YSGOL CREUDDYN

DAWN DYSG DAIONI

The Mathematics Department

10

Measuring Shapes 3

Name:

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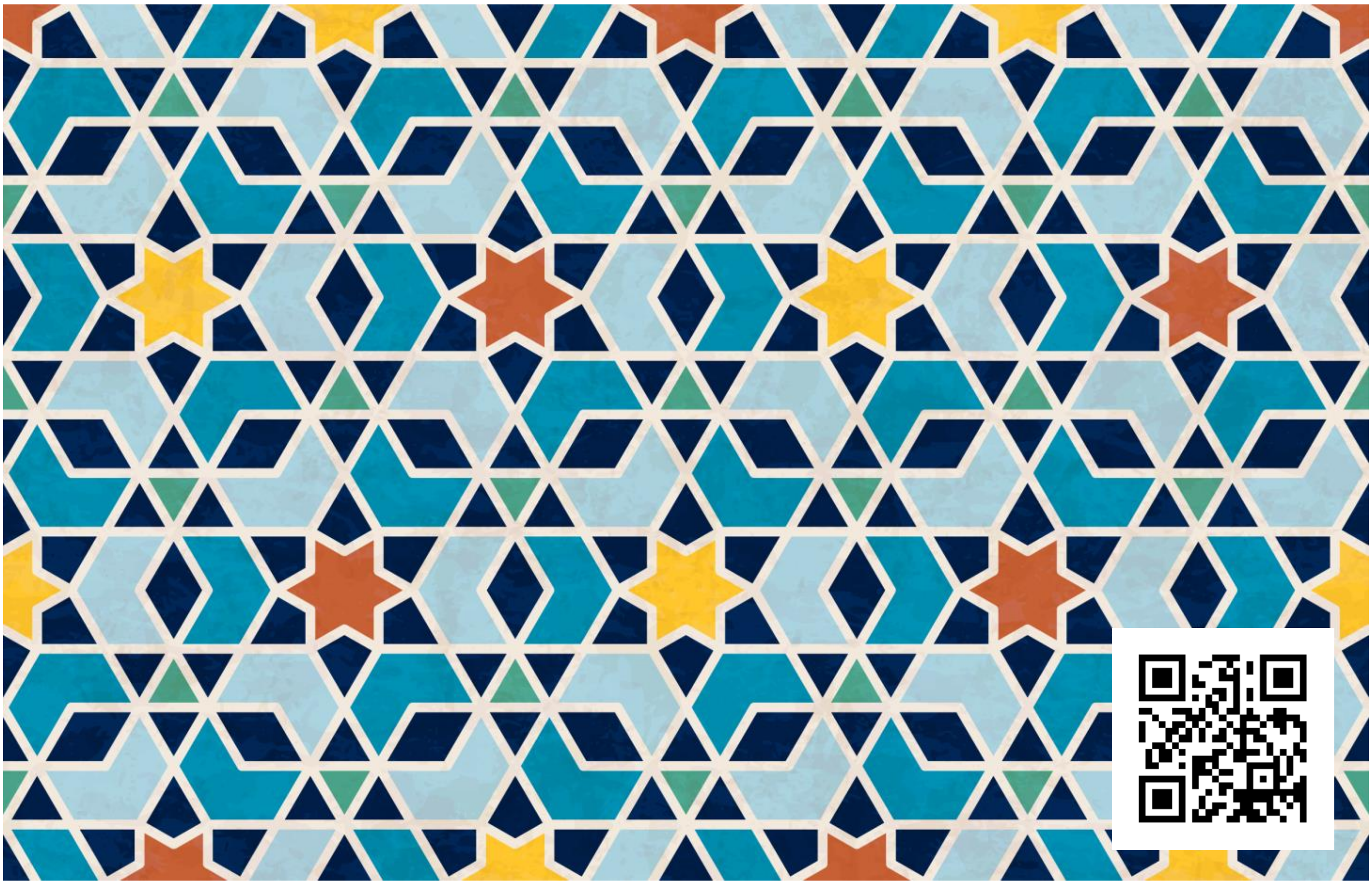
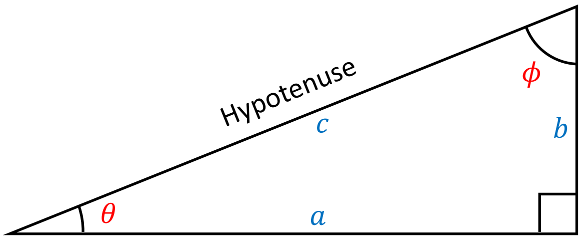


Trigonometry

Right-Angled Triangles

Any right-angled triangle has:

- One angle that is a right angle, or 90°;
- Two acute angles θ and ϕ;
- A hypotenuse c, which is always opposite the right angle;
- Two sides a and b which are shorter than the hypotenuse.



In year 9, we introduced Pythagoras' Theorem, which connects the lengths a, b and c:

$$c^2 = a^2 + b^2$$

Given the length of any two sides in a triangle, we can use Pythagoras' Theorem to calculate the length of the third side.

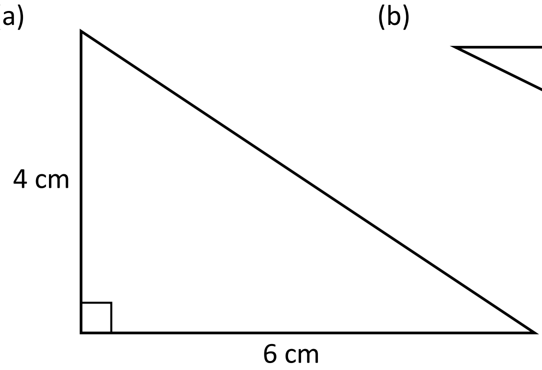
Exercise 1

Revision

I

Use Pythagoras' Theorem to calculate the length of the third side in these right-angled triangles.
Round off your answers to two decimal places.

(a)



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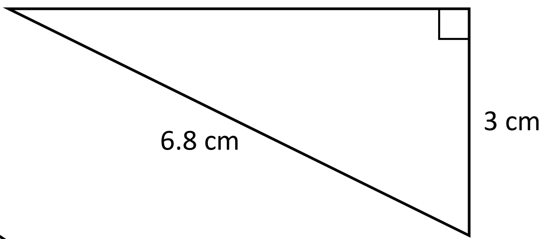

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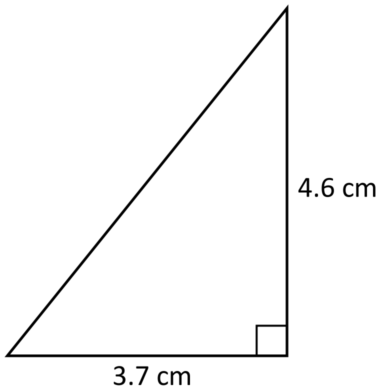
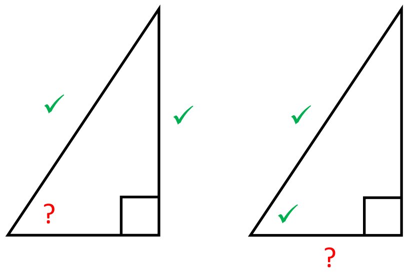


What is trigonometry?

Trigonometry is used for:

- Calculating the size of one of the acute angles in a right-angled triangle, given the length of any two sides;
- Calculating the length of one of the sides in a right-angled triangle given the length of one other side and the size of one acute angle.



How?

Trigonometry uses the relationship between the size of the angles and the lengths of the sides in any right-angled triangle.

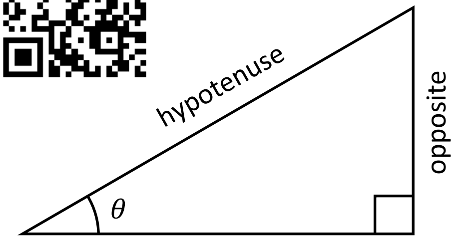
Exercise 2

Applying

Draw any three right-angled triangles where one of the angles measures 30°.
Measure the length of the hypotenuse and the length of the side opposite the 30° angle. What do you notice?

Labelling the sides of a right-angled triangle

Let θ represent the size of one of the acute angles in a right-angled triangle. We follow these conventions when labelling the sides of the triangle.

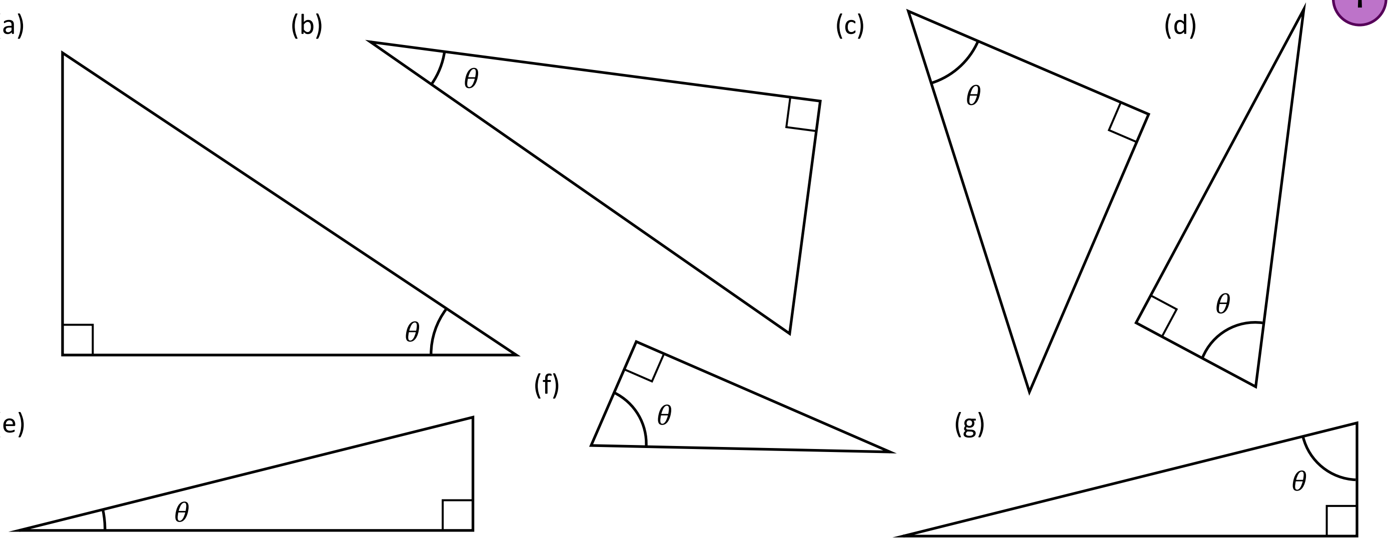


- The **opposite** is the side opposite the angle θ.
- The **hypotenuse** is the side opposite the right angle.
- The **adjacent** is the side left over (it's close to the angle θ).

Exercise 3

Skill I

Label the sides of these triangles using the words "opposite", "hypotenuse" and "adjacent".



Sin, Cos, Tan

For a specific angle θ, we define the **sin**, **cos** and **tan** of the angle as follows.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

How to remember the formula...

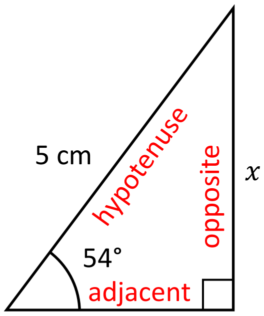
In mathematics, we worship the King SOHCAHTOA.

S O H C A H T O A

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Finding lengths using trigonometry

Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x.



To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we want to calculate the length of the **opposite** (x) side, and we know the length of the **hypotenuse** (5 cm). The trigonometric ratio that uses the words **opposite** and **hypotenuse** is sin, therefore we must use the formula

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

in this question. By substituting values into the formula, we obtain

$$\sin 54° = \frac{x}{5}.$$

By multiplying both sides of the equation by 5, we obtain

$$x = 5 \times \sin 54°.$$

x on **top** of the fraction leads to a **multiplication** sum in the answer.

By typing this sum on a calculator, we find that $x = 4.05$ cm, correct to two decimal places.

Exercise 4

I

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x.

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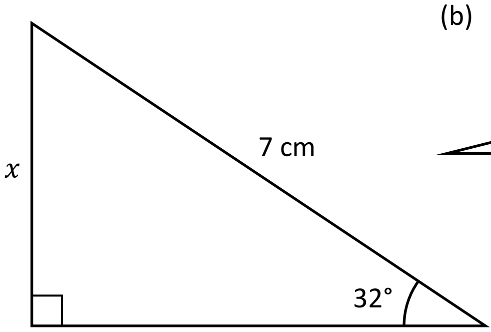

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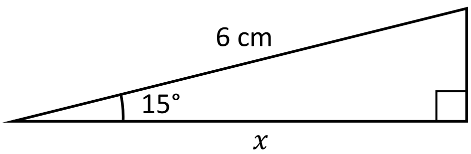

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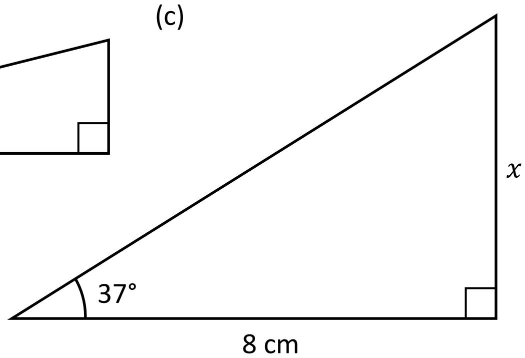

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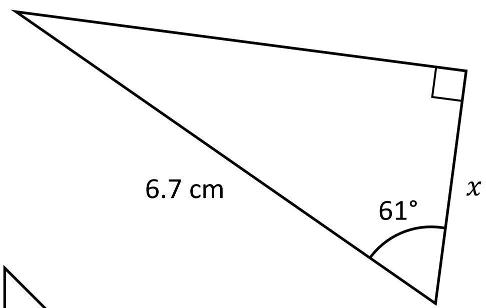

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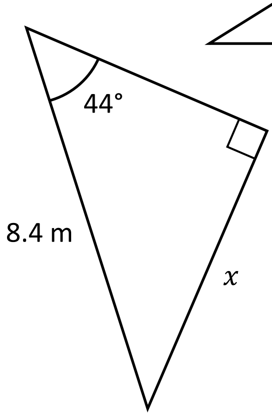

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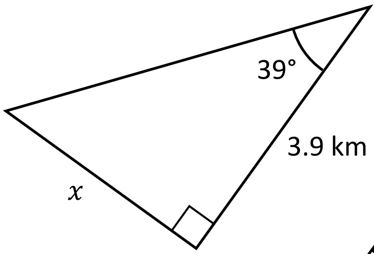

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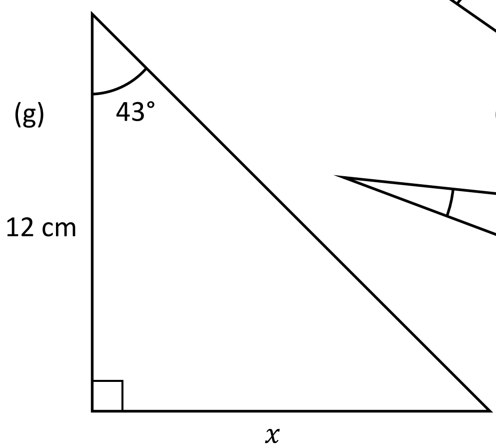

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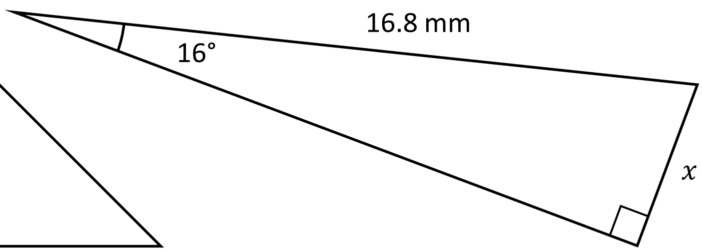

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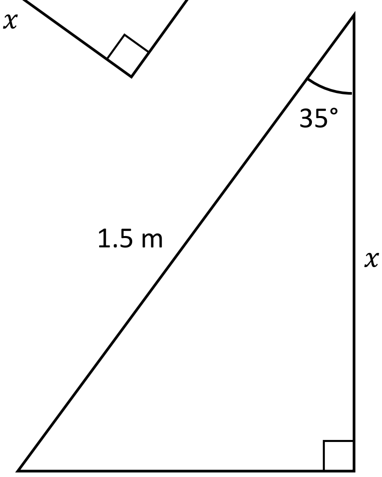
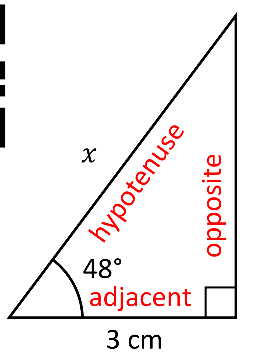


Example



Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x.

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we know the length of the **adjacent** (3 cm), and we want to calculate the length of the **hypotenuse** (x). The trigonometric ratio that uses the words **adjacent** and **hypotenuse** is cos, therefore we must use the formula

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

in this question. By substituting values into the formula, we obtain

$$\cos 48° = \frac{3}{x}.$$

x on the **bottom** of the fraction leads to a **division** sum in the answer.

By multiplying both sides of the equation by x, we obtain

$$x \times \cos 48° = 3.$$

By dividing both sides of the equation by $\cos 48°$, we obtain

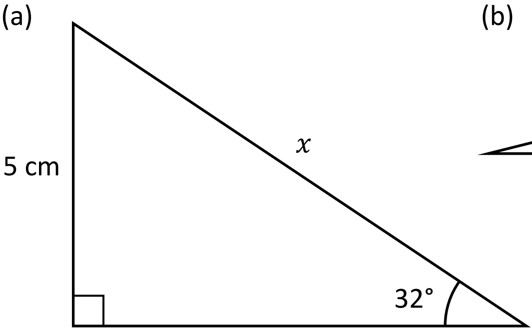
$$x = 3 \div \cos 48°.$$

By typing this sum on a calculator, we find that $x = 4.48$ cm, correct to two decimal places.

Exercise 5 I

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x.

(a)



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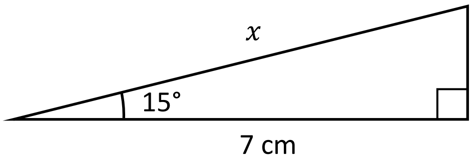

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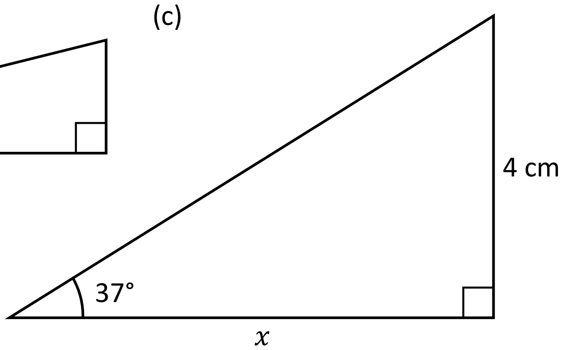

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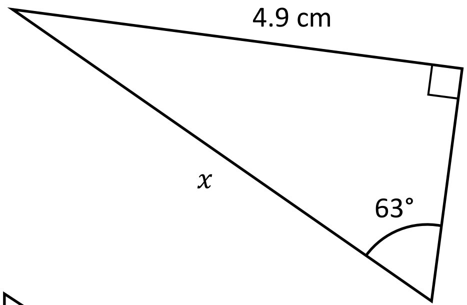

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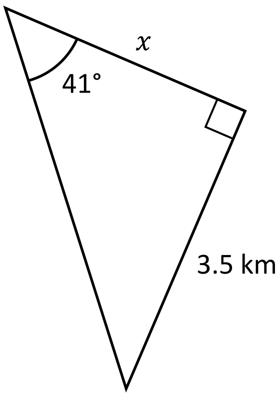

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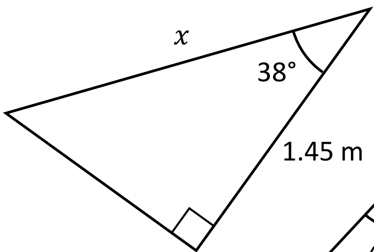

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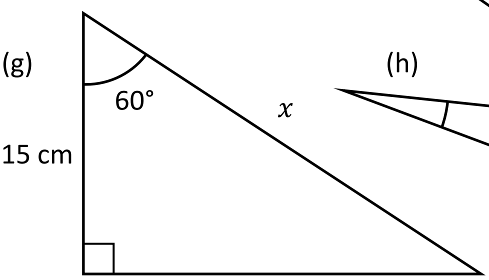

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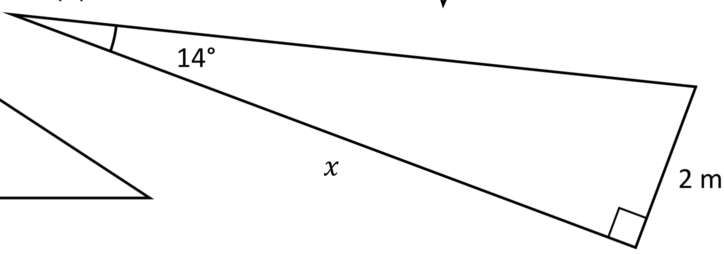

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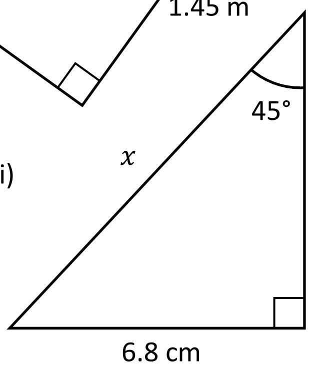
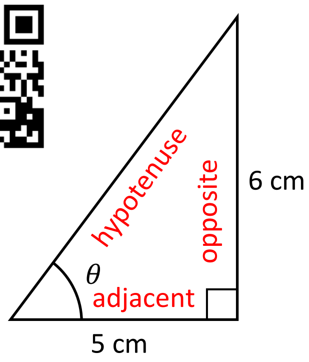


Finding angles using trigonometry



Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the size of the angle θ.

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we know the length of the **opposite** (6 cm) and the length of the **adjacent** (5 cm). The trigonometric ratio that uses the words **opposite** and **adjacent** is tan, therefore we must use the formula

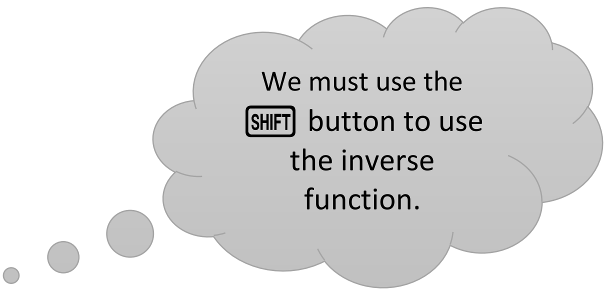
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

in this question. By substituting values into the equation, we obtain

$$\tan\theta = \frac{6}{5}.$$

To find the size of the angle θ we must use the inverse tan function:

$$\theta = \tan^{-1}\left(\frac{6}{5}\right).$$



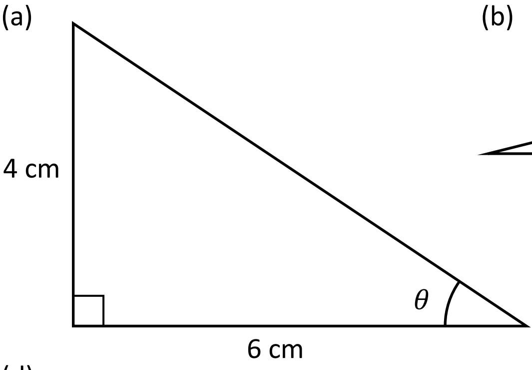
By typing this sum on a calculator, we find that $\theta = 50.19°$, correct to two decimal places.

Exercise 6

I

For the following right-angled triangles, calculate the size of the angle θ.

(a)



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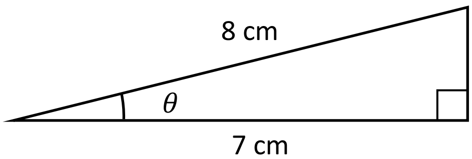

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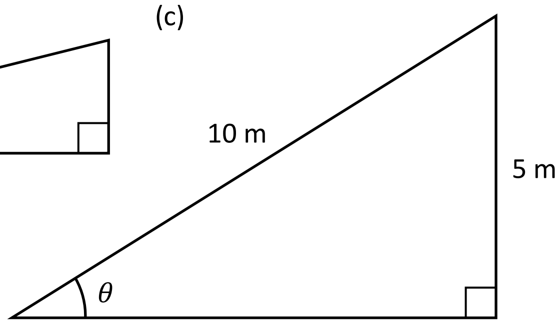

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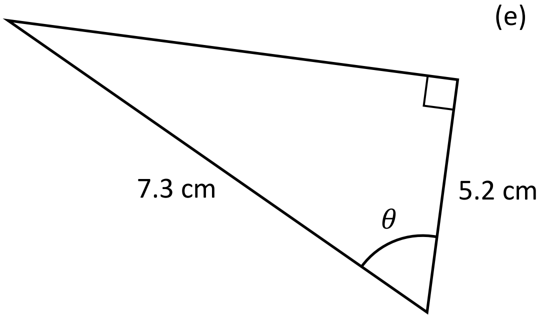

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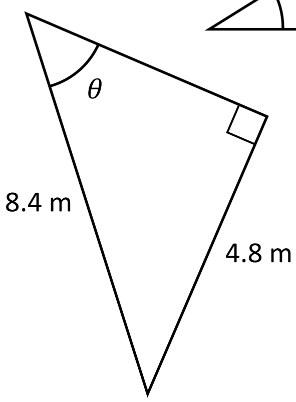

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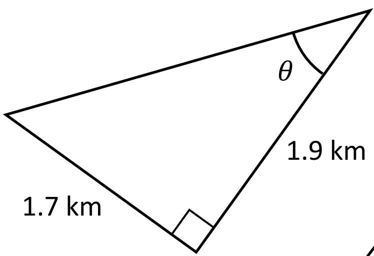

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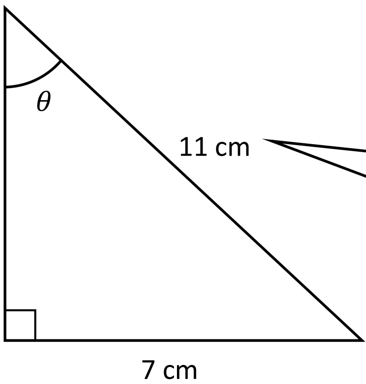

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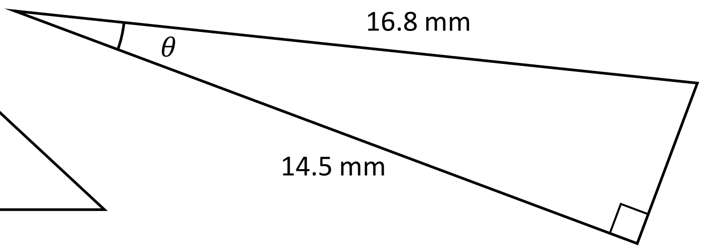

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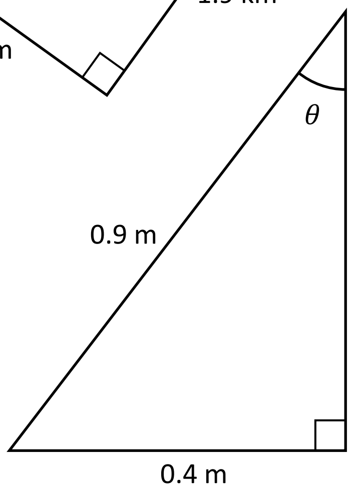
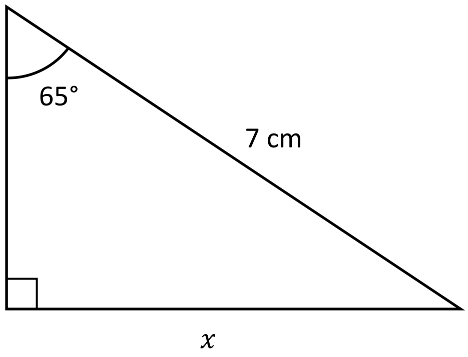


Exercise 7

I

For the following right-angled triangles, calculate the length of the side x, or the size of the angle θ.
Round off your answers correct to two decimal places.

(a)



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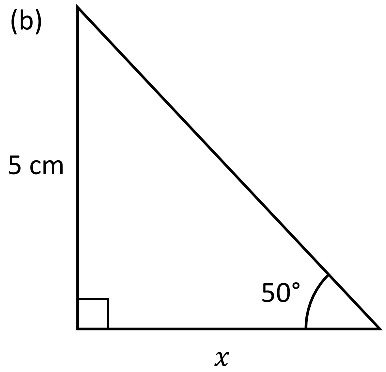

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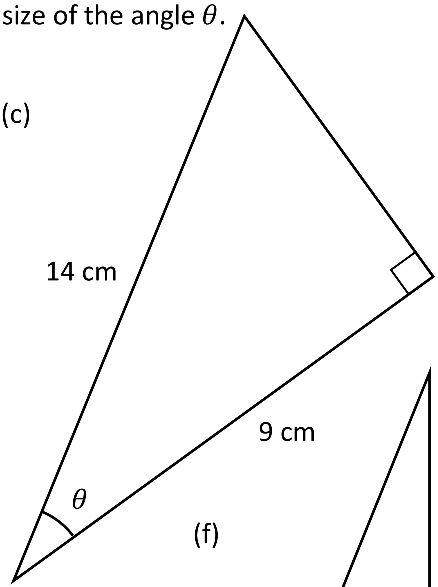

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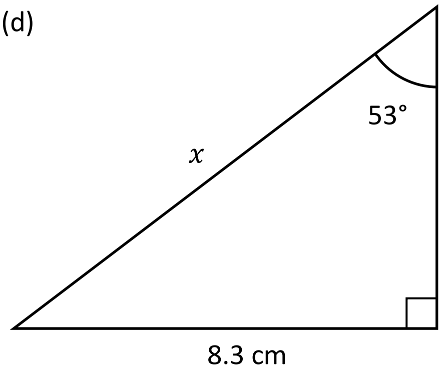

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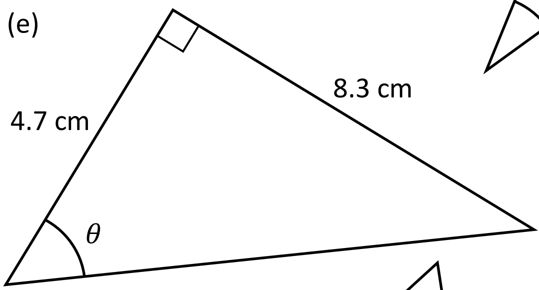

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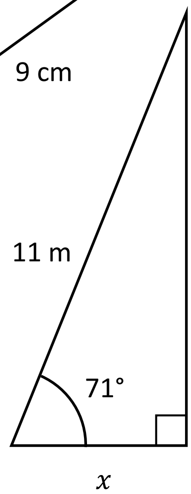

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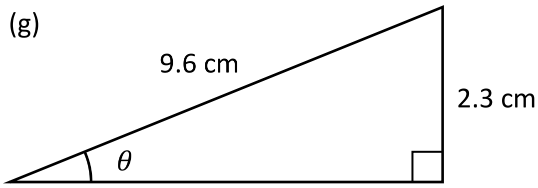

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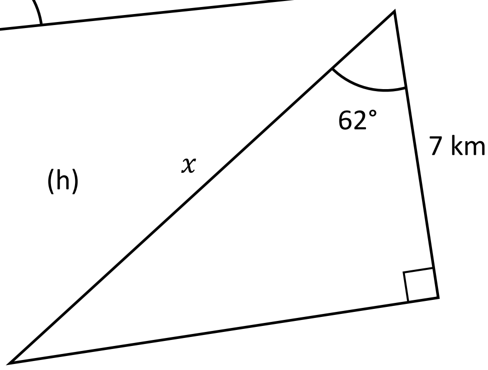

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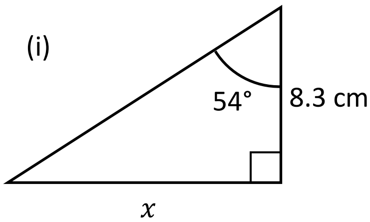
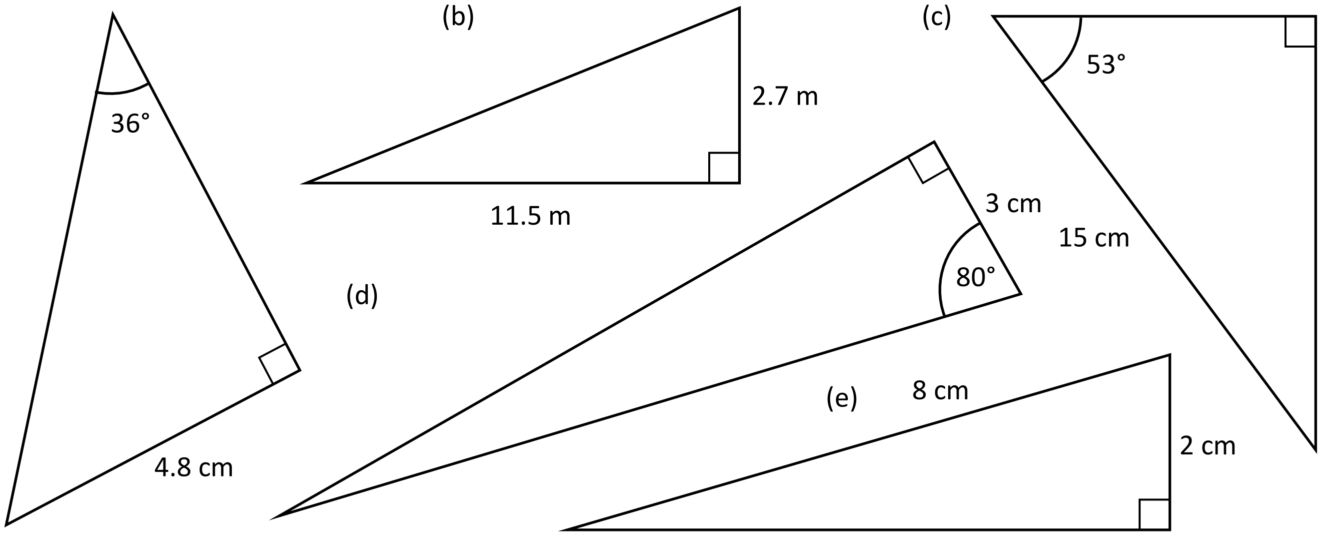


Exercise 8

For the following right-angled triangles, find the size of **every** missing angle and the length of **every** missing side.

(a) (b) (c)



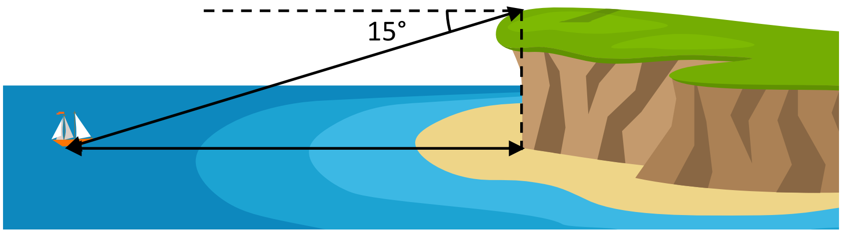
Exercise 9

Applying I

(a)

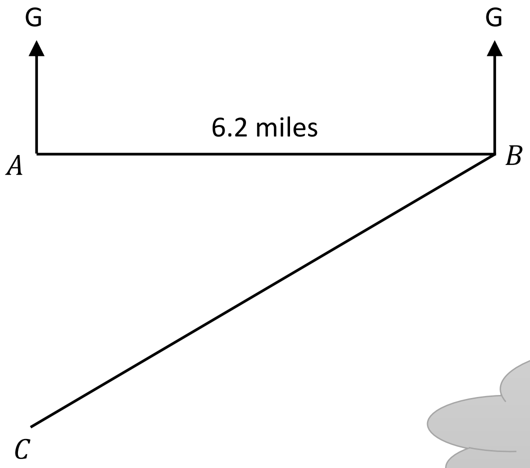
The vertical height of a tree is 3.2 metres. The horizontal distance from point A to the bottom of the tree is 7 metres. Calculate the angle of elevation of the top of the tree from the point A.

(b)



From the top of a vertical cliff, the angle of depression of a sailing boat is 15°. If the sailing boat is 200 m from the bottom of the cliff, calculate the height of the cliff above sea level.

(c)



A ship leaves a port and sails 6.2 miles at a bearing of 090° to reach B. Then it turns and sails at a bearing of 224° until it reaches a point C, which is south of the port A. Calculate the distance between the point C and the port A.

The word trigonometry comes from the Greek language: "*trigon*" means triangle and "*metry*" means measure.

Evaluation

Key Words	Further Questions	What went well?	To reach my target grade I will...
			Grade ☐ Target ☐

Enlargement

Enlargement is one of the four transformations.

Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	**Enlargement**

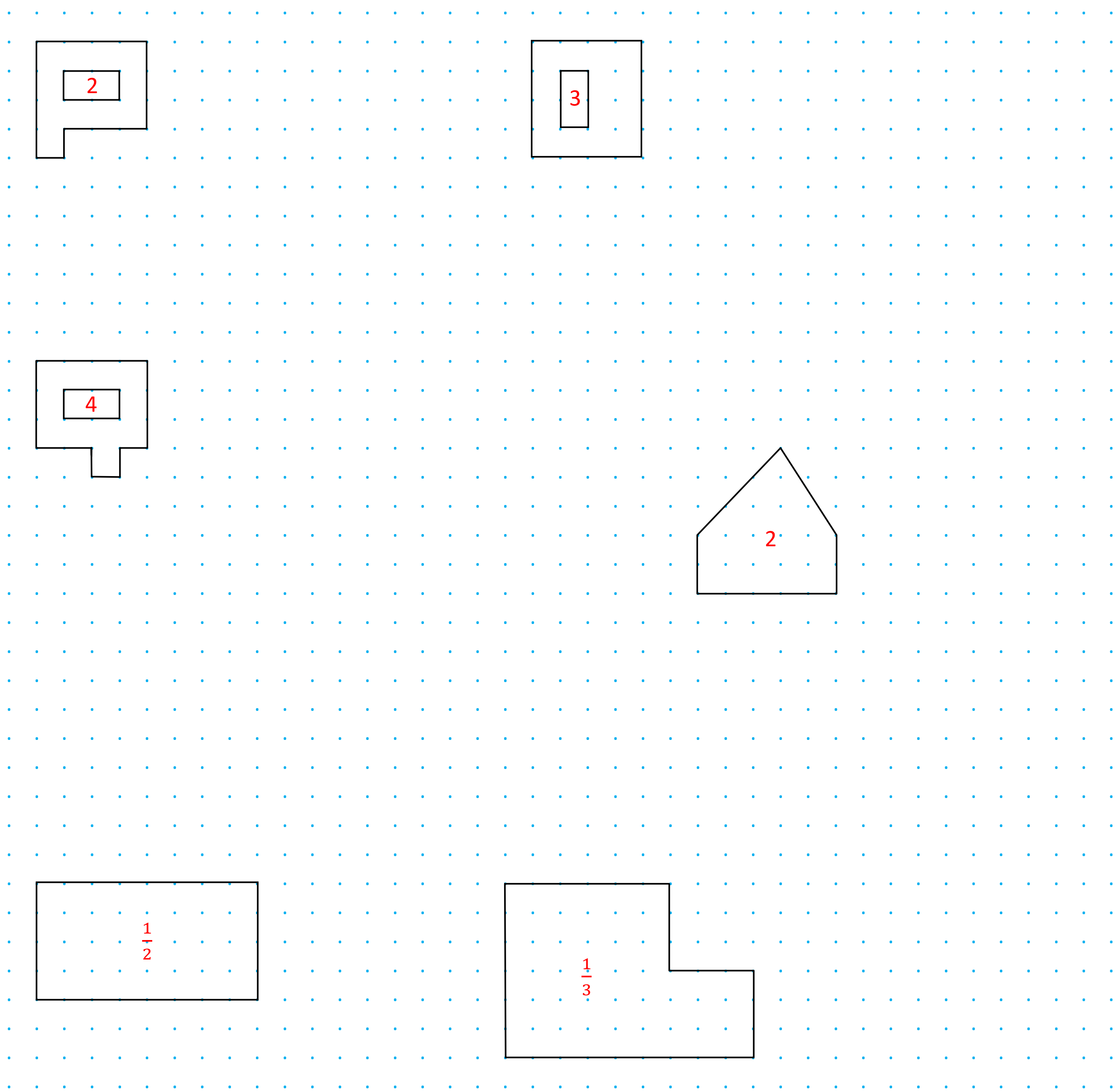
When a shape is enlarged, the **size** of the shape changes. The **scale factor** decides how the shape changes. For example, if the scale factor is 2, then the size of the shape doubles. If the scale factor is $\frac{1}{2}$, then the size of the shape halves.

Skill

Exercise 10

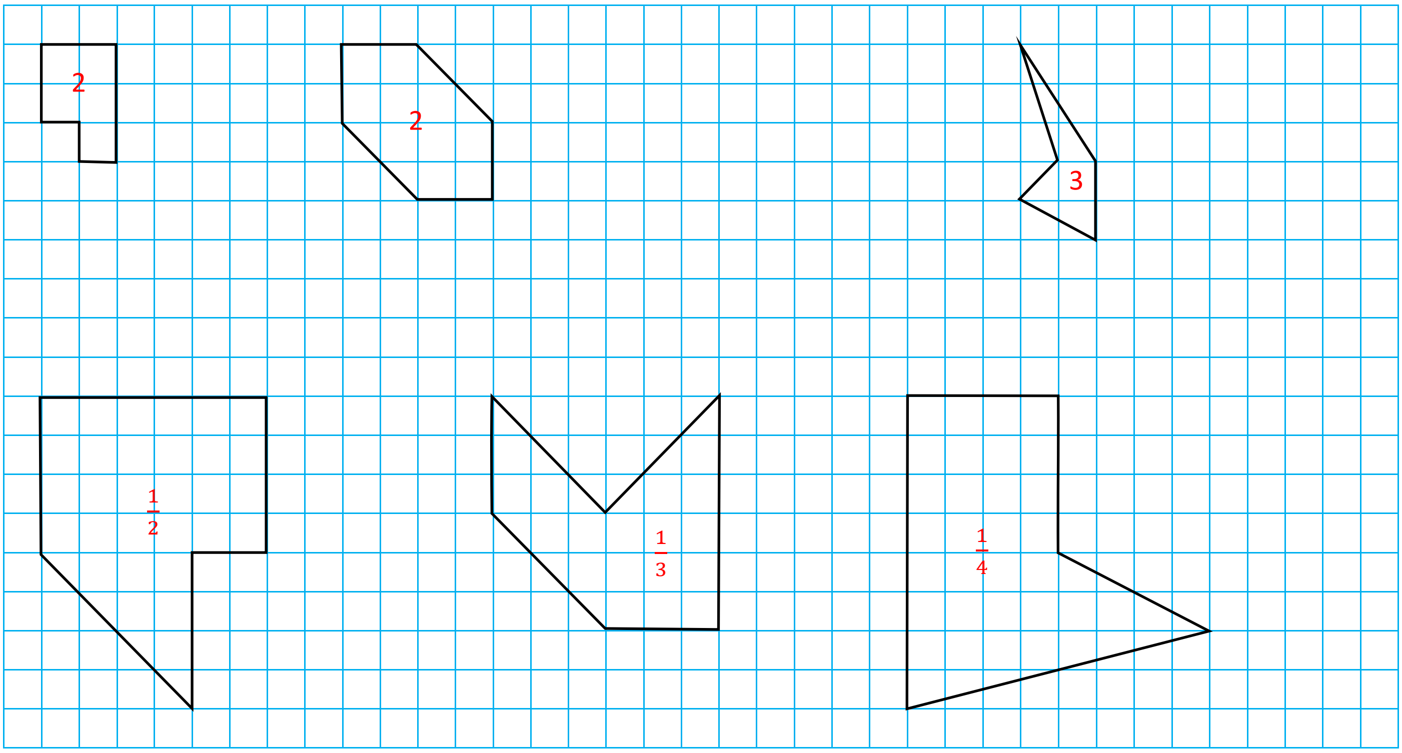
Enlarge the following shapes using the scale factor that is given in the centre of each shape.

I



Exercise 11

Enlarge the following shapes using the scale factor that is given in the centre of each shape.

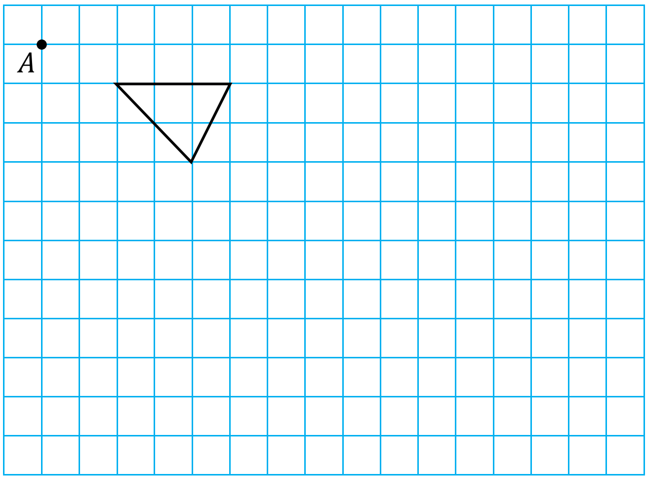


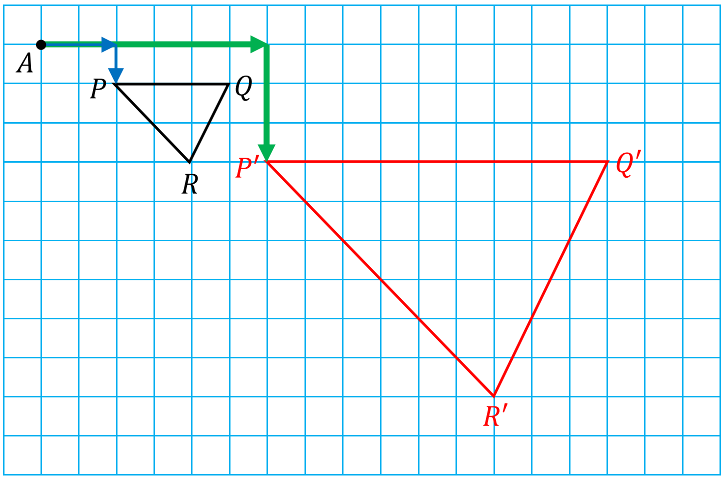
Centre of Enlargement

If a question states a point as the **centre of enlargement,** then the enlargement must appear in a certain location (it cannot appear *anywhere* like in Exercises 10 and 11).

Example

Enlarge the following triangle using a scale factor of 3 and the point A as the centre of enlargement.



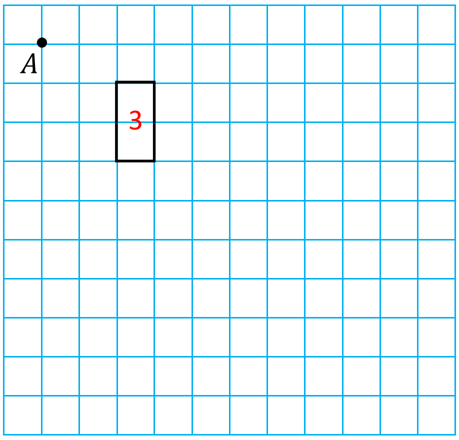


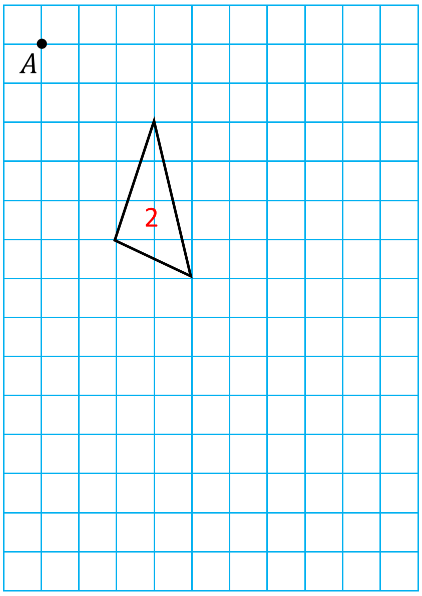
To go from the point A to the vertex P in the original triangle, we must go 2 units to the right and 1 unit down. Since the scale factor is 3, to go from the point A to the vertex P' in the new triangle, we must go $2 \times 3 = 6$ units to the right, and $1 \times 3 = 3$ units down. We can repeat this with the other vertices (Q and R), or you can start at the vertex P' and draw a triangle that is three times larger.

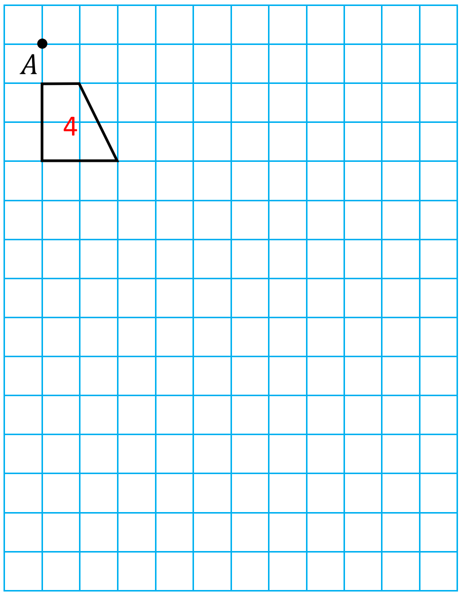
Exercise 12

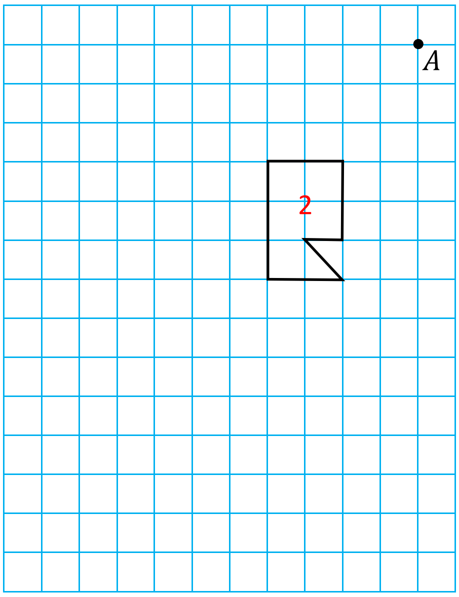
I

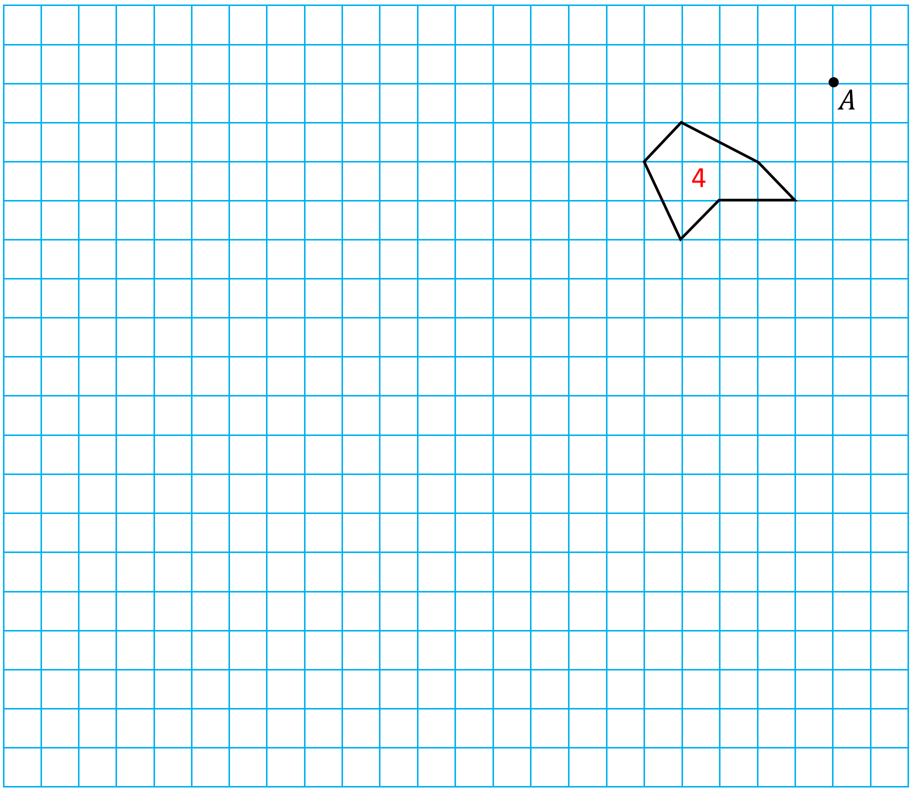
Enlarge the following shapes using the point A as the centre of enlargement and the number in the centre of each shape as the scale factor.

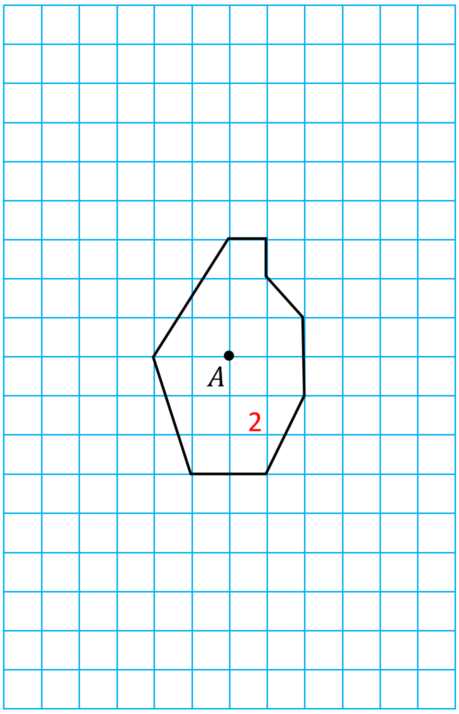


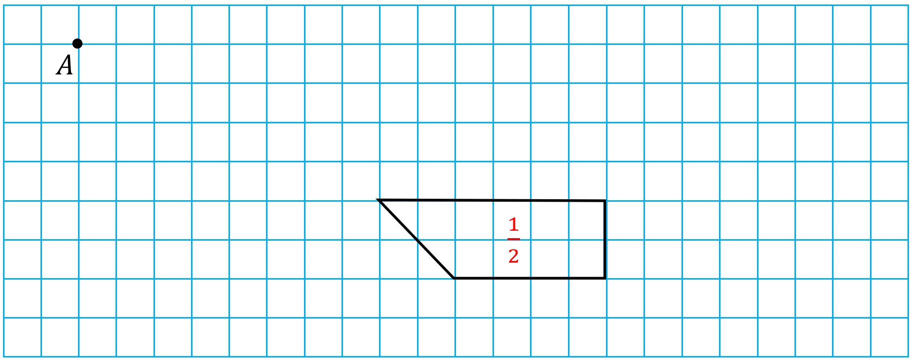




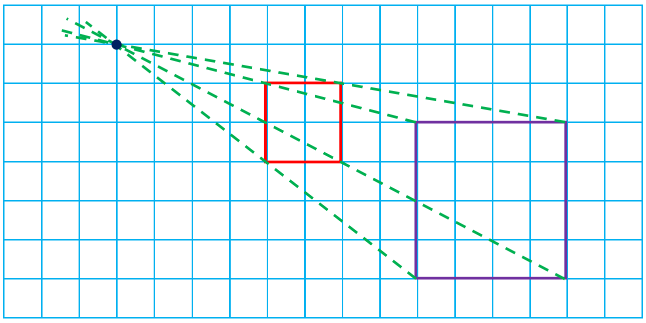








Finding the centre of enlargement

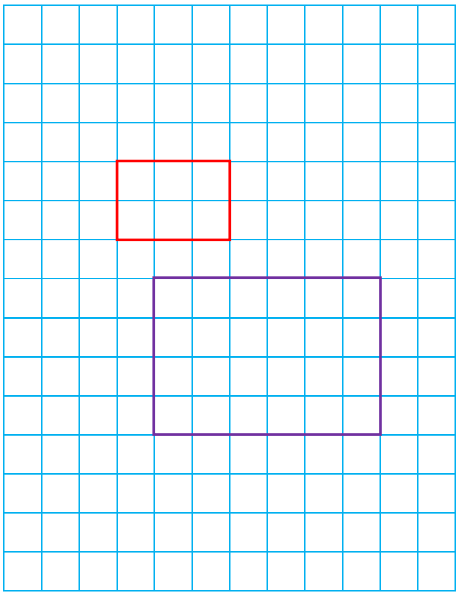


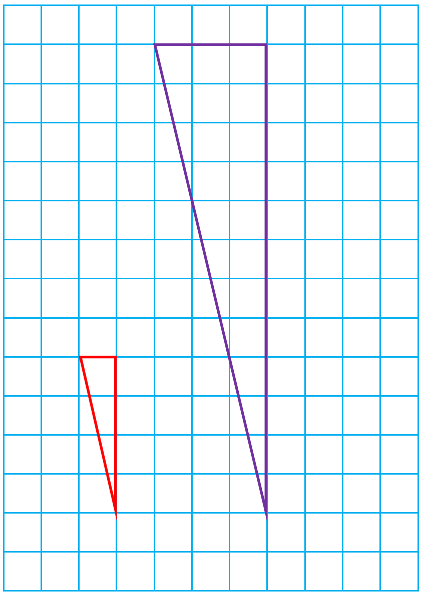
Given a shape and its enlargement, connect the corresponding vertices and extend the lines to find the location of the centre of enlargement.

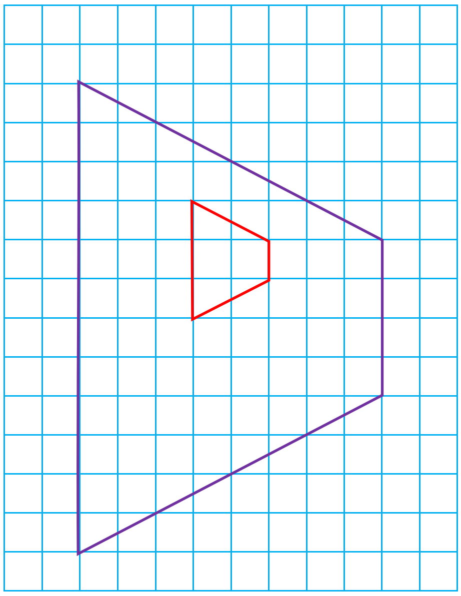
You can find the scale factor by comparing the sizes of the shapes.

Exercise 13

Find the scale factor and the centre of enlargement for the following enlargements.



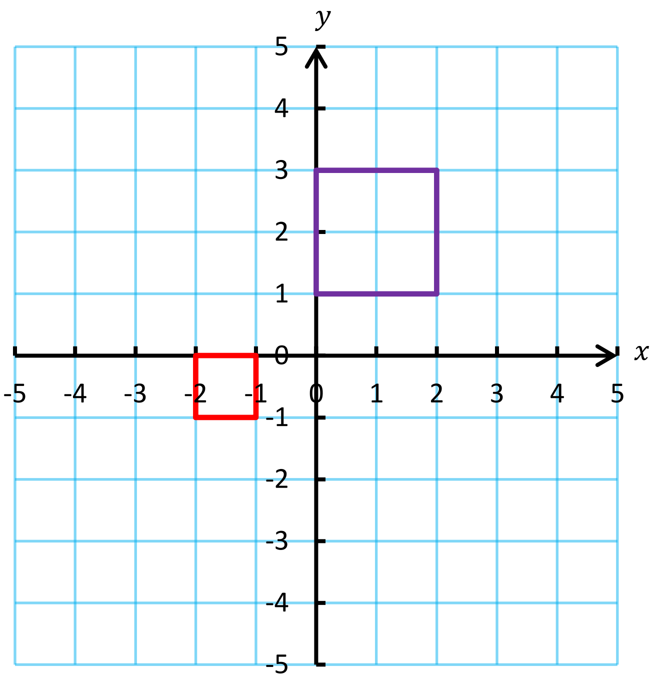




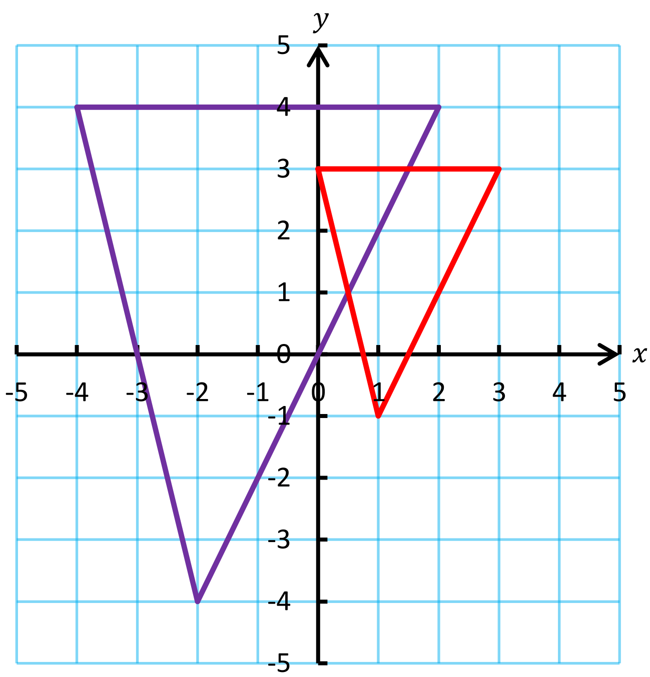
Exercise 14

Find the scale factor and the centre of enlargement for the following enlargements.

(a)



(b)



Negative Scale Factor

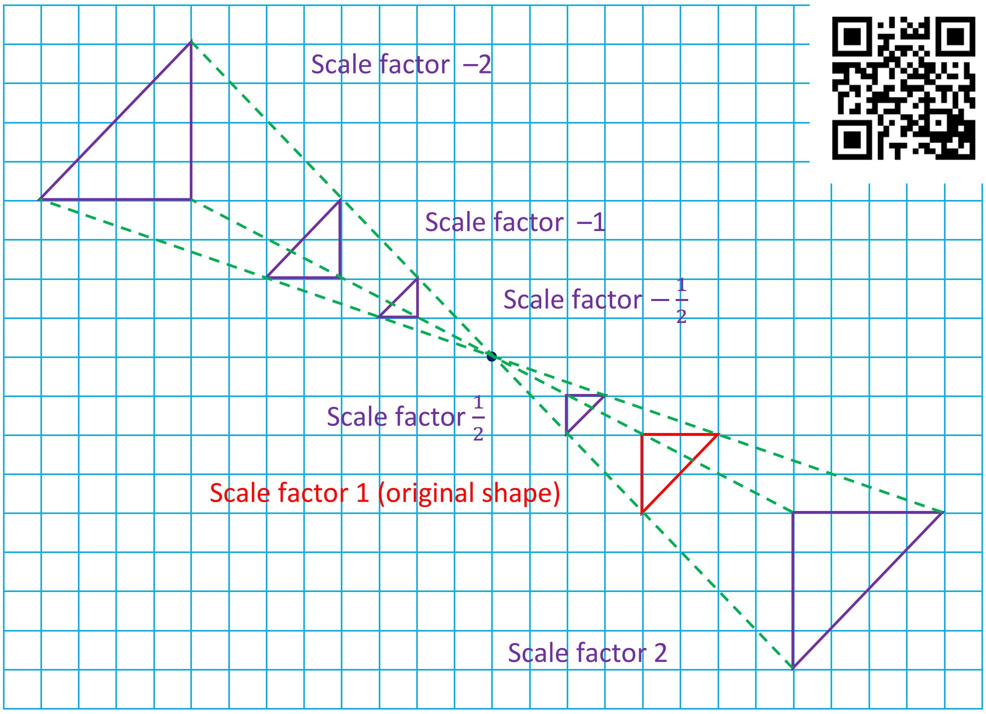
Higher Tier

A **negative** scale factor means working from the centre of enlargement to the opposite direction.

For example, consider the diagram on the right. To go from the centre of enlargement to the top left vertex of the original triangle, we must go 4 units right and 2 units down.

With a scale factor of 2, we must go $4 \times 2 = 8$ units right and $2 \times 2 = 4$ units down.

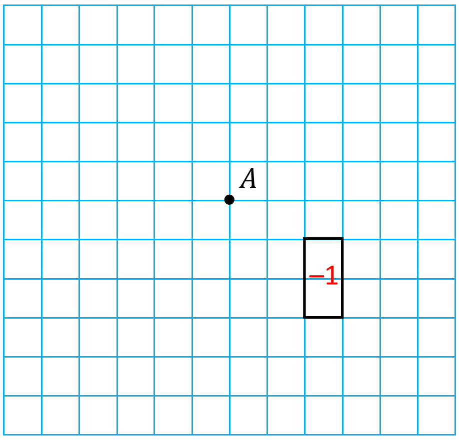
With a scale factor of –2, we must go $4 \times 2 = 8$ units **left** and $2 \times 2 = 4$ units **up**.

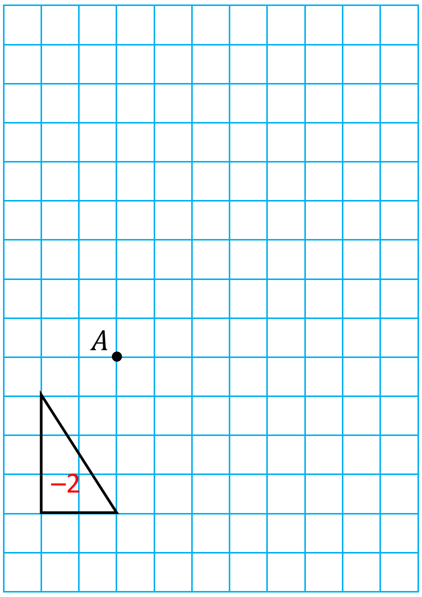


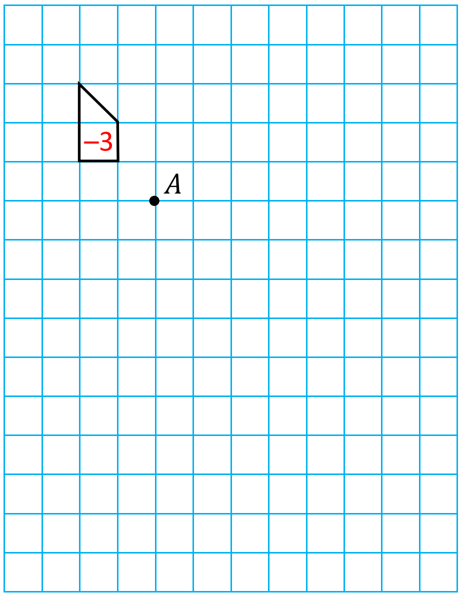
Exercise 15

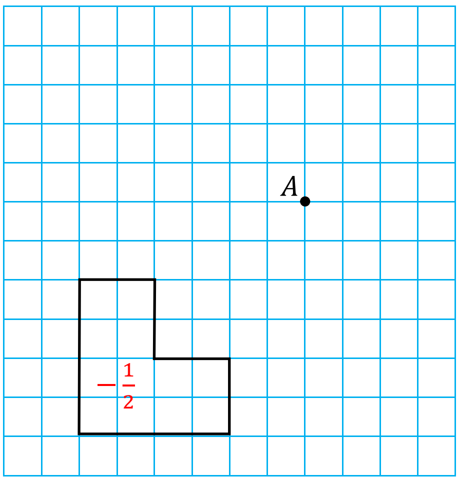
H

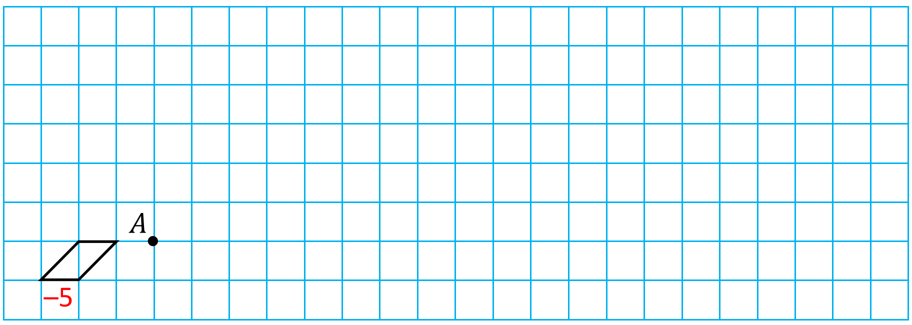
Enlarge the following shapes using the point A as the centre of enlargement and the number in the centre of each shape as the scale factor.







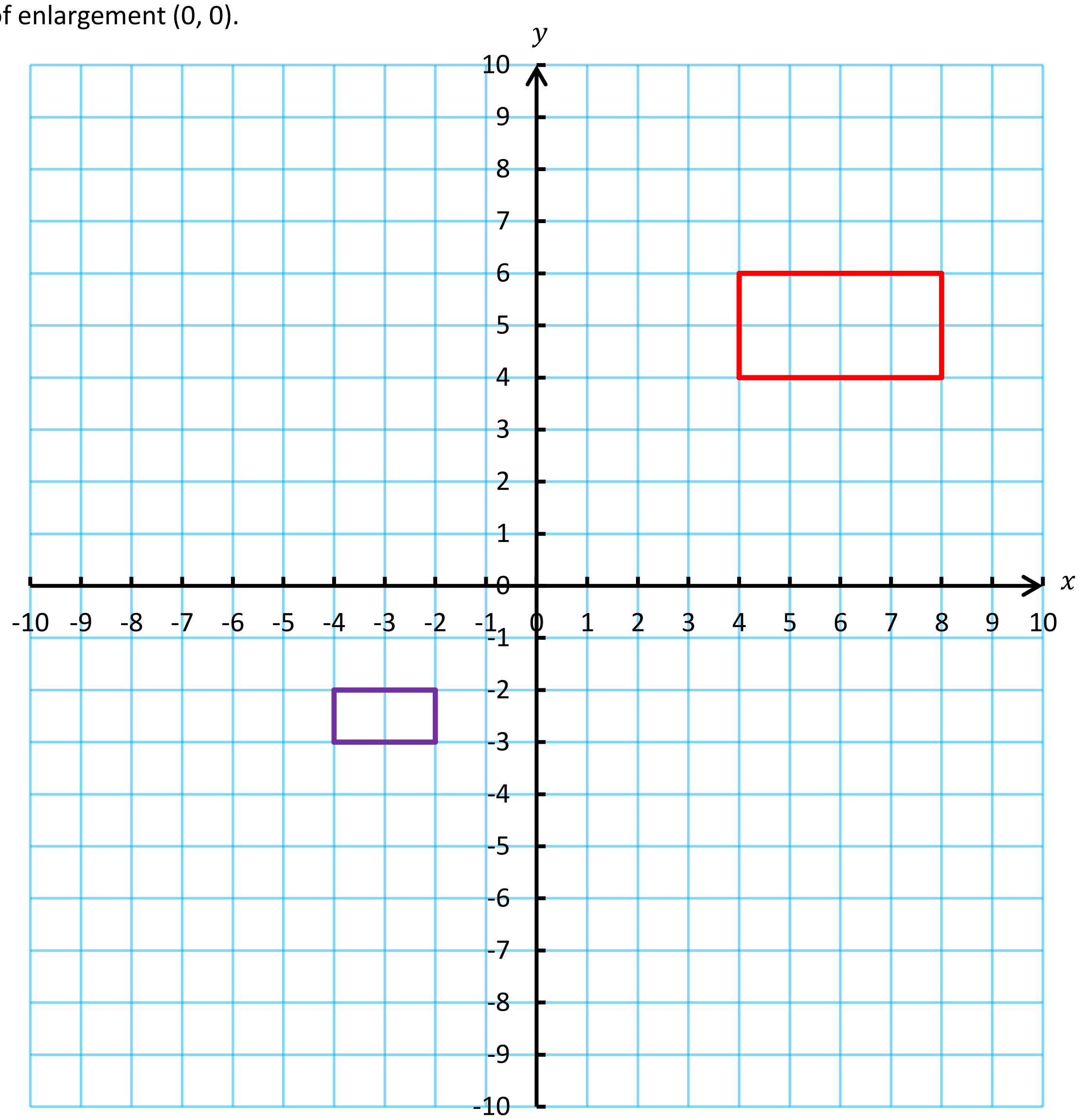




Exercise 16 H

The larger rectangle is transformed to the smaller rectangle. The co-ordinates of the centre of enlargement are (0, 0). Complete the following sentence to fully describe this transformation.

The transformation from the larger rectangle to the smaller rectangle is an enlargement using scale factor ________ and centre of enlargement (0, 0).



Evaluation

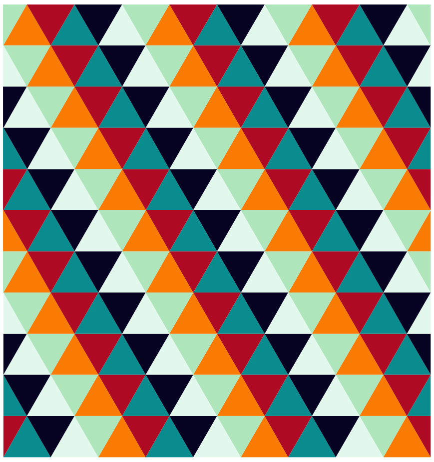
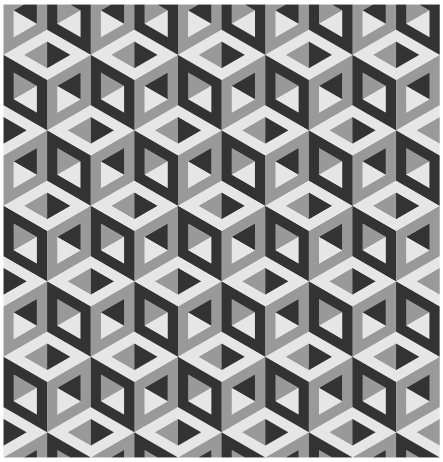
Key Words	Further Questions	What went well?	To reach my target grade I will...
			Grade ☐ Target ☐

Tessellations

A **tessellation** involves repeating a shape (or a number of shapes) so that they fill the space entirely, without leaving any gaps. You can translate, rotate or reflect shapes to create a tessellation.

Example

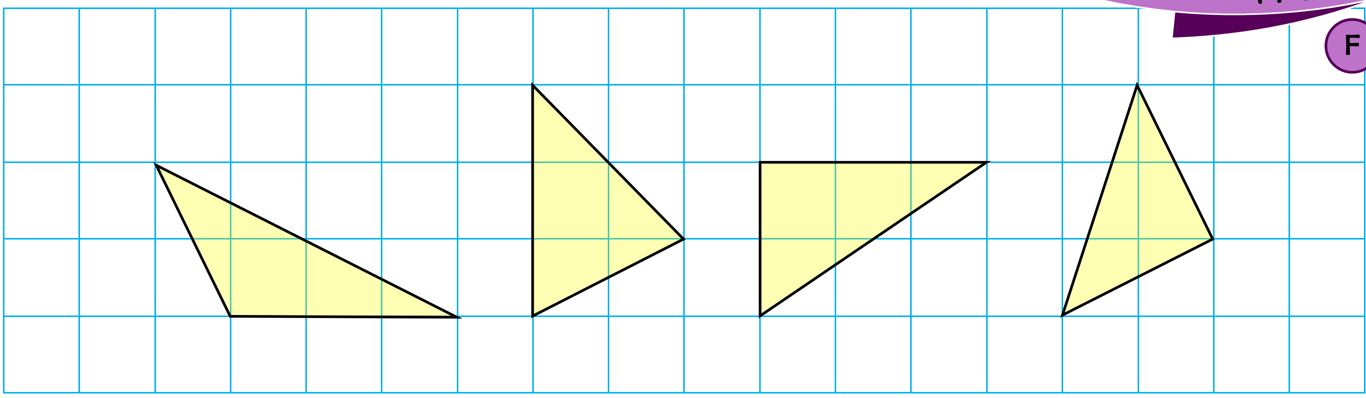
The following pictures show examples of tessellations.

Exercise 17

Applying

F



Choose one of the triangles above. Using the squared paper below, tessellate the triangle to create a tiled pattern. Colour your design, using no more than three colours.

Did you know? The artist M.C. Escher used tessellations in his art.

Extension

Challenge! ⚠

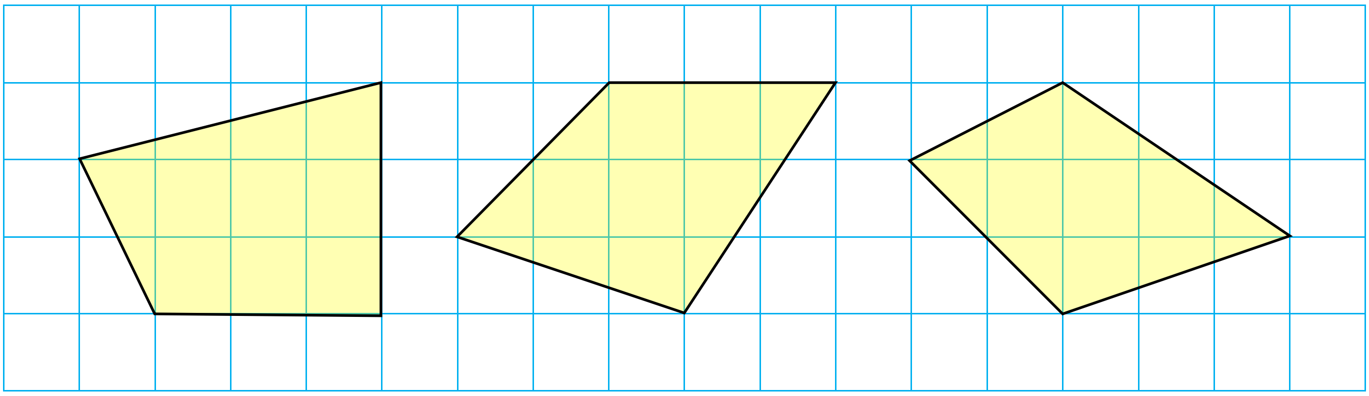
Does every triangle tessellate? If not, give an example of a triangle that does not tessellate. If so, try to show how any triangle will tessellate.

Applying F

Exercise 18

Investigate different tessellations using the MAT tiles.

Exercise 19



Choose one of the above quadrilaterals. Using the squared paper below, tessellate your quadrilateral to form a tiled pattern. Colour your design, using no more than four colours.

Evaluation

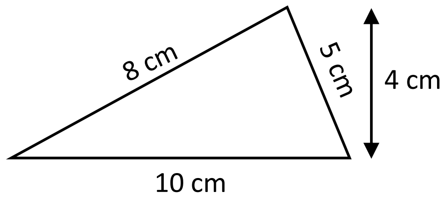
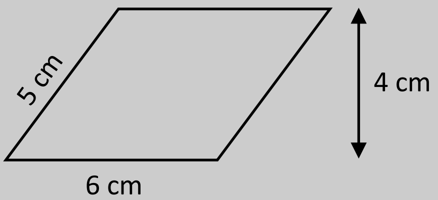
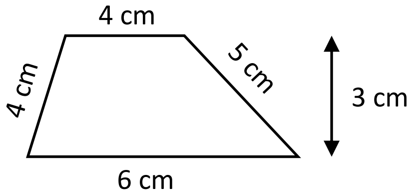
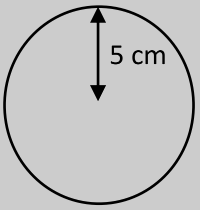
Key Words	Further Questions	What went well?	To reach my target grade I will…
			Grade ☐ Target ☐

Composite Shapes

Revision F

Exercise 20

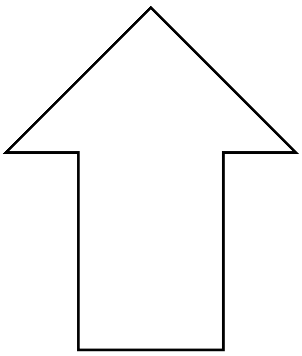
Complete the following table.

Shape	Name of the shape	Formula to find the area of the shape	Calculate the area of the shape
			
			
			
			
			

Exercise 21

Calculate the perimeter of each shape in the above table.

Composite Shapes

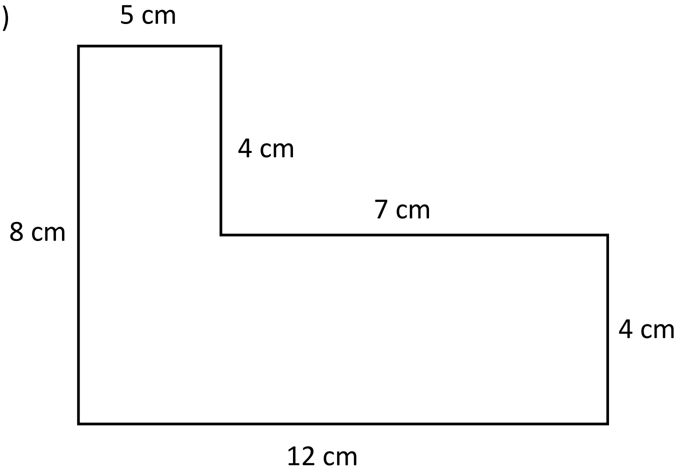


Composite shapes are shapes that you can split into simpler shapes, like the shapes in the above table. For example, the shape on the right is a composite shape – it is possible to split the shape into a rectangle (at the bottom) and a triangle (at the top). We can calculate the area of the composite shape by adding the area of the rectangle to the area of the triangle.

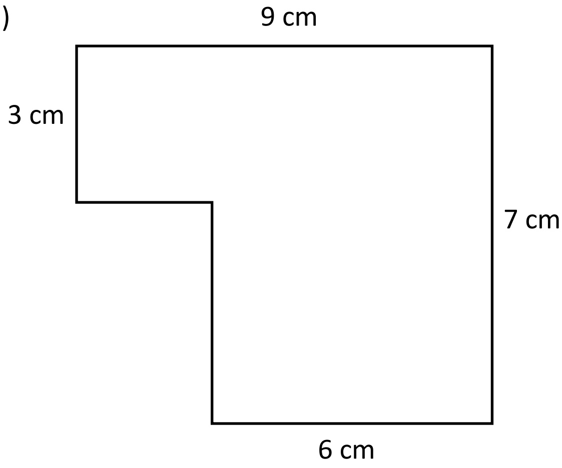
Exercise 22

Calculate the area of each of the following composite shapes.

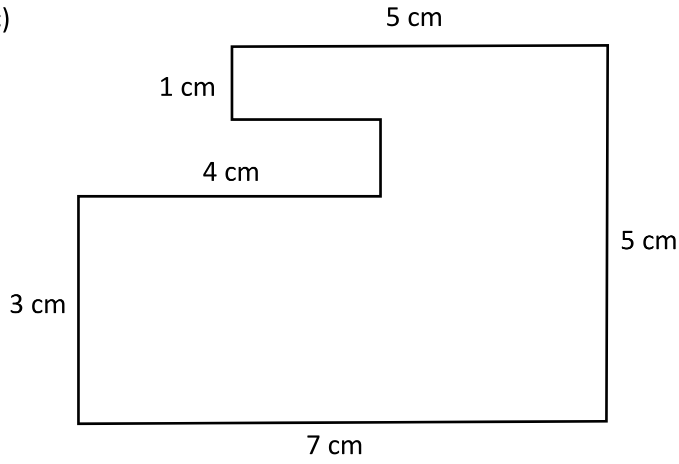
(a)



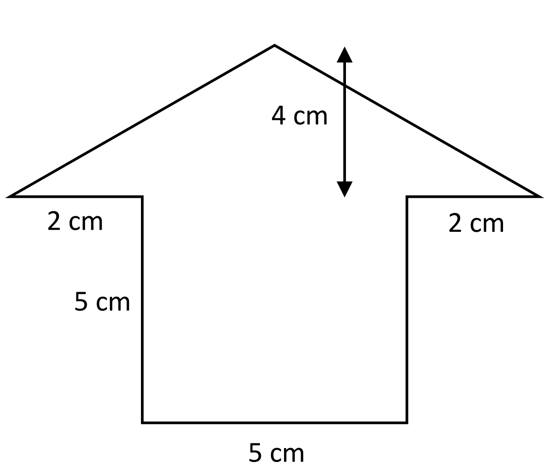
(b)



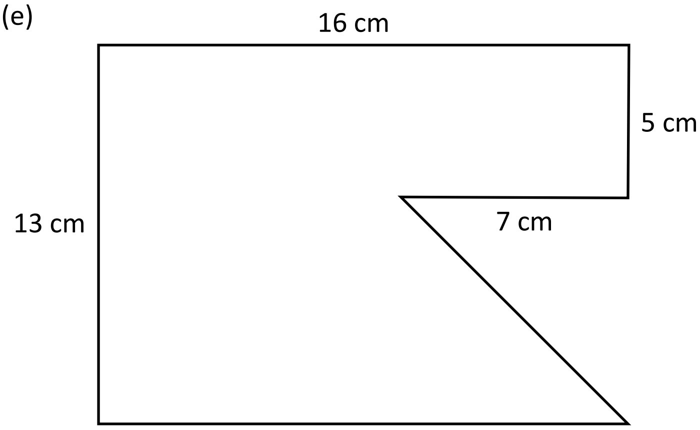
(c)



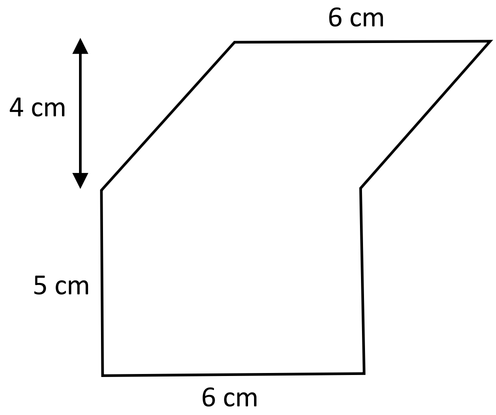
(d)



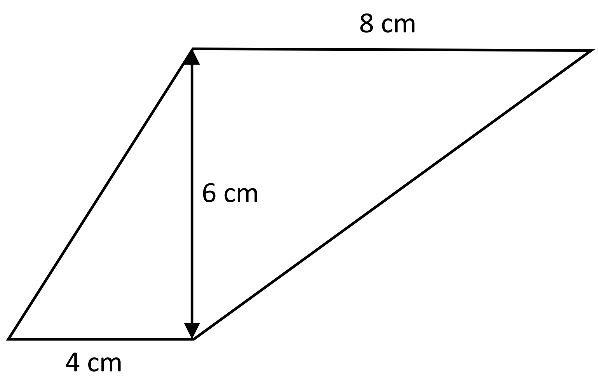
(e)



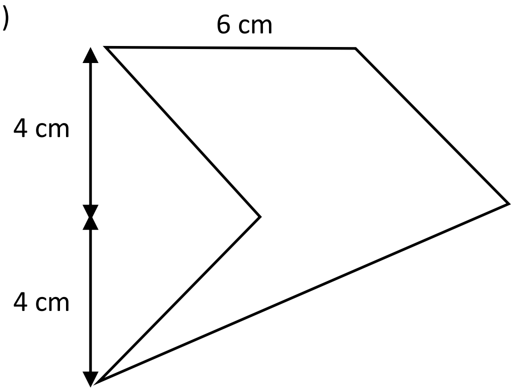
(f)



(g)



(h)

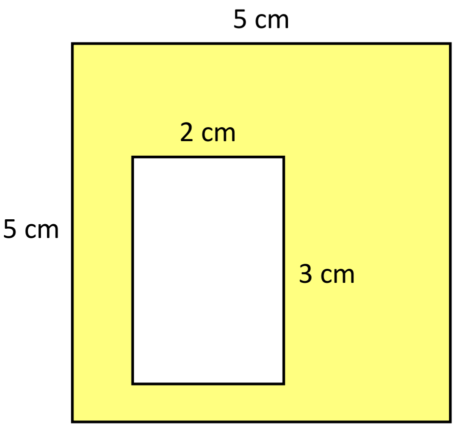


Exercise 23

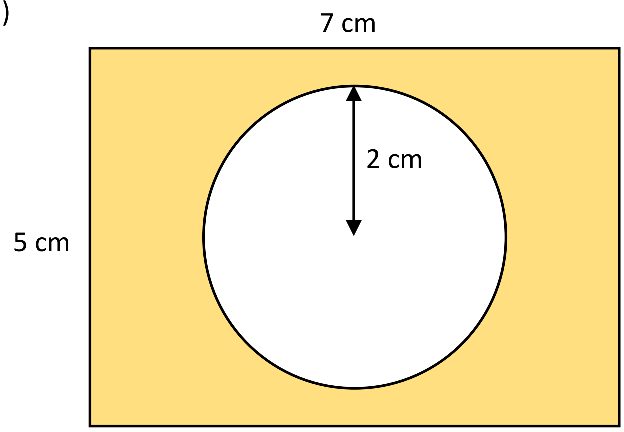
F

Calculate the area of the coloured region.

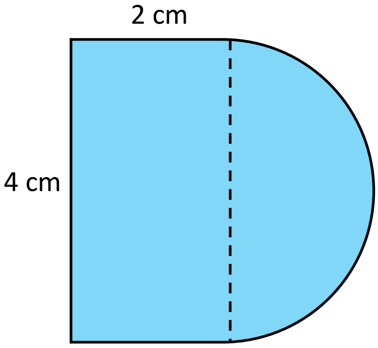
(a)



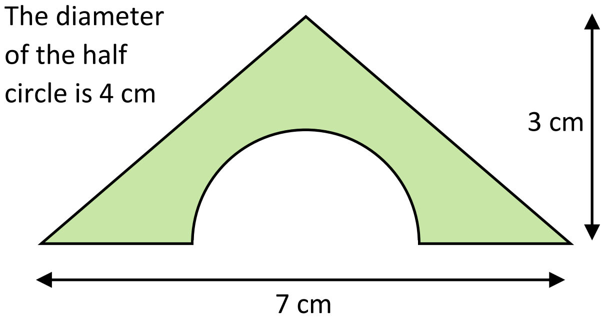
(b)



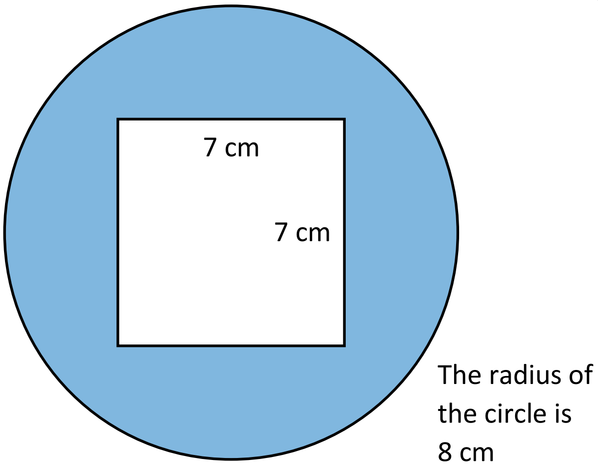
(c)



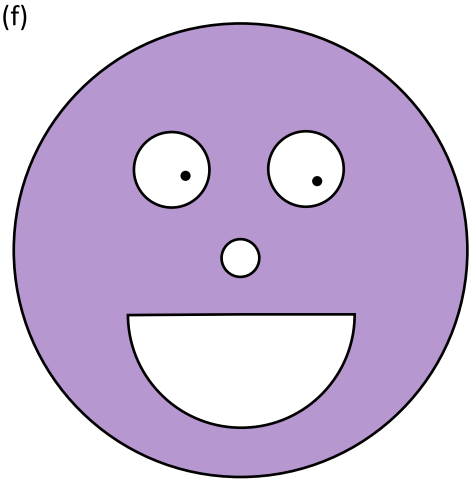
(d)



(e)

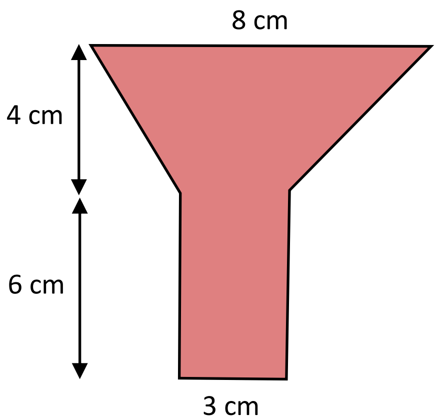


(f)



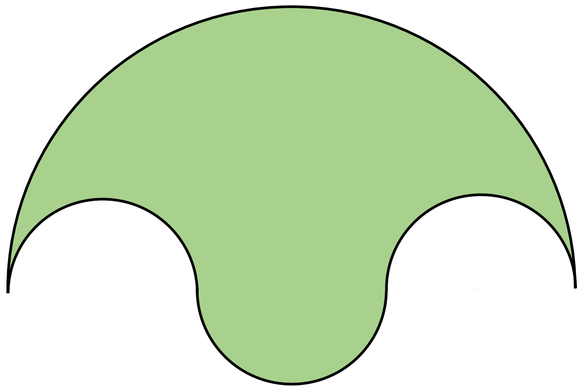
Circle	Radius
Face	6 cm
Eye	1 cm
Nose	0.5 cm
Mouth	3 cm

(g)



(h)

Circle	Radius
Large (1)	7.5 cm
Small (3)	2.5 cm



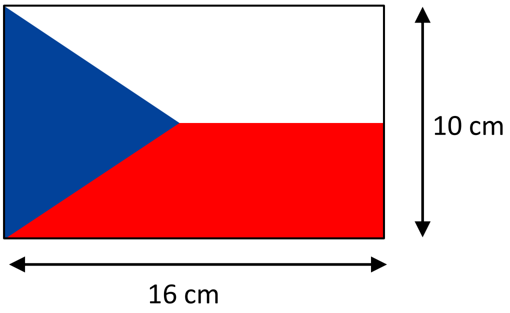
Exercise 24

F

Calculate the area of each colour in the following flags.

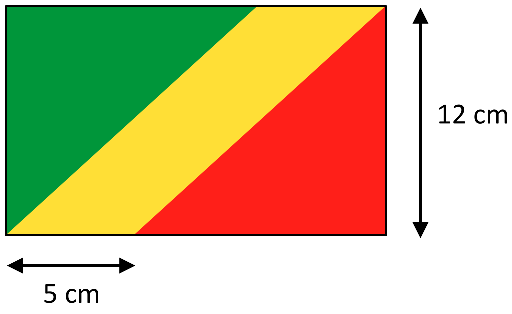
(a) *Czech Republic*

The vertex of the blue triangle is in the centre of the flag.



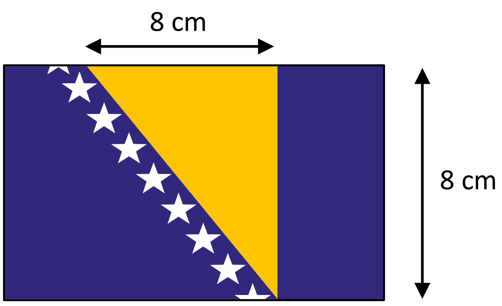
(b) *Republic of the Congo*

The green and red triangles have the same area. The base of the triangle is double the width of the parallelogram.



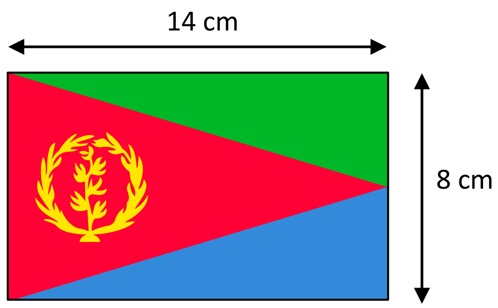
(c) *Bosnia Herzegovina*

The area of each white star is 2 cm². The area of the yellow triangle is $\frac{1}{4}$ of the area of the whole flag.



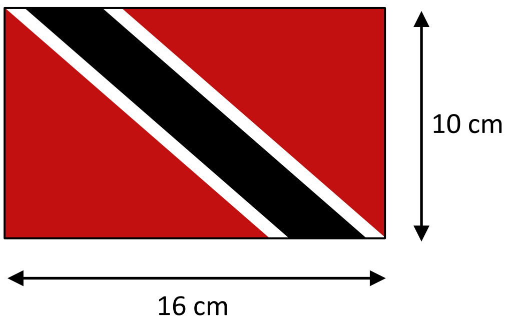
(d) *State of Eritrea*

The green and blue triangles have the same area.
The yellow picture has an area of 12 cm².



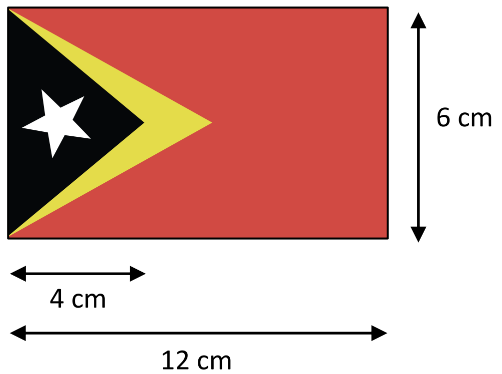
(e) *Republic of Trinidad and Tobago*

The width of the white stripe is 1 cm.
The width of the black stripe is 4 cm.



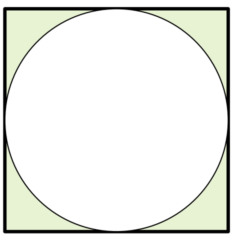
(f) *Democratic Republic of Timor-Leste*

The area of the white star is 4 cm².
The vertex for the yellow triangle is in the centre of the flag.



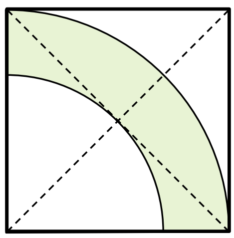
Challenge! ⚠

Which fraction of the square is shaded?



Challenge 2! ⚠

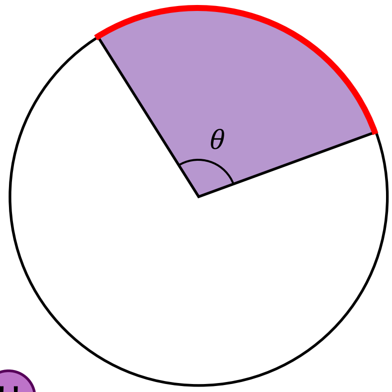
Which fraction of the square is shaded?



Higher Tier

Length of an Arc and the Area of a Sector

The length of an arc is a fraction of the circumference of a circle, whilst the area of a sector is a fraction of the area of the circle.



Arc Length = $\frac{\theta}{360^\circ} \times$ circle circumference

Area of a Sector = $\frac{\theta}{360^\circ} \times$ circle area

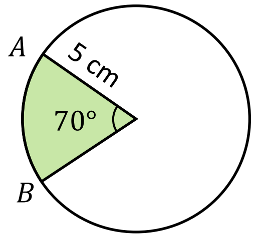
Arc Length = $\frac{\theta}{360^\circ} \times \pi \times$ diameter

Area of a Sector = $\frac{\theta}{360^\circ} \times \pi \times$ radius^2

Exercise 25

H

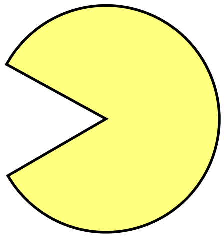
For the circle shown on the right,



(a) Calculate the length of the minor (*smaller*) arc AB.
(b) Calculate the area of the minor sector AB.
(c) Calculate the length of the major (*larger*) arc AB.
(d) Calculate the area of the major sector AB.
(e) What fraction of the circle is shaded in green? Give your answer in its simplest form.

Exercise 26

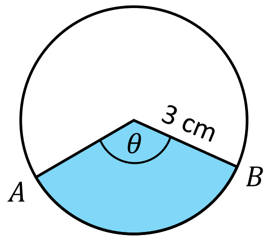
Pacman, the computer game character, is in the shape of a sector.
The angle in the centre of the shape is 300°. If the radius of Pacman is 6 cm,



(a) What is the area of Pacman?
(b) What is the perimeter of Pacman?

Exercise 27

The length of the minor arc AB in the diagram on the right is 7 cm.



(a) What is the size of the angle θ?
(b) Calculate the area of the blue sector.

Evaluation

Key Words	Further Questions	What went well?	To reach my target grade I will…
			Grade ☐ Target ☐