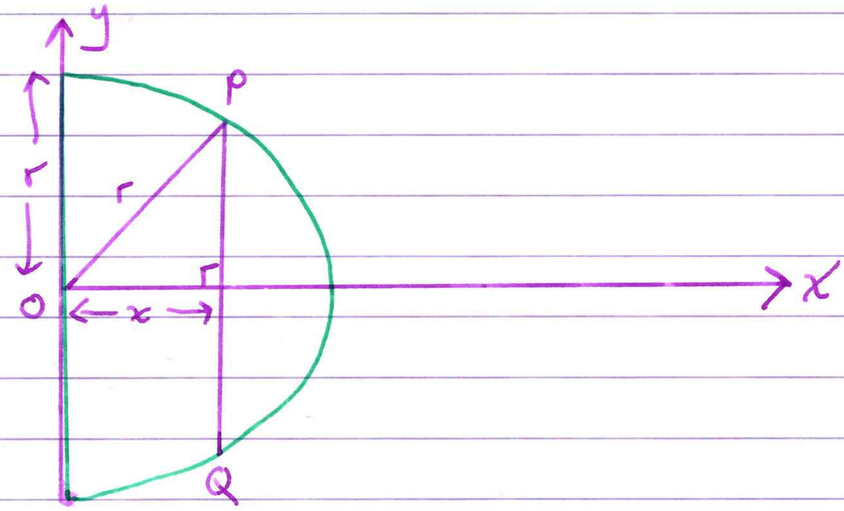
5)




a) Mae'r hemisffêr yn unffurf, fel bod ei ddwysedd $\rho$ yn gyson. ($\rho$ yw'r màs fesul uned cyfaint.)

Trwy gymesuredd, mae'r craidd màs yn gorwedd ar yr echelin-$x$. Gadewch i ni rannu'r hemisffêr yn dafellau yn baralel i'r sylfaen. Ystyriwch dafell PQ, sydd ar bellter $x$ oddi wrth O a thrwch $\delta x$.

Radiws y dafell yw $\sqrt{r^2 - x^2}$ (Theorem Pythagoras).

Màs y dafell = cyfaint × dwysedd

$= \pi r^2 \times$ trwch $\times$ dwysedd

radius y dafell

$= \pi\left(\sqrt{r^2 - x^2}\right)^2 \times \delta x \times \rho$

radius yr hemisffêr

$= \pi(r^2 - x^2)(\delta x)\rho$

Mae hwn yn gweithredu ar bellter $x$ o O.

Mās yr hemisffêr $= \text{Cyfaint} \times \text{dwysedd}$

$$= \frac{2}{3}\pi r^3 \rho$$

Mae hun yn gweithredu ar bellter $\bar{x}$ o O.

Yn cymryd momentau o amgylch yr echelin -y:

$$\bar{x} = \frac{\Sigma(\text{Mās stribed} \times \text{Pellter stribed o O})}{\text{Mās yr hemisffêr}}$$

$$(\text{Mās yr hemisffêr})\,\bar{x} = \Sigma(\text{Mās stribed} \times \text{Pellter stribed o O}).$$

$$\frac{2}{3}\pi r^3 \rho\, \bar{x} = \Sigma\left(\pi(r^2 - x^2)(\delta x)\rho \times x\right)$$

Fel mae $\delta x \to 0$ mae hun yn rhoi

$$\frac{2}{3}\pi r^3 \rho\, \bar{x} = \int_0^r \pi(r^2 - x^2)\rho x \, dx$$

$$\frac{2}{3}\pi r^3 \rho\, \bar{x} = \pi\rho \int_0^r (r^2 - x^2)\, x \, dx$$

$$\frac{2}{3} r^3 \bar{x} = \int_0^r (r^2 - x^2)\, x \, dx$$

$$\frac{2}{3} r^3 \bar{x} = \int_0^r r^2 x - x^3 \, dx$$

$$\frac{2}{3} r^3 \bar{x} = \left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$\frac{2}{3} r^3 \bar{x} = \left(\frac{r^4}{2} - \frac{r^4}{4}\right) - \left(\frac{r^2(0)^2}{2} - \frac{0^4}{4}\right)$$

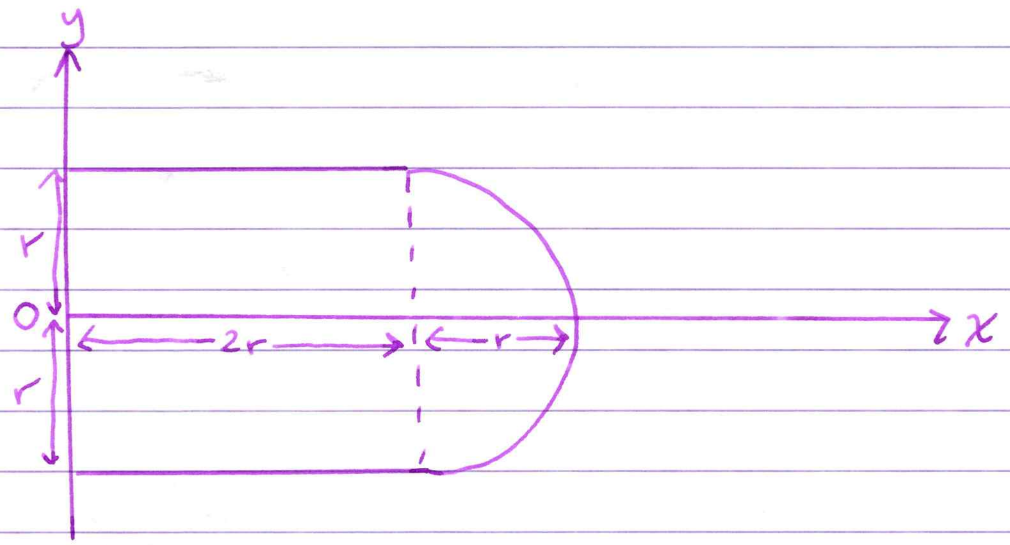
$$\frac{2}{3} r^3 \bar{x} = \frac{r^4}{4}$$

$$\bar{x} = \frac{3 \times \cancel{r^4}^{\,1}}{4 \times 2 \times \cancel{r^3}}$$

$$\bar{x} = \frac{3r}{8}$$

QED

b)




Gadewch i $\rho$ gynrychioli dwysedd y silindr.

| Siâp | Màs | Pellter craidd màs o O |
|---|---|---|
| Silindr | $(\pi r^2 h)\rho = (\pi r^2 \times 2r)\rho$ <br> $= 2\pi r^3 \rho$ | $r$ |
| Hemisffer | $\frac{2}{3}\pi r^3(\rho \times 1.5) = \pi r^3 \rho$ | $2r + \frac{3r}{8} = \frac{19}{8}r$ |
| Solid cyfansawdd | $2\pi r^3\rho + \pi r^3 \rho = 3\pi r^3 \rho$ | $\bar{h}$ |

Yn cymryd momentau o amgylch yr echelin-y:

$$3\pi r^3 \rho(\bar{h}) = 2\pi r^3 \rho(r) + \pi r^3 \rho\left(\frac{19}{8}r\right)$$

$$3\pi r^3 \rho(\bar{h}) = 2\pi r^4 \rho + \frac{19}{8}\pi r^4 \rho$$

$$3\cancel{\pi}\cancel{r^3}\cancel{\rho}(\bar{h}) = \frac{35}{8}\cancel{\pi}r^{\cancel{4}\,1}\cancel{\rho}$$

$$3\bar{h} = \frac{35}{8}r$$

$$\underline{\bar{h} = \frac{35}{24}r \text{ cm}}$$

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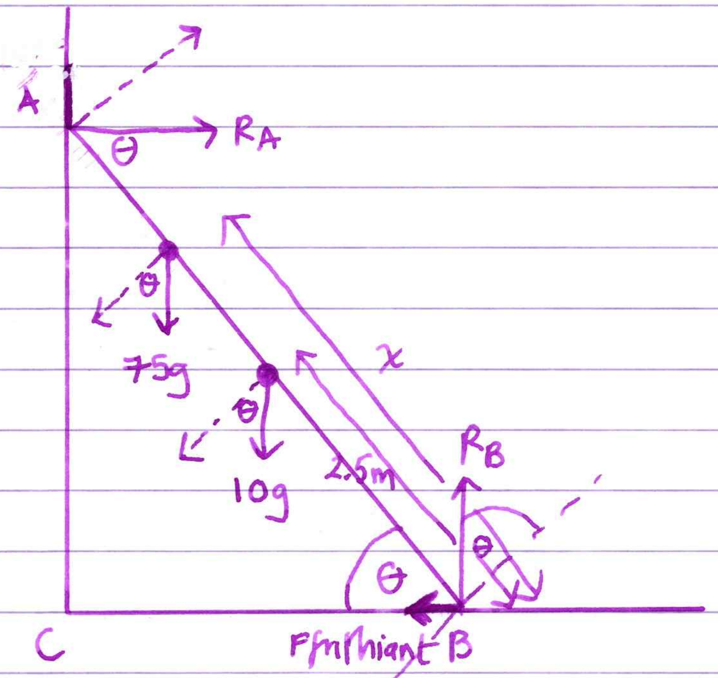
6)




F = Ffrithiant

a) Yn cymryd momentau o amgylch A :

Momentau clocwedd = Momentau gwrthglocwedd

$75g\cos\theta(5-x) + 10g\cos\theta(2.5) + F\sin\theta(5) = R_B\cos\theta(5)$

$75g\cos\theta(5-x) + 25g\cos\theta + 5F\sin\theta = 5R_B\cos\theta$

$15g\cos\theta(5-x) + 5g\cos\theta + F\sin\theta = R_B\cos\theta$

$5(15g\cos\theta) - 15g\cos\theta\, x + 5g\cos\theta + F\sin\theta = R_B\cos\theta$

$75g\cos\theta - 15g\cos\theta\, x + 5g\cos\theta + F\sin\theta = R_B\cos\theta$

$80g\cos\theta - 15g\cos\theta\, x + F\sin\theta = R_B\cos\theta$

$F\sin\theta = R_B\cos\theta + 15g\cos\theta\, x - 80g\cos\theta$

$F = R_B\dfrac{\cos\theta}{\sin\theta} + 15g\dfrac{\cos\theta}{\sin\theta}x - 80g\dfrac{\cos\theta}{\sin\theta}$

$F = \cot\theta(R_B + 15gx - 80g)$

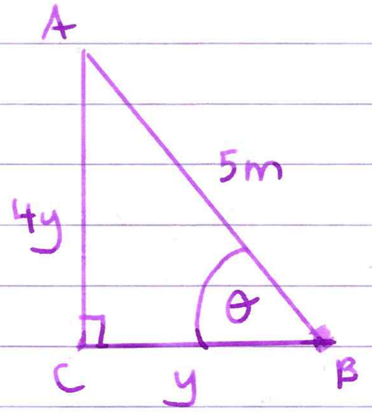
Yn defnyddio grymoedd i fyny = grymoedd i lawr

$R_B = 75g + 10g$

$R_B = 85g$

Felly $F = \cot\theta(85g + 15gx - 80g)$
$F = \cot\theta(5g + 15gx)$
$F = 5g\cot\theta(1+3x)$ ✓

b)




$\cot\theta = \dfrac{\text{agos}}{\text{cyferbyn}}$

$\cot\theta = \dfrac{y}{4y}$

$\cot\theta = \dfrac{1}{4}$

Ffrithiant $\leq \mu R$

$5g\cot\theta(1+3x) \leq \mu R$

$5\cancel{g}\left(\frac{1}{4}\right)(1+3x) \leq 85\cancel{g}\mu$

$\frac{1}{4}(1+3x) \leq 17\mu$

$17\mu \geq \frac{1}{4}(1+3x)$

$\mu \geq \frac{1}{68}(1+3x)$.

Yr uchaf gall y ddynes ddringo yw fel bod $x = 5$m.
Felly rhaid cael $\mu \geq \frac{1}{68}(1+3\times5)$

$\mu \geq \frac{16}{68}$

$\mu \geq \frac{4}{17}$

c) Mae'r ddynes yn cael ei fodelu fel gronyn, fel bod ei phwysau yn gweithredu ar un pwynt yn unig.