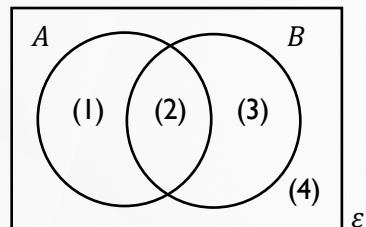


Unit 4 WJEC

Revision Sheet

STATISTICS

Probability



- (1) $P(A \cap B')$.
- (2) $P(A \cap B)$.
- (3) $P(A' \cap B)$.
- (4) $P(A' \cap B')$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$P(A') = 1 - P(A).$$

Independent events:

$$P(A \cap B) = P(A) \times P(B).$$

Mutually exclusive events:

$$P(A \cap B) = 0.$$

Dependent events:

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B).$$

Conditional events:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ (if } P(B) > 0 \text{)}.$$

Tree diagrams: 'AND' $\Rightarrow$ multiplication; 'OR' $\Rightarrow$ addition.

Two-way tables.

Modelling with probability.

Statistical Distributions

The Continuous Uniform Distribution

If $X \sim U[a, b]$ then

$$f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b;$$

$$f(x) = 0 \text{ otherwise.}$$

$$E(X) = \frac{1}{2}(a + b).$$

$$\text{Var}(X) = \frac{1}{12}(b - a)^2.$$

$$P(c \leq X \leq d) = \frac{d-c}{b-a}.$$

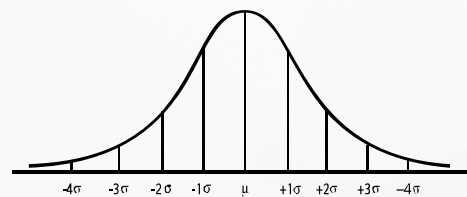
Conditions

- (a) The distribution contains continuous data.
- (b) Any intervals of equal length have the same probability of occurring, between a minimum value a and a maximum value b .

Normal Distribution

If a random variable X has a normal distribution, then $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ is the standard deviation.

The transformation $Z = \frac{X - \mu}{\sigma}$ changes any $X \sim N(\mu, \sigma^2)$ to $Z \sim N(0, 1)$. (Mean 0; Standard Deviation 1.)



$$P(Z < -a) = P(Z > a) = 1 - P(Z < a).$$

You can use a calculator or statistical tables to find the probabilities of the normal distribution.

Conditions

- (a) The distribution contains continuous data.
- (b) The distribution is bell-shaped and symmetrical about the mean, which is the largest value.
- (c) The mean, median and mode are more or less equal.
- (d) The data almost entirely lies within 3 standard deviations of the mean.

Selecting a probability distribution for a context.

- Discrete uniform distribution.
- Binomial distribution.
- Poisson distribution.
- Continuous uniform distribution.
- Normal distribution.

Recognising when a model is appropriate, or not.

Statistical Hypothesis Testing

Correlation Coefficients

- Based on a population: ρ .
- Based on a sample: r .
- The coefficient has a value between -1 (perfect negative correlation) and 1 (perfect positive correlation). 0 means no correlation.

Testing for positive correlation:

$$H_0: \rho = 0 \text{ against } H_1: \rho > 0.$$

Testing for negative correlation:

$$H_0: \rho = 0 \text{ against } H_1: \rho < 0.$$

Testing for any correlation:

$$H_0: \rho = 0 \text{ against } H_1: \rho \neq 0.$$

Mean of a Normal Distribution

If a random variable X follows a normal distribution, then $X \sim N(\mu, \sigma^2)$ and the sample mean, $\bar{X}$, is also normally distributed: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

(n is the size of a sample.)

The test statistic is $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$.

We can use this z -value to find critical values or critical regions.

MECHANICS AND DIFFERENTIAL EQUATIONS

Trigonometry

Problems in mechanics can include trigonometric functions, e.g.

$$a = 12 \sin(3t) - 8 \cos(2t).$$

Differential Equations

If the rate of increase of A at time t is directly proportional to A , then

$$\frac{dA}{dt} \propto +A \text{ or } \frac{dA}{dt} = kA.$$

If the rate of decrease of A at time t is directly proportional to A^2 , then

$$\frac{dA}{dt} \propto -A^2 \text{ or } \frac{dA}{dt} = -A^2.$$

Including links to kinematics.

Quantities and Units

Derived units and quantities for moments.

Kinematics

Displacement: $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$.

$$\text{Distance} = \sqrt{x^2 + y^2}.$$

$$\text{Velocity: } \mathbf{v} = \frac{d\mathbf{r}}{dt} \text{ or } \mathbf{v} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}.$$

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.$$

$$\text{Acceleration: } \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \text{ or}$$

$$\mathbf{a} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j}.$$

Magnitude of acceleration =

$$\sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}.$$

Equations of motion in vector form:

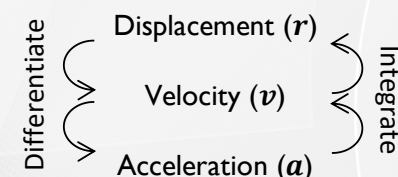
$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

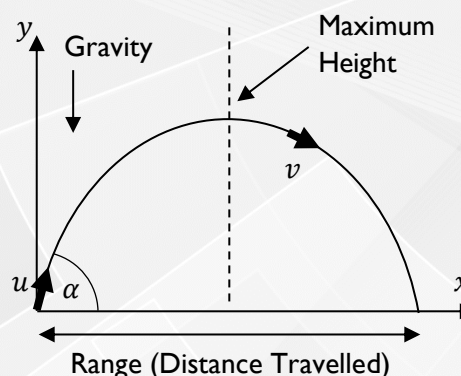
$$\mathbf{r} = \left(\frac{\mathbf{u} + \mathbf{v}}{2}\right)t$$

Care must be taken with motion that does not start at the origin (e.g. perhaps we need to use

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2).$$
 It is safer to differentiate and integrate:



Motion under gravity in two dimensions



Symbol	Horizontal (x)	Vertical (y)
$\mathbf{u}$	$u \cos \alpha$	$u \sin \alpha$
$\mathbf{a}$	0	$-g$
$\mathbf{v}$	$u \cos \alpha$	$u \sin \alpha - gt$
$\mathbf{s}$	$(u \cos \alpha)t$	$(u \sin \alpha)t - \frac{1}{2}gt^2$

$$\text{Maximum height} = \frac{(u \sin \alpha)^2}{2g}.$$

Time at maximum height:

$$t = \frac{u \sin \alpha}{g}.$$

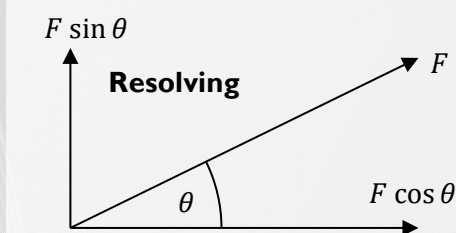
$$\text{Time of flight} = \frac{2u \sin \alpha}{g}.$$

$$\text{Horizontal range} = \frac{u^2 \sin 2\alpha}{g}.$$

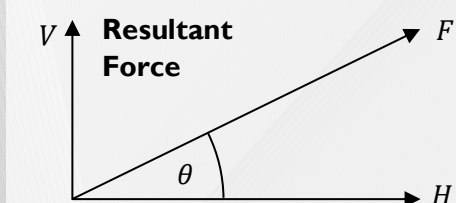
Equation of path:

$$y = x \tan \alpha - \frac{gx^2(1 + \tan^2 \alpha)}{2u^2}.$$

Forces and Newton's Laws



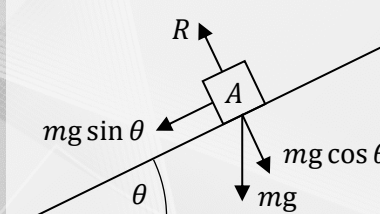
'Cos Contains the angle'.



$$F = \sqrt{V^2 + (H)^2}.$$

$$\theta = \tan^{-1}\left(\frac{V}{H}\right).$$

Motion on an Inclined Plane



'Sin Slides down the slope'.

The motion of particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be on an inclined plane.

Friction

$$\text{Friction} \leq \mu R.$$

If an object is moving, then

$$\text{Friction} = \mu R.$$

The coefficient of friction μ is a number between 0 and 1 without a unit.

Moments

Moment = Force $\times$ Perpendicular Distance.

Units Nm (Newton-metre).

If an object is in equilibrium, then

- 1) Around any point, the total clockwise moments are equal to the total anticlockwise moments;
- 2) The total forces in any two opposite directions (e.g. up, down) are equal.

If an object is *about to turn* about a point, then one of the forces (usually a reaction) is equal to zero.

Vectors

Using three-dimensional vectors.

The Examination

Length: 1 hour 45 minutes.
80 marks.
25% of the A Level qualification

Checklist

- ☐ I have attempted all the past paper questions.
- ☐ I know which formulae appear in the **formulae booklet**.
- ☐ I can check answers using a graphical calculator.

Dr. Gareth Evans

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April 2019

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