Uned 4 Pellach Haf 2019

1) $z = 3 + 4i$

a) Ffurf Trigonometreg:

$r = \sqrt{3^2 + 4^2}$ $\qquad \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$r = \sqrt{25}$ $\qquad \theta = 0.927295218\ldots$

$r = 5$ $\qquad \theta = 0.927$ i 3 lle degol

Felly $z = r(\cos\theta + i\sin\theta)$

$z = re^{i\theta}$

$\underline{z = 5e^{0.927i}}$ i 3 lle degol

b) Gadewch i $z = \sqrt[3]{3+4i}$

fel bod $z^3 = 3 + 4i$

$z^3 = 5(\cos 0.927 + i\sin 0.927)$

$z = \sqrt[3]{5}(\cos 0.927 + i\sin 0.927)^{\frac{1}{3}}$

$$(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \cos\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right] + i\sin\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right]$$

$$(\cos 0.927 + i\sin 0.927)^{\frac{1}{3}} = \cos\left[\frac{0.927}{3} + \frac{2(k-1)\pi}{3}\right] + i\sin\left[\frac{0.927}{3} + \frac{2(k-1)\pi}{3}\right]$$

$\underline{k=1}$: $z = \sqrt[3]{5}\left(\cos\left(\frac{0.927}{3}\right) + i\sin\left(\frac{0.927}{3}\right)\right)$

$z = 1.63 + 0.52i$ i 2 le degol

$K=2$ $\quad z = \sqrt[3]{5}\left(\cos\left(\frac{0.927}{3}+\frac{2\pi}{3}\right)+i\sin\left(\frac{0.927}{3}+\frac{2\pi}{3}\right)\right)$

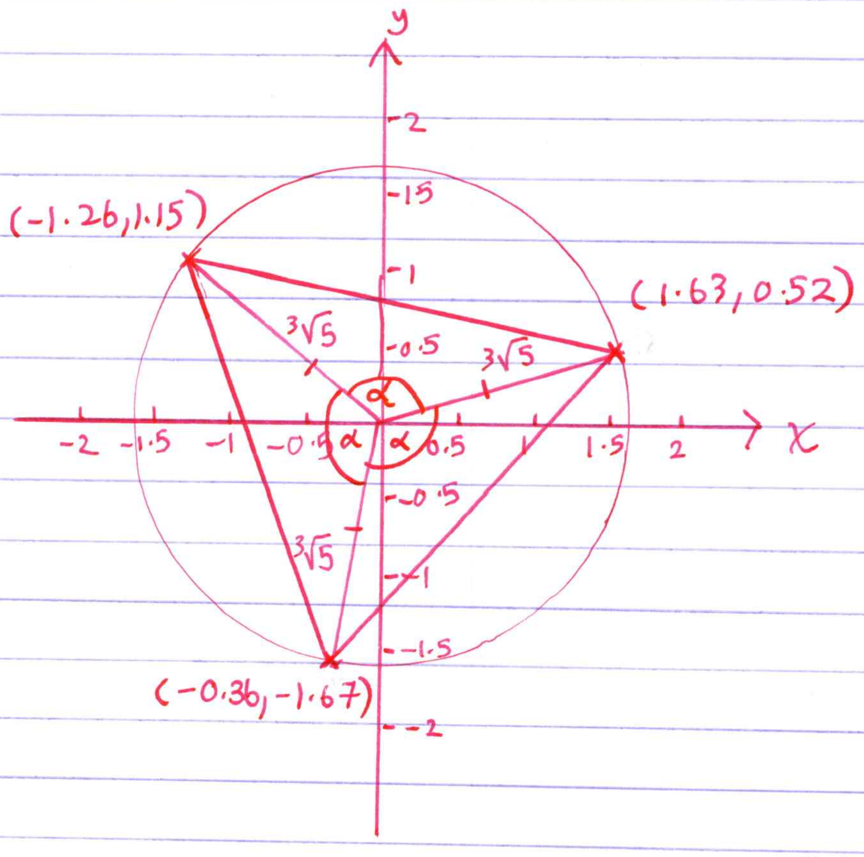
$z = -1.26 + 1.15i$ i 2 le degol

$K=3$ $\quad z = \sqrt[3]{5}\left(\cos\left(\frac{0.927}{3}+\frac{4\pi}{3}\right)+i\sin\left(\frac{0.927}{3}+\frac{4\pi}{3}\right)\right)$

$z = -0.36 - 1.67i$ i 2 le degol.

Cyfesurynnau Cartesaidd fertigau'r triongl yw
(1.63, 0.52), (−1.26, 1.15), (−0.36, −1.67)

ii)



Mae'r triongl yn driongl hafalochrog.

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2) a) $3\sin x + 4\cos x - 2$

Gadewch i $t = \tan\left(\frac{x}{2}\right)$ fel bod $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$.

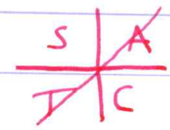
$$3\sin x + 4\cos x - 2$$
$$= 3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) - 2$$
$$= \frac{6t}{1+t^2} + \frac{4-4t^2}{1+t^2} - 2\left(\frac{1+t^2}{1+t^2}\right)$$
$$= \frac{6t+4-4t^2-2-2t^2}{1+t^2}$$
$$= \frac{6t+2-6t^2}{1+t^2} \quad \checkmark$$

b) $3\sin x + 4\cos x - 2 = 3$

$$\frac{6t+2-6t^2}{1+t^2} = 3$$
$$6t+2-6t^2 = 3(1+t^2)$$
$$6t+2-6t^2 = 3+3t^2$$
$$0 = 3+3t^2+6t^2-2-6t$$
$$0 = 9t^2-6t+1$$
$$0 = (3t-1)(3t-1)$$

Felly $3t-1=0$

$$t = \frac{1}{3}$$
$$\tan\left(\frac{x}{2}\right) = \frac{1}{3}$$



$$\frac{x}{2} = \tan^{-1}\left(\frac{1}{3}\right)$$
$$\frac{x}{2} = 0.3217505544 + n\pi$$
$$x = 0.6435011088 + 2n\pi$$

$\underline{x = 0.644 + 2n\pi}$ ar gyfer cyfanrif n. (i 3 ll.d.)

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3) a)

$$\begin{array}{c} \left(\begin{array}{ccc|c} 2 & -7 & 2 & a \\ 0 & 3 & -2 & b \\ -7 & 8 & 4 & c \end{array}\right) \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{cccc} 2 & -7 & 2 & a \\ 0 & 3 & -2 & b \\ 0 & -16.5 & 11 & c+3.5a \end{array}\right) \begin{array}{l} \\ \\ R_3 + 3.5R_1 =: R_4 \end{array}$$

$$\sim \left(\begin{array}{cccc} 2 & -7 & 2 & a \\ 0 & 3 & -2 & b \\ \underbrace{0 \quad 0 \quad 0} & & & c+3.5a+5.5b \end{array}\right) \begin{array}{l} \\ \\ R_4 + 5.5R_2 =: R_5 \end{array}$$

Mae'r tri sero yma'n golygu nad oes datrysiad unigryw i'r hafaliadau.

b)

$$\left(\begin{array}{ccc|c} 1 & 8 & -6 & 5 \\ 2 & 4 & 6 & -3 \\ -5 & -4 & 9 & -7 \end{array}\right) \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\sim \left(\begin{array}{cccc} 1 & 8 & -6 & 5 \\ 0 & -12 & 18 & -13 \\ 0 & 36 & -21 & 18 \end{array}\right) \begin{array}{l} \\ R_2 - 2R_1 =: R_4 \\ R_3 + 5R_1 =: R_5 \end{array}$$

$$\sim \left(\begin{array}{cccc} 1 & 8 & -6 & 5 \\ 0 & -12 & 18 & -13 \\ 0 & 0 & 33 & -21 \end{array}\right) \begin{array}{l} \\ \\ R_5 + 3R_4 =: R_6 \end{array}$$

$R_6 \Rightarrow 33z = -21$

$z = \frac{-21}{33}$

$z = \frac{-7}{11}$

$R_4 \Rightarrow -12y + 18z = -13$

$-12y + 18\left(\frac{-7}{11}\right) = -13$

$-12y = -13 + \frac{126}{11}$

$y = \frac{-\frac{17}{11}}{-12}$

$y = \frac{17}{132}$

$R_1 \Rightarrow x + 8y - 6z = 5$

$$x + 8\left(\frac{17}{132}\right) - 6\left(-\frac{7}{11}\right) = 5$$

$$x = 5 - \frac{34}{33} - \frac{42}{11}$$

$$x = \frac{5}{33}$$

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4) a) $y = \cot^{-1}(x)$

$\cot(y) = \cot(\cot^{-1}(x))$

$\cot(y) = x$

$-\operatorname{cosec}^2(y)\dfrac{dy}{dx} = 1$ Differu mewn perthynas ag x

$\dfrac{dy}{dx} = \dfrac{1}{-\operatorname{cosec}^2 y}$

$\dfrac{dy}{dx} = \dfrac{1}{-(1+\cot^2 y)}$ Defnyddio $\operatorname{cosec}^2 y = 1 + \cot^2 y$

$\dfrac{dy}{dx} = \dfrac{1}{-(1+x^2)}$

$\dfrac{dy}{dx} = \dfrac{-1}{x^2+1}$ ✓

b) $\dfrac{6x^2-10x-9}{(2x+3)(x^2+1)} \equiv \dfrac{A}{2x+3} + \dfrac{Bx+C}{x^2+1}$

$\dfrac{6x^2-10x-9}{(2x+3)(x^2+1)} \equiv \dfrac{A(x^2+1)+(Bx+C)(2x+3)}{(2x+3)(x^2+1)}$

$6x^2-10x-9 \equiv A(x^2+1)+(Bx+C)(2x+3)$

Yn amnewid $x = -\frac{3}{2}$

$6(-\frac{3}{2})^2 - 10(-\frac{3}{2}) - 9 \equiv A((-\frac{3}{2})^2+1) + (Bx+C)(0)$

$13.5 + 15 - 9 \equiv A(3.25)$

$19.5 \equiv 3.25A$

$\underline{A \equiv 6}$

Yn cymharu cyfernodau x^2:

$6 \equiv A + 2B$

$6 \equiv 6 + 2B$

$0 \equiv 2B$

$\underline{B \equiv 0}$

Yn cymharu cysonion:

$-9 \equiv A + 3C$

$-9 \equiv 6 + 3C$

$-15 \equiv 3C$

$\underline{C \equiv -5}$

Felly $\dfrac{6x^2-10x-9}{(2x+3)(x^2+1)} = \dfrac{6}{2x+3} - \dfrac{5}{x^2+1}$

c) $\displaystyle\int \frac{6x^2-10x-9}{(2x+3)(x^2+1)}\,dx = \int \frac{6}{2x+3} - \frac{5}{x^2+1}\,dx$

$\displaystyle = 3\ln|2x+3| - 5\int \frac{1}{x^2+1}\,dx$

$\displaystyle = 3\ln|2x+3| - 5\int \frac{1}{x^2+1^2}\,dx$

$\displaystyle = 3\ln|2x+3| - 5\left(\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right) + K$

$= 3\ln|2x+3| - 5\tan^{-1}(x) + K$

ch) Ni ellir cyfrifo $\displaystyle\int_{-2}^{5} \frac{6x^2-8x-6}{(2x+3)(x^2+1)}\,dx$

gan [illegible] ffwythiant $\dfrac{6x^2-8x-6}{(2x+3)(x^2+1)}$ [illegible]

wedi'i ddiffinio os yw $x = -\frac{3}{2}$

(Mae enwadur y ffracsiwn yn sero).

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5) $\sin\theta - \sin 3\theta$

a) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$\sin\theta - \sin 3\theta = 2\cos\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{\theta-3\theta}{2}\right)$$
$$= 2\cos(2\theta)\sin(-\theta)$$
$$= 2\cos(2\theta)(-\sin(\theta))$$ Etwy gymesuredd graff $\sin\theta$
$$= -2\cos(2\theta)\sin(\theta)$$

Felly $a = -2$, $b = 2$.

b) Gwerth cymedrig $y = 2\cos 2\theta \sin\theta + 7$ rhwng $\theta = 1$ a $\theta = 3$:

$$\frac{1}{3-1}\int_1^3 2\cos 2\theta \sin\theta + 7 \, d\theta$$
$$= \frac{1}{2}\int_1^3 -(\sin\theta - \sin 3\theta) + 7 \, d\theta$$
$$= \frac{1}{2}\int_1^3 \sin 3\theta - \sin\theta + 7 \, d\theta$$
$$= \frac{1}{2}\left[-\frac{1}{3}\cos 3\theta + \cos\theta + 7\theta\right]_1^3$$
$$= \frac{1}{2}\left[\left(-\frac{1}{3}\cos(9) + \cos(3) + 21\right) - \left(-\frac{1}{3}\cos(3) + \cos(1) + 7\right)\right]$$
$$\cancel{= \frac{1}{2}[19.70629742 -}$$
$$= \frac{1}{2}\left[20.31371759 - 7.870299805\right]$$
$$= 6.221708893$$
$$= \underline{\underline{6.22}} \text{ i 2 le degol}$$

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6) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$

Ceisio $y = Ae^{mx}$ fel bod m yn bodloni $am^2 + bm + c = 0$

$a = 1,\ b = -7,\ c = 10$

Hafaliad ategol / Auxiliary equation

$m^2 - 7m + 10 = 0$

$(m-2)(m-5) = 0$

Naill ai $m - 2 = 0$ neu $m - 5 = 0$

$\underline{m = 2}$ $\underline{m = 5}$

Dau ddatrysiad real felly'r ffwythiant cyflenwol (complementary function) yw $y = Ae^{2x} + Be^{5x}$

Does dim angen integryn neilltuol yma gan fod ochr dde yr hafaliad gwreiddiol yn sero.

Os yw $x = 0$, mae $\frac{dy}{dx} = 1$ a $\frac{d^2y}{dx^2} = 8$.

$y = Ae^{2x} + Be^{5x}$

Differu: $\frac{dy}{dx} = 2Ae^{2x} + 5Be^{5x}$

Amnewid: $1 = 2Ae^{(2\times 0)} + 5Be^{(5\times 0)}$

$1 = 2A + 5B$ —— (1)

Differu: $\frac{d^2y}{dx^2} = 4Ae^{2x} + 25Be^{5x}$

Amnewid: $8 = 4Ae^{(2\times 0)} + 25Be^{(5\times 0)}$

$8 = 4A + 25B$ —— (2)

$$
\begin{array}{rrcl}
2 & 8 = & 4A & + 25B \\
2\times 1 & 2 = & 4A & + 10B \\
\hline
 & 6 = & & 15B \\
\hline
\end{array}
$$

$$B = \frac{6}{15}$$

$$B = \frac{2}{5}$$

Yn ôl yn 1: $1 = 2A + 5B$

$$1 = 2A + 5\left(\frac{2}{5}\right)$$

$$1 = 2A + 2$$

$$-1 = 2A$$

$$A = \frac{-1}{2}$$

Datrysiad cyffredinol: $y = \frac{-1}{2} e^{2x} + \frac{2}{5} e^{5x}$

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7) $f(x) = \ln(1-x)$.

a) Cyfres Maclaurin:

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

Nawr $f'(x) = \frac{-1}{1-x}$

$$f''(x) = \frac{d}{dx}\left(\frac{-1}{1-x}\right)$$

$$= \frac{d}{dx} (-1)(1-x)^{-1}$$

$$= +1(1-x)^{-2}(-1)$$

$$= (-1)(1-x)^{-2}$$

$$f'''(x) = \frac{d}{dx} (-1)(1-x)^{-2}$$

$$= +2(1-x)^{-3}(-1)$$

$$= (-2)(1-x)^{-3}.$$

Cyfres Maclaurin:

$$f(x) = \ln(1-0) + x\left(\frac{-1}{1-0}\right) + \frac{x^2}{2!}(-1)(1-0)^{-2} + \frac{x^3}{3!}(-2)(1-0)^{-3} + \cdots$$

$$= \ln(1) + x\left(\frac{-1}{1}\right) + \frac{x^2}{2}(-1)(1)^{-2} + \frac{x^3}{6}(-2)(1)^{-3} + \cdots$$

$$= 0 + x(-1) + \frac{x^2}{2}(-1)(1) + \frac{x^3}{6}(-2)(1) + \cdots$$

$$= -x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots$$

b) $$-2\ln\left(\frac{1-x}{(1+x)^2}\right) = -2\left(\ln(1-x) - \ln(1+x)^2\right)$$
$$= -2\left(\ln(1-x) - 2\ln(1+x)\right)$$
$$= -2\ln(1-x) + 4\ln(1+x).$$

Ar gyfer $g(x) = \ln(1+x)$

$$g'(x) = \frac{1}{1+x}$$
$$g''(x) = \frac{d}{dx}(1+x)^{-1}$$
$$= (-1)(1+x)^{-2}(1)$$
$$= (-1)(1+x)^{-2}$$
$$g'''(x) = \frac{d}{dx}(-1)(1+x)^{-2}$$
$$= (+2)(1+x)^{-3}(1)$$
$$= 2(1+x)^{-3}$$

Cyfres Maclaurin:

$$g(x) = \ln(1+0) + x\left(\frac{1}{1+0}\right) + \frac{x^2}{2!}(-1)(1+0)^{-2} + \frac{x^3}{3!}(2)(1+0)^{-3} + \cdots$$
$$= \ln(1) + x\left(\frac{1}{1}\right) + \frac{x^2}{2}(-1)(1)^{-2} + \frac{x^3}{6}(2)(1)^{-3} + \cdots$$
$$= 0 + x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

Cyfres Maclaurin $-2\ln\left(\frac{1-x}{(1+x)^2}\right)$

$$= -2\left(-x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots\right) + 4\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots\right)$$
$$= 2x + \frac{2x^2}{2} + \frac{2x^3}{3} + \cdots + 4x - \frac{4}{2}x^2 + \frac{4}{3}x^3 + \cdots$$
$$= \underline{6x - x^2 + 2x^3 + \cdots} \quad (\text{felly } a=6, b=-1, c=2)$$

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8) C: $r = \sin 2\theta \qquad (0 < \theta \leq \frac{\pi}{2})$

a) $x = r\cos\theta$

$x = (\sin 2\theta)\cos\theta$

$x = (2\sin\theta\cos\theta)\cos\theta$

$x = 2\sin\theta\cos^2\theta$

$\frac{dx}{d\theta} = 2\sin\theta(2\cos\theta\sin\theta) + 2\cos\theta(\cos^2\theta)$

$= 4\sin^2\theta\cos\theta + 2\cos^3\theta$

$y = r\sin\theta$

$y = (\sin 2\theta)\sin\theta$

$y = (2\sin\theta\cos\theta)\sin\theta$

$y = 2\sin^2\theta\cos\theta$

$\frac{dy}{d\theta} = 2\sin^2\theta(-\sin\theta) + 2(2\sin\theta)\cos\theta(\cos\theta)$

$= -2\sin^3\theta + 4\sin\theta\cos^2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\frac{dy}{dx} = (-2\sin^3\theta + 4\sin\theta\cos^2\theta) \times \frac{1}{4\sin^2\theta\cos\theta + 2\cos^3\theta}$

$\frac{dy}{dx} = \frac{-2\sin^3\theta + 4\sin\theta\cos^2\theta}{4\sin^2\theta\cos\theta + 2\cos^3\theta}$

Os yw'r tangiad yn baralel i'r llinell gychwynnol yna mae

$\frac{dy}{dx} = 0$

$$\frac{-2\sin^3\theta + 4\sin\theta\cos^2\theta}{4\sin^2\theta\cos\theta + 2\cos^3\theta} = 0$$

$$-2\sin^3\theta + 4\sin\theta\cos^2\theta = 0$$

$$2\sin\theta(-\sin^2\theta + 2\cos^2\theta) = 0$$

Naill ai $2\sin\theta = 0$ neu $-\sin^2\theta + 2\cos^2\theta = 0$

$\sin\theta = 0$

S | A
T | C

$\theta = \sin^{-1}(0)$

$\theta = 0, \pi, 2\pi, \ldots$

Dim atebion yn y parth

$2\cos^2\theta = \sin^2\theta$

$2 = \tan^2\theta$

$\tan\theta = \pm\sqrt{2}$

$\tan\theta = \sqrt{2}$

S | A
T | C

$\theta = \tan^{-1}(\sqrt{2})$

$\theta = \underline{\underline{0.9553}}, 4.0969$

$\tan\theta = -\sqrt{2}$

$\theta = \tan^{-1}(-\sqrt{2})$

S | A
T | C

$\theta = -0.955, 2.186$

Dim atebion yn y parth

Hun yw gwerth θ.

$r = \sin 2\theta$

$r = \sin(2 \times 0.9553166181)$

$r = 0.9428090416$

Cyfesurynnau pegynlinol y pwynt ar C ble mae'r tangiad yn baralel i'r llinell gychwynnol yw $(0.943, 0.955)$ (i 3 lle degol).

b) $x = r\cos\theta$

$= 0.9428090416 \times \cos(0.9553166181)$

$= 0.544$ i 3 lle degol

$y = r\sin\theta$

$= 0.9428090416 \times \sin(0.9553166181)$

$= 0.770$ i 3 lle degol

Cyfesurynnau Cartesaidd y pwynt yw $(0.544, 0.770)$ (i 3 lle degol).

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9) a) $y = \sin^{-1}(\cos\theta)$, $0 \leq \theta \leq \pi$

Gadewch i $x = \cos\theta$ fel bod $y = \sin^{-1}(x)$.
o'r llyfryn fformiwlâu, $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$.

$$\frac{dy}{d\theta} = \frac{dy}{dx} \times \frac{dx}{d\theta}$$

$$\frac{dy}{d\theta} = \frac{1}{\sqrt{1-x^2}} \times -\sin\theta$$

$$\frac{dy}{d\theta} = \frac{1}{\sqrt{1-\cos^2\theta}} \times -\sin\theta$$

$$\frac{dy}{d\theta} = \frac{-\sin\theta}{\sqrt{\sin^2\theta}}$$

$$\frac{dy}{d\theta} = \frac{-\sin\theta}{\sin\theta}$$

$\frac{dy}{d\theta} = -1$ fel bod $\underline{K = -1}$

b) $y = x^3 \tan^{-1}(4x)$

Gadewch i $u = 4x$ fel bod gennym $\tan^{-1}(u)$ yn yr hafaliad.

$$\frac{d}{dx}\left(\tan^{-1}(u)\right)$$

$$= \frac{d}{du}\left(\tan^{-1}(u)\right) \times \frac{du}{dx}$$

$$= \frac{1}{1+u^2} \times 4$$

$$= \frac{4}{1+(4x)^2}$$

$$= \frac{4}{1+16x^2}$$

Differu $y = x^3 \tan^{-1}(4x)$

$$\frac{dy}{dx} = x^3\left(\frac{4}{1+16x^2}\right) + 3x^2 \tan^{-1}(4x).$$

$$\frac{dy}{dx} = \frac{4x^3}{1+16x^2} + 3x^2 \tan^{-1}(4x).$$

Os yw $x = \frac{\pi}{2}$, $\frac{dy}{dx} = \frac{4(\frac{\pi}{2})^3}{1+16(\frac{\pi}{2})^2} + 3(\frac{\pi}{2})^2 \tan^{-1}(4 \times \frac{\pi}{2})$

$$\frac{dy}{dx} = 10.84205284$$

$$\frac{dy}{dx} = \underline{10.84} \text{ i 2 ledegol}$$

c) $y = \tanh^{-1}(1-x)$

Gadewch i $u = 1-x$ fel bod $y = \tanh^{-1}(u)$.

$$\frac{dy}{du} = \frac{1}{1-u^2} \quad \text{o'r llyfryn fformiwlâu}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{1-u^2} \times (-1)$$

$$\frac{dy}{dx} = \frac{-1}{1-(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{1-(1-2x+x^2)}$$

$$\frac{dy}{dx} = \frac{-1}{2x-x^2}$$

Os yw $x = 1.7$, $\frac{dy}{dx} = \frac{-1}{2(1.7) - 1.7^2}$

$\frac{dy}{dx} = \frac{-100}{51}$

Felly graddiant y normal yw $\frac{51}{100} = 0.51$ (negatif y cilydd).

Os yw $x = 1.7$, $y = \tanh^{-1}(1 - 1.7)$

$y = \tanh^{-1}(-0.7)$

$y = -0.867300527\,7$

Hafaliad y normal i'r gromlin: $y - y_1 = m(x - x_1)$

$y - -0.8673005277 = 0.51(x - 1.7)$

$y + 0.8673005277 = 0.51x - 0.867$

$y = 0.51x - 1.734300528$

$\underline{y = 0.51x - 1.734}$ i 3 lle degol

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10) $\sec x \dfrac{dy}{dx} + y \operatorname{cosec} x = 2$

$\dfrac{1}{\cos x} \dfrac{dy}{dx} + y \dfrac{1}{\sin x} = 2$

$\dfrac{dy}{dx} + y \dfrac{\cos x}{\sin x} = 2\cos x$

$\dfrac{dy}{dx} + y \cot x = 2\cos x$

Ar gyfer $\dfrac{dy}{dx} + Fy = G$,

ble mae F a G yn ffwythiannau mewn x yn unig,
y ffactor integru yw $I = e^{\int F dx}$

$I = e^{\int \cot x \, dx}$

$I = e^{\ln|\sin x|}$

$I = \sin x$

Felly $\dfrac{dy}{dx} + y \cot x = 2\cos x$

Lluosi bob ochr yr hafaliad â'r ffactor integru:

$\sin x \dfrac{dy}{dx} + y \cot x \sin x = 2\cos x \sin x$

$\int \left(\sin x \dfrac{dy}{dx} + y \dfrac{\cos x}{\cancel{\sin x}} \cancel{\sin x} \right) dx = \int 2\cos x \sin x \, dx$

$(\sin x) y = \int \sin 2x \, dx$

$(\sin x) y = -\frac{1}{2}\cos 2x + K$

Os yw $x = \frac{\pi}{2}$ mae $y = 2$, felly

$(\sin \frac{\pi}{2})(2) = -\frac{1}{2}\cos(2 \times \frac{\pi}{2}) + K$

$$1 \times 2 = -\frac{1}{2}(-1) + K$$

$$2 - \frac{1}{2} = K$$

$$K = \frac{3}{2}$$

Felly $(\sin x)y = -\frac{1}{2}\cos(2x) + \frac{3}{2}$

$$2y\sin(x) = 3 - \cos(2x)$$

Os yw $x = \frac{\pi}{4}$, $2y\sin\left(\frac{\pi}{4}\right) = 3 - \cos\left(2 \times \frac{\pi}{4}\right)$

$$2y\left(\frac{1}{\sqrt{2}}\right) = 3 - 0$$

$$\underline{\underline{y = \frac{3\sqrt{2}}{2}}}$$

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11) a) $\int_0^1 x \sinh(x)\,dx$

Gadewch i $u = x$ $\qquad \frac{dv}{dx} = \sinh(x)$

$\frac{du}{dx} = 1$ $\qquad v = \cosh(x)$.

$$\int_0^1 x\sinh(x) = [x\cosh(x)]_0^1 - \int_0^1 \cosh(x)\,dx$$
$$= [1\times\cosh(1) - 0\times\cosh(0)] - [\sinh(x)]_0^1$$
$$= \cosh(1) - [\sinh(1) - \sinh(0)]$$
$$= \cosh(1) - \sinh(1).$$

Nawr $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$,

$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.

Felly $\int_0^1 x\sinh(x) = \frac{1}{2}(e^1 + e^{-1}) - \frac{1}{2}(e^1 - e^{-1})$
$$= \cancel{\frac{1}{2}e^1} + \frac{1}{2}e^{-1} \cancel{- \frac{1}{2}e^1} + \frac{1}{2}e^{-1}$$
$$= e^{-1}$$
$$= \frac{1}{e}$$

(≈ 0.3679 i 4 lle degol).

b) $\int_0^1 \pi r^2\,dr = \pi\int_0^1 (\cosh 2x)^2\,dx$
$$= \pi\int_0^1 \cosh^2 2x\,dx.$$

Nawr $\cosh 2x = 2\cosh^2 x - 1$

felly $\frac{\cosh 2x + 1}{2} = \cosh^2 x$

Felly'r cyfaint yw $\pi\int_0^1 \frac{\cosh 4x + 1}{2}\,dx$
$$= \frac{\pi}{2}\int_0^1 \cosh 4x + 1\,dx$$

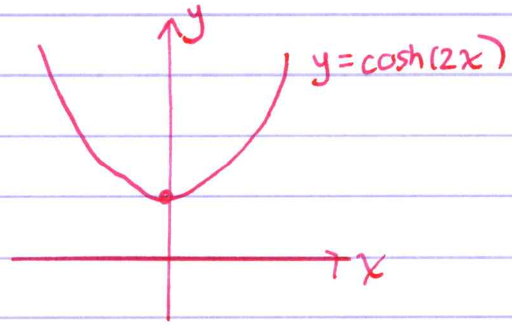
$$= \frac{\pi}{2}\left[\frac{\sinh 4x}{4} + x\right]_0^1$$

$$= \frac{\pi}{2}\left[\left(\frac{\sinh(4)}{4} + 1\right) - \left(\frac{\sinh(0)}{4} + 0\right)\right]$$

$$= \frac{\pi}{2}\left(\frac{\sinh(4)}{4} + 1\right)$$

≈ 12.2875 i 4 lle degol

c)



Trwy gymesuredd graff $\cosh(2x)$

y cyfaint yw $\quad 2 \times \frac{\pi}{2}\left(\frac{\sinh(4)}{4} + 1\right)$

$$= \pi\left(\frac{\sinh(4)}{4} + 1\right)$$

≈ 24.5750 i 4 lle degol

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12) a) $\int_3^4 \frac{1}{\sqrt{x^2-4}}\,dx = \int_3^4 \frac{1}{\sqrt{x^2-2^2}}\,dx$

$= \left[\cosh^{-1}\left(\frac{x}{2}\right)\right]_3^4$

$= \cosh^{-1}\left(\frac{4}{2}\right) - \cosh^{-1}\left(\frac{3}{2}\right)$

$= \cosh^{-1}(2) - \cosh^{-1}\left(\frac{3}{2}\right)$

$= \underline{0.355}$ i 3 lle degol

b) $\int_1^2 \frac{K}{9-x^2}\,dx = \ln\left(\frac{25}{4}\right)$

$K\int_1^2 \frac{1}{3^2-x^2}\,dx = \ln\left(\frac{25}{4}\right)$

$K\left[\frac{1}{2\times 3}\ln\left|\frac{3+x}{3-x}\right|\right]_1^2 = \ln\left(\frac{25}{4}\right)$

$K\left[\frac{1}{6}\ln\left|\frac{5}{1}\right| - \frac{1}{6}\ln\left|\frac{4}{2}\right|\right] = \ln\left(\frac{25}{4}\right)$

$\frac{K}{6}\left[\ln(5) - \ln(2)\right] = \ln\left(\frac{25}{4}\right)$

$\frac{K}{6}\ln\left(\frac{5}{2}\right) = \ln\left(\frac{25}{4}\right)$

$\frac{K}{6} = \frac{\ln\left(\frac{25}{4}\right)}{\ln\left(\frac{5}{2}\right)}$

$\frac{K}{6} = \frac{\ln\left(\left(\frac{5}{2}\right)^2\right)}{\ln\left(\frac{5}{2}\right)}$

$\frac{K}{6} = \frac{2\cancel{\ln\left(\frac{5}{2}\right)}}{\cancel{\ln\left(\frac{5}{2}\right)}}$

$K = 2\times 6$

$\underline{K = 12}$

c) $\displaystyle\int \frac{(\cosh x - \sinh x)^3}{\cosh^2 x + \sinh^2 x - \sinh 2x}\, dx$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$
$\cosh x = \frac{1}{2}(e^x + e^{-x})$.

$$\int \frac{\left(\frac{1}{2}(e^x + e^{-x}) - \frac{1}{2}(e^x - e^{-x})\right)^3}{\left[\frac{1}{2}(e^x + e^{-x})\right]^2 + \left[\frac{1}{2}(e^x - e^{-x})\right]^2 - \frac{1}{2}(e^{2x} - e^{-2x})}\, dx$$

$$= \int \frac{(\cancel{\tfrac{1}{2}e^x} + \frac{1}{2}e^{-x} - \cancel{\tfrac{1}{2}e^x} + \frac{1}{2}e^{-x})^3}{\frac{1}{4}((e^x)^2 + 2(e^x)(e^{-x}) + (e^{-x})^2) + \frac{1}{4}((e^x)^2 - 2(e^x)(e^{-x}) + (e^{-x})^2] - \frac{1}{2}(e^{2x} - e^{-2x})}\, dx$$

$$= \int \frac{(e^{-x})^3}{\frac{1}{4}(e^{2x} + 2e^0 + e^{-2x}) + \frac{1}{4}(e^{2x} - 2e^0 + e^{-2x}) - \frac{1}{2}(e^{2x} - e^{-2x})}\, dx$$

$$= \int \frac{e^{-3x}}{\cancel{\tfrac{1}{4}e^{2x}} + \cancel{\tfrac{2}{4}} + \frac{1}{4}e^{-2x} + \cancel{\tfrac{1}{4}e^{2x}} - \cancel{\tfrac{2}{4}} + \frac{1}{4}e^{-2x} - \cancel{\tfrac{1}{2}e^{2x}} + \frac{1}{2}e^{-2x}}\, dx$$

$$= \int \frac{e^{-3x}}{e^{-2x}}\, dx$$

$$= \int e^{-3x - -2x}\, dx$$

$= \int e^{-x}\, dx$

$= -e^{-x} + c$ ✓