



Mathematics

Years 7-9

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Using the Workbooks



When you see a QR code (like the one on the left), scan it using your mobile device in order to reach a Welsh YouTube video hosted on the following channel.

www.youtube.com/adolygumathemateg

The numbers in circles, for example **3**, show the level of the work in the Welsh national curriculum. The higher the number, the more difficult the work. The levels correspond roughly to the following GCSE grades.

Level	3	4	5	6	7	8
GCSE Grade	G	F	E	D	C	B

All the workbooks contain a variety of exercises, labelled as follows.



Exercises on new topics.



Answering a question in context, or solving a problem.



A more difficult question.



Revision of material from previous workbooks.



There are evaluation boxes at the end of each chapter to revise the completed work.

Supporting Materials:

- Diagnostic Questions
 - A quiz for each workbook on the website www.diagnosticquestions.com.
- Reflection Sheet
 - An opportunity to assess your understanding of a workbook.
- Old WJEC examination questions; worksheets; investigations; Tarsia puzzles.
 - Available for some topics.

The website www.mathemateg.com contains an electronic copy of each workbook, alongside all the supporting materials.



**Level at the start
of the year: _____**

End of KS3
target: _____



Tracking Sheet

Year 7

Exam: %

**Level at the end
of the year: _____**







A pink rectangular card is shown against a light brown background. A green pushpin is pinned to the left edge of the card. In the top left corner of the card, there is a large black percentage sign (%). In the bottom left corner, there are three icons arranged horizontally: two black stars and a black magic wand with a star at its tip. On the right side of the card, the text "Welcome to the school" is written vertically in a bold, black, sans-serif font.

[illegible]

Introducing Angles



The Mathematics Department

7

Welcome to

Ysgol y Creuddyn

Name:

Contents

Chapter	Mathematics	Page Number
The Library	Multiples.	3
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The Library

The library is located above the English block, in block A. The sixth form study in the library.

Each book has a special number, an **ISBN**, which is an acronym for *International Standard Book Number*. Since 2007, ISBN numbers have 13 digits. The final digit is a **check digit** to help bar code readers read an ISBN for a particular book.

Here is the bar code for the book *Gwyddoniadur Cymru*.



To ensure that the ISBN is correct, we must alternatively multiply the digits by 1 and 3, and then find the total of all the multiplication sums.

$$\begin{aligned}
 &1 \times 9 + 3 \times 7 + 1 \times 8 + 3 \times 0 + 1 \times 7 + 3 \times 0 + 1 \times 8 + 3 \times 3 + 1 \times 1 + 3 \times 9 + 1 \times 5 + 3 \times 4 + 1 \times 3 \\
 &= 9 + 21 + 8 + 0 + 7 + 0 + 8 + 9 + 1 + 27 + 5 + 12 + 3 \\
 &= 110
 \end{aligned}$$

Because 110 is a **multiple of 10**, this ISBN is correct. If the total had not been a multiple of 10, then there would have been a mistake somewhere.

Exercise 1

Which of the following are valid ISBN numbers?

- (a) 978-1-8452-7516-7
- (b) 978-1-8485-1997-9
- (c) 978-1-8468-8423-7
- (d) 978-1-7847-5263-7
- (e) 978-1-4088-5565-2
- (f) 978-1-4088-5868-3
- (g) 978-1-3545-5102-4
- (h) 978-0-0074-8831-5
- (i) 978-0-2342-2359-2
- (j) 978-1-7839-0065-7

Challenge!



Extension

For the valid ISBN numbers above, which books give these numbers? (Investigate using the Internet.)

Applying

3



Other Multiples

With ISBN numbers, it was easy to verify whether a total was a multiple of 10, as all multiples of 10 end with a zero. What about other multiples?

Multiples of 2

A number is a multiple of 2 if it is an even number, so ends with either a 2, 4, 6, 8 or 0.

Multiples of 3

A number is a multiple of 3 if the sum of its digits is a multiple of 3. For example, 47268 is a multiple of 3, as $4 + 7 + 2 + 6 + 8 = 27$ is a multiple of 3.

Multiples of 4

A number is a multiple of 4 if half of the number is a multiple of 2. For example, 106 is not a multiple of 4, as halving 106 gives 53, which is an odd number, and not an even number.

Multiples of 5

A number is a multiple of 5 if it ends with either a 5 or a 0.



Skill

3

Exercise 2

(a) Which of the following numbers are multiples of 2?

27 82 192 2828 1293 28 95 8629 2380 29 0 238 92

(b) Which of the following numbers are multiples of 3?

39 512 1569 6495 84652 24155 543 918 64578 9 254 26 54374

(c) Which of the following numbers are multiples of 4?

48 66 75 100 128 150 240 364 601 710 864 912 1052

(d) Which of the following numbers are multiples of 5?

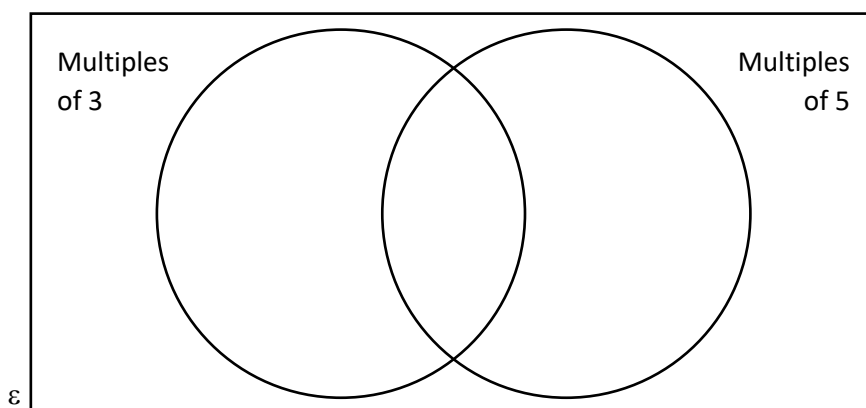
51 165 70 502 3215 5400 214 15 39552 5001 6020 687 654650

Exercise 3

Copy the following Venn diagram into your book. Complete the diagram using the numbers below.

4

25 18 26 30 95 36 60 14 31 45 75 72 100 105



Challenge!

- (a) Which numbers between 1 and 100 are multiples of 3 and 5?
 (b) Which numbers between 1 and 100 are multiples of 4 and 5?
 (c) Which numbers between 1 and 100 are multiples of 2, 3, 4 and 5?

Extension**More Multiples***Multiples of 6*

A number is a multiple of 6 if half of the number is a multiple of 3.

Multiples of 7

There is no quick way to decide whether a number is a multiple of 7. We must consider whether or not the number is in the seven times table.

Multiples of 8

A number is a multiple of 8 if it is possible to halve the number twice and end with an even number.

Multiples of 9

A number is a multiple of 9 if the sum of its digits is a multiple of 9.

Multiples of 10

A number is a multiple of 10 if it ends with a 0.

**Skill****Exercise 4**

- (a) Which of the following numbers are multiples of 6?

18 23 32 42 78 120 130 250 300 301 340 500 600

- (b) Which of the following numbers are multiples of 7?

15 28 35 44 63 76 84 105 119 132 140 217 279

- (c) Which of the following numbers are multiples of 8?

56 88 100 105 140 160 200 250 256 328 360 420 1600

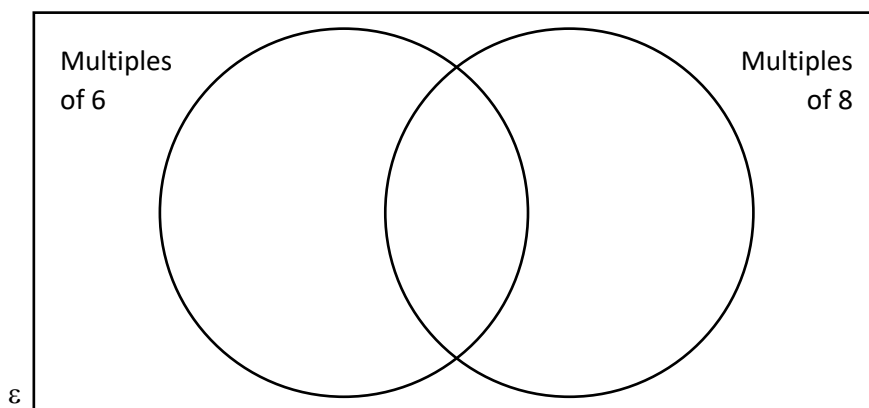
- (d) Which of the following numbers are multiples of 9?

45 65 99 124 351 486 845 909 1542 1978 5229 9571 26946

Exercise 5

Copy the following Venn diagram into your book. Complete the diagram using the numbers below.

3 12 16 20 24 30 38 40 44 48 54 56 58 96



Exercise 6

4

5	8	15	40	21	65	81	96	42	32	48
	6	18	9	7	33	56	4	27	84	28
49	72	24	36	25	63	77	88	99	64	

Which of the numbers in the box are:

- (a) Multiples of 8?
- (b) Multiples of 4?
- (c) Multiples of 7?
- (d) Multiples of 9?
- (e) Multiples of 4 and 7?
- (f) Multiples of 8 and 12?
- (g) Multiples of 4, 6 and 9?

Exercise 7

4

- (a) Which is the first multiple of 7 over 50?
- (b) Which is the first multiple of 9 over 60?
- (c) Which is the first multiple of 4 over 130?
- (d) Which is the first multiple of 6 over 90?
- (e) Which is the first multiple of 11 over 100?

Exercise 8 (Revision)

4

- (a) When is a number a multiple of 3?
- (b) When is a number a multiple of 8?
- (c) Write down the multiples of 6 between 50 and 80.
- (d) Write down the multiples of 9 between 30 and 60.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

The Hall

The hall is located in the centre of the school, by reception. Morning assembly; drama lessons; performances of musicals and much more take place here!

Can you see from the picture the **shape** of the **floor of the hall**?

Exercise 9

Name the shapes with the following number of sides.

- (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- (f) 8
- (g) 9
- (h) 10

Skill

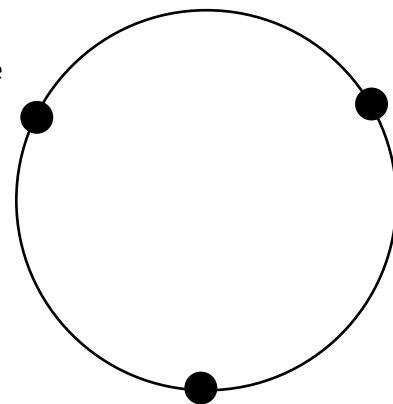
2



Investigation

Using a compass, draw a circle in your book (use a radius of around 4cm). Add three dots to your circle, as shown in the diagram on the right. Using straight lines, connect the dots in every possible way. Verify that this divides the circle into four regions. Repeat with a circle that has four dots, five dots, and so on. How many regions are formed each time? Is there a pattern?

Dots	2	3	4	5	6	7
Regions	2	4				



Regular and Irregular Polygons

A shape which uses straight lines only is called a **polygon**. The polygon is **regular** if its sides all have the same length and its angles are also all equal. If the polygon is not regular, then it is an **irregular** polygon.

Example

(a)

Regular Pentagon

(b)

Irregular Quadrilateral

(c)

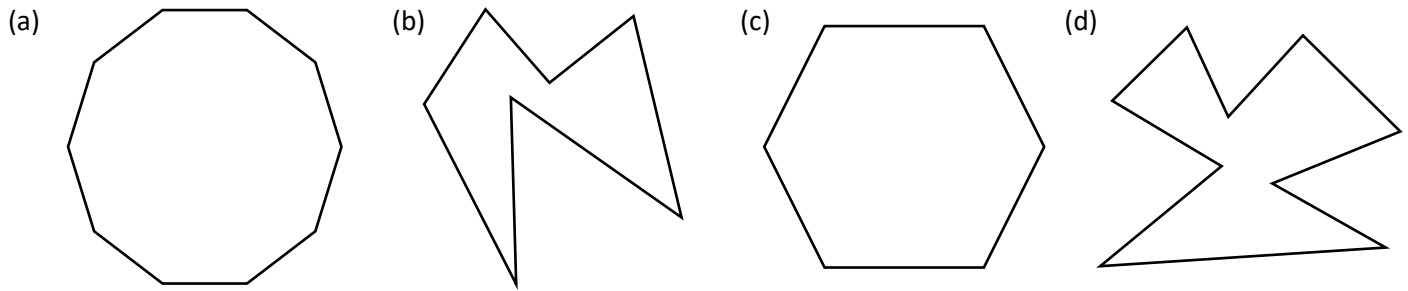
Regular Octagon

Exercise 10

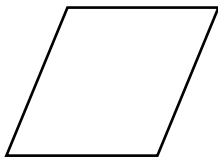
Name the following polygons.

Skill

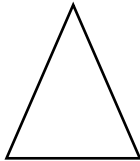
4

(e) Explain why the following shapes are **not** regular polygons.

(i) Rhombus



(ii) Isosceles Triangle



(iii) Rectangle

**Challenge!** 

Try to say the names of the following polygons...

Number of sides	Name	Number of sides	Name
13	Triskaidecagon	39	Triacontakaienneagon
21	Icosikaihenagon	45	Tetracontakaipentagon
25	Icosikaipentagon	100000	Megagon
31	Triacontakaihenagon	10^{100}	Googolgonalmost

Special Lines

Consider the picture shown of the musical "Herspre" (December 2014).

Can you see any **horizontal** lines?Can you see any **vertical** lines?Can you see any **parallel** lines?Can you see any **perpendicular** lines?**Exercise 11**

4

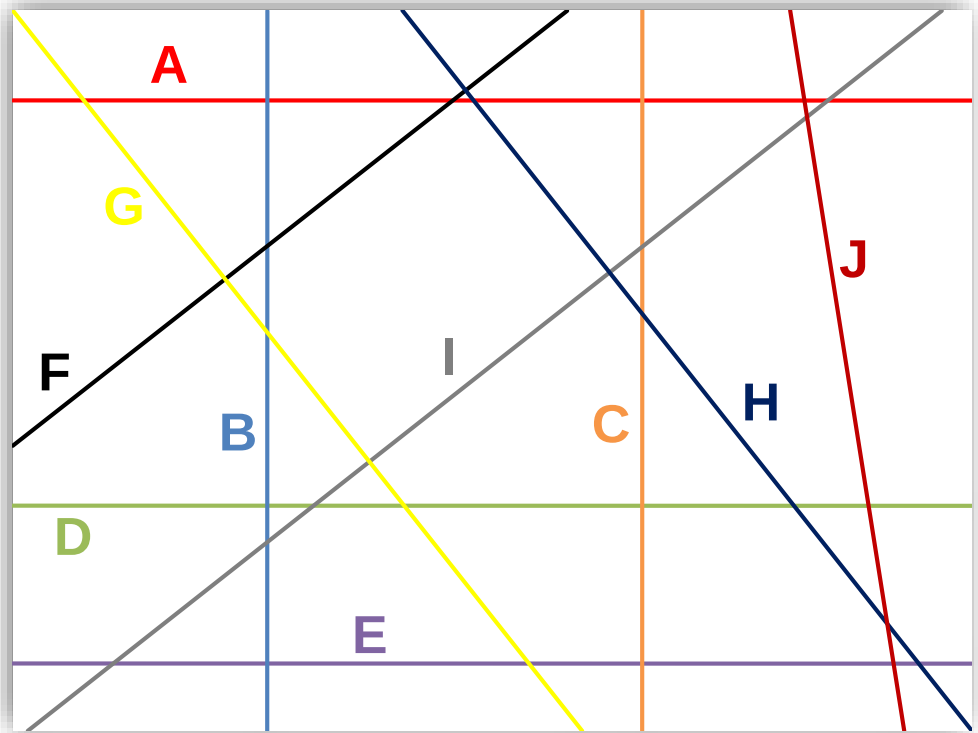
In your book, explain what is meant by a *horizontal* line; a *vertical* line; *parallel* lines; and *perpendicular* lines.



3

(a) the horizontal lines;
(b) the vertical lines;
(c) the pairs of parallel lines;
(d) the pairs of perpendicular lines.

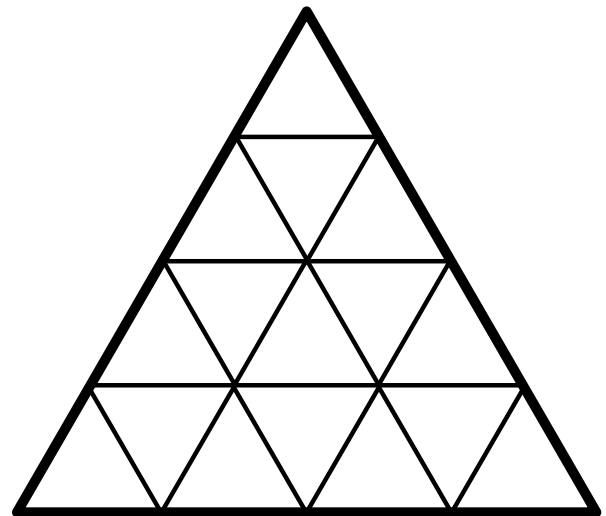
The word “parallel” comes from the Latin word “parallēlus” meaning side-by-side.



How many triangles can be seen in the diagram on the right?

4

- (a) How many sides does a heptagon have?
- (b) Draw a regular quadrilateral.
What is the usual name for this shape?
- (c) Draw a pair of perpendicular lines.
- (d) I have 4 octagons, 3 hexagons and 2 nonagons.
How many sides do I have?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

The Sports Hall

The sports hall was built during 2005-06, with the official opening taking place in the summer term of 2006. The school uses the hall during the day, and the local community at night.

It is possible to modify the hall for the use of many different sports.

Exercise 14

How many players are needed to form a **team** in these sports?

- (a) Netball
- (b) Football
- (c) Basketball
- (d) Cricket
- (e) Rugby league
- (f) Rugby union.

What do you notice about all your answers to Exercise 14?

Exercise 15

- (a) Which of the following numbers are even numbers?

23 86 99 104 158 2603 3547 6712 95420 451245874

- (b) Which of the following numbers are odd numbers?

8 41 59 84 107 245 659 1054 5249 3154287

- (c) List the 5 even numbers after 95.

- (d) List the 5 odd numbers after 36.

- (e) Explain what the difference is between an odd number and an even number.

Exercise 16

- (a) What is the largest odd number that can be made using 7, 1, 6, 4, 8?
- (b) What is the smallest even number that can be made using 6, 5, 2, 1, 9?
- (c) Answer “always”, “sometimes” or “never” to the following questions.

- (i) Adding two even numbers gives an even number.
- (ii) Adding two odd numbers gives an odd number.
- (iii) Multiplying two even numbers gives an even number.
- (iv) Multiplying two odd numbers gives an odd number.
- (v) Multiplying an odd number by an even number gives an odd number.

Challenge!

Think of a sport which requires an even number of players to form a team.



Skill

2



3



Sports Day

Square Numbers

Just as the even numbers form a number pattern, the **square numbers** also form a number pattern. To illustrate this pattern, consider the following sequence of squares.



What will be the next number in the sequence? And the one after that?

It is possible to write the numbers in this sequence as multiplication sums:

1×1 2×2 3×3 4×4

To get 16, which is the fourth number in the sequence, we multiply four by itself. This is written mathematically as 4^2 , and we say that we are calculating “**four squared**”. Therefore, $4^2 = 4 \times 4 = 16$.

Exercise 17

Evaluate the following square numbers.

- (a) 7^2 (b) 5^2 (c) 9^2 (d) 10^2 (e) 8^2 (f) 6^2

Skill

4

Exercise 18

Calculate the following sums.

- (a) $1^2 + 3^2$ (b) $4^2 + 5^2$ (c) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$ (d) $10^2 - 6^2$ (e) $12^2 - 2^2$
 (f) $5^2 - 4^2 - 3^2$ (g) $2^2 - 1^2$ (h) $3^2 \times 4^2$ (i) $1^2 \times 13^2$ (j) $2^2 \times 9^2 \times 1^2$
 (k) $5^2 \times 2^2 \times 1^2$ (l) $10^2 \div 5^2$ (m) $8^2 \div 4^2$ (n) $5^2 \div 1^2$ (o) $5^2 \div 2^2$
 (p) $5^2 \times 2^2 - 1^2$ (q) $4^2 \div 1^2 + 3^2$ (r) $8^2 \times 1^2 \div 2^2$ (s) $7^2 + 4^2 + 10^2 - 9^2$

Challenge!

5

(a) A number is palindromic if it reads the same backwards as it does forwards. For example, 373 is a palindromic number. Find the first three square numbers that are palindromic. (Clue: The answers are greater than 100, but less than 1000.)

(b) Find two square numbers that add to give another square number.



(c) Find three square numbers that add to give another square number.

Extension

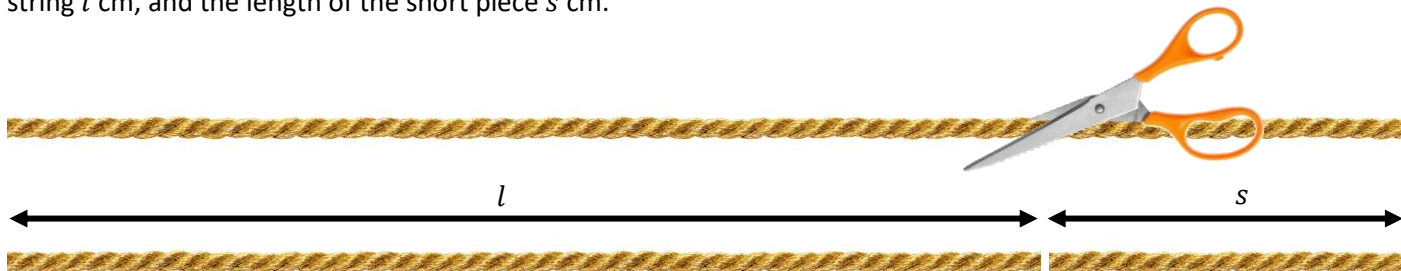


Investigation: The Golden Number**Applying****Problem 1**

When we square any number more than one, we obtain an answer that is greater¹. For example, when squaring 3, we obtain 9; it follows that 3 is six less than its square. Question: Which number is **exactly one less** than its square? Investigate for a few minutes using your calculator...

Problem 2

Consider a piece of string with a length of 1m, or 100cm. You are allowed to cut the string in any place, but not in the middle. This leaves two pieces of string – a long piece and a short piece. Let us call the length of the long piece of string l cm, and the length of the short piece s cm.



Question: Where should the string be cut so that $100 \div l$ is the same as $l \div s$? Investigate for a few minutes using your calculator...

The Answer

It turns out that both problems have a connection to a special number known as the golden number. The symbol for the golden number is Φ ("Phi"). One way of finding Φ is to consider the **Fibonacci sequence**. In this sequence, we obtain the next number by adding the two previous numbers. Starting with 1 and 1, the sequence continues like this:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Exercise 19

2

Find the next five numbers in the Fibonacci sequence.

Consider dividing the number in the Fibonacci sequence like this: $1 \div 1$, $2 \div 1$, $3 \div 2$, $5 \div 3$, $8 \div 5$, ... (can you see the pattern?) As Fibonacci's sequence continues, these division sums give an answer that is closer and closer to the golden number Φ .

Exercise 20

5

Using your answer to Exercise 20, write down your best estimate for Φ . What is the link between Φ and problems 1 and 2 above?

Extension**The Golden Number and the Body**

Calculate the following ratios. What do you notice?

- (a) Distance from the floor to your belly button $\div$ Distance from the floor to your knee.
- (b) Distance from your wrist to your elbow $\div$ Length of your hand.



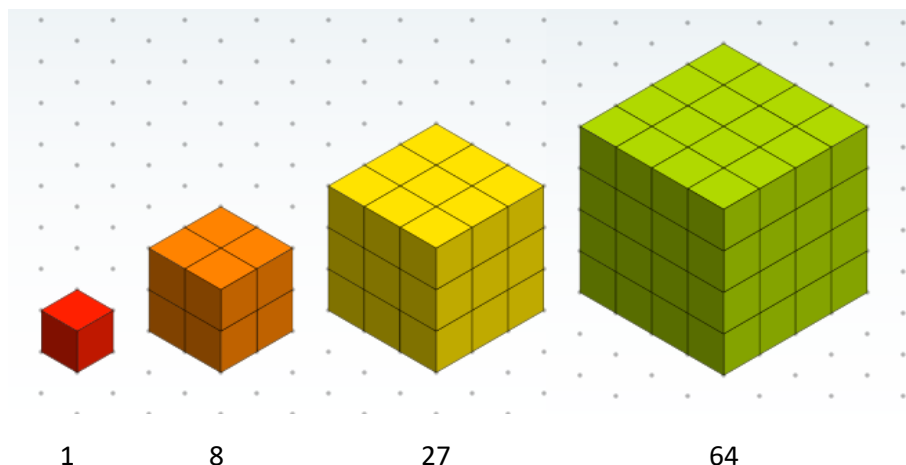
Sports Day

¹ What happens if you square a number between 0 and 1?

Cube Numbers



The **cube numbers** come from the following set of diagrams of cubes.



What will be the next number in the sequence? And the one after that?

It is possible to write the numbers in this sequence as multiplication sums:

$1 \times 1 \times 1$ $2 \times 2 \times 2$ $3 \times 3 \times 3$ $4 \times 4 \times 4$

To get 64, which is the fourth number in the sequence, we multiply four by itself three times. This is written mathematically as 4^3 , and we say that we are calculating “**four cubed**”. Therefore, $4^3 = 4 \times 4 \times 4 = 64$.

Exercise 21

Evaluate the following cube numbers.

- (a) 5^3 (b) 6^3 (c) 7^3 (d) 8^3 (e) 9^3 (f) 10^3

Skill

4

Exercise 22

Calculate the following sums.

- (a) $2^3 + 3^3$ (b) $3^3 + 4^3$ (c) $7^3 - 4^3$ (d) $10^3 \times 1^3$
 (e) $4^3 + 5^3$ (f) $6^3 + 1^3 + 2^3$ (g) $2^3 \times 5^3$ (h) $1^3 \times 5^3$
 (i) $8^3 + 9^3$ (j) $4^3 \div 1^3$ (k) $10^3 \div 2^3$ (l) $10^3 - 2^3$

4

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

The School Timetable

Here is the school timetable, as shown in the diary.

08:55 – 09:15	<i>Registration Period / Assembly</i>
09:15 – 10:05	<i>Lesson 1</i>
10:05 – 10:55	<i>Lesson 2</i>
10:55 – 11:10	<i>Break</i>
11:10 – 12:00	<i>Lesson 3</i>
12:00 – 12:50	<i>Lesson 4</i>
12:50 – 13:45	<i>Dinner Time and Activities</i>
13:45 – 13:50	<i>Warning Bell</i>
13:50 – 14:40	<i>Lesson 5</i>
14:40 – 15:30	<i>Lesson 6</i>



Applying

3

Exercise 23

- What is the length of the registration period, in minutes?
- How many minutes are there between the end of lesson 4 and the end of lesson 6?
- What is the length of the school day, in hours and minutes?
- You are exactly half way into lesson 2. What is the time?
- The headmaster grants an extension of twenty five minutes to the usual break. When will lesson 3 start?
- What is the length of break time in seconds?

The 12 hour clock and the 24 hour clock

There are two methods for telling the time.

- The **12 hour** clock, e.g. 2:45 a.m., 11:40 p.m., 7:45 a.m.
- The **24 hour** clock, e.g. 02:45, 16:42, 12:32.



The 24 hour clock runs from 00:00 to 23:59. The times from 00:00 to 11:59 are a.m. times, and the times from 12:00 to 23:59 are p.m. times.

The letters a.m. stand for Ante Meridiem (from the Latin, meaning “before midday”). The letters p.m. stand for Post Meridiem (again from the Latin, meaning “after midday”).

Exercise 24

Complete the following table.

Skill

3

12 hour clock	24 hour clock
2:34 p.m.	
4:56 a.m.	
9:20 p.m.	
12:00 a.m.	
2:45 a.m.	
11:38 p.m.	
3:30 p.m.	
7:34 a.m.	
8:30 p.m.	
12:00 p.m.	

12 hour clock	24 hour clock
	11:15
	18:31
	00:45
	06:30
	16:54
	22:13
	08:55
	13:04
	01:25
	21:59

**Exercise 25**

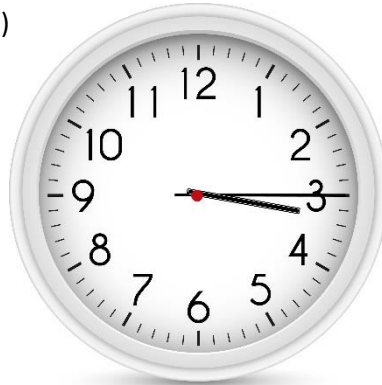
3

The following clocks show the time at different stages during the school day. For each one, write down
(i) The time using the 12 hour clock; (ii) The time using the 24 hour clock; (iii) The time in words.

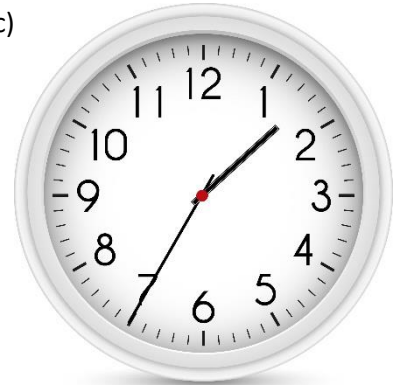
(a)



(b)



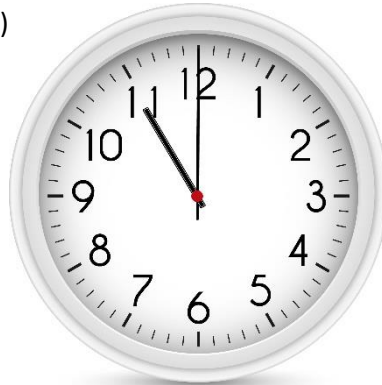
(c)



(d)



(e)



(f)

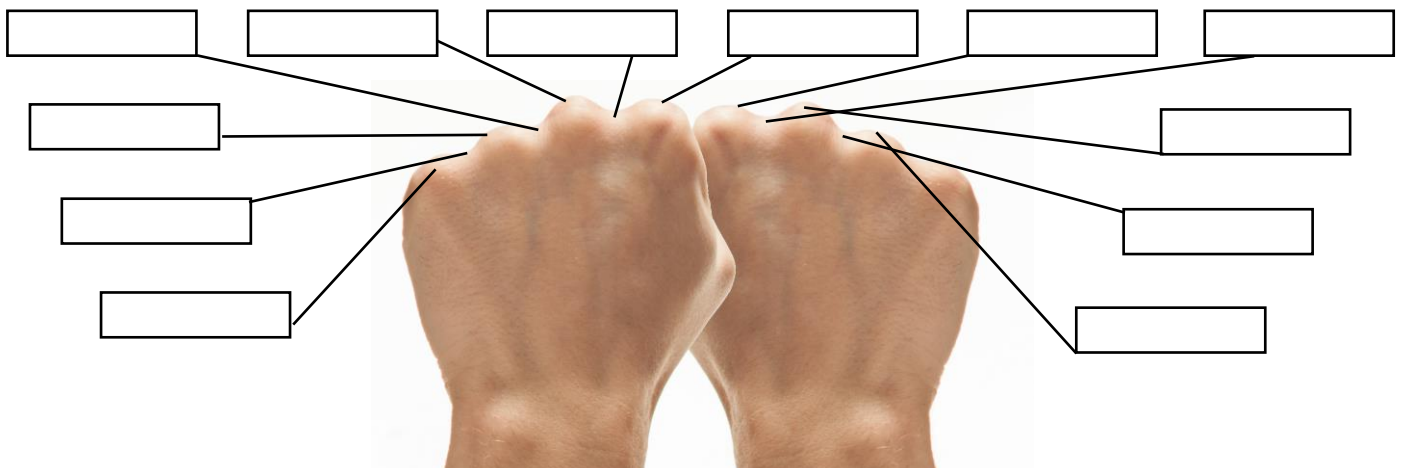
**The Calendar**

There are 12 months in a year, but how many days are there in each month? Try to complete the following table.

Name of the month	Number of days
January	

Name of the month	Number of days
July	

How can the following picture help you remember how many days make up each month?



Exercise 26

Here is a calendar for the year 2020.

Applying

3

January

Mo	Tu	We	Th	Fr	Sa	Su
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

February

Mo	Tu	We	Th	Fr	Sa	Su
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	

March

Mo	Tu	We	Th	Fr	Sa	Su
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

April

Mo	Tu	We	Th	Fr	Sa	Su
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May

Mo	Tu	We	Th	Fr	Sa	Su
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June

Mo	Tu	We	Th	Fr	Sa	Su
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

July

Mo	Tu	We	Th	Fr	Sa	Su
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August

Mo	Tu	We	Th	Fr	Sa	Su
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

September

Mo	Tu	We	Th	Fr	Sa	Su
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October

Mo	Tu	We	Th	Fr	Sa	Su
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November

Mo	Tu	We	Th	Fr	Sa	Su
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

December

Mo	Tu	We	Th	Fr	Sa	Su
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

- Find 1st January 2020. Which day of the week is this?
- How many months in 2020 begin on Saturday?
- Is 2020 a leap year? How do you know?
- How many Saturdays are in September?
- Which month is the first in the year with more Fridays than Thursdays?
- On which day will Fireworks Night be in 2020?
- How many Mondays are there in the year?
- Which day will be the final day of January **2021**?

Challenge!

Are there more days in the first six months of a year or the final six months of a year?

School History

Ysgol y Creuddyn was opened on Wednesday, September 2nd, 1981. There were 218 pupils to begin with, including exactly 100 in year seven. The pupils were welcomed by the headmaster Mr. Roland Jones (Year 3); Mrs. Ellen Kent (Year 1); and Mr. Dafydd Whittall (Year 2).

The official opening took place on Wednesday, October 17th, 1984.

Question: Was 1981 a leap year? What about 1984? How do we go about deciding?



Leap Years

- There is a leap year every four years.
- There is one extra day (February 29th) during a leap year.
- There are 366 days in a leap year.
- Four is a factor of every leap year. It is possible to check whether a particular year is a leap year by dividing the year by four.

$$\begin{array}{r} 0489 \\ 4 \overline{) 1956} \\ \underline{1956} \\ 0 \end{array}$$

Remainder 0 so 1956 was a leap year

$$\begin{array}{r} 04652 \\ 4 \overline{) 1862} \\ \underline{1862} \\ 2 \end{array}$$

Remainder 2 so 1862 was not a leap year



Exercise 27

Decide whether the following years were leap years.

- (a) 1936 (b) 1832 (c) 1873 (d) 1990 (e) 1906 (f) 1740 (g) 1808

Decide whether the following years will be leap years.

- (h) 2056 (i) 2140 (j) 2562 (k) 3483 (l) 4800 (m) 5136 (n) 13276

Timetables

Exercise 28

(a) What is the length of a television programme that starts at 17:55 and ends at 18:40?

(b) The film started at 21:35. If the film lasted 1 hour 40 minutes, when did it finish?

(c) When cooking a turkey, it requires 20 minutes cooking for each pound of weight and then an extra 20 minutes. You have a 12 pound turkey and need it to be ready by 19:30. When should you start cooking the turkey?

(d) Film A starts at 13:52 and finishes at 16:35. Film B starts at 14:33 and finishes at 16:48. Which film is the longest, and by how much?

(e) A bus leaves Llandudno at 11:35 and reaches Bangor at 12:50. How long was the journey?

(f) Look at the following timetable and answer the questions that follow.

Holyhead	0551	0628	0715	0805	0923	1040	1127
Bangor	0618	0706	0802	0902	1002	1107	1200
Llandudno Junction	0635	0722	0823	0923	1023	1123	1222
Colwyn Bay	0642	0731	0831	0931	1031	1131	1229
Rhyl	0653	0741	0841	0941	1041	1141	1240
Prestatyn	0658	0747	0847	0947	1047	1147	1245
Flint	0712	0800	0900	1000	1100	1200	1259
Chester	0726	0815	0914	1015	1115	1216	1313

4

Skill

4

Applying



- (i) What time does the first train reach Llandudno Junction?
- (ii) How many minutes does the 0841 from Rhyl take to reach Chester?
- (iii) Siwan lives in Colwyn Bay. She works in Prestatyn and starts work at 09:00. Which train should Siwan catch in order to reach work on time?
- (iv) Geraint reaches Bangor train station at 10:23. How many minutes does he have to wait until the next train arrives?
- (v) Of the seven trains on the timetable, which is the **slowest** in travelling from Holyhead to Chester?



Challenge!

Which is the largest: the number of seconds in a week, or the number of minutes in a year?



5

Exercise 29 (Revision) 4

- (a) How many minutes are there in 4 hours?
- (b) How many years and months make up 35 months?
- (c) Change 10:34 p.m. to be in the 24 hour clock.
- (d) The clock on the right is three quarters of an hour early. What is the correct time?
- (e) How many days are there in April and October combined?
- (f) Was 1846 a leap year?
- (g) The concert finished at ten minutes to ten at night. The concert's length was 2 hours 35 minutes. When did the concert start?
- 



a.m.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

The Refectory

Each dinner time, the refectory serves hot food; salad; sandwiches; pasta king and different puddings. An extension to the refectory was built during 2014.

Exercise 30

3

Skill

Add symmetry lines to the following foods.

(a)



(b)



(c)



(d)



Challenge!

Can you identify the names of each of the foods above?

Exercise 31

Add symmetry lines to the following diagrams.

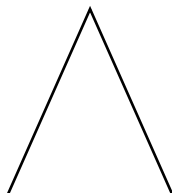


3

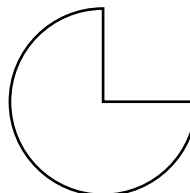
(a)



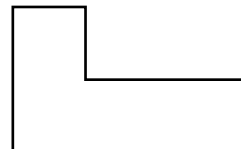
(b)



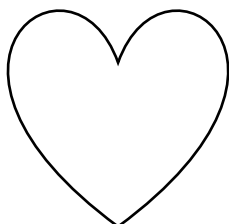
(c)



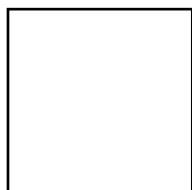
(d)



(e)



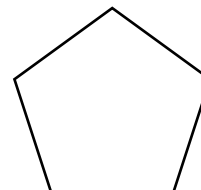
(f)



(g)



(h)



Exercise 32

3

Applying

Classify the following flags according to whether they have (a) vertical symmetry only; (b) horizontal symmetry only; (c) both horizontal and vertical symmetry; (d) no symmetry.

**Challenge!**

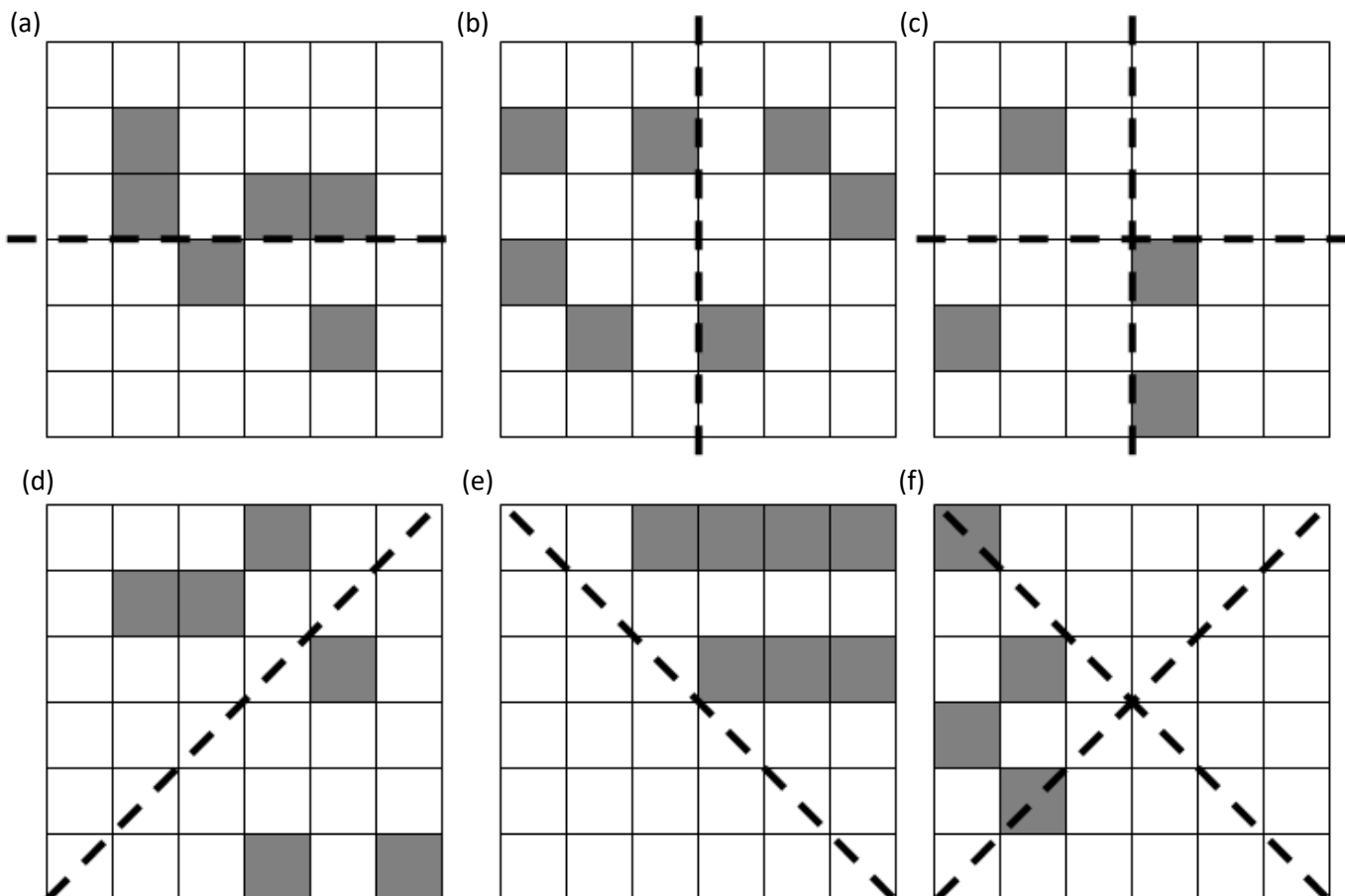
Which countries appear on the above flags?

Exercise 33

4

Skill

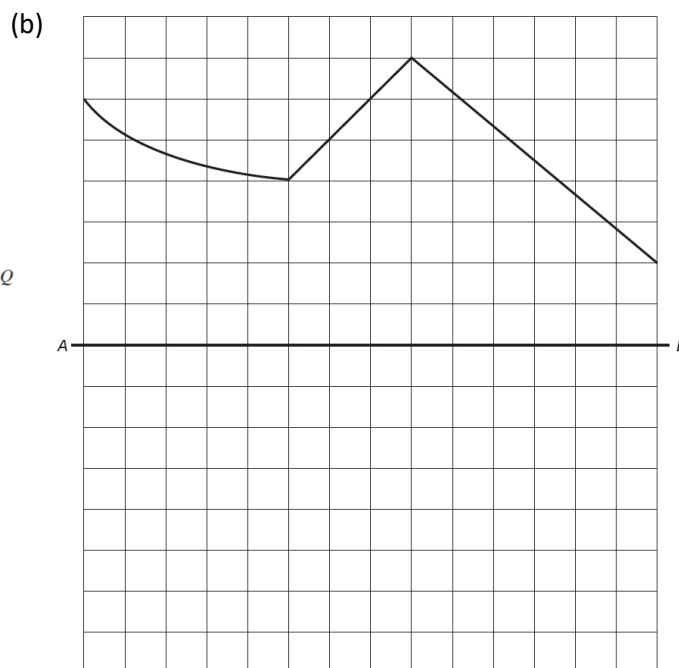
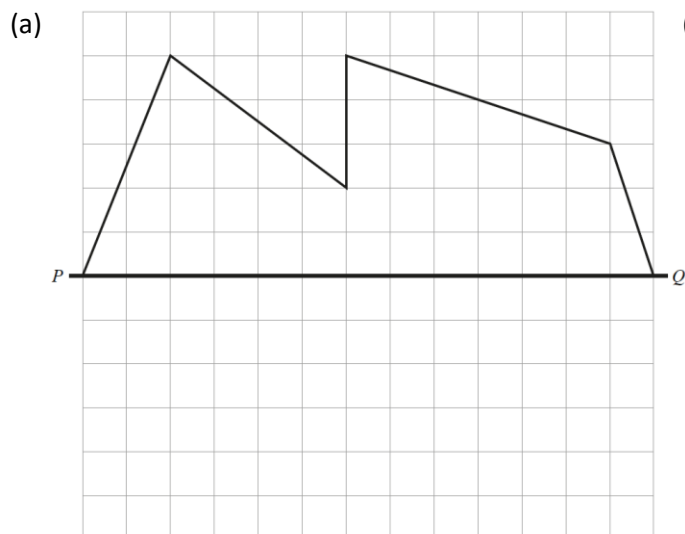
Shade the least number of squares so that the dotted lines are symmetry lines.



Exercise 34

4

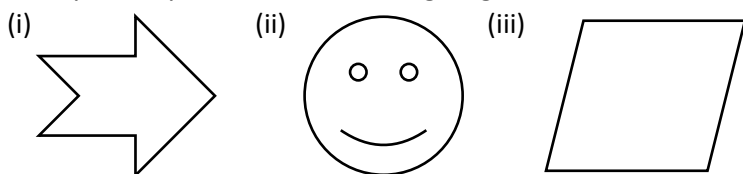
Complete the following shapes so that the horizontal line is a symmetry line.



Exercise 35 (Revision)

4

(a) Add symmetry lines to the following diagrams.



(b) Explain whether France's flag has any symmetry lines.

(c) How many symmetry lines does a regular hexagon have?

(d) How many symmetry lines does a circle have?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

The Eisteddfod

The Eisteddfod is one of the highlights of the school year. The houses **Bodysgallen**, **Penrhyn** and **Gloddaeth** battle against each other in competitions such as the Solo Instrumental; Speaking for a Minute; Solo from a Musical; and Gargling the National Anthem.

Many of the competitions are musical in nature, so it is advantageous to be able to read musical notation. Here are some of the symbols used in musical notation.



Treble clef



Semibreve (4 beats)



Rest



Minim (2 beats)



Flat



Crotchet (1 beat)



Sharp

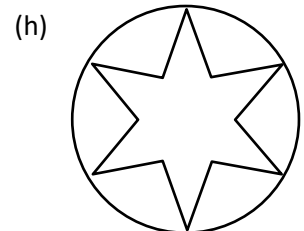
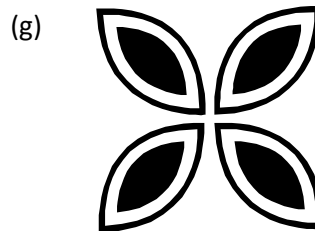
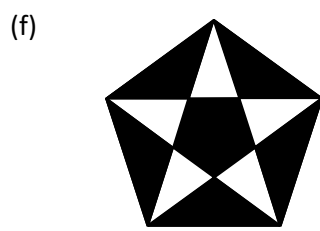
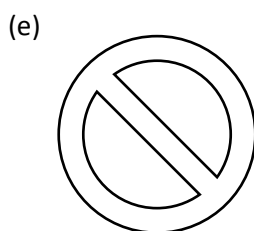
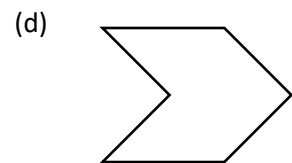
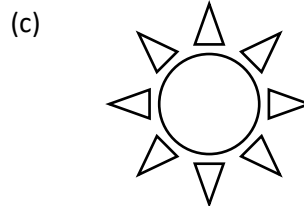
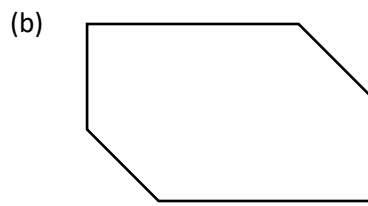
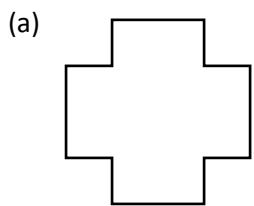
Imagine rotating these symbols. Which symbols will look exactly the same before you finish a complete rotation?

Rotational Symmetry

A shape has **rotational symmetry** if it comes to fit onto itself more than once during a complete rotation. The **order of rotational symmetry** is the number of times the shape comes to fit onto itself during the complete rotation. A shape does not have rotational symmetry if it fits onto itself only once.

Exercise 36

What is the order of rotational symmetry of the following shapes?



(i) Regular Decagon

(j) Square

(k) Rectangle

(l) Circle



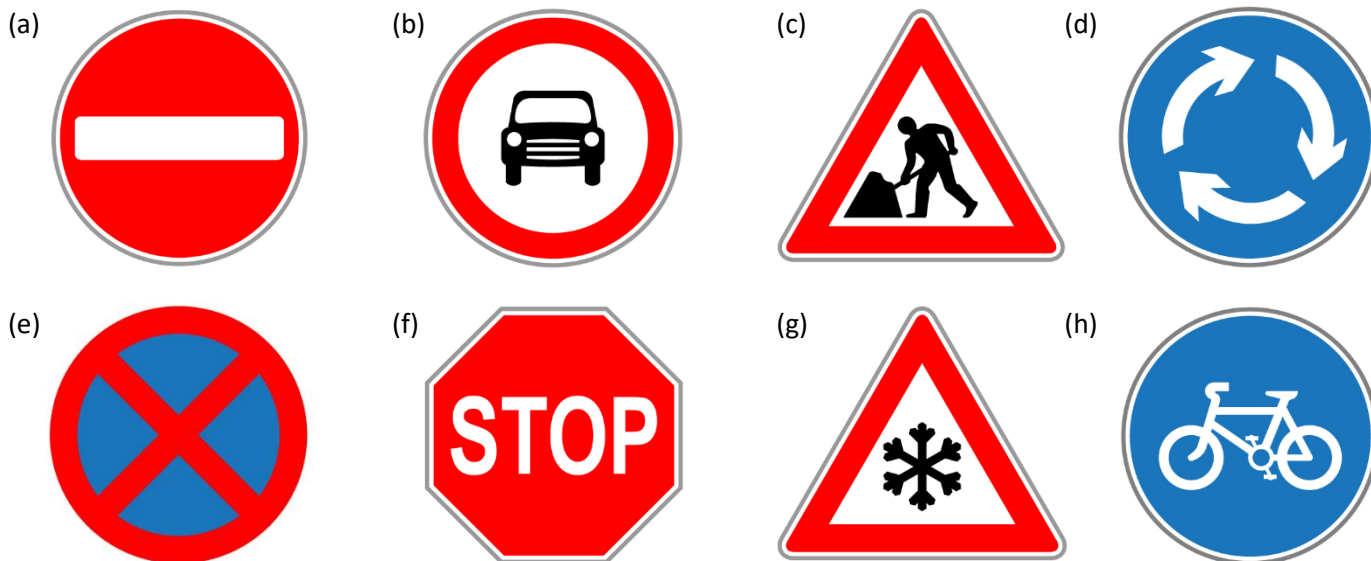
Exercise 37

What is the order of rotational symmetry of the following road signs?



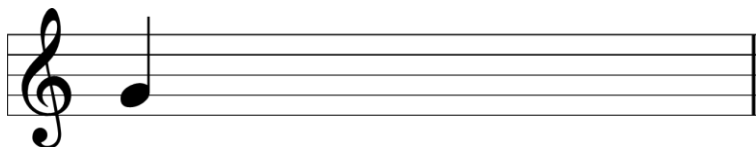
4

Applying

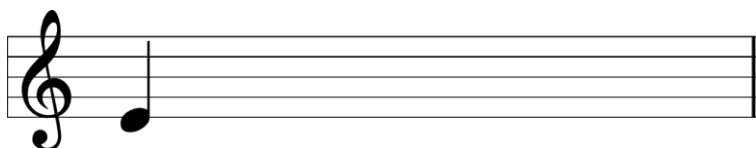


Writing Music

It is possible to write music on a **five line musical stave**. The location of a symbol like a crotchet tells the reader which note to play. For example, the following stave shows a G note.



By moving the crotchet up or down, a different note can be played. For example, moving the crotchet down one line changes the note to be an E.

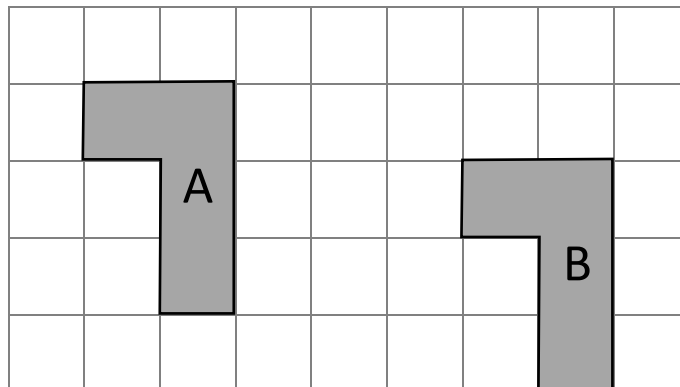


The mathematical name for moving a symbol is **translation**.

Translations

In the diagram on the right, the shape A has been moved 5 squares to the right and 1 square down in order to form the shape B. It is possible to write this **translation** as a **column vector**: $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

The **upper** number in the column vector tells us how to move the shape **horizontally**. (Positive: move to the right; negative: move to the left.) The **lower** number in the column vector tells us how to move the shape **vertically**. (Positive: move up; negative: move down.)



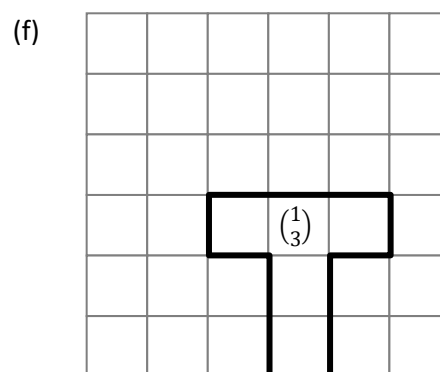
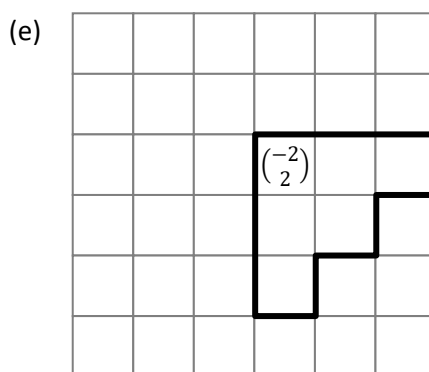
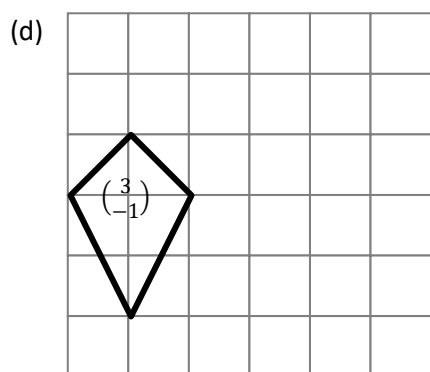
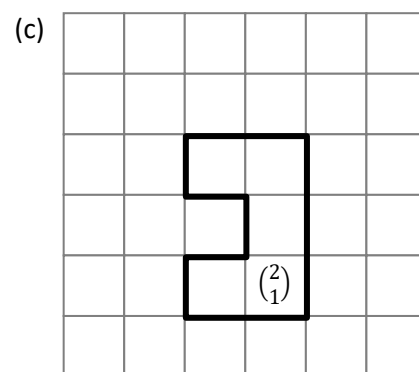
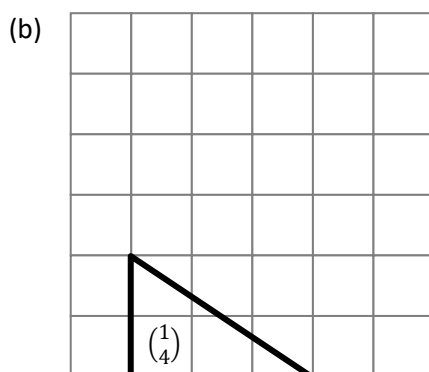
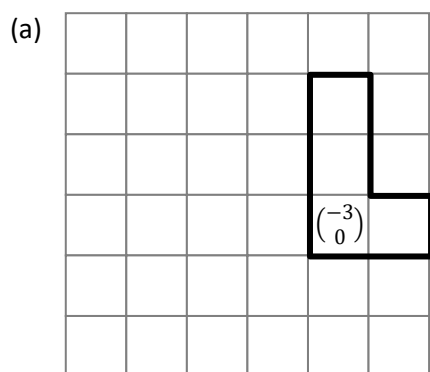
Exercise 38

Write the **column vector** for the following translations.

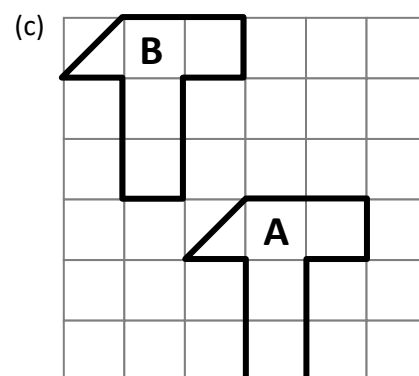
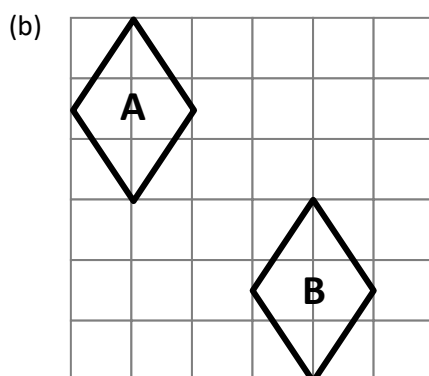
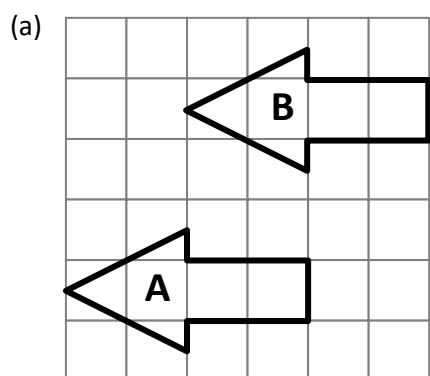
- (a) Move the shape 4 squares to the right and 3 squares up.
- (b) Move the shape 3 squares to the left and 7 squares up.
- (c) Move the shape 8 squares to the right and 2 squares down.
- (d) Move the shape 5 squares up.
- (e) (Care!) Move the shape 2 squares down and 4 squares to the left.

Exercise 39

Translate the following shapes using the shown column vector.

**Exercise 40**

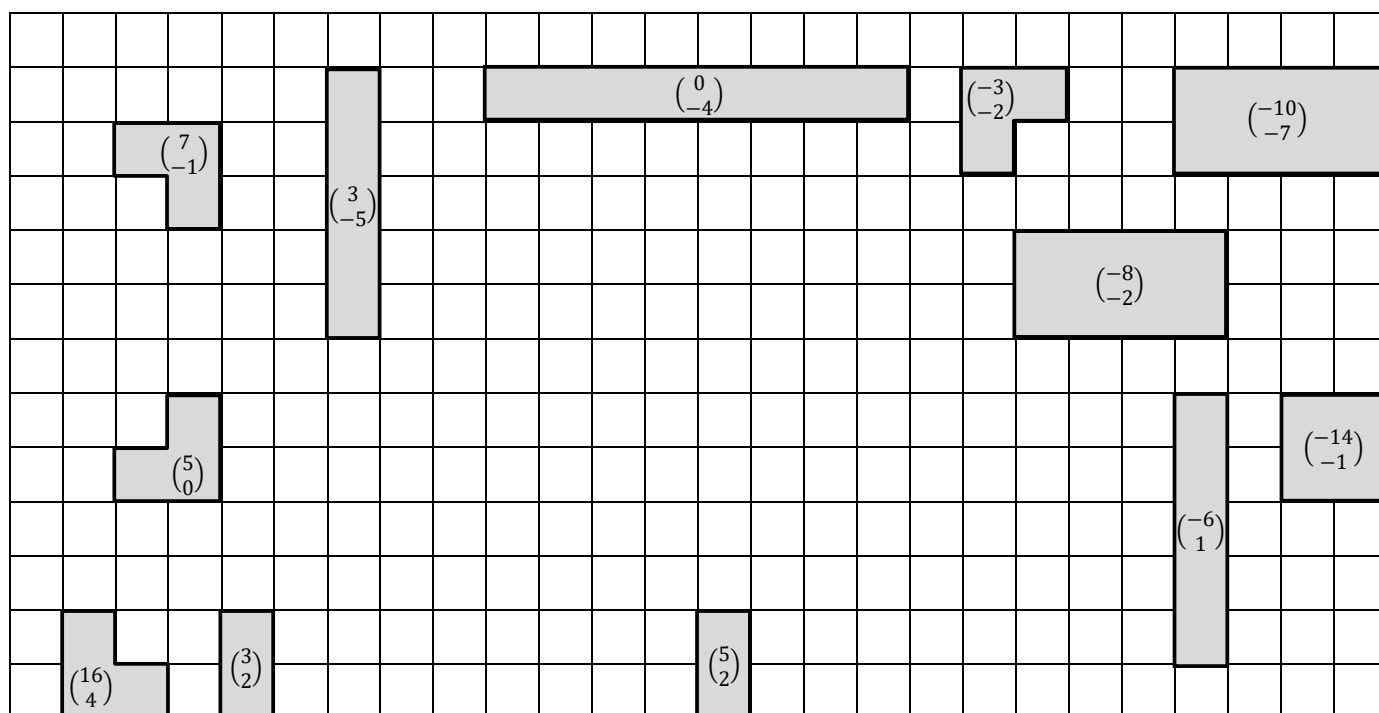
Write down the column vector that translates the shape **A** to the shape **B** in the following diagrams.



Exercise 41

4

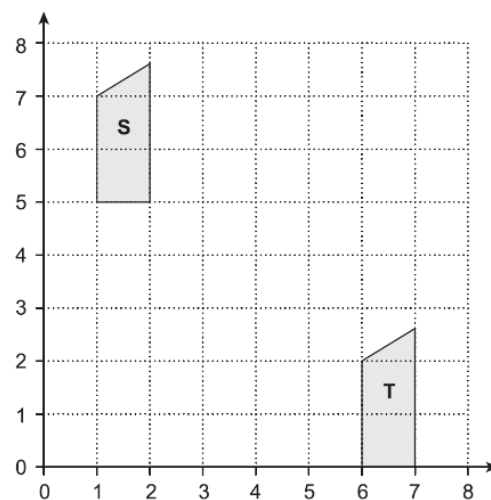
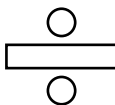
Translate the following shapes to form one big shape...



Exercise 42 (Revision) 4

4

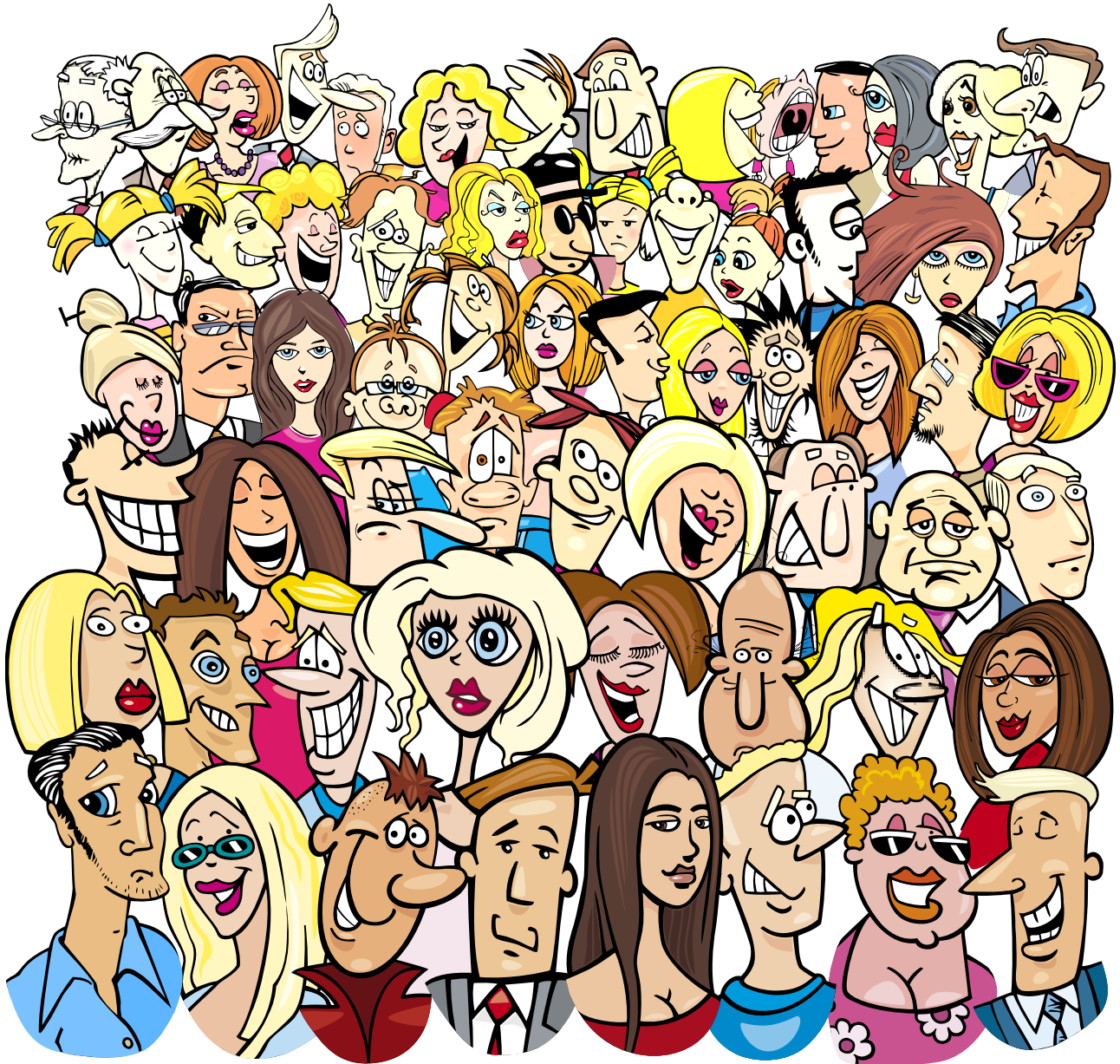
- (a) What is the order of rotational symmetry of the division sign shown on the right?
- 
- (b) How would the column vector $\begin{pmatrix} -4 \\ 7 \end{pmatrix}$ move a shape?
- (c) Write down the column vector that would **undo** the translation from part (b).
- (d) Which column vector translates the shape **S** to the shape **T** in the diagram on the right?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Puzzle: How many people can be seen in the picture?





Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know the meaning of the word ' multiple '.			5, 9	
I know how to decide if a number is a multiple of 2 .			1	
I know how to decide if a number is a multiple of 3 .			5	
I know how to decide if a number is a multiple of 5 .			9	
I know how to decide if a number is a multiple of 7 .			9	
I know how to decide if a number is a multiple of 9 .			12	
I know how to complete a Venn diagram .			12	
I know the names of the shapes with between 3 and 10 sides.			2	
I know the difference between a regular polygon and an irregular polygon .			14	
I know the difference between a vertical line and a horizontal line .			7	
I know the difference between parallel lines and perpendicular lines .			7	
I know the difference between an even number and an odd number .			1, 3	
I can calculate a square number such as 7^2 .			9, 11	
I can calculate a cube number such as 6^3 .			11	
I know the difference between times in the 12-hour clock and times in the 24-hour clock .			4, 5	
I can read the time shown on an analogue clock .			4	
I know the number of days in each month of the year.			5	
I know how to decide whether a year is a leap year .			13	
I can answer questions that use a timetable .			17	
I can add symmetry lines to different shapes.			8	
I can complete a shape to ensure it is symmetrical .			15	
I know how to decide what order of symmetry a shape has.			10	
I know the mathematical meaning of the word ' translation '.			16	
I know how to write a translation as a vector column .			16	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

7

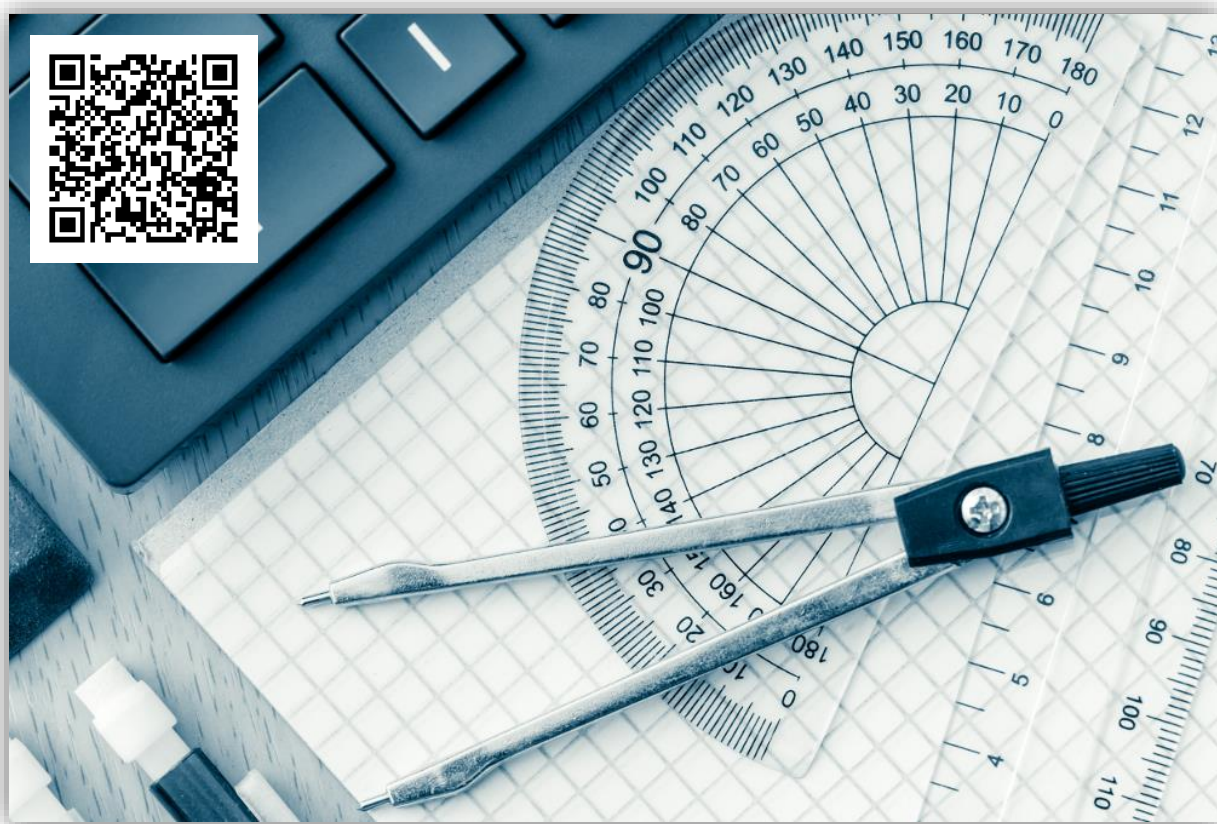
Introducing

Angles

Name:

Contents

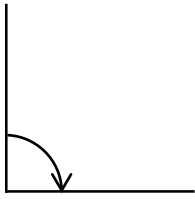
Chapter	Mathematics	Page Number
Types of Angles	Turns. Acute, obtuse and reflex angles. Naming angles.	3
Measuring Angles	Estimating angles. Using a protractor to measure angles.	4
Drawing Angles	Using a protractor to draw angles. Drawing special angles without a protractor.	12
Calculating Angles	Angles around a point. Angles on a straight line. Vertically opposite angles.	14
Angles in Triangles	Angles in a triangle. Types of triangles. The exterior angle of a triangle. Constructing triangles.	18



Types of Angles



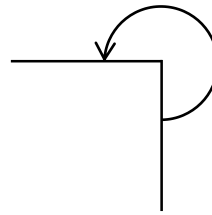
An angle measures **turn**. Here are some examples of different turns.



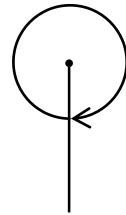
A quarter of a turn clockwise



Half a turn anticlockwise



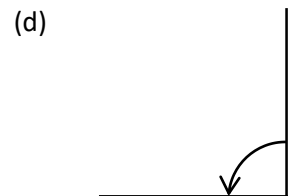
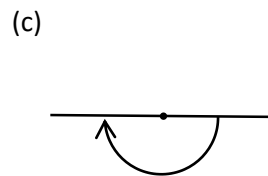
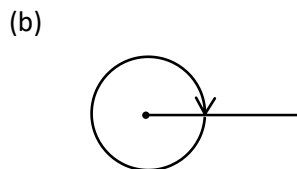
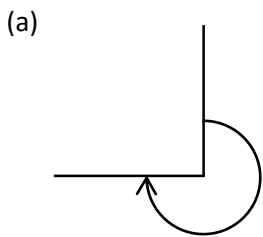
Three quarters of a turn anticlockwise



A full turn clockwise

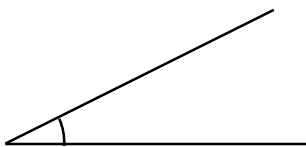
Exercise 1

What **type** of turns do you see below?



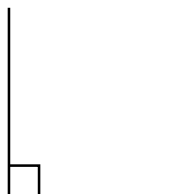
It is possible to measure an angle in degrees. There are **360°** in a full turn, so there are 180° in half a turn; 90° in a quarter of a turn, and so on.

We use the following names for the different types of angles.



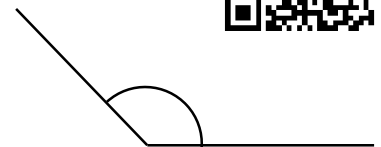
Acute Angle

Any angle between 0° and 90°



Right Angle

An angle that is exactly 90°



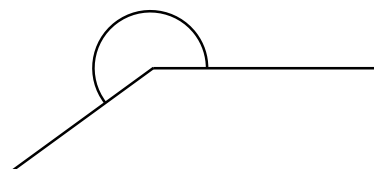
Obtuse Angle

Any angle between 90° and 180°



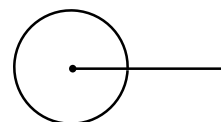
Straight Line

An angle that is exactly 180°



Reflex Angle

Any angle between 180° and 360°



Full Turn

An angle that is exactly 360°

Challenge!

Why do we use 360° for the size of a full turn, and not some other number?

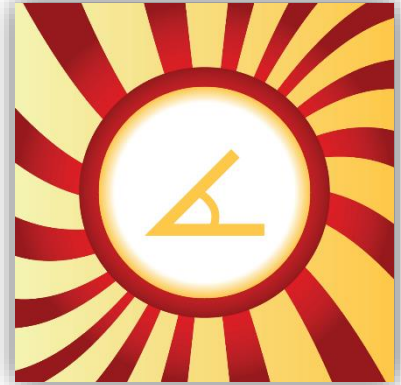
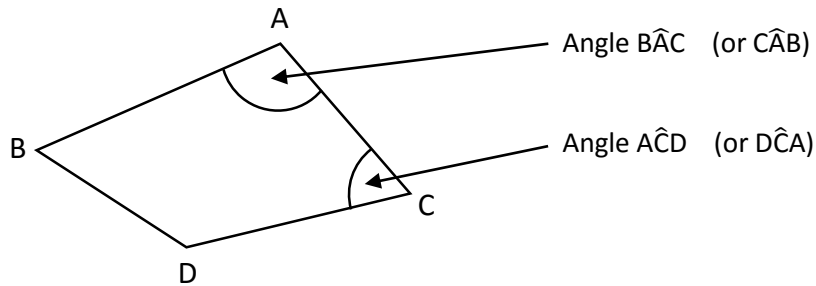


Exercise 2

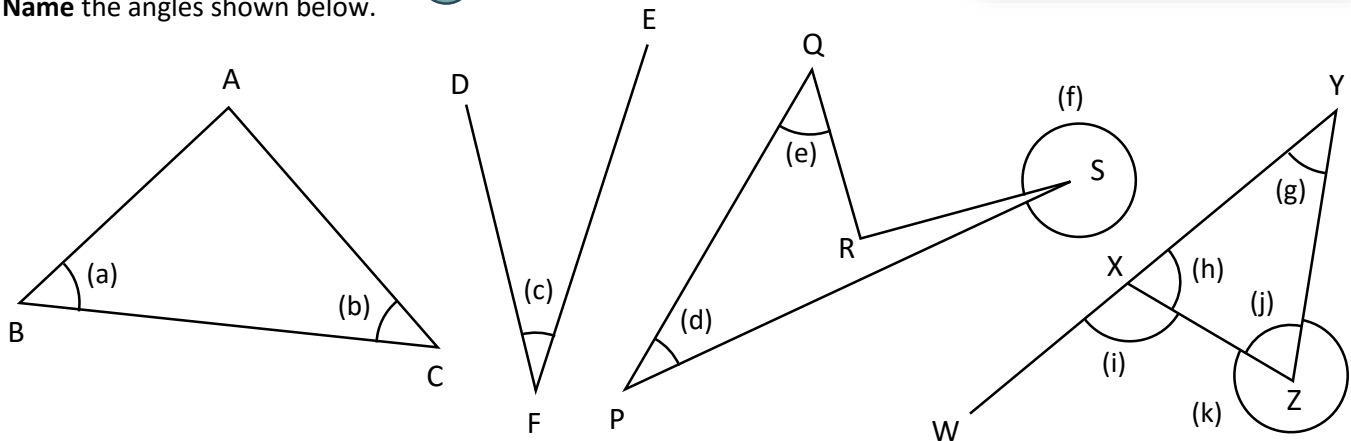
What **type** of angles are the following?

- (a) 140° (b) 14° (c) 280° (d) 104° (e) 325° (f) 94° (g) 180°
 (h) 54° (i) 205° (j) 90° (k) 270° (l) 2° (m) 360° (n) 103°

In order to **name** an angle, we label the vertices in a diagram with capital letters, before using **three letters** to identify a specific angle. (We need to add a **circumflex accent** to the letter in the middle.)

Example**Exercise 3**

Name the angles shown below.

**Exercise 4**

What **type** of angles are the ones shown in Exercise 3?

5

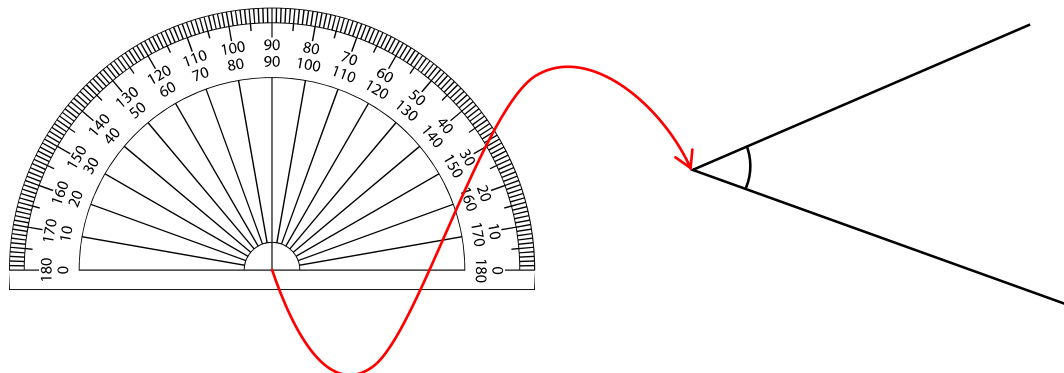
**Evaluation**

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

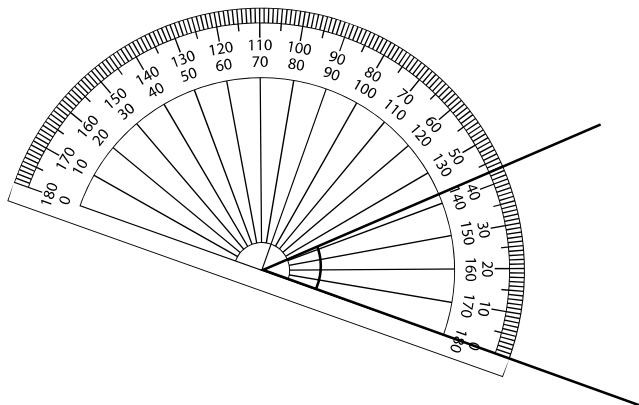
Measuring Angles

We use a **protractor** to measure angles. It is possible to obtain a protractor in the shape of half a circle (measures up to 180°) or in the shape of a full circle (measures up to 360°). To measure an angle:

(i) Put the **centre of the protractor** on the **vertex** (corner) of the angle.



(ii) **Turn** the protractor so that the angle 0° lies on one of the **lines** of the angle.

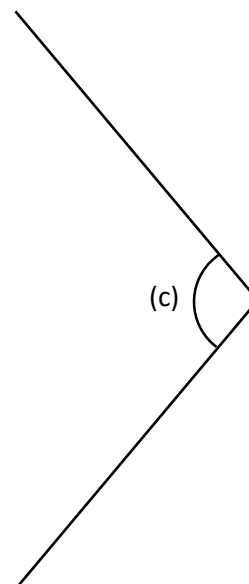
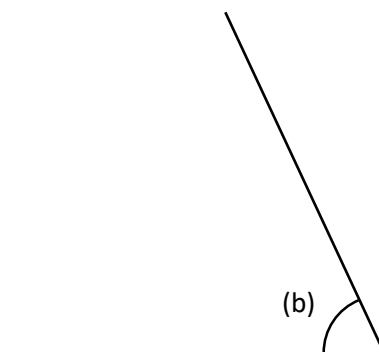
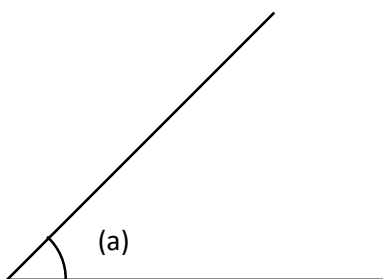


(iii) **Count up from 0°** in order to measure the angle. (It is important to do this as there are two sets of numbers on a protractor.) What angle is shown by the protractor above?

Exercise 5

5

Estimate the size of each of the following angles in degrees. Write your estimate down before **measuring** the angle using a protractor. How accurate were you in estimating the size of the angles?

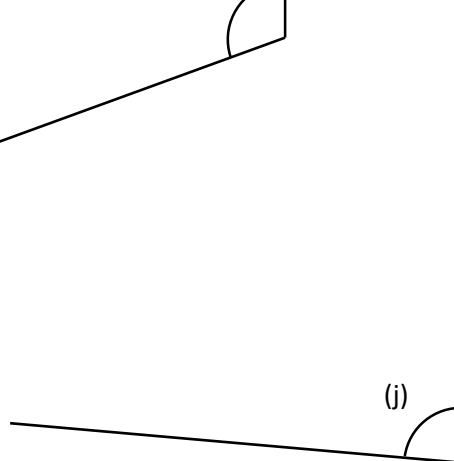
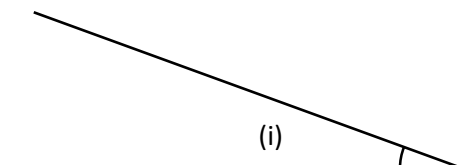
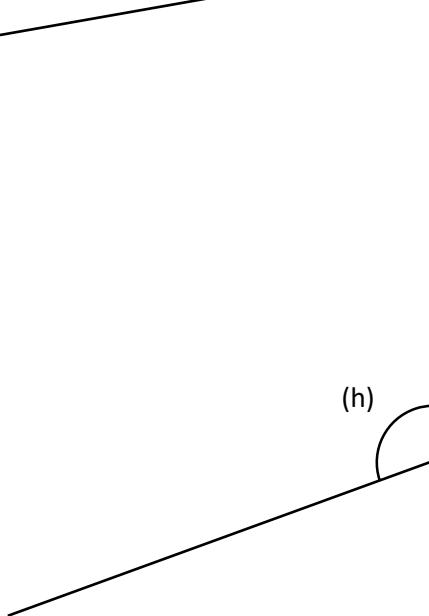
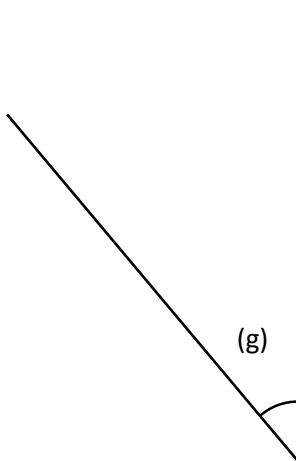
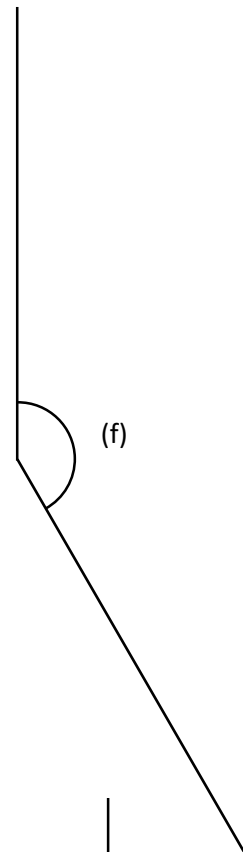
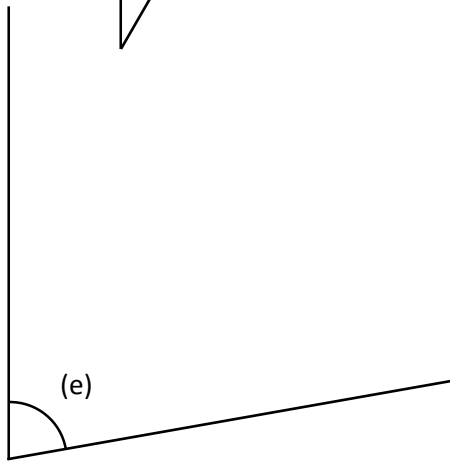
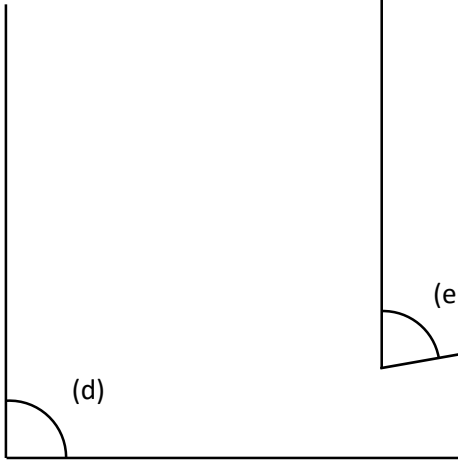
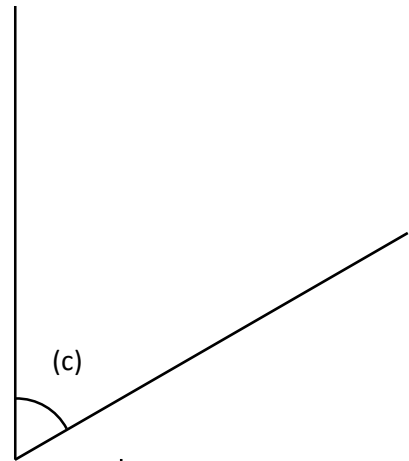
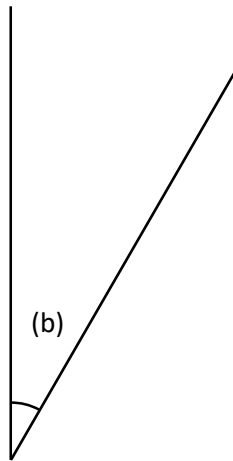
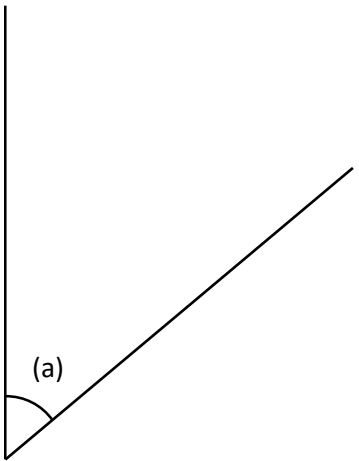


Exercise 6

5

Skill

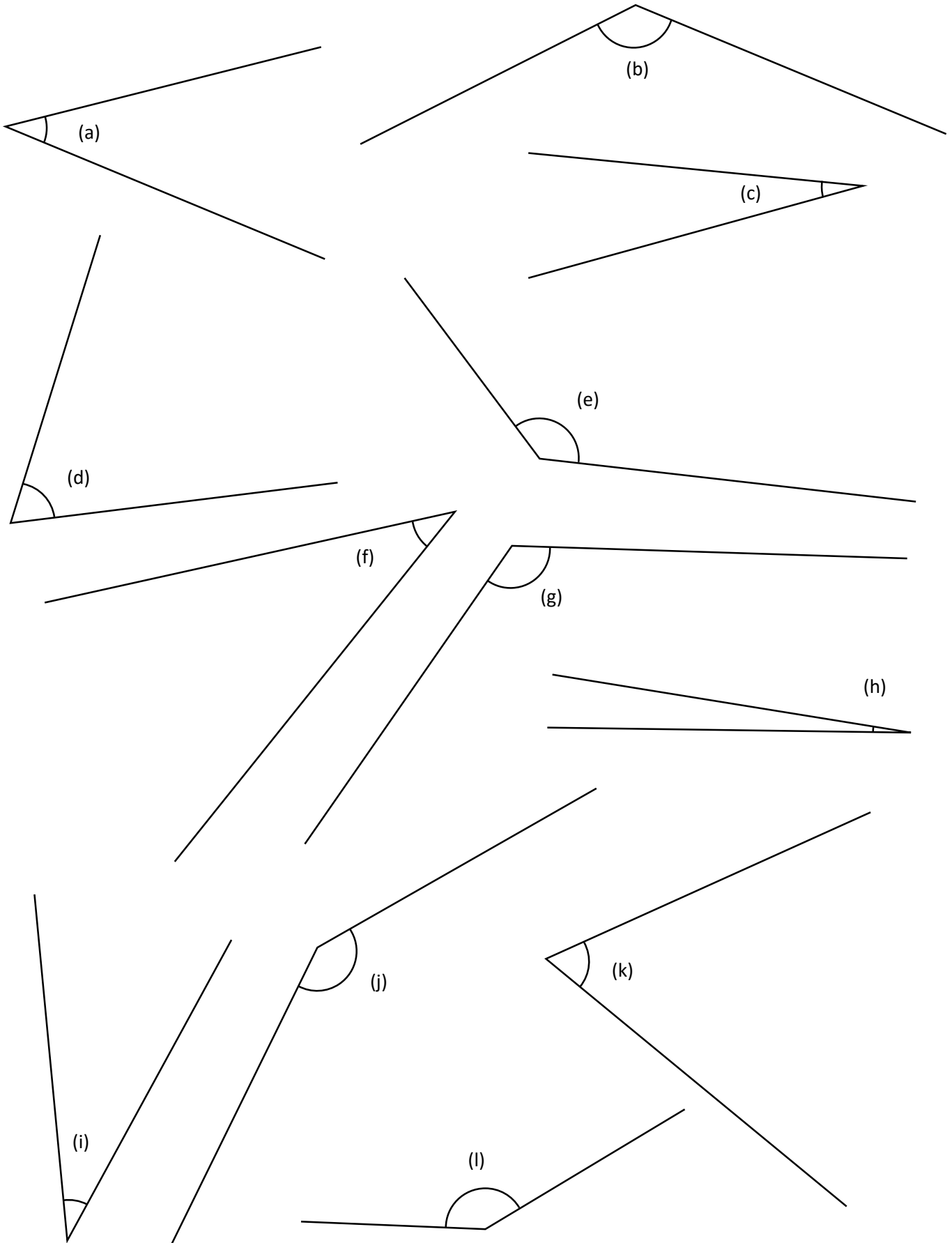
Measure the following angles.



Exercise 7

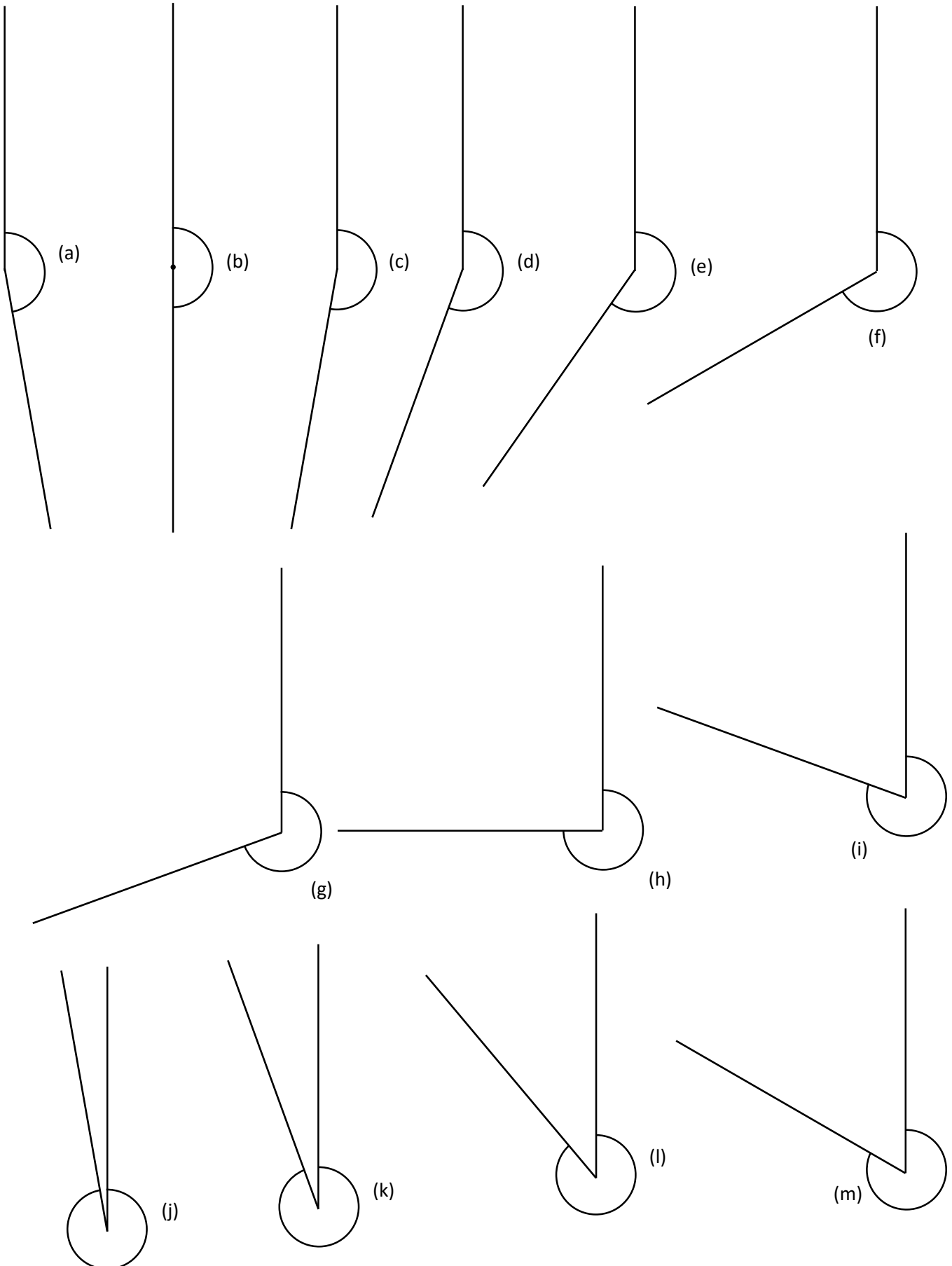
5

Measure the following angles and state what **type** of angles they are.



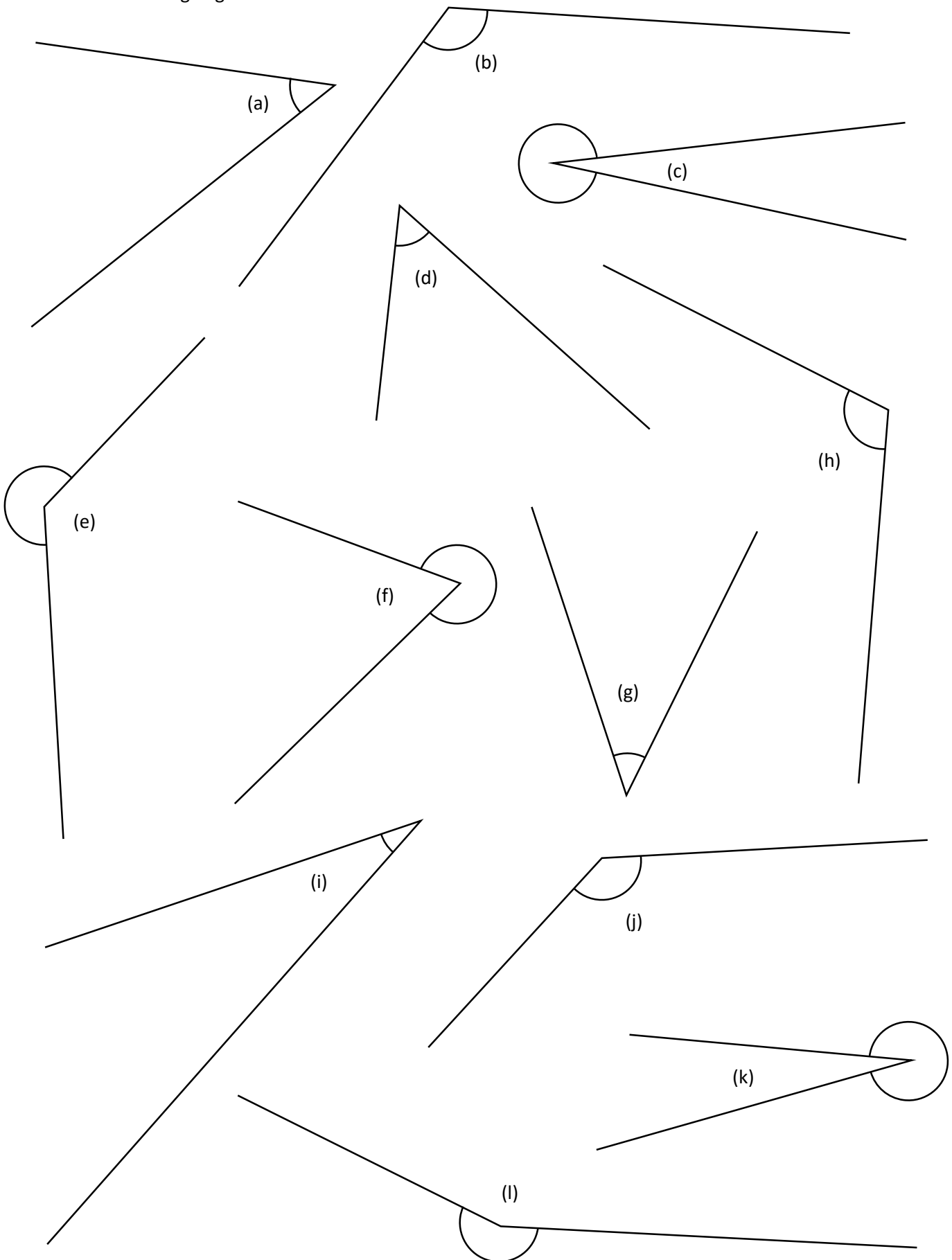
Exercise 8

5

Measure the following angles.

Exercise 9

5

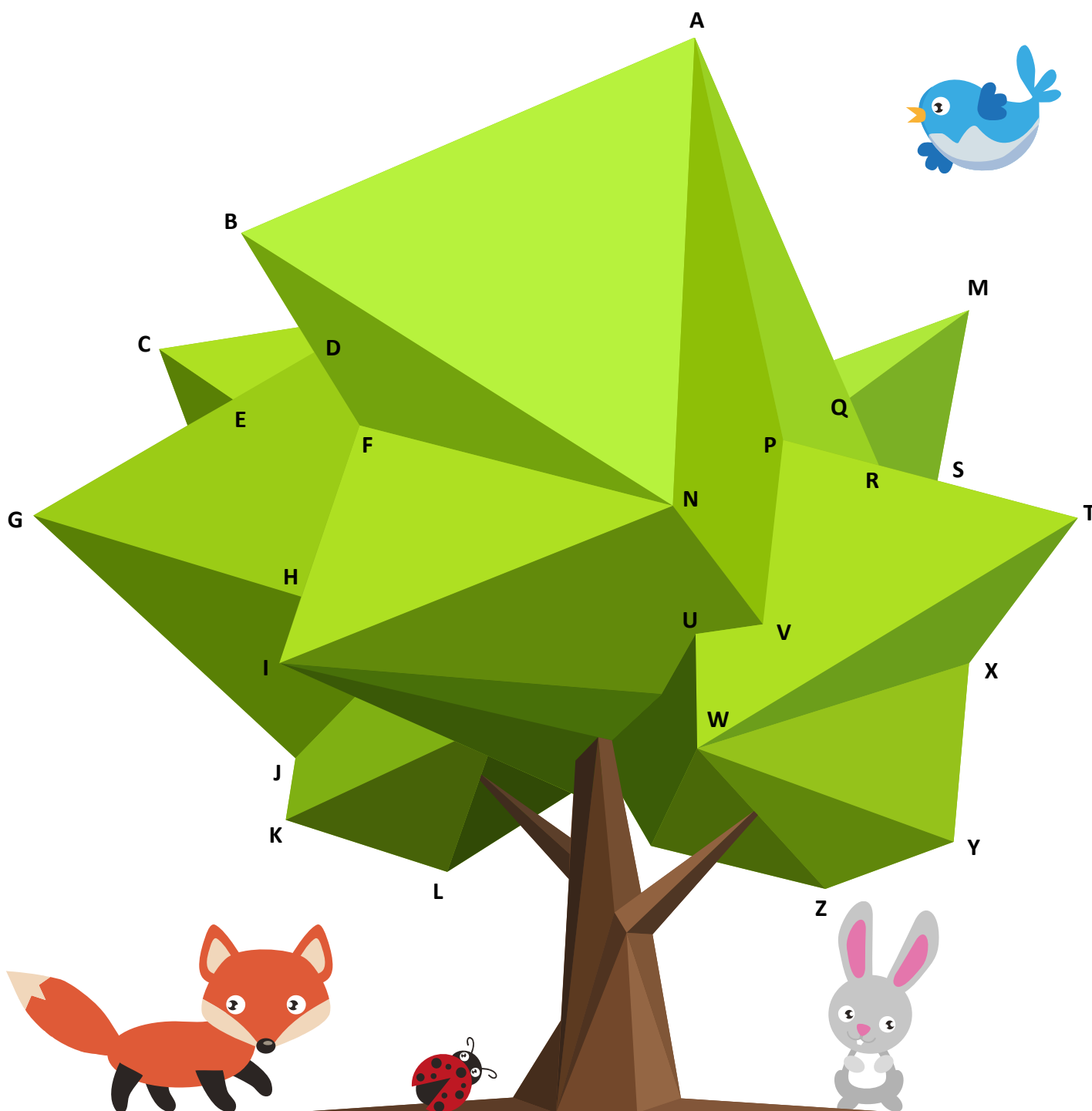
Measure the following angles.

Exercise 10**Measure** the following angles.

5

Applying

- | | | |
|----------------------------------|----------------------------------|----------------------------------|
| (a) $\widehat{BAN}$ | (b) $\widehat{ABN}$ | (c) $\widehat{ANB}$ |
| (d) $\widehat{BFH}$ | (e) $\widehat{HGE}$ | (f) $\widehat{NIF}$ |
| (g) $\widehat{PTW}$ | (h) Reflex angle $\widehat{PTW}$ | (i) $\widehat{QMS}$ |
| (j) $\widehat{TXW}$ | (k) $\widehat{WUV}$ | (l) $\widehat{VNA}$ |
| (m) Reflex angle $\widehat{HGJ}$ | (n) $\widehat{LKJ}$ | (o) Reflex angle $\widehat{ZYX}$ |



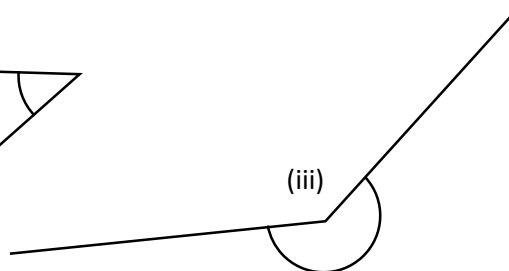
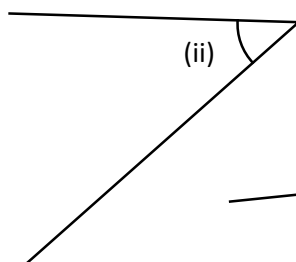
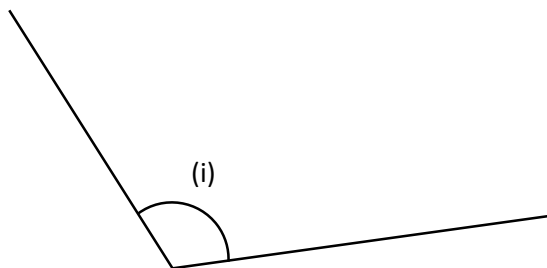
Exercise 11 (Revision)

(a) **Estimate** the size of each of the following angles in degrees.

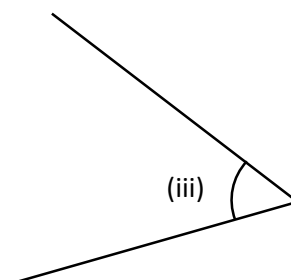
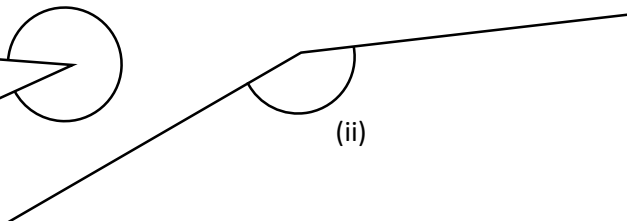
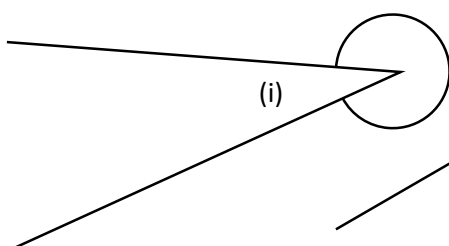
Write down your estimate before **measuring** the size of the angle using a protractor.

5

Skill



(b) **Measure** the following angles and state what **type** of angles they are.

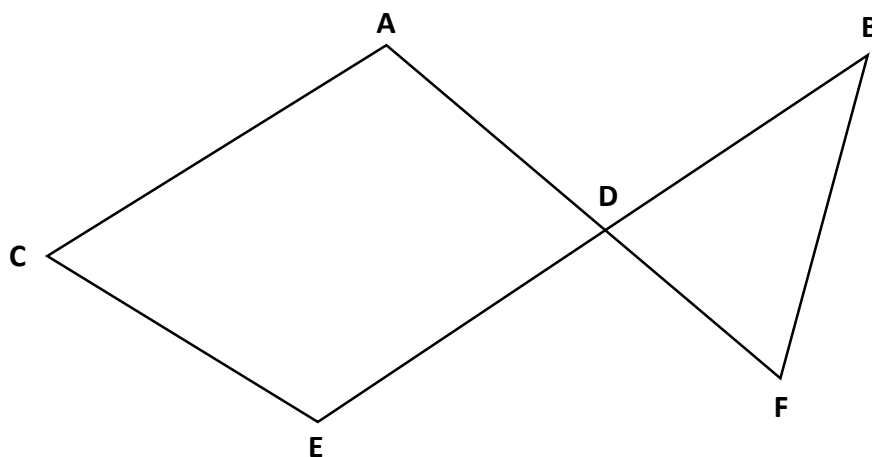


(c) **Measure** the following angles.

(i) $\widehat{ADE}$

(ii) $\widehat{DBF}$

(iii) Reflex angle $\widehat{ACE}$



Evaluation

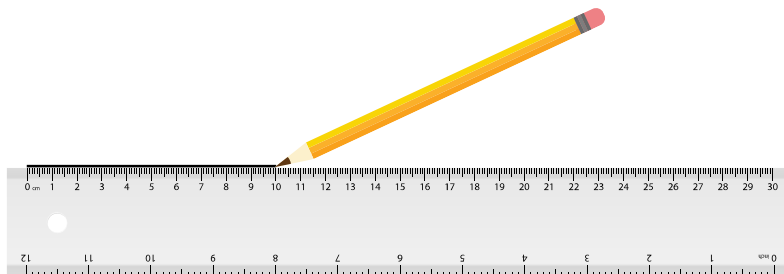
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Drawing Angles

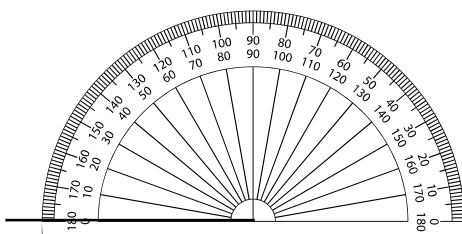
It is possible to draw any angle using a protractor, and it is possible to draw some special angles using a compass.

Drawing Angles Using a Protractor

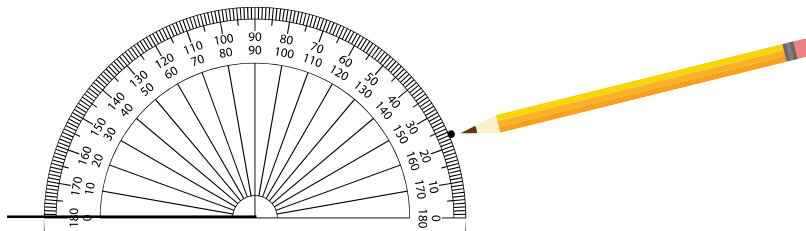
- (1) Draw a straight line using a ruler. (Allow plenty of space around the line in order to draw the angle.)



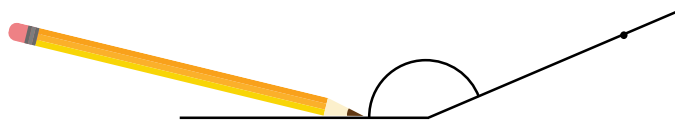
- (2) Place the centre of the protractor on one end of the line. Turn the protractor so that the angle 0° lies on the line.



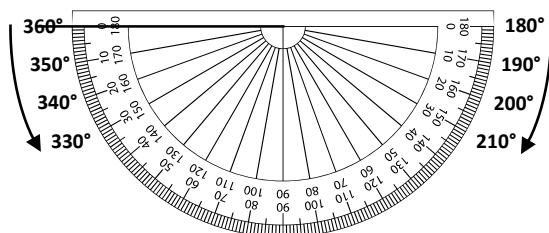
- (3) Count up from 0° to the angle you are attempting to draw. Mark the angle (e.g. 157°) using a pencil.



- (4) Draw a straight line through the dot, starting from where the centre of your protractor was. Draw the arc of the angle.



Note: If you are drawing a **reflex** angle, hold your protractor as shown below during step (2). Then, during step (3), either count up from 180° on the right side of the protractor, or count down from 360° on the left side of the protractor.



Exercise 12

Draw the following angles using a protractor.

- | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| (a) 13° | (b) 40° | (c) 78° | (d) 93° | (e) 107° | (f) 138° | (g) 164° |
| (h) 193° | (i) 216° | (j) 254° | (k) 265° | (l) 284° | (m) 303° | (n) 341° |

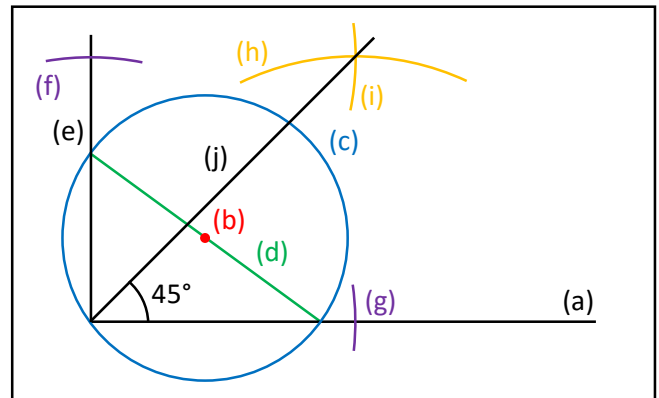
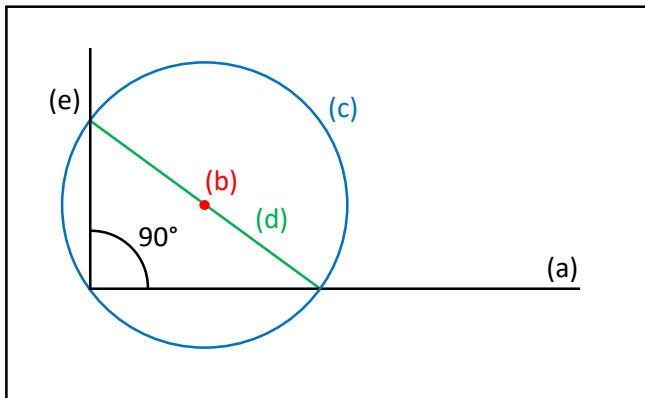
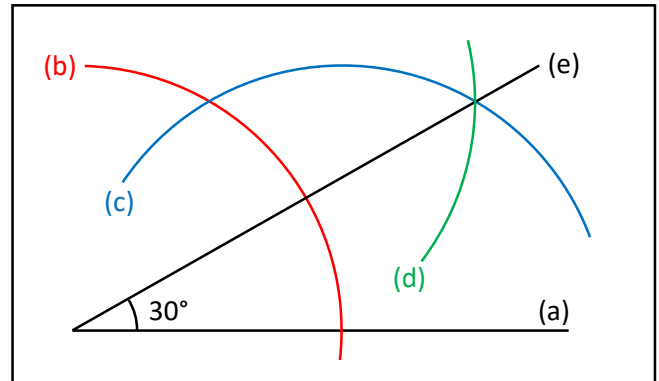
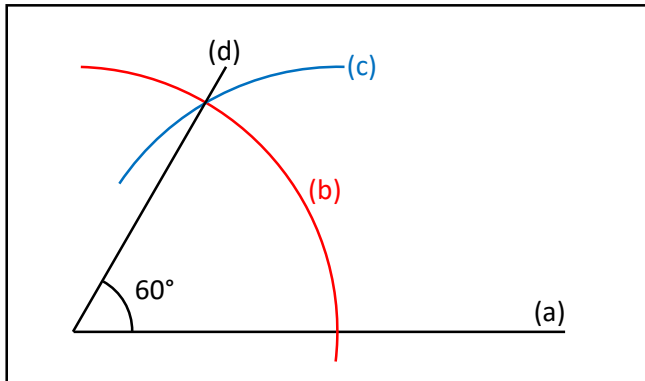
5

Skill

Drawing Angles Using a Compass

It is possible to use a compass and a ruler only to draw the angles 30° , 45° , 60° and 90° .

Do not erase the construction arcs after completing an angle!

**Exercise 13**

Draw the angles 60° , 30° , 90° and 45° using only a compass and a ruler.

5

Exercise 14 (Revision)

(a) Draw the following angles using a protractor.

(i) 25°

(ii) 74°

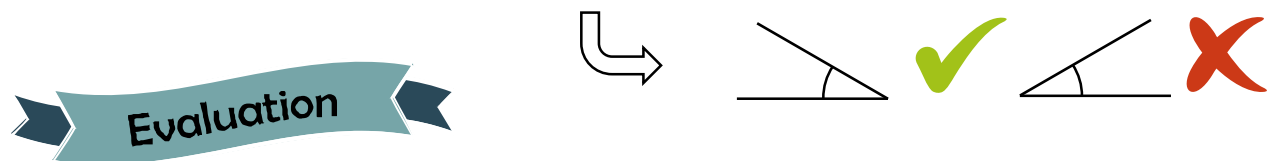
(iii) 172°

(iv) 208°

(v) 270°

(vi) 317°

(b) Repeat Exercise 13, but draw your angles so that the mouth of the angle points to the left, not to the right.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...

My Level ☐ My Target ☐

Calculating Angles

With enough information, it is possible to find the size of some angles without using a protractor, by **calculation**.

Angles Around a Point

From the information on page 3, we know that a complete turn is 360° . It follows that the **angles around any point** must sum to 360° .

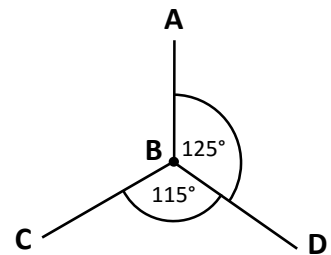
Example

In the diagram on the right, it is possible to find the size of the angle $\hat{A}BC$ without using a protractor, as the sum of $\hat{A}BC$, $\hat{C}BD$ and $\hat{A}BD$ must be 360° .

$$125^\circ + 115^\circ = 240^\circ$$

$$\hat{A}BC = 360^\circ - 240^\circ$$

$$\hat{A}BC = 120^\circ.$$

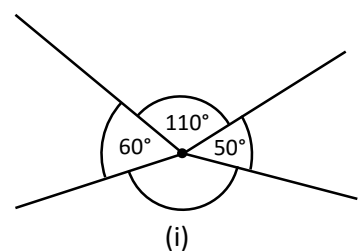
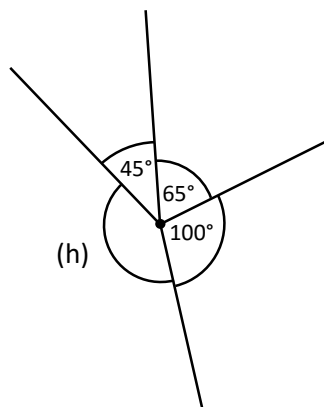
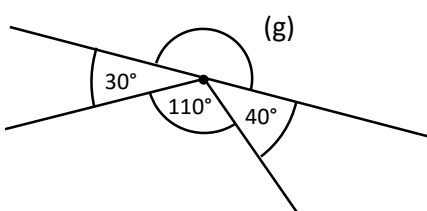
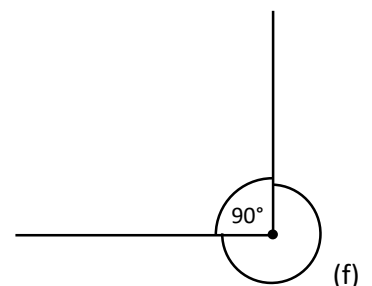
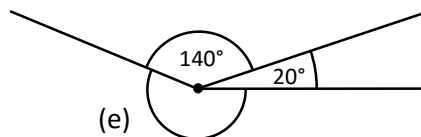
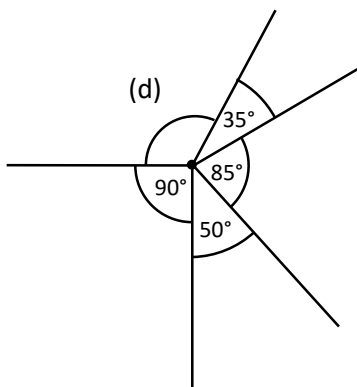
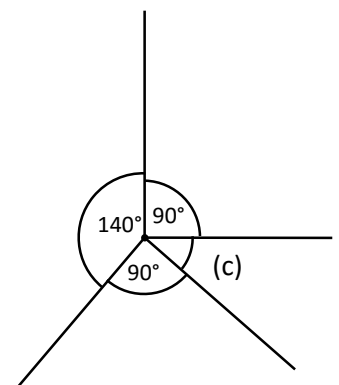
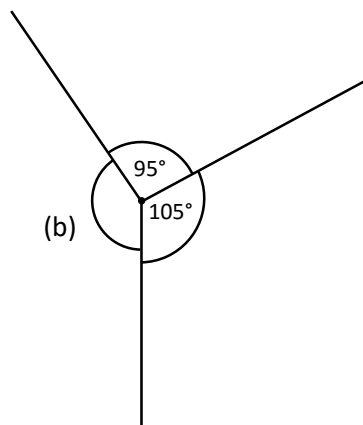
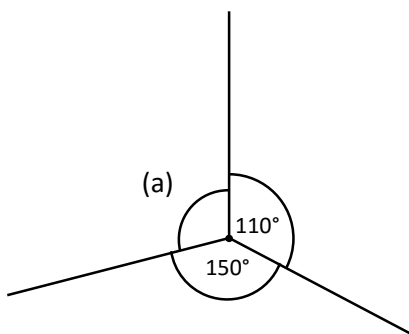


6

Skill

Exercise 15

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Angles on a Straight Line

The angle for half a turn is 180° , so the angles on any **straight line** must sum to 180° .

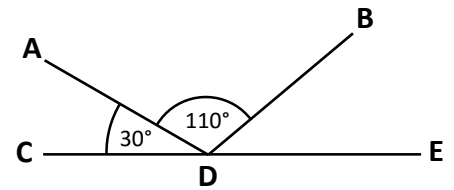
Example

In the diagram on the right, it is possible to find the size of the angle $\widehat{BDE}$ without using a protractor, as the sum of $\widehat{ADC}$, $\widehat{ADB}$ and $\widehat{BDE}$ must be 180° .

$$30^\circ + 110^\circ = 140^\circ$$

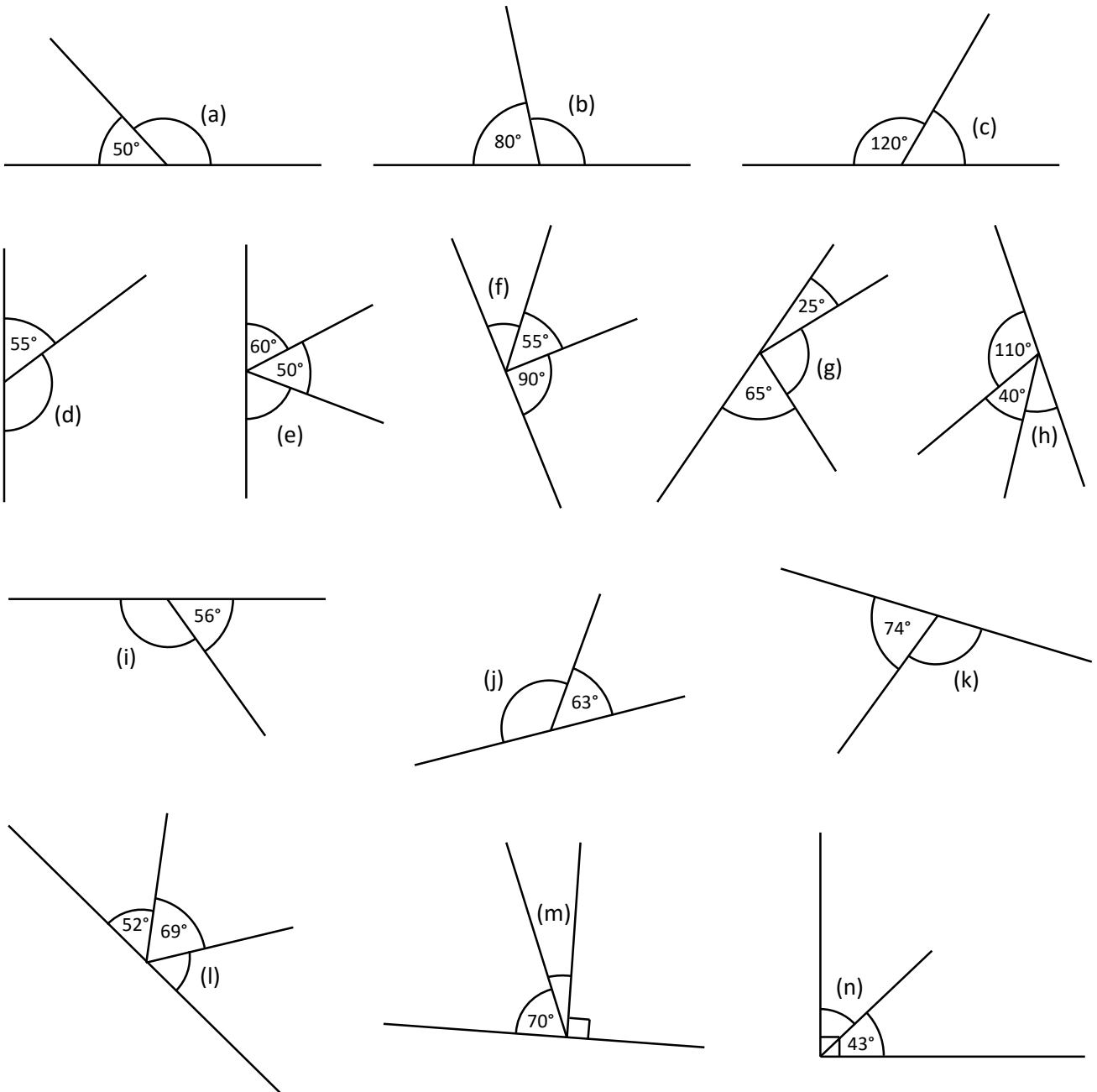
$$\widehat{BDE} = 180^\circ - 140^\circ$$

$$\widehat{BDE} = 40^\circ.$$



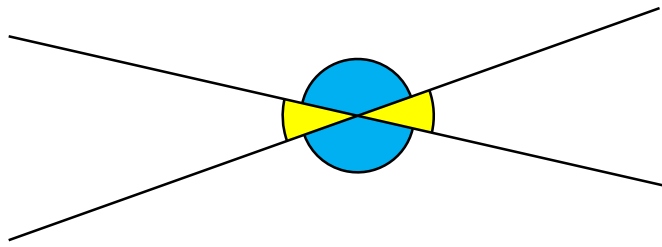
Exercise 16

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Vertically Opposite Angles

Every time that two lines cross, four angles are formed around the vertex. The angles that are opposite each other (not next to each other) are **equal**. We call these angles **vertically opposite angles**.

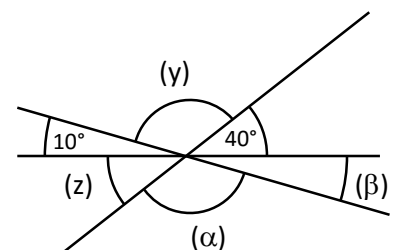
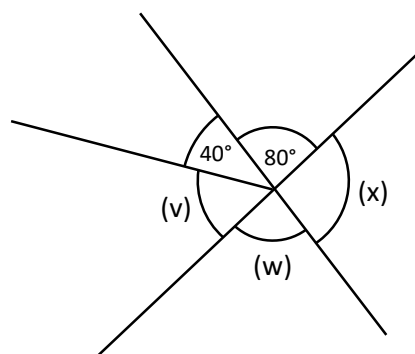
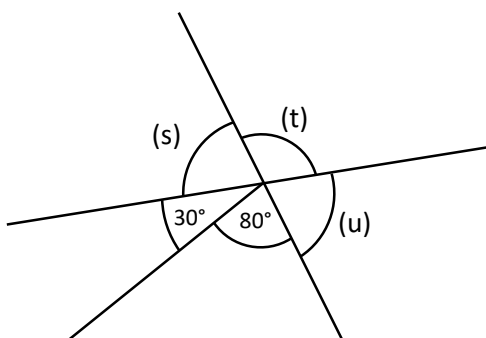
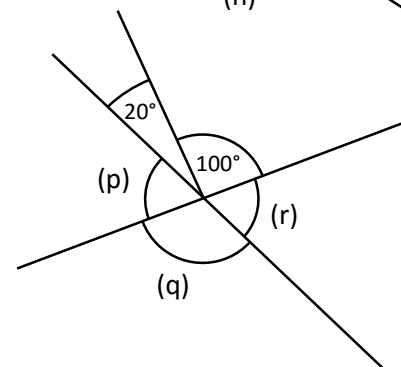
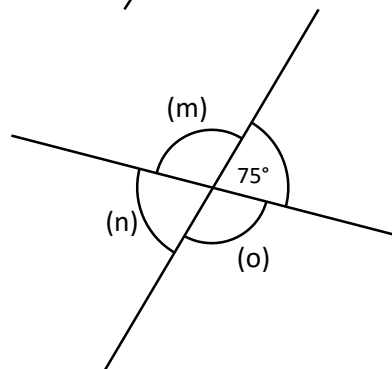
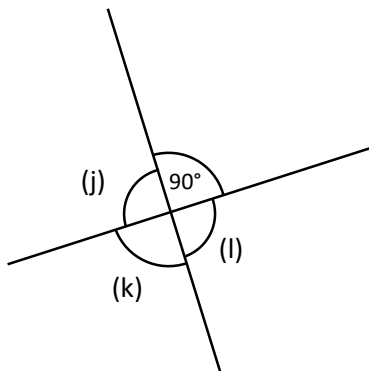
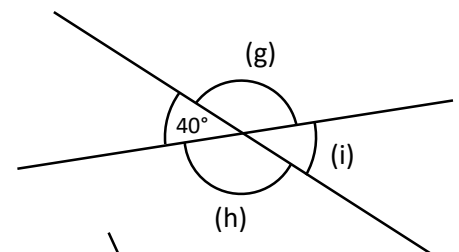
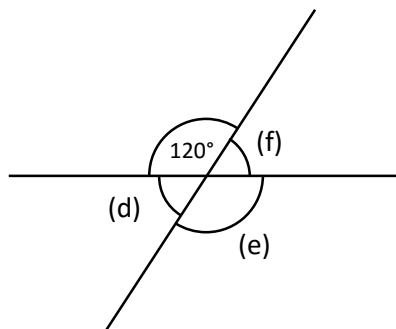
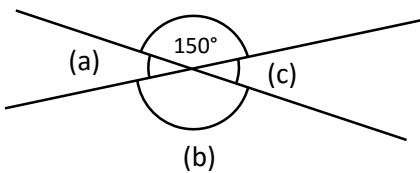


Exercise 17

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

6

Skill



Challenge!

The width of a football goal is 8 yards. The shortest distance from the goal to the penalty spot is 12 yards. Construct a scale drawing in order to find the angle shown in the picture, which is the angle to score a penalty.



Applying

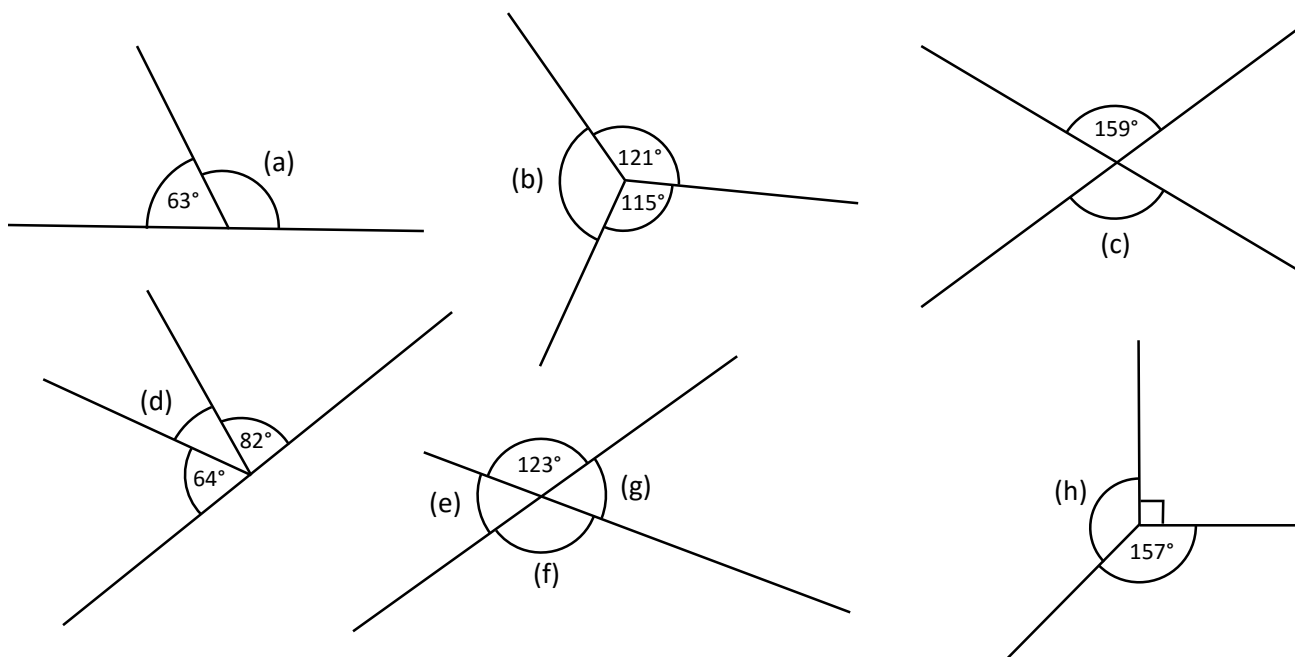


Exercise 18

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

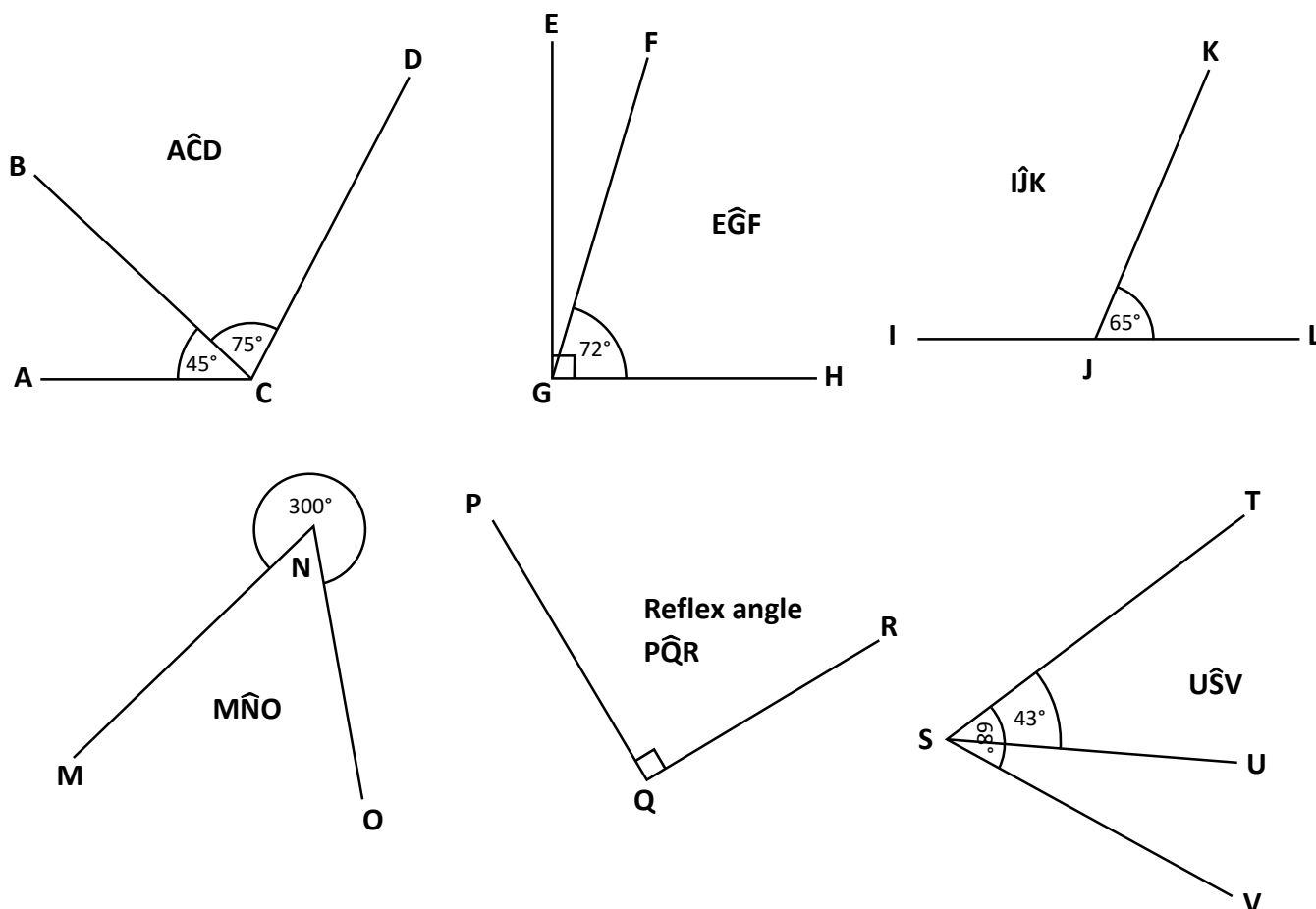
6

Skill

**Exercise 19**

Calculate the **size** of each of the named angles. (The diagrams are not drawn to scale.)

6

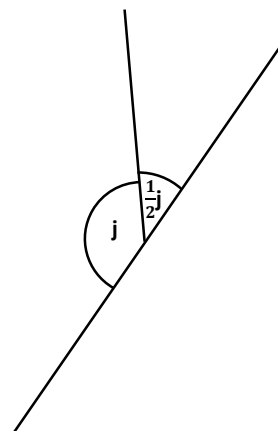
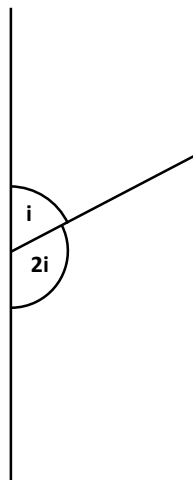
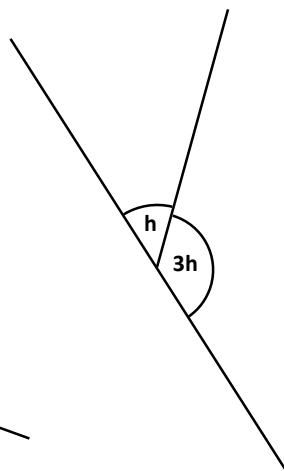
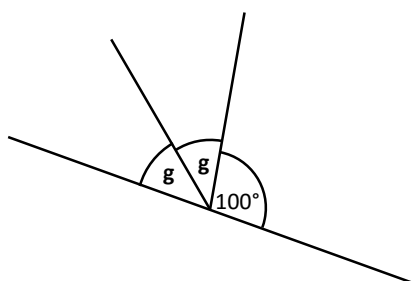
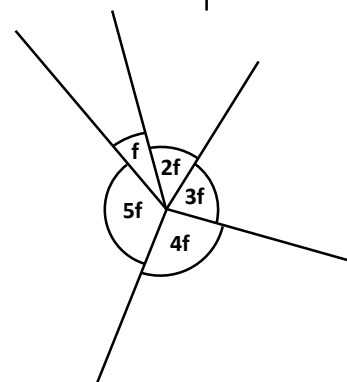
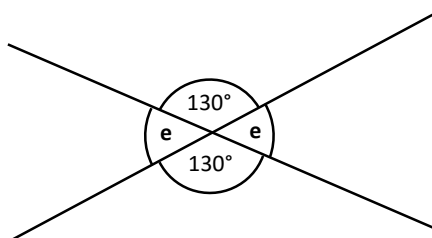
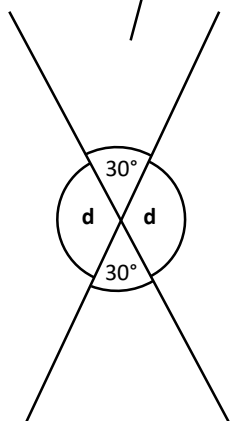
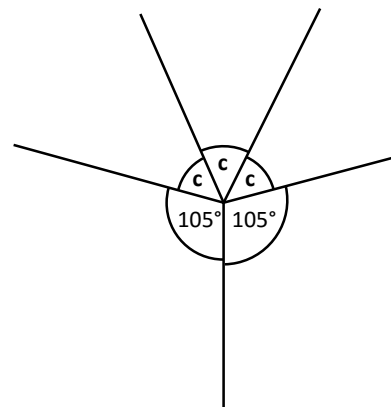
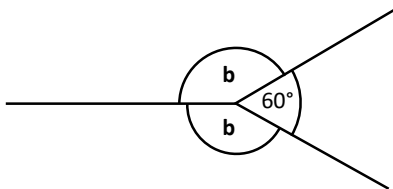
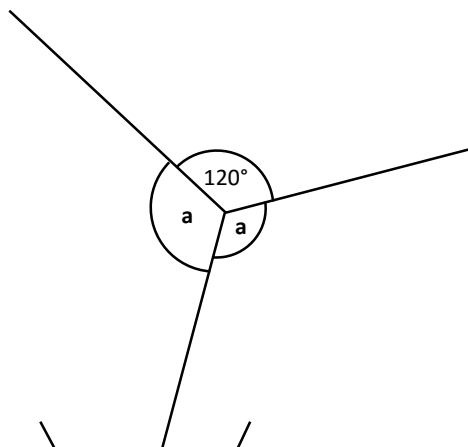


Can you find the following angles? (The diagrams are not drawn to scale.)



6

Extension



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Angles in a Triangle

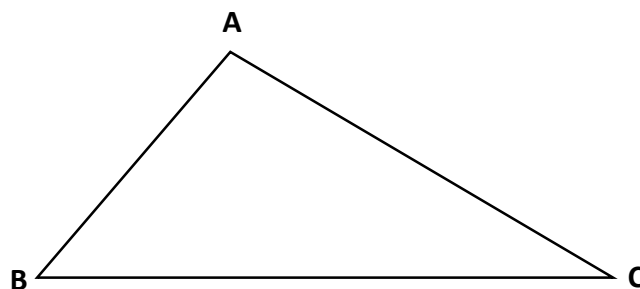
The angles inside a shape are called **interior angles**.

Use your protractor to measure the three interior angles of the triangle shown on the right.

$$\hat{A}BC = \underline{\hspace{2cm}}^\circ$$

$$\hat{B}AC = \underline{\hspace{2cm}}^\circ$$

$$\hat{A}CB = \underline{\hspace{2cm}}^\circ$$



What is the total of your answers?

$$\text{Total} = \underline{\hspace{2cm}}^\circ$$

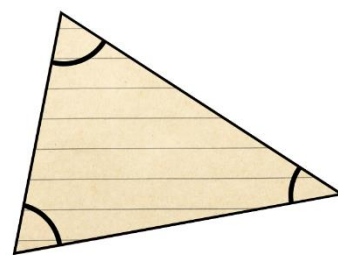
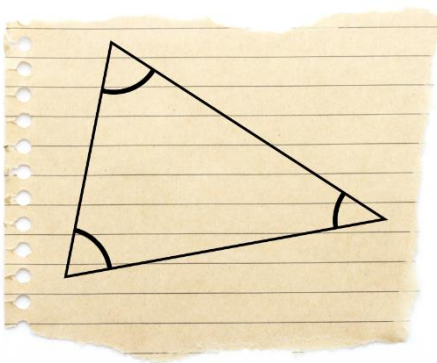
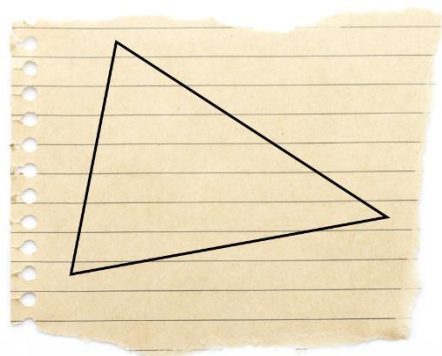
Proving the Sum of the Interior Angles of a Triangle

Follow the steps below to show that the sum of the interior angles of a triangle is always the same.

1. Draw any triangle on a piece of scrap paper. The triangle does **not** need to look like the triangle below.

2. Mark the interior angles of your triangle with thick arcs.

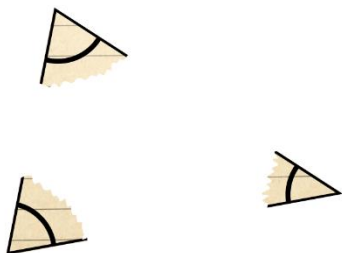
3. Cut your triangle out using a pair of scissors.



4. Tear off the corners of your triangle.

5. Arrange the corners to form a straight line.

6. Glue your corners (in a straight line) in the box below.



7. Complete the following sentence:

The sum of the interior angles of a triangle is always $\underline{\hspace{2cm}}^\circ$



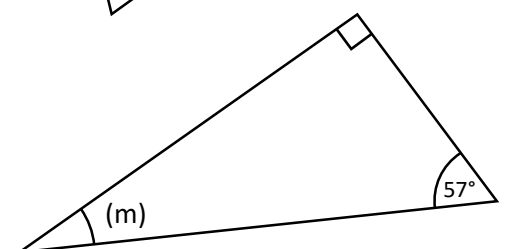
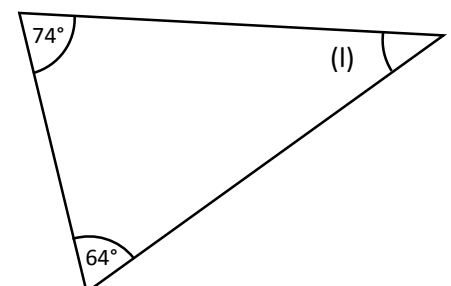
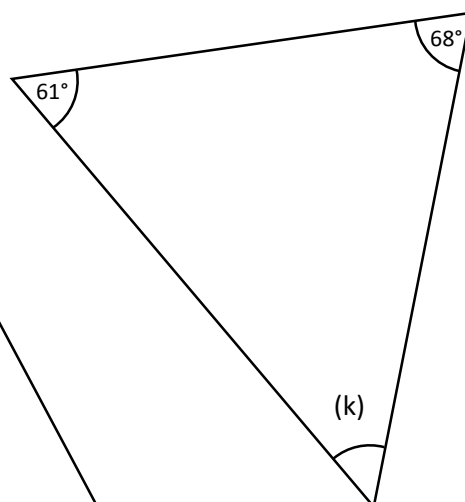
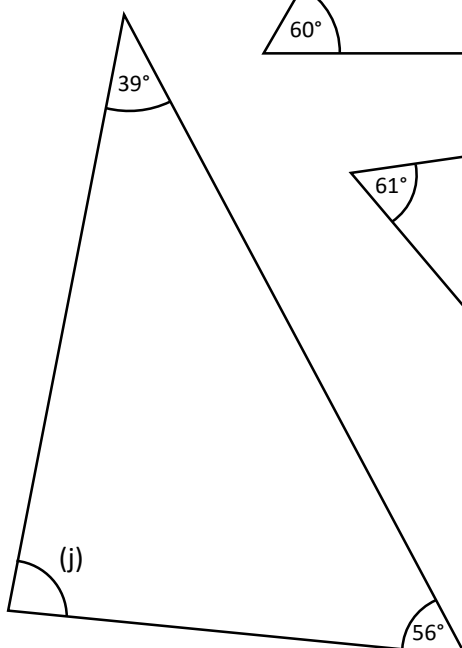
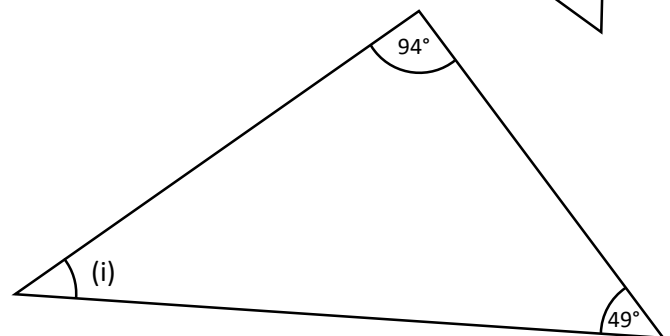
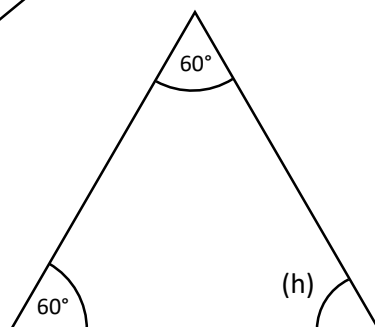
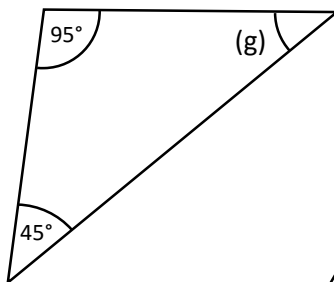
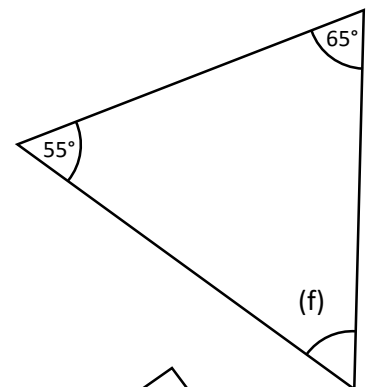
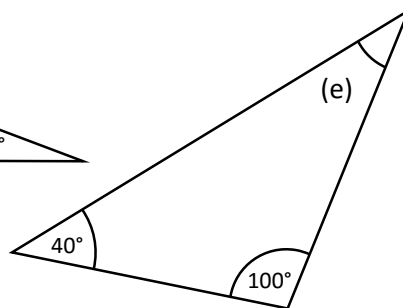
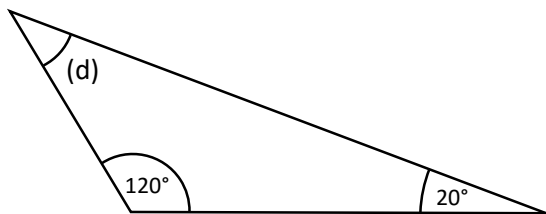
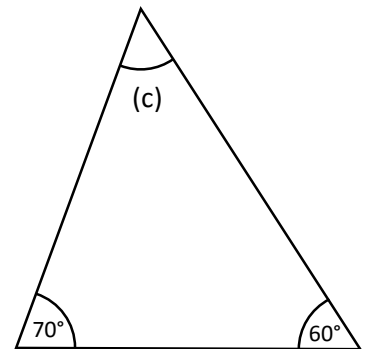
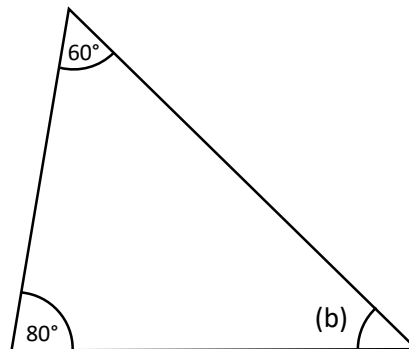
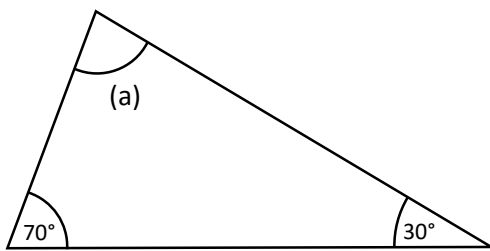
Exercise 20

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



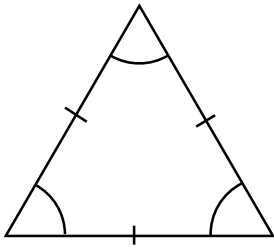
6

Skill

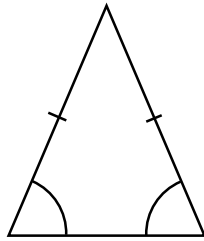


Types of Triangle

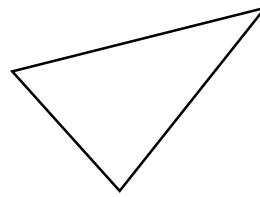
An **equilateral** triangle has three equal sides and three equal angles.



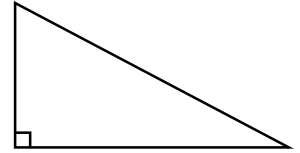
An **isosceles** triangle has two equal sides and two equal angles.



A **scalene** triangle does not have any equal sides or any equal angles.



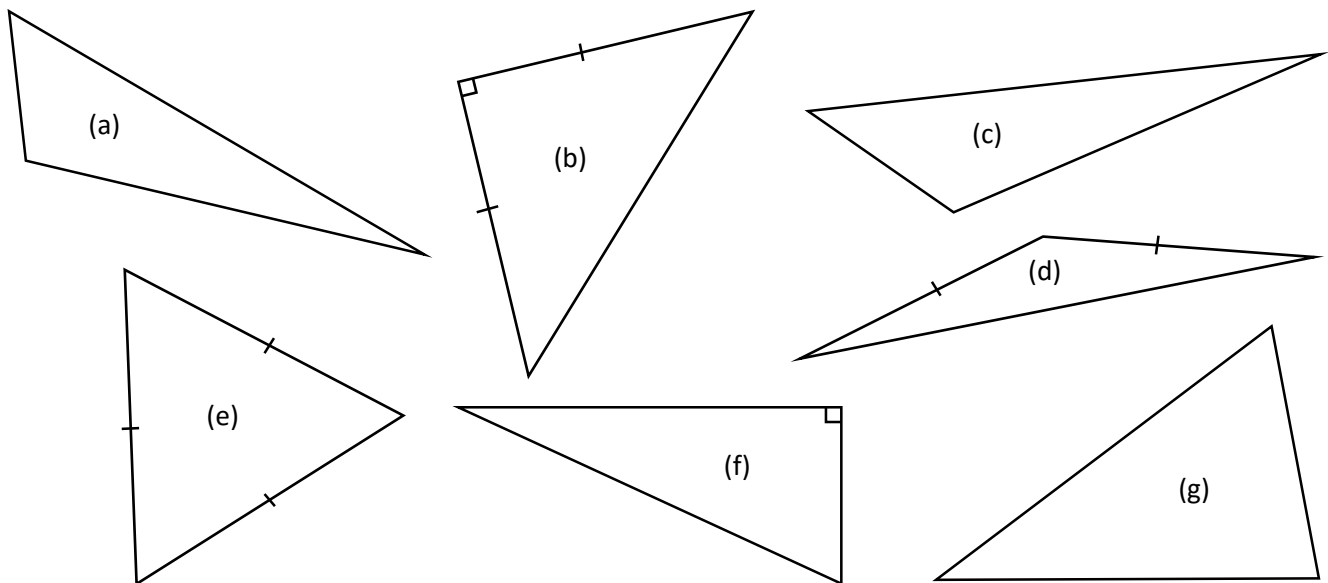
A **right-angled** triangle has one angle that is a right angle, or 90° .

**Exercise 21**

What **type** of triangles are shown below?

6

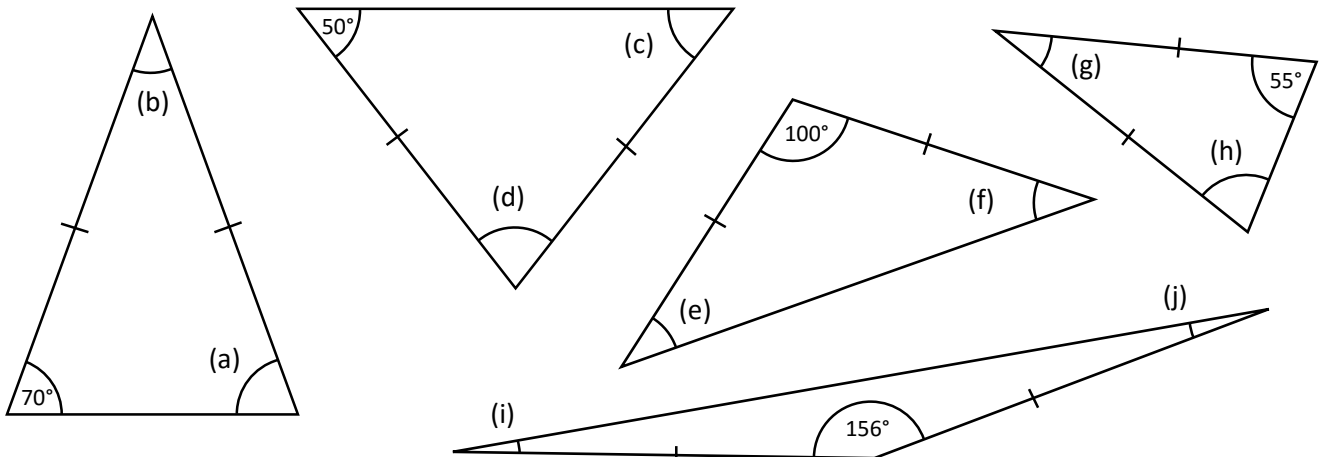
Skill

**Exercise 22**

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



6



Exercise 23

(a) Dewi draws a triangle with angles 50° , 80° and 50° .

What type of triangle is this?

(b) Explain whether or not it is possible to draw a triangle with two obtuse angles.

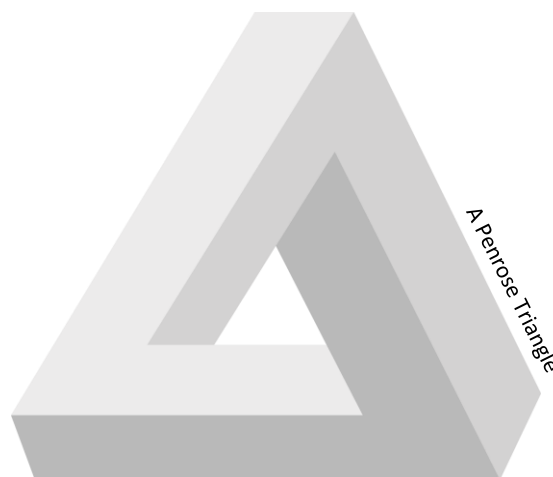
(c) Llinos draws an isosceles triangle that is also a right-angled triangle.

What are the size of the angles in Llinos' triangle?

(d) Explain whether or not it is possible to draw an equilateral triangle that is also a right-angled triangle.

(e) Tomos draws a triangle and measures its angles as 95° , 95° and 170° .

Erin measures the angles of the same triangle and gets 85° , 85° and 10° . Explain who has measured the angles correctly. What mistake has the other person made?

**Investigation**

Draw triangles on the dotted paper below (each corner of a triangle must be placed on a dot).

Identify the type of triangle underneath the dots. How many different triangles are possible?

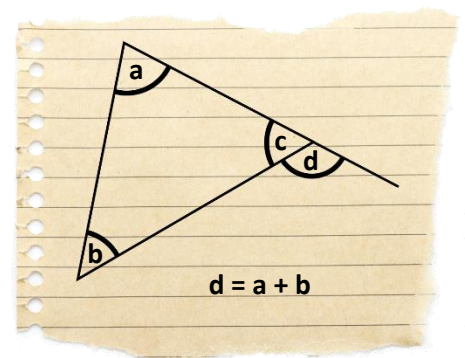
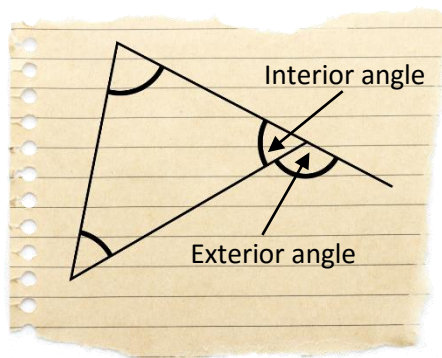
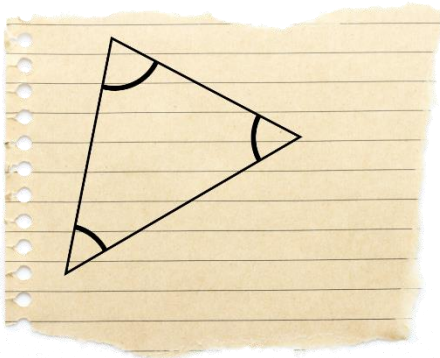
Isosceles; Right-Angled			

The Exterior Angle of a Triangle

Consider any triangle.

By extending one of the sides of the triangle, we create an **exterior angle**.

The size of the exterior angle is the sum of the **two opposite interior angles**.

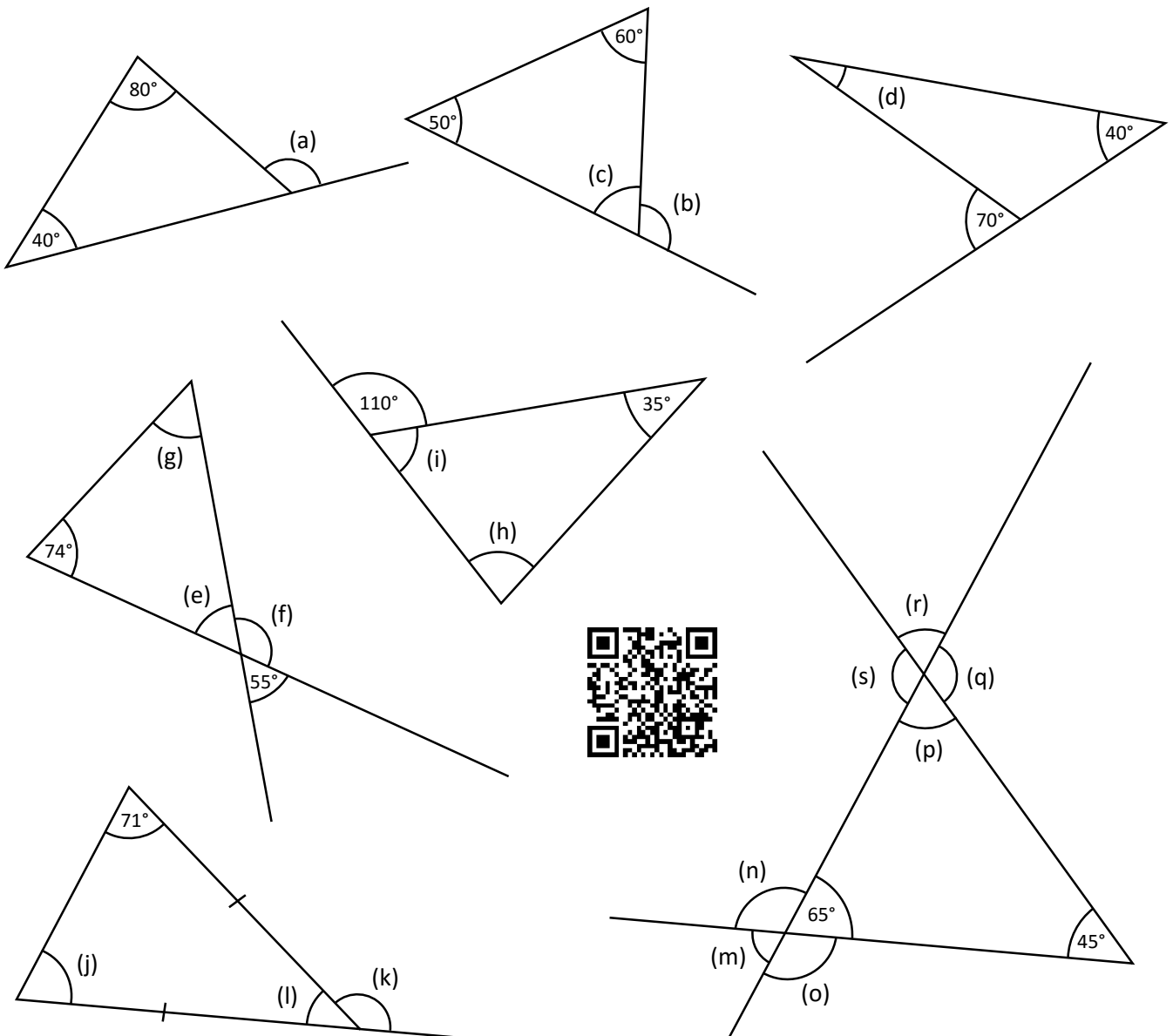


Exercise 24

Calculate the **size** of the marked angles. (The diagrams are not drawn to scale.)

6

Skill



Constructing Triangles

Using a ruler, a protractor and a compass, it is possible to draw any triangle if we know one of the following.

- The length of **one side** of the triangle and the size of **two of the angles**.
- The length of **two sides** of the triangle and the size of the **angle joining** these two sides.
- The length of **all three sides** of the triangle.

Exercise 25

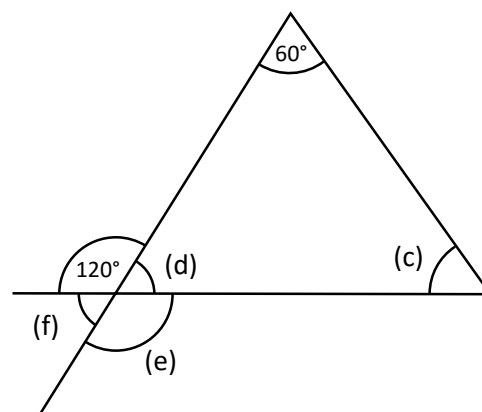
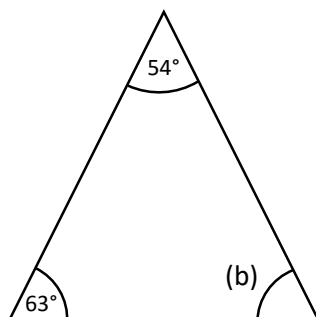
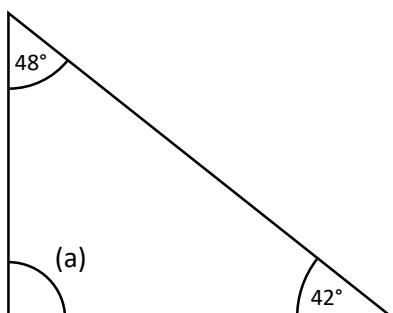
Use a ruler, a protractor and a compass to draw a triangle ABC of the following forms.

- (a) $AB = 5\text{cm}$; $\widehat{BAC} = 40^\circ$; $\widehat{ABC} = 60^\circ$.
 (b) $AB = 5.5\text{cm}$; $\widehat{BAC} = 30^\circ$; $\widehat{ABC} = 80^\circ$.
 (c) $AB = 6.2\text{cm}$; $\widehat{BAC} = 45^\circ$; $\widehat{ABC} = 36^\circ$.
 (d) $AB = 4.8\text{cm}$; $\widehat{BAC} = 27^\circ$; $\widehat{ACB} = 79^\circ$.
 (e) $AB = 6\text{cm}$; $BC = 8\text{cm}$; $\widehat{ABC} = 64^\circ$.
 (f) $AB = 5\text{cm}$; $BC = 6.2\text{cm}$; $\widehat{ABC} = 105^\circ$.
 (g) $AC = 7\text{cm}$; $BC = 6\text{cm}$; $\widehat{ACB} = 90^\circ$.
 (h) $AB = 4.7\text{cm}$; $BC = 4.7\text{cm}$; $AC = 4.7\text{cm}$.
 (i) $AB = 6.7\text{cm}$; $BC = 7.2\text{cm}$; $\widehat{ABC} = 38^\circ$.
 (j) $AB = 4.5\text{cm}$; $BC = 7.3\text{cm}$; $AC = 7.3\text{cm}$.
 (k) $AB = 5.7\text{cm}$; $BC = 8.1\text{cm}$; $AC = 9.2\text{cm}$.
 (l) $AB = 3.5\text{cm}$; $BC = 8\text{cm}$; $AC = 6\text{cm}$.



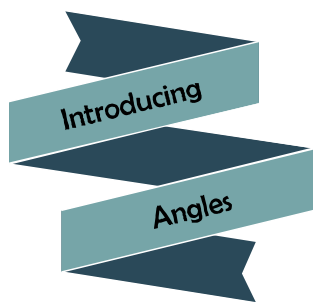
Exercise 26 (Revision)

Find the **size** of the marked angles. What **type** of triangles are shown? (The diagrams are not drawn to scale.)



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know the names for the different types of angle, e.g. acute angle.			1, 10	
I know how to name an angle, e.g. $\hat{ABC}$.			5	
I know how to estimate an angle.				
I know how to measure an angle with a protractor.			3	
I know how to draw an angle using a protractor and ruler .			2	
I know how to draw the angles 60° , 30° , 90° , 45° using only a compass and ruler .			4	
I know what the angles around a point total to.			6, 12	
I know what the angles on a straight line total to.			6, 12	
I know how to use vertically opposite angles to find the size of some angles.			6, 12	
I know the total of the angles in any triangle .			6, 9, 12	
I know what the different types of triangle are, e.g. equilateral triangle.			7, 8	
I know how to use external angles to find the size of some angles.			6	
I know how to draw different triangles using a ruler, compass and protractor.			11	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

7

Data Handling

and Statistics

Name:

Contents

Chapter	Mathematics	Page Number
The Mean	Calculating the mean. Calculating the mean from a table. Comparing two distributions.	3
The Range	Calculating the range. Comparing two distributions.	9
Bar Charts	Types of data. Bar charts. Frequency diagrams. Interpreting charts.	12
Mean of Grouped Data	Estimating the mean from a table.	20
Pie Charts	Drawing pie charts. Interpreting pie charts.	25



The Mean

In the qualifying games for Euro 2016, Wales' football team scored the following number of goals.

2, 0, 2, 0, 3,
1, 1, 0, 0, 2.

Which number would best describe the number of goals scored by Wales per game? When we are looking for a number of this type, we are looking for an **average** for the data. An average is a number which attempts to represent the typical or middle value of the data. The **mean** is an example of an average.



Calculating the Mean

In order to calculate the mean of a set of data:

- (1) Find the **total** of all the data items.
- (2) **Divide** by the number of data items.



For the data about Wales' goals, the total of all the data items would be

$$2 + 0 + 2 + 0 + 3 + 1 + 1 + 0 + 0 + 2 = 11.$$

It follows that the mean is $11 \div 10 = 1.1$. This means that, on average, Wales scored around 1 goal per game.

Exercise 1

Find the mean number of goals scored by the following football teams during the first games of the new football season.

(a) Arsenal	5,	0,	2,	0,	4.
(b) Chelsea	0,	2,	1,	2,	3.
(c) Everton	2,	1,	1,	2,	0.
(d) Liverpool	2,	3,	0,	2,	1.
(e) Manchester United	3,	2,	3,	4,	0.
(f) Tottenham Hotspur	2,	0,	0,	4,	1.

Exercise 2

In Exercise 1, which team has the **largest** mean number of goals?

Exercise 3

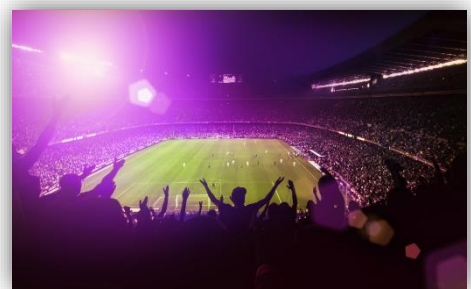
Sara asked her friends how much pocket money they received. She wrote down their responses:

£1, £3, £2, £5, £2, £10, £8, £7.

What is the **mean amount** of pocket money?

5

Skill



Exercise 4

Here is some information about five children.

5

Applying

Elin

Height: 132cm
Weight: 45kg
Shoe size: 3

Hari

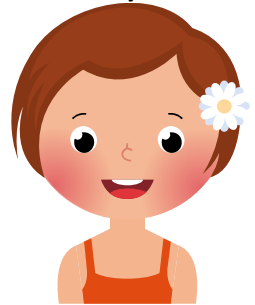
Height: 148cm
Weight: 50kg
Shoe size: 5

Catrin

Height: 136cm
Weight: 39kg
Shoe size: 4

Siôn

Height: 150cm
Weight: 49kg
Shoe size: 5

Esyllt

Height: 150cm
Weight: 52kg
Shoe size: 6

- What is the mean height of these children?
- Which children are taller than the mean height?
- What is the mean weight of these children?
- Which children are lighter than the mean weight?
- What is the mean shoe size of these children?
- Can you buy shoes of mean shoe size?

**Exercise 5**

Here is some information about four babies.

**Carwyn**

Weight: 8kg Height: 69cm

Bethan

Weight: 5kg Height: 70cm

Eryl

Weight: 7kg Height: 75cm

Osian

Weight: 9kg Height: 77cm

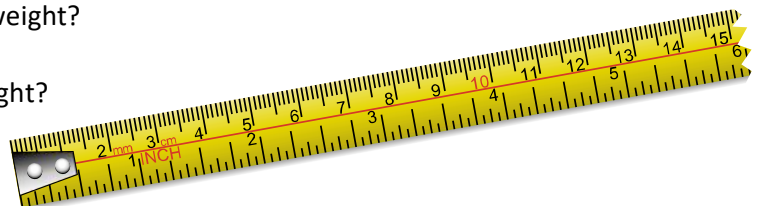
- What is the mean weight of these 4 babies?
- Which of the babies are heavier than the mean weight?
- What is the mean height of these 4 babies?
- Which of the babies are taller than the mean height?

Exercise 6

Here is the weekly shopping cost of 15 families.

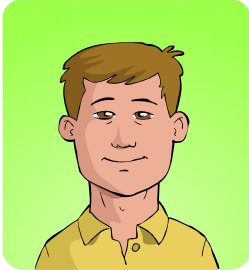
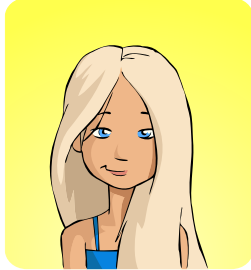
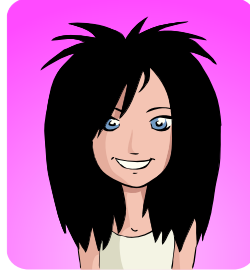
£75 £86 £83 £62 £86 £78 £64 £94 £101 £79 £91 £71 £59 £91 £95

Find the **mean amount** spent, per week, by these families.



Exercise 7

Here is some information about ten adults.

Sophie  Age: 32 Height: 154cm Number of brothers: 2	David  Age: 38 Height: 163cm Number of brothers: 1	Jessica  Age: 18 Height: 145cm Number of brothers: 3	Ben  Age: 24 Height: 165cm Number of brothers: 0	Ffion  Age: 26 Height: 150cm Number of brothers: 1
Jack  Age: 74 Height: 149cm Number of brothers: 1	Jane  Age: 34 Height: 165cm Number of brothers: 0	Iwan  Age: 30 Height: 155cm Number of brothers: 2	Sara  Age: 44 Height: 139cm Number of brothers: 1	Jeff  Age: 48 Height: 153cm Number of brothers: 0

- What is the mean age of these 10 people?
- Who is older than the mean age?
- What is the mean height of these 10 people?
- Who is shorter than the mean height?
- What is the mean number of brothers of these 10 people?
- How many people have less brothers than the mean amount?
- What is the mean age of the males?
- What is the mean age of the females?
- On average, who is older: the males or the females?
- On average, who is taller: the males or the females?

**Exercise 8**

Dafydd asked his friends how much money they had in their pockets. He wrote down their responses:

99p, 50p, 25p, 12p, 52p, 32p, 90p, 80p.

What is the **mean** of this data?

**Exercise 9**

Here are the times, in seconds, that the females in 7C took to run a 100m race.

14.1, 15.3, 15.5, 15.6, 15.0, 14.7, 13.9, 16.1, 15.8, 14.5, 15.7, 14.6, 16.3, 15.7.

- Find the **mean time taken** to complete the race.
- How many females were **faster** than the mean time?

Calculating the mean from a table

Sometimes, when we have a lot of data, it is easier (and quicker) to use a table to present the data.

Example

The following table shows the shoe size of the learners in 7R.

Shoe size	Number of children
3	4
4	5
5	10
6	9
7	2



In order to calculate the mean shoe size for 7R, we first need to find the **total of all the data values**. To do this, we must consider each row in the table in turn. The first row tells us that four pupils in 7R have a shoe size of 3. The total of the shoe sizes of these four pupils would be $3 + 3 + 3 + 3$, or 3×4 . The other rows of the table would also lead to multiplication sums, as shown below.

Shoe size	Number of children	Multiplication
3	4	$3 \times 4 = 12$
4	5	$4 \times 5 = 20$
5	10	$5 \times 10 = 50$
6	9	$6 \times 9 = 54$
7	2	$7 \times 2 = 14$
Total	30	150



The total of all the data values would be the total of the final column, namely $12 + 20 + 50 + 54 + 14 = 150$.

To find the mean, we must divide this total by the **number of data items**. In this example, the number of data items is the number of pupils in 7R, which is the total of the second column of the table.

$$4 + 5 + 10 + 9 + 2 = 30.$$

It follows that the mean shoe size for 7R is $150 \div 30 = 5$.

Exercise 10

100 packets of Smarties are chosen randomly. The following table shows the number of Smarties in each tube.

Number of Smarties in a tube	Number of tubes
35	14
36	24
37	26
38	20
39	16

Find the mean number of Smarties in a tube.

Challenge!

Today, the first 30 customers visiting a shop spent a mean total of £4 each. After the next 50 customers had visited the shop, the mean had risen to £6. What was the mean total spent by these 50 customers?



Exercise 11

The following table shows the number of M&Ms in several big packets of M&Ms.

Number of M&Ms in a big packet	Number of big packets
96	2
97	13
98	8
99	17

- (a) Calculate the mean number of M&Ms in a big packet.
 (b) How many packets contained more than the mean value?
 (c) How many packets contained less than the mean value?

**Exercise 12**

A dice is thrown 100 times.

The scores are listed in the following table.

Score	Number of times the dice was thrown
1	14
2	17
3	16
4	18
5	17
6	18



Calculate the mean score.

Exercise 13

On a golf course, the hole shown on the right is called a Par 3. This means that the majority of the players are expected to complete the hole in three shots.

The table below shows the number of shots taken by some players in completing the hole.

Number of shots	Number of people
1	1
2	18
3	30
4	26
5	25

- (a) Calculate the mean number of shots needed to complete the hole.
 (b) In your opinion, should this hole be called a Par 3? Explain your answer.

**Challenge!**

The mean age of a pop group with 3 members is 21. What will be the mean age of the group if a new, 25-year-old member joins the group?



5

Applying

Test 1:	42	55	47	61	59	71	49	53	65	61
Test 2:	54	49	63	63	58	48	68	61	48	58

(b) In 8 games, the netball team of Ysgol y Creuddyn scored the following number of goals:

In 6 games the netball team of Ysgol John Bright scored the following number of goals:

(c) The following data shows the golf scores of two players.

Morris:	81	76	79	85	89	78	74	83	74	87
Parry:	79	88	87	76	91	73	74	76		

(d) The teachers in the physical education department measure, in seconds, the times taken by the football and rugby teams to complete a 100m race.

The rugby team: 12, 13, 14, 11, 12, 14, 15, 13, 15, 14, 11, 16, 14, 15, 13.

Challenge!

The mean of 20 numbers is 20. If the mean of the first 4 numbers is 4, what is the mean of the final 16 numbers?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

The Range



Arwyn and Megan travel to school on different school buses. Over a period of a fortnight, Arwyn and Megan wrote down the times the buses arrived to take them to school.

Arwyn's Bus

08:13 08:16 08:14 08:15 08:14
08:15 08:14 08:17 08:14 08:15

Megan's Bus

08:29 08:27 08:33 08:28 08:28
08:33 08:26 08:31 08:34 08:30



Looking at the above times, can you tell which bus is **most reliable**?

Measures of Spread

In order to discuss which bus is most reliable, we need to measure the **spread** of the data. A data set has **low data spread** if the data values are all close to each other. We could depend on a bus with low data spread to reach the bus stop at the same time each day, or close to this time. A bus with **high data spread** wouldn't be as reliable.

The Range

The **range** is one measure of data spread. To calculate the range for a specific data set, find the **difference** between the greatest data item and the least data item. Usually, this means **taking away the least number from the greatest number**.

Arwyn and Megan

For Arwyn's bus, the latest time is 08:17, and the earliest time is 08:13. The difference between these two times is 4 minutes, so the range for Arwyn's bus is 4 minutes.

For Megan's bus, the latest time is 08:34, and the earliest time is 08:26. The difference between these two times is 8 minutes, so the range for Megan's bus is 8 minutes.

Because the range of Megan's bus is more than the range of Arwyn's bus, we can say that the **data spread** of Megan's bus is higher, and so Megan's bus is **less reliable** than Arwyn's bus.

Exercise 15

Find the range of the following data sets.

- (a) 4 8 9 10 11 15 16
- (b) 1 2 4 8 16 17 19 22
- (c) 10,800 15,000 15,500 18,300 21,300
- (d) 2.8 3.1 4.9 8.6 14.3 21.6
- (e) 7.2 1.3 8.4 2.1 3.6 4.7 9.3 8.4
- (f) 5 4 0 2 4
- (g) 150 150 141 137 146
- (h) 70 69 69 51 80

5

Skill

Exercise 16

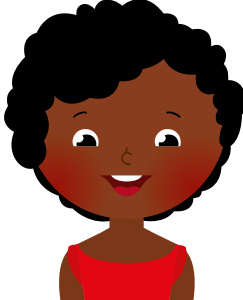
- (a) The range of a data set is 16. If the largest data value is 26, what is the **smallest** data value?
- (b) Peter collected data from 10 of his friends concerning the weekly pocket money they received. He forgot to write down the final value in his list.

£1 £1.20 £1.15 £1.25 £1 £1.40 £1.90 70c 90c

He remembered that the range of values was £1.40. What could be the **final** data value?

Exercise 17

Here is some information about five children.

Huw	Erin	William	Lili	Iestyn
				
Height: 150cm Weight: 52kg Shoe size: 4	Height: 158cm Weight: 52kg Shoe size: 6	Height: 150cm Weight: 50kg Shoe size: 5	Height: 149cm Weight: 46kg Shoe size: 4	Height: 146cm Weight: 45kg Shoe size: 3

- (a) Who is the tallest pupil?
- (b) Who is the shortest pupil?
- (c) What is the range of the heights of the pupils?
- (d) What is the range of the weights of the pupils?
- (e) What is the range of the shoe sizes of the pupils?

Exercise 18

Alys and Hywel travel to school on different school buses. Over a period of a fortnight, Alys and Hywel wrote down the times the buses arrived to take them to school.

Alys' Bus

08:25 08:27 08:24 08:29 08:31
08:30 08:25 08:32 08:26 08:26

Hywel's Bus

08:09 08:14 08:11 08:12 08:10
08:10 08:09 08:12 08:13 08:10

- (a) What is the range of Alys' times?
- (b) What is the range of Hywel's times?
- (c) In your opinion, which bus is most reliable? Explain your answer.

5

Applying



Exercise 19

Two groups of 6 people took part in a quiz.

(a) The 6 members of group A had the following scores.

52 29 78 56 24 37

What was the range of group A's scores?

(b) The scores for the 6 members of group B are shown in the following table.

Score	Number of people
22	2
25	2
26	1
28	1



Which group had the biggest range for their scores? You must give a reason for your answer.

Exercise 20 (Revision)

Here are the ages (in years) of 8 members of an evening class:

36 28 45 24 31 34 27 47

- Find the range of the ages of the members of the evening class.
- What was the range of the ages exactly one year ago? Give a reason for your answer.
- Find the mean age of the members of the evening class.
- What was the mean age of the ages exactly one year ago? Give a reason for your answer.
- A new, 40-year-old member joins the evening class. Without doing any further calculations, state
 - Will the range of the evening class increase, decrease or stay the same?
 - Will the mean of the evening class increase, decrease or stay the same?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Bar Charts

We use bar charts to display data. The type of bar chart depends on the type of data that has been collected.

Types of Data

It is possible to place data into two categories: **qualitative** and **quantitative**.

Qualitative Data	Quantitative Data
Deals with descriptions . Data that can be observed but not measured. <i>Examples:</i> Method of travelling to school; eye colour; favourite pet; mobile phone manufacturer.	Deals with numbers . Data that can be measured. <i>Examples:</i> The height of a person; temperature; number of brothers; time to complete a race.

Exercise 21

State whether the following data is **qualitative** or **quantitative**.

- (a) Favourite board game (e.g. Monopoly; Scrabble).
- (b) The weight of a cake.
- (c) The name of your dentist.
- (d) The number of learners in your registration class.
- (e) The daily rainfall in Llandudno.
- (f) The day of the week (e.g. Monday) that you were born on.



4

Skill

Quantitative Data

It is possible to place quantitative data into two categories: discrete and continuous.

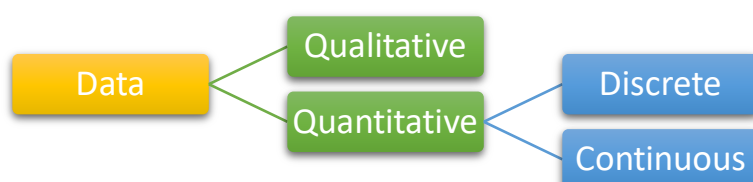
Discrete Data	Continuous Data
Only takes specific values . <i>Examples:</i> Shoe size; number of songs on an album; the salary of a nurse; number of sisters.	Takes any values within a specific range. <i>Examples:</i> Length of your foot; the temperature at midday; the time taken to cycle from Rhyl to Abergel.

Exercise 22

State whether the following quantitative data is **discrete** or **continuous**.

- (a) Population of the United Kingdom.
- (b) The length of your thumb.
- (c) The age of women when they marry.
- (d) The amount of money earnt for babysitting one night.
- (e) The number of cars on sale at a showroom.
- (f) The weight of the letters in a postman's sack.

Summary



Drawing a Bar Chart

We can draw a **bar chart** for the following types of data.

- Qualitative data.
- Discrete quantitative data that is not grouped.



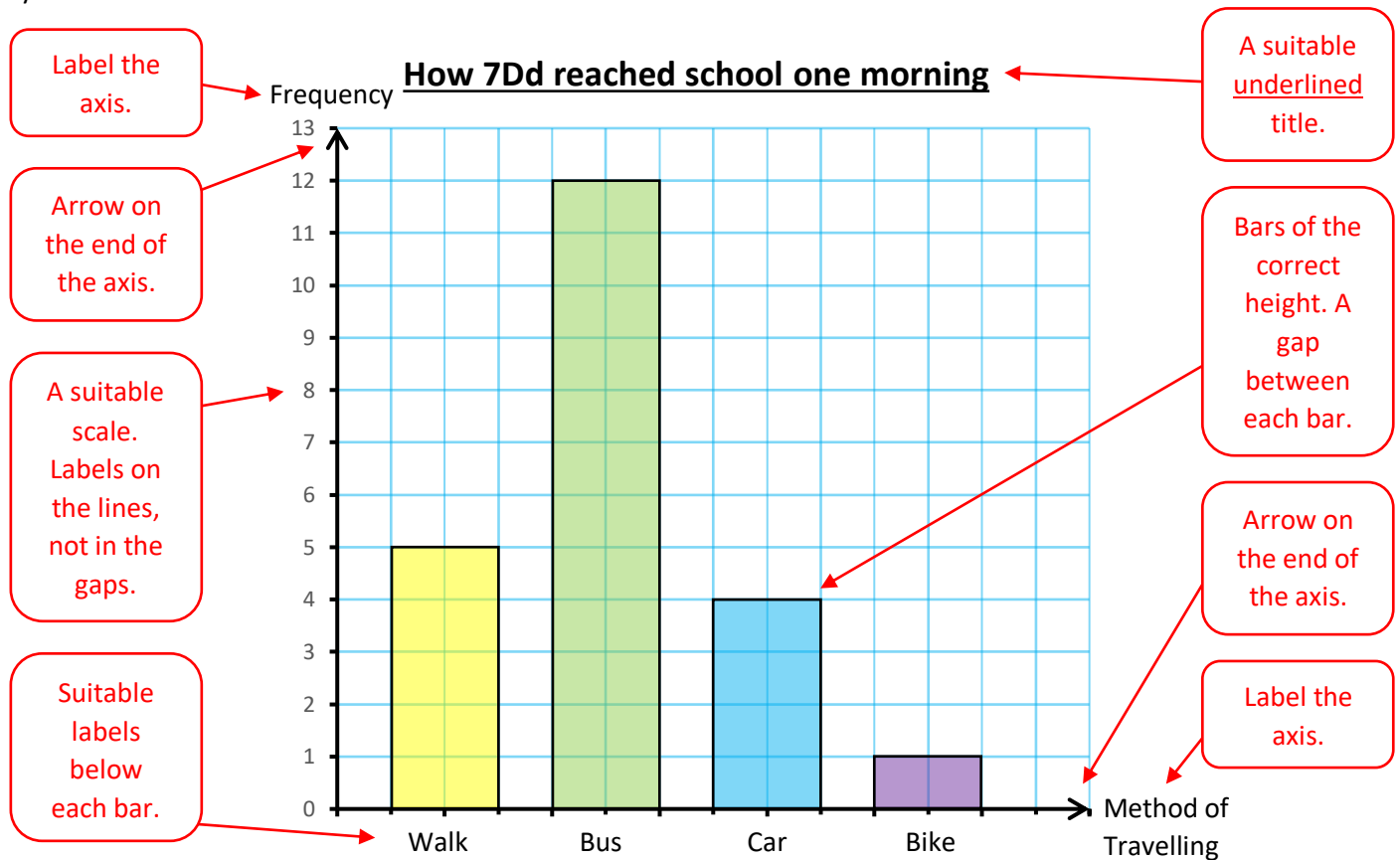
We need to include a **gap** between each bar in a bar chart.

Example

Dylan asked each member of 7Dd how they got to school one morning. Here are the results:

Walk 5 Bus 12 Car 4 Bike 1

Dylan drew a bar chart for the data.



Exercise 23

On a piece of squared paper, draw a **bar chart** for the following sets of data.

(a) Number of hours of sunshine in Colwyn Bay during one week.

Monday: 3 hours Tuesday: 4 hours Wednesday: 4 hours Thursday: 3 hours
Friday: 8 hours Saturday: 7 hours Sunday: 6 hours

(b) 7C's favourite pet.

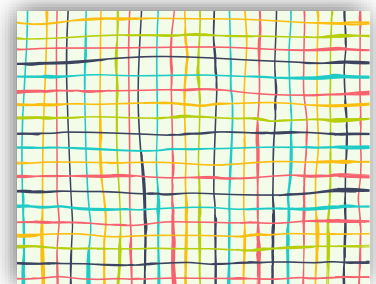
Dog: 8 Cat: 7 Rabbit: 4 Guinea Pig: 2 Snake: 1

(c) 7R's eye colour.

Blue: 6 Brown: 15 Green: 2

(d) The number of brothers of each member of 7E.

No brothers: 5 One brother: 10 Two brothers: 7 Three brothers: 2



3

Skill

Drawing a Frequency Diagram

We can draw a frequency diagram for the following types of data.

- Discrete quantitative data that has been grouped.
- Continuous quantitative data.



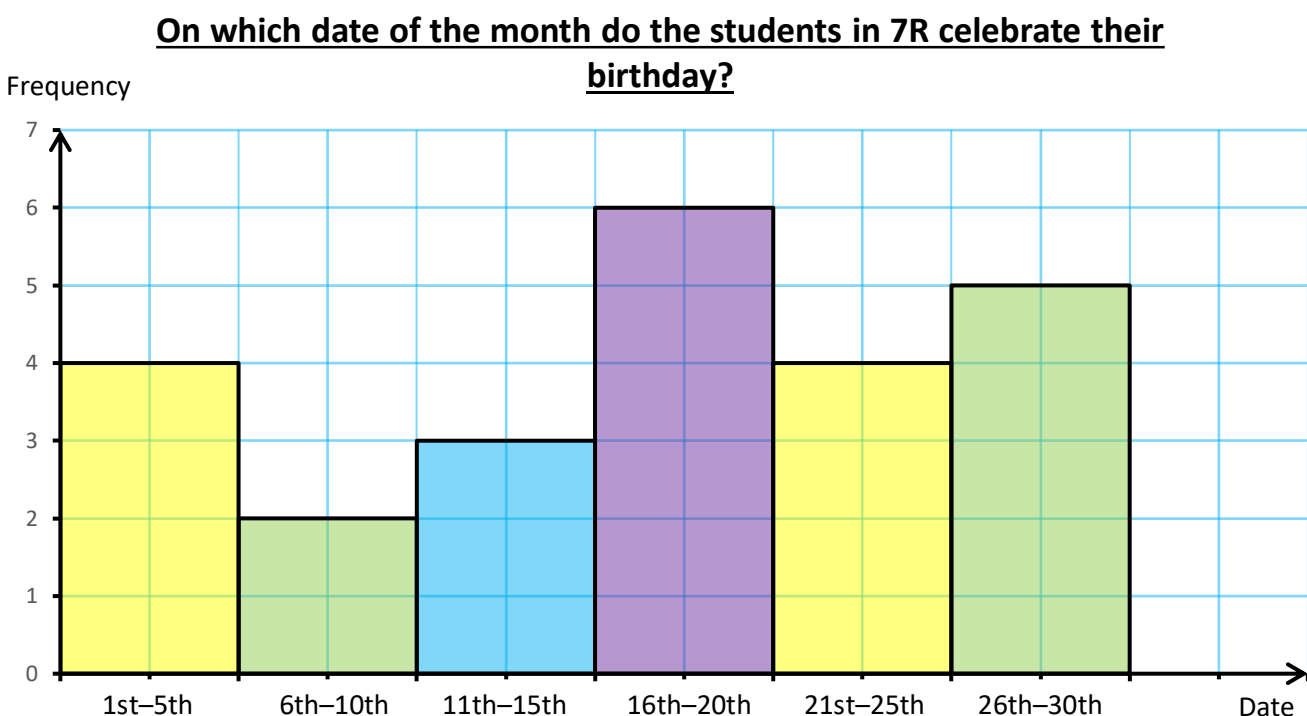
There are no gaps between the bars in a frequency diagram, but the **width** of each bar must be **equal**.

Example

Siwan asked each member of 7R which date of the month their birthday fell on. She received the following responses.

1st–5th: 4 6th–10th: 2 11th–15th: 3 16th–20th: 6 21st–25th: 4 26th–30th: 5

Siwan drew the following frequency diagram to show the data.



Exercise 24

On a piece of squared paper, draw a **frequency diagram** for the following sets of data.

(a) The number of books the pupils in 7E have bought during the past year.

Zero to four books: 12 Five to nine books: 5 Ten to fourteen books: 6 Fifteen to nineteen books: 1

(b) The number of minutes the dentist spends with each patient.

1–5 minutes: 2 6–10 minutes: 4 11–15 minutes: 9 16–20 minutes: 5
 21–25 minutes: 3 26–30 minutes: 3 31–35 minutes: 0 36–40 minutes: 1

(c) The number of days the pupils in 7C were absent from school last term.

0–4 days: 11 5–9 days: 8 10–14 days: 6 15–19 days: 0 20–24 days: 5

(d) Number of words in each sentence during the first 50 sentences of a book.

1–10 words: 2 11–20 words: 9 21–30 words: 14 31–40 words: 7
 41–50 words: 4 51–60 words: 8 61–70 words: 6



Grouping Discrete Data

In Exercise 24, we saw that we must use grouped data in order to draw a frequency diagram. We must use a **frequency table** to group data that is not yet grouped.

Example

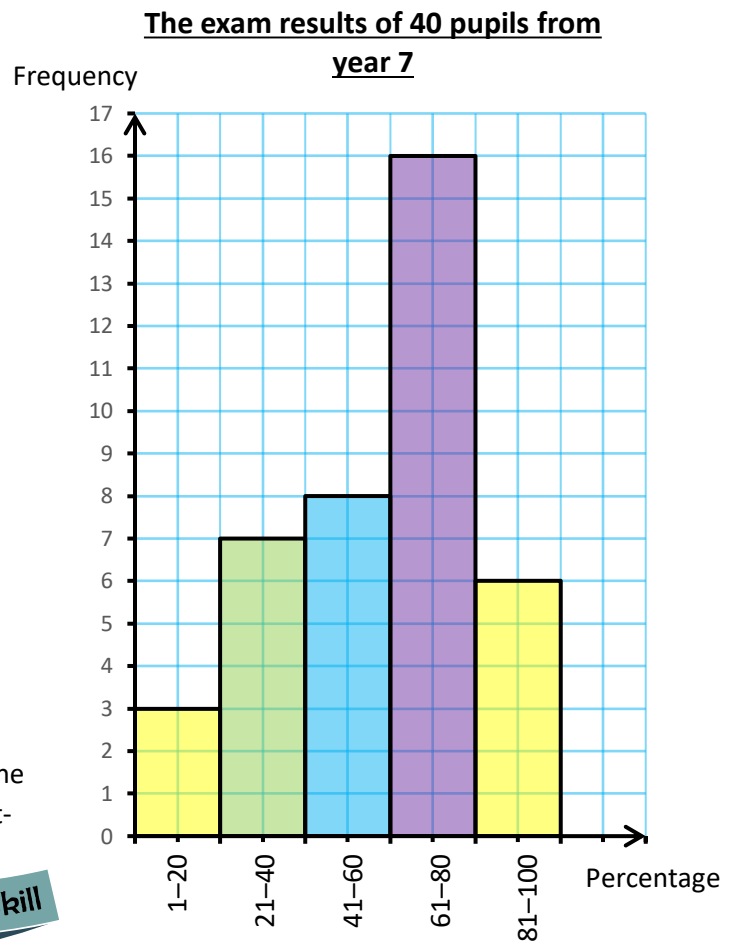
Here are the exam percentages of 40 pupils in year 7.

56	14	61	67	90	38	47	67
72	75	80	41	21	54	92	44
68	85	43	71	69	19	39	64
83	80	65	42	20	35	27	81
70	75	39	36	71	88	50	79

Let us group this data using the groups
1–20, 21–40, 41–60, 61–80, 81–100.

Group	Tally Marks	Frequency
1–20		3
21–40		7
41–60		8
61–80		16
81–100		6
Total		40

After forming the above frequency table, we can now draw the frequency diagram for the data. This can be seen on the right-hand side of this page.



Exercise 25

6

Skill

(a) Here are the times taken by the pupils in 7E to come to school today, correct to the nearest minute.

15	5	8	22	12	10	30	25	27	22	35	8	18	17	22
15	44	35	45	55	12	28	35	48	48	30	40	19	25	50

Draw a **frequency table** for the data, using the groups 1–10, 11–20, 21–30 up to 51–60.

Draw a **frequency diagram** to show the data.

(b) A class measured the arm span of each pupil. Here are the results, to the nearest cm.

132	129	147	127	128	124	134	139	138	136	144
143	133	141	139	130	142	143	140	137	148	136
146	151	123	131	135	135	130	140	138	133	



Draw a **frequency table** for the data, using the groups 120–124, 125–129, 130–134 up to 150–154.

Draw a **frequency diagram** to show the data.

(c) 7C decided to see how good they were at estimating the length of a minute. Here are the results, correct to the nearest second.

72	70	52	48	52	48	63	71	63	70	61	62	59	62	50
64	46	65	69	65	58	64	61	48	63	68	67	73	55	51

Draw a **frequency table** for the data, using the groups 46–50, 51–55, 56–60 up to 71–75.

Draw a **frequency diagram** to show the data.

Grouped Continuous Data

The following frequency table shows the length of the right thumb of the students of a class.

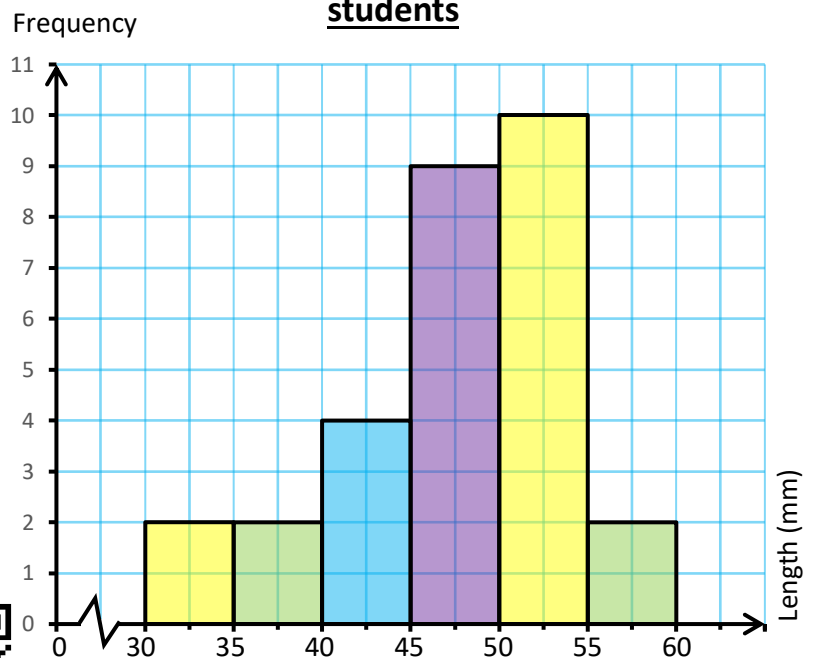
Class interval (l mm)	Frequency
$30 \leq l < 35$	2
$35 \leq l < 40$	2
$40 \leq l < 45$	4
$45 \leq l < 50$	9
$50 \leq l < 55$	10
$55 \leq l < 60$	2

We read $30 \leq l < 35$ as " l is at least 30 but is less than 35" or " l lies between 30 and 35, but can be equal to 30".

The above information is shown in the frequency diagram on the right. Notice the jump in the graph between 0 and 30.



The length of the right thumb of some students

**Exercise 26**

Draw **frequency diagrams** for the following sets of data.

- (a) How much weight the members of a weight loss club lost in 6 months.

Weight (w kg)	Frequency
$0 \leq w < 6$	4
$6 \leq w < 12$	11
$12 \leq w < 18$	12
$18 \leq w < 24$	7
$24 \leq w < 30$	3

- (b) The height of 60 pupils.

Height (h cm)	Frequency
$168 \leq h < 172$	2
$172 \leq h < 176$	6
$176 \leq h < 180$	17
$180 \leq h < 184$	22
$184 \leq h < 188$	10
$188 \leq h < 192$	3

- (c) The noise of 60 items of electrical equipment.

Noise (n db)	Frequency
$15 \leq n < 20$	4
$20 \leq n < 25$	12
$25 \leq n < 30$	15
$30 \leq n < 35$	6
$35 \leq n < 40$	8
$40 \leq n < 45$	3
$45 \leq n < 50$	12

Exercise 27

The following figures give the time, correct to the nearest minute, taken by 50 cross-country runners to complete a race.

30	37	43	55	52	47	49	36	44	40
41	49	52	53	39	41	46	42	50	49
39	53	54	57	43	59	34	38	40	42
48	53	50	52	37	36	45	53	48	42
52	39	41	46	50	52	38	58	57	46

Time (t minutes)	Tally Marks	Frequency
$30 \leq t < 35$		
$35 \leq t < 40$		
$40 \leq t < 45$		
$45 \leq t < 50$		
$50 \leq t < 55$		
$55 \leq t < 60$		



- (a) Complete the frequency table for the data. (Remember that 35 minutes would appear in the class $35 \leq t < 40$, not $30 \leq t < 35$.)

- (b) Draw a **frequency diagram** for the data.

Exercise 28

The following data gives the weights of books on a shelf in a library, in grams.

742 241 332 478 1245 832 746 291 160 635 1042 368 760 590
 748 800 564 1340 824 683 174 960 236 746 328 387 823 894

(a) Complete a **frequency table** for the data. (Use the following classes for the weight, w :

$0 \leq w < 200$, $200 \leq w < 400$, $400 \leq w < 600$, ..., $1200 \leq w < 1400$.)

(b) Draw a **frequency diagram** for the data.

Exercise 29

The following data shows the hand width of several 13-year-old girls, in mm.

171 182 202 164 198 211 164 173 177 173 192 205 232 223
 189 186 191 194 197 205 231 207 169 173 184 209 176 181

Draw a **frequency diagram** for the data. (You will need to complete a suitable **frequency table** first.)

Challenge!

What is wrong with the following frequency table, which shows the length of leaves that fell from a tree one morning? What would you have to do to create a frequency diagram for the data?

Length (l cm)	Tally Marks	Frequency
$0 \leq l < 5$		7
$5 \leq l < 10$		11
$10 \leq l < 20$		9
$20 \leq l < 25$		3
$25 \leq l < 30$		8

**Interpreting Charts****Exercise 30**

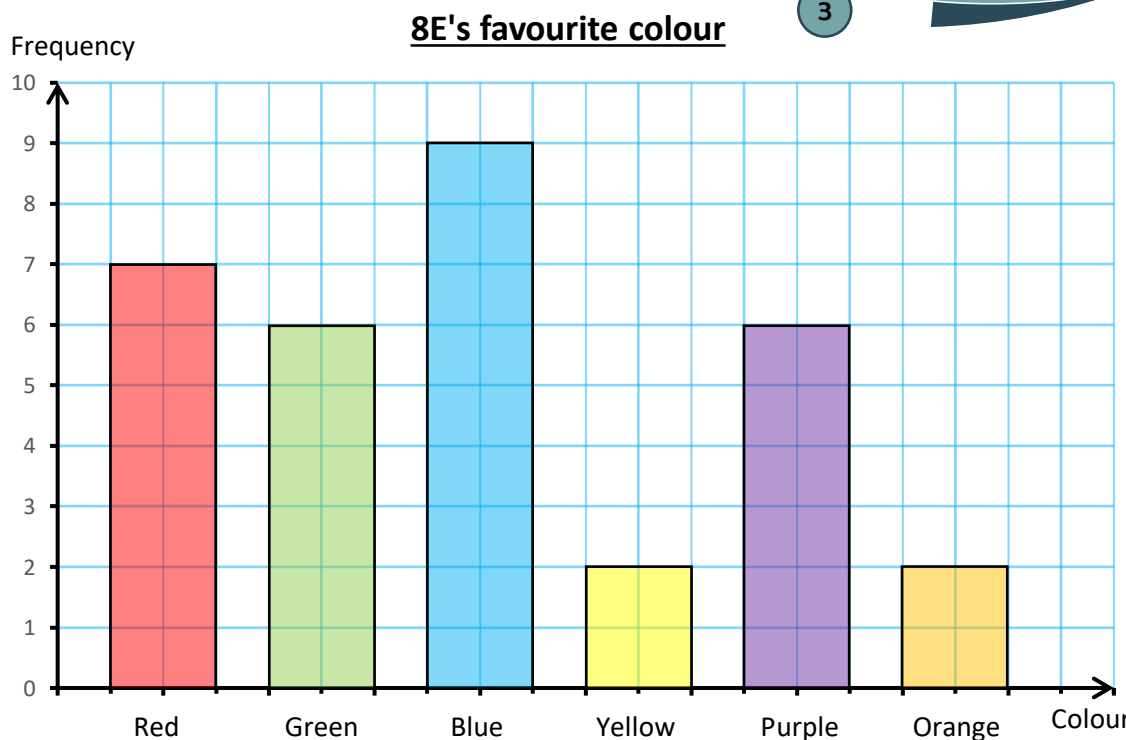
(a) How many pupils chose **red**?

(b) How many pupils chose **yellow**?

(c) Which colour was the **most popular**?

(d) How many pupils are in class 8E?

(e) How many pupils in 8E did **not** like purple?



3

Applying

Exercise 31

6

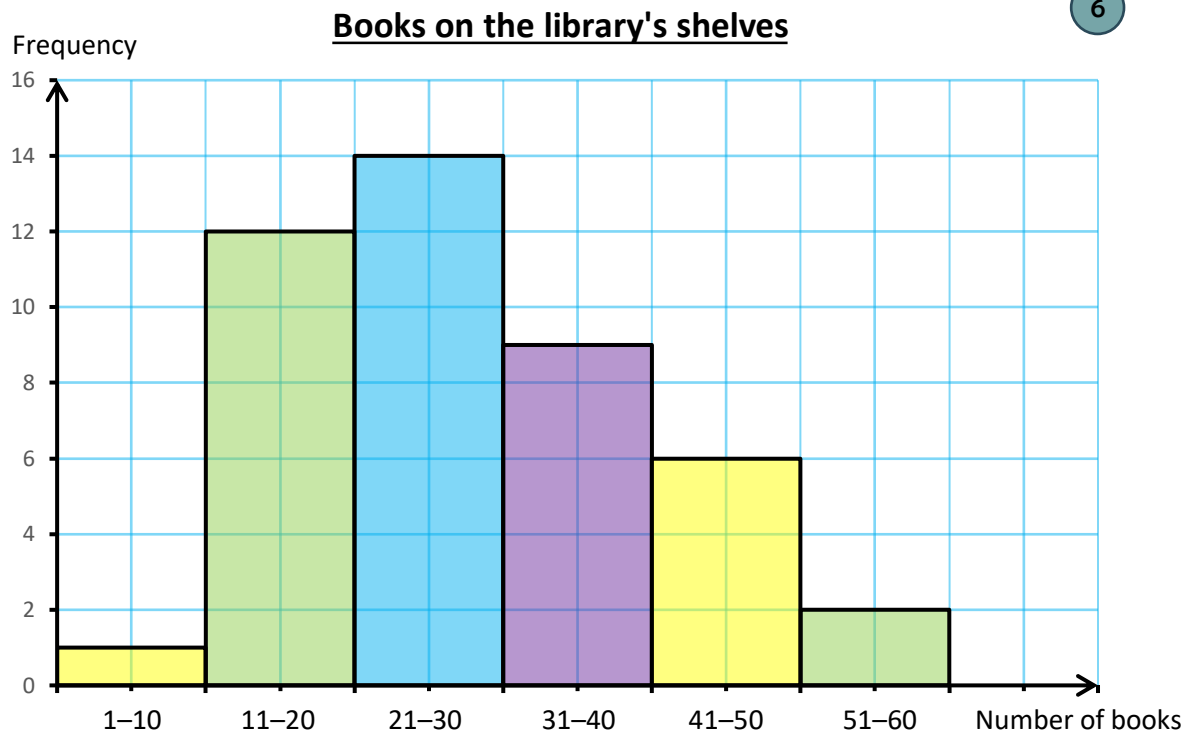
(a) How many shelves contain between **11 and 20** books?

(b) How many shelves contain between **31 and 40** books?

(c) How many shelves contain **more than 30** books?

(d) **How many shelves** are there in total in the library?

(e) Based on this information, can you state how many shelves in the library contain **exactly 20 books**? Explain your answer.

**Exercise 32**

5

(a) How many pupils got **7 marks**?

(b) How many pupils got **less than 5 marks**?

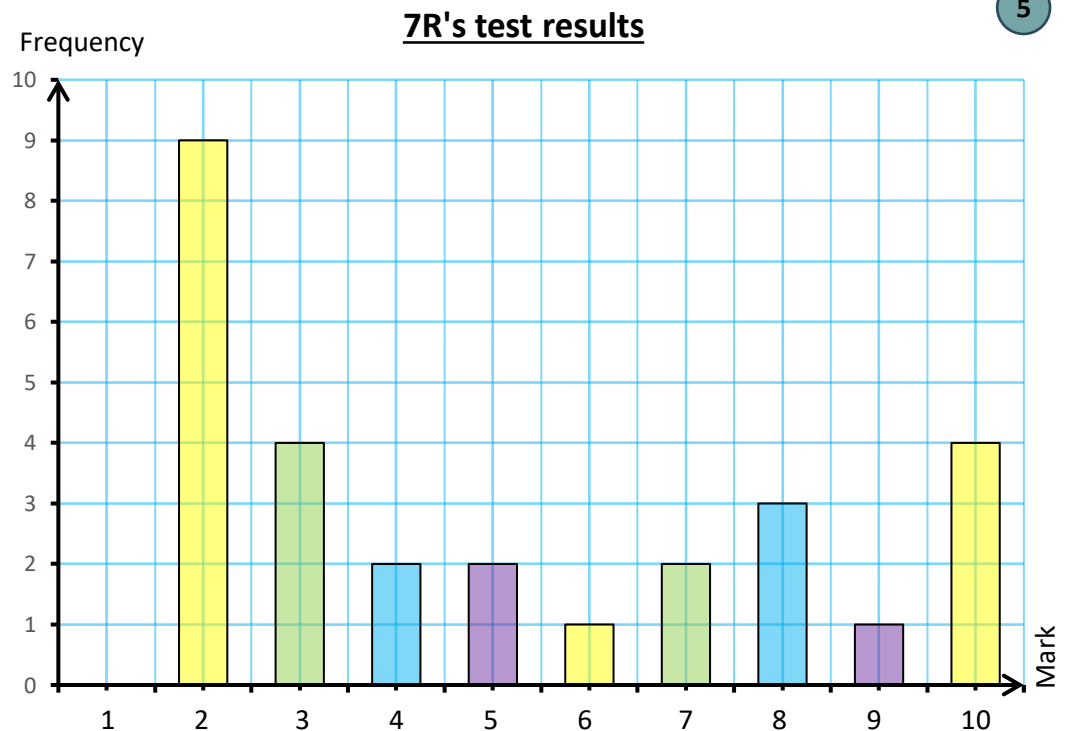
(c) How many pupils got **more than 6 marks**?

(d) **How many pupils** are there in the class?

(e) What was the **total** of the class' marks?

(f) What was the **mean** of the class' marks?

(g) What was the mark **3 of the pupils** got?

**Exercise 33**

6

(a) What **type of data** is shown in Exercises 30, 31 and 32?

(b) Were the graphs in Exercises 30–32 **bar charts** or **frequency diagrams**?

Exercise 34

6

The length of the phone calls in an office one afternoon

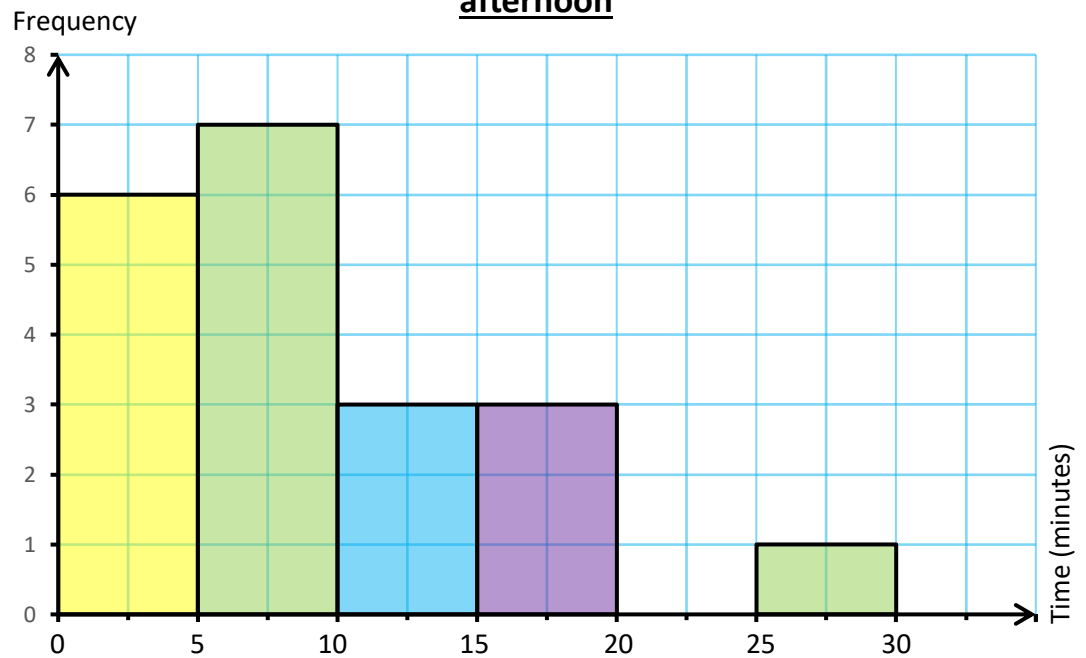
(a) How many phone calls lasted between **10 and 15** minutes?

(b) How many phone calls lasted **at least 15** minutes?

(c) How many phone calls were there **in total** during the afternoon?

(d) What **type** of data is shown here?

(e) Alwen phoned the office during the afternoon. The length of her phone call was 590 seconds. Which bar in the frequency diagram shows this phone call?



Exercise 35 (Revision)

The following list shows the **number of complete games** a squad of football players played during one season.

16 10 9 3 21 30 4 12 0 36 20 14 12 5 16
 0 5 6 14 23 0 0 18 1 38 15 18 3 12 16

(a) What **type** of data is this?

(b) Draw a **frequency table** for the data, using the groups 0–4, 5–9, 10–14 up to 35–39.

(c) Draw a **frequency diagram** for the data.

(d) How many players played more than 25 complete games during the season?

(e) How many players played less than 10 complete games during the season?

Did you know?

The first person to draw a bar chart was William Playfair, in 1786, to show Scottish exports.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Mean of Grouped Data

In the previous chapter, we used a **frequency table** to group data into equal classes. Given a table of this sort, is it possible to calculate the **mean** of the original data?

Example

The following table shows the number of music albums the pupils of 10R have bought during November.

Number of albums	Frequency
0–4	11
5–9	8
10–14	5
15–19	1



What is the **mean number** of music albums the pupils of 10R have bought during November? Without access to the original data (*before it is grouped*), it is **impossible to state** what is the exact mean, because the frequency table does not give us complete data. For example, the table notes that one person has bought between 15 and 19 albums, but how many exactly? It could be 15, but it could also be 16, 17, 18 or 19. Without this information, we cannot calculate the mean exactly. We can, however, **estimate** the mean using the frequency table, by using the following method.

(1) Find the **mid-point** of each class by adding the two numbers that form the class and halving. For example, the numbers 5 and 9 form the class 5–9. By adding these numbers, we obtain 14. Half of 14 is 7, so the mid-point of the class 5–9 is 7.

Number of albums	Mid-point	Frequency
0–4	2	11
5–9	7	8
10–14	12	5
15–19	17	1

(2) Because the mid-point of each class gives a single value that **represents** the class, we can now use the method from page **Error! Bookmark not defined.** to estimate the mean from the table.

To begin, **multiply** each mid-point by the frequency of the class.

Number of albums	Mid-point	Frequency	Multiplication
0–4	2	11	$2 \times 11 = 22$
5–9	7	8	$7 \times 8 = 56$
10–14	12	5	$12 \times 5 = 60$
15–19	17	1	$17 \times 1 = 17$

(3) Find the **total** of the final two columns.

Number of albums	Mid-point	Frequency	Multiplication
0–4	2	11	$2 \times 11 = 22$
5–9	7	8	$7 \times 8 = 56$
10–14	12	5	$12 \times 5 = 60$
15–19	17	1	$17 \times 1 = 17$
Total		25	155

(4) Divide the total of the 'multiplication' column by the total of the 'frequency' column:
 $155 \div 25 = 6.2$. This is the estimate for the mean of this set of data.

Exercise 36Find an **estimate** for the mean of the following data sets.

7

Skill

(a)

Number of cars sold in a day	Number of days
0–4	18
5–9	11
10–14	6
15–19	4
20–24	1

(b)

Number of flowers per m ²	Number of square metres
0–14	43
15–29	21
30–44	13
45–59	2
60–74	1

(c)

Height of a horse chestnut tree, in metres	Frequency
0–3	13
4–7	17
8–11	9
12–15	4
16–19	5
20–23	0
24–27	2

(d)

Number of books sold in a week	Frequency
100–124	1
125–149	9
150–174	8
175–199	5
200–224	2

(e)

Number of customers in a café	Frequency
70–74	3
75–79	21
80–84	30
85–89	28
90–94	14
95–99	4

(f)

Number of sweets in a jar	Frequency
300–324	4
325–349	15
350–374	27
375–399	63
400–424	74
425–449	32
450–474	11
475–499	2



The Range

Let us consider again the data about music from page 20.

Number of albums	Mid-point	Frequency
0–4	2	11
5–9	7	8
10–14	12	5
15–19	17	1

Is it possible to find the **range** of the data set using the above table? As with the mean, it is not possible to calculate the range exactly, as the table does not tell us the maximum and minimum data values. The best we can do is to state that the least value is *between 0 and 4*; and that the greatest value is *between 15 and 19*. We must use the **mid-point** of the classes to find an **estimate** for the range, by taking away the lower mid-point from the higher mid-point. For this data, our estimate for the range is $17 - 2 = 15$.

Exercise 37

Find an **estimate for the range** of the data sets from Exercise 36.

Continuous Grouped Data

The methods of estimating the mean and range also work with continuous data that is grouped.

Example

A golfer hits 40 golf balls using one of his clubs. He records the distance that each ball has travelled. The following frequency table shows his results.

Distance travelled, d , in yards	Frequency
$75 \leq d < 80$	4
$80 \leq d < 85$	13
$85 \leq d < 90$	17
$90 \leq d < 95$	6

We can find an estimate for the **mean** of this data by finding the class mid-points; multiplying by the frequency; and finding the total of the final two columns.

Distance travelled, d , in yards	Mid-point	Frequency	Multiplication
$75 \leq d < 80$	77.5	4	$77.5 \times 4 = 310$
$80 \leq d < 85$	82.5	13	$82.5 \times 13 = 1072.5$
$85 \leq d < 90$	87.5	17	$87.5 \times 17 = 1487.5$
$90 \leq d < 95$	92.5	6	$92.5 \times 6 = 555$
Total		40	3425

The estimate for the **mean** is $3425 \div 40 = 85.625$ yards.

The estimate for the **range** is $92.5 - 77.5 = 15$ yards.



7



Exercise 38

7

Find an estimate for the **mean** and **range** of each of the following data sets.

(a)

The length of the cars, l , that are parked in a supermarket car park, in metres	Frequency
$3 \leq l < 4$	34
$4 \leq l < 5$	41
$5 \leq l < 6$	5

(b)

The time, t , taken to complete a race, in minutes	Frequency
$20 \leq t < 25$	5
$25 \leq t < 30$	11
$30 \leq t < 35$	19
$35 \leq t < 40$	12
$40 \leq t < 45$	3

(c)

The head circumference, c , of people in a factory	Frequency
$50 \leq c < 54$	10
$54 \leq c < 58$	31
$58 \leq c < 62$	16
$62 \leq c < 66$	3

(d)

The thickness, t , of books in a library, in mm	Frequency
$0 \leq t < 10$	5
$10 \leq t < 20$	16
$20 \leq t < 30$	41
$30 \leq t < 40$	32
$40 \leq t < 50$	6

(e)

The length of phone calls, l , in minutes	Frequency
$0 \leq l < 5$	32
$5 \leq l < 10$	42
$10 \leq l < 15$	24
$15 \leq l < 20$	12
$20 \leq l < 25$	7
$25 \leq l < 30$	2



Exercise 39

Over a period of 50 days, Bethan and Lowri recorded how many minutes per day they spent on their mobile phones. The data is given in the following frequency tables.

Bethan's time, t , in minutes	Frequency
$0 \leq t < 30$	14
$30 \leq t < 60$	17
$60 \leq t < 90$	10
$90 \leq t < 120$	9
$120 \leq t < 150$	0
$150 \leq t < 180$	0

Lowri's time, t , in minutes	Frequency
$0 \leq t < 30$	23
$30 \leq t < 60$	14
$60 \leq t < 90$	5
$90 \leq t < 120$	3
$120 \leq t < 150$	2
$150 \leq t < 180$	3

- (a) Calculate an estimate for **Bethan's mean time** and **Lowri's mean time**.
 (b) Calculate an estimate for **Bethan's range** and **Lowri's range**.
 (c) Who would you say uses their mobile phone **the most**? Explain your answer.
 (d) Who would you say is **most consistent** in using their mobile phone? Explain your answer.

Exercise 40 (Revision)

7

The height of 25 Year 7 learners was measured. The results are given below, correct to the nearest inch.

57	53	59	62	61
66	51	58	59	62
60	62	54	64	59
58	62	64	53	67
61	63	57	48	60

- Calculate the **mean height** of the learners.
- Calculate the **range** of the heights.
- What **type** of data is this?
- Complete the following **frequency table** in order to group the data.

Height, h , in inches	Tally Marks	Frequency
$45 \leq h < 50$		
$50 \leq h < 55$		
$55 \leq h < 60$		
$60 \leq h < 65$		
$65 \leq h < 70$		

- (e) Draw a **frequency diagram** for the data.
- (f) Use the frequency table to find an **estimate for the mean and range**.
- (g) How close were your estimates to the true values?

Challenge!

Repeat the above exercise using the heights of the learners in your class.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Pie Charts

It is possible to display data by dividing a circle into **sectors** of appropriate sizes. We form a **pie chart** by doing this.

Example

Here is the weather for the 30 days in June.

<i>Rainy days</i>	4
<i>Cloudy days</i>	8
<i>Sunny days</i>	18

In order to draw a pie chart for this data, we need to start by dividing 360° (which is the angle around a point in the centre of a circle) by 30 (which is the number of data items).

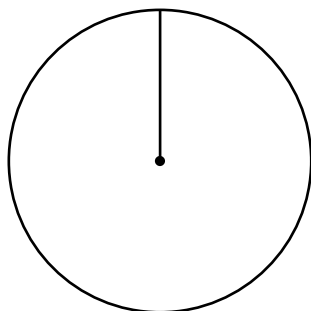
$$360^\circ \div 30 = 12^\circ$$

Next, we multiply the 12° (which is the angle for one day) by the number of days for a particular type of weather.

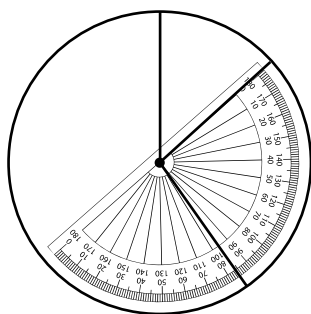
<i>Rainy days</i>	$4 \times 12^\circ = 48^\circ$
<i>Cloudy days</i>	$8 \times 12^\circ = 96^\circ$
<i>Sunny days</i>	$18 \times 12^\circ = 216^\circ$

To finish, we construct the pie chart itself.

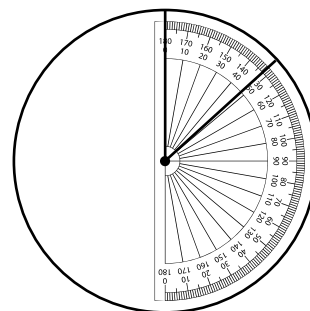
- (a) Draw a **circle** using a compass. Mark the centre of the circle with a **dot**. Draw a **vertical line** from the centre of the circle to the top of the circle.



- (c) Draw the next angle (96°) using a protractor.

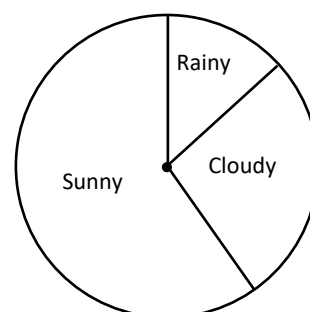


- (b) Measure the first angle (48°) using a protractor. Mark the angle using a pencil before drawing a straight line **from the centre of the circle to your mark**.



- (d) Label the sectors and add a suitable title to the pie chart.

June's weather



Exercise 41

Draw a **pie chart** for the following sets of data.

(a) 8C's favourite fruit.

<i>Apple</i> : 10	<i>Banana</i> : 8
<i>Orange</i> : 4	<i>Strawberry</i> : 2

(b) Year 9's holiday destinations, for last summer.

<i>Europe</i> : 40	<i>Britain</i> : 70
<i>U.S.A.</i> : 10	

(c) Tomos' typical day (24 hours).

<i>Sleeping</i> : 8 hours	<i>Eating</i> : 1 hour
<i>Working</i> : 8 hours	<i>Leisure</i> : 6 hours
<i>Travelling</i> : 1 hour	

(d) The foods that were eaten in a hotel.

<i>Beef</i> : 7	<i>Lamb</i> : 26
<i>Chicken</i> : 18	<i>Vegetarian</i> : 5
<i>Fish</i> : 14	<i>Duck</i> : 2

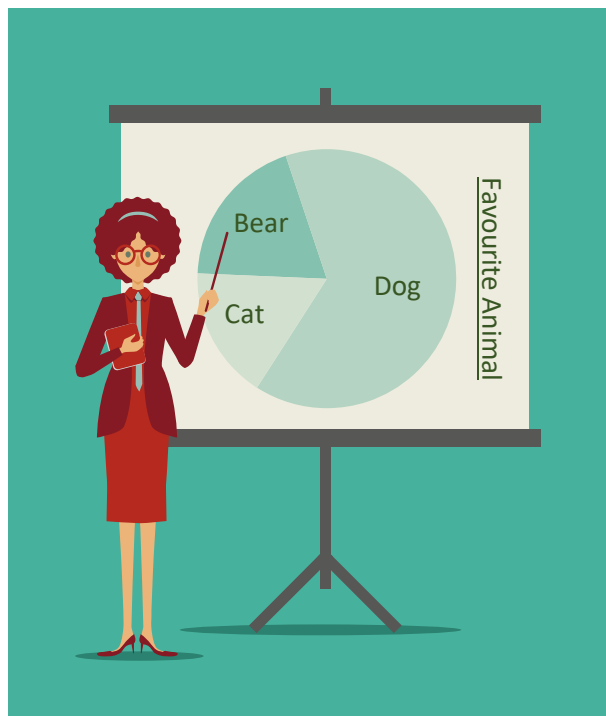
(e) Methods of heating a house.

<i>Coal</i> : 7	<i>Electricity</i> : 9
<i>Gas</i> : 11	<i>Oil</i> : 3



6

Skill



(f) How do the pupils of Ysgol y Creuddyn reach school?

<i>Bus</i> : 500	<i>Car</i> : 150
<i>Bike</i> : 50	<i>Walk</i> : 100

(g) The votes in a recent election.

<i>Conservatives</i> : 13,000	<i>Labour</i> : 9,000
<i>Plaid Cymru</i> : 7,000	<i>UKIP</i> : 4,000
<i>Liberal Democrats</i> : 3,000	

(h) The colour of the cars in a car park.

<i>Red</i> : 8	<i>Blue</i> : 4
<i>Green</i> : 5	<i>Yellow</i> : 7

(i) 8E's favourite sport.

<i>Football</i> : 13	<i>Rugby</i> : 5
<i>Golf</i> : 2	<i>Other</i> : 3

(j) How many sisters do the learners of 9E have?

<i>None</i> : 7	<i>One</i> : 12
<i>Two</i> : 4	<i>Three</i> : 3

**Exercise 42**

What **type** of data is given in the questions of Exercise 41?

4

Interpreting Pie Charts

Exercise 43

5

Applying

(a) The pie chart shows how 7C come to school.

(i) What is the **most popular method** of coming to school?

8 of the pupils come by car.

(ii) How many pupils come by **bus**?

(iii) How many pupils **walk** to school?



(b) Paul has drawn a pie chart. It shows when pupils like to complete their homework.

(i) Which time is the **most popular**?

(ii) What **fraction** of the pupils like to complete their homework after supper?

There were 20 pupils in the survey.

(iii) How many pupils like to complete their homework **before** supper?

(iv) How many pupils chose **dinner time**?

(c) Siân asked her class, 9C, what type of **bike** they had. There are 32 pupils in Siân's class.

(i) What is the **most popular** bike?

(ii) How many pupils **do not** have a bike?

(iii) Complete the following table.

Bike	Mountain	Racing	Normal	No Bike
Number of pupils				

(d) This pie chart shows the **favourite colour** of 40 boys.

(i) Which **fraction** liked **red**?

(ii) How many boys liked **red**?

(iii) Which fraction liked **blue**?

(iv) How many boys liked **blue**?

(v) How many boys did **not** choose green?

(e) Siôn spends 4 hours on different activities at night.

How much time is spent on

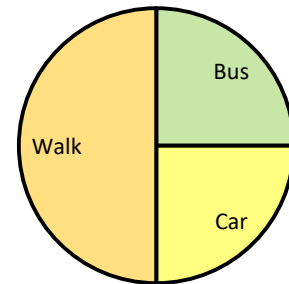
(i) **sports**;

(ii) watching **television**;

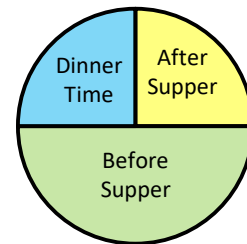
(iii) **reading**;

(iv) **homework**?

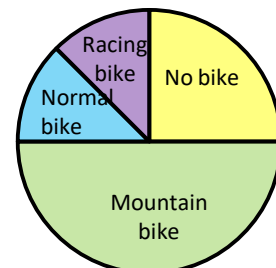
How do 7C come to school?



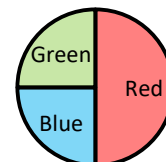
Favourite time for doing homework



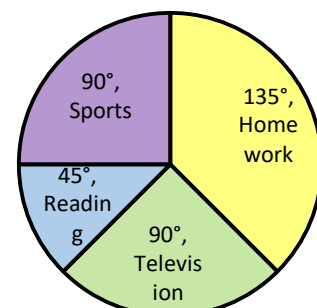
The type of bike that the pupils of Siân's class have



40 boys' favourite colour



Siôn's Activities



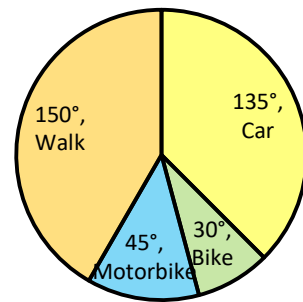
Exercise 44

(a) This pie chart shows how 24 people travelled to town.

Find how many people

- (i) **walked** to town;
- (ii) came in a **car**;
- (iii) came on a **motorbike**;
- (iv) came on a **bike**.

How did 24 people travel to town?



(b) Llinos surveyed 210 pupils in years 7 and 8.

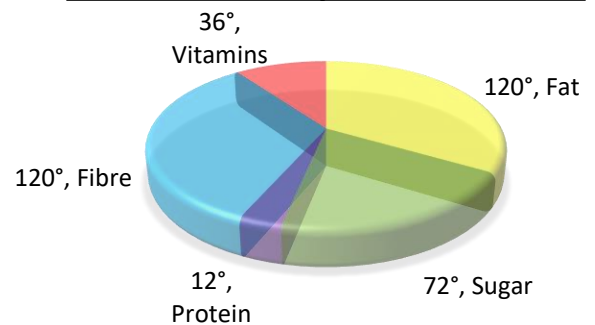
She asked the pupils which of the following, in their opinion, was most important in their diet.

Fat; Sugar; Protein; Fibre; Vitamins.

The pie chart shows her results.

How many pupils are in each category?

What is most important in a diet?

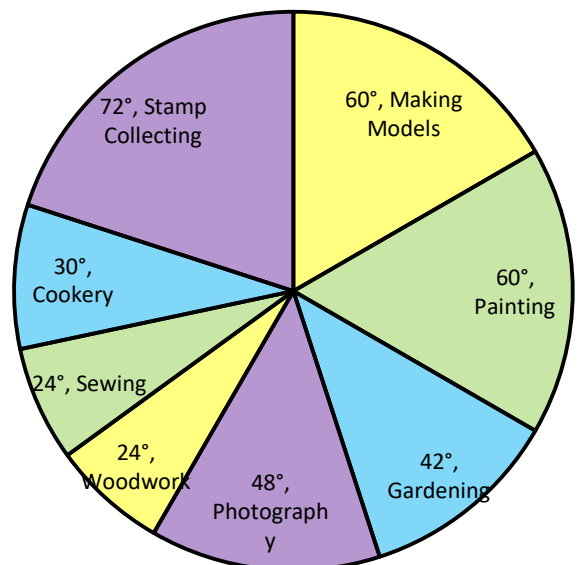


(c) This pie chart shows what several people do in their leisure time.

12 people collect stamps. How many people:

- (i) were surveyed **in total**?
- (ii) like **gardening**?
- (iii) enjoy **photography**?
- (iv) Which two hobbies had the same number of people partaking in them?

Leisure time activities

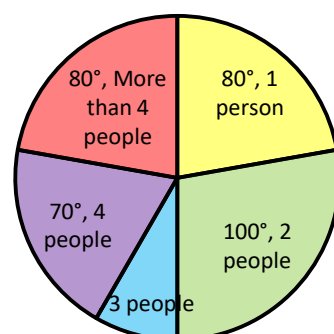


(d) This pie chart shows how many people were in a car when surveyed by Siwan.

Siwan looked at a total of 180 cars.

- (i) How many cars had **2 people** in them?
- (ii) How many cars had **3 people** in them?
- (iii) In how many cars were there **4 people**?

How many people were in the car?



- How much of the crisps is made up of **carbohydrate**?
- How much of the crisps is made up of **fat**?
(Use your protractor to help.)
- How much of the crisps is **protein**?

6

- (i) Daniel looked at the chart and said "*Most of the people had spent their holiday in England*". Jessica said that this was not true. Explain why Daniel is **wrong**.
- (ii) Dafydd looked at the chart and said "*Because 16 people have been in Scotland, it must be that 1% represents 4 people*". Is Dafydd **correct**?
- (iii) How many people had been to **Spain** on their holiday?

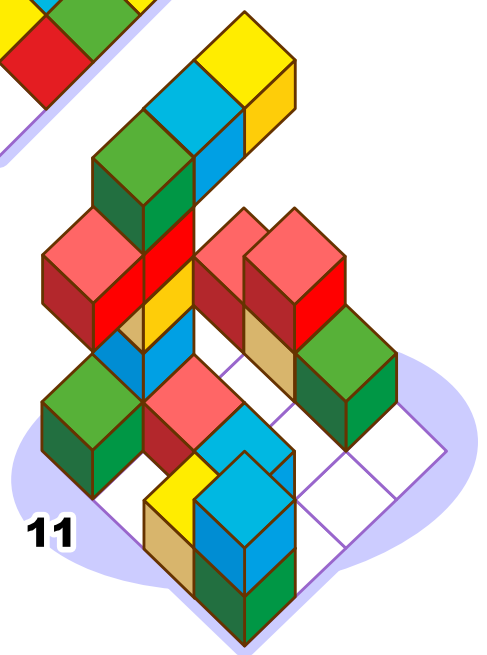
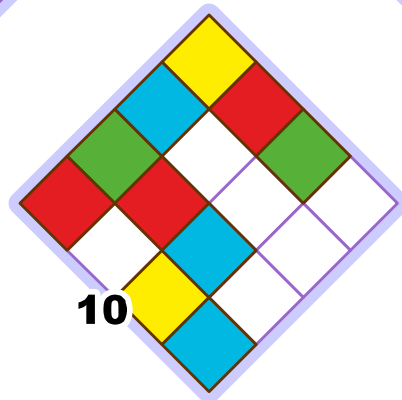
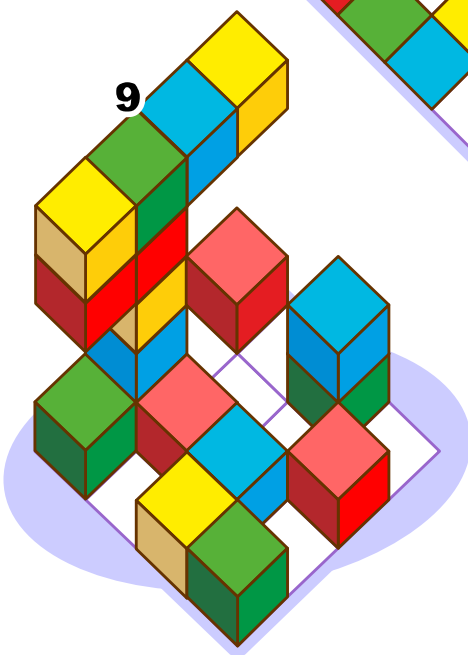
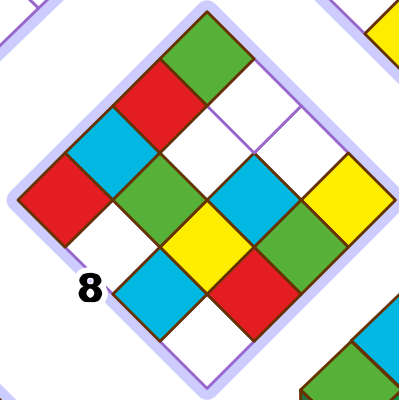
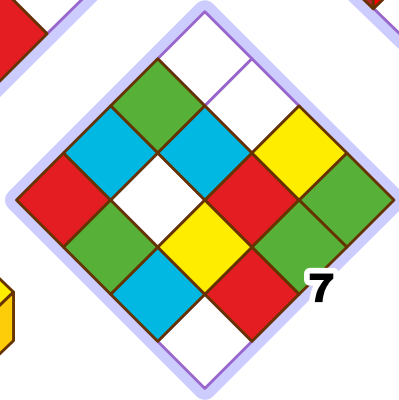
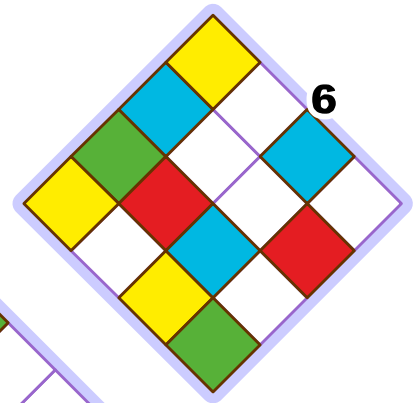
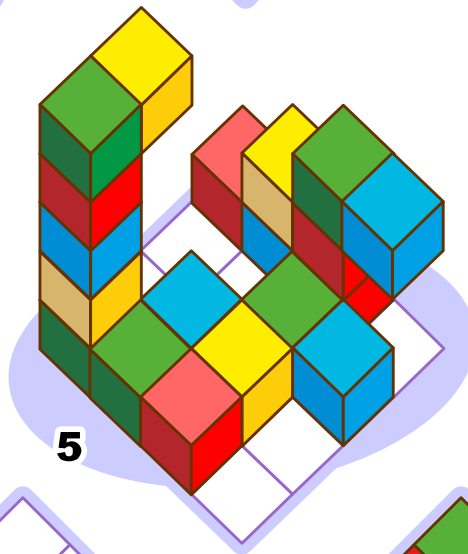
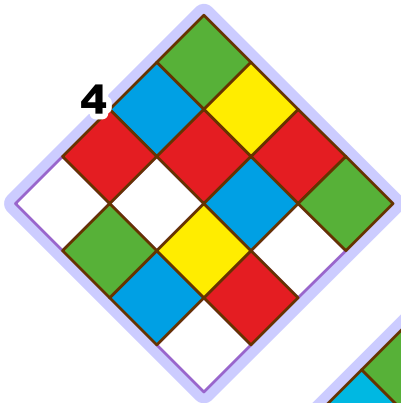
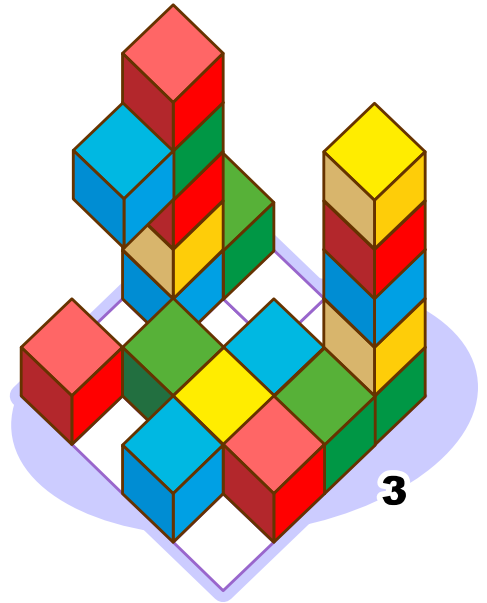
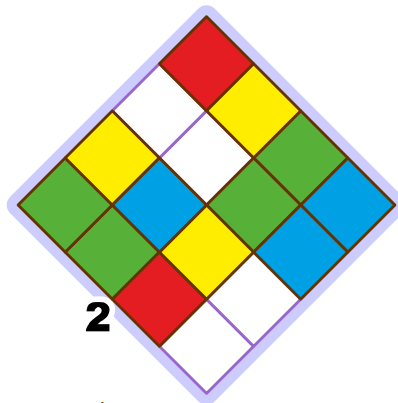
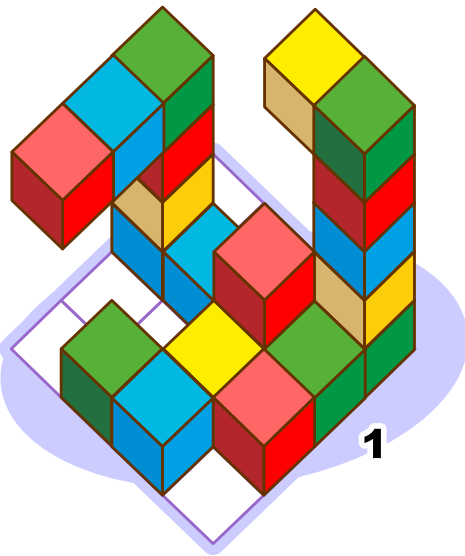
Evaluation

A pie chart illustrating the composition of food. The chart is divided into five segments: a large yellow segment labeled 'Carbohydrate' (45%), a large orange segment labeled 'Fat' (35%), a purple segment labeled 'Protein' (10%), a blue segment labeled 'Fibre' (5%), and a small green segment labeled 'Salt' (5%).

Country	Percentage
England	42%
Spain	30%
Other	10%
Wales	14%
Scotland	4%

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Puzzle: Pair the solid to its top view.





Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to calculate the mean of a set of data.			3	
I know how to calculate the mean of a set of data given in a table .			5	
I know how to calculate the range of a set of data.			3	
I know how to compare two data sets .			3	
I know the difference between qualitative data and quantitative data, and the difference between discrete quantitative data and continuous quantitative data.			2	
I know how to draw a bar chart for qualitative data or for discrete quantitative data that is ungrouped.			1	
I know how to draw a frequency diagram for discrete quantitative data that is grouped or for continuous quantitative data.			7	
I know how to estimate the mean for grouped data.			9	
I know how to estimate the range for grouped data.			9	
I know how to draw a pie chart for qualitative data.			6	
I know how to interpret bar charts, frequency diagrams and pie charts.			4, 8	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

7

Introducing

Percentages

Name:

Contents

Chapter	Mathematics	Page Number
Preparation	Place value. Writing numbers in words and in figures. Multiplying and dividing by a power of 10. Multiplying a decimal by a whole number less than ten. Adding and subtracting decimals.	3
Calculating Percentages	Percentages of shapes. Simple percentages. Calculating a percentage without a calculator. Calculating a percentage with a calculator.	10
Using Percentages	Profit and loss. Simple interest. Salaries. Percentage in a test.	13





Place Value

When we write any number, we use the **decimal system**. The place value of each digit is **ten times more** than the place value of the digit that is immediately to the right.

It is possible to write any number on a **place value chart**. Here is the chart for the number 7289134:

Millions			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
		7	2	8	9	1	3	4

We read the number 7289134 as “seven million, two hundred and eighty-nine thousand, one hundred and thirty-four. It is possible to use commas to make reading the number easier; the number 7289134 is written as 7,289,134. (We group the digits into groups of three.)

Here is the place value chart for the number 457.208:

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
4	5	7	.	2	0	8

We read the number 457.208 as “four hundred and fifty-seven point two zero eight”, or “four hundred and fifty-seven and two hundred and eight thousandths”.

Exercise 1

Write the following numbers on the place value chart shown below.

- (a) 47289 (b) 40.25 (c) 8344903 (d) 91837.234 (e) 0.208
 (f) 139847431986 (g) 30003.001 (h) 300 (i) 31298129381398 (j) 200904.435



	Trillions			Billions			Millions			Thousands			Decimal Point	Tenths	Hundredths	Thousandths
	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units				
(a)																
(b)																
(c)																
(d)																
(e)																
(f)																
(g)																
(h)																
(i)																
(j)																

Exercise 2

Write the numbers from Exercise 1 in words.

Challenge!

Use the internet to investigate names for numbers more than a trillion, or less than a thousandth.

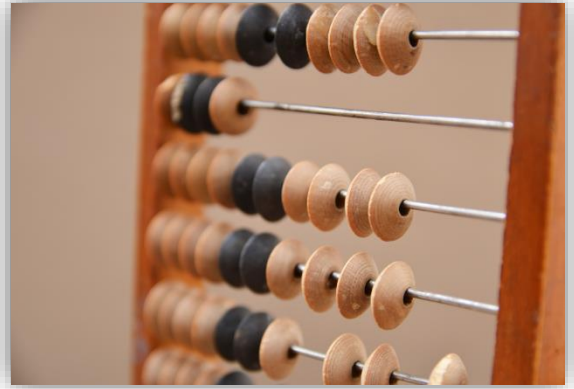
@mathemateg



Exercise 3

Write the following numbers in **words**.
(Hint: use commas to help you.)

- | | | |
|---------------|-------------|---------------|
| (a) 3008 | (b) 7521 | (c) 43978 |
| (d) 12045 | (e) 662907 | (f) 250018 |
| (g) 12650 | (h) 34800 | (i) 1590307 |
| (j) 25482134 | (k) 5062011 | (l) 923010520 |
| (m) 744300042 | (n) 280403 | (o) 2900302 |

**Exercise 4**

Write the following numbers in **figures**.

- | | |
|---|--|
| (a) ten thousand, nine hundred and sixty-four | (b) one hundred and forty-one |
| (c) fifty-two thousand and sixty-six | (d) four million |
| (e) twenty thousand and six hundred | (f) ninety thousand and forty-five |
| (g) nine hundred and eighty thousand, four hundred and five | (h) three billion and seventy-nine |
| (i) forty and four tenths | (j) nine hundredths |
| (k) eight thousand, four hundred and fifty-three | (l) nine thousand and three & fourteen thousandths |

Example

In the number 675, the 7 is in the tens column. The place value of the 7 is therefore seven **tens**. In the number 870031, the 7 is in the tens of thousands column. The place value of the 7 is therefore seven **tens of thousands**.

Exercise 5

What is the **place value** of the 8 in the following numbers?

- | | | | | |
|-------------|-------------|---------------|-------------|--------------|
| (a) 34859 | (b) 584392 | (c) 12346.248 | (d) 4348213 | (e) 38050712 |
| (f) 1800904 | (g) 7342814 | (h) 81234620 | (i) 5263087 | (j) 234.08 |

Egyptian Numbers

In the past, the following system was used in Egypt for writing numbers.

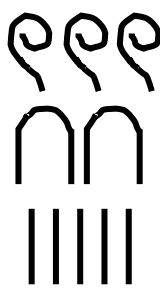
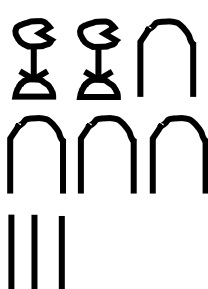
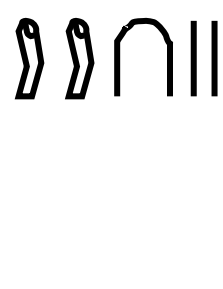
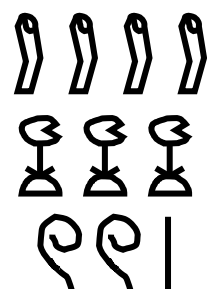
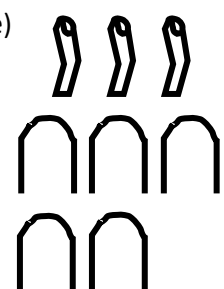
Number	1	10	100	1,000	10,000
Symbol		∩	🌀	👤	👤

**Exercise 6**

Write the following numbers in **words**.

4

Applying

- | | | | | |
|---|---|---|---|---|
| (a)  | (b)  | (c)  | (d)  | (e)  |
|---|---|---|---|---|

Multiplying and Dividing by a Power of 10

Look at the following place value chart for the numbers 5, 23.65, and 284,340,630.2.

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
														5				
													2	3	.	6	5	
						2	8	4	3	4	0	6	3	0	.	2		

By multiplying each of these numbers by 10, we obtain the numbers 50, 236.5, and 2,843,406,302.

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
													5	0				
												2	3	6	.	5		
					2	8	4	3	4	0	6	3	0	2				

Discuss the effect that multiplying by 10 has had on the place value chart. What would be the effect of multiplying by 100 or 1,000?

Exercise 7

- (a) 34×10

(e) 450×10

(i) 328×100

(m) 0.24×100

(q) 82×1000

(u) 0.0003729×1000

(y) 7293.1×10

(b) 23.56×10

(f) 0.24×10

(j) 280×100

(n) 0.0345×100

(r) 928.26332×1000

(v) 2.88×1000

(z) 0.0048×10

(c) 2837×10

(g) 0.00323×10

(k) 234.232×100

(o) 0.00302×100

(s) 2.3×1000

(w) 0.0006929×1000

(α) 2.829×100

(d) 2.3×10

(h) 7.8×10

(l) 2.31×100

(p) 9383.2134×100

(t) 0.0983×1000

(x) 3.1×1000

(β) 0.204×100

5

Skill

By dividing the numbers 5, 23.65, and 284,340,630.2 by 10, we obtain the numbers 0.5, 2.365 and 28,434,063.02.

Trillions			Billions			Millions			Thousands									
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
														0	.	5		
														2	.	3	6	5
							2	8	4	3	4	0	6	3	.	0	2	

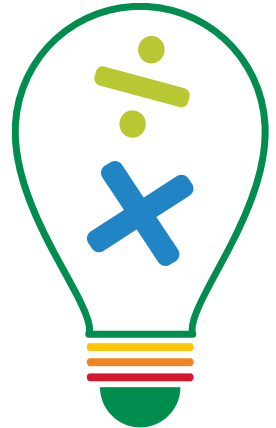
Discuss the effect that dividing by 10 has had on the place value chart. What would be the effect of dividing by 100 or 1,000?

Exercise 8

- | | | | |
|---------------------------|----------------------------|---------------------------|----------------------------|
| (a) $520 \div 10$ | (b) $45 \div 10$ | (c) $2 \div 10$ | (d) $2.3 \div 10$ |
| (e) $4534 \div 10$ | (f) $0.83 \div 10$ | (g) $0.00267 \div 10$ | (h) $124654 \div 10$ |
| (i) $745 \div 100$ | (j) $48000 \div 100$ | (k) $32 \div 100$ | (l) $0.3 \div 100$ |
| (m) $0.86 \div 100$ | (n) $0.0325 \div 100$ | (o) $0.0005 \div 100$ | (p) $346356.4554 \div 100$ |
| (q) $460000 \div 1000$ | (r) $5657.43232 \div 1000$ | (s) $2.3 \div 1000$ | (t) $6 \div 1000$ |
| (u) $0.0003475 \div 1000$ | (v) $5.82 \div 1000$ | (w) $0.0008821 \div 1000$ | (x) $8.5 \div 1000$ |
| (y) $345 \div 10$ | (z) $0.0024 \div 10$ | (α) $7.433 \div 100$ | (β) $0.8 \div 100$ |

**Exercise 9**

- | | | | |
|--------------------------|------------------------|---------------------------|-----------------------|
| (a) 436×10 | (b) $256 \div 100$ | (c) 2.43×1000 | (d) $0.002 \div 10$ |
| (e) 0.0026×100 | (f) $1829 \div 1000$ | (g) 928×100 | (h) $0.239 \div 100$ |
| (i) 0.23×10 | (j) $9 \div 100$ | (k) 0.00238×1000 | (l) $23 \div 10$ |
| (m) 0.000202×10 | (n) $2839 \div 10$ | (o) 2.04×10 | (p) $29 \div 1000$ |
| (q) 900×100 | (r) $1.92 \div 100$ | (s) 8×1000 | (t) $0.0342 \div 100$ |
| (u) 0.08×100 | (v) 78×100 | (w) $0.023 \div 1000$ | (x) $7 \div 100$ |
| (y) 13.45×10 | (z) $19293.2 \div 100$ | (α) 2×10 | (β) $0.004 \div 10$ |

**Multiplying a Decimal by a Whole Number less than 10**

To calculate a sum like 75.82×6 , follow the steps shown below.

1. Set out the sum so that the 6 is below the 2.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline \end{array}$$

2. Working from right to left, start by working out $2 \times 6 = 12$.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 12 \\ 1 \end{array}$$

3. Next, we calculate $8 \times 6 = 48$, and add the 1 to obtain 49.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 92 \\ 4 \quad 1 \end{array}$$

4. We must ensure that the decimal points are lined up.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline .92 \\ 4 \quad 1 \end{array}$$

5. The next sum is $5 \times 6 = 30$, and add 4 to obtain 34.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 4.92 \\ 3 \quad 4 \quad 1 \end{array}$$

6. To finish we do $7 \times 6 = 42$, and add the 3 to obtain 45.

$$\begin{array}{r} 75.82 \\ \times \quad 6 \\ \hline 454.92 \\ 3 \quad 4 \quad 1 \end{array}$$

Puzzle

Awel, Bob and Catrin are counting their money. Awel and Bob have a total of £4.60. Bob and Catrin have a total of £4.97. Awel and Catrin have a total of £6.63. How much money do they have, all together?

Exercise 10

- | | | | |
|------------------------|-------------------------|------------------------|------------------------|
| (a) 43.61×3 | (b) 2.53×4 | (c) 9.46×5 | (d) 2.3×7 |
| (e) 93.68×9 | (f) 102.4×6 | (g) 29.437×2 | (h) 20.31×5 |
| (i) 9.3×8 | (j) 128.432×7 | (k) 26.43×8 | (l) 65.43×6 |
| (m) 23.392×7 | (n) 245.3×4 | (o) 925.32×6 | (p) 902.1×8 |
| (q) 7256.42×3 | (r) 294.04×5 | (s) 92.239×7 | (t) 1002.3×6 |
| (u) 1039.24×8 | (v) 293.9×4 | (w) 23.4×9 | (x) 2849.23×5 |
| (y) 3289.2×6 | (z) 3409.223×4 | (α) 239.44×10 | (β) 2839×6 |

5

Skill



Adding Decimals

To calculate a sum like $24.52 + 387.963$, follow the steps shown below.

1. Set out the sum so that the decimal points line up.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline \end{array}$$

2. Working from right to left, we start by calculating $0 + 3 = 3$.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 3 \end{array}$$

3. Next, we do $2 + 6 = 8$ and then $5 + 9 = 14$.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 483 \end{array}$$

4. Remember to insert the decimal point in the correct position.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline .483 \\ 1 \end{array}$$

5. We now continue with the addition sums: $4 + 7 = 11$, and add one to obtain 12.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 2.483 \\ 1 \quad 1 \end{array}$$

6. We complete the final two additions to arrive at the final solution.

$$\begin{array}{r} 24.52 \\ + 387.963 \\ \hline 412.483 \\ 1 \quad 1 \quad 1 \end{array}$$

Exercise 11

- | | | | |
|----------------------|----------------------|-------------------------|---------------------------|
| (a) $24.67 + 83.49$ | (b) $8.28 + 3.66$ | (c) $23.384 + 98.324$ | (d) $9.3987 + 3.2893$ |
| (e) $4.87 + 23.23$ | (f) $432.2 + 2.7$ | (g) $54.23 + 293.234$ | (h) $85.23 + 829.385$ |
| (i) $839.28 + 2.394$ | (j) $23.2 + 284.284$ | (k) $48.3 + 2.95$ | (l) $87.23 + 7.49$ |
| (m) $9.2839 + 54.23$ | (n) $2.34 + 21.492$ | (o) $204.93 + 48.943$ | (p) $92.3 + 24.503$ |
| (q) $452.95 + 21.2$ | (r) $2004.32 + 3.87$ | (s) $45.828 + 58834.2$ | (t) $2883.23 + 2948.3939$ |
| (u) $0.234 + 2.3$ | (v) $0.0283 + 5.43$ | (w) $256.37 + 0.092$ | (x) $25 + 0.32$ |
| (y) $38.5 + 924.85$ | (z) $0.43 + 0.068$ | (α) $2982.323 + 399.21$ | (β) $399.2 + 299.3567$ |

5



Puzzle

One of the following numbers has been **typed incorrectly**.

9, 7, 4, 6, 3, 5, 2, 4, 1, 3

Which number is incorrect, and why?



Subtracting Decimals

To calculate a sum like $53.8 - 1.46$, follow the steps shown below.

1. Set out the sum so that the decimal points line up.

$$\begin{array}{r} 53.8 \\ - 1.46 \\ \hline \end{array}$$

2. If required, add zeroes after the decimal point so that an equal number of digits appear after the decimal point.

$$\begin{array}{r} 53.80 \\ - 1.46 \\ \hline \end{array}$$

3. It is not possible to do $0 - 6$, so we borrow from the next column to the left: 8 changes to 7 and 0 changes to 10.

$$\begin{array}{r} ^7 ^1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline \end{array}$$

4. Subtracting, we calculate $10 - 6 = 4$; and $7 - 4 = 3$.

$$\begin{array}{r} ^7 ^1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline 34 \end{array}$$

5. Remember to insert the decimal point in the correct position.

$$\begin{array}{r} ^7 ^1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline .34 \end{array}$$

6. We complete the final two subtractions to arrive at the final solution.

$$\begin{array}{r} ^7 ^1 \\ 53.\cancel{8}0 \\ - 1.46 \\ \hline 52.34 \end{array}$$

Exercise 12

- | | | | |
|----------------------|---------------------|----------------------|--------------------------|
| (a) $25.78 - 13.24$ | (b) $7.8 - 6.4$ | (c) $128.76 - 45.5$ | (d) $95.9 - 58.6$ |
| (e) $8.3 - 5$ | (f) $19.8 - 12.9$ | (g) $23.9 - 4.67$ | (h) $48.4 - 7.84$ |
| (i) $256.8 - 135.78$ | (j) $14 - 7.2$ | (k) $23 - 9.7$ | (l) $34 - 4.8$ |
| (m) $689 - 23.6$ | (n) $45.1 - 9.83$ | (o) $208.5 - 123.88$ | (p) $700.34 - 216.74$ |
| (q) $45.67 - 23.895$ | (r) $128.2 - 2.32$ | (s) $27.7 - 23.53$ | (t) $2983 - 2837.3$ |
| (u) $23.4 - 14.528$ | (v) $56.7 - 43.238$ | (w) $87.5 - 23.282$ | (x) $74.2 - 17.854$ |
| (y) $25 - 12.447$ | (z) $90 - 43.245$ | (α) $100 - 23.4329$ | (β) $101010 - 94738.282$ |

5



Exercise 13

- (a) Steffan cuts a piece of string into three pieces. The length of the pieces are 4.24cm; 7.58cm; and 3.5cm. What was the length of the string **before** Steffan made the cuts?
- (b) Emma cycles to work, a journey of 7.8 miles. One morning, on the way to work, Emma stops to buy a newspaper from a shop. If the shop is 2.9 miles from Emma's home, how far is the shop from Emma's place of work?
- (c) In a gymnastics competition, Lisa scores 12.6 on vault; 11.9 on bars; 12.7 on beam; and 12.9 on floor. What was **Lisa's total score**?
- (d) Dewi's height is 1.45m. Dewi has grown 0.12m in 18 months. What was Dewi's height **18 months ago**?



Applying

Investigation

Imagine that you have a pair of fair dice, as shown in the picture. When rolled, we can combine the numbers on the dice to make **two decimals that are less than 10.**

For example, if the two numbers on the dice are 6 and 2, then the two decimals are 6.2 and 2.6.

By rounding off the two decimals, we obtain two whole numbers. In the example, the whole numbers are 6 (rounding off 6.2) and 3 (rounding off 2.6).

Investigate to see if it is possible to obtain two **equal** whole numbers after rounding the decimals. If this is possible, list the different ways of obtaining two equal whole numbers.



Exercise 14 (Revision)

- (a) Write the number 2439245.2 in **words**.
- (b) Write the number three million, seventy thousand and sixty in **figures**.
- (c) Calculate the answer to the following.
- | | |
|---------------------------|---------------------------|
| (i) 45.7×100 | (ii) $2.56 \div 10$ |
| (iii) 0.03245×10 | (iv) $24 \div 1000$ |
| (v) 245.6×9 | (vi) 0.234×4 |
| (vii) $123.54 + 29.829$ | (viii) $23.29 + 9283.928$ |
| (ix) $245.32 - 25.17$ | (x) $45 - 27.54$ |
- (d) Alun bought six gas canisters for a camping holiday. If each of the canisters weighed 0.56kg, what was the **total weight** of the canisters?

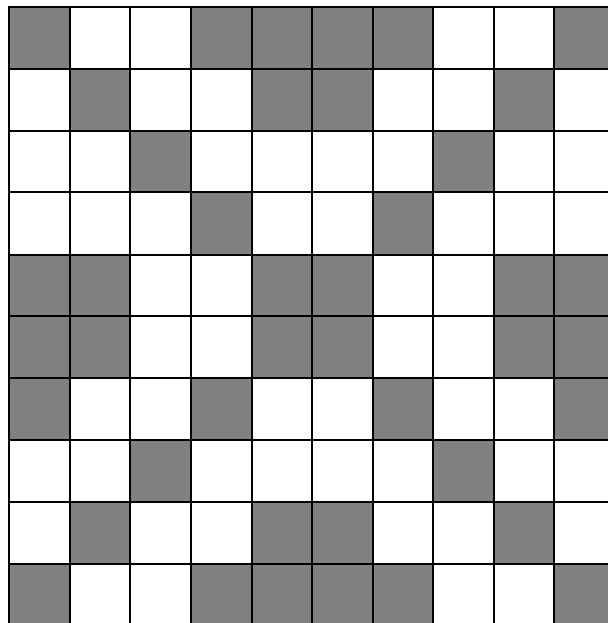
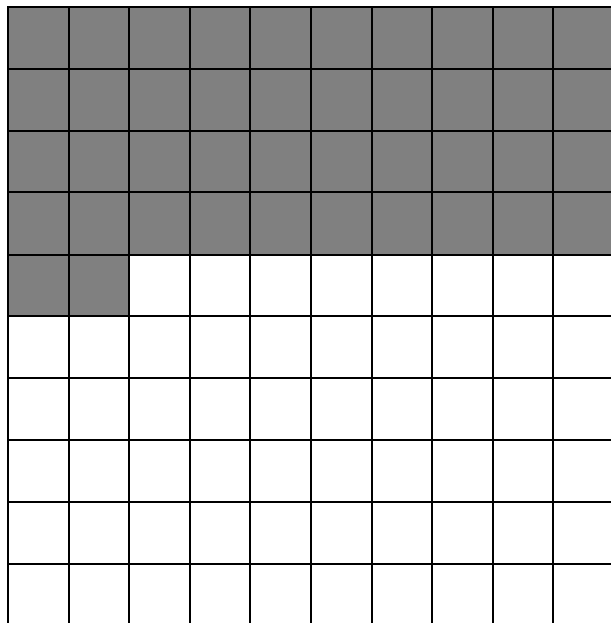


Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Calculating Percentages

A percentage represents a fraction out of 100. For example, 42% is **equivalent** to $\frac{42}{100}$, or 42 out of 100. We read 42% as “forty-two percent”. In each of the following diagrams, 42% of the diagram has been shaded.



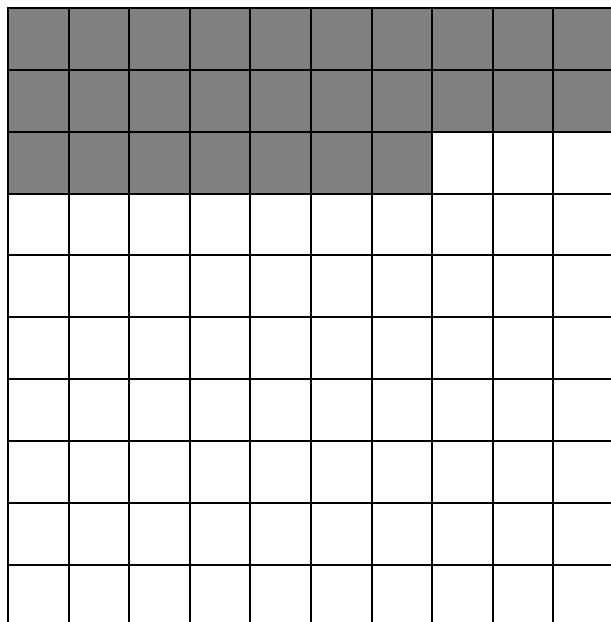
Exercise 15

4

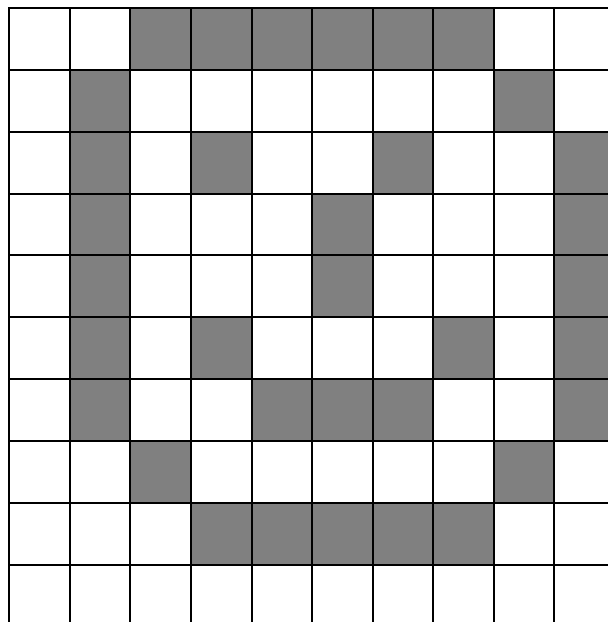
Skill

(a) What percentage of the following shapes have been shaded?

(i)



(ii)



(b) What percentage of the above shapes have **not** been shaded?

Simple Percentages

Some percentages are easier to calculate than others. Try to complete the following table which explains how to calculate some simple percentages.

Percentage	Fraction	To calculate this percentage:
50%	$\frac{1}{2}$, because 50% fits into 100% twice.	Divide by 2, or halve.
25%		
10%		
20%		
5%		
1%		



Exercise 16

4

Shade in the shown percentage.

(a) 50%

(b) 25%

(c) 20%

(d) 10%

(e) 50%

(f) 5%

(g) 60%

(h) 25%

(i) 40%

--	--	--	--	--

Exercise 17

Calculate the shown percentage.

(a) 50% of 80

(b) 25% of 24

(c) 10% of 70

(d) 20% of 30

(e) 50% of 12

(f) 25% of 36

(g) 10% of 160

(h) 5% of 80

(i) 1% of 600

(j) 20% of 35

(k) 50% of 7

(l) 25% of 30

(m) 20% of 65

(n) 10% of 85

(o) 50% of 19

(p) 50% of £48

(q) 25% of £180

(r) 10% of £2500

(s) 20% of £50

(t) 1% of £190

(u) 5% of £120

(v) 25% of £34

(w) 20% of £45

(x) 10% of £19

(y) 40% of £200



4

Calculating a Percentage without a Calculator

To calculate 10% of a number, we divide the number by 10. For example, 10% of £38 is $£38 \div 10 = £3.80$.

Exercise 18

5

Calculate **10%** of the following values.

- | | | | | |
|----------|-----------|----------|------------|-----------|
| (a) 80 | (b) 170 | (c) 600 | (d) 18 | (e) 49 |
| (f) £28 | (g) £187 | (h) £16 | (i) £286 | (j) £182 |
| (k) \$24 | (l) \$920 | (m) 25.4 | (n) £78.50 | (o) £0.70 |

To calculate 1% of a number, we divide the number by 100. For example, 1% of £38 is $£38 \div 100 = £0.38$.

Another way of calculating 1% would be to calculate 10% first, and then divide this answer by 10. For example, 10% of £38 is $£38 \div 10 = £3.80$, so 1% of £380 is $£3.80 \div 10 = £0.38$.

Exercise 19

Calculate **1%** of the following values.

- | | | | | |
|----------|-----------|----------|----------|-----------|
| (a) 200 | (b) 260 | (c) 400 | (d) 17 | (e) 53 |
| (f) £58 | (g) £276 | (h) £12 | (i) £253 | (j) £176 |
| (k) \$26 | (l) \$298 | (m) 45.8 | (n) 64.3 | (o) 97.25 |

Knowing 10% and 1% of a number, it is possible to calculate **any whole number percentage** between 1% and 99%.

Example

What is 37% of £85?

The first step is to calculate **10%** and **1%** of the number.

Next, we calculate **30%** of the number by multiplying the 10% by 3; and calculate **7%** of the number by multiplying the 1% by 7.

Finally, we **add** the 30% to the 7% to obtain the final solution.

10%
 $£85 \div 10 = £8.50$
1%
 $£8.50 \div 10 = £0.85$

$\begin{array}{r} 8.50 \\ \times 3 \\ \hline 25.50 \\ 1 \end{array}$	$\begin{array}{r} 0.85 \\ \times 7 \\ \hline 5.95 \\ 5 \quad 3 \end{array}$
--	---

$$\begin{array}{r} 25.50 \\ + 5.95 \\ \hline £31.45 \\ 1 \quad 1 \end{array}$$

Exercise 20

Calculate the following percentages.

- | | | | |
|-----------------|-----------------|-------------------|-------------------|
| (a) 48% of £75 | (b) 73% of £94 | (c) 42% of £36 | (d) 93% of £71 |
| (e) 16% of £82 | (f) 24% of £68 | (g) 66% of £84 | (h) 79% of £52 |
| (i) 46% of £168 | (j) 32% of £262 | (k) 74% of £375 | (l) 95% of £186 |
| (m) 22% of 450 | (n) 47% of 820 | (o) 81% of 1042 | (p) 15% of 2,400 |
| (q) 6% of £28 | (r) 99% of £80 | (s) 28% of £5,400 | (t) 12% of £8,400 |
| (u) 2% of \$240 | (v) 41% of \$95 | (w) 88% of \$88 | (x) 90% of \$740 |
| (y) 38% of £94 | (z) 46% of £143 | (α) 33% of £400 | (β) 87% of £2,145 |



Calculating a Percentage using a Calculator

Every scientific calculator has a percentage button (%). Sometimes the **SHIFT** or the **2nd Fn** button must be pressed to access the percentage button.

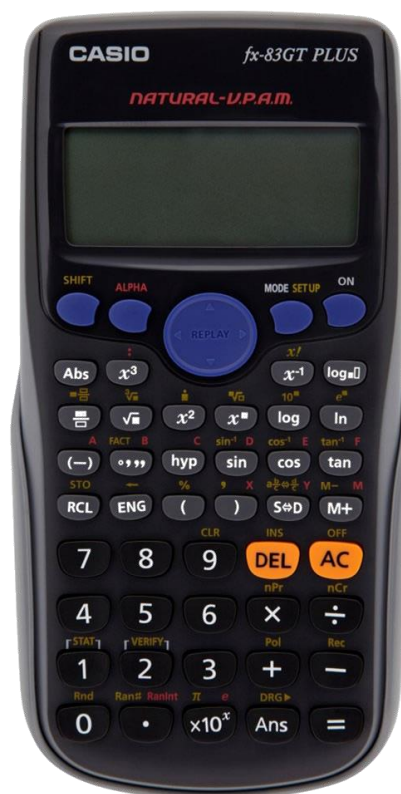
To calculate 37% of £85 using the calculator shown on the right, the following buttons must be pressed.

8 5 × 3 7 SHIFT (=

The screen of the calculator should show 31.45.

(Press the button **S** ⇌ **D** if you see $\frac{629}{20}$.)

Maybe your calculator is different. In the box below, write down the buttons you must press to calculate 43% of £97.



Exercise 21

5

Use your calculator to calculate the following percentages.

- | | | | |
|------------------|-----------------|-------------------|-------------------|
| (a) 74% of £24 | (b) 28% of £57 | (c) 35% of £96 | (d) 4% of £125 |
| (e) 18% of £74 | (f) 27% of £57 | (g) 68% of £15 | (h) 73% of £4 |
| (i) 42% of 285 | (j) 35% of 685 | (k) 77% of 457 | (l) 98% of 35.2 |
| (m) 23% of \$875 | (n) 2% of \$241 | (o) 74% of €1,095 | (p) 35% of €7,800 |



Exercise 22 (Revision)

5

- (a) Draw a circle with a compass. Colour in 25% of the circle.
 (b) Calculate the following **without** using a calculator.
 (i) 50% of 38 (ii) 25% of 500 (iii) 10% of £12 (iv) 20% of \$58 (v) 48% of £76 (vi) 87% of €146
 (c) Calculate the following using a calculator.
 (i) 24% of 57 (ii) 73% of £25 (iii) 83% of \$1,950

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Using Percentages

5% COMMISSION!

COTTON 30%
POLYESTER 70%



Percentages can be seen in many situations, as shown above. Can you think of more examples where a percentage is used?

Profit and Loss

Profit and loss are important in business. In making a profit we are receiving **more** money, and in making a loss we are **losing** money.

Example

A car dealer buys a car for £7,800. If the dealer wants to make a profit of at least 12% in selling the car, what price should the dealer place on the car?

Answer: 10% of £7,800 is $£7,800 \div 10 = £780$,
and 1% of £7,800 is $£780 \div 10 = £78$.

It follows that 2% of £7,800 is $£78 \times 2 = £156$,
and 12% of £7,800 is $£780 + £156 = £936$.

So the dealer should sell the car for at least
 $£7,800 + £936 = £8,736$. (It is probable that the dealer will place a sticker like £8,750 or £9,000 on the car.)



Exercise 23

(a) A car dealer buys a car for £9,200. If the dealer wishes to make a profit of **at least 15%** on selling the car, what price should the dealer place on the car?

(b) A furniture company holds a '**40% off**' sale. Complete the following table showing the prices in the sale.

Item	Original price	Discount	Price in the sale
Sofa	£800		
Table	£250		
Cushion	£15		
Wardrobe	£550		
4 Chairs	£160		

Applying

5



(c) Deiniol went into a clothes shop and saw a pair of trousers with a marked price of £40. If there was a '20% off' sale in the shop, how much did Deiniol **pay** for the trousers at the till?

(d) An estate agent received 3% commission on selling a house for £185,000. **How much commission** did the estate agent receive?

(e) After advertising on social media, a company saw a **16% increase** in their profit. If the company's profit was £2,400 a week before advertising started, what was the company's profit (per week) after advertising started?

(f) A jeweller bought a ring for £1,200, hoping to sell it for **20% profit**. Unfortunately, there was no interest in the ring, so the jeweller had to sell it in a sale, making a **loss of 14%**.

(i) At what price was the jeweller hoping to sell the ring?

(ii) At what price did the jeweller sell the ring in the end?

(g) Megan changed £400 into euros. The bank claimed **2% commission**. How much was the commission?

(h) A shop holds a '**70% off**' closing down sale. Siwan buys a pair of shoes in the sale, usually costing £60. How much did Siwan pay for the shoes?



Simple Interest

When borrowing money from a bank, the bank charges you **interest**. You must pay back this interest on top of the money you have borrowed. On the other hand, if you **invest** money into a bank – which is to place money in a bank for safe keeping – the bank pays you interest. The **interest rate** is usually given as a percentage. There are several types of interest; in year 7 we will consider one type, namely **simple interest**.



Example

Rheinallt borrows **£12,000** from Barclays Bank at a simple interest rate of **3% a year**. Rheinallt wants to pay back all the money after **four years**. How much money will Rheinallt have to pay back after four years?

Answer: With a simple interest rate of 3% a year, the bank will charge 3% of £12,000 for each year that Rheinallt borrows the money.

1) Calculate 3% of £12,000.

2) Multiply the £360 by the number of years that Rheinallt wants to borrow the money for, namely 4 years.

3) Add the simple interest (£1,440) to the amount borrowed (£12,000) to obtain the final solution.

1%
 $£12,000 \div 100 = £120$
3%
 $£120 \times 3 = £360$

$$\begin{array}{r} 360 \\ \times 4 \\ \hline £1440 \\ 2 \end{array}$$

$$\begin{array}{r} 12000 \\ + 1440 \\ \hline £13440 \end{array}$$

Exercise 24

5

The following table shows the simple interest rate for different sums to borrow over a specific period of time. In each case, calculate the amount of money that must be **paid back** to the bank at the end of the period of borrowing.

	Sum to borrow	Simple interest rate, per year	Period to borrow
(a)	£15,000	3%	4 years
(b)	£18,000	4%	3 years
(c)	£9,500	2%	3 years
(d)	£13,500	5%	6 years
(e)	£2,500	7%	2 years
(f)	£26,000	6%	5 years
(g)	£14,750	3%	2 years
(h)	£50,000	4%	3 years



Example

Siriol wants to invest **£8,000** in HSBC Bank at a simple interest rate of **4% a year**. Siriol wants to withdraw all the money from the bank after **five years**. How much money will Siriol be able to withdraw after five years?

Answer: With a simple interest rate of 4% a year, the bank will add 4% of £8,000 for each year that Siriol keeps the money in the bank.

1) Calculate 4% of £8,000.

2) Multiply the £320 by the number of years Siriol wants to keep the money in the bank, namely 5 years.

3) Add the simple interest (£1,600) to the original sum (£8,000) to obtain the final solution.

1%
 $£8,000 \div 100 = £80$
4%
 $£80 \times 4 = £320$

$$\begin{array}{r} 320 \\ \times 5 \\ \hline £1600 \\ 1 \end{array}$$

$$\begin{array}{r} 8000 \\ + 1600 \\ \hline £9600 \end{array}$$

Exercise 25

5

The following table shows the simple interest rate for different sums to invest over a specific period of time. In each case, calculate how much money is available to **withdraw** from the bank at the end of the period of investment.

	Sum to invest	Simple interest rate, per year	Period to invest
(a)	£5,000	4%	3 years
(b)	£8,500	6%	2 years
(c)	£12,000	3%	4 years
(d)	£1,500	4%	5 years
(e)	£2,800	2%	3 years
(f)	£100,000	7%	2 years
(g)	£24,600	1%	5 years
(h)	£6,540	3%	8 years



Personal Finance

Percentages often appear when considering how much money comes into and out of a home.

Salaries

When receiving a salary, you are not allowed to keep every penny of what your employer gives you. Everyone starts off with a **gross salary**, out of which items such as *income tax*; *national insurance*; and *pension* are taken. We call the amount that is left a **net salary**, and this is what reaches your bank account.

Example

Elin receives a gross salary of £450 a week. Elin must **deduct** 26% of her salary as income tax; national insurance and pension. What is Elin's net salary, per week?

Answer: Using a calculator, 26% of £450 is

$$450 \times 26\% = £117.$$

Elin's net salary is therefore $£450 - £117 = £333$ per week.



Exercise 26

5

Use a calculator to complete the following table. (The above example forms the first row of the table.)

	Gross salary (per week)	Percentage of income tax; national insurance and pension	Contributions (or Deductions)	Net salary (per week)
	£450	26%	£117	£333
(a)	£350	22%		
(b)	£720	27%		
(c)	£290	19%		
(d)	£210	15%		
(e)	£596	26%		
(f)	£740	31%		
(g)	£185	12%		
(h)	£245	16%		
(i)	£539	29%		
(j)	£1,050	38%		

Challenge!

Mr. Jones' gross salary is £26,624 per year.

Can you work out Mr. Jones' **net salary** using the following information?



7

Extension

1. Income Tax Basic Rate: 20% on your yearly earnings between £10,600 and £31,785 Higher Rate: 40% on your yearly earnings between £31,786 and £150,000.	2. National Insurance Contributions Mr. Jones pays Class 1 National Insurance Contributions. The rates are: 12% on weekly earnings between £155 and £815 2% on weekly earnings over £815	3. Pension The rates for pension contributions are: 7.5% on your yearly earnings between £0 and £25,999.99 8.5% on your yearly earnings between £26,000 and £34,999.99.
---	---	--

Bills

When you pay for goods, some of the money goes to the government as a **tax** on what you have bought. The name of this tax is **Value Added Tax (VAT)**. At the current time, the rate of VAT is 20%, but some goods use a different rate.

- The rate for *energy bills; child car seats; and mobility aids for the elderly* is 5%.
- The rate for *children's clothes and shoes; books; magazines; and bike helmets* is 0%.



Example

The price of a television before adding VAT is £250. What is the price of the television after adding VAT?

Answer: The VAT rate for a television is 20%. Using a calculator, 20% of £250 is $£250 \times 20\% = £50$. It follows that the price of a television after adding VAT is $£250 + £50 = £300$.

Exercise 27

Complete the following table showing the prices of items **after** adding VAT. (The above example forms the first row of the table.)

5

Applying

	Item	Price before adding VAT	VAT Rate	Amount of VAT	Price after adding VAT
	Television	£250	20%	£50	£300
(a)	Mobile Phone	£275	20%		
(b)	Kettle	£30	20%		
(c)	Electricity Bill	£1,400	5%		
(d)	Sofa	£600	20%		
(e)	Chocolate Bar	£1.25	20%		
(f)	Gas Bill	£740	5%		
(g)	Magazine	£3.60	0%		
(h)	Washing Machine	£187.50	20%		
(i)	Children's Shoes	£7.50	0%		
(j)	Toy	£35	20%		

Special Offers

Exercise 28

5



(a) Two shops sell the same mobile phone. The first shop sells the mobile phone for £120. The second shop sells the mobile phone for £150, but today there is a special offer available: "25% off". Which shop sells the mobile phone for the **cheapest** price today?

(b) Two shops sell the same television. The first shop sells the television for £425. The second shop sells the television for £500, but today there is a special offer available: "20% off". Which shop sells the television for the **cheapest** price today?

(c) Two shops sell the same hairdryer. The first shop sells the hairdryer for £25. The second shop sells the hairdryer for £35, but today there is a special offer available: "30% off". Which shop sells the hairdryer for the **cheapest** price today?

(d) Two shops sell the same vacuum cleaner. The first shop sells the vacuum cleaner for £199. The second shop sells the vacuum cleaner for £350, but today there is a special offer available: "40% off". Which shop sells the vacuum cleaner for the **cheapest** price today?

Percentage in a Test

If Hari has received 26 out of 30 in a test, what **percentage** has Hari received in the test?

Answer: To change Hari's mark to a percentage, we need to multiply the fraction $\frac{26}{30}$ by 100%. On a calculator, the buttons that need to be pressed to do this are

$$2 \quad 6 \quad \div \quad 3 \quad 0 \quad \times \quad 1 \quad 0 \quad 0 \quad =$$

The number 86.66666667 should appear on the screen of the calculator. (Perhaps you must press **S** $\leftrightarrow$ **D** to see the number.) This is Hari's percentage in the test. Rounded off to the nearest whole number, Hari's percentage in the test is **87%**.



Exercise 29

□

Change the following test results to **percentages**. Give your answers to the nearest percentage.

- (a) 18 out of 25 (b) 11 out of 20 (c) 23 out of 28 (d) 5 out of 18
(e) 45 out of 63 (f) 123 out of 150 (g) 4 out of 7 (h) 32 out of 70
(i) 25 out of 80 (j) 14 out of 23 (k) 241 out of 300 (l) 47 out of 100



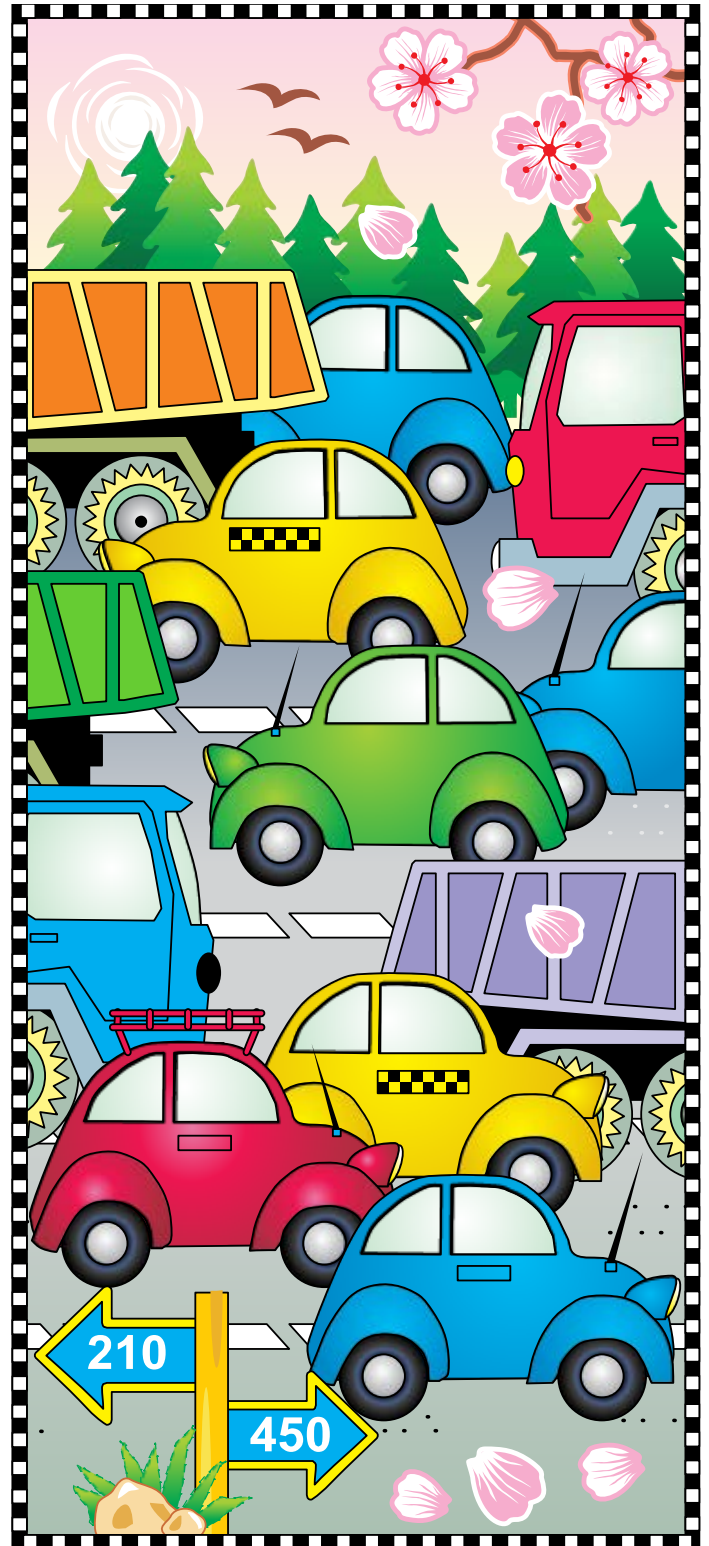
Exercise 30 (Revision)

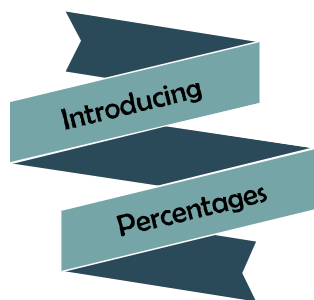
- John changes £500 to dollars. The post office take **3% commission**. How much is the commission?
- Imogen wants to invest **£6,000** into Lloyds Bank at a simple interest rate of **3% a year**. Imogen wants to take all the money out of the bank after **two years**. How much money will Imogen be able to withdraw after two years?
- Sean receives a **gross salary** of £450 per week. Sean must deduct 24% of his salary as income tax; national insurance and pension. What is Sean's **net salary**, per week?
- The price of a computer **before** adding VAT (at 20%) is £400. What is the price of the computer **after** adding VAT?
- Emma scores 28 out of 43 in an English test. What **percentage** did Emma score in the test?
- Out of all 650 pupils in Ysgol y Creuddyn, 381 are female. What **percentage** of the school's pupils are **female**?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Puzzle: Find the ten differences between the pictures.





Reflection Sheet

Name:

Percentage in the test:

I know this. 	I need to revise this. 	Question in the test: Without a calculator With a calculator	Correct in the test?
I know how to write numbers in words and in figures .		1, 2	
I know the place value of different digits in numbers, e.g. the place value of the 8 in the number 28,105.		3	
I know how to multiply and divide numbers by 10 , 100 or 1,000 .		7	
I know how to multiply a decimal by a whole number less than 10 , e.g. 83.548×7 .		8	
I know how to add decimals .		9	
I know how to subtract decimals , using "borrowing" if required.		9	
I know how to decide which percentage of a shape is shaded .		4, 5	
I know how to calculate simple percentages quickly, e.g. 25% of £80.		6	
I know how to calculate any percentage without using a calculator , e.g. 73% of £170.		10	
I know how to calculate any percentage using a calculator .		1	
I know the meaning of profit and loss .		11, 2	
I know how to use simple interest when borrowing from or investing into a bank.		3	
I know how to use percentages in questions involving salaries , bills or special offers .		4, 5, 6, 8	
I know how to use a calculator to calculate the percentage obtained in a test .		7	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

7

Co-ordinates

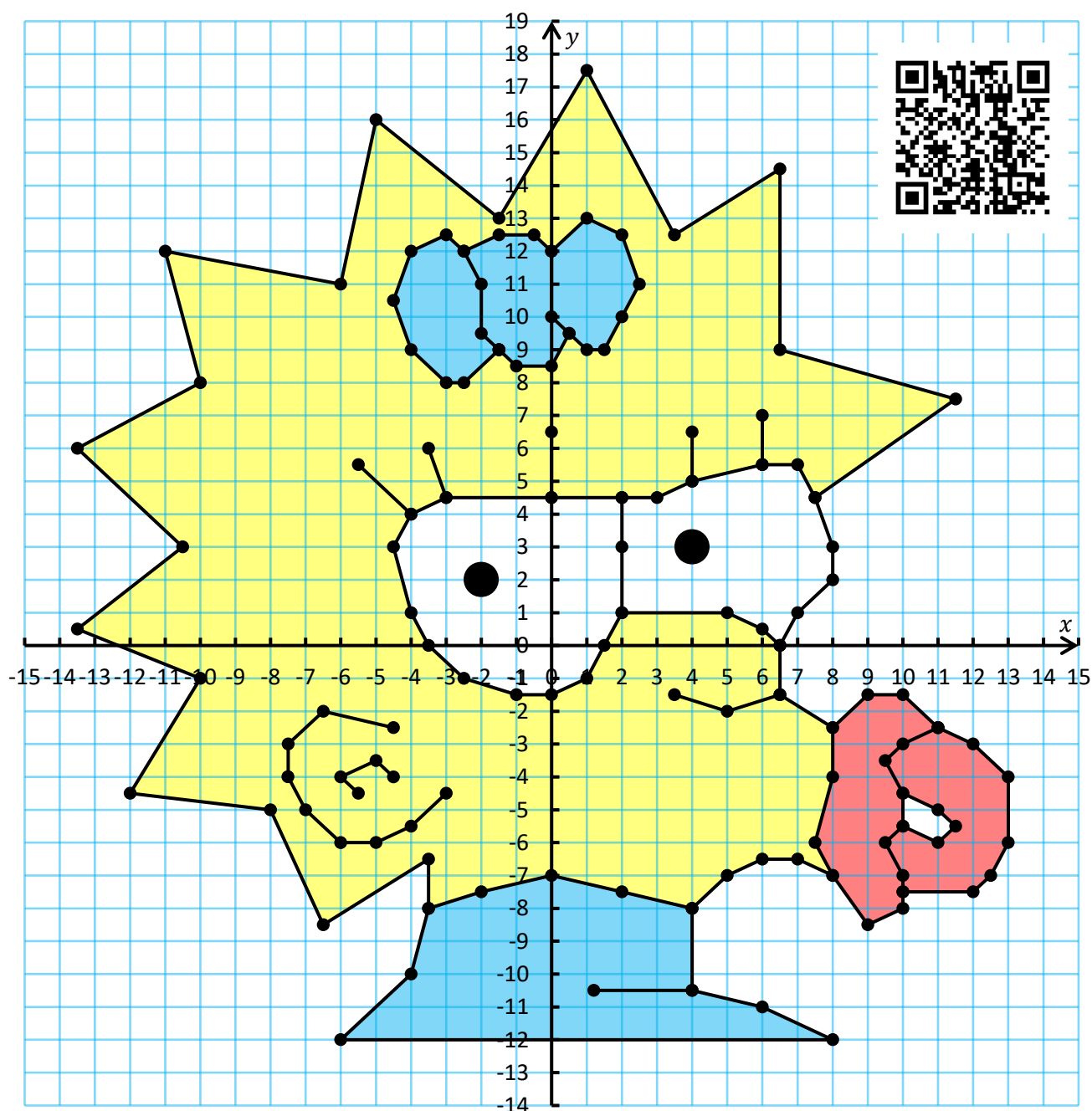
in the Four

Quadrants

Name:

Contents

Chapter	Mathematics	Page Number
The First Quadrant	Plotting co-ordinates in the first quadrant.	3
The Four Quadrants	The four quadrants. Plotting co-ordinates in the four quadrants.	7
Mid-point of a Line	The first quadrant. Adding negative numbers. The four quadrants.	11
Using Co-ordinates	Missing co-ordinates. Translations using a column vector.	14



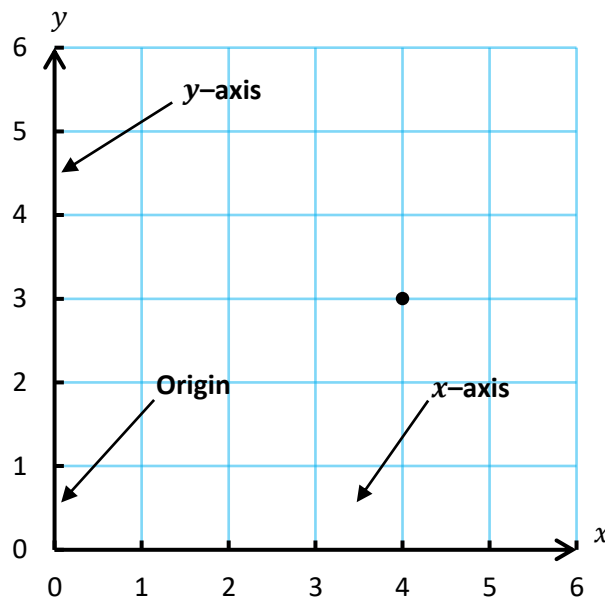
The First Quadrant

One way of locating a point is to use a pair of **numbered, perpendicular lines**. These lines are called **axes**.

- The **horizontal** axis is the **x -axis**.
- The **vertical** axis is the **y -axis**.
- The **origin** is the point where the two axes **meet**.
- To **plot a point** on the axes:
 - Start at the origin.
 - Move **across** to begin with.
 - Then move **up**.

We use **co-ordinates** to represent points on a set of axes. The co-ordinate (4, 3) is shown on the axes to the right. For this particular co-ordinate,

- The **x -coordinate** is 4. This is the distance we move **across**.
- The **y -coordinate** is 3. This is the distance we move **up**.



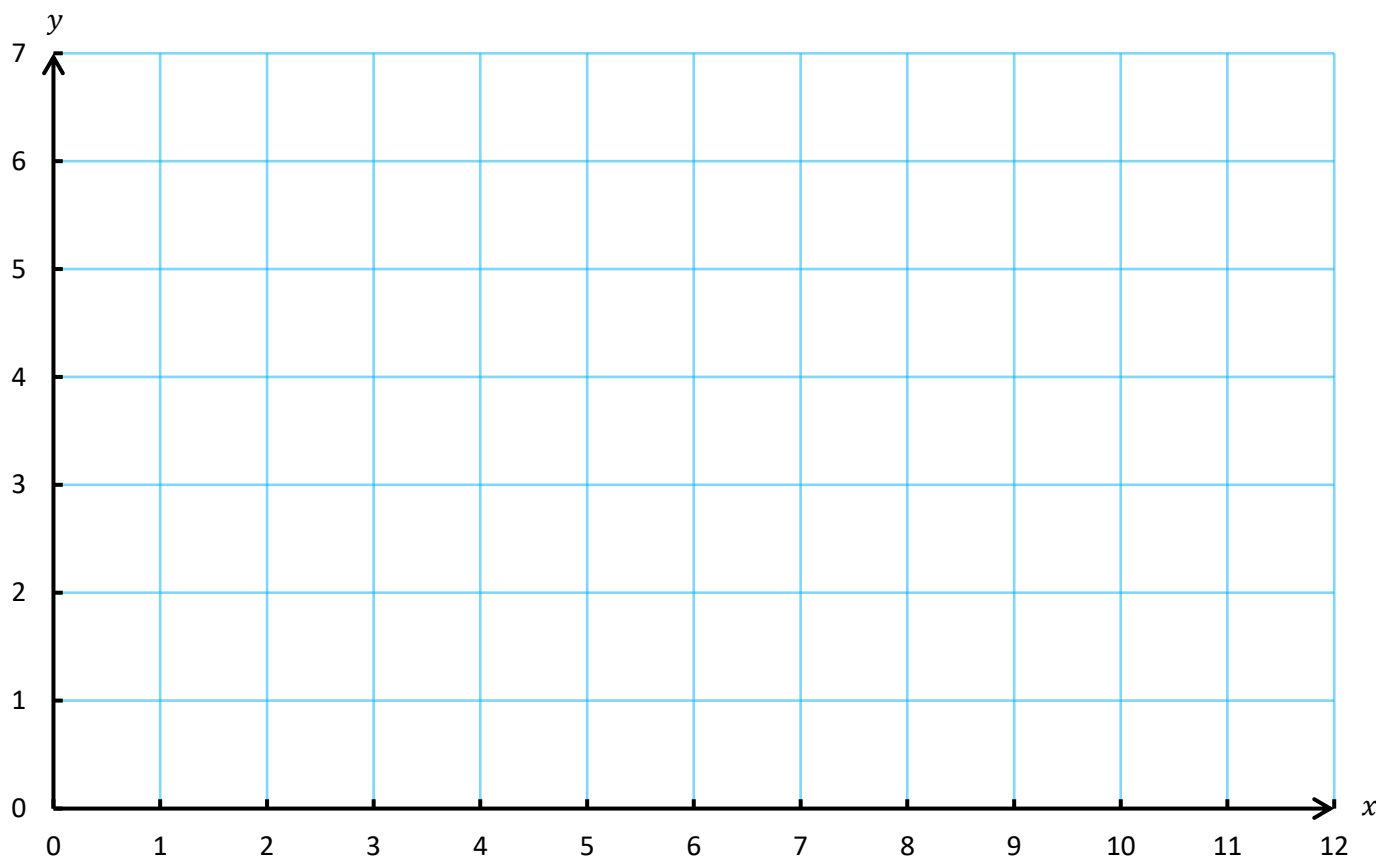
4

Skill

Exercise 1

Plot the following points on the axes shown below.

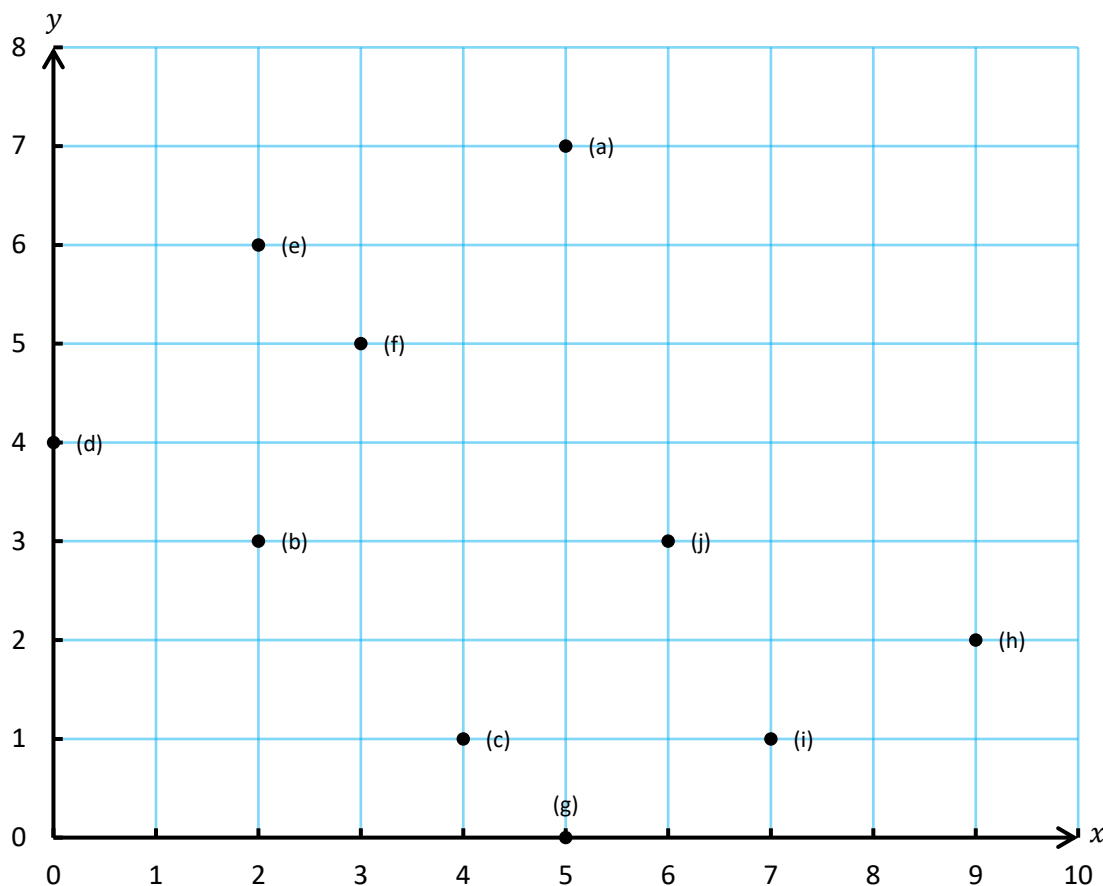
- | | | | | |
|------------|------------|-------------|--------------|----------------|
| (a) (4, 5) | (b) (8, 1) | (c) (11, 4) | (d) (1, 5) | (e) (6, 2) |
| (f) (9, 2) | (g) (0, 4) | (h) (6, 0) | (i) (2.5, 3) | (j) (6.5, 4.5) |



Exercise 2

4

Write down the co-ordinates of each of the points (a), (b), ..., (j) on the axes shown below.



(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

Exercise 3

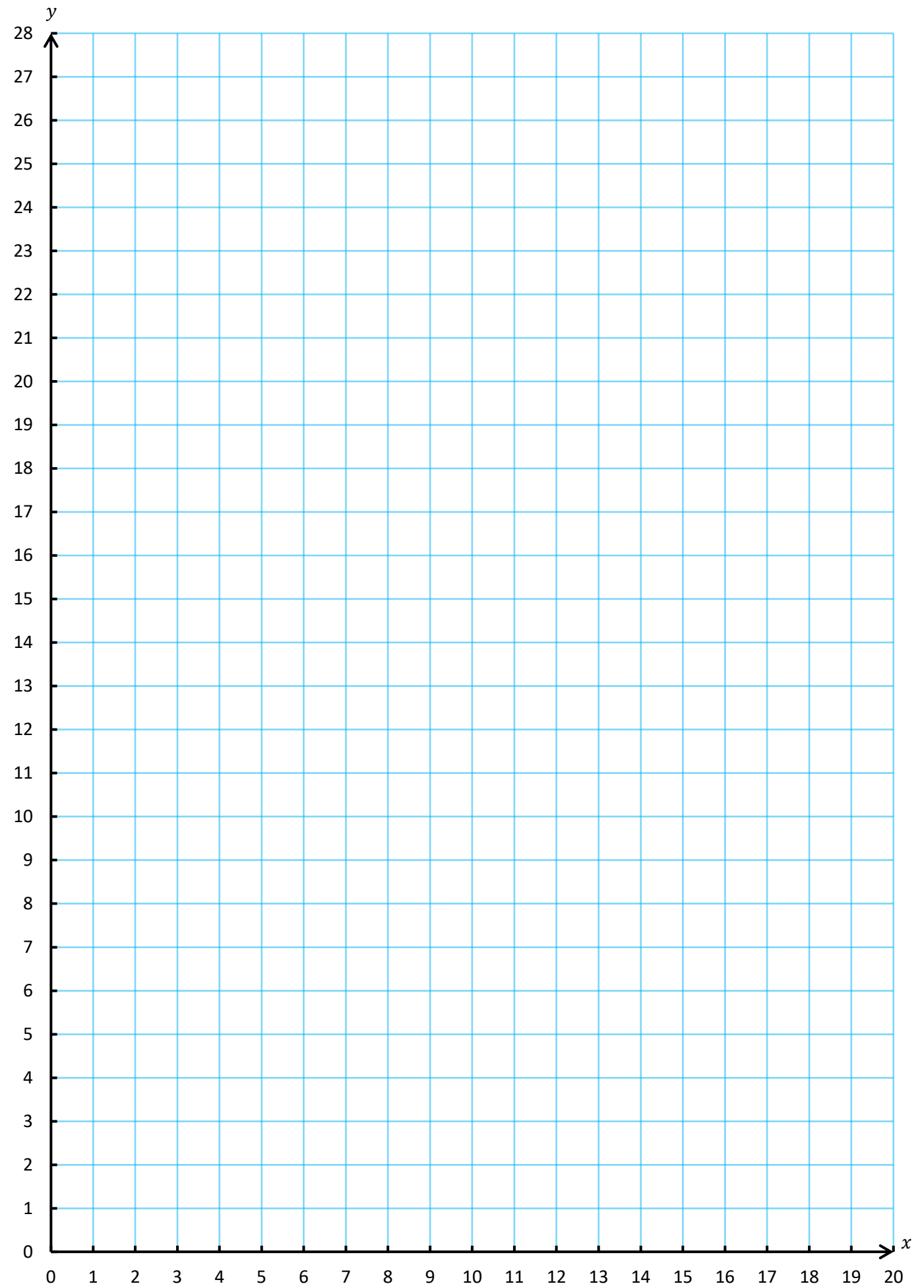
Plot the following co-ordinates on the next page to draw the **face of a famous person**...

Connect the co-ordinates as you go on, starting a new line for each new bullet point.

For the **red** co-ordinates, you must **double** the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (5, 8), then you must plot the point (10, 16).

For the **blue** co-ordinates, you must **subtract three** from the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (7, 11), then you must plot the point (4, 8).

- **Face:** (10, 0), (13, 6), (8, 3), (6, 5), (2, 5), (1, 6), (2, 6), (2, 11), (1, 12), (1, 16), (2, 17), (2, 26), (3, 25), (2, 13), (5, 25), (6, 26), (10, 28), (8, 26), (9, 25), (10, 26), (11, 25), (12, 26), (13, 25), (7, 13), (15, 25), (16, 26), (17, 25), (18, 26), (18, 15), (17, 4), (16, 3), (19, 3)
- **Right Eye:** (8, 11), (6, 13), (6, 16), (7, 18), (12, 22), (11, 19), (13, 18), (7, 8), (13, 12), (10, 11), (9, 11)
- **Nose:** (6, 9), (5, 8), (4, 8), (3, 9), (3, 10), (7, 14), (5, 11), (3, 6), (7, 12)
- **Eyebrow:** (6, 17), (5, 18), (2, 9), (2, 17)
- **Left Eye:** (3, 11), (5, 14)
- **Mouth:** (7, 5), (13, 8), (11, 6), (13, 6), (14, 5)
- **Mouth:** (13, 7), (13, 6)
- **Ear:** (17, 11), (9, 6), (19, 11), (19, 9), (9, 4), (17, 8), (16, 9)
- **Place a Big Dot at (4, 15) and at (14, 18)**



Exercise 4

Write down the letter that is next to each of the following points. After you finish, you should have a Welsh sentence. What is the sentence?

(4, 9) (8, 1)

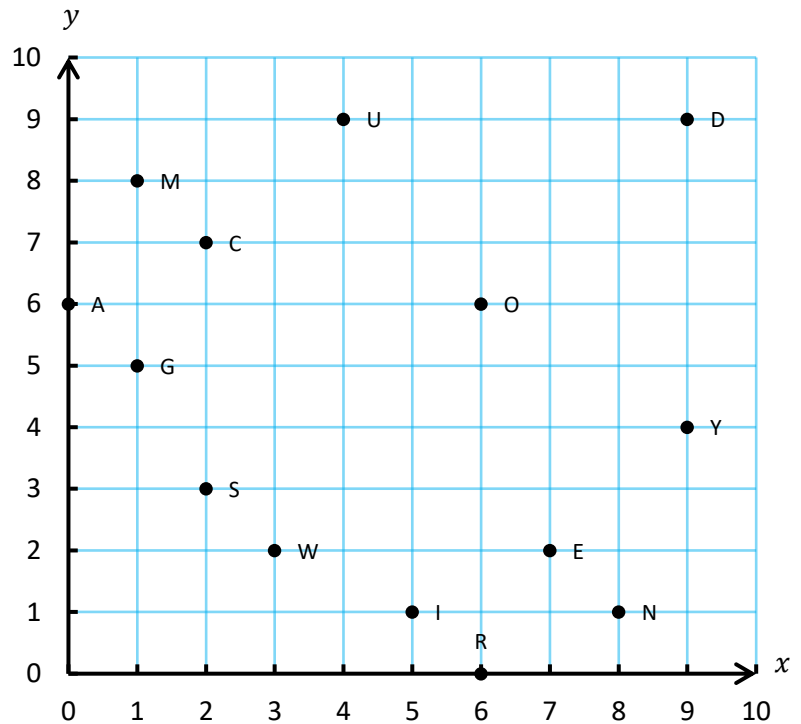
(2, 7) (0, 6) (1, 8) (1, 5) (9, 4) (1, 8)

(7, 2) (6, 0) (5, 1) (0, 6) (9, 9)

 $(3, 2) \quad (8, 1) \quad (0, 6)$

(8, 1) (7, 2) (1, 5) (7, 2) (2, 3)

(3, 2) (5, 1) (6, 0) (5, 1) (6, 6) (8, 1)



Exercise 5 (Revision)

Draw x and y axes on a piece of squared paper. Number the axes from 0 to 18.

Plot the following points, connecting them with straight lines as you go on. (Work row-by-row.)

(4, 8),	(6, 7),	(8, 7),	(9, 6),	(8, 5),	(6, 5),	(5, 4),
(5, 3),	(6, 3),	(6, 4),	(9, 4),	(10, 5),	(13, 4),	(14, 2),
(13, 1),	(13, 0),	(14, 0),	(14, 1),	(15, 2),	(14, 5),	(13, 5),
(13, 7),	(12, 9),	(17, 8),	(14, 9),	(17, 9),	(14, 10),	(17, 11),
(15, 11),	(17, 12),	(13, 11),	(12, 9),	(10, 9),	(8, 11),	(8, 12),
(5, 15),	(4, 15),	(3, 16),	(3, 15),	(1, 13),	(1, 12),	(2, 12),
(4, 13),	(5, 11),	(4, 10),	(2, 11),	(0, 9),	(0, 8),	(1, 8),
(1, 9),	(2, 10),	(3, 9),	(2, 7),	(4, 5),	(5, 5),	(5, 6),
(4, 6),	(3, 7),	(4, 8).				

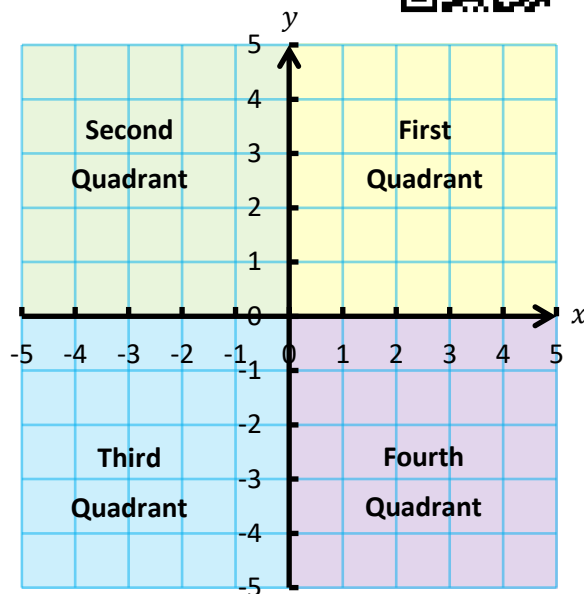
Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

The Four Quadrants



In the previous chapter, only co-ordinates with **positive** numbers were used. To plot co-ordinates that use **negative** numbers, we must extend the x -axis to the left and the y -axis downwards. By doing this, the x and y axes divide the **plane** into **four quadrants**. These quadrants are numbered in an anti-clockwise direction, as shown in the diagram on the right.



Exercise 6

6

Skill

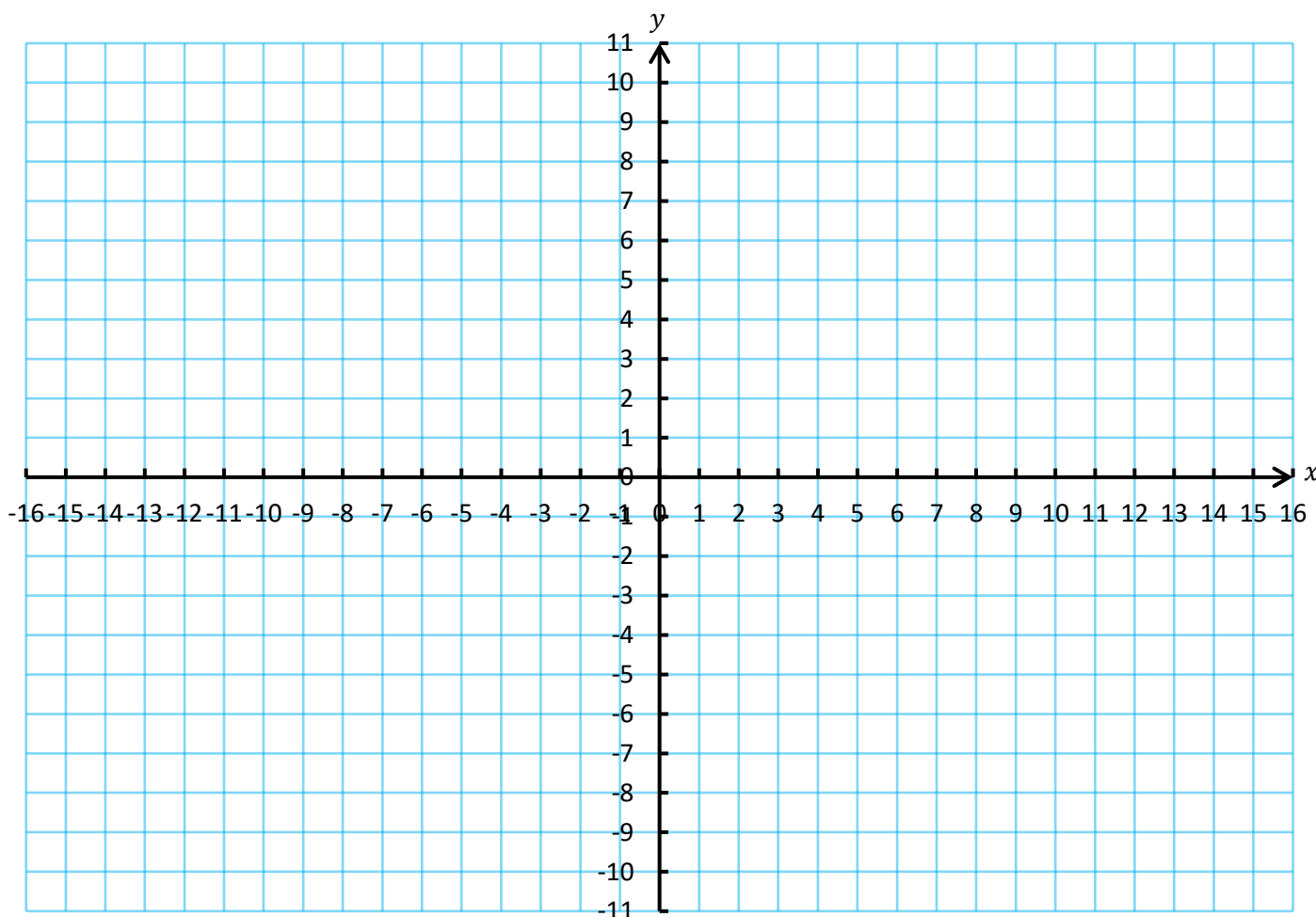
In which quadrant do the following points appear?

- | | | | |
|--------------|------------------|-------------|------------------|
| (a) (2, 5) | (b) (-2, 3) | (c) (4, -2) | (d) (-4, 1) |
| (e) (-4, -3) | (f) (2, 8) | (g) (1, -6) | (h) (-2, 4.5) |
| (i) (5, 0) | (j) (-9, -2) | (k) (-4, 3) | (l) (5, -1) |
| (m) (0, -2) | (n) (-3.5, -1.5) | (o) (0, 0) | (p) (-0.5, 3.25) |

Exercise 7

Plot the following points on the axes shown below.

- | | | | | | | |
|--------------|--------------|---------------|----------------|---------------|-------------------|----------------|
| (a) (13, 7) | (b) (-3, 5) | (c) (8, -2) | (d) (-10, -4) | (e) (-14, 7) | (f) (6, -3) | (g) (0, -6) |
| (h) (13, 10) | (i) (4.5, 8) | (j) (-5.5, 3) | (k) (4.5, -10) | (l) (-7.5, 0) | (m) (-11.5, -6.5) | (n) (14.5, -3) |



Exercise 8

6

(a) Write down the letter that is next to each of the following points. After you finish, you should have a Welsh sentence. What is the sentence?

(7, 3), (6, 5), (1, 1), (6, 5), (-4, -6), (2, 2),

(4, 5), (7, -1)

(6, 5)

(-4, -6), (6, 5), (3, 3), (-2, -2), (0, -4), (-4, 0),

(-4, 0), (7, 5), (1, 5)

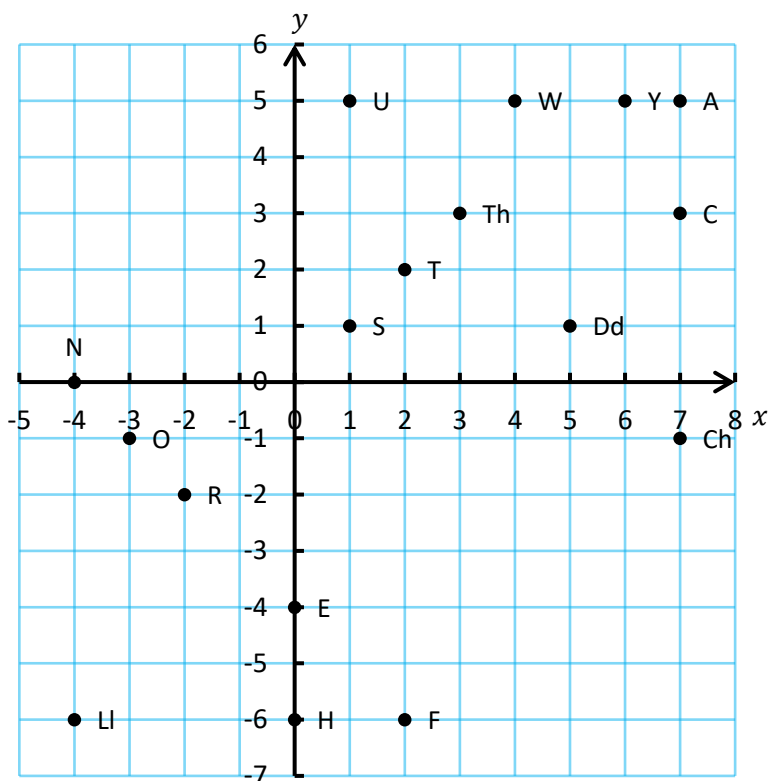
(6, 5), (-4, 0)

(-4, 0), (0, -6), (-2, -2), (0, -4), (2, -6), (-4, 0)

(6, 5), (-2, -2)

(4, 5), (6, 5), (5, 1), (-3, -1), (-2, -2)

(b) Do what is asked of you in part (a). What is the result?



Exercise 9

Plot the following co-ordinates on the next page to draw the **face of a famous person**...

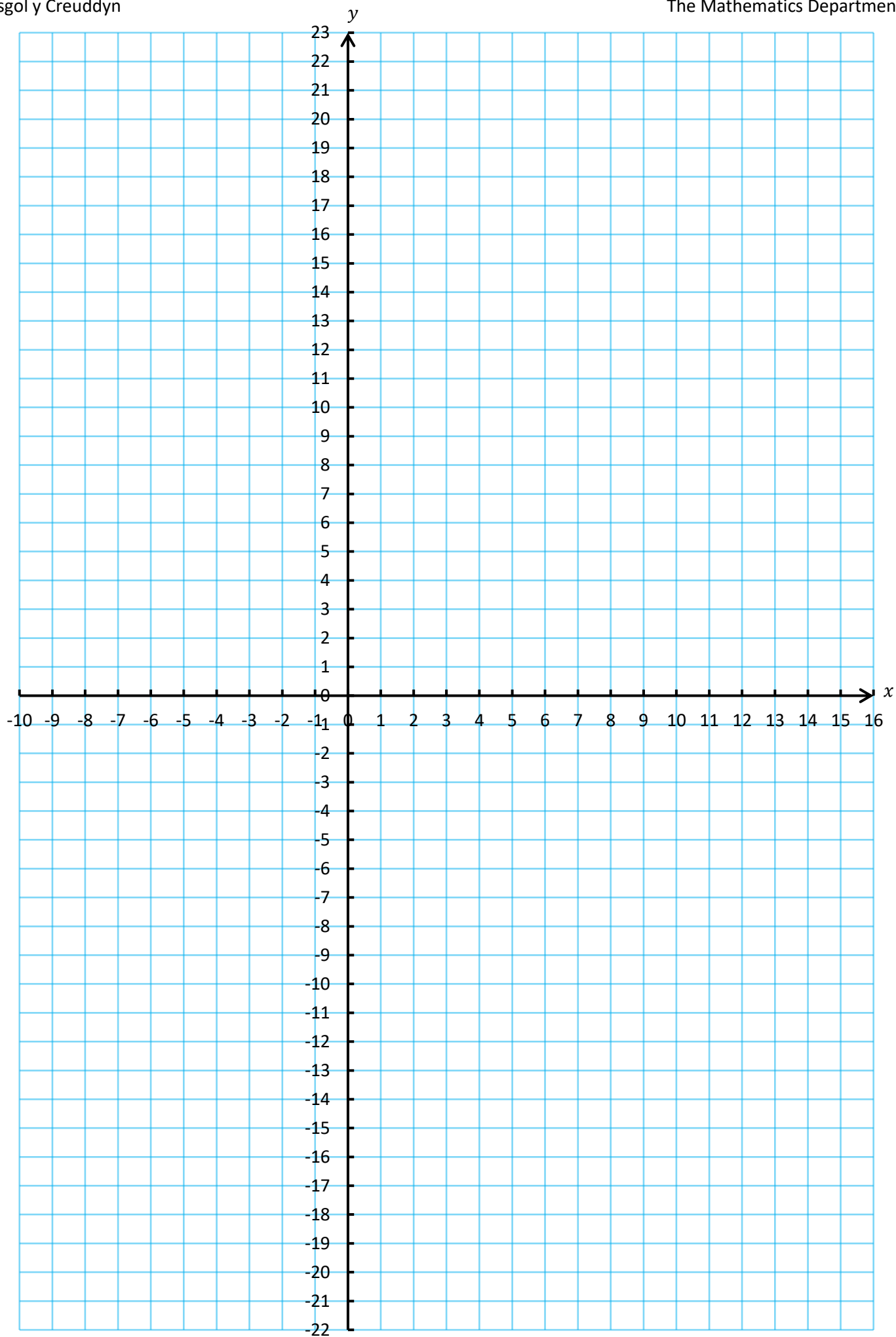
Connect the co-ordinates as you go on, starting a new line for each new bullet point.

For the **red** co-ordinates, you must **halve** the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (8, 10), then you must plot the point (4, 5).

For the **blue** co-ordinates, you must **add four** to the numbers to arrive at the correct co-ordinate. For example, if you see the co-ordinate (2, 5), then you must plot the point (6, 9).

- **Mouth:** (11, -11), (11, -12), (9, -14), (5, -16), (3, -16), (0, -15), (-2, -11), (-2, -9), (-1, -7), (2, -4), (4, -3), (12, -3), (14, -4), (15, -6), (15, -9), (11, -11), (8, -12), (5, -11), (2, -10)
- **Lips:** (0, -11), (4, -9)
- **Neck:** (-9, -20), (-7, -12), (-5, -7), (-7, -7), (-9, -5), (-9, -4), (-8, -3), (-6, -2), (-5, -3)
- **Ear:** (-7, -5), (-7, -4), (-6, -4)
- **Neck:** (8, -21), (7, -19), (7, -17), (8, -15), (9, -14)
- **Hair:** (-9, -3), (-9, 3), (-7, -2), (-6, 3), (-4, -2)
- **Left Eye:** (12, -3), (13, -2), (14, 0), (12, 1), (6, 1), (4, 0), (3, -1), (1, -1), (-1, 0), (-2, 1), (-3, 4), (-1, 7), (4, 16), (4, 8), (2, 2), (7, 4), (6, 1)
- **Right Eye:** (12, 1), (13, 2), (10, 0), (13, 6), (24, 14), (9, 8), (7, 7), (6, 6)
- **Head:** (-7, 0), (-9, 6), (-9, 12), (-8, 15), (-6, 17), (-3, 19), (0, 20), (2, 20), (1, 15), (7, 17), (9, 15), (20, 26), (11, 11), (11, 9), (9, 5), (12, 7)
- **Hair:** (-8, 12), (-9, 15), (-8, 18), (-6, 20), (-4, 20), (-3, 19)
- **Hair:** (-6, 14), (-7, 17), (-6, 20), (-4, 22), (-1, 22), (1, 21), (2, 20)
- **Place a Big Dot at (0, 3) and at (20, 6).**





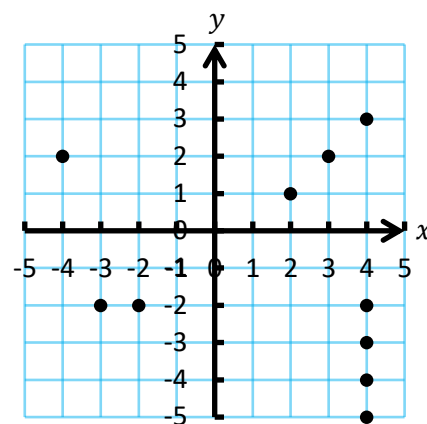
Battleships (a game for two players)

Each player will need the worksheet for the game.

Place the following ships on your graph without the other player seeing:

1 submarine (1 dot); **1 destroyer** (2 dots); **1 cruiser** (3 dots); **1 battleship** (4 dots). An example is shown to the right.

Each player names a co-ordinate in turn. The other player announces “*hit*” or “*miss*”. When each of the dots representing a ship have been hit, the opponent announces, “*ship destroyed*”. The first player to lose **all** of his/her ships is the loser.



Exercise 10 (Revision)

6

Draw x and y axes on a piece of squared paper. Number the x -axis from -18 to 18 , and the y -axis from -26 to 16 . Plot the following points, connecting them with straight lines as you go on. (Work row-by-row.)

$(-13, -25)$	$(-11, -22)$	$(-8, -18)$	$(-6, -15)$	$(-5, -12)$	$(-5, -11)$	$(-6, -10)$
$(-7, -10)$	$(-8, -9)$	$(-7, -7)$	$(-6, -7)$	$(-5, -8)$	STOP	$(-7, -9)$
$(-6, -8)$	STOP	$(-6.5, -8.5)$	$(-5, -10)$	STOP	$(-7, -25)$	$(-6, -23)$
$(-4, -21)$	STOP	$(-5, -22)$	$(-3, -23)$	$(1, -23)$	$(4, -21)$	STOP
$(6, -25)$	$(4, -21)$	$(4, -17)$	$(5, -14)$	$(7, -13)$	$(10, -12)$	$(13, -10)$
$(13, -9)$	$(11, -6)$	STOP	$(-2, -11)$	$(2, -13)$	$(7, -13)$	STOP
$(-3, -12)$	$(-1, -10)$	STOP	$(6, -6)$	$(10, -6)$	$(11, -7)$	$(11, -8)$
$(10, -9)$	$(8, -9)$	STOP	$(6, -2)$	$(7, -1)$	$(9, 0)$	$(11, 0)$
$(13, -2)$	$(13, -4)$	$(12, -6)$	$(11, -6)$	STOP	$(4, -8)$	$(0, -8)$
$(-2, -6)$	$(-2, -4)$	$(-1, -2)$	$(2, 0)$	$(4, 0)$	$(6, -2)$	$(7, -4)$
$(6, -6)$	$(4, -8)$	STOP	$(-2, -5)$	$(-4, -4)$	STOP	$(-1, -2)$
$(-3, 0)$	STOP	$(2, 0)$	$(1, 2)$	STOP	$(4, 0)$	$(5, 2)$
STOP	$(9, 0)$	$(8, 2)$	STOP	$(11, 0)$	$(12, 2)$	STOP
$(12, -1)$	$(14, 0)$	STOP	$(13, -3)$	$(15, -2)$	STOP	$(-5, -12)$
$(-9, -13)$	$(-10, -9)$	$(-14, -8)$	$(-11, -4)$	$(-15, -1)$	$(-10, 2)$	$(-12, 7)$
$(-6, 7)$	$(-3, 13)$	$(1, 9)$	$(6, 14)$	$(8, 9)$	$(14, 10)$	$(12, 4)$
$(17, 2)$	$(13, -2)$	A big dot at $(1, -6)$ and at $(9, -5)$.				

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Mid-point of a Line

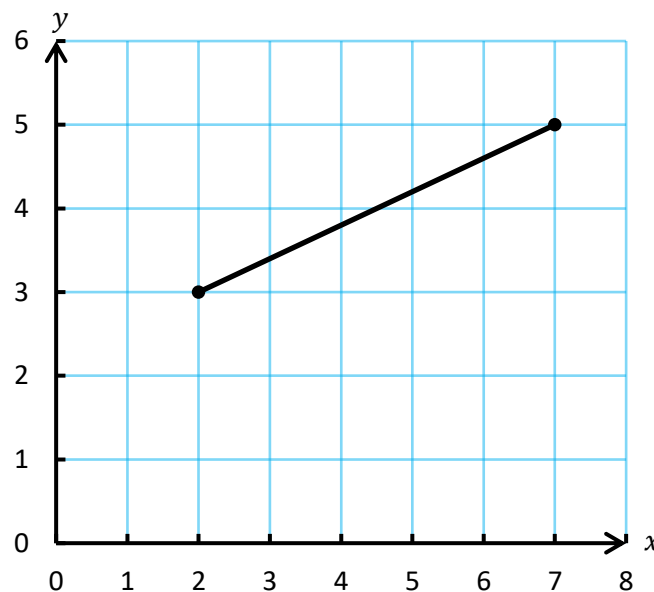
It is possible to use a straight line to connect any two different co-ordinates. What is the **mid-point** of a straight line of this type?

Example

Let us consider the two coordinates (2, 3) and (7, 5), as shown in the graph on the right.

We can see from the graph that the mid-point of the line joining (2, 3) to (7, 5) is (4.5, 4), but it is possible to calculate this *without* using the graph. To do this, we use the **mean** of the *x*-coordinates and the mean of the *y*-coordinates.

- The *x*-coordinates are 2 and 7. The total of 2 and 7 is 9, so the **mean of the *x*-coordinates** is $9 \div 2 = 4.5$.
- The *y*-coordinates are 3 and 5. The total of 3 and 5 is 8, so the **mean of the *y*-coordinates** is $8 \div 2 = 4$.
- It follows that the mid-point of the line joining (2, 3) to (7, 5) is (4.5, 4).



Exercise 11

Find the mid-point of the straight line joining the following pairs of co-ordinates.

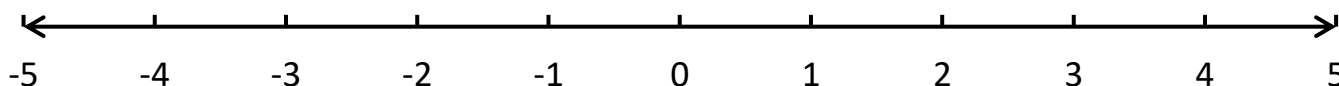
- | | | | |
|-------------------------|---------------------------|---------------------------|---------------------------|
| (a) (2, 6) and (8, 2) | (b) (4, 5) and (2, 3) | (c) (9, 6) and (3, 10) | (d) (11, 7) and (9, 1) |
| (e) (2, 4) and (3, 8) | (f) (4, 1) and (10, 6) | (g) (1, 2) and (2, 1) | (h) (10, 4) and (5, 12) |
| (i) (1, 6) and (12, 12) | (j) (24, 10) and (14, 18) | (k) (34, 2) and (26, 5) | (l) (0, 4) and (6, 4) |
| (m) (9, 9) and (8, 8) | (n) (18, 4) and (20, 0) | (o) (13, 17) and (14, 16) | (p) (45, 67) and (83, 24) |

Adding Negative Numbers

In Exercise 11, the co-ordinates all came from the first quadrant. To find the mid-point of lines joining co-ordinates in other quadrants, we must first learn how to add negative numbers together.

The Number Line

Think of a number line whilst doing the following questions. Remember that you must move **right** when **adding**, and move **left** when **subtracting**.



Exercise 12

- | | | | |
|-----------------|-----------------|------------------|------------------|
| (a) $4 + 7$ | (b) $-4 + 7$ | (c) $4 - 7$ | (d) $-4 - 7$ |
| (e) $-4 + 2$ | (f) $-4 - 2$ | (g) $5 - 8$ | (h) $-5 - 8$ |
| (i) $-8 + 10$ | (j) $-6 + 2$ | (k) $12 - 16$ | (l) $-12 + 16$ |
| (m) $-9 + 5$ | (n) $9 - 5$ | (o) $-9 - 5$ | (p) $-5 + 9$ |
| (q) $-12 + 1$ | (r) $-1 + 12$ | (s) $-1 - 12$ | (t) $-12 - 1$ |
| (u) $9 + 3$ | (v) $-9 + 3$ | (w) $3 - 9$ | (x) $-3 - 9$ |
| (y) $2.5 + 3.5$ | (z) $2.5 - 3.5$ | (α) $-2.5 + 3.5$ | (β) $-2.5 - 3.5$ |



Consider the sums shown in the box to the right. Going down the rows, the number we add to three decreases by one each time. What should be the answer to the final sum?

To continue the pattern, the answer to the final sum should be 2. This means that **adding a negative number is the same as subtraction**, so that the sum $3 + -1$ is the same as the sum $3 - 1$.

Example

$$(a) 8 + -3 = 8 - 3 \\ = 5$$

$$(b) -5 + -2 = -5 - 2 \\ = -7$$

$$(c) 7.5 + -1.5 = 7.5 - 1.5 \\ = 6$$

$$\begin{aligned} 3 + 4 &= 7 \\ 3 + 3 &= 6 \\ 3 + 2 &= 5 \\ 3 + 1 &= 4 \\ 3 + 0 &= 3 \\ 3 + -1 &= ? \end{aligned}$$

Exercise 13

$$(a) 9 + -2$$

$$(b) 14 + -3$$

$$(c) 7 + -6$$

$$(d) 25 + -6$$

$$(e) -3 + -2$$

$$(f) -8 + -4$$

$$(g) -2 + -6$$

$$(h) -9 + -5$$

$$(i) 5 + -9$$

$$(j) 3 + -4$$

$$(k) 17 + -20$$

$$(l) 6 + -6$$

$$(m) -11 + -4$$

$$(n) -15 + -10$$

$$(o) -21 + -7$$

$$(p) -28 + -11$$

$$(q) 8.5 + -1.5$$

$$(r) 11.5 + -2$$

$$(s) 8 + -3.5$$

$$(t) -2.5 + -4.5$$

$$(u) 14 + -1$$

$$(v) -5 + -5$$

$$(w) 3.5 + -1$$

$$(x) -14 + -2$$

$$(y) 45 + -21$$

$$(z) 63 + -68$$

$$(\alpha) -78 + -13$$

$$(\beta) -14.5 + -17$$

Exercise 14 (Mixture)

$$(a) 7 - 9$$

$$(b) -7 + 9$$

$$(c) 9 - 7$$

$$(d) 7 + -9$$

$$(e) 9 + -7$$

$$(f) -7 + -9$$

$$(g) -9 - 7$$

$$(h) -9 + -7$$

$$(i) 6 + -8$$

$$(j) -7 + 12$$

$$(k) -3 + -4$$

$$(l) 9 - 13$$

$$(m) 14 + -2$$

$$(n) -5 - 11$$

$$(o) 10 + -11$$

$$(p) -2 + -3$$

$$(q) -9 + 5$$

$$(r) -3 - 8$$

$$(s) -2 + -6$$

$$(t) 2.5 + -4.5$$

$$(u) -2 + -5.5$$

$$(v) -9.5 - 0.5$$

$$(w) 6 - 7.5$$

$$(x) -2 + -1.5$$

$$(y) 26 + -39$$

$$(z) -26 - 39$$

$$(\alpha) -26 + 39$$

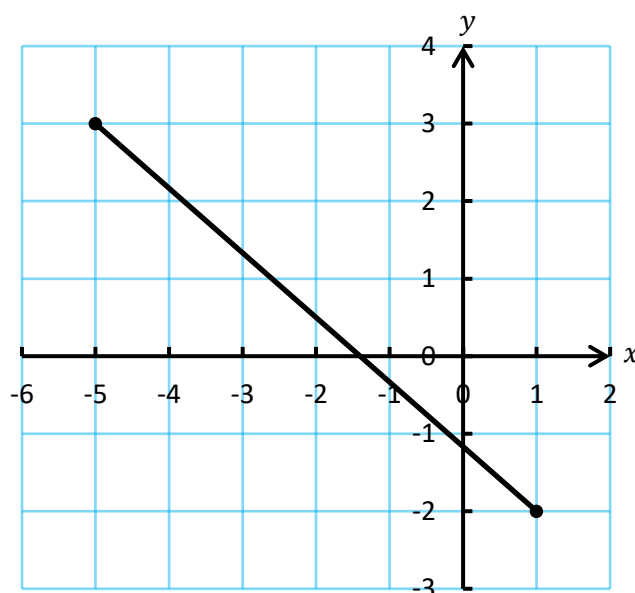
$$(\beta) -39 + 26$$

Mid-point of Lines in the Four Quadrants

Let us consider the two co-ordinates $(1, -2)$ and $(-5, 3)$, as shown in the graph on the right.

We can see from the graph that the mid-point of the line joining $(1, -2)$ to $(-5, 3)$ is $(-2, 0.5)$, but it is possible to calculate this *without* using the graph. To do this, we use (as on the previous page) the **mean** of the x -coordinates and the mean of the y -coordinates.

- The x -coordinates are 1 and -5 .
The total of 1 and -5 is $1 + -5 = 1 - 5 = -4$,
so the **mean of the x -coordinates** is $-4 \div 2 = -2$.
- The y -coordinates are -2 and 3.
The total of -2 and 3 is $-2 + 3 = 1$, so the mean of the y -coordinates is $1 \div 2 = 0.5$.
- It follows that the mid-point of the line joining $(1, -2)$ to $(-5, 3)$ is $(-2, 0.5)$.



Exercise 15

6

Skill

Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) (4, -2) and (2, 8) (b) (8, 4) and (-2, 6) (c) (9, -4) and (5, 2) (d) (-4, -6) and (10, 12)
 (e) (4, 2) and (-10, -6) (f) (-8, 5) and (1, 3) (g) (-6, -2) and (11, 15) (h) (34, -5) and (-4, 10)
 (i) (-8, 14) and (2, -6) (j) (-4, -6) and (-2, -4) (k) (-5, 4) and (-7, -12) (l) (-14, 0) and (-6, -8)
 (m) (15, 13) and (11, -1) (n) (-25, -20) and (7, -4) (o) (-1, 0) and (0, -1) (p) (-12.5, -3.5) and (0.5, -1.5)

Exercise 16

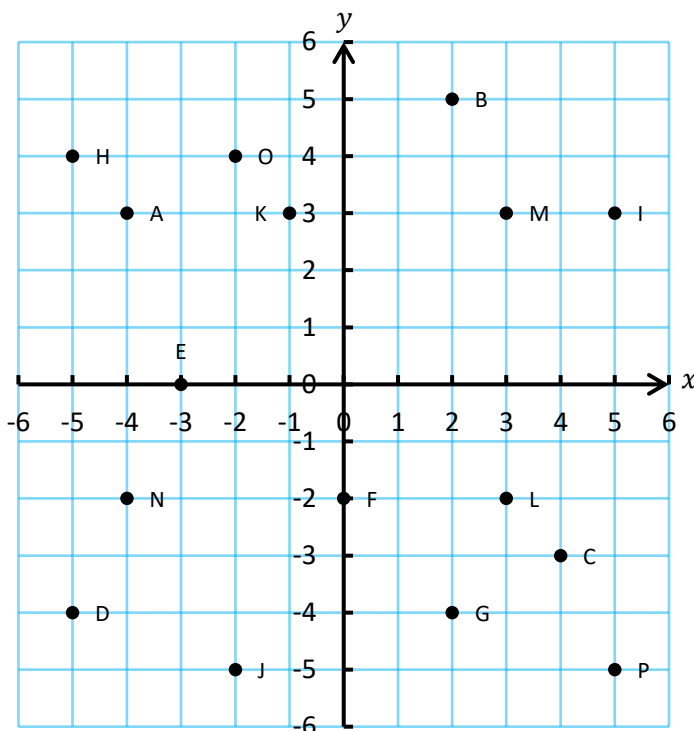
Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) A and B (b) C and D
 (c) E and F (d) G and H
 (e) I and J (f) K and L
 (g) M and N (h) O and P

Exercise 17 (Revision)

Find the mid-point of the straight line joining the following pairs of co-ordinates.

- (a) (9, 3) and (1, 9) (b) (9, -3) and (1, 9)
 (c) (-9, 3) and (1, 9) (d) (-9, -3) and (-1, -9)
 (e) (12, 6) and (5, 18) (f) (-4, 5) and (18, 12)
 (g) (-6, -6) and (20, -20) (h) (0, 4) and (-3, 0)
 (i) (0, 0) and (-9, 28) (j) (11.5, 23) and (4.5, 3)
 (k) (-9, 3) and (-17, -10) (l) (77, 44) and (33, 11)
 (m) (-24, 12) and (-5, 9) (n) (35, -41) and (-3, -5)
 (o) (-0.5, 6) and (-1.5, -1) (p) (250, -135) and (-350, 425)

**Evaluation**

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Using Co-ordinates

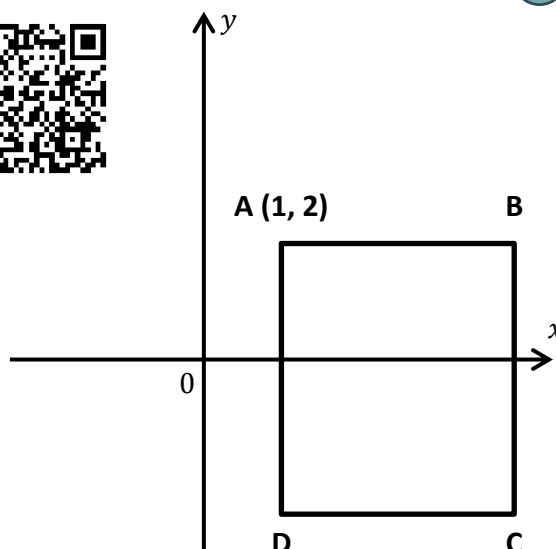
Applying

6

Missing Co-ordinates

Exercise 18

In the diagram on the right, the length of the sides of the square **ABCD** is 6 units. Find the co-ordinates of the points **B**, **C** and **D**.



Exercise 19

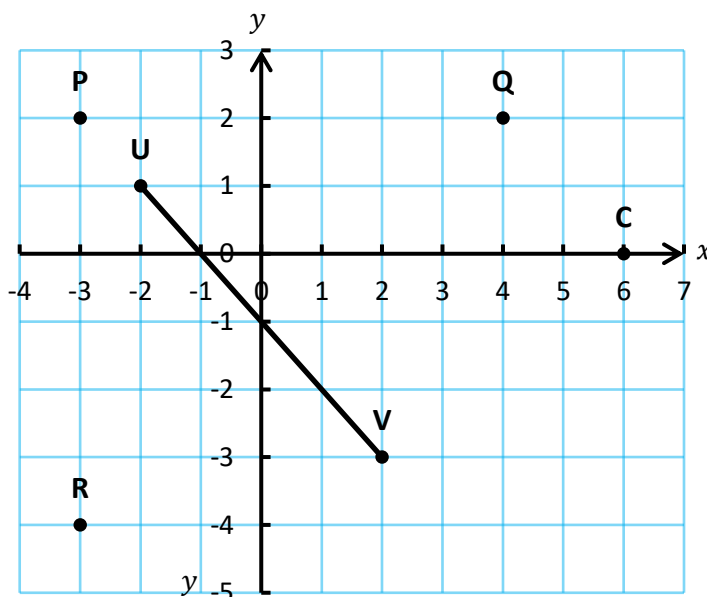
Hayley plots a square on graph paper. The co-ordinates of three of the corners are $(-4, 4)$, $(-1, 4)$ and $(-4, 1)$. What is the co-ordinate of the **fourth corner** of the square?

Exercise 20

Guto plots a rectangle on graph paper. The co-ordinates of three of the corners are $(2, -1)$, $(5, -1)$ and $(5, -5)$. What is the co-ordinate of the **fourth corner** of the rectangle?

Exercise 21

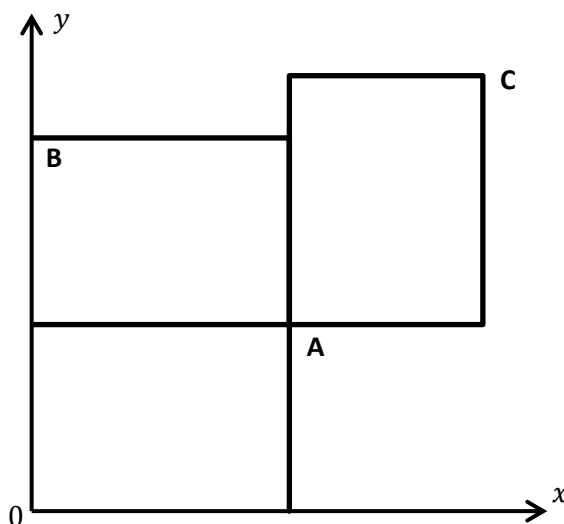
- Write down the co-ordinates of the point **P**.
- The point **W** lies on the line **UV**. The y -coordinate of **W** is -2 . What is **W**'s x -coordinate?
- What are the co-ordinates of the mid-point of the line **UV**?
- The co-ordinates of the point $(1, 3)$ add to 4. What number do you get if you add the co-ordinates of any point on the line **UV**?
- Write down the co-ordinates of the point **S** so that **PQRS** forms a rectangle.
- CD** is a vertical line. What would be the co-ordinates of the point **D** if **CD** was 4 units in length and in the fourth quadrant?



Exercise 22

The diagram on the right shows 3 rectangles, each of them 4 units by 3 units.

- Find the co-ordinates of the points **A**, **B** and **C**.
- What would be the co-ordinates of the mid-point of the line joining **A** to **B**?
- What would be the co-ordinates of the mid-point of the line joining **A** to **C**?

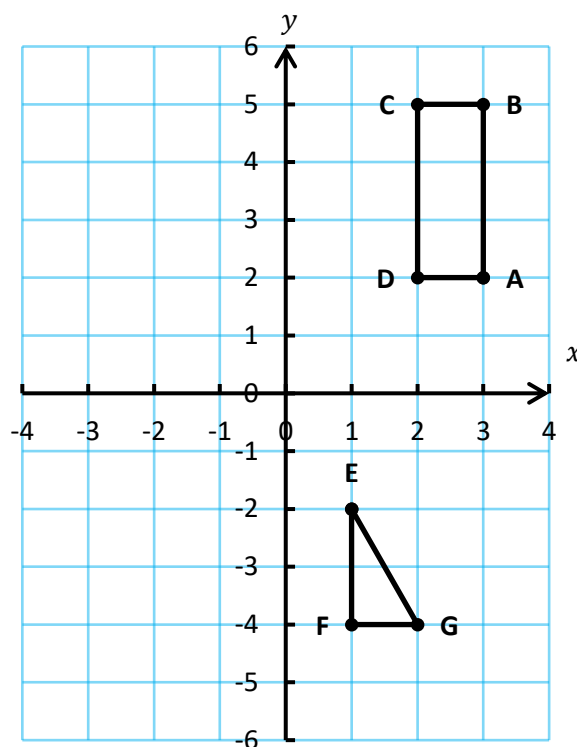


Translation Using a Vector Column

6

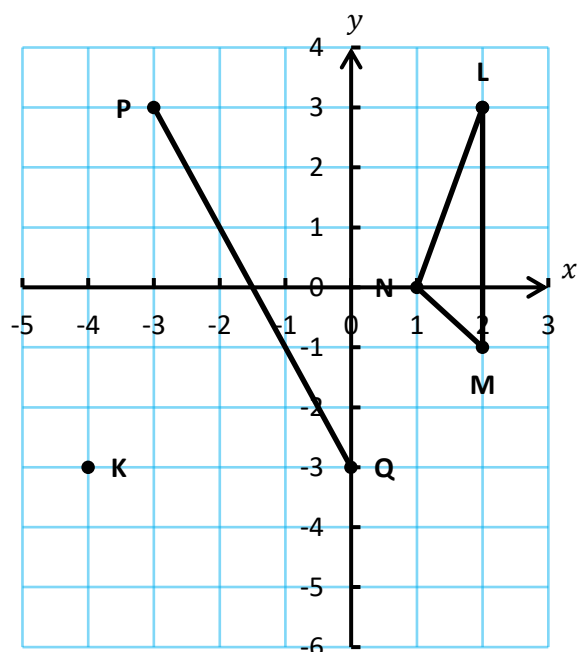
Exercise 23

- (a) The rectangle **ABCD** is translated using the vector column $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$. Write down the co-ordinates of the new vertices.
- (b) The rectangle **ABCD** is translated using the vector column $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$. Write down the co-ordinates of the new vertices.
- (c) The rectangle **ABCD** is translated so that its vertices move to the co-ordinates (0, 1), (1, 1), (1, 4), (0, 4). What vector column represents this translation?
- (d) The triangle **EFG** is translated using the vector column $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$. Write down the co-ordinates of the new vertices.



Exercise 24 (Revision)

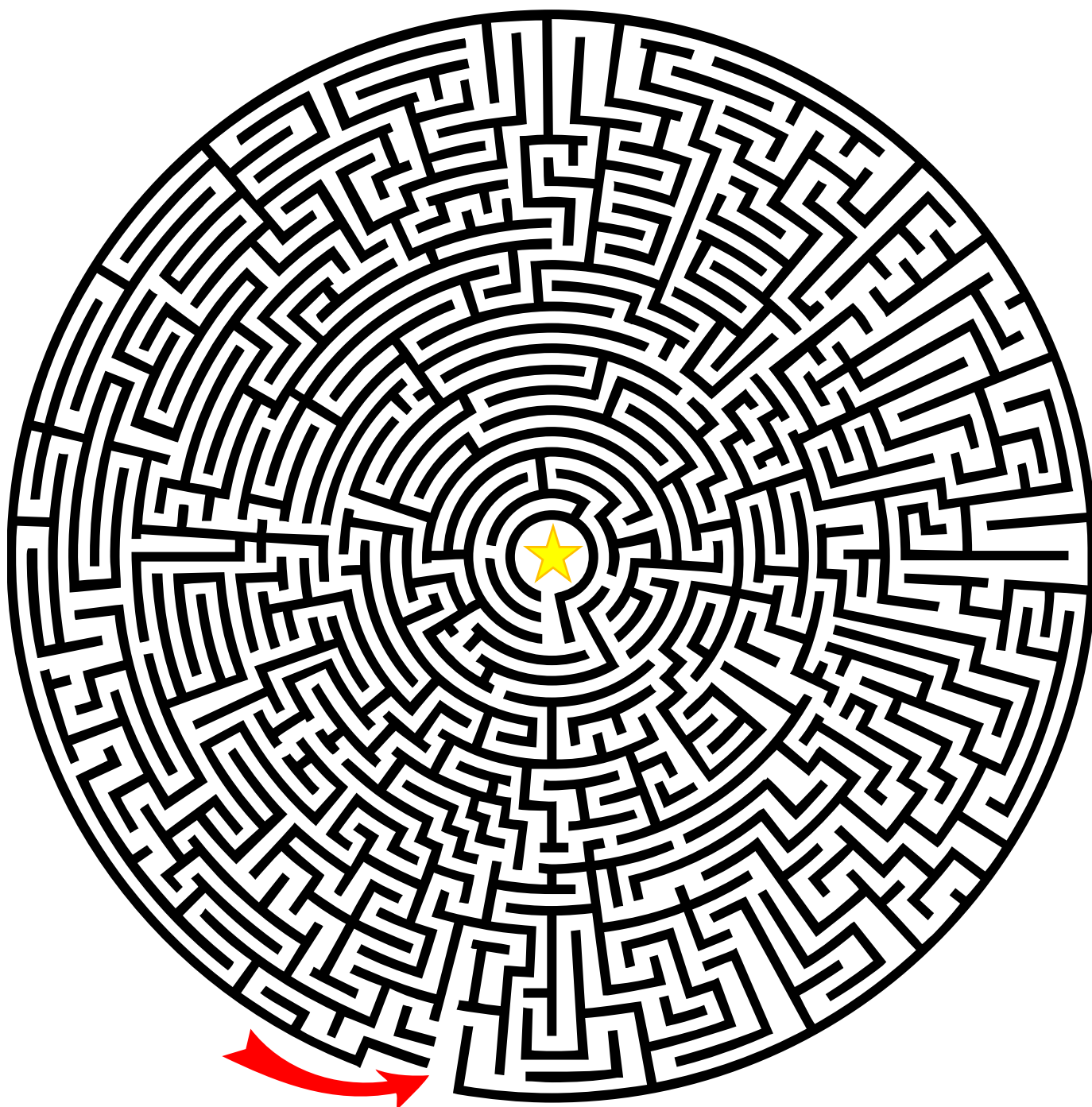
- Write down the co-ordinates of the point **K**.
- The point **R** lies on the line **PQ**. The y -coordinate of **R** is 1. What is the x -coordinate of **R**?
- What are the co-ordinates of the mid-point of the line **PQ**?
- The shape **LMN** is translated using the vector column $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$. Write down the co-ordinates of the new vertices.
- The line **PQ** forms one side of a square. What could be the co-ordinates of the other two corners?

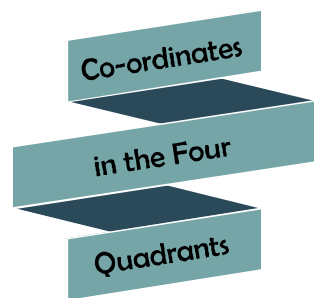


Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Puzzle: Find a path from the outside to the star in the centre.





Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to plot co-ordinates in the first quadrant , e.g. (5, 2).			2	
I know how to recognise co-ordinates that have been plotted in the first quadrant .			1	
I know what the co-ordinate of the origin is.			5	
Given a co-ordinate such as (7, 3), I know what the x-coordinate is and what the y-coordinate is.			3, 4	
I know how to plot co-ordinates in all four quadrants , e.g. (-7, 3).			7	
I know how to recognise co-ordinates that have been plotted in all four quadrants .			11	
I know how to recognise in which quadrant a given co-ordinate appears.			8	
I know how to add a negative number , e.g. $5 + -2$.			6	
I can find the mid-point of a straight line that connects a pair of co-ordinates.			9, 11	
I can find missing co-ordinates in diagrams.			11	
I can write co-ordinates for translations that use vector columns .			10	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐

Factors and Multiples

★ ★ ✨

%

Introducing Algebra

★ ★ ✨

%

Level at the start
of the year: _____

End of KS3
target: _____

Tracking Sheet Year 8

Exam: %

Level at the end
of the year: _____

Measuring Shapes

★ ★ ✨

%

Data Handling and
Statistics 2

★ ★ ✨

%



The Mathematics Department

8

Factors and

Multiples

Name:

Contents

Chapter	Mathematics	Page Number
Square Root and Cube Root	Revision of Square Numbers and Cube Numbers from Year 7 work. Square Root, Cube Root.	3
Factors and Prime Numbers	Listing common factors. Defining a prime number. Recognising a prime number.	6
Venn Diagrams	Two circle Venn diagrams. Three circle Venn diagrams. Set notation.	9
Index Form	Writing a number as a product of its prime factors. Writing numbers in index form. Square numbers in index form.	14
Highest Common Factor, Lowest Common Multiple	Finding the Highest Common Factor of two numbers. Finding the Lowest Common Multiple of two numbers.	19



Square Root and Cube Root

In year 7, you learnt how to calculate different square numbers and cube numbers.

Example

$$5^2 = 5 \times 5 \\ = 25$$

$$9^3 = 9 \times 9 \times 9 \\ = 81 \times 9 \\ = 729$$

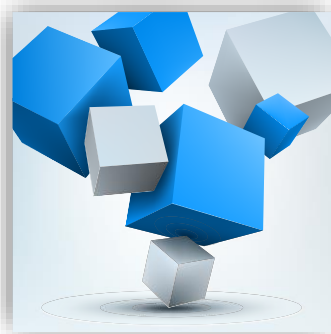


Revision

Exercise 1

- (a) 4^2 (b) 7^2 (c) 3^2 (d) 9^2 (e) 8^2
 (f) 2^3 (g) 4^3 (h) 3^3 (i) 5^3 (j) 8^3
 (k) $6^2 + 2^2$ (l) $4^2 \div 2^3$ (m) $6^3 + 6^2$ (n) $10^3 - 1^2$ (o) $7^3 + 11^2$
 (p) Which numbers between 1 and 1,000 are both square numbers and cube numbers?

4



Square Root

The square root of a number is the number that **squares** to give the original number. For example, the square root of 25 is 5 because 5^2 is 25. The symbol for square root is $\sqrt{\quad}$, therefore $\sqrt{25} = 5$.

Exercise 2

- (a) $\sqrt{9}$ (b) $\sqrt{36}$ (c) $\sqrt{49}$ (d) $\sqrt{100}$ (e) $\sqrt{16}$
 (f) $\sqrt{1}$ (g) $\sqrt{64}$ (h) $\sqrt{4}$ (i) $\sqrt{81}$ (j) $\sqrt{144}$

4

Skill

Cube Root

The cube root of a number is the number that **cubes** to give the original number. For example, the cube root of 729 is 9 because $9^3 = 729$. The symbol for cube root is $\sqrt[3]{\quad}$, therefore $\sqrt[3]{729} = 9$.



Exercise 3

- (a) $\sqrt[3]{8}$ (b) $\sqrt[3]{64}$ (c) $\sqrt[3]{1000}$ (d) $\sqrt[3]{125}$ (e) $\sqrt[3]{27}$
 (f) $\sqrt[3]{1}$ (g) $\sqrt[3]{216}$ (h) $\sqrt[3]{512}$ (i) $\sqrt[3]{0}$ (j) $\sqrt[3]{343}$

4

Exercise 4

- (a) $\sqrt{16} + \sqrt{25}$ (b) $\sqrt[3]{8} \times \sqrt{25}$ (c) $\sqrt{81} + \sqrt[3]{125}$ (d) $\sqrt[3]{512} \div \sqrt{64}$ (e) $\sqrt{1} + \sqrt[3]{1}$
 (f) $\sqrt{49} \times \sqrt{64}$ (g) $\sqrt[3]{64} - \sqrt{4}$ (h) $\sqrt{121} + \sqrt[3]{216}$ (i) $\sqrt[3]{343} \times \sqrt{49}$ (j) $\sqrt[3]{27} \times \sqrt{100}$

4

Exercise 5

- (a) $5^2 - \sqrt{25}$ (b) $5^3 - \sqrt{1}$ (c) $6^3 - \sqrt[3]{8}$ (d) $5^2 + \sqrt{49}$ (e) $7^2 + \sqrt{169}$
 (f) $3^3 \times \sqrt{100}$ (g) $2^2 - \sqrt[3]{1}$ (h) $5^2 \times \sqrt[3]{8}$ (i) $2^3 + 3^2$ (j) $8^2 - \sqrt{49}$
 (k) $8^3 \times \sqrt{4}$ (l) $3^3 - 4^2 - \sqrt{1}$ (m) $\sqrt{49} + \sqrt[3]{343}$ (n) $\sqrt[3]{216} \times 1^3$ (o) $2^3 + 5^2$
 (p) $10^3 \div \sqrt{100}$ (q) $\sqrt{81} \times \sqrt[3]{1000}$ (r) $5^2 + 5^3$ (s) $5^3 - 5^2 + 5$ (t) $\sqrt{81} + (3^2 \times 2^3)$

4

Challenge!

Can you calculate the following sums?

- (a) 2^4 (b) $\sqrt[4]{81}$ (c) 3^5 (d) $\sqrt[6]{64}$ (e) 1^{2500}

Exercise 6: Answer these using a calculator.

4

(a) $\sqrt{49}$

(b) 8^2

(c) $\sqrt[3]{8}$

(d) 4^3

(e) $\sqrt{9} + 3^2$

Exercise 7: Answer these using a calculator.

4

(a) 26^2

(b) 33^2

(c) $20^2 - 9^2$

(d) $18^2 - 6^2 - 3^2$

(e) $3^2 + 19^2$

(f) $24^2 - 5^2 + 3^2$

(g) $40^2 - 16^2 - 4^2 - 7^2$

(h) $\sqrt{625} - \sqrt{196}$

(i) $\sqrt{441} - \sqrt{36}$

(j) $\sqrt{361} + \sqrt{121}$

(k) $\sqrt{900} - \sqrt{400}$

(l) $\sqrt{961} + \sqrt{3600}$

(m) $\sqrt{289} + \sqrt{196}$

(n) $\sqrt{1600} - \sqrt{144}$

(o) $7^3 - 5^3$

(p) $1^3 + 2^3 + 3^3$

(q) $9^3 - 4^3$

(r) $6^3 + 2^3 - 3^3$

(s) $8^3 - 3^3 - 1^3$

(t) $10^3 + 5^3$

(u) $12^3 - 6^3 - 4^3$

(v) $\sqrt[3]{27} + \sqrt[3]{216}$

(w) $\sqrt[3]{1728} - \sqrt[3]{1000}$

(x) $\sqrt[3]{2744} - \sqrt[3]{729}$

(y) $\sqrt[3]{3375} + \sqrt[3]{27}$

(z) $\sqrt[3]{5832} - \sqrt[3]{2744}$

(\alpha) $\sqrt[3]{15625} + \sqrt[3]{216}$

(\beta) $\sqrt[3]{32768} - \sqrt[3]{1000}$

(\chi) $\sqrt[3]{46656} + \sqrt[3]{9261} - \sqrt[3]{2197}$

Exercise 8: Answer these using a calculator.

4

(a) $7^2 + 3^3 + \sqrt{49}$

(b) $\sqrt{121} + \sqrt{81} - \sqrt{64}$

(c) $5^3 - \sqrt{100} + \sqrt{36}$

(d) $8^3 + 3^2 - \sqrt{64}$

(e) $12^2 + \sqrt[3]{2197}$

(f) $\sqrt[3]{5832} - \sqrt{169}$

(g) $\sqrt[3]{32768} - \sqrt{441}$

(h) $\sqrt[3]{35937} - 5^2 + 13^2$

(i) $7^2 + \sqrt[3]{19683}$

(j) $\sqrt{276676}$

Exercise 9

4

(a) Using a calculator, write the answer to $\sqrt{50}$.

(b) Draw a picture of a 5 cm by 5 cm square.

(c) Measure the length of one of the square's diagonals. What do you notice about your answer?

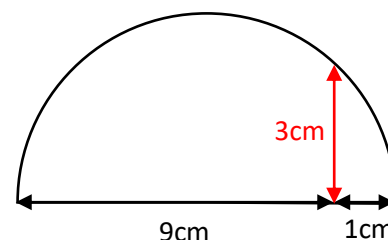
Descartes Investigation: Finding square roots with a semi-circle

René Descartes was born on the 15th of March 1596, in France. He used graphs to connect algebra with geometry. Graphs that use an x -axis and a y -axis are called Cartesian graphs, after him.

As he lay in bed one morning (he usually didn't get up until after midday!), he thought about the following method for calculating the square root of any number.



- Draw a **straight line** of the number you want to find the square root of e.g. 9 cm.
- Add **one unit extra** to the length.
- Draw a **semi-circle** over the whole line.
- Draw a **vertical line** from the point where the original line finished.
- The length of the vertical line will be the square root of the additional number, in this case 3 cm. This method works for any initial number!

**Exercise 10**

Use Descartes' method to calculate the square roots of the following numbers. Measure the length of the vertical line as accurately as possible. Compare your answer with the correct answer from the calculator.

(a) 8

(b) 10

(c) 5.6

(d) 7.8

(e) 6.5



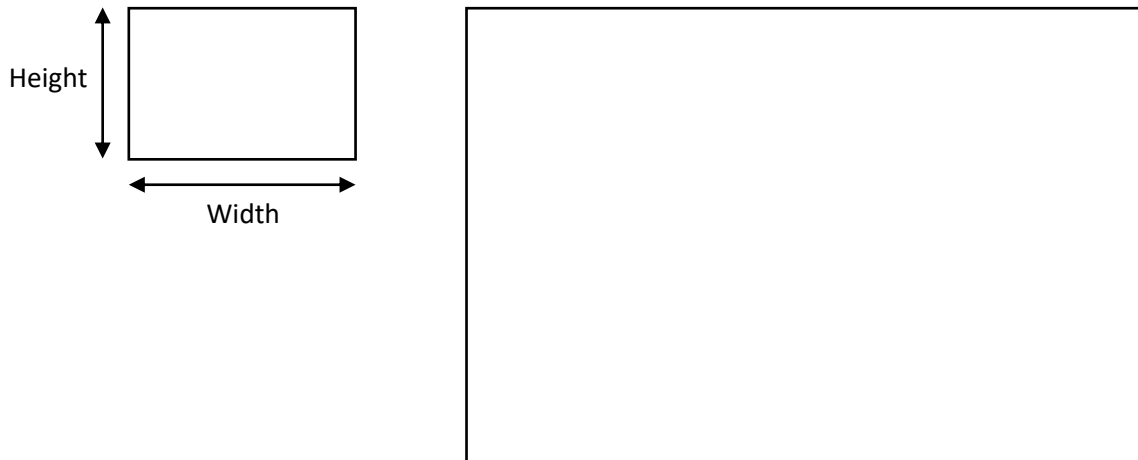
Applying

5

Similar Shapes Investigation**Set 1 only**

Two shapes are **similar** if they are the same shape but have different size.

Consider the two similar rectangles below.

**Exercise 11****8**

- Use a ruler to measure the height and width of the two rectangles.
- How many times larger** is the height of the large rectangle compared to the height of the small rectangle?
- Calculate the **area** of the small rectangle and the area of the large rectangle.
- How many times larger** is the area of the large rectangle compared to the area of the small rectangle?
- What is the **connection** between your answers to (b) and (d)?
- Now consider a new rectangle that has a length and width **four times** the size of the small rectangle.
 - What is the height and width of the new rectangle?
 - How many times larger is the area of the new rectangle compared to the area of the small rectangle?
- What if another rectangle had a length and width **seven times** the size of the small rectangle? How about **eight** times larger? How about x times larger?

**Exercise 12 (Revision)****4**

- | | | | | |
|-------------------------|-----------------|---------------------------------|--------------------|-----------------------|
| (a) 6^2 | (b) 4^3 | (c) $\sqrt{64}$ | (d) $\sqrt[3]{27}$ | (e) $5^2 + \sqrt{16}$ |
| (f) $\sqrt[3]{8} + 8^2$ | (g) $8^2 - 2^3$ | (h) $\sqrt{81} \times \sqrt{4}$ | (i) 15^2 | (j) 11^3 |

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Factors and Prime Numbers

A number is a **factor** of another number if it **divides exactly** into that number. For example, 3 is a factor of 12 because 3 divides exactly into 12. (Or, we can say that 12 is in the 3 times table.) 5 is not a factor of 12 as 5 does not divide exactly into 12. (Or, we can say that 12 is not in the 5 times table.)

To list all the factors of a given number, we can think of all the multiplication sums that give that number. For example, to find all the factors of 16, we can list the multiplication sums that give 16.

$$16 = 1 \times 16$$

$$16 = 2 \times 8$$

$$16 = 4 \times 4$$



So, the factors of 16, ordered from smallest to largest, are 1, 2, 4, 8 and 16.

Exercise 13: List all the factors of the following numbers.

- | | | | | | | |
|--------|--------|--------|--------|---------|---------|---------|
| (a) 4 | (b) 12 | (c) 20 | (d) 26 | (e) 30 | (f) 36 | (g) 42 |
| (h) 50 | (i) 54 | (j) 60 | (k) 72 | (l) 75 | (m) 84 | (n) 100 |
| (o) 1 | (p) 8 | (q) 48 | (r) 90 | (s) 120 | (t) 128 | (u) 360 |

Exercise 14

- | | |
|--|--------------------------------|
| (a) Which numbers in this list are factors of 24? | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. |
| (b) List all the factors of 96. | |
| (c) Which numbers in this list are both factors of 60 and factors of 96? | 3, 8, 10, 6, 18, 12, 24, 4. |
| (d) Which numbers in this list are both factors of 36 and factors of 48? | 6, 9, 12, 4, 8, 3, 16, 18. |
| (e) True or False? 1 is a factor of every whole number. | |
| (f) Which numbers in this list are factors of 45, 75 and 120? | 2, 10, 5, 20, 3, 12, 15, 9 |
| (g) Which numbers in this list are factors of 28, 84 and 126? | 4, 7, 2, 12, 14, 21, 6, 9 |
| (h) Write the odd numbers that are factors of 36. | |
| (i) Write every even number that is a factor of 40. | |
| (j) Apart from 42, what is the highest factor of 42? | |

Exercise 15

Here is a list of numbers.

2,	4,	6,	10,	15,	16,	20,	27,	32,	42,	48,	50,	75
----	----	----	-----	-----	-----	-----	-----	-----	-----	-----	-----	----

Using only numbers from the list above, write

- | | | | |
|-------------------------|----------------------------|--------------------|---|
| (a) the factors of 100 | (b) a square number | (c) a cube number | (d) multiples of 3 |
| (e) the cube root of 64 | (f) the square root of 225 | (g) factors of 20 | (h) two numbers that add to 63 |
| (i) even numbers | (j) factors of 32 | (k) multiples of 5 | (l) two numbers with a difference of 17 |

Challenge!

A person goes shopping and buys a loaf of bread for £1.50; a drink for £1.20 and two other items. Using a calculator, the person multiplies the cost of the items instead of adding. At the till the person says “So, I need to pay £7.11?” and the till operator (that has added the prices correctly) agrees! What was the price of the other two items?

Prime Numbers

A number is a **prime number** if it has **exactly two factors**: 1 and the number itself. For example, 5 is a prime number, because the factors of 5 are 1 and 5 (exactly two factors). 6 is not a prime number, because the factors of 6 are 1, 2, 3 and 6 (more than 2 factors).



Exercise 16: Are the following numbers prime numbers?

4

- | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|
| (a) 3 | (b) 4 | (c) 7 | (d) 9 | (e) 11 | (f) 12 | (g) 15 |
| (h) 16 | (i) 19 | (j) 23 | (k) 27 | (l) 29 | (m) 39 | (n) 41 |
| (o) 45 | (p) 49 | (q) 59 | (r) 65 | (s) 67 | (t) 71 | (u) 77 |

The Sieve of Eratosthenes

Eratosthenes was a Greek mathematician born in 276 B.C. He created an **algorithm** to find all the prime numbers that exist. Use the algorithm to find all the prime numbers between 1 and 100, using the hundred square on the right.

- (1) Put a **cross** through 1 as it is not a prime number (one factor only, not two).
- (2) **Circle** the number 2 as it is a prime number (the factors of 2 are 1 and 2).
- (3) Cross out any numbers that are **multiples of 2** (4, 6, 8, 10, ...) as these are not prime numbers.
- (4) The next **empty** number is 3. Circle this number to show that it is a prime number.
- (5) Cross out numbers that are **multiples of 3** (6, 9, 12, ...).
- (6) **Repeat** this (circle the next number that is empty; cross out any of the multiples) until every number is either circled or crossed out.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Applying

Factors Investigation

- Give a number to each member of the class.
- Everyone to start with their hands down.
- If 1 is a factor of your number, raise your hand. (What should happen here?)
- If 2 is a factor of your number, put your hand down.
- If 3 is a factor of your number, change the position of your hand. (If your hand is up, put it down. If your hand is down, put it up.)
- Repeat this with the rest of the numbers, changing the position of your hand when needed.



At the end, whose hands are raised?

Have another go at the investigation to see if the answer is the same (watch out for people making mistakes!)

Can you explain the end result?

Exercise 17

- List all the prime numbers between 50 and 60.
- What is the next prime number after 23?
- What is the next prime number after 100?
- List all the prime numbers between 110 and 120.
- How many prime numbers are there between 1 and 100?

Exercise 18

What number am I?

- I'm greater than 8. I'm less than 15. I'm an odd number. I'm not a prime number.
- I'm a factor of 60. I'm a two-digit number. I'm an odd number.
- I'm a square number. I'm less than 100. I have 9 factors.
- I'm a prime number. I'm less than 30. I'm one greater than a multiple of 7.
- I'm a multiple of 5. I'm a three-digit number. I'm less than 120. I'm a multiple of 3.

Exercise 19

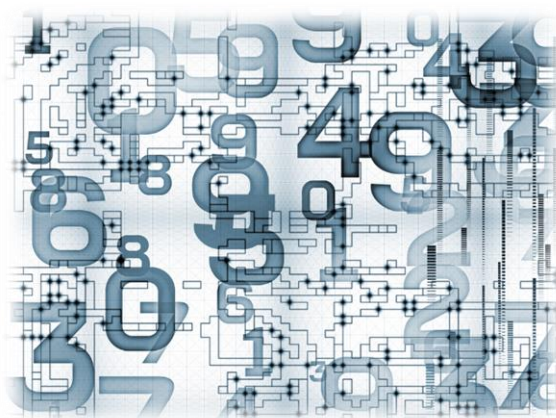
Use the internet to research how prime numbers keep the internet safe. Write half a page presenting your research.

Challenge!

It is possible to write some square numbers as the sum of two prime numbers. For example, $4 = 2 + 2$; $9 = 2 + 7$; $16 = 5 + 11$. What is the smallest square number (apart from 1) that is impossible to write as the sum of two prime numbers?

Exercise 20 (Revision)

- List all the factors of 24.
- List all the factors of 80 that are even numbers.
- Apart from 91, what is the highest factor of 91?
- True or False? Every prime number is odd.
- List all the prime numbers between 60 and 70.
- List all the prime numbers between 100 and 110.



Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Venn Diagrams

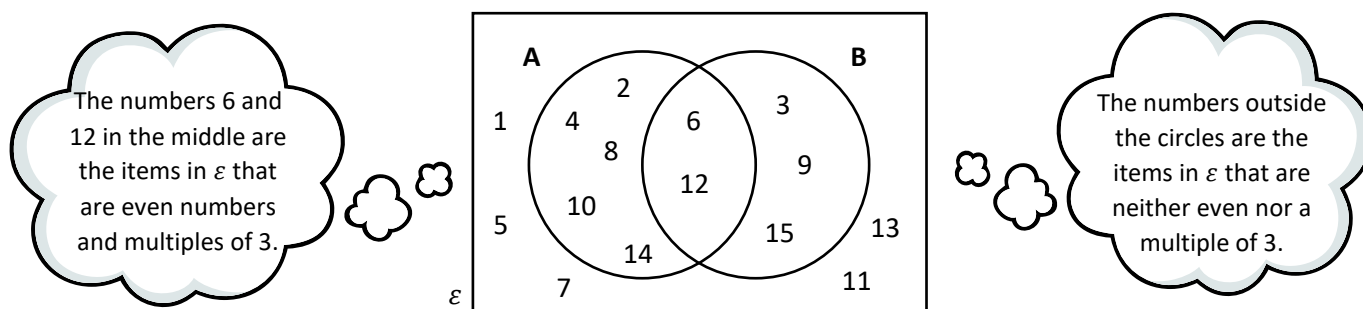
In 1880, John Venn introduced a method for displaying information using overlapping circles.

A Venn diagram always includes:

- A **universal set** ε , a rectangle that contains all the information under inspection.
- One or more **circles**, each with a specific **label**.
- Items from the universal set placed into the correct location.



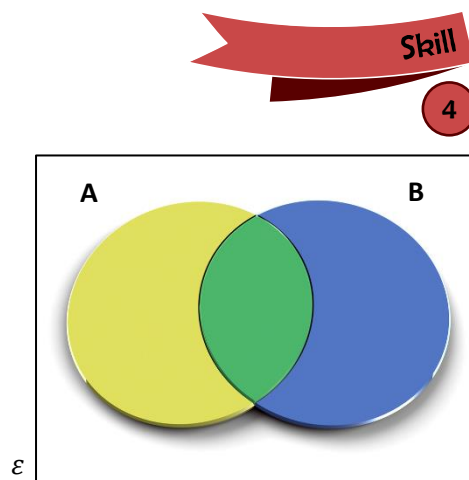
For example, let the universal set ε be the numbers between 1 and 15. Let the circle **A** represent the even numbers, whilst the circle **B** represents the multiples of 3. It is possible to construct a Venn diagram to represent this situation.



Exercise 21

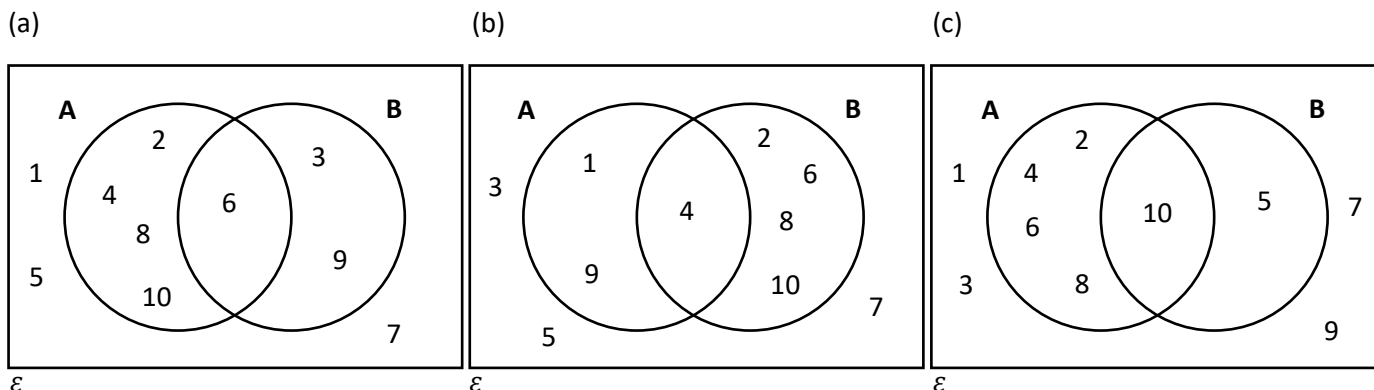
Construct a Venn diagram for the following situations.

- $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = multiples of 5.
- $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = even numbers; **B** = multiples of 4.
- $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = even numbers.
- $\varepsilon = \{1, 2, 3, \dots, 15\}$; **A** = factors of 15; **B** = factors of 5.
- $\varepsilon = \{1, 2, 3, \dots, 15\}$; **A** = factors of 20; **B** = even numbers.
- $\varepsilon = \{1, 2, 3, \dots, 15\}$; **A** = prime numbers; **B** = odd numbers.
- $\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = factors of 8; **B** = multiples of 8.
- $\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = multiples of 3; **B** = factors of 60.
- $\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = square numbers; **B** = odd numbers.



Exercise 22

In the following Venn diagrams, the universal set ε consists of the numbers between 1 and 10. What could the sets **A** and **B** represent?



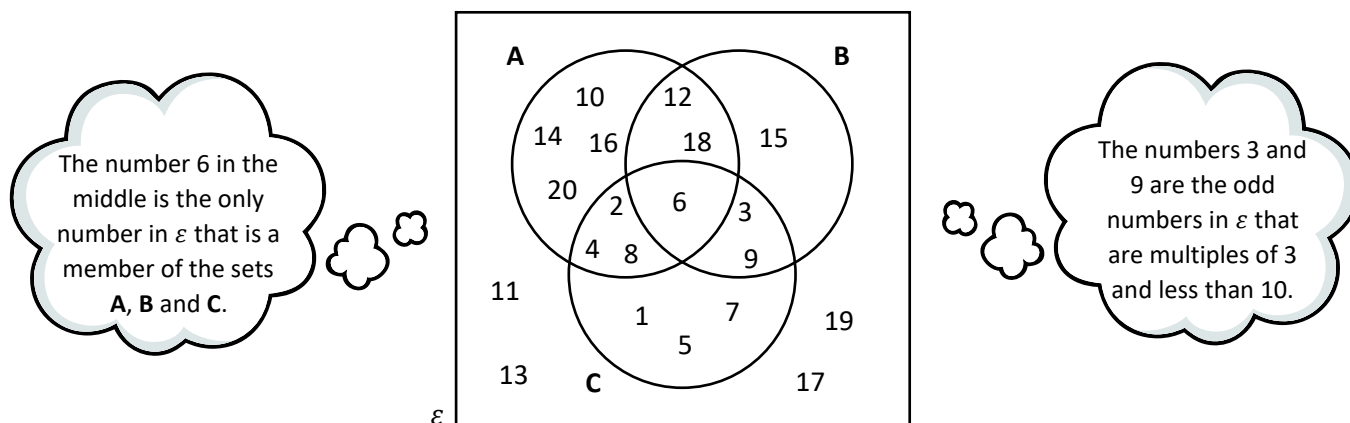
Challenge! Construct a Venn diagram for the following situation.



$\varepsilon = \{1, 2, 3, \dots, 40\}$; **A** = factors of 72; **B** = multiples of 3.

Example

$\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = even numbers; **B** = multiples of 3; **C** = numbers less than 10.

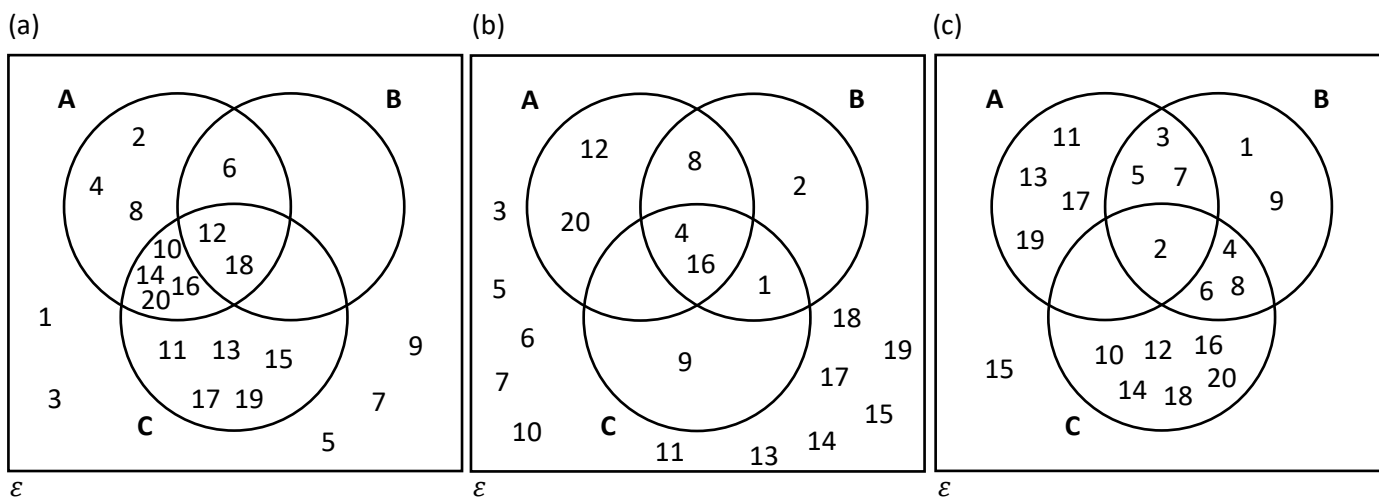
**Exercise 23**

Construct a Venn diagram for the following situations.

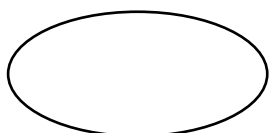
- (a) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = even numbers; **B** = multiples of 4; **C** = numbers less than 5.
 (b) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; **A** = odd numbers; **B** = prime numbers; **C** = numbers greater than 6.
 (c) $\varepsilon = \{1, 2, 3, \dots, 15\}$; **A** = even numbers; **B** = multiples of 3; **C** = multiples of 5.
 (d) $\varepsilon = \{1, 2, 3, \dots, 15\}$; **A** = square numbers; **B** = factors of 16; **C** = numbers less than 7.
 (e) $\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = multiples of 2; **B** = multiples of 3; **C** = multiples of 9.
 (f) $\varepsilon = \{1, 2, 3, \dots, 20\}$; **A** = prime numbers; **B** = factors of 24; **C** = two-digit numbers.

**Exercise 24**

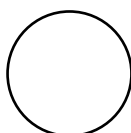
In the following Venn diagrams, $\varepsilon = \{1, 2, 3, \dots, 20\}$. What could the sets **A**, **B** and **C** represent?

**Challenge!**

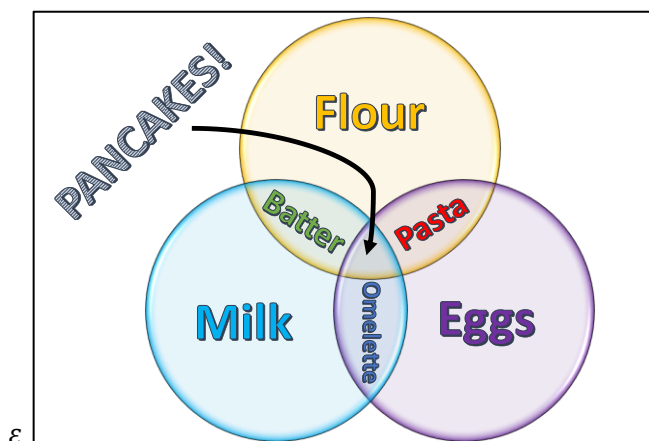
Try to construct a Venn diagram for four sets **A**, **B**, **C** and **D**.
 Hint: You will need to use ellipses (not circles) to show each region.



Ellipse



Circle



Set Notation

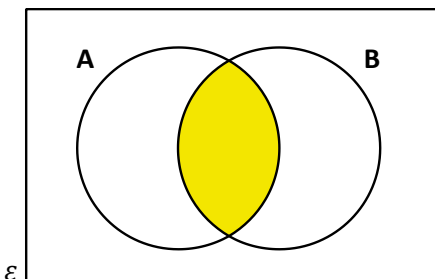


There are special symbols that are used to describe different regions in a Venn diagram.

$$A \cap B$$

"A intersection B".

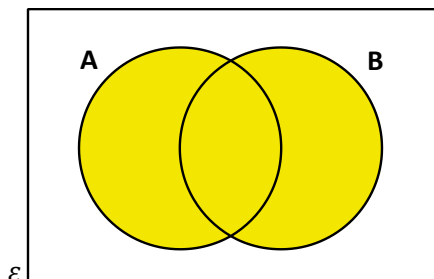
The items in set A **and** in Set B.



$$A \cup B$$

"A union B".

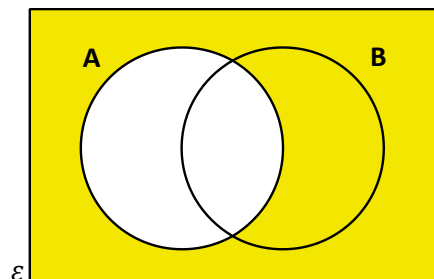
The items in set A **or** in Set B.



$$A'$$

"Complement of A".

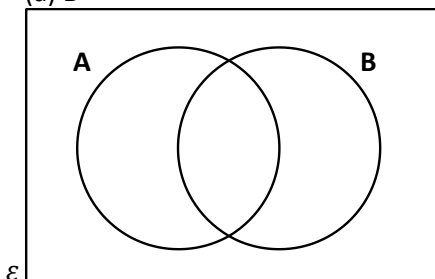
The items that are **not** in set A.



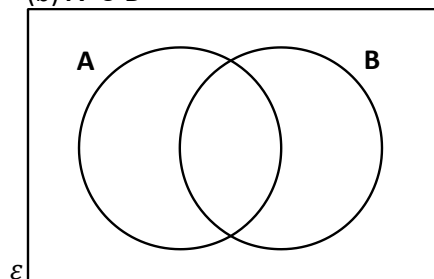
Exercise 25: Colour the following regions.

6

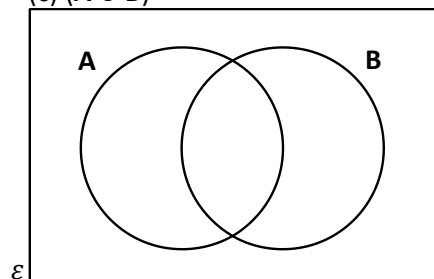
(a) B'



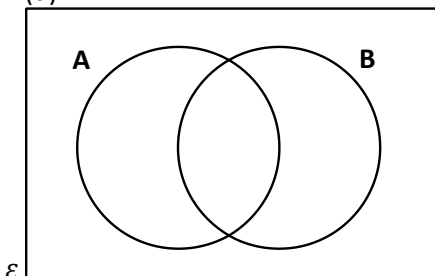
(b) $A' \cup B'$



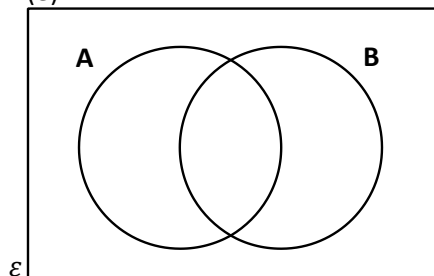
(c) $(A \cup B)'$



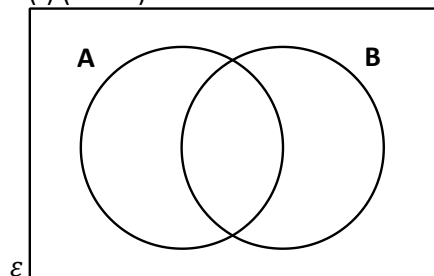
(d) $A \cap B$



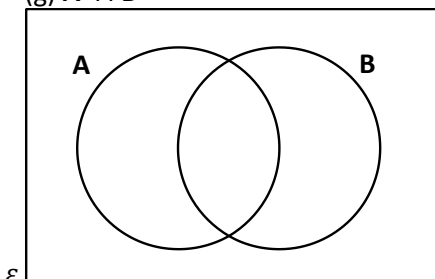
(e) A'



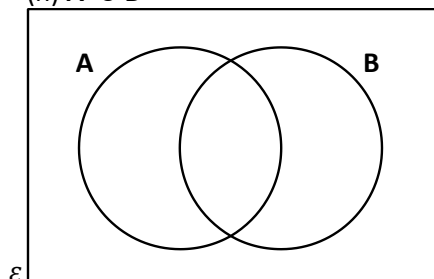
(f) $(A \cap B)'$



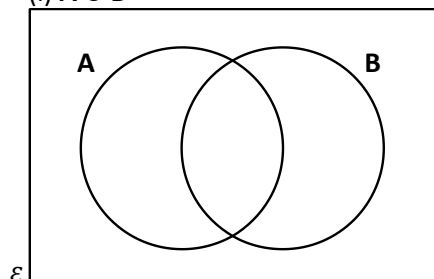
(g) $A' \cap B'$



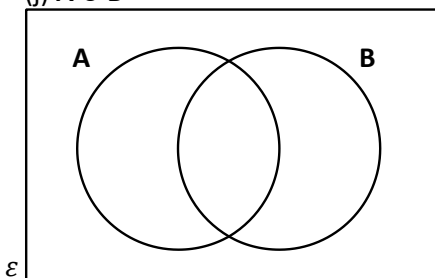
(h) $A' \cup B$



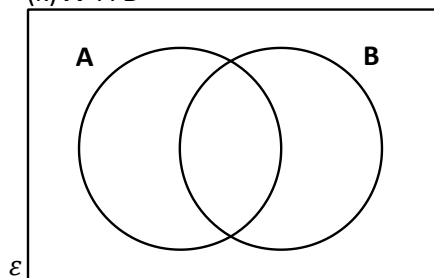
(i) $A \cup B'$



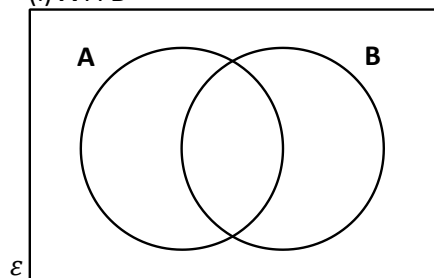
(j) $A \cup B$



(k) $A' \cap B$

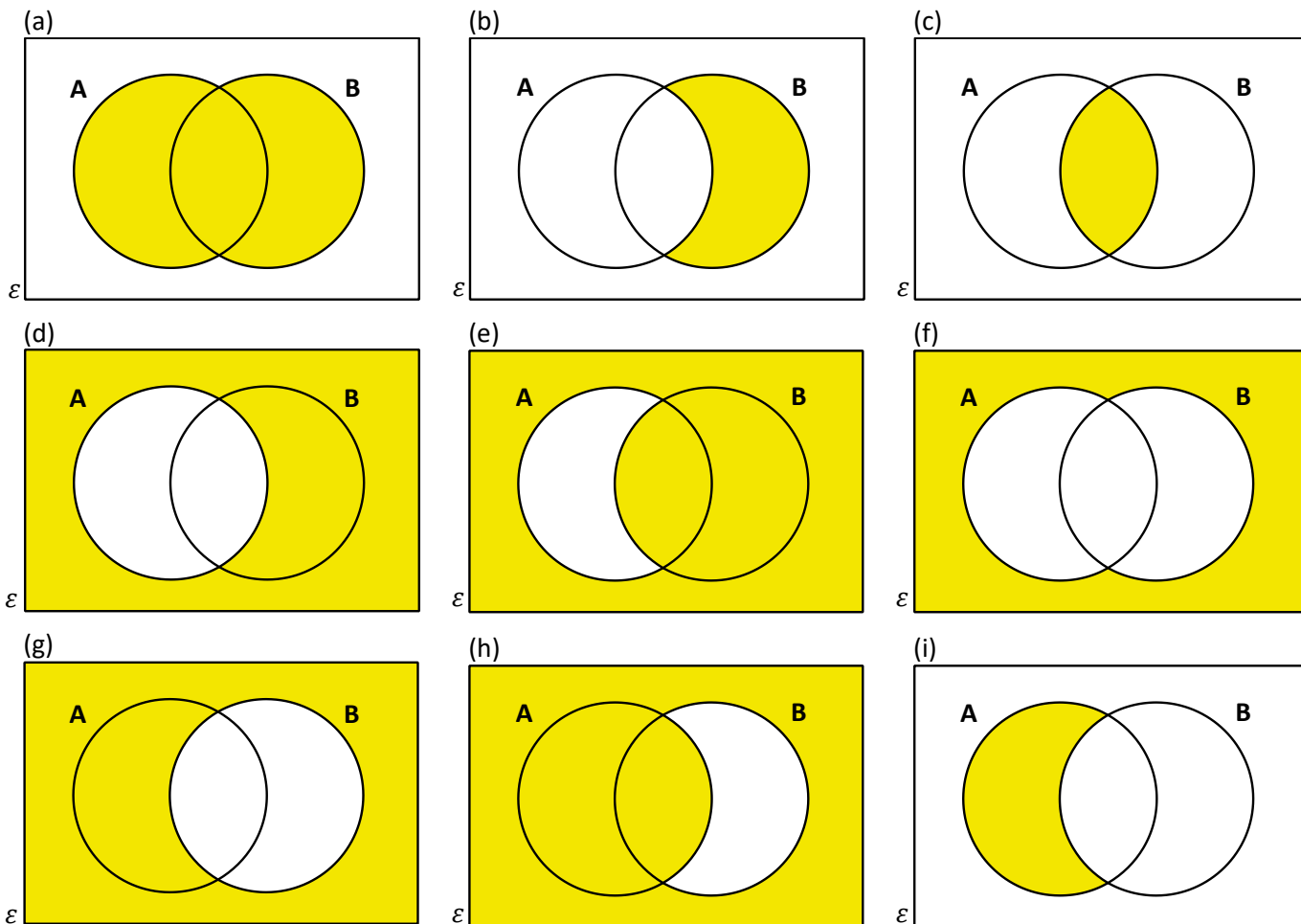


(l) $A \cap B'$



Exercise 26: Which region has been coloured?

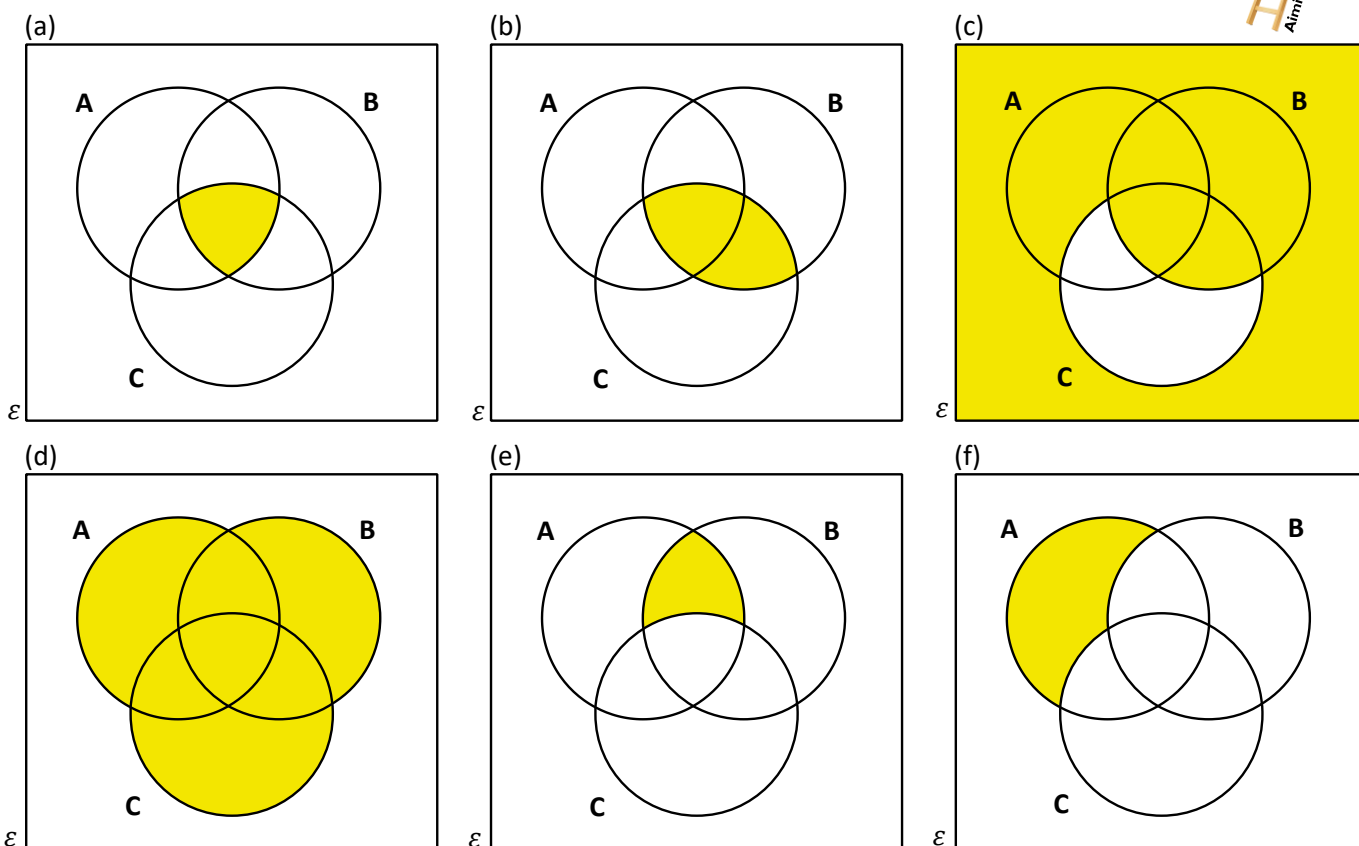
6



Exercise 27: Which region has been coloured?



6

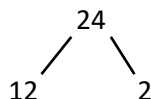


Index Form

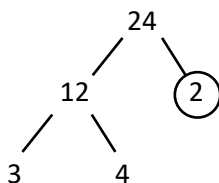
It is possible to write any whole number greater than one as a **product of its prime factors**. This means writing the number as a multiplication sum using only prime numbers. We can use a **factor tree** to help us write a number as a product of its prime factors.

Example

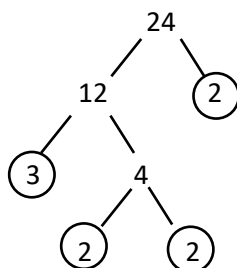
Consider the number 24. Think of a multiplication sum that makes 24, for example 12×2 . Write 24 and then split 24 using the numbers 12 and 2.



As 2 is a prime number, we circle it; however as 12 is not a prime number, we continue to split the number, using (for example) $12 = 3 \times 4$.



To complete the factor tree, we continue to circle prime numbers, and split numbers that are not prime numbers, until all numbers have been considered. Notice that 1 will never appear in a factor tree, as 1 is not a prime number.



After completing the factor tree, we can now write 24 as a product of its prime factors. We must consider all the prime numbers that are circled and write a multiplication sum using these numbers. It is good practice to write these numbers in order, from smallest to largest. We can therefore write 24 as a product of its prime factors as

$$24 = 2 \times 2 \times 2 \times 3$$

It does not matter how you split the numbers in a factor tree; the same numbers will be circled in the end.

Exercise 30

Write the following numbers as a product of their prime factors.

- | | | | | |
|---------|---------|----------|----------|---------|
| (a) 6 | (b) 15 | (c) 20 | (d) 25 | (e) 28 |
| (f) 36 | (g) 42 | (h) 48 | (i) 55 | (j) 56 |
| (k) 60 | (l) 64 | (m) 70 | (n) 72 | (o) 75 |
| (p) 78 | (q) 80 | (r) 84 | (s) 92 | (t) 96 |
| (u) 100 | (v) 115 | (w) 120 | (x) 152 | (y) 180 |
| (z) 256 | (α) 630 | (β) 1080 | (χ) 1225 | |

Skill

4

Challenge!

Write 415,800 as a product of its prime factors.

Index Form

You will remember from work in Year 7 that 5^2 (“five squared”) means 5×5 , whilst 5^3 (“five cubed”) means $5 \times 5 \times 5$. Following this pattern, 5^4 (“five to the power of four”) means the sum $5 \times 5 \times 5 \times 5$, whilst 5^6 (“five to the power of six”) means the sum $5 \times 5 \times 5 \times 5 \times 5 \times 5$. When writing a sum such as $7 \times 7 \times 7 \times 7$ in the form 7^4 , we say that we have written the sum in **index form**.

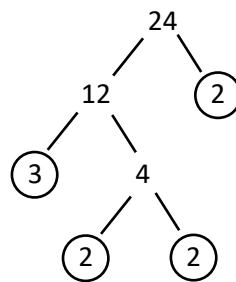
Exercise 31

Write the following sums in index form.

- (a) $5 \times 5 \times 5$ (b) 8×8 (c) $9 \times 9 \times 9 \times 9$
(d) $12 \times 12 \times 12$ (e) $2 \times 2 \times 2 \times 2 \times 2$ (f) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
(g) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ (h) $20 \times 20 \times 20 \times 20$ (i) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
(j) $11 \times 11 \times 11 \times 11 \times 11 \times 11$ (k) $5 \times 5 \times 5 \times 5 \times 5$ (l) $1 \times 1 \times 1 \times 1 \times 1 \times 1$
(m) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ (n) $8 \times 8 \times 8 \times 8$ (o) 14

Writing a number in index form

Earlier in the chapter, we wrote the number 24 as a product of its prime factors, using the following factor tree.



$$24 = 2 \times 2 \times 2 \times 3$$

We can now write 24 in index form using our work above.

$$24 = 2^3 \times 3$$

(Notice that it would also be possible to write 24 as $24 = 2^3 \times 3^1$, however as 3 to the power of 1 (3^1) is the same as 3 on its own, we do not usually write the power of 1.)

Exercise 32

The following numbers have been written as a product of their prime factors. Re-write these numbers in index form.

- (a) $16 = 2 \times 2 \times 2 \times 2$ (b) $45 = 3 \times 3 \times 5$ (c) $90 = 2 \times 3 \times 3 \times 5$
(d) $225 = 3 \times 3 \times 5 \times 5$ (e) $704 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11$ (f) $26325 = 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 13$

Exercise 33

Use a factor tree to help you write the following numbers in index form.

- | | | | | |
|---------|---------|---------|---------|----------|
| (a) 8 | (b) 12 | (c) 18 | (d) 27 | (e) 30 |
| (f) 32 | (g) 40 | (h) 52 | (i) 63 | (j) 88 |
| (k) 98 | (l) 125 | (m) 128 | (n) 160 | (o) 212 |
| (p) 288 | (q) 360 | (r) 500 | (s) 648 | (t) 1125 |

Index Form using a Calculator

Casio calculators have a “**FACT**” button that can be used to show numbers in index form. For example, to write 280 in index form, press the following buttons.

2 8 0 = SHIFT 0.99



You should see $2^3 \times 5 \times 7$ on the screen; this is the index form for 280.

Exercise 34

5

Use your calculator to write the following numbers in index form. (If you do not have a “**FACT**” button, use your calculator to help you construct a factor tree.)

- | | | | | |
|----------|-----------|-----------|------------|-------------|
| (a) 175 | (b) 200 | (c) 1296 | (d) 1500 | (e) 2048 |
| (f) 7225 | (g) 36015 | (h) 83980 | (i) 312500 | (j) 1306368 |

Square Numbers in Index Form

When we write a square number in index form, we notice something special about the powers.

Exercise 35

5

Use your calculator or a factor tree to write the following square numbers in index form.

- | | | | | |
|--------|--------|--------|---------|---------|
| (a) 4 | (b) 9 | (c) 16 | (d) 25 | (e) 36 |
| (f) 49 | (g) 64 | (h) 81 | (i) 100 | (j) 121 |

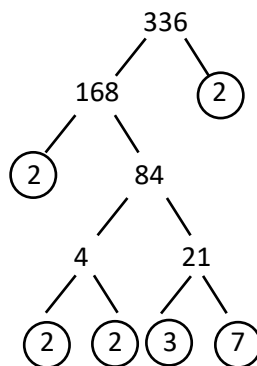
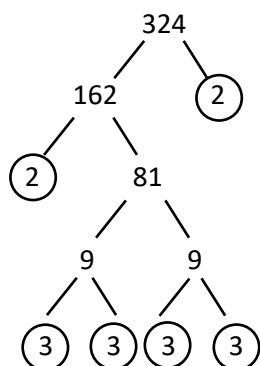
If you completed the exercise correctly, you should notice that the powers are always **even numbers**. This is always true for square numbers.

Fact: When writing a square number in index form, all the powers are always even numbers.

We can use the above fact to decide whether or not a number is a square number.

Example: Are the numbers 324 and 336 square numbers?

To start, let's construct factor trees for 324 and 336.



As a product of its prime factors, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
In index form, $324 = 2^2 \times 3^4$

and $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$.
and $336 = 2^4 \times 3 \times 7$.

For 324, the powers in the index form are all even numbers, so 324 is a square number.

For 336, the powers in the index form are not all even numbers (remember that 3 on its own is the same as 3^1). Therefore 336 is not a square number.

Exercise 36

6

Write the following numbers in index form. Hence, decide whether or not the numbers are square numbers.

- | | | | | |
|---------|---------|---------|---------|---------|
| (a) 144 | (b) 200 | (c) 324 | (d) 400 | (e) 460 |
| (f) 576 | (g) 600 | (h) 729 | (i) 800 | (j) 900 |

Finding the square root of a square number

It is possible to use the index form to find the square root of any square number.

For example, consider again the number 324 from the previous page. We saw that 324, when written in index form, is $324 = 2^2 \times 3^4$. To find the square root of 324, we **halve the powers** in the index form. In this case, we halve the powers 2 and 4 to obtain 1 and 2 so that

$$\begin{aligned}\sqrt{324} &= 2^1 \times 3^2 \\ \sqrt{324} &= 2 \times 9 \\ \sqrt{324} &= 18\end{aligned}$$



(So $18^2 = 324$.)

Rule: To find the square root of a square number, halve the powers in the index form.

Exercise 37

6

Find the square root of the following square numbers, which have been written in index form.

- | | | |
|--|--|-----------------------------|
| (a) $1444 = 2^2 \times 19^2$ | (b) $2304 = 2^8 \times 3^2$ | (c) $2500 = 2^2 \times 5^4$ |
| (d) $3136 = 2^6 \times 7^2$ | (e) $3600 = 2^4 \times 3^2 \times 5^2$ | (f) $4096 = 2^{12}$ |
| (g) $4900 = 2^2 \times 5^2 \times 7^2$ | (h) $5184 = 2^6 \times 3^4$ | (i) $5625 = 3^2 \times 5^4$ |

Multiplying a number to make a square number

If a number is **not** a square number, what is the **smallest** whole number that we need to **multiply** our number by to make a square number?



Again, consider the number 336 from the previous page. We saw that 336, when written in index form, is $336 = 2^4 \times 3 \times 7$. As the powers are not all even, 336 is not a square number. We could multiply 336 with itself to make a square number, $336 \times 336 = 112,896 = 336^2$. But we can also multiply 336 by 21 to make a square number: $336 \times 21 = 7056 = 84^2$.

Why 21? Again, consider the index form for 336, $336 = 2^4 \times 3 \times 7$. The numbers that have odd powers are 3 and 7. By multiplying 336 by 3 and 7, the index form for the new number is $2^4 \times 3^2 \times 7^2$. This index form **does** represent a square number as all the powers are even. So, by multiplying 336 by $3 \times 7 = 21$, we have made a square number. (There is not a number less than 21 that works as the powers in the index form would not all be even.)

Rule: To multiply a number to make a square number, (a) write the number in index form; (b) multiply the number by the numbers in the index form that do not have even powers.

Exercise 38

6

What is the smallest whole number that **multiplies** with the following numbers to make a square number?

- | | | |
|----------------------------------|-------------------------------------|-------------------------------------|
| (a) $60 = 2^2 \times 3 \times 5$ | (b) $72 = 2^3 \times 3^2$ | (c) $308 = 2^2 \times 7 \times 11$ |
| (d) $405 = 3^4 \times 5$ | (e) $540 = 2^2 \times 3^3 \times 5$ | (f) $1617 = 3 \times 7^2 \times 11$ |
| (g) 18 | (h) 44 | (i) 156 |
| (j) 252 | (k) 720 | (l) 850 |



If a number is **not** a square number, what is the **smallest** whole number that we need to **divide** our number by to make a square number?

Again, consider the number $336 = 2^4 \times 3 \times 7$. We could divide 336 by itself to make a square number: $336 \div 336 = 1$, and 1 is a square number (1^2). But we can also divide 336 by 21 to make a square number: $336 \div 21 = 16 = 4^2$.

Why 21? Again, the numbers in the index form without even powers are important. By dividing by these numbers, the numbers will either disappear from the index form (which happens with 336), or the power becomes one less, and change to be even. In each case, the powers in the index form for the new number will all be even numbers, so we end up with a square number.

Rule: To divide a number to make a square number, (a) write the number in index form; (b) divide the number by the numbers in the index form that do not have even powers.

Exercise 39

What is the smallest whole number that **divides** the following numbers to make a square number?

- (a) $68 = 2^2 \times 17$ (b) $84 = 2^2 \times 3 \times 7$ (c) $260 = 2^2 \times 5 \times 13$
(d) $567 = 3^4 \times 7$ (e) $792 = 2^3 \times 3^2 \times 11$ (f) $4114 = 2 \times 11^2 \times 17$
(g) 20 (h) 27 (i) 42
(j) 288 (k) 350 (l) 940

Exercise 40 (Revision)

- (a) Write the following numbers as a product of their prime factors.
- (i) 68 (ii) 140 (iii) 304
- (b) Using a factor tree, write the following numbers in index form.
- (i) 92 (ii) 196 (iii) 1200
- (c) Using a calculator, write the following numbers in index form.
- (i) 952 (ii) 5325 (iii) 1658880
- (d) Are the following numbers square numbers? If they **are** square, find the square root of the square number. If they are **not** square, find the smallest whole number that **multiplies** with the number to make a square number.
- (i) 484 (ii) 888 (iii) 1600

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

HCF, LCM

The **Highest Common Factor (HCF)** of two numbers is the largest number that is a factor of both numbers.
The **Lowest Common Multiple (LCM)** of two numbers is the smallest number that is a multiple of both numbers.

Example

What is the Highest Common Factor and Lowest Common Multiple of 24 and 36?

Method 1

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

The **largest** number that appears in **both** lists is 12.

Therefore, 12 is the Highest Common Factor of 24 and 36.

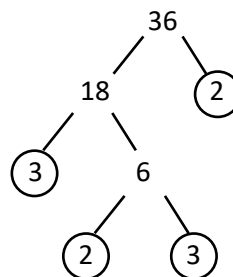
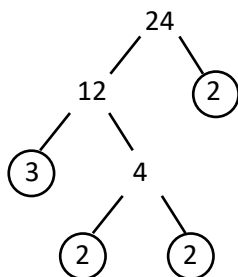
The multiples of 24 are 24, 48, 72, 96, 120, ...

The multiples of 36 are 36, 72, 108, 144, 180, ...

The **smallest** number that appears in both lists is 72. Therefore, 72 is the Lowest Common Multiple of 24 and 36.

Method 2

Here are some factor trees for 24 and 36.



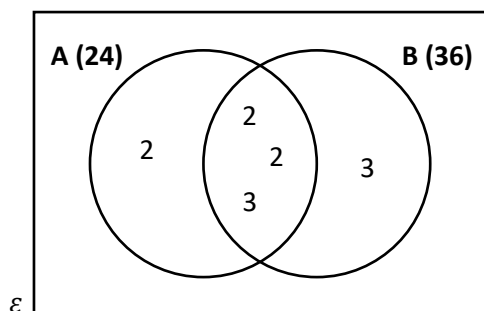
As a product of their prime factors,

$$24 = 2 \times 2 \times 2 \times 3$$

and

$$36 = 2 \times 2 \times 3 \times 3.$$

We can construct the following Venn diagram for the prime numbers that form the product of prime factors for 24 and 36. ε = Prime factors of 24 and 36; **A** = Prime factors of 24; **B** = Prime factors of 36.



To find the Highest Common Factor of 24 and 36, we multiply all the numbers that appear in the centre of the Venn diagram (in the **intersection**). Therefore, the Highest Common Factor of 24 and 36 is $2 \times 2 \times 3 = 12$.

To find the Lowest Common Multiple of 24 and 36, we multiply all the numbers that appear in the **union** of the Venn diagram (inside the circles). Therefore, the Lowest Common Multiple of 24 and 36 is $2 \times 2 \times 2 \times 3 \times 3 = 72$.

Note: In this method, if no numbers appear in the middle of the Venn diagram (in the intersection), then the Highest Common Factor of the two numbers is 1 (remember that 1 is always a factor of any whole number).

Exercise 41

Skill

5

Find the Highest Common Factor and Lowest Common Multiple of the following numbers.
(Suggestion: Use a factor tree and Venn diagram if the numbers are large.)

- (a) 6 and 8 (b) 8 and 18 (c) 8 and 9 (d) 36 and 48 (e) 25 and 55 (f) 33 and 55 (g) 54 and 72
(h) 30 and 40 (i) 45 and 63 (j) 24 and 50 (k) 27 and 63 (l) 20 and 49 (m) 48 and 84 (n) 50 and 64
(o) 42 and 49 (p) 84 and 96 (q) 120 and 125 (r) 128 and 160 (s) 180 and 200 (t) 75 and 315 (u) 125 and 144
(v) 240 and 250 (w) 256 and 280 (x) 340 and 380 (y) 350 and 400 (z) 640 and 800 (α) 700 and 750 (β) 1296 and 1536

Challenge! (Suggestion: You will need a Venn diagram with three circles.)

What is the Highest Common Factor and Lowest Common Multiple of 24, 84 and 126?

Exercise 42

Applying

6

- (a) Dewi and John are cycling around a bicycle track. Dewi takes 50 seconds to complete a lap of the track. John takes 80 seconds to complete a lap of the track. Dewi and John start cycling at the same time from the start line. When is the next time they will they **both** be at the start line?

(b) Buses to Llandudno leave the station every 18 minutes. Buses to Abergele leave the same station every 16 minutes. A bus to Llandudno and a bus to Abergele leave the station at 08:20. When will be the next time that buses leave to the two places at the same time?

(c) Lydia pays £6 for a ticket every time she goes to the cinema. Megan goes to a different cinema, and she pays £8 for a ticket each time. Lydia and Megan meet and realise that they have spent the same amount on cinema tickets this year. Megan has not spent more than £40. How many times have they each been to the cinema this year?

(d) Fiona is organising a party. She would like to buy sausage rolls, small pork pies and cakes for the guests. The table below shows what is available to buy in the local shop.

	Sausage Rolls	Small Pork Pies	Cakes
Amount in each pack	12	6	9
Cost per pack	£2.40	£1.80	£2.10

Fiona wants to be sure that each person in the party has at least one sausage roll, one pork pie and one cake each. Fiona has £30 to spend on food for the party. Does Fiona have enough money to buy the required food?

Exercise 43 (Revision)

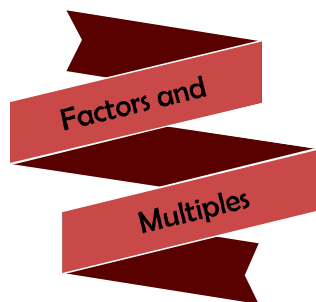
5

Find the Highest Common Factor and Least Common Multiple of the following numbers.

- (a) 12 and 16 (b) 32 and 40 (c) 35 and 45 (d) 100 and 150 (e) 392 and 400

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to find the square root of a square number.			1	
I know how to find the cube root of a cube number.			1	
I can perform calculations that include squaring, cubing, square root and cube root .			1	
I can list the factors of any whole number.			2	
I know the meaning of the terms sum, difference, multiple and even number .			5	
I know how to decide whether or not a number is a prime number .			3	
I can list all the prime numbers in a certain range, e.g. between 70 and 80.			4	
I know how to draw a Venn diagram that uses two circles .			5	
I know how to draw a Venn diagram that uses three circles .			8	
I know the meaning of the terms " intersection ", " union " and " complement ".			9	
I can shade regions of Venn diagrams using set notation .			9	
I can answer worded problems using Venn diagrams.			11	
I can write a number as a multiple of its prime factors .			7	
I can write numbers in index form .			6	
I can use a calculator to find the index form of a given number.				
I know what's special about square numbers when written in index form.			10	
I can recognise square numbers using index form.			10	
I can use index form to find the square root of a square number .			10	
I can use index form to decide what to multiply/divide by in order to change a number into a square number .			10	
I can find the Highest Common Factor of two numbers.			7	
I can find the Lowest Common Multiple of two numbers.			7	
I can answer worded problems using the Highest Common Divisor or Lowest Common Multiple.			12	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

8

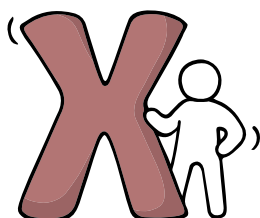
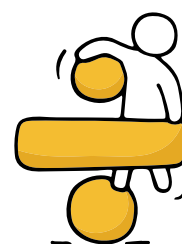
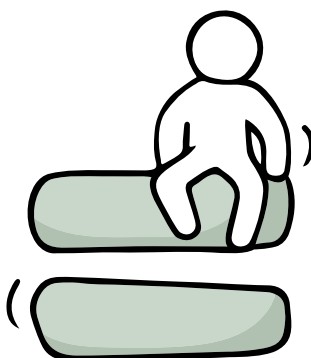
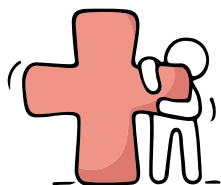
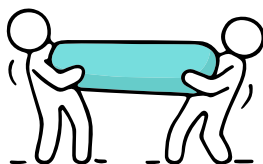
Introducing

Algebra

Name:

Contents

Chapter	Mathematics	Page Number
Sequences	Number patterns. Function machines.	3
Forming Expressions	Rules of algebra. Writing algebraic expressions.	8
Substitution	Substituting into algebraic expressions. Substituting into formulae.	10
Collecting Like Terms	Collecting like terms using one variable. Collecting like terms using more than one variable.	13
Solving Equations	Solving one step equations. Solving more complex equations. Solving problems with equations.	14



Sequences

A **sequence**, or a **number pattern**, is a list of numbers where it's possible to describe how the pattern continues. For example, consider the following list of numbers.

5, 8, 11, 14, 17,

To get the next number in the pattern, we must add three to the previous number. Therefore, the next number in the pattern will be $17 + 3 = 20$, and the next one will be $20 + 3 = 23$.



Other examples

5, 10, 20, 40, 80,

Double, or multiply by two.

The next number is $80 \times 2 = 160$.

54, 48, 42, 36, 30,

Subtract 6.

The next number is $30 - 6 = 24$.

1, 4, 9, 16, 25,

Square numbers.

The next number is $6^2 = 36$.

1, 3, 6, 10, 15,

The triangle numbers –

add one more each time.

The next number is $15 + 6 = 21$.



Exercise 1

Find the next two numbers in the following sequences.

Addition

(a) 6, 8, 10, 12, 14,,

(d) 1, 4, 7, 10, 13,,

(b) 12, 16, 20, 24, 28,,

(e) 31, 38, 45, 52, 59,,

(c) 5, 10, 15, 20, 25,,

(f) 1.5, 2, 2.5, 3, 3.5,,

Subtraction

(g) 48, 44, 40, 36, 32,,

(j) 5, 4, 3, 2, 1,,

(h) 65, 58, 51, 44, 37,,

(k) 75, 64, 53, 42, 31,,

(i) 128, 119, 110, 101, 92,,

(l) 7, 6.75, 6.5, 6.25, 6,,

Multiplication

(m) 1, 2, 4, 8, 16,,

(p) 2, 8, 32, 128, 512,,

(n) 1, 3, 9, 27, 81,,

(q) 1, 5, 25, 125, 625,,

(o) 3, 6, 12, 24, 48,,

(r) 2, 6, 18, 54, 162,,

Division

(s) 128, 64, 32, 16, 8,,

(v) 729, 243, 81, 27, 9,,

(t) 320, 160, 80, 40, 20,,

(w) 1458, 486, 162, 54, 18,,

(u) 800, 400, 200, 100, 50,,

(x) 10000, 1000, 100, 10,,

Something Else...

(y) 1, 2, 4, 7, 11,,

(β) 1, 3, 7, 13, 21,,

(z) 2, 3, 7, 14, 24,,

(χ) 1, 5, 2, 6, 3, 7, 4, 8,,

(α) 40, 39, 37, 34, 30,,

Investigation

Who was **Fibonacci**? What sequence was named after him?
How do you find the next number in Fibonacci's sequence?
What's the connection between this sequence and nature?



Exercise 2

Find the next two numbers in the following sequences.

- | | | |
|------------------------------|-------------------------------|-------------------------|
| (a) 7, 12, 17, 22, 27, | (b) 50, 44, 38, 32, 26, | (c) 3, 6, 12, 24, |
| (d) 3, 6, 9, 12, 15, | (e) 1, 4, 9, 16, 25, | (f) 32, 16, 8, 4, |
| (g) 5, 9, 13, 17, 21, | (h) 1, 8, 27, 64, 125, | (i) 48, 24, 12, |

Exercise 3

Explain, **in words**, how it's possible to find the next number in the following sequences.

- | | | |
|-------------------------------------|--------------------------------|---------------------------------------|
| (a) 15, 17, 19, 21, 23, | (b) 67, 64, 61, 58, 55, | (c) 4, 8, 16, 32, 64, |
| (d) 1200, 600, 300, 150, 75, | (e) 7, 7.5, 8, 8.5, 9, | (f) 146, 139, 132, 125, 118, |
| (g) 4, 12, 36, 108, 324, 972, | (h) 405, 135, 45, 15, 5, | (i) 73, 72.75, 72.5, 72.25, 72, |

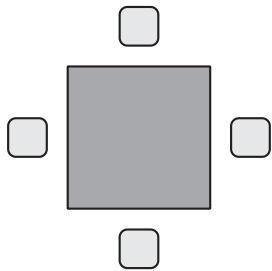
Exercise 4

Find the next two numbers in the following sequences.

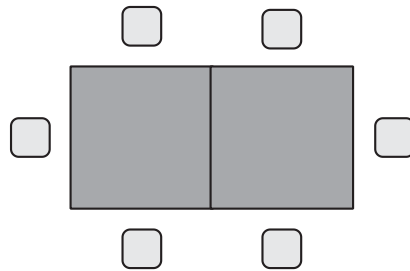
- | | | |
|-----------------------------|-----------------------------|--------------------------------------|
| (a) 5, 8, 12, 17, 23, | (b) 2, 5, 10, 17, 26, | (c) 4, 9, 19, 34, |
| (d) 52, 42, 33, 25, | (e) 3, 7, 15, 31, 63, | (f) 8, 10, 11, 13, 14, 16, 17, |
| (g) 2, 5, 11, 20, | (h) 3, 6, 4, 7, 5, 8, | (i) 8, 6, 12, 10, 20, 18, |

Exercise 5: Setting tables and chairs in a row

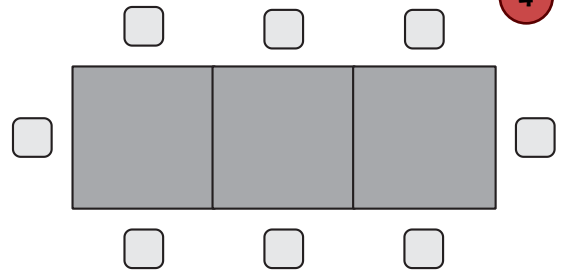
One table



Two tables



Three tables



Applying

4

- How many chairs are needed if you have three tables?
- How many chairs are needed if you have four tables?
- How many chairs are needed if you have ten tables?
- How many tables are needed if you have six chairs?
- How many tables are needed if you have twenty chairs?

Exercise 6: Building a card tower

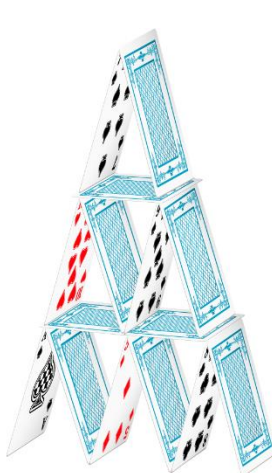
One tier



Two tiers



Three tiers



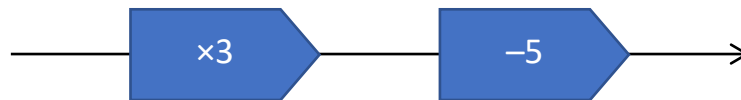
- How many cards are needed to create a two-tier tower?
- How many cards are needed to create a three-tier tower?
- How many cards are needed to create a four-tier tower?
- How many cards are needed to create a ten-tier tower?



Additional challenge! Can you build a card tower containing four tiers?

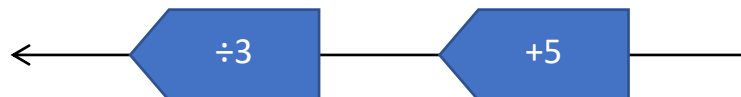
Function Machines

To use a function machine, we put a number into the machine as input, and then use the mathematical sums in the machine to find the output. For example, here is a function machine with two sums: multiply by three then subtract five.



If 7 goes into the machine, we must multiply by 3 first, to obtain 21, then subtract 5 to obtain 16.

Question: What number went into the machine if the output was 28? Here we must work backwards through the machine, undoing any sums the machine has done. You can do this in your head, or by drawing the following function machine to help you.



If 28 came out of the original machine, then to find the input, we must add 5 first (to obtain 33) and then divide by 3 to obtain 11. This is the input of the original function machine to obtain 28 as the output.

Exercise 7

4

Complete the following table to show how to undo different mathematical sums. (The first one has been done for you.)

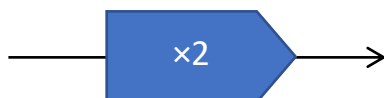
Sum	How to undo?
Add	Subtract
Subtract	
Multiply	
Divide	
Square	
Square root	
Cube	
Cube root	

Exercise 8

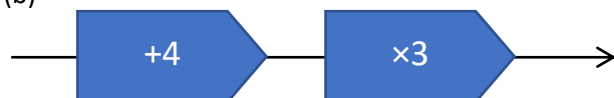
Use the following function machines to complete the input and output tables.



(a)



(b)



4

Input	Output
5	
13	
24	
50	
134	
	8
	14
	26
	46
	142

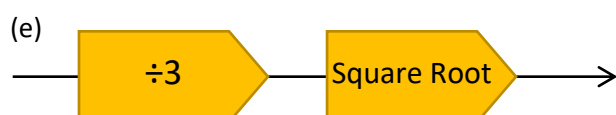
Input	Output
3	
6	
7	
11	
20	
	18
	27
	36
	42
	60



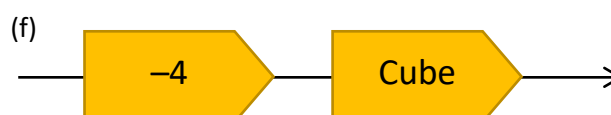
Input	Output
10	
14	
20	
34	
100	
	8
	10
	15
	21
	34



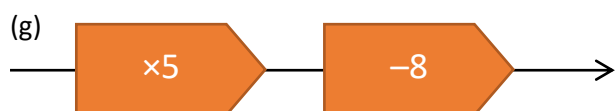
Input	Output
3	
4	
7	
10	
11	
	8
	29
	40
	68
	148



Input	Output
12	
27	
75	
300	
3	
	4
	6
	7
	9
	0



Input	Output
6	
8	
11	
13	
14	
	27
	125
	216
	512
	1



Input	Output
2	
4	
8	
11	
15	
	7
	22
	37
	52
	492

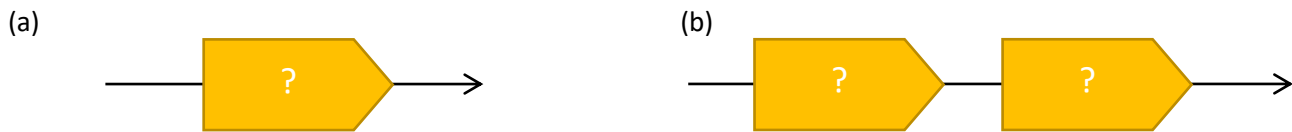


Input	Output
2	
5	
7	
12	
47	
	9
	15
	24
	36
	72

Exercise 9

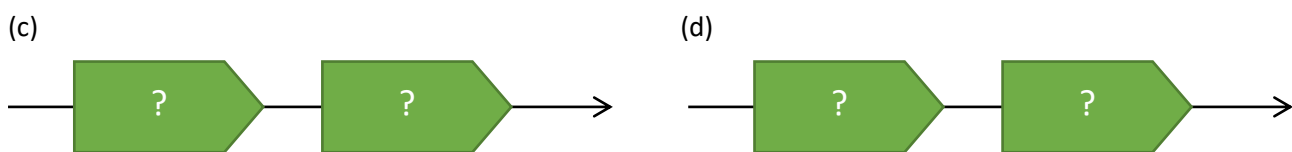
4

What must appear in these function machines to give these inputs and outputs?



Input	Output
3	9
4	12
6	18
9	27
14	42

Input	Output
2	14
3	16
5	20
8	26
10	30



Input	Output
4	4
5	8
6	12
7	16
10	28

Input	Output
2	10
3	15
4	22
5	31
10	106

Exercise 10 (Revision)

4

(a) Find the next two numbers in the following sequences.

(i) 24, 28, 32, 36, 40,,

(ii) 5, 10, 20, 40, 80,,

(iii) 76, 73, 70, 67, 64,,

(iv) 5, 6, 8, 11, 15, 20,,

(v) 2, 5, 10, 17, 26,,

(vi) 7, 14, 15, 30, 31, 62,,

(b) Deio puts the numbers 1, 2, 3, 4 and 5 into a function machine and gets 4, 10, 16, 22 and 28 as output. What could Deio's function machine be?



Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Forming Expressions

Rules of Algebra

In algebra, we use a variable to represent a number. For example, consider the pile of coins in the picture to the right. Without counting, we don't know how many coins are in the pile. We can use a variable to represent how many coins are in the pile, say the letter x .



- If someone **adds** three of the same coins to the pile, there will be $x + 3$ coins in the pile.
- If someone **removes** four coins from the pile, there will be $x - 4$ coins in the pile.
- If someone **doubles** the number of coins in the pile, then there will be $2x$ coins in the pile. (Note that we don't write $2 \times x$, since the multiplication symbol $\times$ is too similar to the letter x .)
- If it is possible to **half** the number of coins in the pile (when would this be possible?), then there would be $\frac{x}{2}$ coins in the pile. (Note that we use a fraction to write the answer here, and not the division sum $x \div 2$.)

In the above examples, we have written **algebraic expressions** to explain the change in the pile of coins.

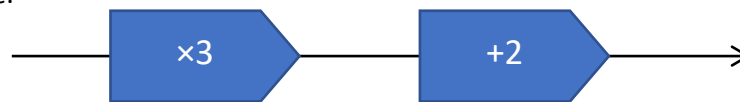
Exercise 11

5

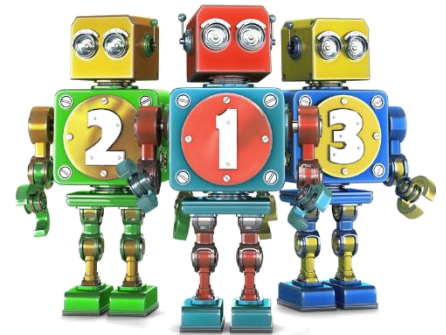
- (a) There are x coins in a pile. Write an expression for the number of coins in a pile that includes
- 5 extra coins
 - 8 fewer coins
 - three times as many coins
- (b) Siriol thinks of a number and uses the letter y to represent the number. Write an expression for the following.
- The sum of 4 and the number
 - Two less than the number
 - Two times the number
 - Three more than the number
 - Half the number
 - Six times the number
- (c) Jars like the one in the picture are available in many different heights. The height of the jar in the picture is h cm. Write an expression for the height of a jar if its height is
- three centimetres taller than the one shown
 - seven centimetres shorter than the one shown
 - three times the height of the one shown
 - a third of the height of the one shown
 - eight centimetres more than double the height of the one shown.
- (d) Bethan has x pounds. Rhys has 5 pounds more than Bethan. Kieran has three times Bethan's amount. Ceidiog has four times Kieran's amount. Write expressions for how many pounds the following people have.
- Rhys
 - Kieran
 - Ceidiog
- (e) A magazine costs £ m . Write, in terms of m , the cost of 6 magazines.
- (f) Louise's weight is x kg. Sara is 4 kg lighter. Write, in terms of x , Sara's weight.
- (g) David's height is h cm. Eysyllt is seven centimetres taller than David. Write, in terms of h , Eysyllt's height.
- (h) There are 30 seats in each row in a cinema. How many seats are there in
- 2 rows
 - 6 rows
 - n rows?
- (i) Susie is 12 years old. What will her age be in
- 5 years
 - 14 years
 - x years?



(j) Here is a function machine.



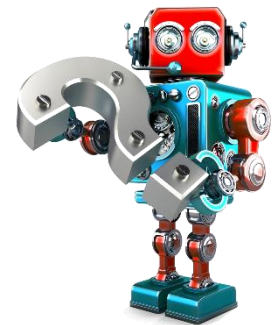
- What is the output if 7 is the input?
- What is the output if 20 is the input?
- What is the output if x is the input?
- What is the input if 17 is the output?
- What is the input if 38 is the output?
- What is the input if z is the output?



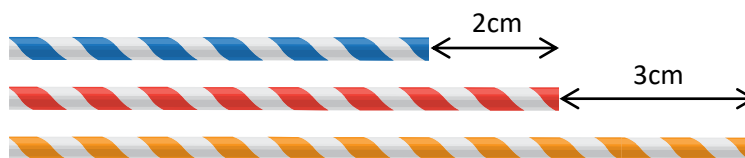
(k) Here is a function machine.



- What is the output if 4 is the input?
- What is the output if 9 is the input?
- What is the output if x is the input?
- What is the input if 4 is the output?
- What is the input if 59 is the output?
- What is the input if z is the output?



(1)



The length of the red straw in the middle is x cm. Write an expression for the length of

- (i) the blue straw; (ii) the yellow straw.

Exercise 12 (Revision)

5

- (a) Steven's car is l cm long. Megan's car is 14 cm longer than Steven's. Write, in terms of l , an expression for the length of Megan's car.
- (b) Pencils cost x pence each. Delyth buys four pencils. What was the cost of the pencils bought by Delyth?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Substitution

Given an algebraic expression, it's possible to calculate the **value** of the expression if we know the value of the variables that make up the expression. For example, consider the following expression.

$$5x - 2y$$

Given the values of x and y , we can calculate the value of $5x - 2y$.

If the value of x is 3 and the value of y is 4, then the value of $5x - 2y$ is

$$\begin{aligned} & 5 \times 3 - 2 \times 4 \\ &= 15 - 8 \\ &= 7 \end{aligned}$$

On the other hand, if $x = 7$ and $y = 6$, then the value of $5x - 2y$ is

$$\begin{aligned} & 5 \times 7 - 2 \times 6 \\ &= 35 - 12 \\ &= 23 \end{aligned}$$

When finding the value of $5x - 2y$ given the values of x and y , we say that we are **substituting** the values of x and y into $5x - 2y$.

Exercise 13

In the following questions, calculate the value of the expression by substituting the values of the relevant variables into the expression.

Skill
5

- | | | | |
|-------------------------|---------------------------|----------------------------|------------------------------|
| (a) $4x + 3y$ | (i) $x = 2, y = 5$ | (ii) $x = 5, y = 6$ | (iii) $x = 8, y = 2$ |
| (b) $6x + 2z$ | (i) $x = 3, z = 4$ | (ii) $x = 8, z = 3$ | (iii) $x = 2, z = 12$ |
| (c) $8x - 3y$ | (i) $x = 4, y = 2$ | (ii) $x = 7, y = 6$ | (iii) $x = 10, y = 11$ |
| (d) $6u + 5v$ | (i) $u = 5, v = 6$ | (ii) $u = 4, v = 2$ | (iii) $u = 8, v = 9$ |
| (e) $7a - 3b$ | (i) $a = 2, b = 3$ | (ii) $a = 6, b = 2$ | (iii) $a = 11, b = 13$ |
| (f) $9c - 5d$ | (i) $c = 4, d = 3$ | (ii) $c = 6, d = 8$ | (iii) $c = 3, d = 6$ |
| (g) $\frac{1}{2}x + 3y$ | (i) $x = 8, y = 4$ | (ii) $x = 12, y = 3$ | (iii) $x = 20, y = 7$ |
| (h) $4y + z$ | (i) $y = 3, z = 3$ | (ii) $y = 9, z = 4$ | (iii) $y = 2, z = -2$ |
| (i) $2x + 5y + 3z$ | (i) $x = 3, y = 4, z = 6$ | (ii) $x = 2, y = 6, z = 8$ | (iii) $x = 5, y = 7, z = 9$ |
| (j) $5x - 3y + 2z$ | (i) $x = 4, y = 2, z = 5$ | (ii) $x = 8, y = 4, z = 3$ | (iii) $x = 10, y = 5, z = 8$ |

More substitution

Here's how to substitute the values $x = 3, y = 4, z = 5$ into the following algebraic expressions.

- | | | | | |
|--------------------|--------------------------------------|-------------------|-------------------------------------|---|
| (a) x^2 | (b) $3y^2$ | (c) xy | (d) $7yz$ | (e) $\frac{xz+5}{y}$ |
| $3^2 = 3 \times 3$ | $3 \times 4^2 = 3 \times 4 \times 4$ | $3 \times 4 = 12$ | $7 \times 4 \times 5 = 7 \times 20$ | $\frac{3 \times 5 + 5}{4} = \frac{15 + 5}{4}$ |
| $= 9$ | $= 3 \times 16$ | | 140 | $= \frac{20}{4}$ |
| | $= 48$ | | | $= 5$ |

Exercise 14

Substitute the values $x = 3, y = 2, z = 4$ into the following algebraic expressions.

- | | | | | |
|-----------|------------|--------------|----------------|----------------|
| (a) x^2 | (b) $3y^2$ | (c) xy | (d) $7yz$ | (e) xyz |
| (f) z^2 | (g) $7x^2$ | (h) xz | (i) $9y + 2xy$ | (j) $3xyz$ |
| (k) y^3 | (l) $5x^3$ | (m) $xz - 7$ | (n) $8x + 3z$ | (o) $2x^2 + 5$ |

Exercise 15

5

Substitute the values $a = 7$, $b = 3$, $c = 9$ into the following algebraic expressions.

- | | | | | |
|----------------|----------------------|-----------------|---------------|----------------------|
| (a) a^2 | (b) $2c^2$ | (c) ac | (d) $4bc$ | (e) abc |
| (f) b^3 | (g) $a + 23$ | (h) $7c - 2b$ | (i) $ab + 7c$ | (j) $\frac{c}{b}$ |
| (k) $5c + b^2$ | (l) $\frac{3c+1}{a}$ | (m) $5b + 2a^2$ | (n) $13 + 8c$ | (o) $\frac{2a+4}{c}$ |

Challenge!

Substitute the values $p = 8$, $q = 6$, $r = 2$ into the following algebraic expressions.

- | | | | | |
|-------------------|--------------------------|------------------------|-------------------------------|-----------------|
| (a) $4q^3 + 2p^3$ | (b) $r^4 + \frac{pq}{r}$ | (c) $\frac{4p+8q}{2r}$ | (d) $\frac{q+\frac{p}{r}}{r}$ | (e) $p^r - q^r$ |
|-------------------|--------------------------|------------------------|-------------------------------|-----------------|

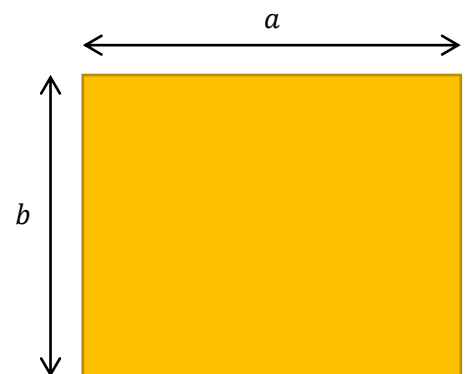
Formulae

A **formula** tells us how to calculate the value of something.

For example, the formula $P = 2a + 2b$ tells us how to calculate the perimeter, P , of the rectangle shown on the right. (The perimeter of a shape is the distance around the shape.)

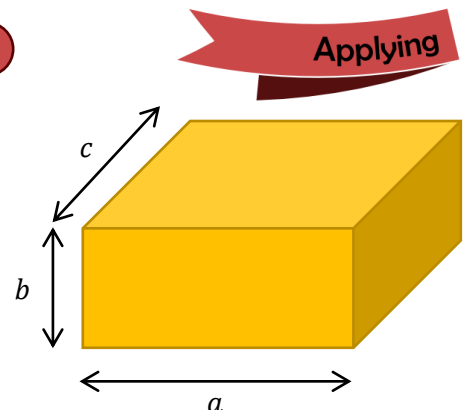
If $a = 4$ cm, $b = 7$ cm, then the perimeter of the rectangle is

$$\begin{aligned} P &= 2 \times 4 + 2 \times 7 \\ P &= 8 + 14 \\ P &= 22 \text{ cm} \end{aligned}$$

**Exercise 16**

The formula $C = abc$ gives the volume of any cuboid with the measurements shown on the diagram to the right. Use this formula to find the volume of cuboids with the following measurements.

- (a) $a = 3$ cm, $b = 4$ cm, $c = 2$ cm.
 (b) $a = 4$ cm, $b = 5$ cm, $c = 6$ cm.
 (c) $a = 7$ cm, $b = 3$ cm, $c = 4$ cm.

**Exercise 17**

It is believed that the formula $s = \frac{28-a}{2}$ gives the number of hours of sleep a child needs. s is the number of hours of sleep, and a is the age of the child, in years.

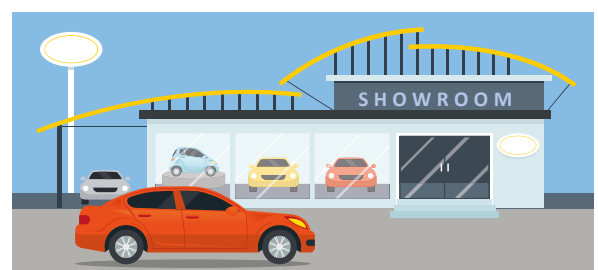
Copy and complete the table below.

a	2	4	6	8	10	12
s						

**Exercise 18**

The basic salary for a car salesperson is £300 a week plus £100 for every car sold. The formula $C = 300 + 100n$ gives the weekly salary, where n represents the number of cars sold. Calculate the weekly wage if the car salesperson sells

- (a) 4 cars
 (b) 3 cars
 (c) No cars
 (d) 1 car?



Exercise 19

It is possible to stack coffee cups on top of each other as shown to the right.
The height of n cups stacked on top of each other is given by the formula
 $H = 1.2n + 8$. The formula gives the height, H , in centimetres.

Use the formula to find the height of

- (a) 2 coffee cups (b) 5 coffee cups (c) 25 coffee cups
stacked on top of each other.

- (d) Jane has a pile of coffee cups stacked on top of each other. The height of the stack is 21.2 cm.
How many coffee cups are in the stack?

**Exercise 20**

Albert Einstein's equation for the amount of Energy E equivalent to a mass m is

$$E = mc^2$$

where c represents the speed of light, which is 299,792,458 metres per second.

Use your calculator to calculate how much energy is equivalent to a 2.5 kg block of gold.
(The units of energy are Joules, J.)

**Exercise 21 (Revision)**

- (a) Substitute the values $x = 5$, $y = 4$, $z = 15$ into the following algebraic expressions.

(i) $3x + 2y$

(ii) $3x^2$

(iii) $\frac{z}{x}$

(iv) $3xy$

(v) $2z - 3x$

- (b) The formula

$$\text{Cost} = 175 + 90n$$

gives the cost (in £) for renting each of the cars that Steffan will need for his wedding. n is the number of hours needed to borrow the cars. What will be the price to rent each car for 3 hours?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Collecting Like Terms

Sometimes it is possible to **simplify** an expression written in algebra, so that we write the expression in a more concise or more simple manner. One way to simplify an algebraic expression is to **collect like terms**.



Collecting like terms including one variable

Consider the following two algebraic expressions.

$$2x + 3x$$

$$5x$$

The values of $2x + 3x$ and $5x$ will be the same, no matter what the value of x is. For example, if $x = 4$, then

$$\begin{aligned} 2x + 3x &= 2 \times 4 + 3 \times 4 & \text{and} & & 5x &= 5 \times 4 \\ &= 8 + 12 & & & &= 20 \\ &= 20 \end{aligned}$$

It follows that the expressions $2x + 3x$ and $5x$ are **equivalent** expressions, and since $5x$ is more concise than writing $2x + 3x$, we say that $5x$ is the simpler version. Therefore, every time we see $2x + 3x$ written in algebra, it is possible to **simplify** it to $5x$. We say that we **collect the terms** $2x$ and $3x$ to give $5x$, and write

$$2x + 3x = 5x$$

Other examples of collecting like terms

$$5n + 7n = 12n$$

$$\begin{aligned} 4x + x &= 4x + 1x \\ &= 5x \end{aligned}$$

$$8y - 3y = 5y$$

$$\begin{aligned} 8z + 3z - 5z &= 11z - 5z \\ &= 6z \end{aligned}$$

Exercise 22

Simplify the following expressions.

(a) $2x + 4x$

(d) $7x + x$

(g) $7z + 2z + 8z$

(j) $a + a + a + a + a + a$

(m) $28y + 4y - 2y$

(p) $18h + 4h + 5h - 6h + h$

(s) $5k + 3k - 2k - 5k$

(v) $9x^2 - 4x^2$

(y) $83v - 26v + 17v$

(b) $8x - 3x$

(e) $8y - y$

(h) $10s + 5s - 2s$

(k) $d + 2d + d + d + 3d - d$

(n) $2x - 5x$

(q) $18t + t - 4t + 2t + t - 3t$

(t) $3x - 3x$

(w) $2xy + 3xy$

(z) $x + x + x - x + x - x + x - x$

(c) $3y + 4y$

(f) $4c + 3c + 2c$

(i) $23r + 2r + 9r$

(l) $3e + e + e + 2e + e$

(o) $12q - 3q + 6q$

(r) $2p + 4p - p + 3p - p$

(u) $4x^2 + 3x^2$

(x) $23g + 5g - 21g - g$

(α) $5y - 3y + 99y - 9y + 14y$

Skill

5

Collecting like terms including more than one variable

It is possible to simplify algebraic expressions that include more than one variable, but we must keep the terms with **different variables separate**. For example, consider the expression $4x + 3y + 5x + 7y$. Here, it is possible to collect the x terms to give $4x + 5x = 9x$, and collect the y terms to give $3y + 7y = 10y$. The final answer, after simplifying, is

$$\begin{aligned} 4x + 3y + 5x + 7y &= 4x + 5x + 3y + 7y \\ &= 9x + 10y \end{aligned}$$

Other examples for collecting like terms

$$\begin{aligned} 3x + 7y + x - 4y &= 3x + 1x + 7y - 4y \\ &= 4x + 3y \end{aligned}$$

$$\begin{aligned} 5p - 4q - p + 6q &= 5p - 1p - 4q + 6q \\ &= 4p + 2q \end{aligned}$$

Exercise 23

5

Choose the correct answer for simplifying the following algebraic expressions.

- | | | | |
|-------------------------|-----------------|-----------------|------------------|
| (a) $4x + 2x + 3y + 5y$ | (i) $14xy$ | (ii) $6x + 8y$ | (iii) $8x + 15y$ |
| (b) $4x + 2n + 2n + 6x$ | (i) $10x + 4n$ | (ii) $24x + 4n$ | (iii) $14xn$ |
| (c) $9b + 5a - 6a - 3b$ | (i) $12b + 11a$ | (ii) $6b + 11a$ | (iii) $6b - a$ |
| (d) $4b - 4c + 3c - b$ | (i) $3b - c$ | (ii) $3b - 7c$ | (iii) $3b + c$ |
| (e) $7x - 4a - 6x - 3a$ | (i) $13x + 7a$ | (ii) $x + 7a$ | (iii) $x - 7a$ |
| (f) $4n - 3d + 8n - 3d$ | (i) $12n + 6d$ | (ii) $12n - 6d$ | (iii) $12n$ |
| (g) $6a + 3b - 4a - 2b$ | (i) $2a + b$ | (ii) $10a + 5b$ | (iii) $2a - b$ |

Exercise 24

5

Simplify the following algebraic expressions.

- | | | |
|----------------------------------|-------------------------------------|---------------------------------|
| (a) $4x + 5x + 3y + 8y$ | (b) $6x + 2y + 3x + 4y$ | (c) $7x + 3y + x + 2y$ |
| (d) $10x + 5y - 3x - 2y$ | (e) $2x + 3y + 4x + 7x$ | (f) $8x + 10y - 7x + 2y$ |
| (g) $4g + 3d + 6g + 18d$ | (h) $12f + 3g + 5h + 4f + 6g + 9h$ | (i) $2p + 4q + p + p + q$ |
| (j) $34u + 2v - 10u + 9v$ | (k) $23x - 3y + 4x + 9y$ | (l) $x + y + x + x + y + y$ |
| (m) $12a + 6b + c + 2a - 4b - c$ | (n) $12d + d + 3d - 2d$ | (o) $4x^2 + 3y^2 + 2x^2 + 7y^2$ |
| (p) $4xy + 3xz + 2xy + 5xz$ | (q) $8r + 8s - 4r + 4s$ | (r) $-3x - 4y + 5x + 9y$ |
| (s) $-6x - 7y - 2x + 2y$ | (t) $-6x - 20y - 4x - 5y$ | (u) $10m + 14n - 4n + 5m$ |
| (v) $14x + 2y + 3x + 5y$ | (w) $14x + 3 + 3x$ | (x) $5x + 4y + 3z$ |
| (y) $2 + 4x + 4 + 2x$ | (z) $5x^2 + 3x + 5 + 2x^2 + 5x + 3$ | (α) $-y^2 + y^2$ |

Exercise 25 (Revision)

5

Simplify the following algebraic expressions.

- (a) $4x + 3x$
 (d) $8x^2 + 3x^2$
 (g) $7x + 3y + 8x + 2y$
 (j) $a + a + b + b + a$
 (m) $10s - 3t + 2s + 7t$
 (p) $16x + 4y - 12x + 3y - 4x$
 (s) $8d - 7e + 10d + 15e - d$
 (v) $4x + 3 + 6y - 2 + 9y + 6x$
 (y) $14xy + 10x + 3$
- (b) $2y + 5y + y$
 (e) $6ab + 4ab$
 (h) $4x + 3y + x + 2y$
 (k) $d + e + d + f + e + d + f + d$
 (n) $-2g - 2h - 4g - 6h$
 (q) $15x + 12y + 2x + 5y - 7y$
 (t) $-10u + 14v + 14u - 10v$
 (w) $4x^2 - 5x + 2 + 2x^2 - 7x - 8$
 (z) $18c + 6b + 2a - 7b - 4a - 3c$
- (c) $5z + 2z - 3z$
 (f) $ha + ha$
 (i) $6x + 2y - 2x + 8y$
 (l) $12p + 4q + 3p - 2q$
 (o) $12x - 3y - 15x - 5y$
 (r) $14p + 3p + 20q + 6q$
 (u) $5 + 4x + 4 + 9x$
 (x) $3x - 3x + 3x - 3x + 3x - 3x$
 (α) $2x + 7x + 3x - 11x$

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Solving Equations

In a **formula** like $P = 2a + 2b$, we can find several different values for P , depending upon the values of a and b . In an **equation** like $2x + 4 = 16$, there is only one possible value for x , and it's possible to **solve** the equation to find this value.

Examples of formulae

$$P = 2a + 2b$$

$$C = abc$$

$$E = mc^2$$

Examples of equations

$$2x + 4 = 16$$

$$5x + 7 = 3x + 27$$

$$\frac{x}{2} = 8$$

Notice that several variables appear in a formula, whereas only one variable appears in the above equations.



Solving simple equations

It is possible to solve some equations using only one calculation step. This is true for the examples below.

$$x + 4 = 9$$

$$x - 3 = 6$$

$$3x = 21$$

$$\frac{x}{2} = 4$$

"Which number add four gives the answer nine?"

This number is $9 - 4 = 5$, therefore the solution to this equation is

$$x = 5$$

"Which number take away three gives the answer six?"

This number is $6 + 3 = 9$, therefore the solution to this equation is

$$x = 9$$

"Three multiplied by which number gives the answer twenty-one?"

This number is $21 \div 3 = 7$, therefore the solution to this equation is

$$x = 7$$

"Which number divided by two gives the answer four?"

This number is $2 \times 4 = 8$, therefore the solution to this equation is

$$x = 8$$

Exercise 26

Solve the following equations.

(a) $x + 6 = 8$

(b) $x - 2 = 4$

(c) $4x = 24$

(d) $\frac{x}{2} = 6$

(e) $x + 13 = 20$

(f) $x - 4 = 9$

(g) $3x = 12$

(h) $\frac{x}{3} = 9$

(i) $x + 3 = 18$

(j) $x - 7 = 15$

(k) $5x = 35$

(l) $\frac{x}{2} = 10$

(m) $y + 2 = 7$

(n) $y - 4 = 10$

(o) $6y = 18$

(p) $\frac{y}{4} = 8$

(q) $z + 1 = 8$

(r) $z - 3 = 23$

(s) $2z = 18$

(t) $\frac{z}{5} = 10$

(u) $x + 5 = 17$

(v) $x - 10 = 40$

(w) $7x = 56$

(x) $\frac{x}{6} = 2$

(y) $x + 0.5 = 5$

(z) $x - 2.5 = 6$

(α) $2x = 15$

(β) $\frac{x}{2} = 5$



Exercise 27

Write an equation for each of the following. In each case, let x represent the number to be found.

(a) When a number is multiplied by 3, the answer is 15.

(b) When a number is divided by 5, the answer is 2.

(c) Two less than a number is 7.

(d) Three more than a number is 15.

(e) Two times the length of a table is 120 cm.

(f) Half the weight of a cake is 400 g.



Exercise 28

6

Solve the equations you wrote in Exercise 27.

Solving More Complex Equations

How can we solve the following equation?

$$2x + 3 = 11$$

Method 1: The Balancing Method

We can think of the above equation as being set out on the scales shown on the right. The scales are balanced, which shows that the left-hand side of the equation ($2x + 3$) is equal to the right-hand side of the equation (11). The problem is to find the weight of one of the green boxes, which is the value of x .

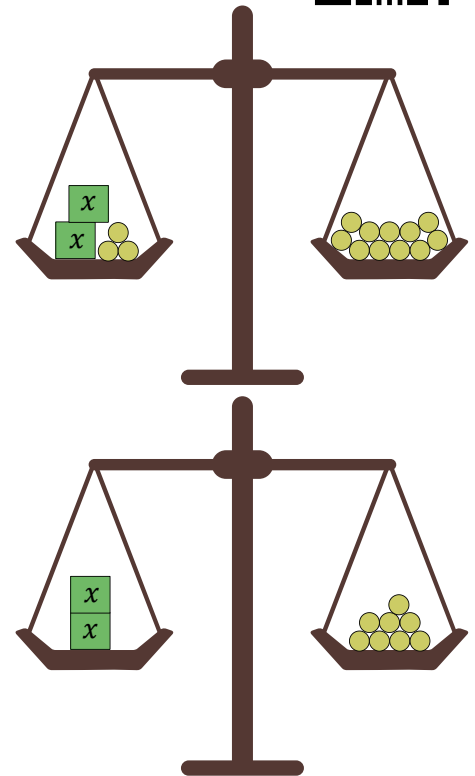
To keep the scales balanced (and keep the equation true), we must do **the same thing** to both sides of the scales. Therefore, if we want to take away three yellow circles from the left side of the scales, we must also remove three yellow circles from the right side. By doing this, the scale will still be balanced, **and more importantly** the equation will be simpler.

$$2x = 8$$

This equation is of a similar type to the ones seen on the previous page. We can solve it by asking “two times which number gives the answer eight?” The answer is 4, so the solution to the equation is

$$x = 4$$

On the scales, we can say that one of the green boxes weighs the same as four of the yellow circles.



Fact: An equation remains true if the same number is added (or subtracted) from both sides or if both sides are multiplied or divided by the same number.

Method 2: The Jumping Method

In this method, we move one of the terms in an equation to the other side of the equation, to reach a situation where it is possible to simplify the equation. For the equation in the example, $2x + 3 = 11$, it is possible to move the 3 on the left side of the equation to be on the right side of the equation. When doing this, we must remember to **change the term's sign**, so that the +3 on the left side of the equation becomes -3 on the right side of the equation.

$$2x = 11 - 3$$

We can now see there is a way to simplify the equation, since $11 - 3 = 8$. Therefore the equation simplifies to be

$$2x = 8$$

which can be solved (like before) to give

$$x = 4$$



Rule: When jumping terms in an equation from one side to the other, you must change the sign. “Change sides, change sign”.

Another ExampleSolve the equation $3x + 5 = 29$.**The Balancing Method**

$$3x + 5 = 29$$

$$3x = 24 \quad [\text{Subtract 5 from both sides}]$$

$$x = 8 \quad [\text{Divide by 3}]$$

The Jumping Method

$$3x + 5 = 29$$

$$3x = 29 - 5 \quad [\text{Jump the +5 to be -5}]$$

$$3x = 24 \quad [\text{Simplify}]$$

$$x = 8 \quad [\text{Divide by 3}]$$

Exercise 29

Solve the following equations.

(a) $2x + 4 = 16$

(d) $3x + 8 = 44$

(g) $2x + 1 = 16$

(j) $7x + 3 = 45$

(m) $10x + 3 = 73$

(p) $2y + 3 = 5$

(s) $\frac{x}{2} + 3 = 7$

(v) $14 = 2x + 2$

(b) $2x + 5 = 19$

(e) $4x + 1 = 21$

(h) $5x + 4 = 34$

(k) $8x + 3 = 35$

(n) $11x + 2 = 79$

(q) $2y + 6 = 13$

(t) $\frac{x}{3} + 5 = 8$

(w) $34 = 3x + 1$

(c) $3x + 2 = 20$

(f) $4x + 3 = 31$

(i) $6x + 9 = 57$

(l) $9x + 5 = 77$

(o) $12x + 1 = 61$

(r) $5y + 7 = 17$

(u) $\frac{x}{4} + 9 = 17$

(x) $50 = 5x + 10$

ExampleSolve the equation $2x - 6 = 26$.**The Balancing Method**

$$2x - 6 = 26$$

$$2x = 32 \quad [\text{Add 6 to both sides}]$$

$$x = 16 \quad [\text{Divide by 2}]$$

The Jumping Method

$$2x - 6 = 26$$

$$2x = 26 + 6 \quad [\text{Jump the -6 to be +6}]$$

$$2x = 32 \quad [\text{Simplify}]$$

$$x = 16 \quad [\text{Divide by 2}]$$

Exercise 30

Solve the following equations.

(a) $2x - 1 = 15$

(d) $3x - 4 = 17$

(g) $6x - 9 = 51$

(j) $9x - 14 = 22$

(m) $2y - 4 = 24$

(p) $\frac{x}{2} - 4 = 6$

(s) $42 = 2x - 4$

(v) $20 = \frac{x}{2} - 2$

(b) $2x - 6 = 22$

(e) $4x - 5 = 23$

(h) $7x - 7 = 56$

(k) $10x - 25 = 75$

(n) $3z - 5 = 34$

(q) $\frac{x}{2} - 6 = 12$

(t) $6 = 3x - 18$

(w) $15 = \frac{y}{3} - 4$

(c) $2x - 9 = 11$

(f) $5x - 2 = 43$

(i) $8x - 2 = 46$

(l) $11x - 2 = 64$

(o) $4w - 8 = 44$

(r) $\frac{x}{3} - 2 = 9$

(u) $19 = 5x - 16$

(x) $70 = \frac{z}{4} - 30$

Exercise 31

Solve the following equations.

(a) $2x + 5 = 29$

(d) $3x - 9 = 6$

(g) $4y - 1 = 55$

(j) $\frac{z}{3} - 4 = 2$

(b) $2x - 3 = 41$

(e) $4x + 3 = 27$

(h) $5y + 2 = 57$

(k) $\frac{w}{2} + 3 = 5$

(c) $2x + 7 = 37$

(f) $5x - 9 = 36$

(i) $6y - 19 = 17$

(l) $14 = \frac{x}{5} + 3$

Example

Solve the equation $5x + 2 = 2x + 29$.

The Balancing Method

$$\begin{aligned} 5x + 2 &= 2x + 29 \\ 5x &= 2x + 27 && \text{[Subtract 2 from both sides]} \\ 3x &= 27 && \text{[Subtract } 2x \text{ from both sides]} \\ x &= 9 && \text{[Divide by 3]} \end{aligned}$$

The Jumping Method

$$\begin{aligned} 5x + 2 &= 2x + 29 \\ 5x &= 2x + 29 - 2 && \text{[Jump the } +2 \text{ to be } -2\text{]} \\ 5x &= 2x + 27 && \text{[Simplify]} \\ 5x - 2x &= 27 && \text{[Jump the } 2x \text{ to be } -2x\text{]} \\ 3x &= 27 && \text{[Simplify]} \\ x &= 9 && \text{[Divide by 3]} \end{aligned}$$

Exercise 32

Solve the following equations.

- | | | |
|-------------------------|----------------------------|----------------------------|
| (a) $5x + 3 = 2x + 21$ | (b) $4x + 1 = 2x + 19$ | (c) $6x + 3 = 4x + 35$ |
| (d) $5x + 9 = 3x + 39$ | (e) $5x + 9 = x + 25$ | (f) $7x + 10 = 4x + 34$ |
| (g) $10x + 8 = 4x + 44$ | (h) $11x + 2 = 6x + 47$ | (i) $2x + 3 = x + 9$ |
| (j) $5y + 4 = 3y + 26$ | (k) $7z + 9 = 4z + 39$ | (l) $23w + 3 = 13w + 79$ |
| (m) $6x + 6 = 4x + 11$ | (n) $8x + 2.5 = 3x + 17.5$ | (o) $4.6y + 2 = 2.6y + 46$ |

Example

Solve the equation $7x - 3 = 5x + 19$.

The Balancing Method

$$\begin{aligned} 7x - 3 &= 5x + 19 \\ 7x &= 5x + 22 && \text{[Add 3 to both sides]} \\ 2x &= 22 && \text{[Subtract } 5x \text{ from both sides]} \\ x &= 11 && \text{[Divide by 2]} \end{aligned}$$

The Jumping Method

$$\begin{aligned} 7x - 3 &= 5x + 19 \\ 7x &= 5x + 19 + 3 && \text{[Jump the } -3 \text{ to be } +3\text{]} \\ 7x &= 5x + 22 && \text{[Simplify]} \\ 7x - 5x &= 22 && \text{[Jump the } 5x \text{ to be } -5x\text{]} \\ 2x &= 22 && \text{[Simplify]} \\ x &= 11 && \text{[Divide by 2]} \end{aligned}$$

Exercise 33

Solve the following equations.

- | | | |
|---------------------------|--------------------------|----------------------------|
| (a) $7x - 2 = 5x + 24$ | (b) $6x - 3 = 4x + 5$ | (c) $9x - 2 = 6x + 13$ |
| (d) $7x - 4 = 3x + 20$ | (e) $9x - 5 = 2x + 23$ | (f) $10x - 2 = 2x + 38$ |
| (g) $12y - 3 = 2y + 77$ | (h) $14z - 12 = 2z + 24$ | (i) $8z - 1 = z + 6$ |
| (j) $20x - 13 = 13x + 22$ | (k) $15y - 4 = 4y + 51$ | (l) $19z - 14 = 4z + 31$ |
| (m) $6x - 2 = 4x - 2$ | (n) $2x - 1.4 = x + 2.6$ | (o) $5.6z - 3 = 0.6z + 12$ |

Exercise 34

Solve the following equations.

- | | | |
|---------------------------|------------------------|----------------------------|
| (a) $6x + 2 = 4x + 6$ | (b) $5x - 2 = 3x + 20$ | (c) $6x - 1 = 3x + 20$ |
| (d) $8x + 4 = 3x + 54$ | (e) $2x - 5 = x + 16$ | (f) $8x + 15 = 6x + 17$ |
| (g) $4x + 6 = 2x + 7$ | (h) $10x - 2 = 6x + 6$ | (i) $9x + 1 = 5x + 2$ |
| (j) $12z - 3 = 9z + 39$ | (k) $4p - 6 = 2p - 2$ | (l) $100e + 20 = 80e + 60$ |
| (m) $78t - 4 = 28t + 146$ | (n) $y + 7 = 0.5y + 9$ | (o) $64v - 3 = 24v + 117$ |

Exercise 35

6

Solve the following equations.

(a) $4x = 2x + 6$

(b) $7x = 3x + 32$

(c) $9x = 3x + 66$

(d) $10x = 2x + 56$

(e) $11x = 4x + 63$

(f) $3x = x + 18$

(g) $5x = 2x + 39$

(h) $14x = 8x + 54$

(i) $13x = 4x + 72$

(j) $5x - 6 = 3x$

(k) $7x - 20 = 3x$

(l) $10x - 35 = 5x$

(m) $12x - 40 = 10x$

(n) $11x - 10 = 10x$

(o) $15x - 40 = 10x$

Exercise 36

6

Write an equation for each of the following.

In each case, let x represent the number to be found.

(a) Two times a number plus three is equal to 15.

(b) The result of subtracting 5 from three times a number is 19.

(c) Subtracting 4 from double a number is the same as adding 7 to the number.

(d) The result of adding 8 to three times a number is the same as adding 12 to two times the same number.

(e) Six times a number subtract seven is equal to 59.

Exercise 37

Solve the equations you wrote in Exercise 36.

Exercise 38: Problem Solving

(a) In a boat race, the length of the second leg of the race is twice the length of the first leg. The total length of both legs is 204 km. What is the length of the first leg?

(b) Alaw got 17 less marks in her History exam than in her Maths exam. If Alaw got a total of 151 marks in the two exams, how many marks did she get in the Maths exam?

(c) Two lorries arrived to move a load of pebbles. One of the lorries can carry three times more pebbles than the other. Together the two lorries can carry a total of 16 m^3 pebbles. How many pebbles can the smaller lorry carry?

(d) The diagram on the right shows a square. Find the length of the square's side.

(e) The lengths, in centimetres, of the five sides of a pentagon are

x

$x + 2$

$2x$

$3x + 5$

$4x$

The perimeter of the pentagon is 95 centimetres.

Form an equation in terms of x and solve it to find the value of x .(f) In a quiz night, one of the questions was a puzzle to find the values of the letters a , b and c . The letters represent different values and the sum of each row is given in the last column. Using the information given in the table, find the values of each of the letters.

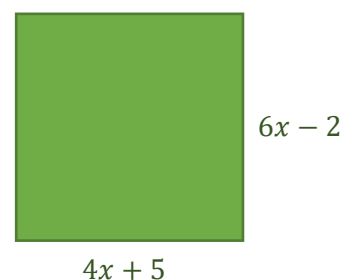
	a	a	18
	a	b	20
a	b	c	17



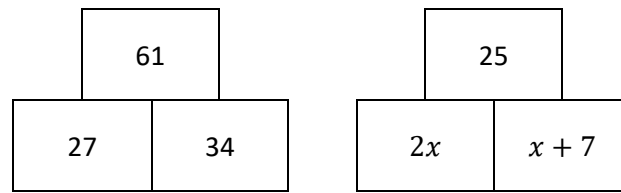
6

Applying

6

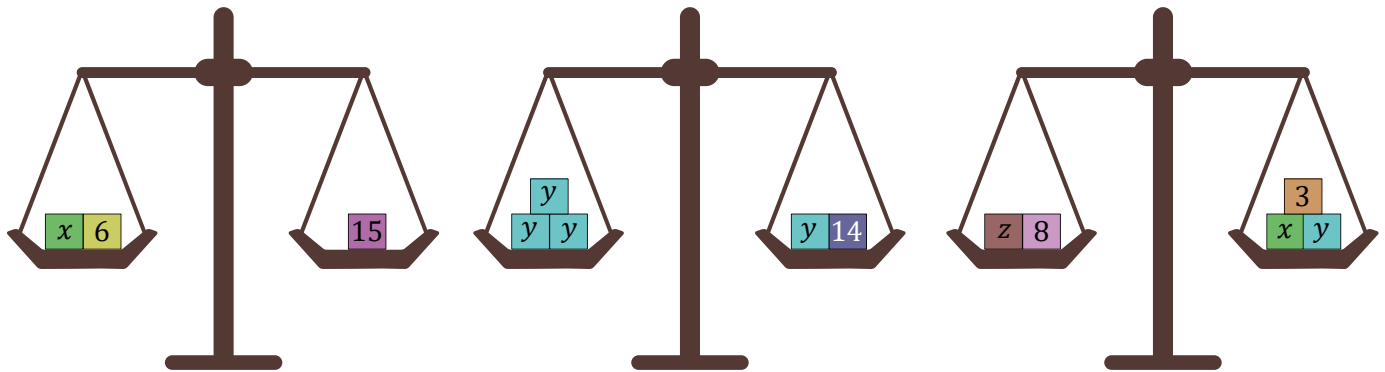


(g)



In the above diagrams, the value of the higher box is found by finding the total of the two boxes beneath. For the second diagram, form and solve an equation to find the value of x .

(h)



Each diagram represents scales where the sum of the weight on both sides is equal. Find the values of x , y and z .

Exercise 39 (Revision)

6

(a) Solve the following equations.

(i) $x + 13 = 19$

(ii) $3x = 27$

(iii) $y - 3 = 20$

(iv) $\frac{z}{4} = 12$

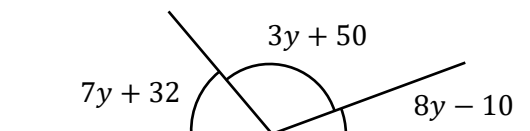
(v) $2x + 7 = 35$

(vi) $6y - 2 = 40$

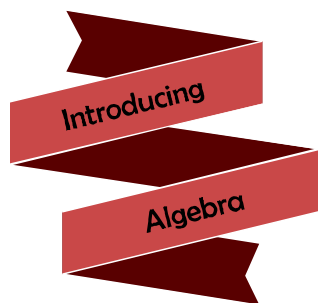
(vii) $7x + 2 = 3x + 46$

(viii) $8y - 5 = 3y + 45$

(b) All of the angles are measured in degrees. Find the size of each of the three angles.

**Evaluation**

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to find the next number in a number sequence .			1	
I know how to explain, in words , how to find the next number in a number sequence.			2	
I know how to form a number sequence from a series of pictures .			3	
I know how to use a number machine .			4	
I know how to draw the reverse of a number machine.			4	
I can form algebraic expressions .			5	
I can write the output of a number machine if the input is a variable such as x .			4	
I can substitute values into an algebraic expression.			6	
I can use a mathematical formula .			7	
I can collect like terms using one variable.			8	
I can collect like terms using more than one variable.			8	
I can solve simple equations .			9	
I can solve more complex equations using either the balancing method or the jumping method .			10	
I can form a mathematical equation and solve it.			11	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

8

Measuring

Shapes

Name:

Contents

Chapter	Mathematics	Page Number
Measures	Measures of Length, Mass and Volume. Reading scales. Conversion graphs.	3
Quadrilaterals	Square; Rectangle; Parallelogram; Rhombus; Trapezium; Kite; Arrowhead. Angles in a quadrilateral.	12
Perimeter and Area	Perimeter of shapes. Area of triangles. Area of quadrilaterals. Estimating the area of irregular shapes.	17
Solids	Names of solids. Nets of solids. Volume of a cuboid.	27



Measures

When measuring different shapes, we must use **units** to be able to compare them.

There have been many unit systems throughout the centuries, including a Welsh system, but by now we use two different systems in this country, the **metric** system and the **Imperial** system.



The metric system:

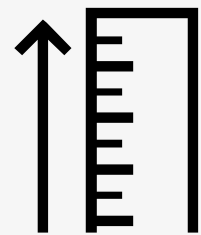
- Published in 1960.
- Used across the world.
- Easy to convert between one unit and another.

The Imperial system:

- Published in 1824.
- Used in the British Empire.
- Being gradually replaced by the metric system.

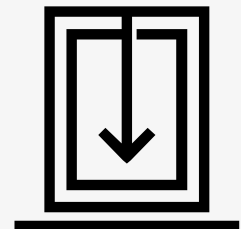
Measures of Length

METRIC	IMPERIAL
• 10 millimetres (mm) = 1 centimetre (cm) • 100 centimetres = 1 metre (m) • 1,000 metres = 1 kilometre (km)	• 12 inches (") = 1 foot (') • 3 feet = 1 yard (yd) • 1,760 yards = 1 miles (mi)
CONVERSIONS	
• 1 inch $\approx$ 2.5 cm • 1 yard $\approx$ 0.9 m • 1 mile $\approx$ 1.6 km (therefore 5 miles $\approx$ 8 km)	



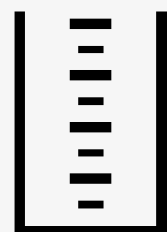
Measures of Mass

METRIC	IMPERIAL
• 1,000 milligrams (mg) = 1 gram (g) • 1,000 grams = 1 kilogram (kg) • 1,000 kilograms = 1 tonne or metric ton (t)	• 16 ounces (oz) = 1 pound (lb) • 14 pounds = 1 stone (st) • 160 stone = 1 ton
CONVERSIONS	
• 1 ounce $\approx$ 30 g • 2.2 pounds $\approx$ 1 kg • 1 ton $\approx$ 1 tonne	



Measures of Volume

METRIC	IMPERIAL
• 1,000 millilitres (ml) = 1 litre (l) • 100 centilitres (cl) = 1 litre	• 20 fluid ounces (fl. oz) = 1 pint (pt) • 8 pints = 1 gallon
CONVERSIONS	
• 1 pint $\approx$ 0.57 litres (therefore 1 litre $\approx$ 1.75 pints) • 1 gallon $\approx$ 4.5 litres	



Exercise 1

Draw a circle around the amount which is the most appropriate estimate for the following.

(a) Length of a football pitch	120 km	120 m	120 mm	120 cm
(b) Weight of a man	80 kg	80 g	80 mg	800 kg
(c) Capacity of a cup	2 litres	10 ml	200 ml	1 ml
(d) Distance from Bangor to Cardiff	270 mm	270 cm	270 m	270 km
(e) Height of the Eiffel Tower	324 mm	324 cm	324 m	324 km
(f) Weight of an apple	90 mg	90 g	90 kg	90 tons
(g) Capacity of an Olympic swimming pool	250 ml	25 litres	2,500 litres	2,500,000 litres
(h) Weight of a car	2 mg	2 g	2 kg	2 tonnes
(i) Diameter of a strand of hair	0.1 mm	0.1 cm	0.1 m	0.1 km

Skill

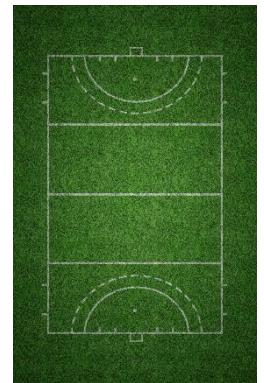
4

Exercise 2

Write the best **metric** unit for measuring

- | | |
|-------------------------------------|--|
| (a) the length of a hockey pitch | (b) the fuel tank capacity of a car |
| (c) the distance from Paris to Rome | (d) the weight of a big dog |
| (e) the volume of water in a bath | (f) the weight of a pocket calculator |
| (g) the diameter of a circular coin | (h) how much liquid a tablespoon holds |

4

**Exercise 3**

Write the best **Imperial** unit for measuring everything in Exercise 2.

4

Exercise 4

Connect each statement with the appropriate measure.

4

Statement
Length of a pen
Height of a door
Weight of a soup tin
Capacity of a can of fizzy drink
Weight of a bag of sugar
How much liquid a teaspoon holds

Measure
1 kg
14 cm
330 ml
420 g
5 ml
2 m

**Puzzle**

A snail who climbs up a 10 m tall building climbs 3 m every night and slips down 2 m every day. After how many nights will the snail have reached the top?

**Challenge!**

Use the internet to find out what the “Gimli Glider” was. What role did measures play in its story?

Exercise 5

5

Convert the units in each of the following.

- | | | |
|-----------------------|----------------------|--------------------------|
| (a) 300 cm in m | (b) 5,000 m in km | (c) 40 mm in cm |
| (d) 700 cm in m | (e) 8,000 m in km | (f) 120 mm in cm |
| (g) 250 cm in m | (h) 7,300 m in km | (i) 75 mm in cm |
| (j) 5 m in cm | (k) 2 km in m | (l) 6 cm in mm |
| (m) 4.5 m in cm | (n) 4.5 km in m | (o) 3.2 cm in mm |
| (p) 3 feet in inches | (q) 4 yards in feet | (r) 2 miles in yards |
| (s) 24 inches in feet | (t) 27 feet in yards | (u) 5,280 yards in miles |



5

Exercise 6

Convert the units in each of the following.

- | | | |
|-------------------------|-------------------------|--------------------------|
| (a) 4,000 g in kg | (b) 5,000 kg in t | (c) 7,000 mg in g |
| (d) 2,000 kg in t | (e) 8,000 g in kg | (f) 10,000 mg in g |
| (g) 6,500 g in kg | (h) 3,200 kg in t | (i) 9,100 mg in g |
| (j) 9 kg in g | (k) 3 t in kg | (l) 4 g in mg |
| (m) 8.2 kg in g | (n) 1.9 t in kg | (o) 2.9 g in mg |
| (p) 3 pounds in ounces | (q) 4 stone in pounds | (r) 5 tons in stones |
| (s) 64 ounces in pounds | (t) 28 pounds in stones | (u) 1,600 stones in tons |



5

Exercise 7

Convert the units in each of the following.

- | | | |
|-----------------------------|------------------------------|-------------------------------|
| (a) 7,000 ml in l | (b) 500 cl in l | (c) 3,000 ml in l |
| (d) 900 cl in l | (e) 4,500 ml in l | (f) 350 cl in l |
| (g) 6 l in ml | (h) 7 l in cl | (i) 7.4 l in ml |
| (j) 3 pints in fluid ounces | (k) 3 gallons in pints | (l) 120 fluid ounces in pints |
| (m) 40 pints in gallons | (n) 30 fluid ounces in pints | (o) 4 pints in gallons |



5

Exercise 8

Fill in the blanks.

- | | | |
|------------------------------------|--------------------------------------|--|
| (a) 2 inches $\approx$ _____ cm | (b) 4 yards $\approx$ _____ m | (c) 2 miles $\approx$ _____ km |
| (d) 7.5 cm $\approx$ _____ inches | (e) 2.7 m $\approx$ _____ yards | (f) 12.8 km $\approx$ _____ miles |
| (g) 4 ounces $\approx$ _____ g | (h) 4 kg $\approx$ _____ pounds | (i) 3 tons $\approx$ _____ tonnes |
| (j) 60 g $\approx$ _____ ounces | (k) 1.1 pounds $\approx$ _____ kg | (l) 15 tonnes $\approx$ _____ tons |
| (m) 3 pints $\approx$ _____ litres | (n) 3 gallons $\approx$ _____ litres | (o) 5 pints $\approx$ _____ litres |
| (p) 2 litres $\approx$ _____ pints | (q) 9 litres $\approx$ _____ gallons | (r) 0.5 gallons $\approx$ _____ fluid ounces |

Puzzle

A motorbike has travelled 30,000 km. Three tyres were used the same amount over this distance. How many kilometres did each tyre travel?

Challenge!

Write an estimate for each of the following.

- (a) 4.3 km in inches
 (b) 1 ton in grams
 (c) 1 gallon in millilitres



Exercise 9: Problem Solving

(a) A biking club arranged a race which took place over two weeks. In the race, the competitors travelled 320 miles in England and 480 kilometres in France. In which country did they travel furthest? What was the difference between these distances?

(b) Jack weighs 122 lb. Find Jack's weight in stones and pounds.

(c) Wayne weighs 10 stone 4 lb. Find Wayne's weight in pounds.

(d) Laura has her own car. In April

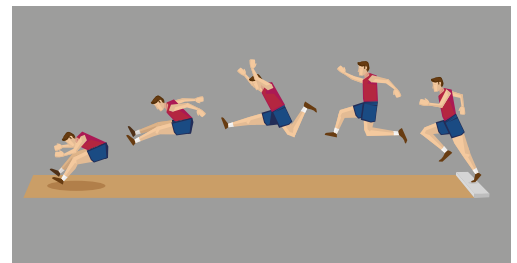
- Laura drove a total of 560 miles in her car.
- Her fuel consumption was 37.8 mpg (miles per gallon).
- The cost of the petrol was £1.48 per litre.

Calculate how much Laura spent on petrol during April.

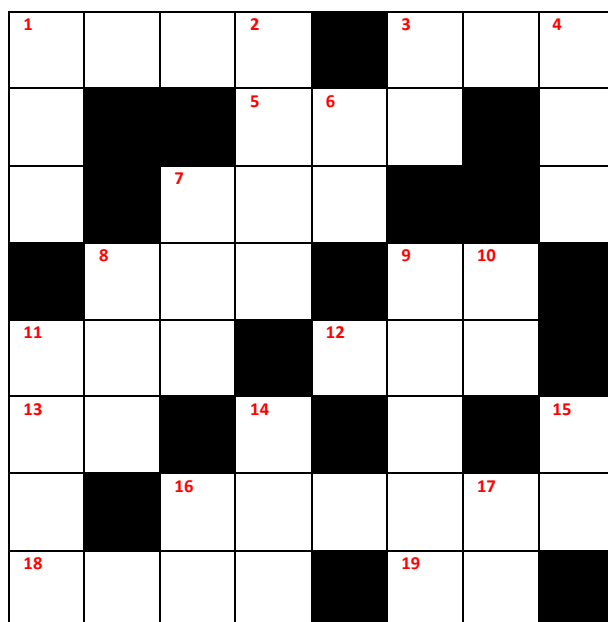
(e) Which is the better bargain? 3 kg of potatoes for 80p, or 5 lb of potatoes for 80p?

(f) Huw is the last competitor in a long jump competition. His best jump so far is 6 metres. He has one jump left. The competitor in the lead has jumped a distance of 6.3 metres. Huw says, "If I jump 4 cm further than my best jump so far, I will win". Is Huw correct or not? You must explain your answer.

(g) Worktops for a kitchen are measured in mm. The length of a worktop is 4,500 mm. What is this in metres?

**Exercise 10**

Complete this crossnumber. If decimal points are required, use a whole square for each one.

**Across**

1. cm in 57.32 m
3. mm in 65.2 cm
5. g in 0.673 kg
7. m in 4 yd
8. mm in 16.7 cm
9. lb in 20 kg
11. cm in 3"
12. litres in 176 gallons
13. cl in 0.26 l
16. yd in 272 mi
18. mg in 9.325 g
19. inches in 8'

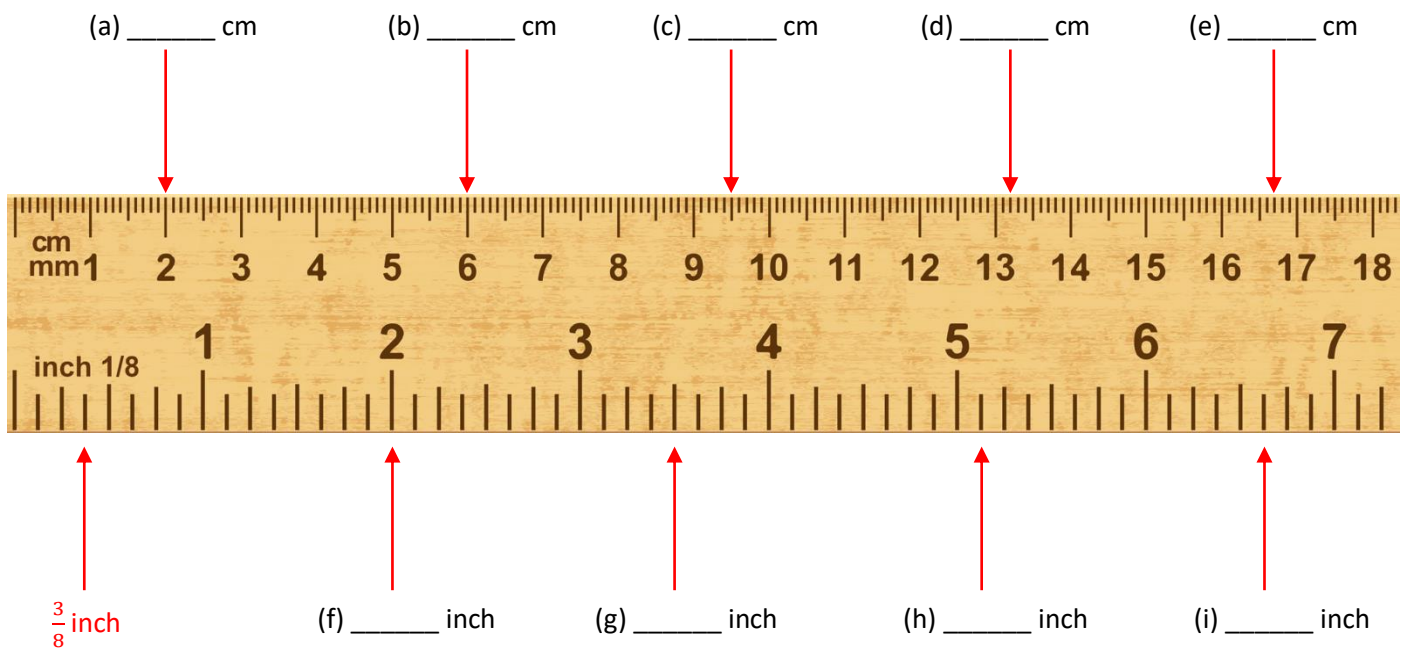
Down

1. kg in 5,300 g
2. cl in 0.267 l
3. feet in 21 yd
4. oz in 14 lb
6. ml in 0.076 l
7. kg in 0.365 t
8. km in 1 mi
9. mg in 49.879 g
10. mm in 4.2 cm
11. kg in 7.279 t
14. m in 0.675 km
15. fl. oz in 2 pt
16. lb in 3 st
17. g in 0.026 kg

Reading Scales

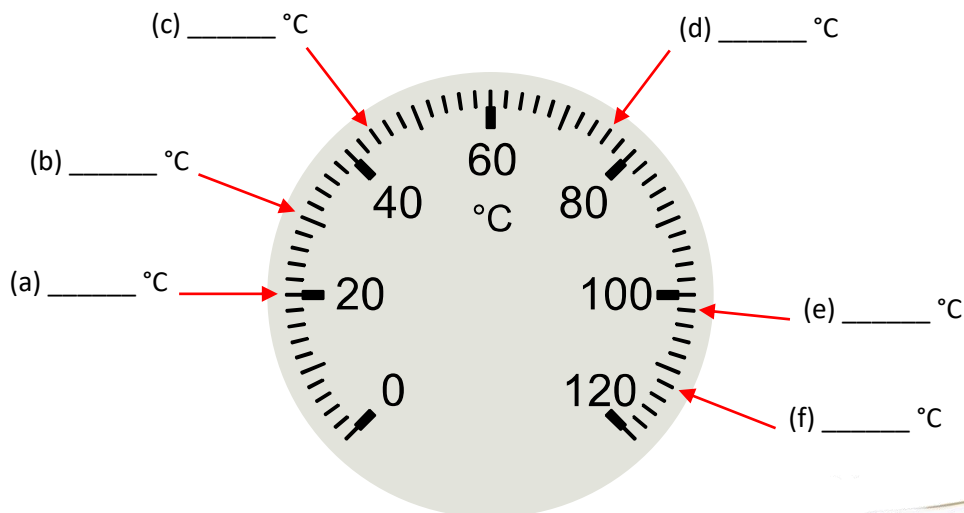
Exercise 11

To which number is the arrow pointing?



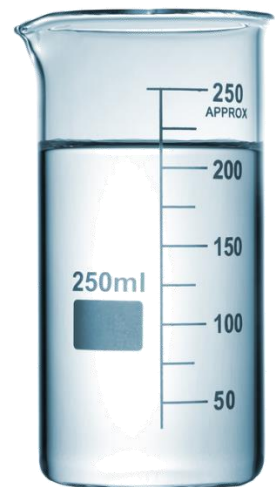
Exercise 12

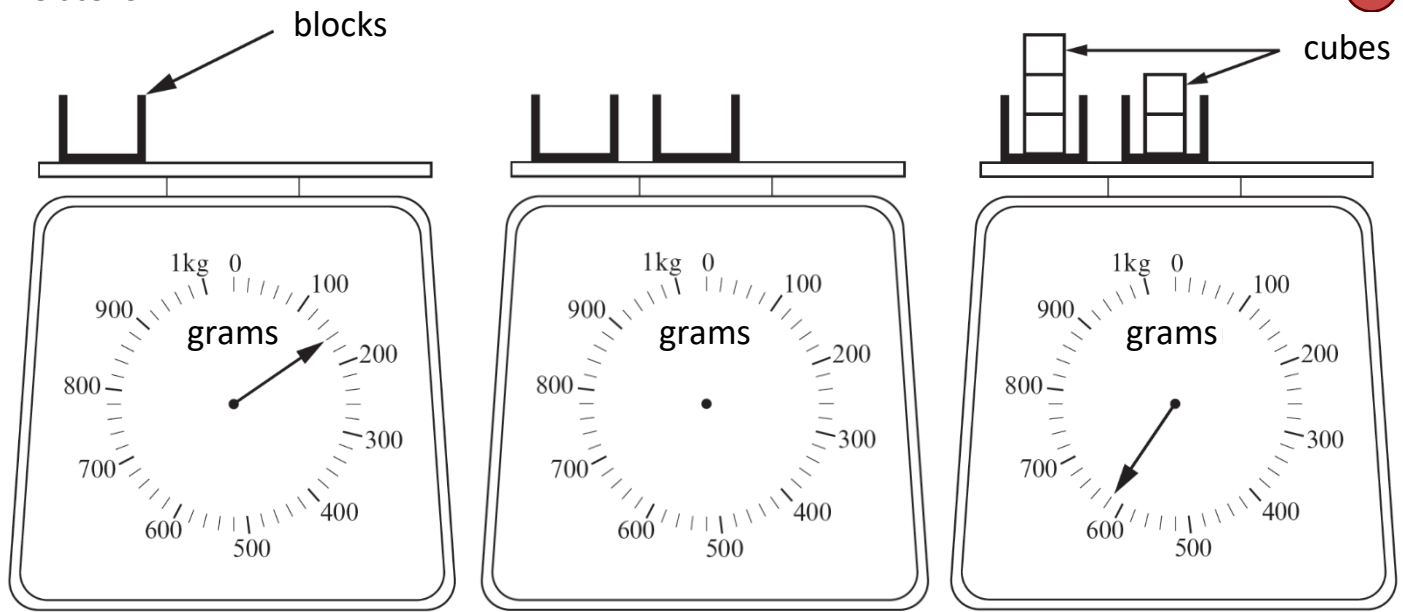
To which number is the arrow pointing?



Exercise 14

- (a) How many millilitres of oil has Sali put in the jug?
- (b) How many fluid ounces of oil is in the jug?
- (c) In reality, Sali put too much oil into the jug. Only a quarter of a litre was required. How many millilitres of oil need to be poured out from the jug?
- (d) Use the jug to estimate how many fluid ounces are in a full litre.
- (e) Which costs more? A litre of milk or two pints of milk?



Exercise 15**5**

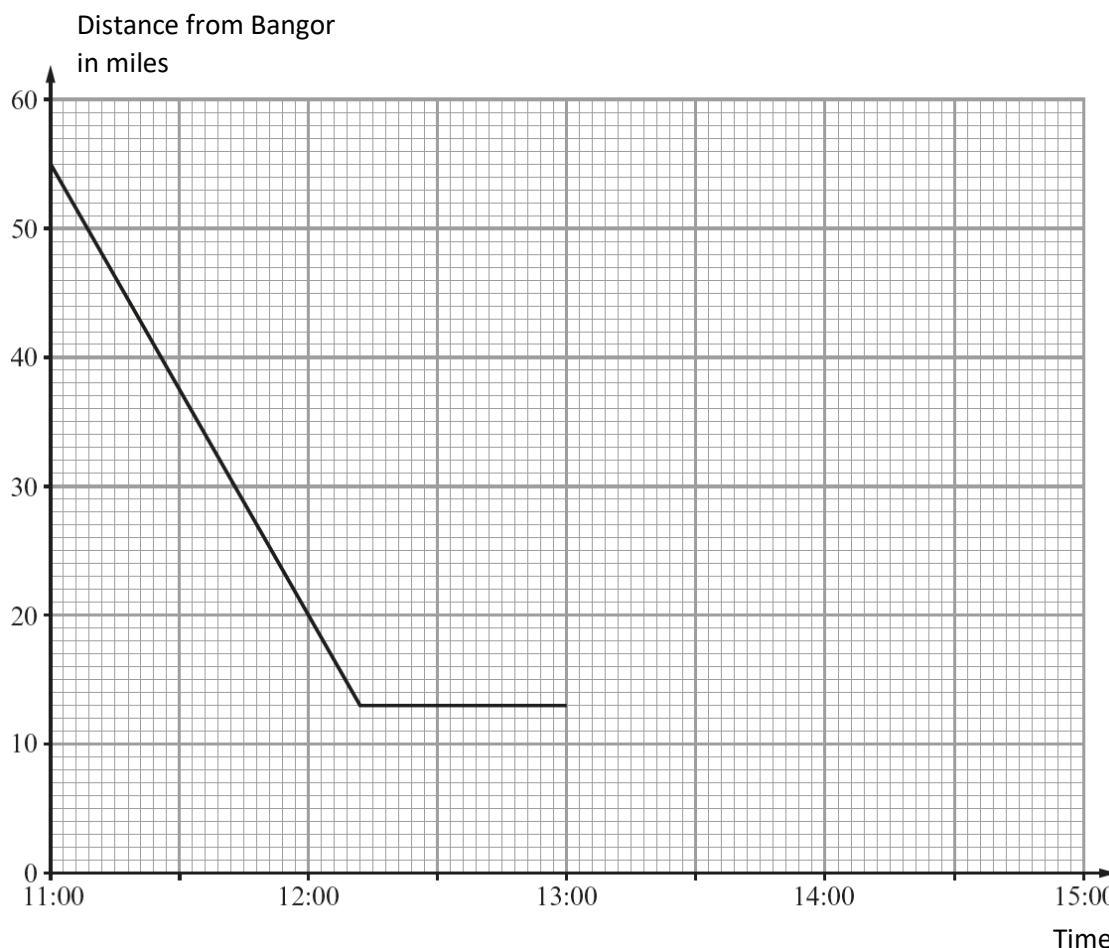
A container is placed onto scales, as shown in the first diagram.

(a) Now two of these containers are placed on the scales, as shown in the second diagram. Draw a pointer on the scales to show how much they weigh.

(b) Now 5 identical cubes are placed in the containers, as shown in the third diagram. Find how much one cube weighs.

Exercise 16**5**

The travel graph below represents part of Nigel's journey.



(a) How far away from Bangor was Nigel at the beginning of his journey?

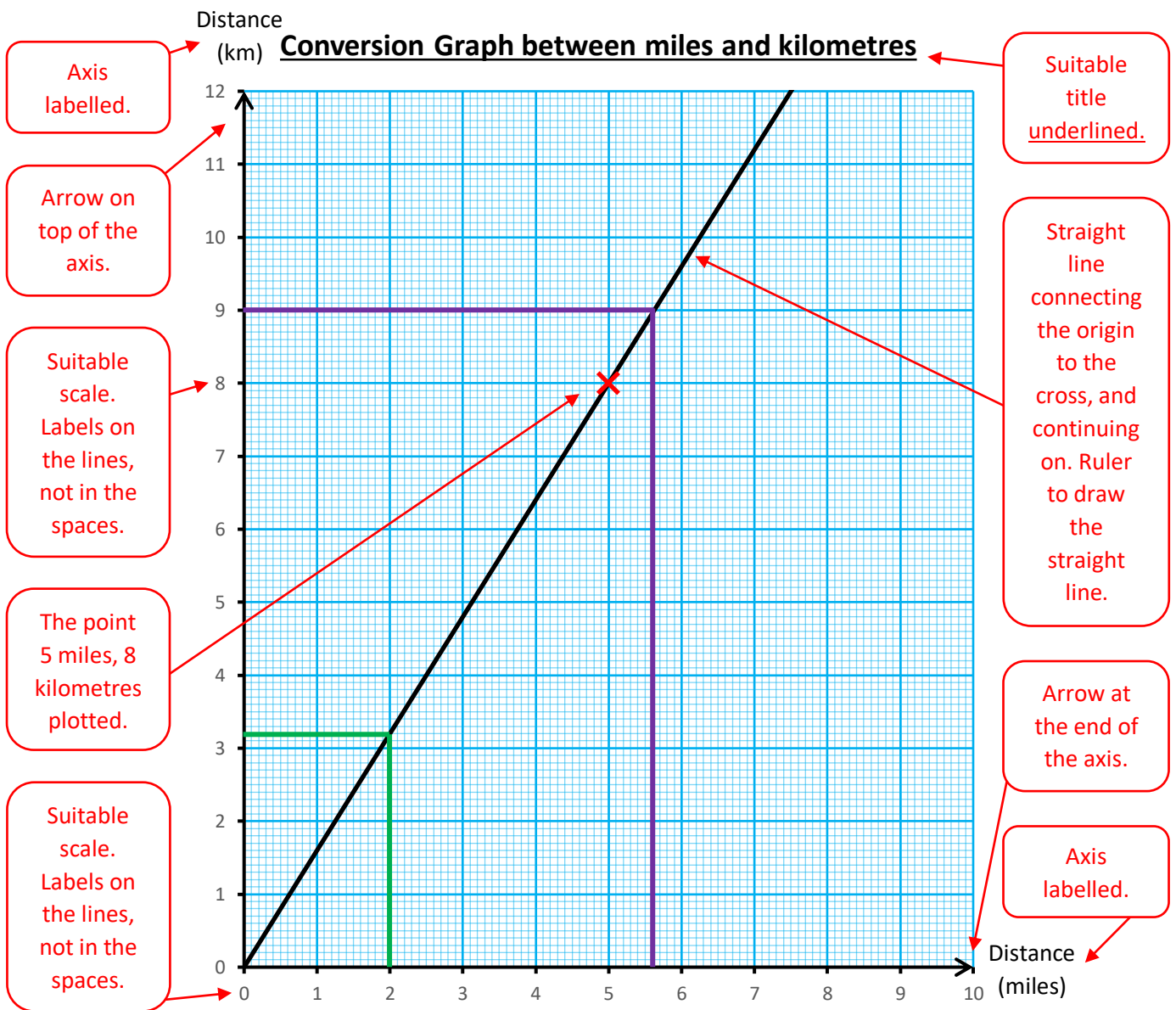
(b) How far did he travel in the first hour?

(c) For how many minutes did he stop during his journey?

(d) After his stop, he continued his journey at a steady speed. By 14:18 he reached a point 48 miles from Bangor. Draw the graph for this part of the journey.

Conversion Graphs

It is possible to draw **conversion graphs** to change from one unit of measure to another. For example, consider changing the measurement of length from miles to kilometres. Earlier, we saw that 5 miles is about 8 km. It is possible to use this fact to draw the following conversion graph.



Once the conversion graph is drawn, we can change between miles and kilometres. For example, the **green** lines show that 2 miles is about 3.2 km, whilst the **purple** lines show that 9 km is about 5.6 miles.

Exercise 17

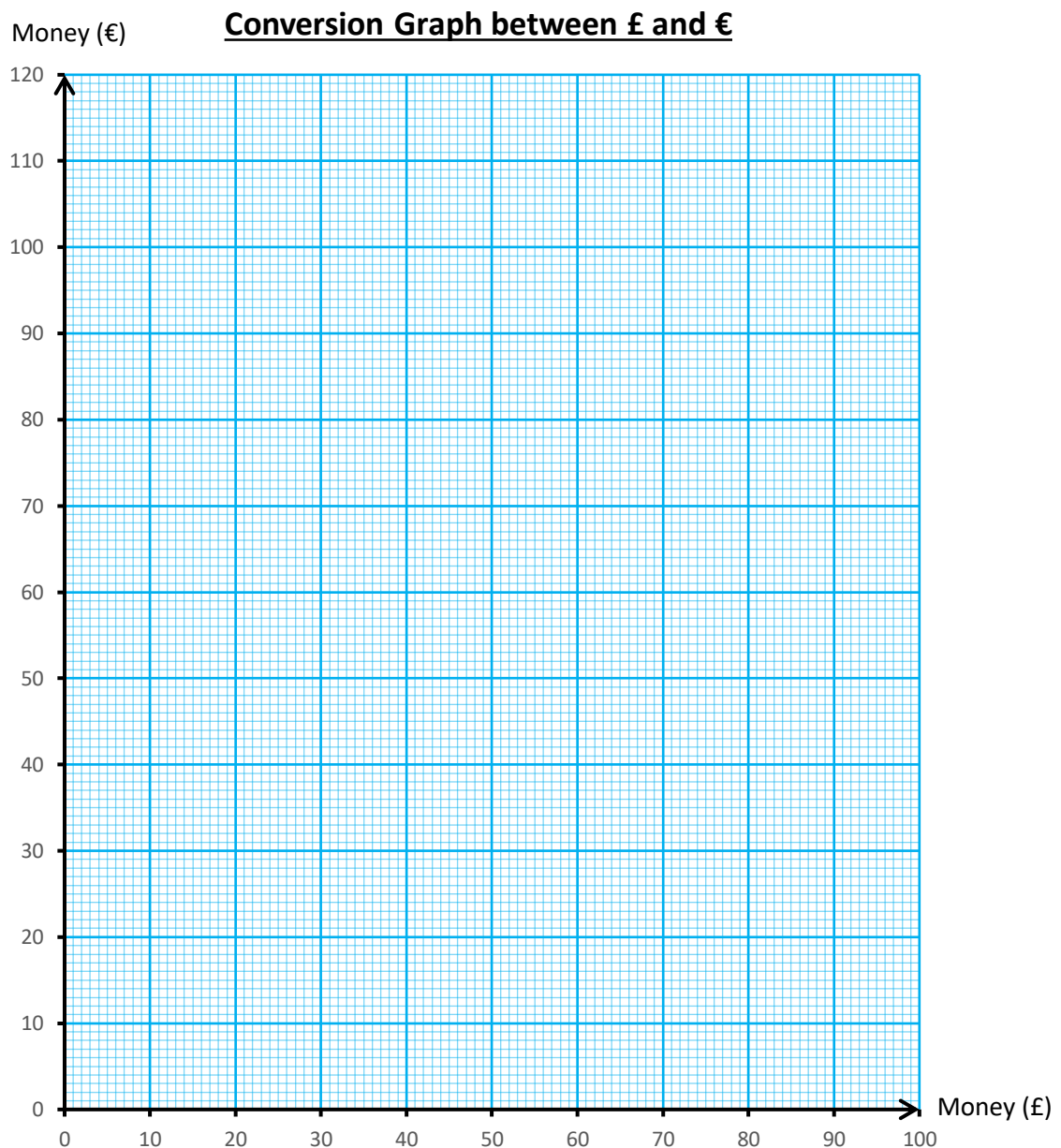
Use the above conversion graph to answer the following questions.

- The distance from Bow Street to Borth Zoo is 3.2 miles.
How far is this in km?
- Catrin's mobile phone shows that she walked 11 km in a day.
How far is this in miles?
- Which is the longest distance: 4 miles or 6.5 km?
- How far is 20 km in miles?



Exercise 18

Jessica is going on holiday to France this summer. Complete the following conversion graph to help Jessica on her holiday, using the exchange rate $\text{£}1 = \text{€}1.18$ (so that $\text{£}50 = \text{€}59$).



Use the above conversion graph to answer the following questions.

- Jessica changes $\text{£}90$ into Euros to spend on her holiday. How many Euros did she get?
- Whilst in France, Jessica bought a train ticket for $\text{€}32$. How much was this in pounds?
- Jessica sees a perfume for $\text{€}24$ in a shop. She knows that the perfume costs $\text{£}18$ at home.
What advice would you give to Jessica?
- After returning home, Jessica had $\text{€}15$ left. She went to the post office to change it back into pounds. How much money did she get?



Exercise 19

5

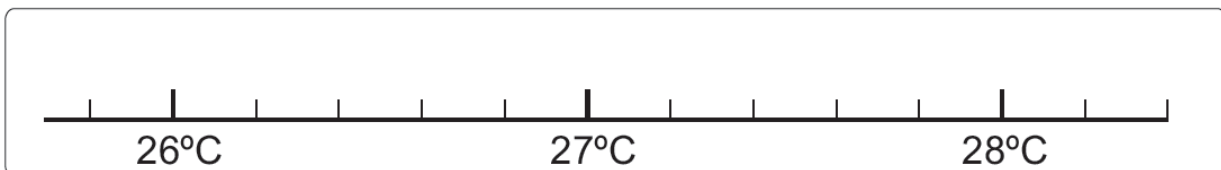
Use graph paper to draw the following conversion graphs.

- (a) Change between height in feet and inches (up to 7 feet) and height in cm (up to 200 cm).
- (b) Change between weight in stones and pounds (up to 12 stone) and weight in kilograms (up to 80 kg).
(Use 10 stone $\approx$ 64 kg to help.)
- (c) Change between money in pounds (up to £80) and money in American dollars (up to \$120).
Use the exchange rate £1 = \$1.30 (so that £50 = \$65).
- (d) Change between temperature in Celsius and temperature in Fahrenheit.
Use the conversions $0^{\circ}\text{C} = 32^{\circ}\text{F}$ and $100^{\circ}\text{C} = 212^{\circ}\text{F}$.

5

**Exercise 20 (Revision)**

- (a) Write the best metric unit for measuring the width of a table.
- (b) Convert the units in each of the following.
- | | | |
|-----------------------|-----------------------|------------------------------|
| (i) 6 m in cm | (ii) 4 kg in g | (iii) 9 l in ml |
| (iv) 4 feet in inches | (v) 3 stone in pounds | (vi) 2 pints in fluid ounces |
| (vii) 45 mm in cm | (viii) 7,250 mg in g | (ix) 40 cl in l |
- (c) Fill in the blanks.
- | | | |
|---------------------------------|-----------------------------------|--|
| (i) 4 miles $\approx$ _____ km | (ii) 5 ounces $\approx$ _____ g | (iii) 6 gallons $\approx$ _____ litres |
| (iv) 10 yards $\approx$ _____ m | (v) 6.6 pounds $\approx$ _____ kg | (vi) 3 litres $\approx$ _____ pints |
- (d) The temperature is measured to be 27.2°C . Draw an arrow on the scale below to show this temperature.



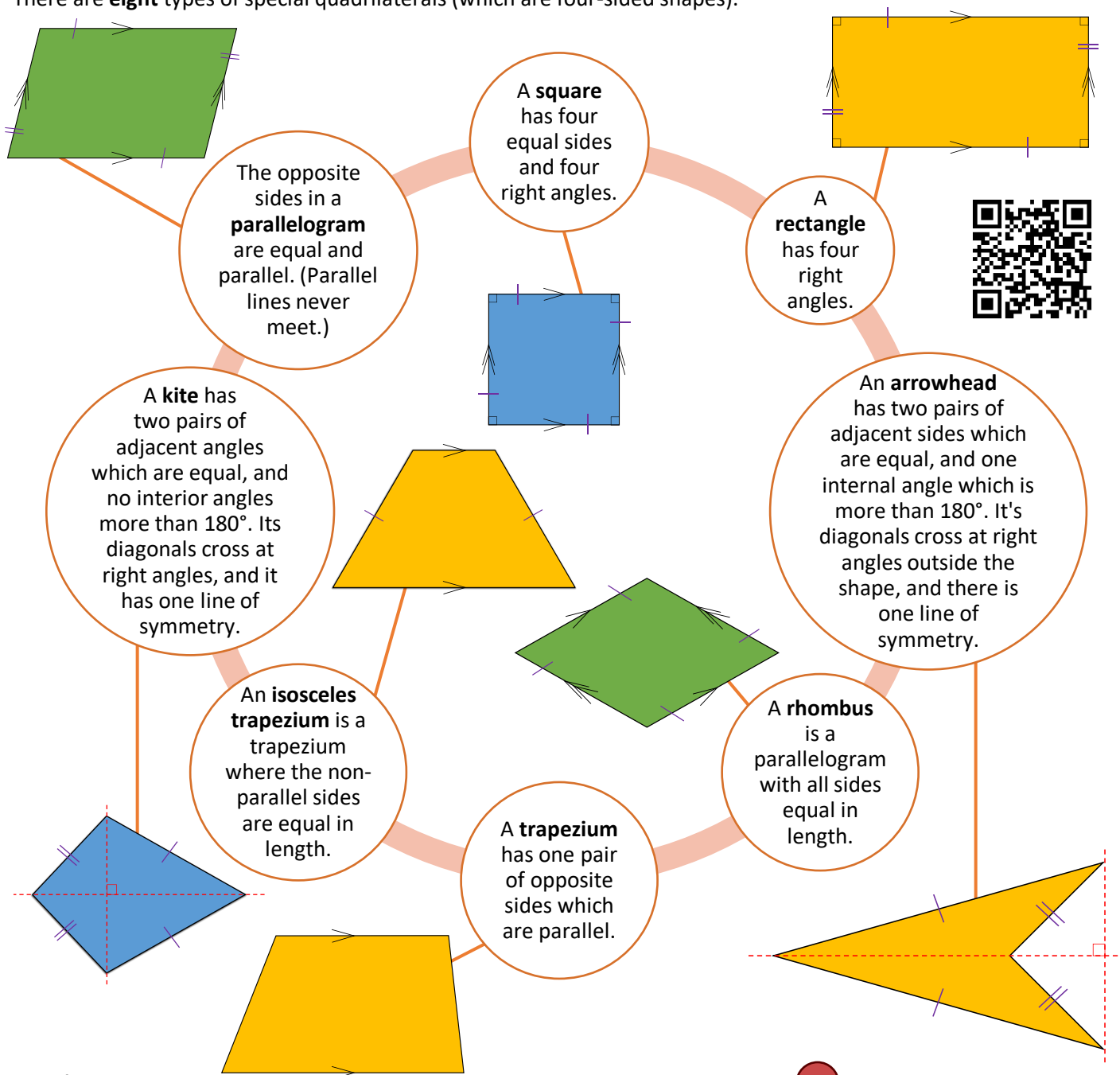
- (e) On graph paper, draw a conversion graph for the change between measuring lengths in yards (up to 6 yards) and measuring lengths in metres (up to 6 metres).

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Quadrilaterals

There are **eight** types of special quadrilaterals (which are four-sided shapes).



Exercise 21

6

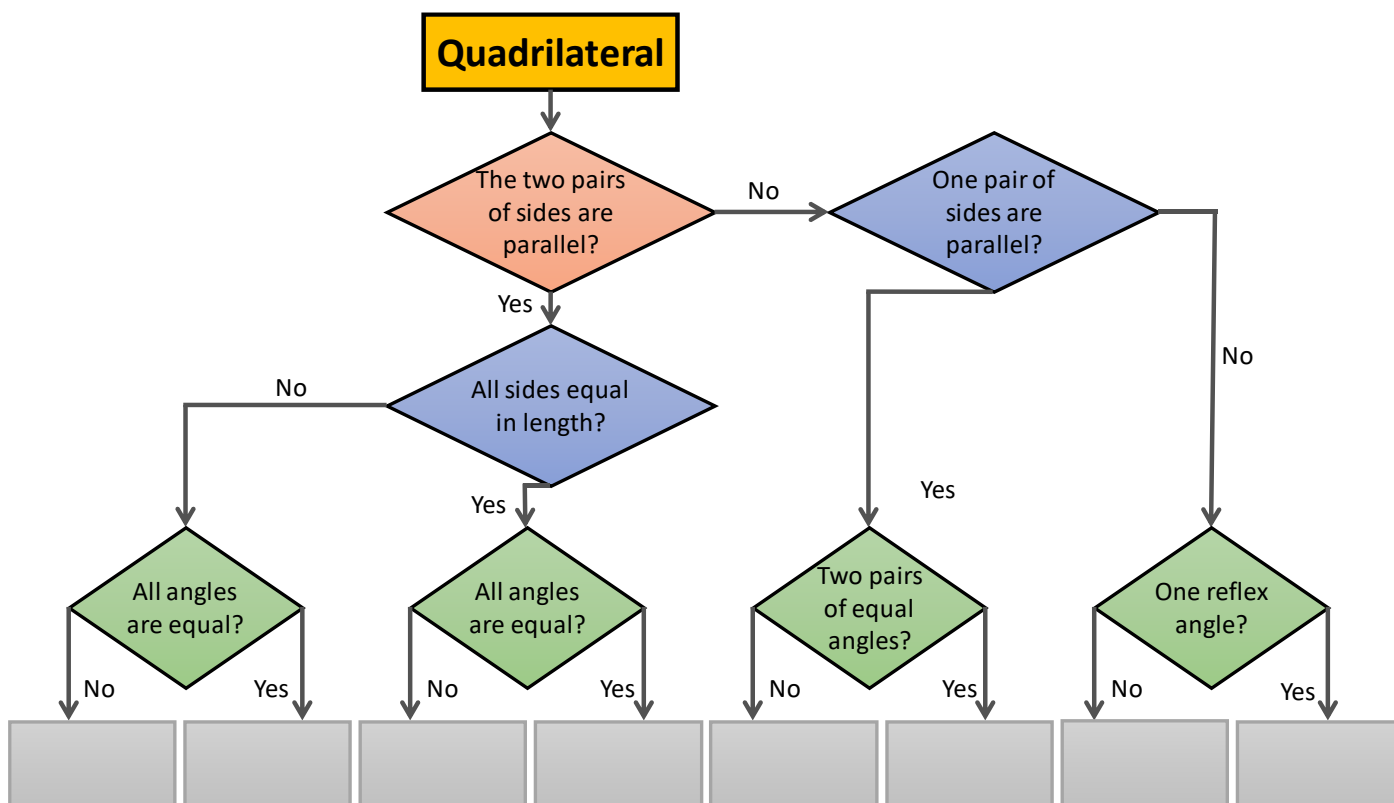
Complete the following table by putting a tick (✓) in any space where the given statement is true.

Statement	Square	Parallelogram	Trapezium
The length of the diagonals is equal.			
The opposite angles are equal.			
Only one pair of opposite sides are parallel.			
The diagonals are lines of symmetry.			

Exercise 22

6

Complete the following flow chart.

**Exercise 23**

6

Complete the following table by putting a tick (✓) in any space where the given statement is true.

Statement	Rectangle	Parallelogram	Rhombus
The diagonals are not equal.			
Every angle is equal.			
The opposite angles are equal.			
There are no lines of symmetry.			

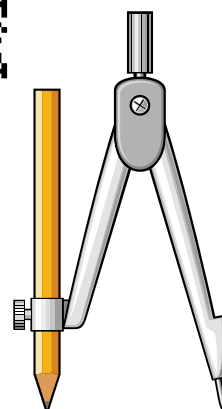
Exercise 24

Use a compass and ruler to draw the following shapes.

- A rhombus with lengths 6 cm and the length of the longest diagonal is 10 cm.
- A kite with lengths 6 cm and 8 cm, and the length of the longest diagonal is 10 cm.
- An arrowhead with lengths 6 cm and 3 cm, and the length of the diagonal outside the shape is 5 cm.
- A rhombus with lengths 5.4 cm and the length of the longest diagonal is 8 cm.
- A kite with lengths 7.5 cm and 2.3 cm, and the length of the longest diagonal is 9 cm.
- An arrowhead with lengths 5.4 cm and 4.1 cm, and the length of the diagonal outside the shape is 7cm.

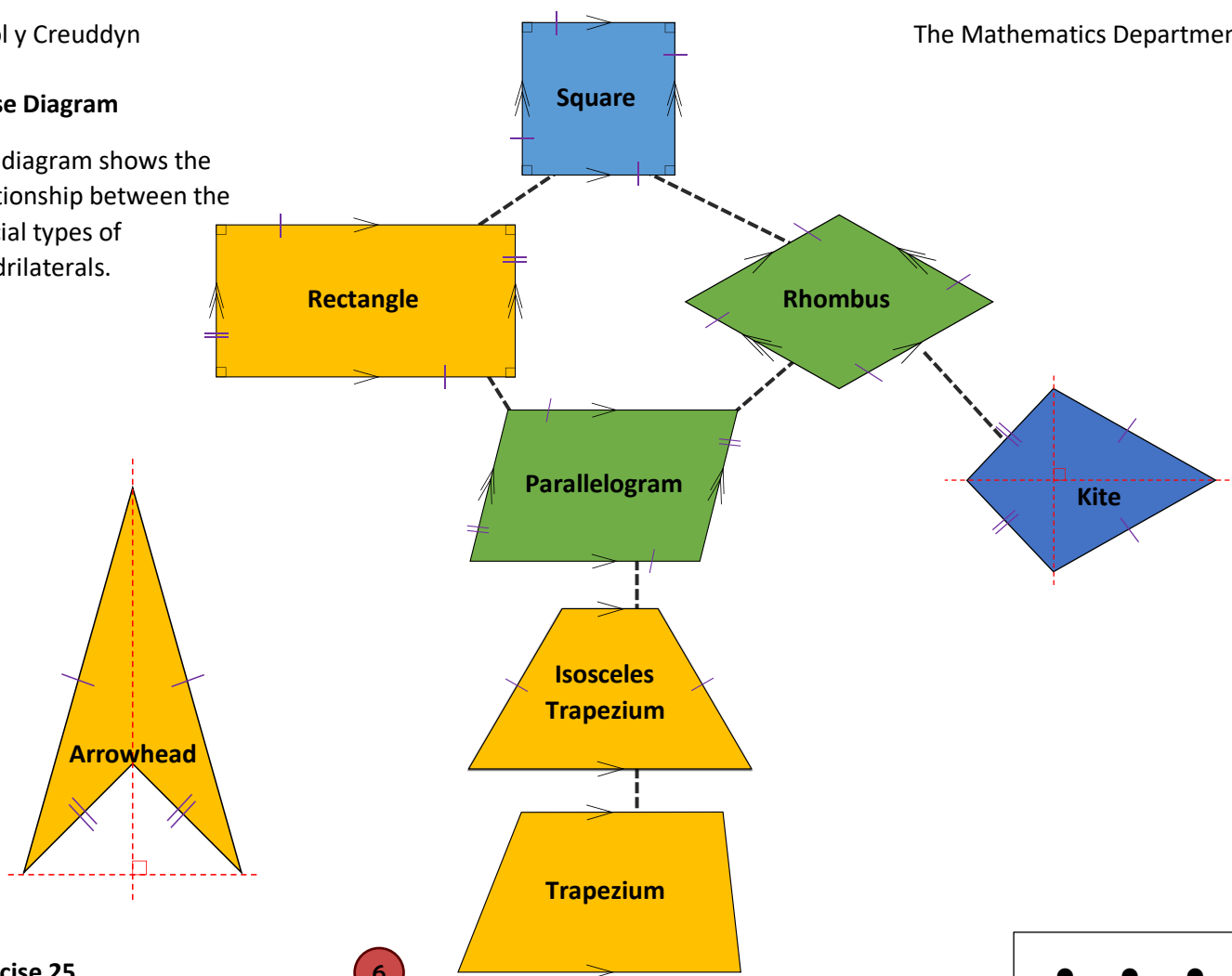


6



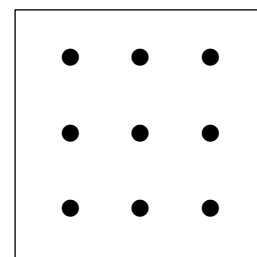
Hasse Diagram

This diagram shows the relationship between the special types of quadrilaterals.

**Exercise 25**

6

How many different quadrilaterals can be drawn using a 3×3 pegboard?
Can you give special names to all of them?

**Exercise 26**

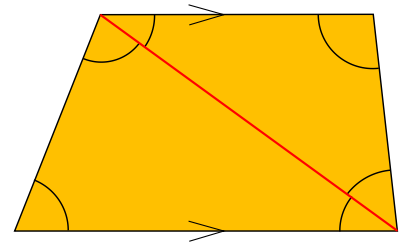
6

Complete the following table.

Shape	Number of pairs of parallel lines	Number of pairs of equal lines	Number of lines of symmetry	Order of rotational symmetry	Two pairs of equal angles?	Diagonals crossing at right angles?
Rectangle	2					
Square		2				
Parallelogram						
Rhombus						
Trapezium				No rotational symmetry		
Isosceles Trapezium						
Kite						
Arrowhead			1			

Angles

It is possible to split any quadrilateral into two triangles. Since the total interior angles of every triangle is 180° , it follows that the total internal angles of any quadrilateral is $2 \times 180^\circ = 360^\circ$.

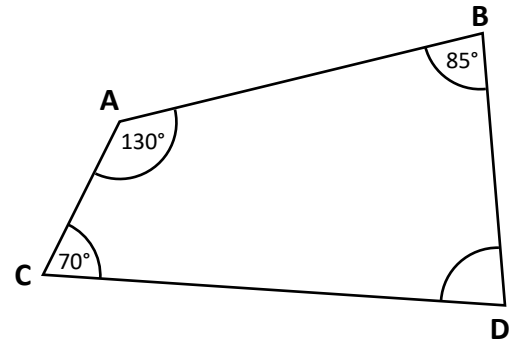
**Example**

In the diagram on the right, it is possible to calculate the size of the angle $\widehat{BDC}$ without using a protractor, since the sum of $\widehat{BDC}$, $\widehat{ACD}$, $\widehat{CAB}$ and $\widehat{ABD}$ must be 360° .

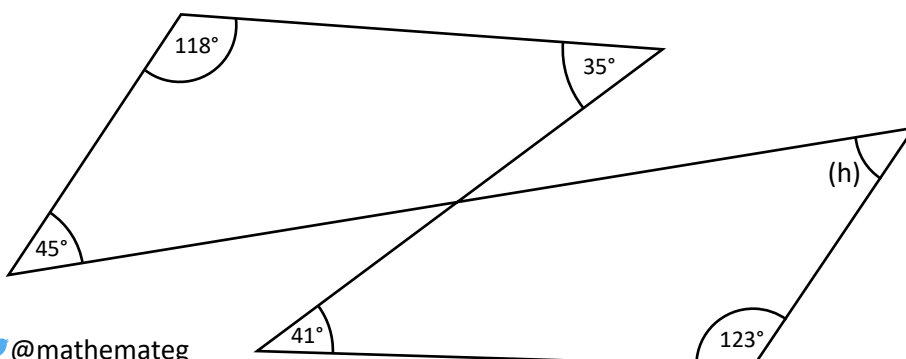
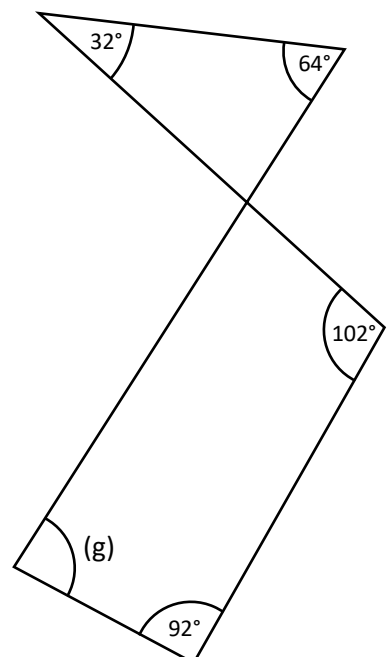
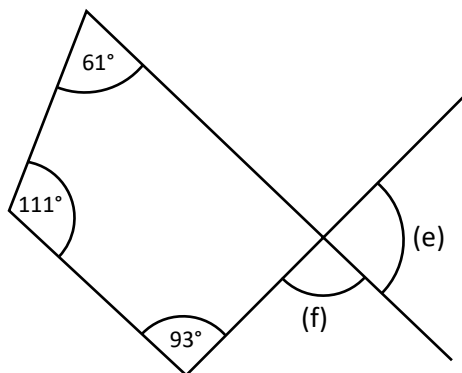
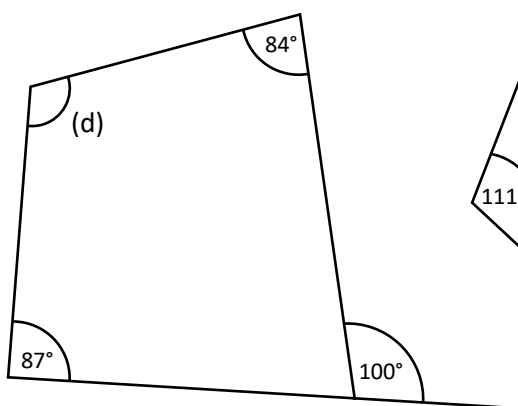
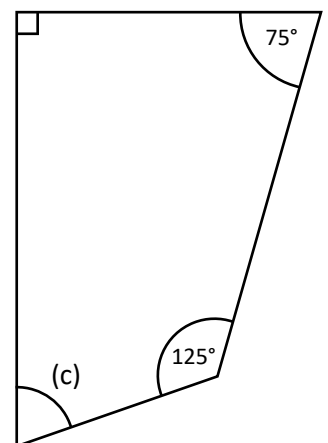
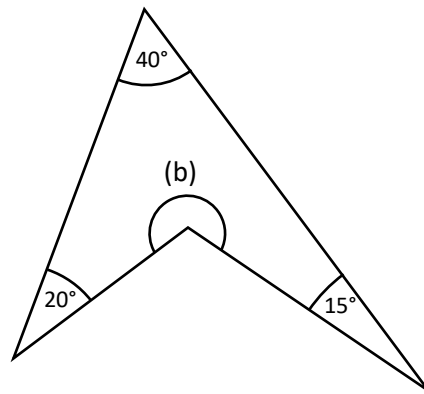
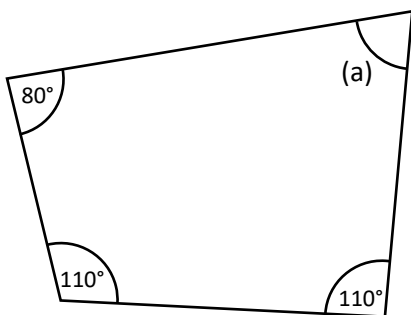
$$70^\circ + 130^\circ + 85^\circ = 285^\circ$$

$$\widehat{BDC} = 360^\circ - 285^\circ$$

$$\widehat{BDC} = 75^\circ.$$

**Exercise 27**

Find the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Perimeter and Area

The **perimeter** of a shape is the distance around the shape.

To find the perimeter, we add the length of **all** of the shape's sides.

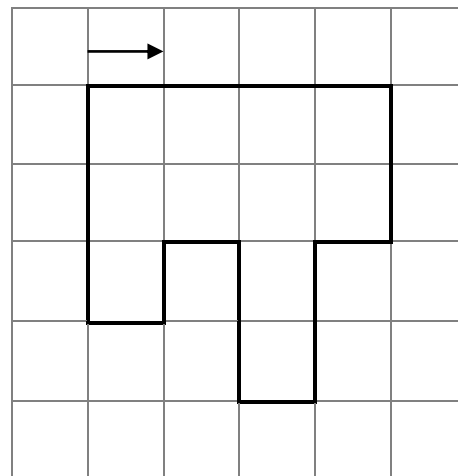
The **area** of a shape is the size of the surface of the shape.

Area is measured in square units.

Example

The perimeter of the shape on the right is $4\text{ cm} + 2\text{ cm} + 1\text{ cm} + 2\text{ cm} + 1\text{ cm} + 2\text{ cm} + 1\text{ cm} + 1\text{ cm} + 1\text{ cm} + 3\text{ cm} = 18\text{ cm}$.

By counting squares, the area of the shape on the right is 11 cm^2 .

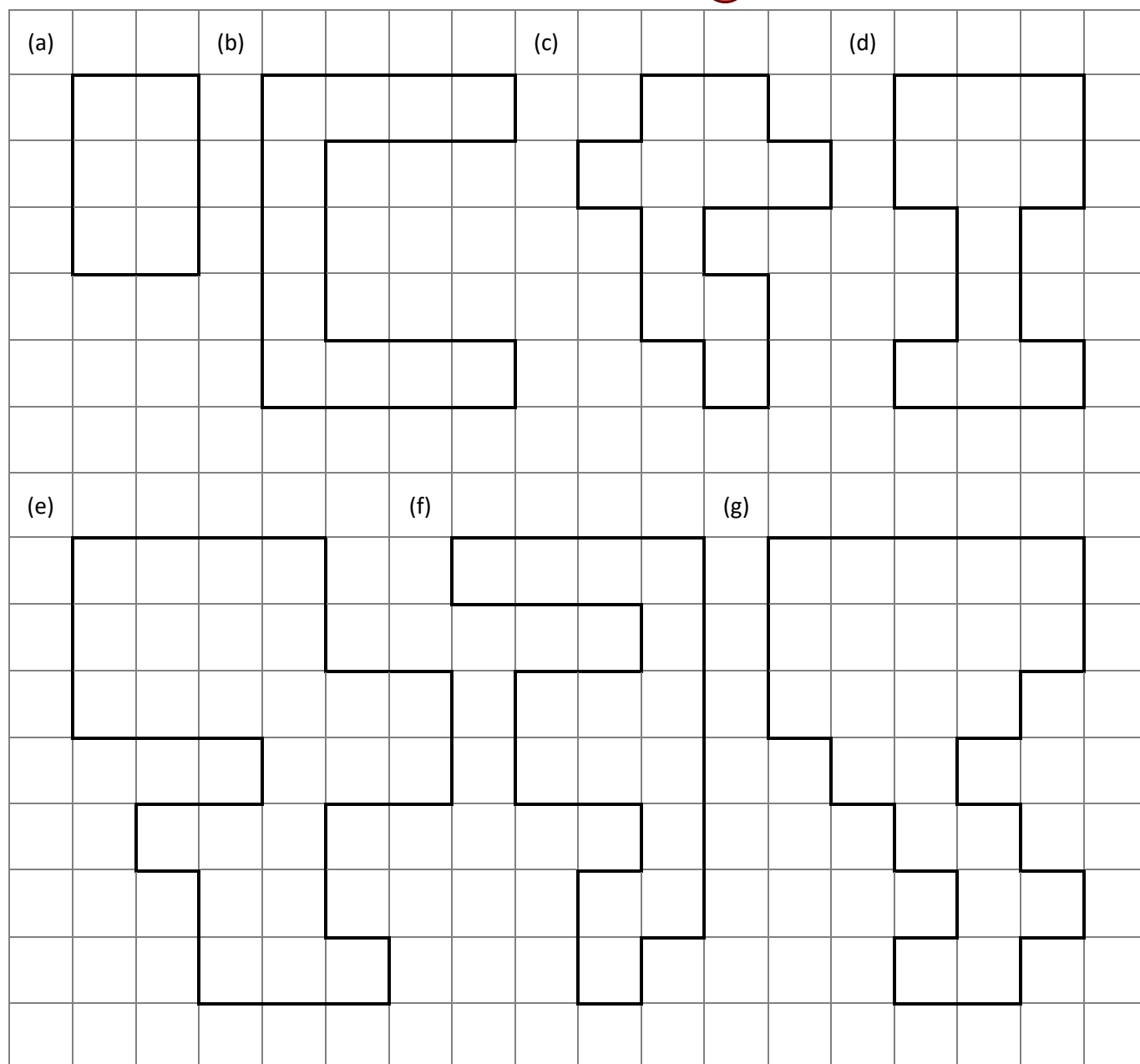


Exercise 31

Calculate the perimeter and area of the following shapes.

Skill

4

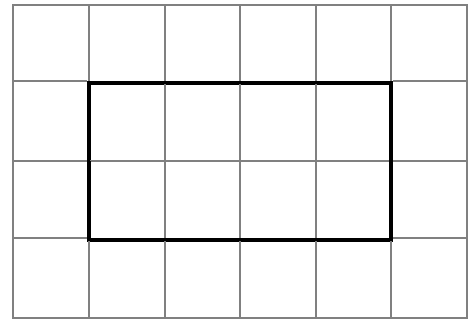


Rectangle

$$\text{Area of a Rectangle} = \text{Length} \times \text{Width}$$

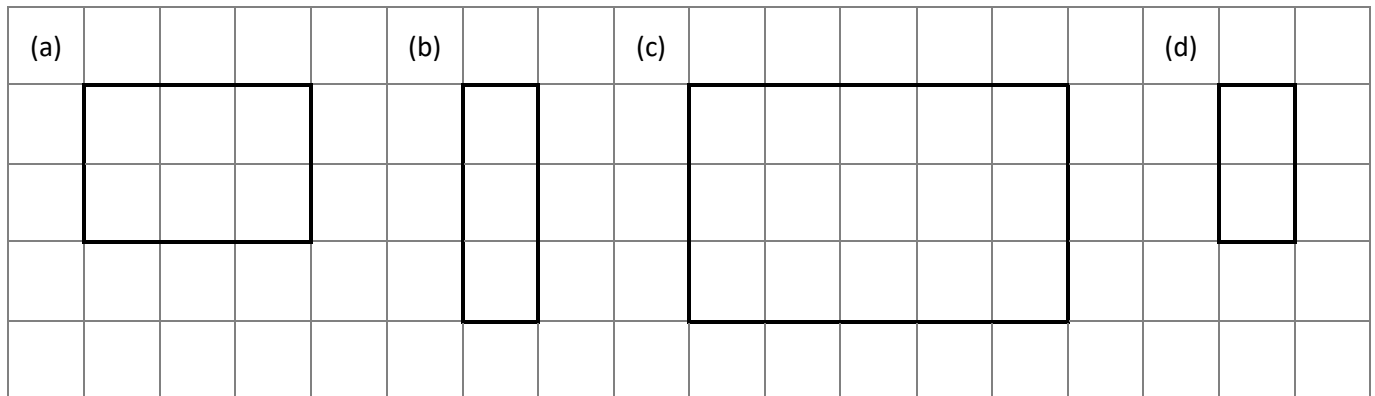
Example

The perimeter of the rectangle on the right is $4 \text{ cm} + 2 \text{ cm} + 4 \text{ cm} + 2 \text{ cm} = 12 \text{ cm}$. The area of the rectangle on the right is $4 \text{ cm} \times 2 \text{ cm} = 8 \text{ cm}^2$.

**Exercise 32**

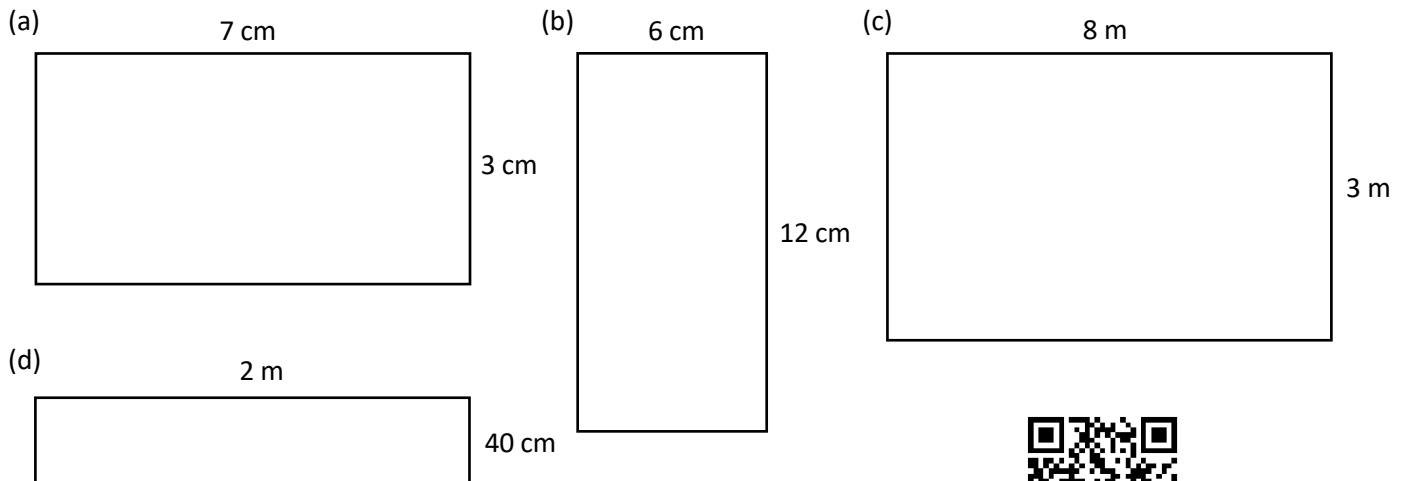
5

Find the perimeter and area of the following rectangles.

**Exercise 33**

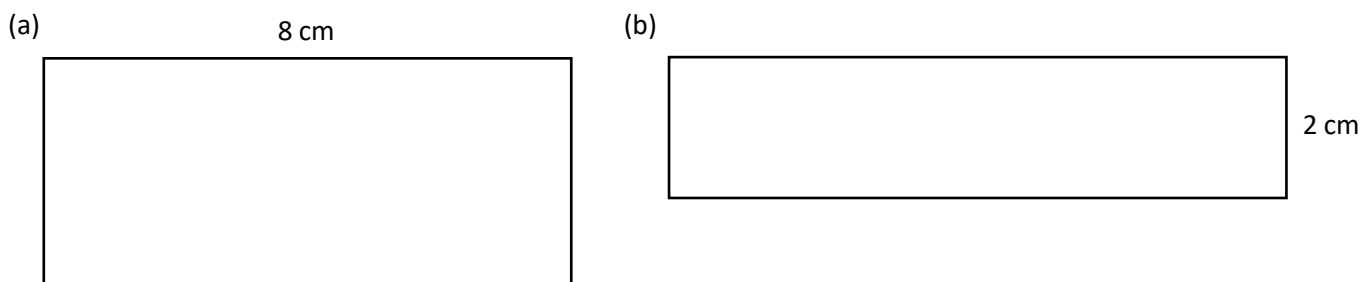
5

Find the perimeter and area of the following rectangles. (The diagrams are not drawn to scale.)

**Exercise 34**

5

The perimeter of each rectangle below is 24 cm. What is the area of each rectangle?

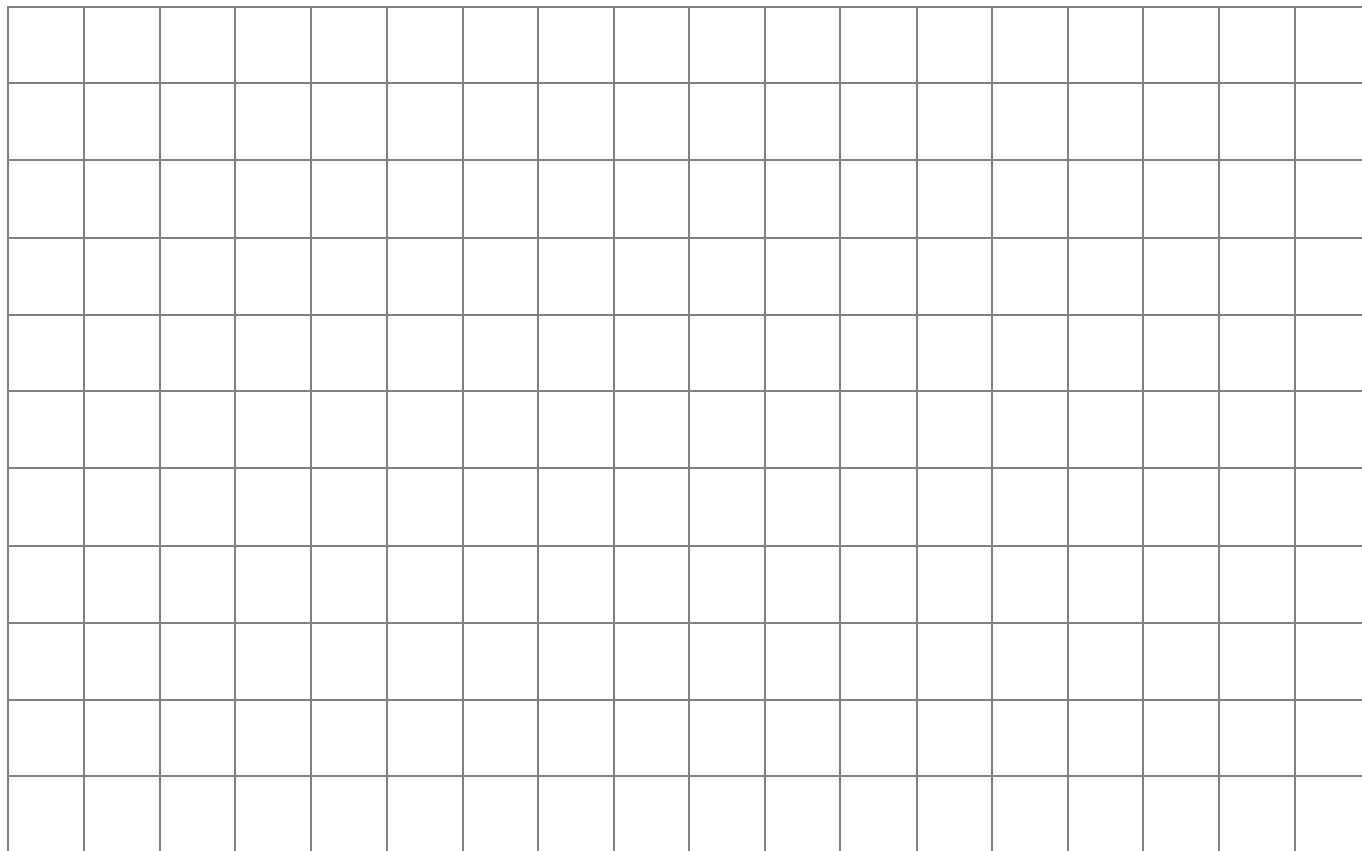


Exercise 35

5

On the squared paper below, draw shapes with **perimeter 14 cm**, and area

- (a) 6 cm^2 (b) 7 cm^2 (c) 8 cm^2 (d) 9 cm^2 (e) 10 cm^2 .

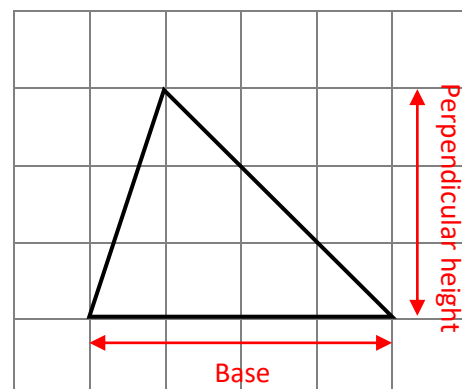


Triangle

$$\text{Area of a Triangle} = \frac{\text{Base} \times \text{Height}}{2}$$

To find the area of any triangle, we must choose one side to be the **base**. Then, we must measure the **perpendicular height** of the triangle from the base.

- To calculate the area: (1) Multiply the length of the base by the height.
(2) Halve the answer.



Example

For the triangle shown, the length of the base is 4 cm. The perpendicular height is 3 cm.

Therefore the area is $\frac{4 \times 3}{2} = \frac{12}{2} = 6 \text{ cm}^2$.

Exercise 36

5



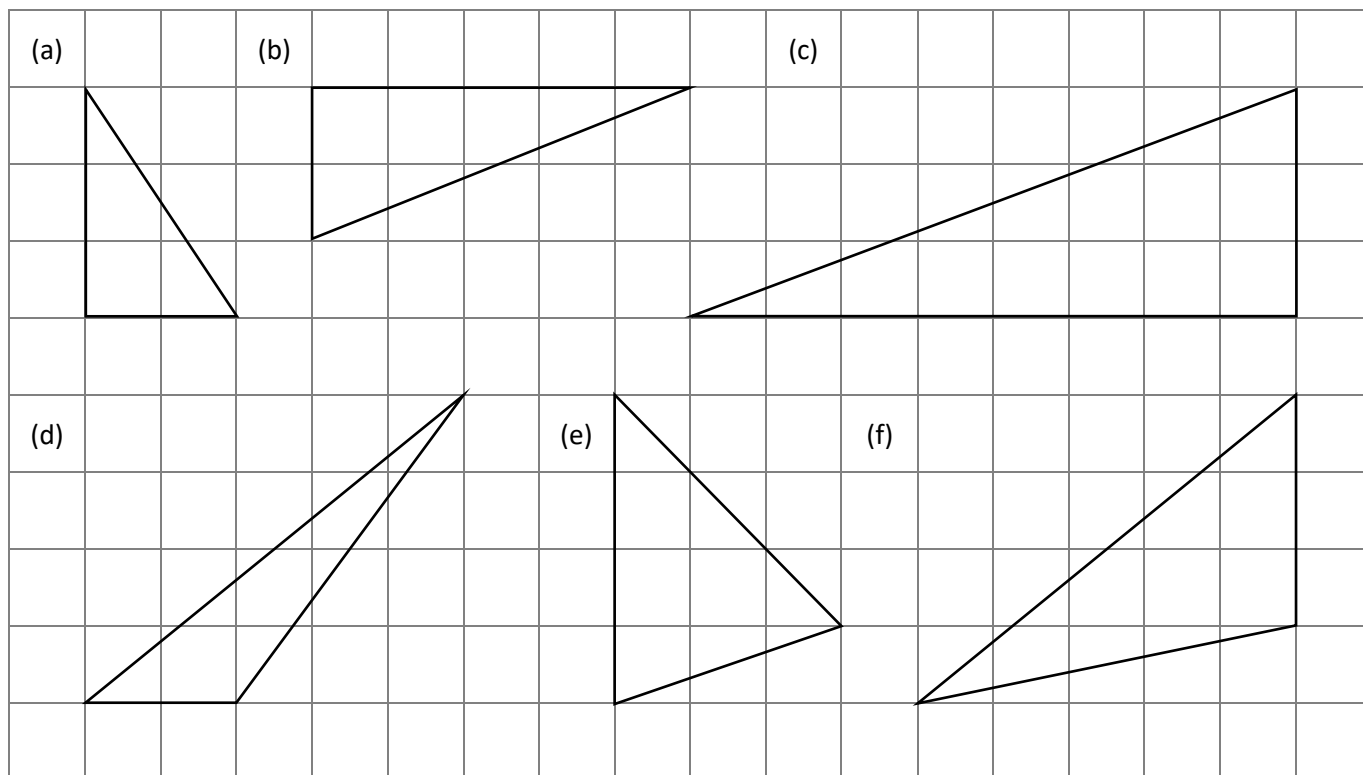
Calculate the area of the triangles with the following measurements.

- | | | |
|------------------------------|------------------------------|-------------------------------|
| (a) Base 4 cm, height 6 cm. | (b) Base 7 cm, height 4 cm. | (c) Base 3 cm, height 9 cm. |
| (d) Base 10 cm, height 5 cm. | (e) Base 2 cm, height 3 cm. | (f) Base 3 cm, height 3 cm. |
| (g) Base 5 cm, height 3 cm. | (h) Base 7 cm, height 3 cm. | (i) Base 9 cm, height 4 cm. |
| (j) Base 2 cm, height 15 cm. | (k) Base 3 cm, height 12 cm. | (l) Base 10 cm, height 10 cm. |
| (m) Base 4 m, height 4 m. | (n) Base 6 m, height 3 m. | (o) Base 7 m, height 8 m. |

Exercise 37

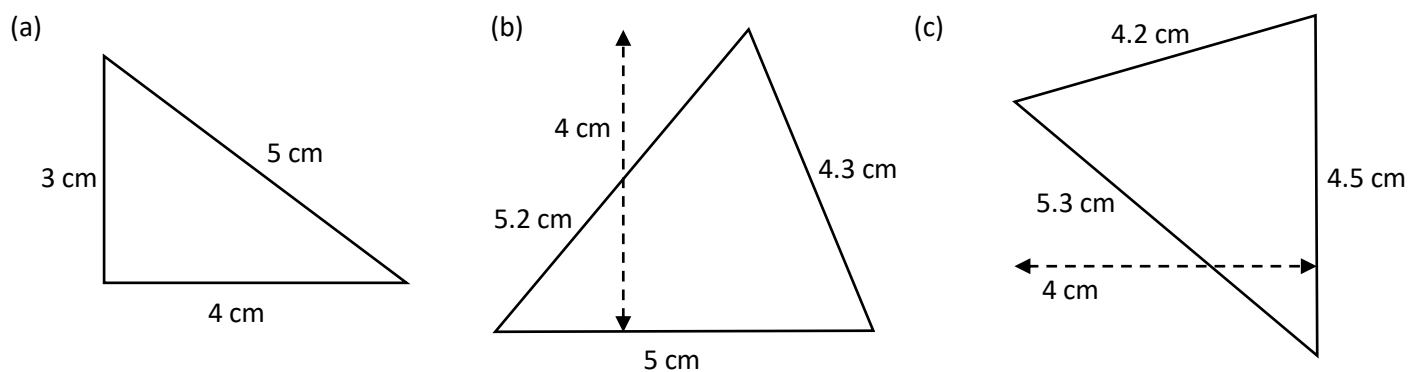
5

Calculate the area of the following triangles.

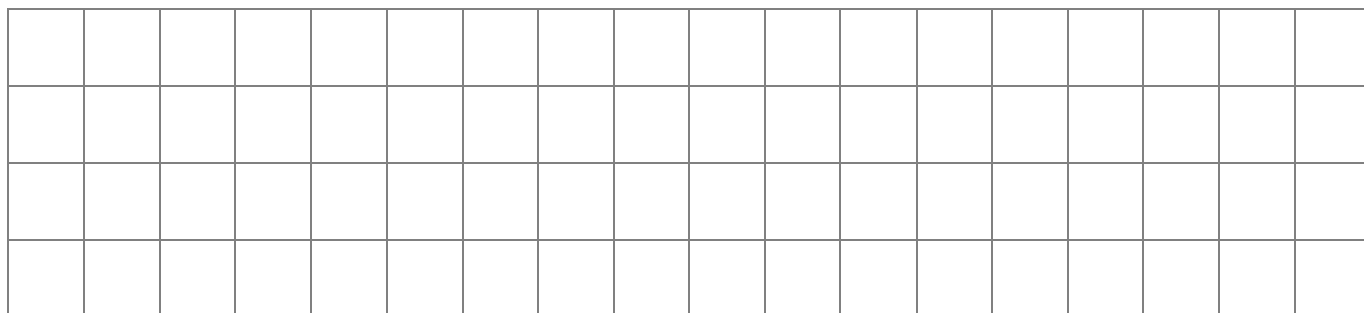
**Exercise 38**

5

Calculate the perimeter and area of the following triangles.

**Exercise 39**

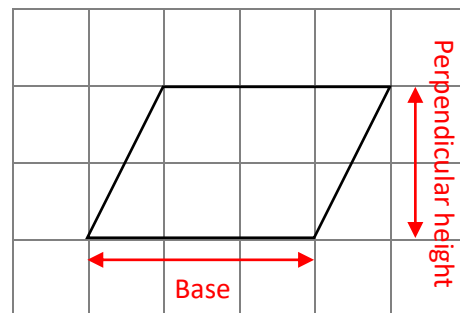
5

On the squared paper below, draw three different triangles with area 3 cm^2 and base 3 cm.

Parallelogram

$$\text{Area of a Parallelogram} = \text{Base} \times \text{Height}$$

To find the area of any parallelogram, we must choose one side to be the **base**. Then, we must measure the **perpendicular height** of the parallelogram from the base. To calculate the area, we multiply the base by the height.

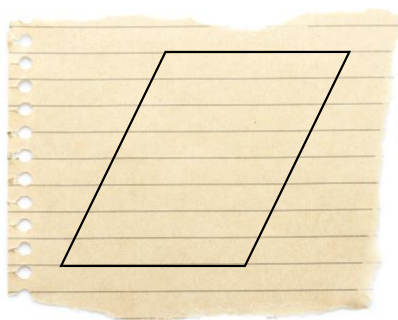


Example

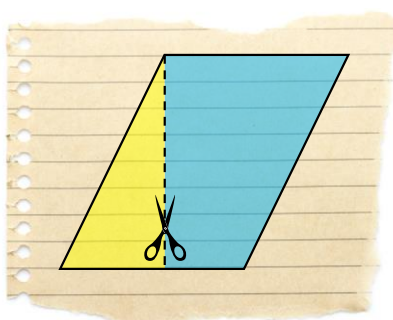
For the parallelogram shown, the length of the base is 3 cm. The perpendicular height of the parallelogram is 2 cm. Therefore, the area is $3 \times 2 = 6 \text{ cm}^2$.



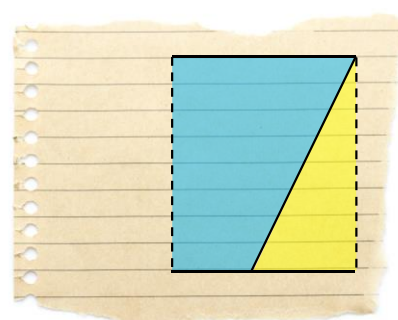
Why Base $\times$ Height?



(1) Start with any **parallelogram**.



(2) Split the parallelogram as shown. Cut along the dotted line.



(3) Move the yellow piece to the right. Now we have a **rectangle**. The area of a rectangle = Length $\times$ Width, therefore the area of the original parallelogram is its base times its height.

Exercise 40

Calculate the area of the parallelograms with the following measurements.

(a) Base 6 cm, height 5 cm.

(b) Base 8 cm, height 2 cm.

(c) Base 5 cm, height 5 cm.

(d) Base 9 cm, height 8 cm.

(e) Base 10 cm, height 8 cm.

(f) Base 9 cm, height 6 cm.

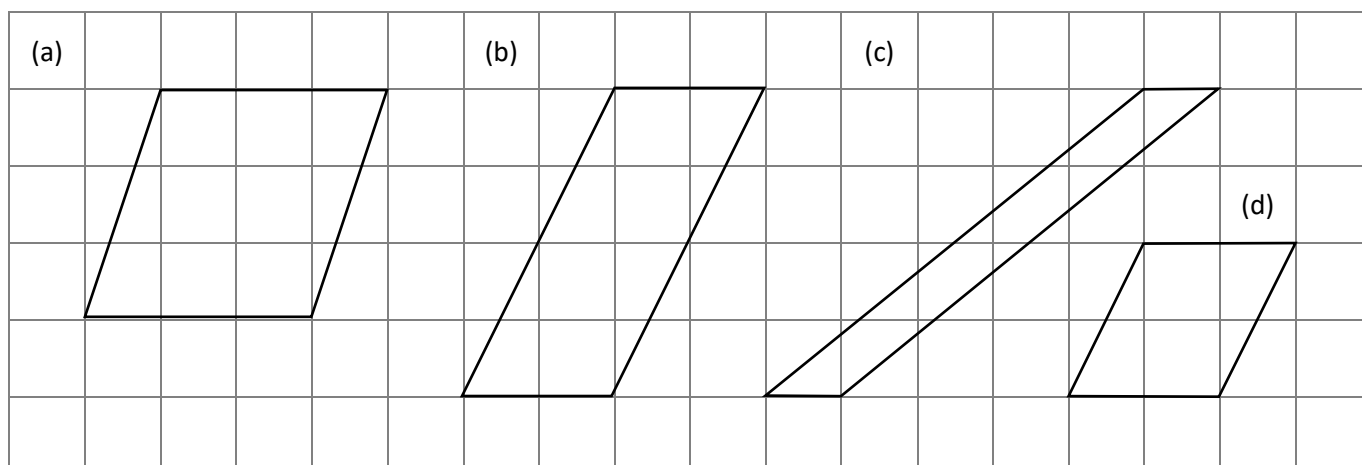
(g) Base 8 m, height 4 m.

(h) Base 11 m, height 6 m.

(i) Base 5 km, height 7 km.

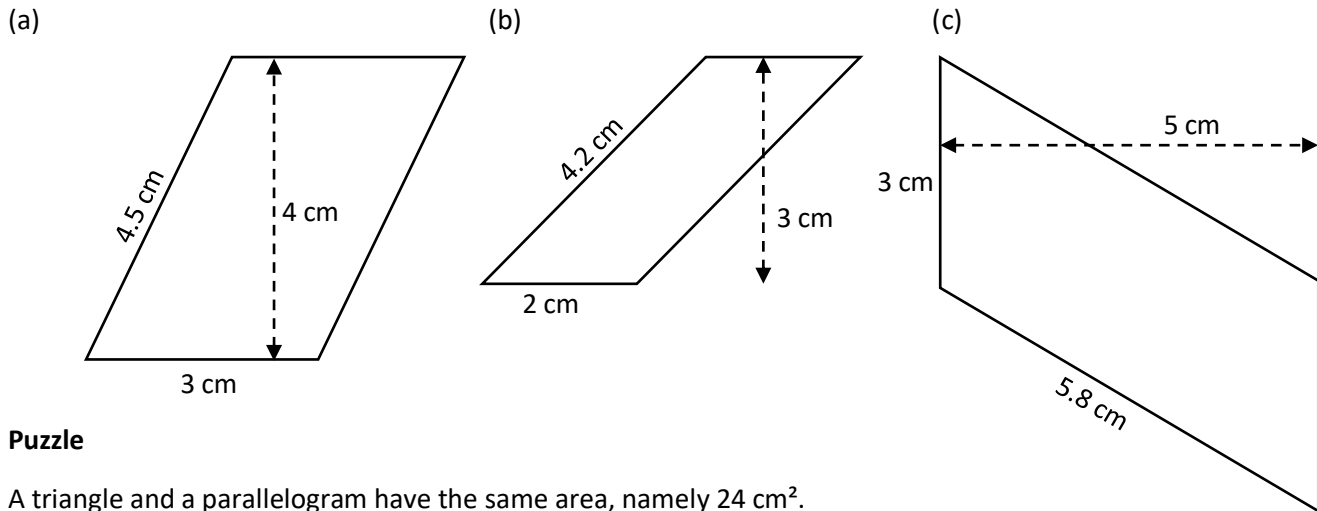
Exercise 41

Calculate the area of the following parallelograms.



Exercise 42

Calculate the perimeter and area of the following parallelograms.

**Puzzle**

A triangle and a parallelogram have the same area, namely 24 cm^2 .

The triangle and the parallelogram share the same base.

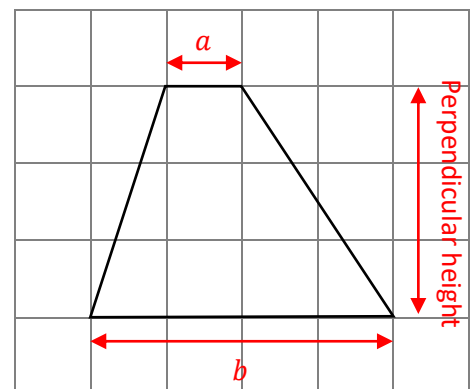
If the height of the triangle is 4 cm, what is the height of the parallelogram?

Trapezium

 **Area of a Trapezium = $\frac{1}{2}(a + b) \times \text{Height}$** 

To find the area of any trapezium, measure the lengths of the parallel sides (a and b) and the perpendicular height between the parallel sides.

To calculate the area: (1) Add the lengths of the parallel sides, $a + b$.
(2) Multiply by the height.
(3) Halve the answer.

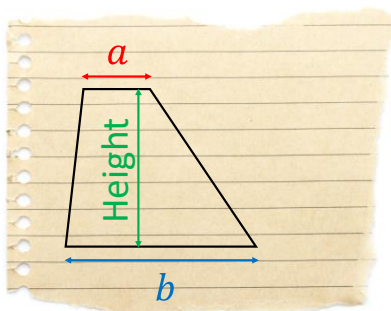
**Example**

For the trapezium shown, the lengths of the parallel sides are 1 cm and 4 cm, and the perpendicular height is 3 cm. To calculate the area:

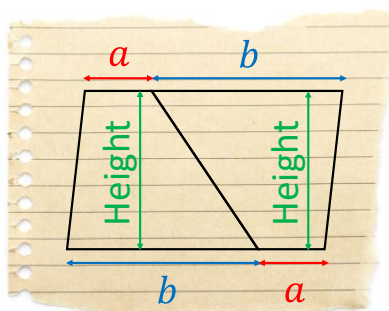
$$\begin{aligned} 1 \text{ cm} + 4 \text{ cm} &= 5 \text{ cm} \\ 5 \text{ cm} \times 3 \text{ cm} &= 15 \text{ cm}^2 \\ 15 \text{ cm}^2 \div 2 &= 7.5 \text{ cm}^2. \end{aligned}$$



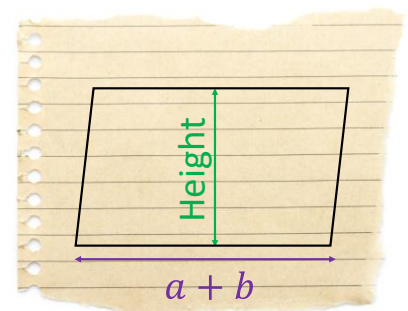
Why $\frac{1}{2}(a + b) \times \text{Height}$?



(1) Start with a trapezium with the above measurements.



(2) Add a second copy of the trapezium to the side of the first. This makes a parallelogram.



(3) The area of this parallelogram is $(a + b) \times \text{Height}$. The area of the original trapezium is half of this, which is $\frac{1}{2}(a + b) \times \text{Height}$.

Exercise 43

6

Calculate the area of the trapezia with the following measurements.

(a) $a = 4$ cm, $b = 6$ cm, height = 5 cm.

(b) $a = 3$ cm, $b = 5$ cm, height = 2 cm.

(c) $a = 6$ cm, $b = 9$ cm, height = 4 cm.

(d) $a = 4$ cm, $b = 11$ cm, height = 3 cm.

(e) $a = 12$ cm, $b = 4$ cm, height = 5 cm.

(f) $a = 14$ cm, $b = 6$ cm, height = 7 cm.

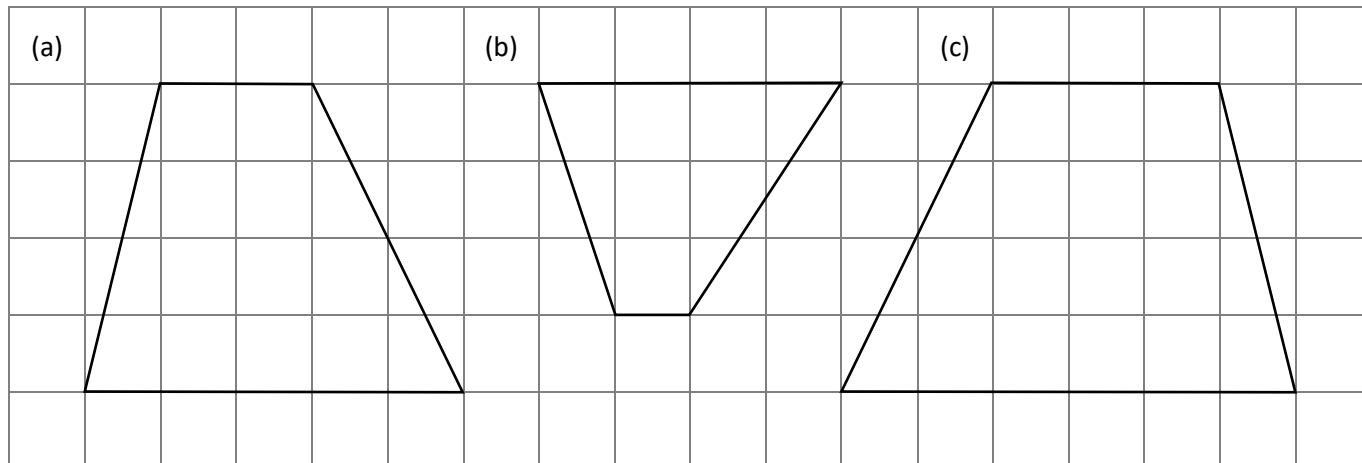
(g) $a = 6$ m, $b = 8$ m, height = 10 m.

(h) $a = 2$ mm, $b = 1$ mm, height = 16 mm.

Exercise 44

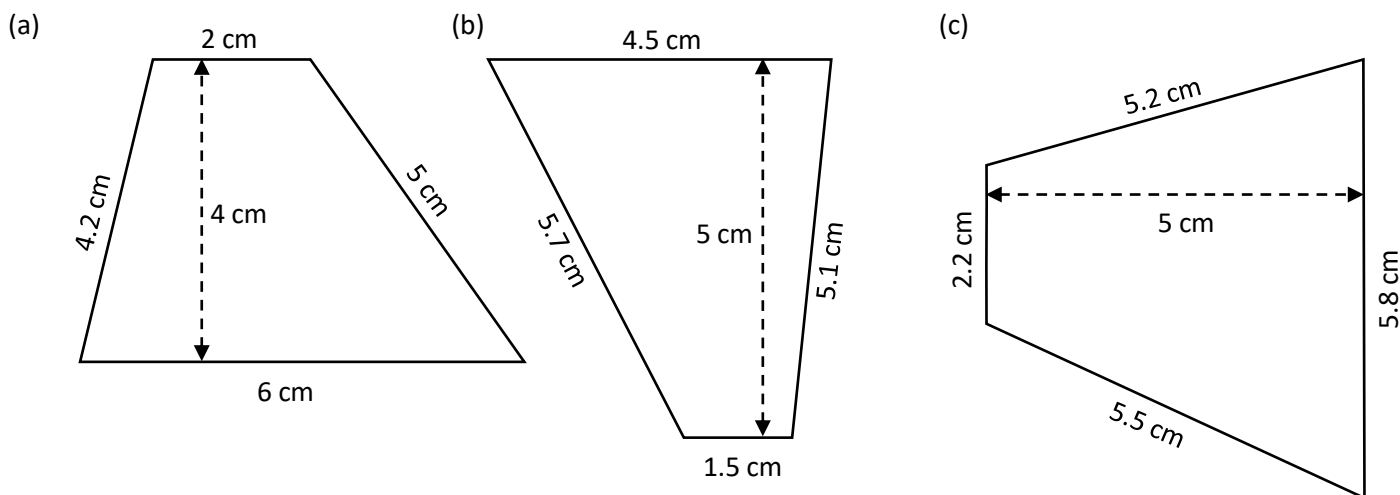
6

Calculate the area of the following trapezia.

**Exercise 45**

6

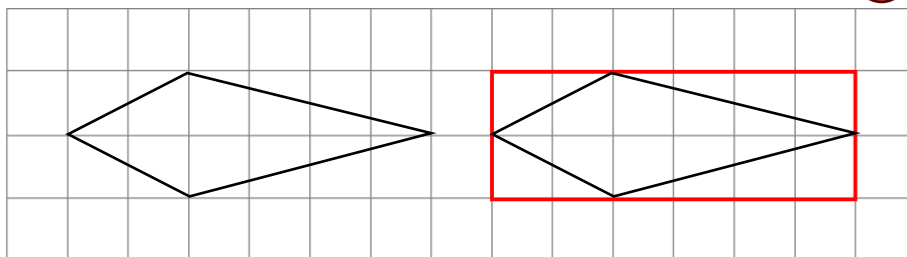
Calculate the perimeter and area of the following trapezia.

**Exercise 46**

Set 1 only

7

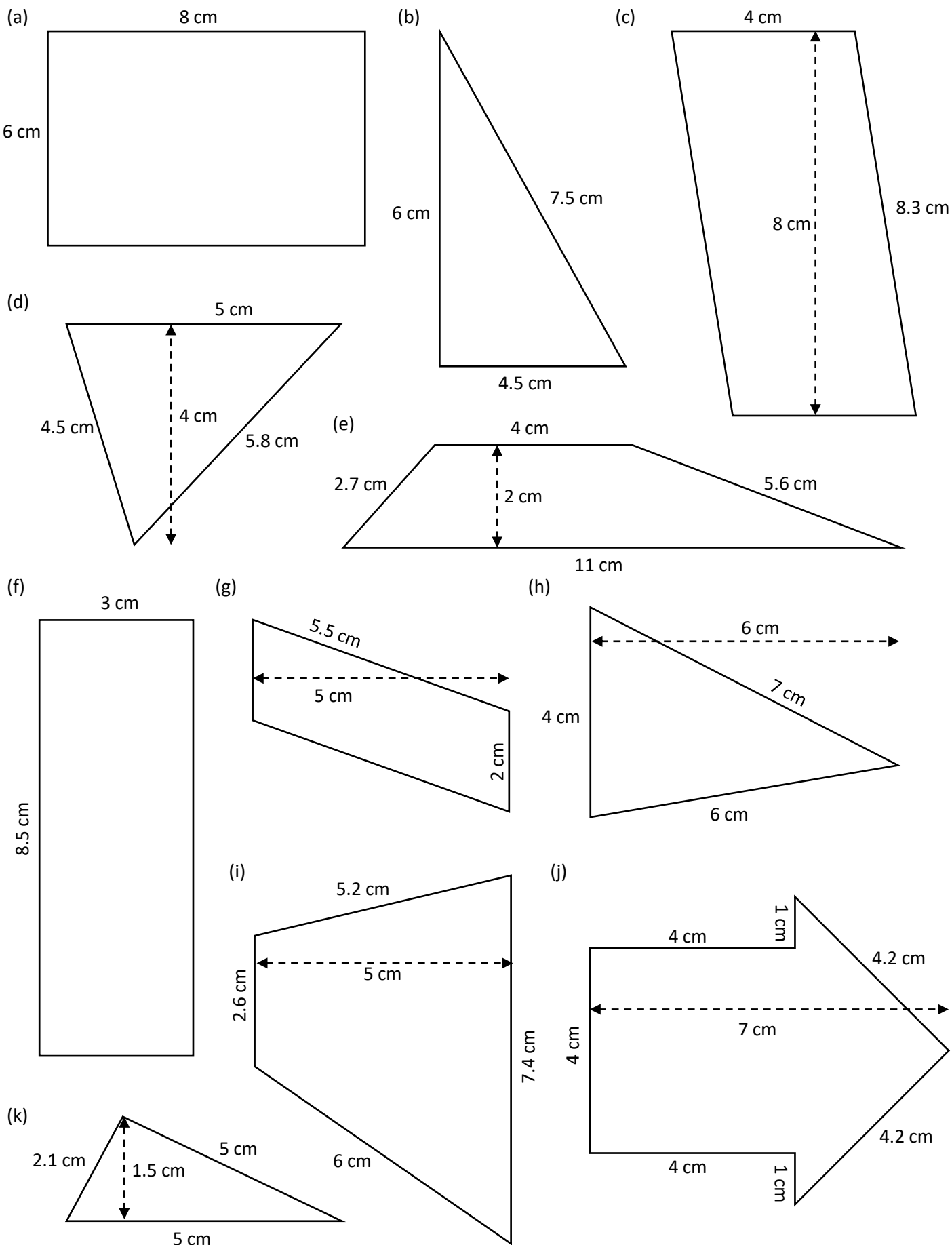
Discuss the relationship between the area of a kite and the area of the rectangle which surrounds the kite. Use this to find a formula for the **area of a kite**. Does this same formula work for any rhombus? What about any arrowhead?



Exercise 47

6

Calculate the perimeter and area of the following shapes.



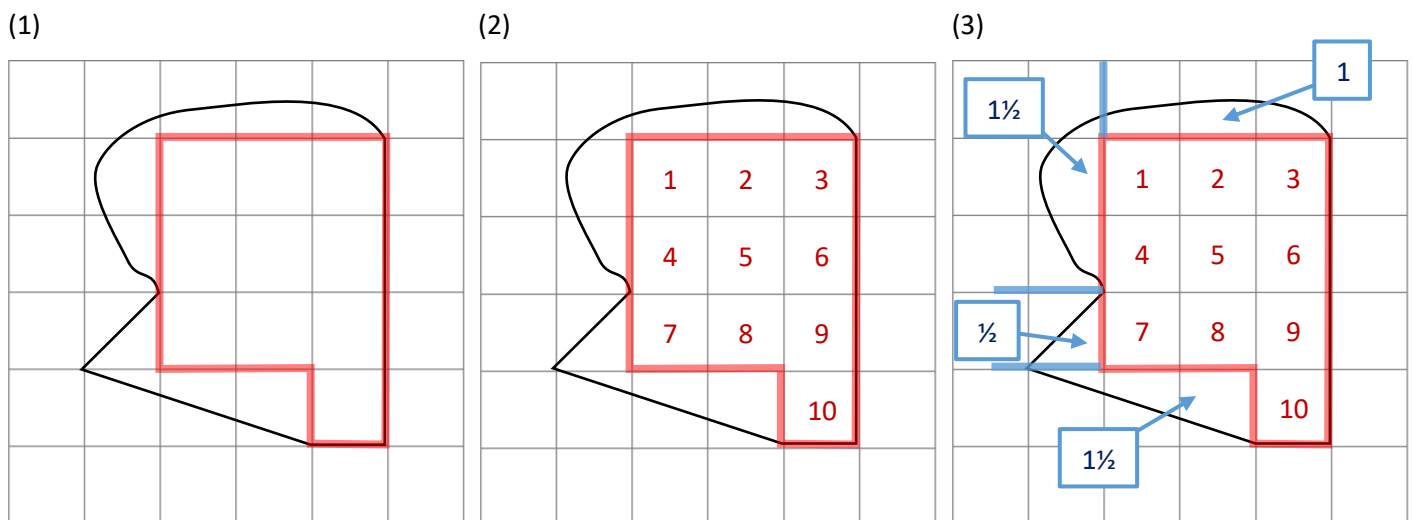
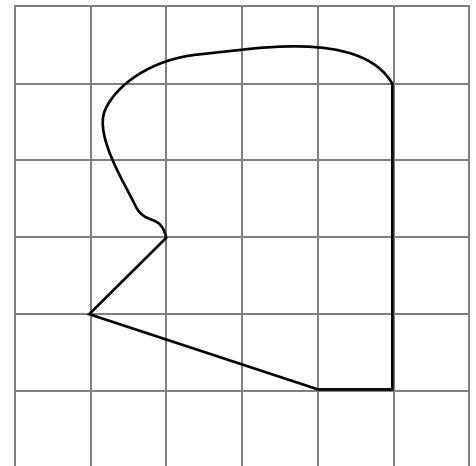
Estimating Area

There isn't a formula to calculate the area of all shapes. However, it is possible to **estimate** the area of some shapes by counting squares.

Example

Consider the shape shown on the right. To estimate its area:

- (1) Draw a **grid** inside the shape to include all the complete squares.
- (2) **Number** the complete squares.
- (3) **Split** the pieces outside of the grid into smaller pieces. **Estimate the area** of each piece.
- (4) Add the total number of complete squares to the total area of the pieces outside of the grid.

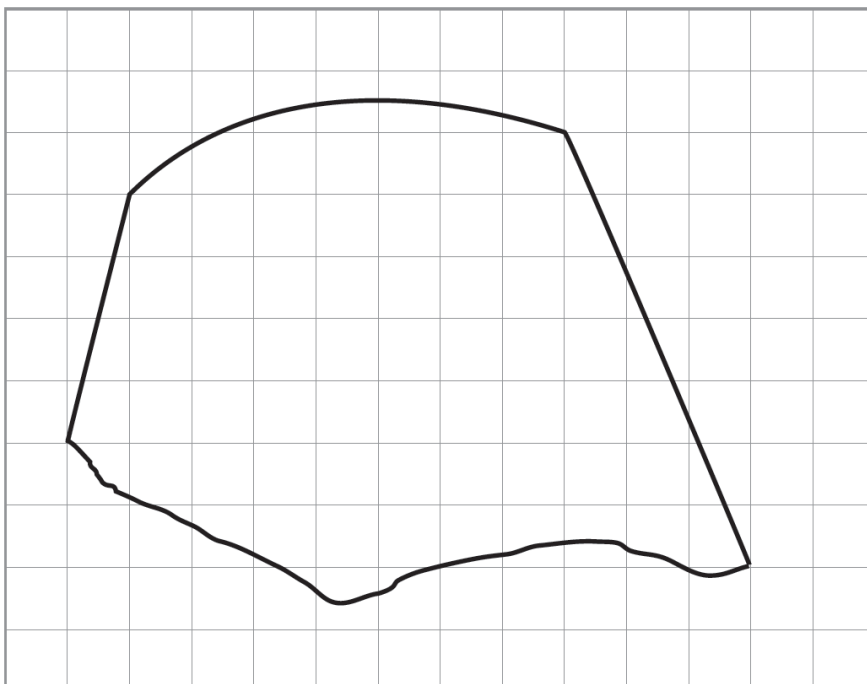


(4) $10 + (1 + 1\frac{1}{2} + \frac{1}{2} + 1\frac{1}{2}) = 10 + 4\frac{1}{2} = 14\frac{1}{2} \text{ cm}^2$.

Exercise 48

4

Find an estimate for the area of the following shape. (Each square represents an area of 1 cm^2 .)



6

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Solids

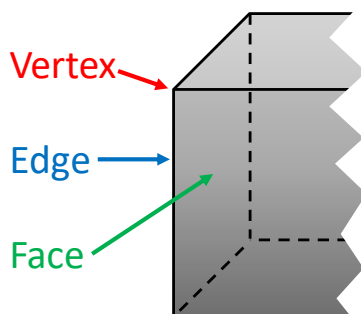


In the previous chapter, all of the shapes (e.g. rectangle, rhombus) were shapes in **two dimensions** (2-D). In this chapter, the solids we will be considering will be shapes in **three dimensions** (3-D).

Exercise 50

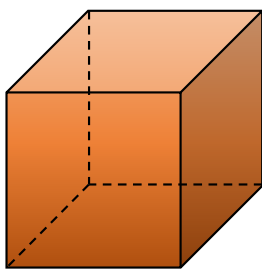
5

Complete the cards to describe the properties for the following solids.

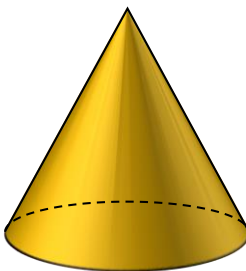


(a) 

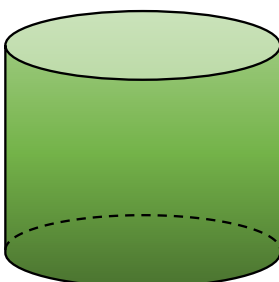
Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(b) 

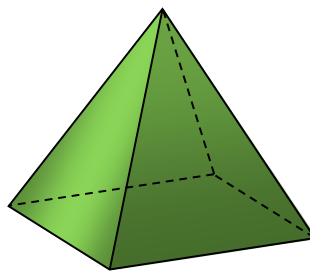
Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(c) 

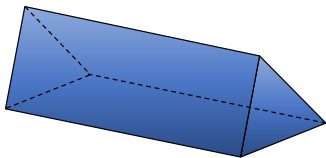
Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(d) 

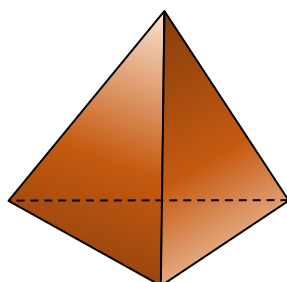
Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(e) 

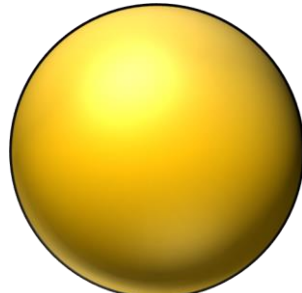
Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(f) 

Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(g) 

Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

(h) 

Name: _____
 Vertices: _____
 Edges: _____
 Faces: _____

Exercise 51

5

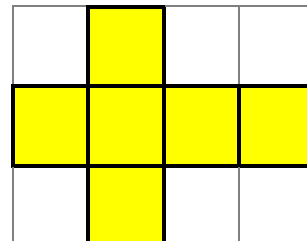
It is possible to calculate **Euler's characteristic** for any solid using the equation

$$\text{Euler's characteristic} = \text{Vertices} + \text{Faces} - \text{Edges.}$$

Investigate Euler's characteristic for the solids on the previous page. Can you explain the pattern that you see?

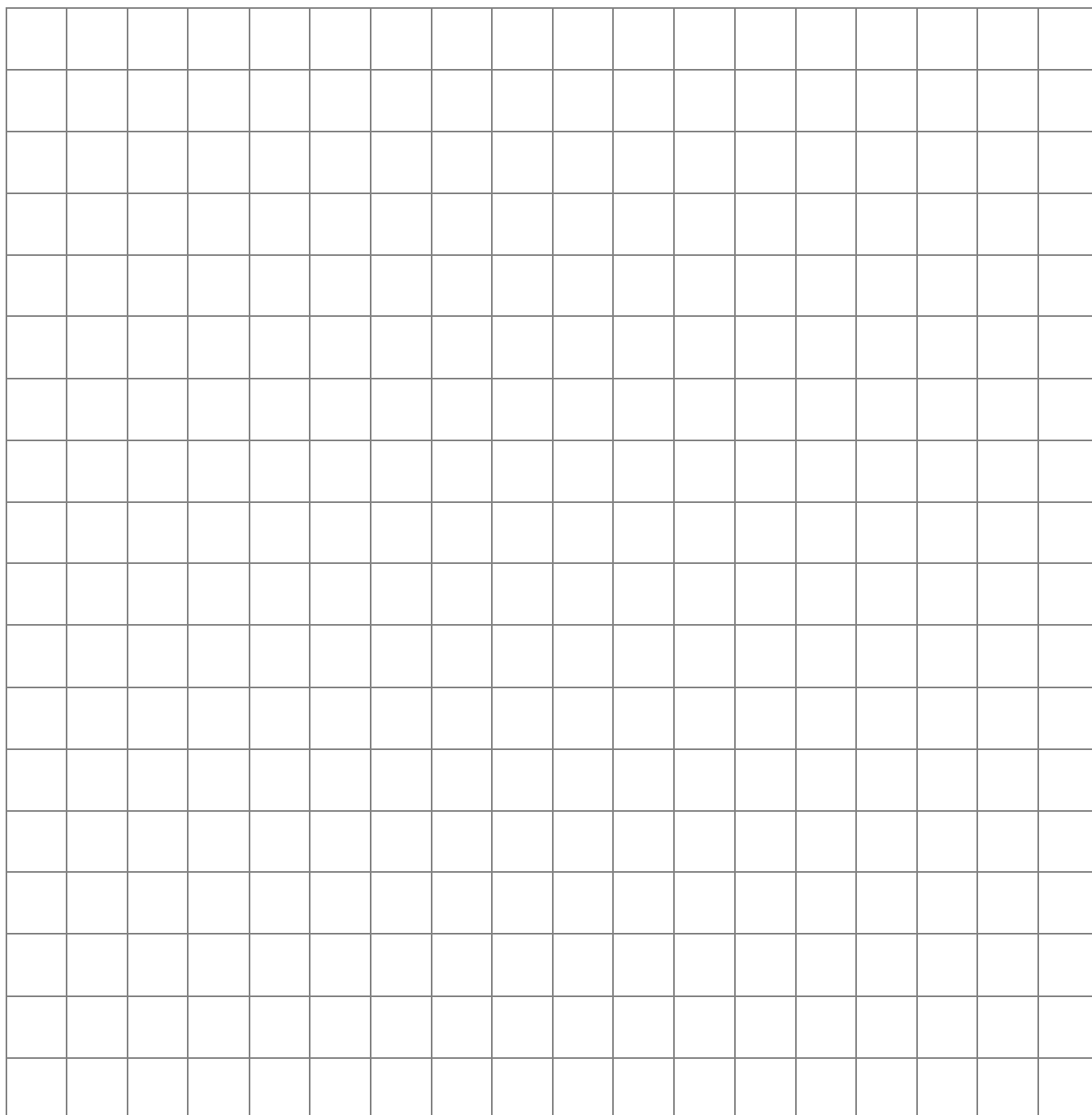
Nets

A **net** is a two-dimensional shape that can be **folded** to create a three-dimensional shape. For example, the shape on the right is a net for a cube.

**Exercise 52**

5

Use the squared paper below to draw the **11 different nets** of a cube.



Exercise 53

5

On a piece of squared paper, use a ruler, pencil and compass to draw nets for the following solids.

- (a) A cube with edges of length 3 cm.
- (b) A cuboid with edges 3 cm, 4 cm, 5 cm.
- (c) A cuboid with edges 4 cm, 4 cm, 7 cm.
- (d) A square-based pyramid where all edges are 5 cm.
- (e) A triangular prism where the length of the prism is 8 cm and the edges of the triangle are 5 cm.
- (f) A tetrahedron where every edge is 7 cm.

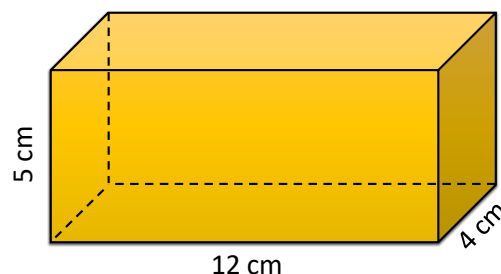
**Exercise 54**

5

Is it possible to draw a net for a cylinder, a cone or a sphere? Use the internet to investigate.

Volume of a Cuboid

$$\text{Volume of a Cuboid} = \text{Length} \times \text{Width} \times \text{Height}$$



Volume measures how much space a solid takes, or uses. For any cuboid, we must multiply the length by the width by the height to find the volume.

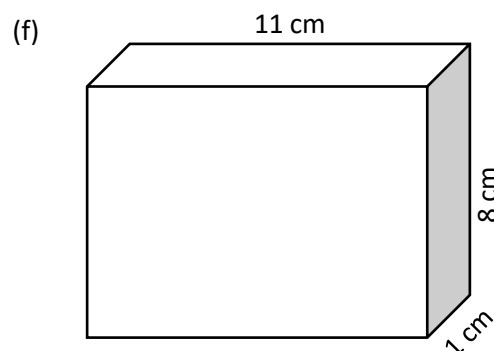
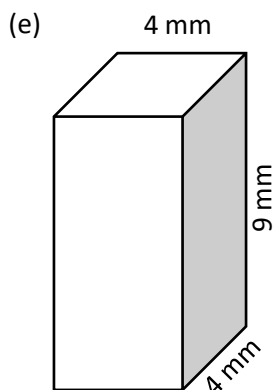
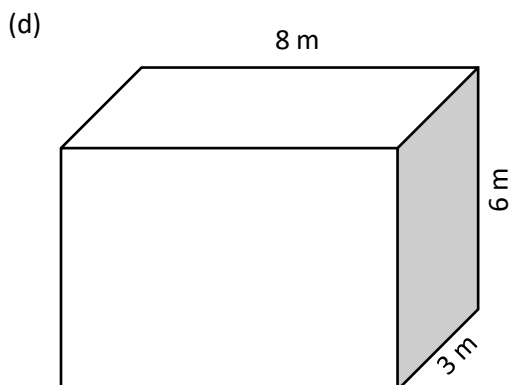
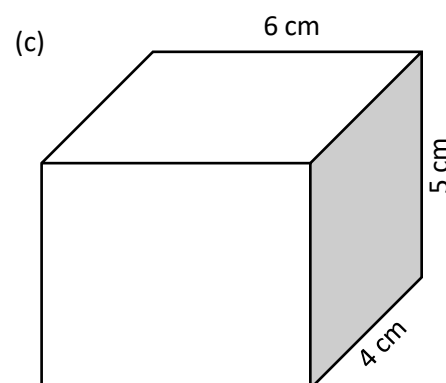
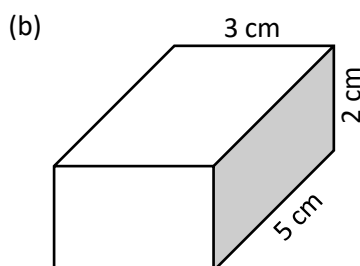
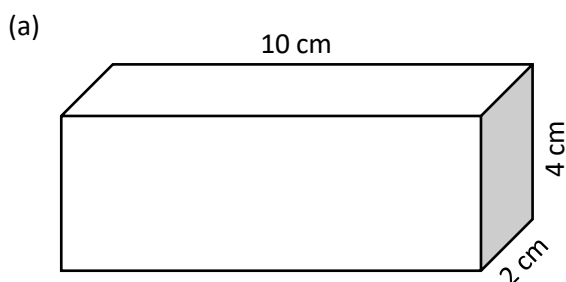
For the cuboid shown on the right, the volume is $4 \times 5 \times 12 = 240 \text{ cm}^3$.

Notice that volume is measured in **cube units**.

**Exercise 55**

5

Find the volume of the following cuboids.



Exercise 56

(a) Dewi wants to put water into the tank shown on the right. Using the fact that **1 ml of water takes 1 cm³ of space**, calculate how much water, in ml, can be put into the tank.

(b) How many litres of water can be put into the tank?

(c) Dewi puts 49 litres of water into the tank. How high does the water reach in the tank?

Exercise 57

Plastic 1 cm cubes are used to make the solid shown on the right.

All of the plastic cubes are rearranged to make a cuboid.

By showing your method, find possible dimensions for the cuboid.

Length = cm Width = cm Height = cm

Exercise 58 (Revision)

(a) How many vertices does a tetrahedron have?

(b) How many faces does a cylinder have?

(c) Sketch a picture of a hexagonal prism. How many vertices, edges and faces does this solid have?

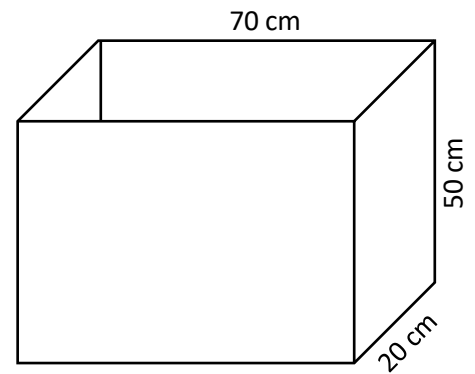
(d) Draw an accurate net for a cuboid with edges 5 cm, 6 cm, 3 cm.

(e) (i) Calculate the volume of the cuboid shown on the right.

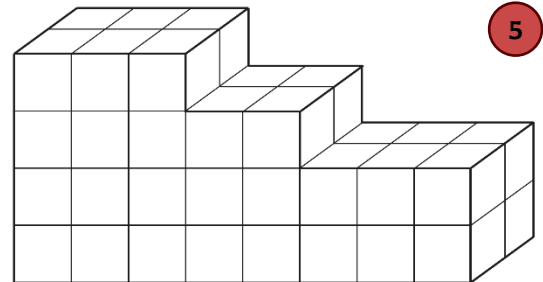
(ii) If 63 ml of water is poured into the cuboid, how high will the water reach?

Applying

5



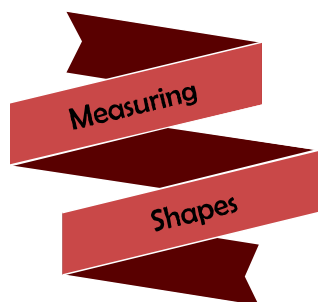
5



5

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I can recall all the facts about metric and Imperial measures .			10	
I can choose the best unit to measure something.			1, 2, 3	
I can convert between different metric and Imperial measurements.			10	
I can read scales accurately.			5	
I can draw a conversion graph .			9	
I can use a conversion graph .			9	
I know the names of the special types of quadrilaterals .			14, 15	
I know the properties of the special types of quadrilaterals .			14, 15	
I can draw a rhombus; kite or arrowhead using a compass and ruler .			16	
I know the total interior angles of any quadrilateral.			18	
I know how to calculate the perimeter and area of simple shapes drawn on squared paper .			6	
I know how to calculate the perimeter and area of a rectangle .			7	
I know how to calculate the perimeter and area of a triangle .			7	
I know how to calculate the perimeter and area of a parallelogram .			17	
I know how to calculate the perimeter and area of a trapezium .			17	
I can estimate the area of any shape drawn on squared paper.			4	
I know the names of eight special types of solid .				
I know how many vertices, edges and faces the eight special types of solid have.			11, 12	
I can recognise whether or not a given shape is a net .			13	
I can draw nets for cubes; cuboids; square-based pyramids; triangular prisms; and tetrahedra .			13	
I know how to calculate the volume of a cuboid .			8	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



Yr Adran Fathemateg

8

Trin Data ac

Ystadegaeth 2

Enw:

Cynnwys

Pennod	Mathemateg	Rhif y Dudalen
Y Modd	Cyfrifo'r modd. Cyfrifo'r modd o dabl. Dosbarth modd. Cymharu dau ddosraniad.	3
Pictogramau	Llunio pictogramau. Dehongli pictogramau.	9
Diagramau Llinellau Fertigol	Adolygu mathau o ddata. Llunio diagramau llinellau fertigol. Dehongli diagramau llinellau fertigol.	12
Graffiau Llinell	Llunio graffiau llinell. Dehongli graffiau llinell. Adolygu arddangos data.	14
Tebygolrwydd Geiriol	Geirfa tebygolrwydd. Graddfa tebygolrwydd. Tebygolrwydd fel rhif.	17



Y Modd

Ym mlwyddyn 7, fe welsom fod hi'n bosib cyfrifo'r **cymedr** ar gyfer set o ddata er mwyn ffeindio **cyfartaledd** ar gyfer y data.

Ymarfer 1

Cyfrifwch y cymedr ar gyfer y setiau canlynol o ddata.

- (a) 2, 3, 1, 6, 3.
- (b) 9, 1, 3, 1, 8, 2, 5, 5, 4, 2.
- (c) 1.9, 2.4, 0.3, 1.6, 3.8.
- (ch) 56, 32, 80.
- (d) 3, 2, 5, 1, 3, 2, 1, 2, 1, 3, 4, 2, 0, 2, 4, 1, 5, 4, 2, 3.

Mae'r **modd** yn gyfartaledd gwahanol ar gyfer set o ddata. Er mwyn cyfrifo'r **modd**, rhaid edrych am y gwerth data sy'n ymddangos **mwyaif aml**. Mae'n bosib bydd gan set o ddata **mwyaif nag un** modd, neu **dim modd** o gwbl os yw'r gwerthoedd data i gyd yn wahanol.

Enghraifft

- | | |
|------------------|--|
| 8, 3, 2, 4, 5, 3 | Y modd yw 3 gan fod 3 yn ymddangos dwywaith yn y data ac mae pob rhif arall yn ymddangos unwaith yn unig. |
| 3, 4, 1, 3, 2, 4 | Y modd yw 3 a 4 gan fod y rhifau yma'n ymddangos dwywaith yn y data ac mae pob rhif arall yn ymddangos unwaith yn unig. |
| 8, 2, 4, 5, 1, 6 | Does dim modd gan fod pob rhif yn ymddangos unwaith yn unig. |

Ymarfer 2

Cyfrifwch y modd ar gyfer y setiau canlynol o ddata.

- (a) 7, 3, 2, 4, 3, 1, 8
- (b) 24, 32, 21, 23, 28, 24, 23
- (c) 5, 2, 8, 9, 1, 4, 8, 2, 4, 8
- (ch) 16, 19, 13, 12, 12, 14, 19, 13, 15, 17
- (d) 34, 21, 18, 42, 58, 43
- (dd) 1, 1, 2, 2, 2, 3, 3, 3, 4, 5, 5, 5, 6, 6, 7, 8, 8, 8, 9, 9

Ymarfer 3

Cyfrifwch y cymedr ar gyfer y setiau data o Ymarfer 2.

Ymarfer 4

Cyfrifwch y modd ar gyfer y setiau canlynol o ddata.

- (a) Coch, Glas, Melyn, Du, Gwyrdd, Glas, Piws, Pinc.
- (b) Lerpwl, Manceinion, Llundain, Caerdydd, Abertawe.
- (c) Bob, Elfed, Mari, Ceinwen, Mair, Dewi, Mari, Sophie, Mair, Seren, Dewi.

(ch) 😎, 😂, 😜, 😄, 😍, 😬.

(d) Tost, Rice Krispies, Wy, Corn Flakes, Frosties, Rice Krispies, Wy, Coco Pops.

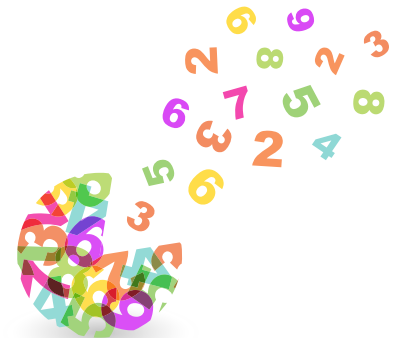
(dd) Fiat, Audi, Ford, Vauxhall, Seat, BMW, Mini, Peugeot, Ford, Mercedes, Ferrari, Seat, Toyota, Honda.

Ymarfer 5

Eglurwch pam nad yw'n bosib cyfrifo'r cymedr ar gyfer y setiau data yn Ymarfer 4.

Adolygu

5



Sgîl

4



5

4



5

Ymarfer 6

Defnyddio

5



(a) Deintydd yw Dylan. Mae angen iddo adrodd yn ôl i'w rheolwr ar nifer cyfartalog y llenwadau (*fillings*) mae pob cwsmer yn ei dderbyn tra'n ymweld â'r deintydd. Dros gyfnod o ddeuddydd, mae Dylan yn cofnodi'r nifer y llenwadau mae pob cwsmer yn ei dderbyn ganddo.

2, 1, 0, 0, 3, 1, 3, 1, 1, 2, 1, 1, 4, 1, 0, 2, 1, 2, 0, 4

Cyfrifwch y modd a'r cymedr ar gyfer data Dylan. Pa gyfartaledd dylai Dylan ddefnyddio yn ei adroddiad, o wybod bod Dylan eisiau defnyddio'r cyfartaledd isaf posib?

(b) Mae Heledd yn hyfforddi'r tîm pêl-rwyd lleol, ac eisiau gwella eu ffitrwydd. Mae'n dod â'r tîm i'r gampfa ac yn gofyn iddynt wneud gymaint o fyrfreichiau (*press-ups*) ac y gallent mewn munud. Dyma'r canlyniadau.

40, 34, 28, 40, 38, 42, 30

Cyfrifwch y modd a'r cymedr ar gyfer data Heledd. Mae Heledd yn dymuno perswadio rheolwr y tîm y dylai'r tîm hyfforddi mwy. Pa gyfartaledd dylai Heledd ddewis er mwyn gwneud hyn?



(c) Mae Geraint yn yrrwr fan ddosbarthu. Dros gyfnod o 10 diwrnod, mae'n cofnodi'r nifer o barseli mae'n dosbarthu'n ddyddiol i gwsmeriaid.

94, 104, 120, 98, 112, 120, 96, 113, 98, 105

Mae Geraint yn cyfrifo'r modd ar gyfer ei ddata.

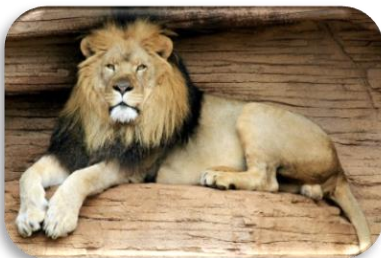
(i) Beth yw modd Geraint?

(ii) Pam y gall Geraint gael ei ddrysus gan ei fodd? Beth y dylai ei wneud i geisio delio â hyn?

(ch) Mae Sarah yn yrrwr ambiwlans sydd angen adrodd yn ôl i'r papur newydd lleol am amser ymateb cyfartalog y gwasanaeth ambiwlans. Dyma amser ymateb Sarah (mewn munudau) ar gyfer ei chwe galwad diwethaf.

24, 30, 18, 38, 24, 40

Rhowch gyngor i Sarah ar beth i ddweud wrth y papur newydd lleol.



(d) Mae Eleri yn gweithio mewn sw. Mae ei rheolwr yn meddwl bod Eleri yn gor-fwydo'r llewod dan ei gofal. Dyma bwysau'r llewod dan ofal Eleri, mewn cilogramau.

256, 278, 265, 262, 241, 259, 274, 230, 265, 270

Pa gyfartaledd dylai Eleri ddefnyddio er mwyn ceisio perswadio'i rheolwr nad yw'n gor-fwydo'r llewod?

(dd) Gofynnodd Arwyn i'w ffrindiau faint o arian poced roedden nhw'n ei dderbyn bob wythnos. Ysgrifennodd eu hatebion:

£2, £4, £2, £5, £4, £2, £2, £3

Mae Arwyn yn dymuno perswadio'i rieni y dylai dderbyn mwy o arian poced bob wythnos. Pa gyfartaledd dylai ei ddefnyddio i geisio gwneud hyn?



Ymarfer 7

5

Dyma ychydig o wybodaeth am saith person.



Roger 42 oed	Bethan 35 oed	Elis 24 oed	Steve 26 oed	Beca 23 oed	Will 35 oed	Catrin 25 oed
Maint esgid: 8	Maint esgid: 7	Maint esgid: 9	Maint esgid: 7	Maint esgid: 6	Maint esgid: 6	Maint esgid: 4
Pwysau: 50 kg	Pwysau: 43 kg	Pwysau: 52 kg	Pwysau: 58 kg	Pwysau: 38 kg	Pwysau: 62 kg	Pwysau: 40 kg

- Beth yw oed moddol y 7 person yma?
- Pwy sydd yn hynach na'r oed moddol?
- Sawl person sydd yn ieuengach na'r oed moddol?
- Beth fydd oed moddol y 7 person yma ymhen blwyddyn union?
- Beth yw maint esgid moddol y 7 person yma?
- Beth yw amrediad maint esgid y 7 person yma?
- Beth yw modd pwysau'r 7 person yma?
- Beth yw cymedr pwysau'r 7 person yma?
- Pa berson sydd yn pwysu agosaf at y pwysau cymedrig?
- Sawl person sydd yn pwysu'n fwy na'r pwysau cymedrig?
- Mae pawb yn bwyta gormod dros y Nadolig ac yn rhoi 3 kg o bwysau ymlaen. Beth yw cymedr newydd pwysau'r 7 person yma?

Cyfrifo'r modd o dabl

Enghraifft

Mae'r tabl canlynol yn dangos y nifer o chwirydd sydd gan ddosbarth o 28 o blant.

Nifer o chwirydd	Amllder
0	9
1	10
2	6
3	3



Y modd ar gyfer y nifer o chwirydd yw 1, gan fod gan fwy o blant un chwaer nag unrhyw werth arall.



Ymarfer 8

4

Mae'r tabl canlynol yn dangos maint esgid dysgwyr 8C.

Maint esgid	Nifer o blant
3	2
4	5
5	6
6	9
7	3
8	0
9	1



- (a) Beth yw maint esgid moddol y dosbarth?
 (b) Sawl person yn y dosbarth sydd efo maint esgid mwy na'r maint esgid moddol?

Ymarfer 9

5

Holwyd 50 person ar y stryd sawl gwaith oeddent wedi bod i'r sinema yn ystod y chwe mis diwethaf.

Nifer o weithiau	Amllder
0	18
1	13
2	11
3	3
4	5



- (a) Beth yw'r modd ar gyfer y nifer o ymweliadau i'r sinema?
 (b) A yw'r mwyafrif (mwy na hanner) o bobl yn yr arolwg wedi bod i'r sinema yn ystod y chwe mis diwethaf?
 (c) Beth yw'r cymedr ar gyfer y nifer o ymweliadau i'r sinema?
 (ch) Mae rheolwr sinema eisiau defnyddio'r data i geisio dangos bod mynd i'r sinema yn weithgaredd poblogaidd. A fydddech yn awgrymu i'r rheolwr ddefnyddio'r cymedr neu'r modd? Eglurwch eich ateb.

Ymarfer 10

Holwyd 40 person mewn archfarchnad pa dîm pêl-droed oeddent yn eu cefnogi.

Tîm pêl-droed	Amllder
Lerpwl	12
Manceinion Unedig	14
Everton	3
Chelsea	2
Wrecsam	1
Ddim yn hoffi pêl-droed	8

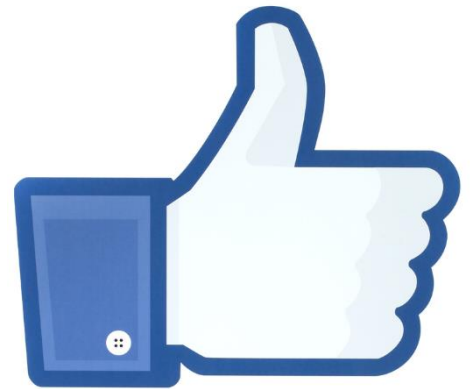
- (a) Pa dîm pêl-droed oedd yr un mwyaf poblogaidd ymysg y 40 person yma?
 (b) Pa gyfartaledd wnaethoch ddewis ar gyfer ateb rhan (a) o'r cwestiwn?
 (c) Sut fyddai canlyniad yr arolwg yn wahanol pe bai'n cael ei gynnal tu allan i stadiwm pêl-droed, yn hytrach na mewn archfarchnad?



Dosbarth Modd

Mae'r tabl canlynol yn dangos sawl "ffrind" sydd gan set o rieni ar [facebook](#).

Ffrindiau (f)	Amllder
$1 \leq f \leq 50$	23
$51 \leq f \leq 100$	45
$101 \leq f \leq 150$	112
$151 \leq f \leq 200$	24
$201 \leq f \leq 250$	18
$251 \leq f \leq 300$	6



Y dosbarth efo'r amllder mwyaf yw'r dosbarth $101 \leq f \leq 150$, felly hwn yw'r **dosbarth modd**.

Ymarfer 11

7

Darganfyddwch y dosbarth modd ar gyfer y setiau canlynol o ddata.

(a) Pwysau'r parseli mewn fan ddosbarthu ddoe.

(b) Taldra 50 o ddisgyblion.

(c) Nifer y llawrlwythiadau yr awr o ap penodol dros gyfnod o 24 awr.

Pwysau (p kg)	Amllder
$0 \leq p < 5$	23
$5 \leq p < 10$	10
$10 \leq p < 15$	5
$15 \leq p < 20$	6
$20 \leq p < 25$	2

Taldra (t cm)	Amllder
$168 \leq t < 172$	5
$172 \leq t < 176$	9
$176 \leq t < 180$	12
$180 \leq t < 184$	16
$184 \leq t < 188$	7
$188 \leq t < 192$	1

Llawrlwythiadau (l)	Amllder
$0 \leq l < 1,000$	2
$1,000 \leq l < 2,000$	4
$2,000 \leq l < 3,000$	3
$3,000 \leq l < 4,000$	4
$4,000 \leq l < 5,000$	5
$5,000 \leq l < 6,000$	5
$6,000 \leq l < 7,000$	1

Ymarfer 12

Darganfyddwch amcangyfrif ar gyfer y cymedr ar gyfer y setiau data yn Ymarfer 11.

Ymarfer 13

Mae'r ffigurau hyn yn rhoi'r amser, i'r eiliad agosaf, a gymerodd 40 o redwyr i gwblhau ras 400 m.

62	67	82	73	65	59	71	83	68	82
73	68	73	58	93	86	79	80	73	64
68	74	94	86	84	71	70	64	68	62
72	64	71	78	83	80	64	73	79	84

Amser (a eiliad)	Marciau Rhifo	Amllder
$50 \leq a < 60$		
$60 \leq a < 70$		
$70 \leq a < 80$		
$80 \leq a < 90$		
$90 \leq a < 100$		

(a) Cwblhewch y **tabl amllder** ar gyfer y data.

(Cofiwch byddai eitem data 70 eiliad yn mynd i'r dosbarth $70 \leq a < 80$, nid $60 \leq a < 70$.)

(b) Cyfrifwch y dosbarth modd ar gyfer y data.

(c) Lluniwch ddiagram amllder ar gyfer y data.

Set 1 yn unig

7



Cymharu dau ddosraniad

Ymarfer 14

(a) Dyma'r nifer y goliau y sgoriodd dau dîm pêl-droed mewn gemau ar ddechrau'r tymor.

Lerpswl	2	1	1	0	2	3	2	0
Everton	0	1	2	1	0	0	1	3



Defnyddiwch y modd i gymharu perfformiad y ddau dîm pêl-droed ar ddechrau'r tymor.

(b) Mae Dilys a Gwenda yn cymryd rhan mewn cwis wythnosol ble mae'n rhaid ateb 20 cwestiwn gwahanol.

Dyma faint o gwestiynau y gwnaeth y ddwy gael yn gywir dros y deg wythnos diwethaf.

Dilys	18	14	16	11	17	18	13	15	20	18
Gwenda	19	13	16	20	17	15	16	18	17	16

Defnyddiwch y modd i geisio penderfynu pwy yw'r person gorau am ateb cwestiynau yn y cwis.

(c) Aeth Janet a Steffan i ddwy archfarchnad gwahanol i holi pobl faint fyddent yn fodlon talu am pizza 10 modfedd.

Dyma’u canlyniadau, wedi’u rhoi i’r £1 agosaf.

Janet	£3	£4	£2	£5	£4	£2	£4	£4
Steffan	£3	£2	£1	£2	£1	£3	£2	£4

(i) Cyfrifwch y modd ar gyfer data Janet ac ar gyfer data Steffan.

(ii) Gwanethpwyd un o'r holiaduron yn archfarchnad Lidl, a'r llall yn archfarchnad Marks and Spencer. Defnyddiwch eich atebion i (i) i geisio dweud pwy oedd ym mha archfarchnad. Eglurwch eich ateb.

Sialens!



Mae Jake yn meddwl am 6 rhif cyfan.

- Modd rhifau Jake yw 5 ag 8.
- Cymedr rhifau Jake yw 5.
- Mae pob un o rhifau Jake yn fwy na sero.

Pa chwe rhif mae Jake wedi meddwl amdanynt?



Gwerthuso

Geirfa Allweddol	Cwestiynau Pellach	Beth aeth yn dda?	I gyrraedd fy lefel darged mi wnâi...
			Fy Lefel ☐ Fy Nharged ☐

Pictogramau

Mae **pictogram** yn arddangos data gan ddefnyddio **symbol** i gynrychioli'r data. Rhaid cofio cynnwys **allwedd** i nodi faint o ddata mae'r symbol yn ei gynrychioli.

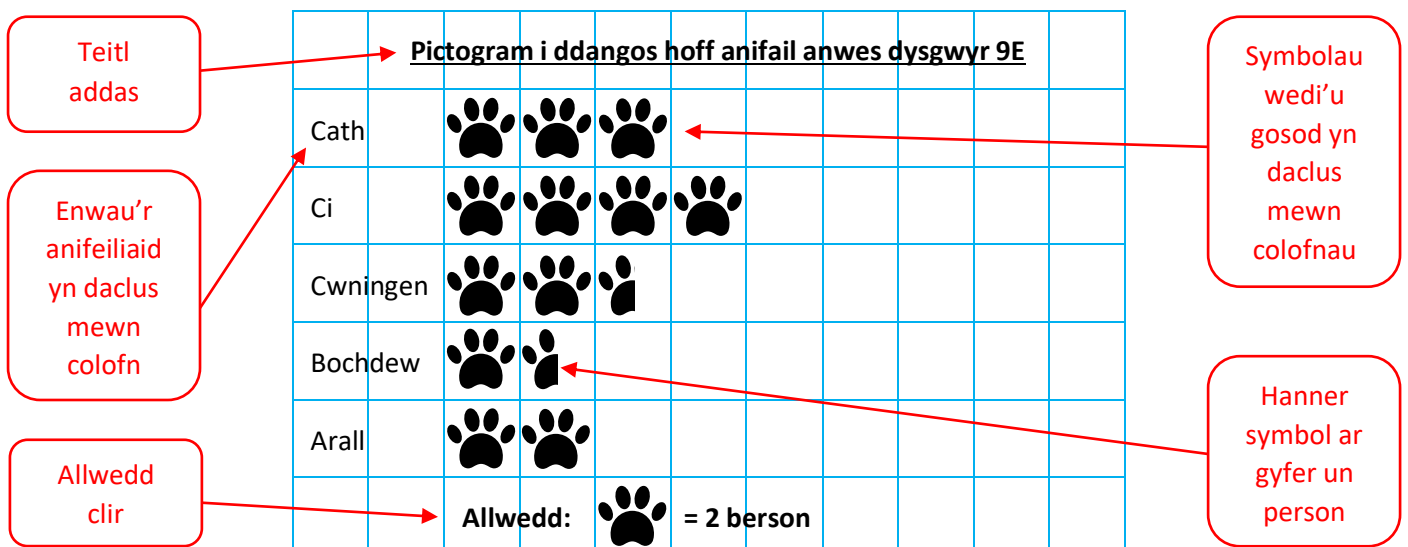
Enghraifft

Mae'r data canlynol yn dangos hoff anifail anwes dysgwyr 9E.

Anifail	Amllder
Cath	6
Ci	8
Cwningen	5
Bochdew	3
Arall	4



Dyma bictogram i arddangos y data.



Ymarfer 15

Lluniwch bictogramau ar gyfer y setiau o ddata canlynol.

(a) Hoff ddiod 10R

Diod	Amllder
Coke	6
Dŵr	7
Llefrith	3
Sudd Oren	2
Diod Arall	5

Coke											
Dŵr											
Llefrith											
Sudd Oren											
Diod Arall											
Allwedd:				= 2 berson							

Sgîl

3

(b) Lliw y ceir a yrrodd heibio siop mewn awr.

Lliw	Amllder
Coch	16
Gwyn	12
Glas	10
Llwyd	13
Gwyrdd	19
Arall	6

Allwedd:  = 4 car

(c) Hoff fath o lyfr 9C.

Math o lyfr	Amllder
Antur	12
Ffuglen Wyddonol	11
Rhamant	3
Ffeithiol	1

Allwedd:  = 2 berson

(ch) Nifer y myfyrwyr oedd yn absennol o'r ysgol wythnos diwethaf.

Diwrnod	Amllder
Dydd Llun	20
Dydd Mawrth	22
Dydd Mercher	16
Dydd Iau	13
Dydd Gwener	19

Allwedd:  = 4 person

Lluniwch, yn eich llyfr, bictogramau ar gyfer y setiau o ddata canlynol. (Dewiswch allwedd addas ar gyfer y data.)

(c) Aelodaeth clybiau 8C.

Clwb	Amllder
Gymnasteg	8
Nofio	6
Pêl-droed	10
Drama	7

Edrychwch ar y pictogram canlynol.

<u>Pictogram i ddangos tywydd Llandudno yn ystod un mis yn 2017</u>									
Heulog									
Cymylog									
Glawog									
Eira									
		Allwedd:		= 2 ddiwrnod					



(d) Beth oedd y tywydd moddol yn ystod y mis yma?

Gwerthuso

Geirfa Allweddol	Cwestiynau Pellach	Beth aeth yn dda?	I gyrraedd fy lefel darged mi wnâi...
			Fy Lefel ☐ Fy Nharged ☐

Diagramau Llinellau Fertigol

Ymarfer 18

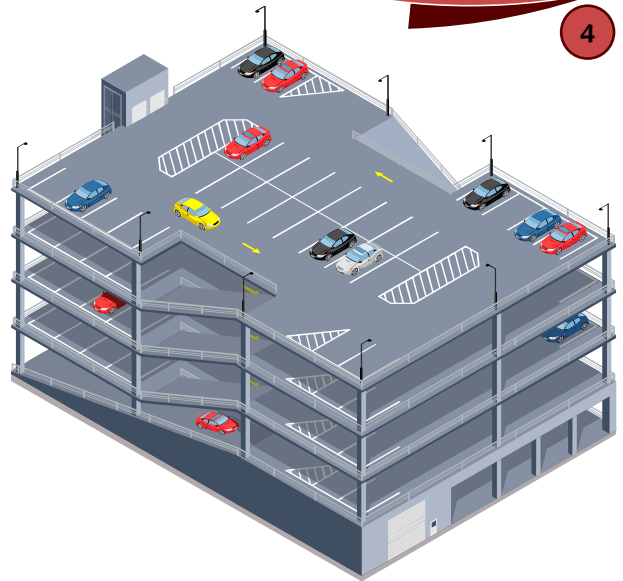
Pa fath o ddata yw'r canlynol?

- (a) Enw canol y myfyrwyr mewn dosbarth.
- (b) Taldra blodau mewn planhigfa.
- (c) Nifer y lluniau ar ddeg ffôn symudol.
- (ch) Mâs ugain bloc o gaws.
- (d) Gwneuthurwr y ceir mewn maes parcio.
- (dd) Nifer y llythrennau mewn geiriau ar dudalen llyfr.



Adolygu

4



Llunio Diagram Llinellau Fertigol

Rydym yn llunio **diagram llinellau fertigol** ar gyfer y mathau canlynol o ddata.

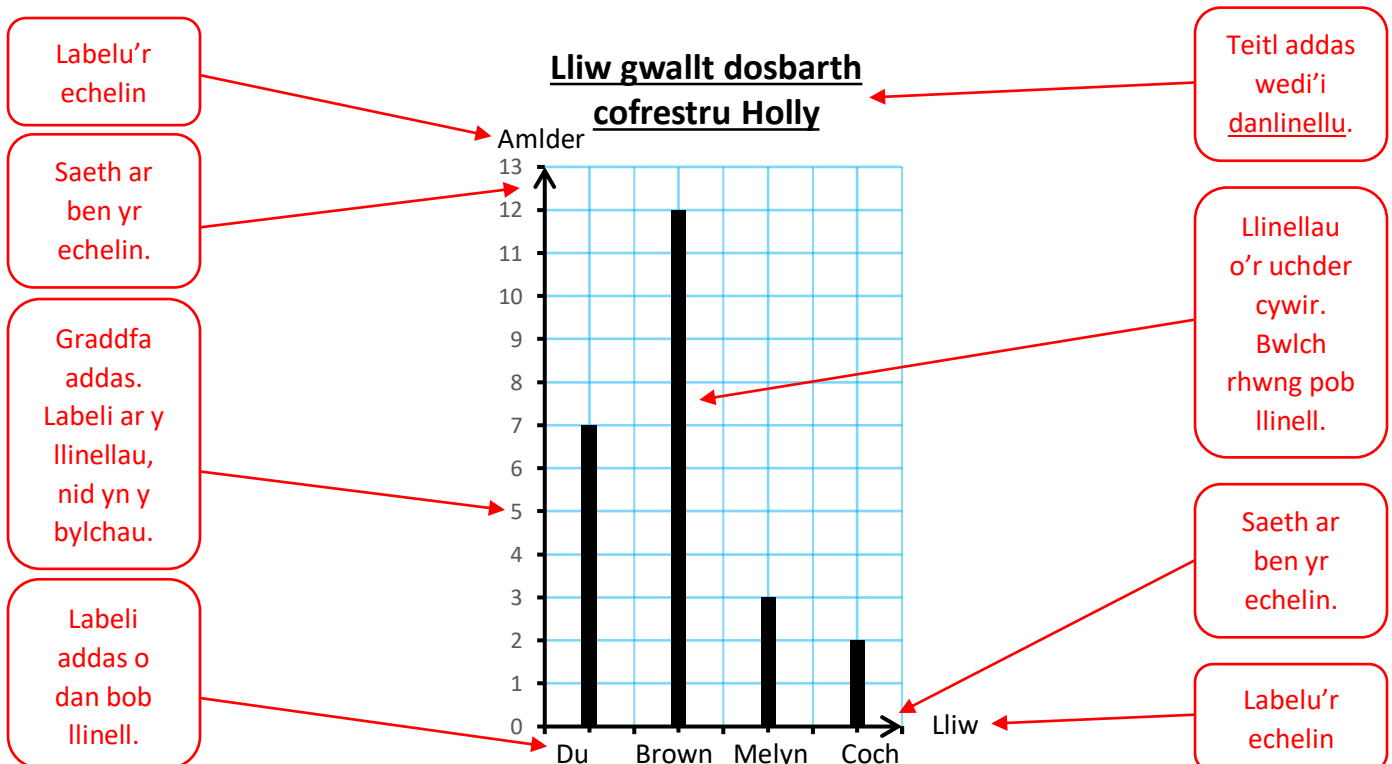
- Data Ansoddol.
- Data Meintiol Arwahanol heb ei grwpio.

Enghraifft

Gwnaeth Holly arolwg o liw gwalltiau ei dosbarth cofrestru. Dyma'r canlyniadau:

Du 7 Brown 12 Melyn 3 Coch 2

Lluniodd Holly ddiagram llinellau fertigol ar gyfer ei data.



Sgìl

4

Celf: 2

Pedwar: 2

Jenga: 3

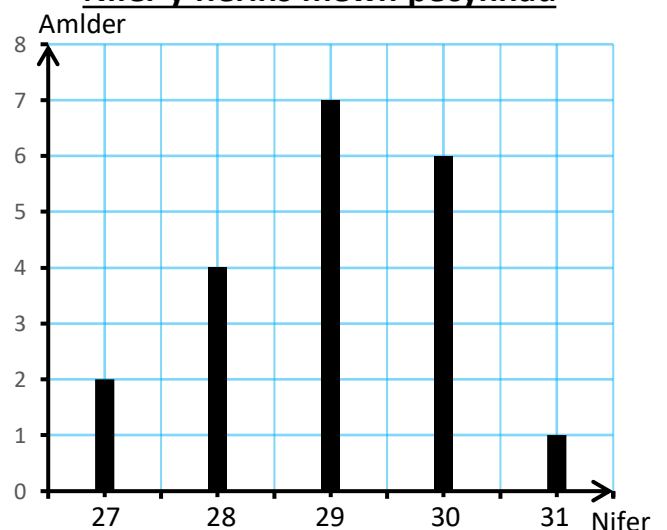
Mwy na tri: 3

Coctel corgimwch: 10

Defnyddio

5

(f) Beth oedd yr amrediad ar gyfer nifer o fferins mewn
paced?



Gwerthuso

Geirfa Allweddol	Cwestiynau Pellach	Beth aeth yn dda?	I gyrraedd fy lefel darged mi wnâi...

Graffiau Llinell

Llunio Graffiau Llinell

Rydym yn llunio **graft llinell** ar gyfer y mathau canlynol o ddata.

- Data Ansoddol.
- Data Meintiol Arwahanol heb ei grwpio.



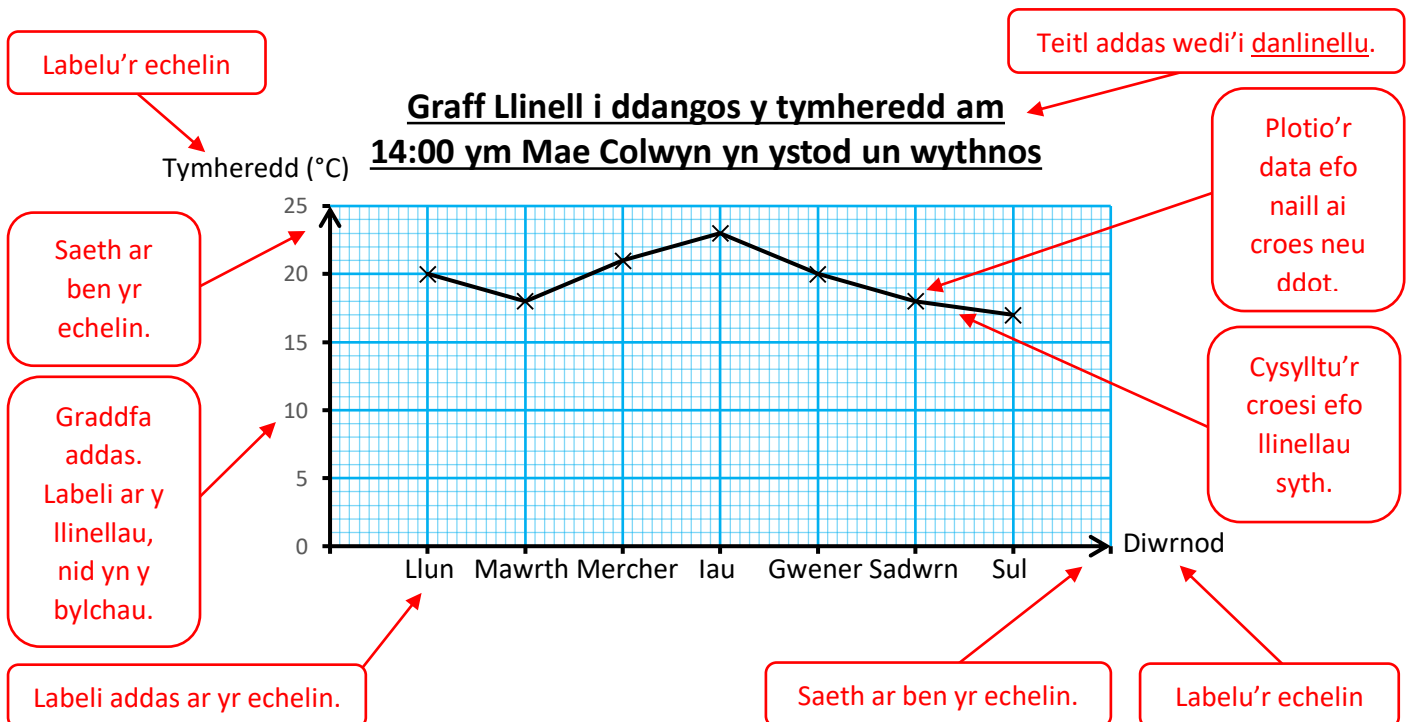
Caiff graft llinell ei lunio trwy blotio pwyntiau ac yna'u cysylltu â phren mesur.

Enghraifft

Mae Nel yn cofnodi'r tymheredd ym Mae Colwyn am 14:00 bob dydd am wythnos. Dyma'i chanlyniadau.

Diwrnod	Llun	Mawrth	Mercher	Iau	Gwener	Sadwrn	Sul
Tymheredd	20°C	18°C	21°C	23°C	20°C	18°C	17°C

Mae Nel yn penderfynu dangos y data mewn graft llinell.



Ymarfer 21

Lluniwch graffiau llinell ar gyfer y setiau data canlynol.

(a) Y tymheredd ym Metws-y-coed am 12:00 bob dydd yn ystod un wythnos.

Diwrnod	Llun	Mawrth	Mercher	Iau	Gwener	Sadwrn	Sul
Tymheredd	14°C	12°C	13°C	17°C	17°C	16°C	12°C

(b) Estyniad sbring o'i gymharu â'r màs a roddwyd arno.

Màs a roddwyd ar y sbring (kg)	0	1	2	3	4	5	6
Hyd y sbring (cm)	2	3	4	5	6	7	8

Sgîl

4

(c) Taldra planhigyn Lowri.

Wythnos	0	1	2	3	4	5	6
Taldra (mm)	0	10	28	32	40	44	46

(ch) Buanedd y gwynt yn ystod un diwrnod.

Amser	6am	7am	8am	9am	10am	11am	12pm	1pm	2pm	3pm	4pm	5pm	6pm
Buanedd y gwynt (not)	4	2	4	8	12	16	22	24	26	28	24	20	18

(d) Cyfanswm nifer y gwylwyr ar fideo newydd ar sianel YouTube yr ysgol.

Diwrnod	1	2	3	4	5	6	7	8	9	10
Cyfanswm nifer y gwylwyr	4	26	38	46	48	50	50	54	55	55

(dd) Nifer y goliau sgoriodd Gareth Bale mewn 8 gêm ar ddechrau'r tymor.

Gêm	1	2	3	4	5	6	7	8
Goliau	0	1	0	2	1	0	3	1

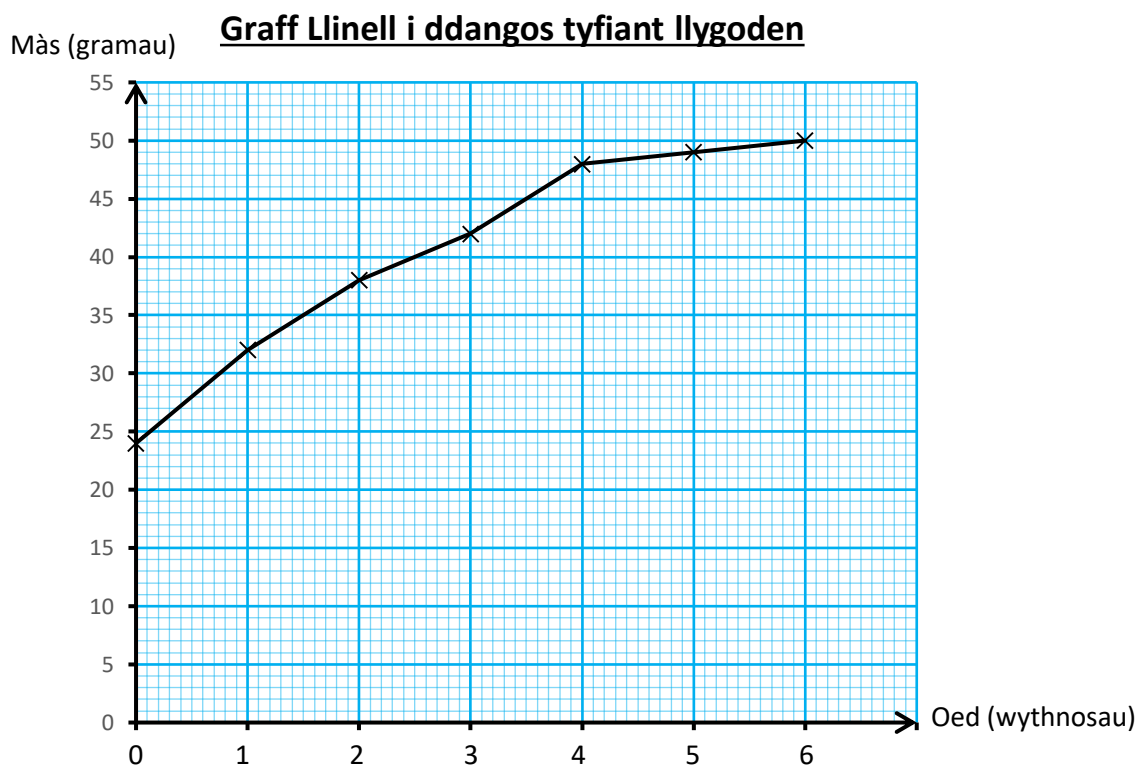
Dehongli Graffiau Llinell

Ymarfer 22

Edrychwch ar y graff llinell ganlynol, sy'n dangos tyfiant llygoden.

Defnyddio

5



(a) Beth oedd màs y llygoden ar ddiwedd 3 wythnos?

(b) Beth oedd oed y llygoden pan oedd ei màs yn 49 gram?

(c) Rhwng pa ddwy wythnos oedd y cynnydd mwyaf mewn màs? Beth oedd y cynnydd yma?

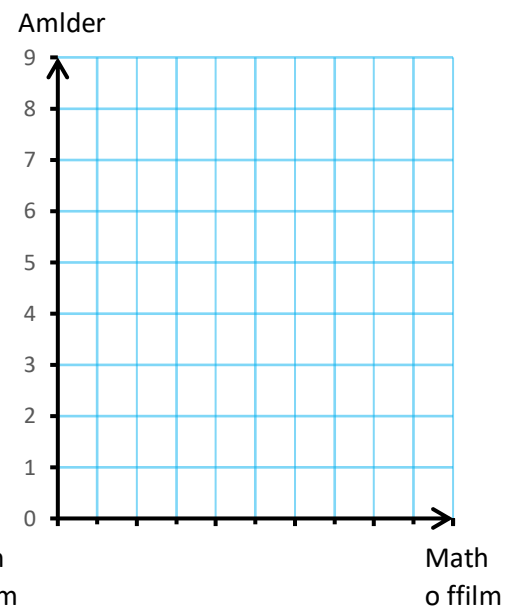
(ch) Amcangyfrifwch fàs y llygoden ar ôl $3\frac{1}{2}$ wythnos.

Adolygu

5

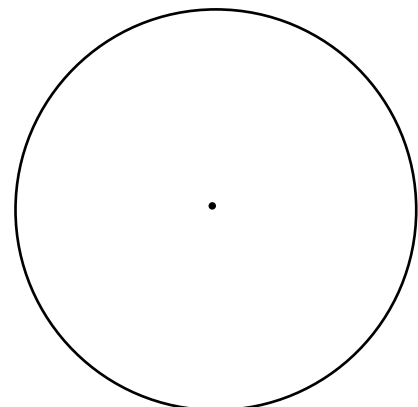
Arswyd: 3

Siart Bar i ddangos hoff
fath o ffilm 15 o bobl



Siart Cylch i ddangos
hoff fath o ffilm 15 o bobl

	<u>Pictogram i ddangos hoff fath o ffilm 15 o bobl</u>									
Animeiddiad										
Comedi										
Arswyd										
				Allwedd:		= 2 berson				



Gwerthuso

Geirfa Allweddol	Cwestiynau Pellach	Beth aeth yn dda?	I gyrraedd fy lefel darged mi wnâi...
			Fy Lefel ☐ Fy Nharged ☐

Tebygolrwydd Geiriol



Mae **tebygolrwydd** yn dweud wrthym pa mor debygol yw rhywbeth o ddigwydd.

Mae'n bosib defnyddio'r eirfa ganlynol i ddisgrifio tebygolrwydd penodol.



Ymarfer 24

Trafodwch efo partner sut fydddech yn trefnu'r eirfa uchod, o'r lleiaf tebygol i'r mwyaf tebygol.

Ymarfer 25

Defnyddiwch "dim siawns"; "siawns isel"; "siawns deg"; "siawns dda" neu "sicr" i ddisgrifio tebygolrwydd y digwyddiadau canlynol.

Defnyddio

- (a) Bydd dydd Nadolig ar Ragfyr 25 y flwyddyn nesaf.
- (b) Bydd y Pennaeth yn byw i fod yn 200 oed.
- (c) Bydd o leiaf un diwrnod o wyliau'r haf yn heulog.
- (ch) Cael "7" wrth daflu dis.
- (d) Bydd y person nesaf i chi gyfarfod yn rhannu'r un pen-blwydd â chi.
- (dd) Byddwch yn cael "Pen" wrth daflu darn arian.
- (e) Byddwch yn cael sglodion i swper heno.
- (f) Bydd o leiaf un diwrnod sych yn ystod mis Mai flwyddyn nesaf.
- (ff) Bydd y gloch yn canu am 15:30 heddiw.
- (g) Bydd y person nesaf i chi weld ar ôl gadael yr ystafell ar ddiwedd y wers yn fachgen.
- (ng) Bydd estronwyr (*aliens*) yn glanio ar gae'r ysgol amser cinio yfory.
- (h) Bydd Lerpwl yn ennill yr Uwch Gynghrair eleni.
- (i) Mae'n bosib cael sgôr o 61 ar fwrdd dartiau gan ddefnyddio dim ond un dart.

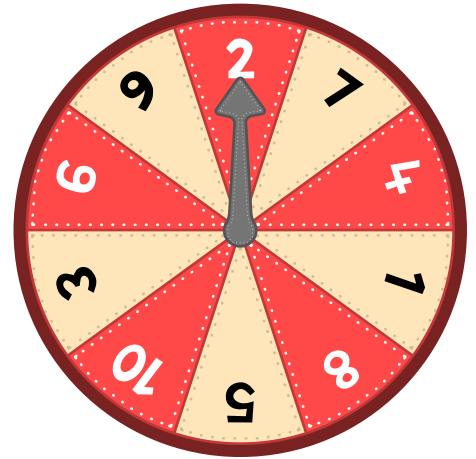


Ymarfer 26

Dewiswch y gair gorau, o'r rhai sydd wedi'u rhoi isod, i ddisgrifio'r siawns y bydd pob un o'r digwyddiadau canlynol yn digwydd.

amhosibl annhebygol siawns deg tebygol sicr

- (a) Pan gaiff 3 dis eu taflu bydd sgôr o 1 yn cael ei dangos ar bob dis.
- (b) Bydd tu chwith (*tails*) yn cael ei ddangos pan gaiff darn 2 geiniog ei daflu unwaith.
- (c) Pan gaiff y troellwr ar y dde ei droi unwaith bydd yn glanio ar 10.
- (ch) Pan gaiff y troellwr ar y dde ei droi unwaith bydd yn glanio ar 0.
- (d) Pan gaiff y troellwr ar y dde ei droi unwaith bydd yn glanio ar eilrif.
- (dd) Mehefin fydd o leiaf un o'r misoedd yn y 14 mis nesaf.
- (e) Ennill raffl pan fyddwch wedi prynu pump o'r 300 o docynnau sydd wedi'u gwerthu.
- (f) Bydd y babi cyntaf i gael ei eni yfory, mewn ysbyty, yn fachgen.
- (ff) Byddwch chi'n cael deg pan fydd dis 6-ochr teg â'r rhifau 1 i 6 arno yn cael ei rolïo.
- (g) Ni chafodd person sy'n cael ei ddewis ar hap ei eni ar benwythnos.

**Ymarfer 27**

Mae gan Anna braidd (*flock*) o 70 o ddefaid. Mae gan 35 ohonyn nhw wynebaw gwyn ac mae gan 35 ohonyn nhw wynebaw du. Mae Anna yn dewis un o'i defaid o'r praid hwn ar hap.

Dewiswch yr ymadrodd gorau o'r rhai sydd wedi'u rhoi isod i ddisgrifio'r siawns bod Anna yn dewis dafad sydd ag wyneb du.

amhosibl annhebygol siawns gyfartal tebygol sicr

**Ymarfer 28**

Mae gan Owen flwch sy'n cynnwys 30 o beli. Mae 10 ohonyn nhw'n lliw oren ac mae 20 yn lliw gwyn. Mae Owen yn dewis un bêl ar hap o'r blwch hwn.

Dewiswch yr ymadrodd gorau o'r rhai sydd wedi'u rhoi isod i ddisgrifio'r siawns bod Owen yn dewis pêl lliw oren.

dim siawns annhebygol siawns deg tebygol sicr

**Ymarfer 29**

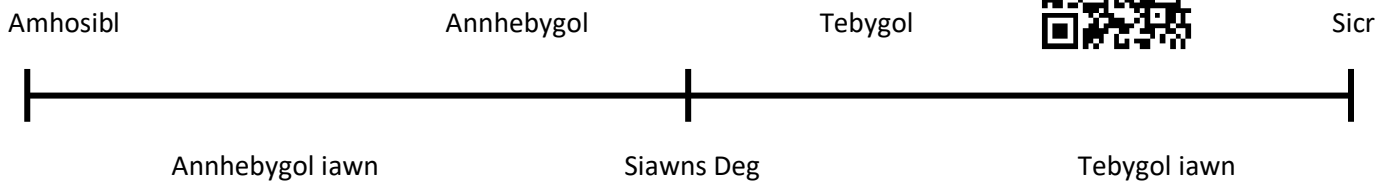
Ysgrifennwch dri chwestiwn ar gyfer bob un o'r tebygolrwyddau canlynol. (Cychwynnwch bob cwestiwn efo'r geiriau "Beth yw'r tebygolrwydd...".)

- (a) Amhosibl
- (b) Annhebygol iawn
- (c) Siawns Deg
- (ch) Tebygol
- (d) Sicr



Graddfa Tebygolrwydd

Mae'n bosib dangos tebygolrwyddau ar **raddfa tebygolrwydd**.

**Ymarfer 30**

Lluniwch raddfa tebygolrwydd. Marciwch y pwyntiau a, b, c, ch i ddangos pa mor debygol, yn eich tyb chi, yw pob un o'r canlynol.

- (a) Bydd hi'n bwrw eira ym Mhegwn y Gogledd (*The North Pole*) yfory.
- (b) Byddwch yn cael tu blaen (*heads*) wrth daflu darn arian.
- (c) Byddwch yn cael y rhif 4 wrth daflu dis teg cyffredin.
- (ch) Bydd hi'n glawio yn ystod y dydd yfory.



5

Ymarfer 31

Lluniwch raddfa tebygolrwydd. Marciwch y pwyntiau a, b, c, ch i ddangos pa mor debygol, yn eich tyb chi, yw pob un o'r canlynol.

- (a) Bydd y disgybl nesaf y byddwch yn ei weld dros 2 fetr o daldra.
- (b) Bydd o leiaf un diwrnod heulog ym mis Mehefin yn Llundain.
- (c) Byddwch yn llwyddo eich prawf gyrru cyn bod yn 18 mlwydd oed.
- (ch) Bydd eira ar ben mynydd Everest ym mis Rhagfyr.

**Ymarfer 32**

Lluniwch raddfa tebygolrwydd. Marciwch y pwyntiau a, b, c, ch i ddangos pa mor debygol, yn eich tyb chi, yw pob un o'r canlynol.

- (a) Bydd y disgybl nesaf y byddwch yn ei weld yn hŷn na chi.
- (b) Bydd y disgybl nesaf y byddwch yn ei weld yn un o efeilliaid.
- (c) Bydd y disgybl nesaf y byddwch yn ei weld yn llaw dde.
- (ch) Bydd y disgybl nesaf y byddwch yn ei weld yn gwisgo sgert.

**Ymarfer 33**

Lluniwch raddfa tebygolrwydd. Marciwch y pwyntiau a, b, c, ch i ddangos pa mor debygol, yn eich tyb chi, yw pob un o'r canlynol.

- (a) Byddwch yn dal annwyd cyn diwedd y flwyddyn.
- (b) Bydd y trydan yn diffodd yn eich tŷ (*power cut*) cyn diwedd yr wythnos.
- (c) Bydd mwy na hanner y dosbarth yn gwylio rhaglen nesaf Coronation Street.
- (ch) Bydd yn cael rhif llai na 7 wrth rollo dis cyffredin.



Tebygolrwydd fel Rhif

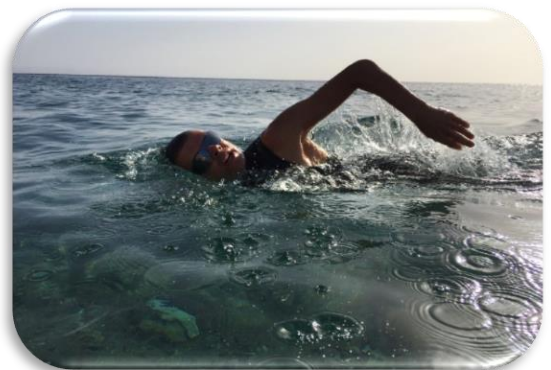
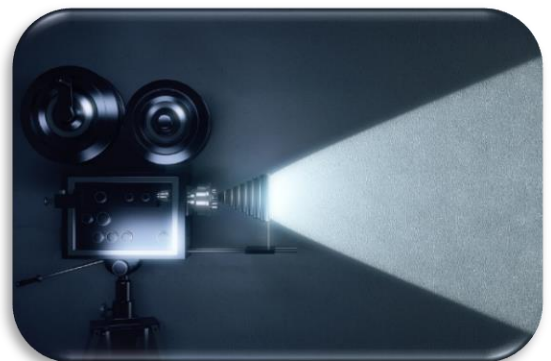
Mae hefyd yn bosib ysgrifennu tebygolrwydd fel rhif. Mae 0 yn cynrychioli amhosibl, ac mae 1 yn cynrychioli sicr. Mae pob tebygolrwydd yn rif rhwng 0 ac 1.



Ymarfer 34

Pa mor debygol yw'r canlynol? (Atebwch efo rhif rhwng 0 ag 1.)

- (a) Byddwch yn derbyn neges destun cyn diwedd y diwrnod.
- (b) Byddwch yn derbyn A am ymdrech ym mhob un o'ch pynciau yn eich adroddiad nesaf.
- (c) Byddwch yn priodi cyn i chi fod yn 20 oed.
- (ch) Byddwch yn cael eilrif wrth daflu dis teg cyffredin.
- (d) Bydd cwmni Apple yn rhyddhau iPhone arall cyn mis Ebrill flwyddyn nesaf.
- (dd) Bydd y pennaeth yn dod i'r dosbarth yn ystod y wers yma.
- (e) Bydd fflapjacs ar gael yn y ffreutur amser cinio yfory.
- (f) Bydd hi'n glawio yn ystod amser egwyl yfory.
- (ff) Bydd oed y person nesaf i chi ei weld yn 12.
- (g) Byddwch yn gwyllo ffilm heno.
- (ng) Aston Villa fydd enillwyr nesaf Cwpan yr F.A.
- (h) Rhywbryd yn eich bywyd, byddwch yn nofio ar draws y môr o Loegr i Ffrainc.
- (i) Merch fydd yn gyrru'r car nesaf i chi ei weld.
- (j) Bydd rhywun yn eich dosbarth yn derbyn llythyr yfory.
- (l) Bydd Cymru yn ennill Cwpan y Byd (rygbi) nesaf.
- (ll) Byddwch angen prynu pensil newydd cyn diwedd y flwyddyn.



Gwerthuso

Geirfa Allweddol	Cwestiynau Pellach	Beth aeth yn dda?	I gyrraedd fy lefel darged mi wnâi...



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to calculate the mode for a set of data.			2	
I know how to find the modal class for a set of grouped data.			10	
I know how to compare two distributions using the mode.			7	
I know how to use the mode and the mean to decide on the best average to use.			5	
I know how to draw a pictogram for a set of data.			1	
I know how to interpret a pictogram .				
I know how to draw a vertical line diagram for a set of data.			4	
I know how to interpret a vertical line diagram .			4, 9	
I know how to draw a line graph for a set of data.			4	
I know how to interpret a line graph .			4	
I know what vocabulary to use then answering questions involving probability.			3	
I know how to use a probability scale .			6	
I can write probability as a number .			8	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

8

The End

of Year 8

Name:

Contents

Chapter	Mathematics	Page Number
Directed Numbers	Ordering Directed Numbers. 'Less than' and 'More than'. Calculating with Directed Numbers (Addition; Subtraction; Multiplication; Division).	3
Decimals	Adding and Subtracting Decimals. Multiplying and Dividing Decimals by a Whole Number Less Than 10. Ordering Decimals. Converting between Percentages and Decimals. Rounding Off to a Specified Number of Decimal Places.	7
Angles 2	Revision of Year 7 work. Angles with Parallel Lines.	12
Transformations: Rotation	Rotating a Shape Around a Specific Point.	14



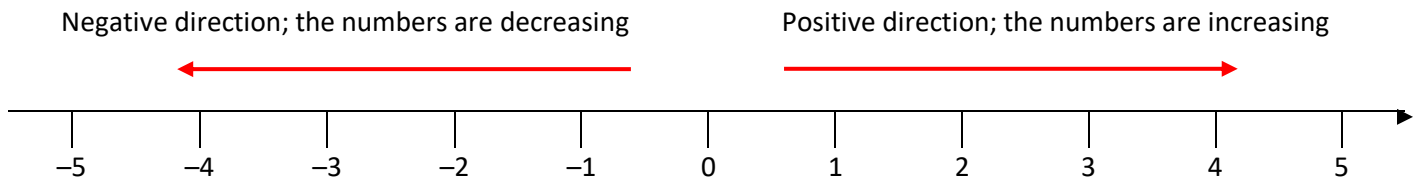
Directed Numbers

Numbers with a negative sign before them are **negative numbers**.

Other numbers (with the exception of zero) are **positive numbers**. Sometimes we write positive numbers with a positive sign before them, e.g. **+5**.

Zero is neither positive nor negative.

We can show positive and negative numbers on a number line:



Exercise 1

Write the following numbers in order, from least to greatest.

- | | | |
|------------------------------|---------------------------|------------------------------|
| (a) 4, -3, 1, -5, 2 | (b) -1, 4, 0, -2, 3 | (c) -2, 4, -5, 5, 3 |
| (d) -4, 7, -8, 9, -3 | (e) 0, -12, 14, 1, -3 | (f) -25, 27, -21, 29, -23 |
| (g) 102, -94, -120, 96, -100 | (h) 35, -42, 17, -54, 26 | (i) -35, -32, -38, -37, -33 |
| (j) -3, 5, -9, -1, 4 | (k) -14, 17, -12, 13, -15 | (l) 276, -156, 623, -312, 76 |

Skill

5



'Less than' and 'Greater than'

It is possible to write the symbols $<$ or $>$ between two different numbers.

The symbol $<$ means 'less than'. The symbol $>$ means 'greater than'.

Example

7 is less than 10, so we can write $7 < 10$.

3 is greater than -4, so we can write $3 > -4$.

The arrow always points towards the **least** number.

Exercise 2

Write the symbol $<$ or $>$ between the following pairs of numbers.

- | | | | |
|----------------|-----------------|-------------------|-------------------|
| (a) 8 ____ 18 | (b) 17 ____ 11 | (c) 4 ____ -2 | (d) 3 ____ -5 |
| (e) -2 ____ -4 | (f) -5 ____ -1 | (g) -3 ____ 0 | (h) -4 ____ 2 |
| (i) -5 ____ -8 | (j) 7 ____ -8 | (k) -102 ____ -98 | (l) 14 ____ -16 |
| (m) -4 ____ 8 | (n) -12 ____ -3 | (o) 87 ____ 165 | (p) -87 ____ -165 |



Exercise 3

True or False?

- | | | | |
|----------------|------------------|------------------|-----------------|
| (a) $8 > 5$ | (b) $3 > 9$ | (c) $-4 > 2$ | (d) $-8 < -5$ |
| (e) $-10 > -7$ | (f) $-4 < 8$ | (g) $9 > -14$ | (h) $-21 > -19$ |
| (i) $-12 < 10$ | (j) $180 < -190$ | (k) $-120 > -90$ | (l) $-4 < -3$ |
| (m) $-4 > -7$ | (n) $0 < -4$ | (o) $8 > 0$ | (p) $-9 > -11$ |



Challenge!

Search for "Cool Cash Confusion" on the internet. Write a short report on your findings.

 @mathemateg



Calculating with Directed Numbers

Let us imagine playing a board game where a **green card** is worth 5 points and a **red card** is worth -3 points.

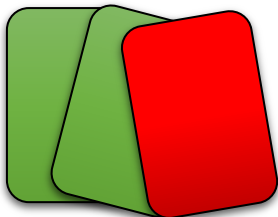
Example

If Dewi has three **green** cards and two **red** cards, Dewi's score is $5 + 5 + 5 - 3 - 3 = 9$.

Exercise 4

How many points do the following people have?

(a) Eryl



(b) Siwan



(c) Megan

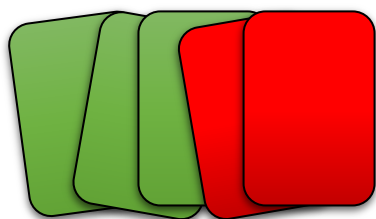


Applying

3

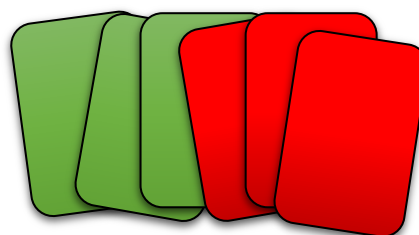
Adding a Negative Number

If Dewi receives an extra **red** card as part of the game, Dewi's score would change to be 6.



Score $5 + 5 + 5 - 3 - 3 = 9$

Receive an extra
red card



Score $9 - 3 = 6$

Because **receiving** a red card has **reduced** the score, it must mean that **adding a negative number is the same as subtraction**.

$$\begin{aligned} 9 + -3 &= 9 - 3 \\ &= 6 \end{aligned}$$

Subtracting a Negative Number

If Dewi, on the other hand, loses a red card as part of the game, then Dewi's score would change to be 12.



Score $5 + 5 + 5 - 3 - 3 = 9$

Lose a red card



Score $9 + 3 = 12$

Because **removing** a red card has **increased** the score, it must mean that **subtracting a negative number is the same as addition**.

$$\begin{aligned} 9 - -3 &= 9 + 3 \\ &= 12 \end{aligned}$$

Example

$$(a) 9 + -2 = 9 - 2 = 7 \quad (b) -4 + -3 = -4 - 3 = -7 \quad (c) 14 - -2 = 14 + 2 = 16 \quad (d) -4 - -1 = -4 + 1 = -3$$

Exercise 5 (Adding a Negative Number)

$$\begin{array}{llll} (a) 4 + -2 & (b) 13 + -2 & (c) 9 + -4 & (d) 28 + -9 \\ (e) -5 + -2 & (f) -9 + -2 & (g) -3 + -3 & (h) -10 + -7 \\ (i) 6 + -8 & (j) 2 + -5 & (k) 19 + -21 & (l) 7 + -7 \\ (m) -12 + -3 & (n) -18 + -8 & (o) -24 + -8 & (p) -38 + -5 \end{array}$$

Exercise 6 (Subtracting a Negative Number)

$$\begin{array}{llll} (a) 6 - -4 & (b) 16 - -4 & (c) 24 - -3 & (d) 65 - -11 \\ (e) -8 - -2 & (f) -12 - -3 & (g) -25 - -9 & (h) -40 - -9 \\ (i) -2 - -5 & (j) -10 - -15 & (k) -1 - -2 & (l) -8 - -8 \\ (m) -10 - -5 & (n) -2 - -7 & (o) 14 - -8 & (p) -101 - -4 \end{array}$$

Exercise 7 (Mixture)

$$\begin{array}{llll} (a) 8 + -3 & (b) 8 - -3 & (c) -8 + -3 & (d) -8 - -3 \\ (e) 14 - -2 & (f) 2 - -14 & (g) 14 + -2 & (h) 2 + -14 \\ (i) -7 + -9 & (j) -7 - -9 & (k) -9 + -7 & (l) -9 - -7 \\ (m) 24 - -2 & (n) -12 + -2 & (o) -9 - -12 & (p) 3 + -8 \\ (q) 2.5 + -0.3 & (r) 5.4 - -1.2 & (s) -2.1 - -0.5 & (t) -4.2 + -0.7 \\ (u) 25 + -2 + -3 & (v) 25 + -2 - -3 & (w) 25 - -2 - -3 & (x) 25 - -2 + -3 \end{array}$$

Challenge!

What should the answers be to the following?

$$(a) 16 + +4 \quad (b) 16 - +4 \quad (c) 1 + -2 - -3 + -4 - -5$$

Exercise 8

(a) In a golf championship, players complete four rounds of golf on the golf course.

Out of the following four players, who has the best score? (In golf, the lower the score, the better the score.)

Name	Round 1	Round 2	Round 3	Round 4
Jordan Speith	+2	-3	-1	+1
Rickie Fowler	-2	-1	+4	-3
Rory McIlroy	0	-3	+1	-2
Justin Rose	-3	+2	+4	-2

(b) The temperature at midnight in Llanrwst was recorded each day during one week in January.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
-3°C	-4°C	1°C	-3°C	-2°C	0°C	4°C

What was the **mean** temperature at midnight in Llanrwst during this week?

(c) At the end of May, Trefor had £200 in his bank account. He spent £800 during the first week of June. He was paid £500 during the second week of June. In the third week, he spent £400. During the final week of June, he was paid £900. How much money has Trefor got in his bank account at the end of June?

Skill**5****Applying****5**

Multiplying and Dividing Directed Numbers**Exercise 9 (Revision)**

- | | | | |
|------------------|-------------------|-------------------|-------------------|
| (a) 5×6 | (b) 7×2 | (c) 3×8 | (d) 6×7 |
| (e) 9×4 | (f) 2×9 | (g) 4×7 | (h) 7×7 |
| (i) 8×7 | (j) 9×6 | (k) 8×9 | (l) 10×8 |
| (m) 5×9 | (n) 12×6 | (o) 8×11 | (p) 9×7 |

Skill

4

Remember! The **ttrockstars** website will improve your times tables...

Multiplying Directed NumbersWhat is 5×-3 ?

We can think of 5×-3 as 'five times negative three', or $-3 + -3 + -3 + -3 + -3$.

Because adding a negative number is the same as subtraction, we can write this as $-3 - 3 - 3 - 3 - 3$, which is -15 . Therefore, $5 \times -3 = -15$.

What is -3×4 ?

The order of the numbers is not important when multiplying, so we can think of -3×4 as 4×-3 , or 'four times negative three'. We can write this as $-3 + -3 + -3 + -3$, which is $-3 - 3 - 3 - 3 = -12$. Therefore, $-3 \times 4 = -12$.

What is -6×-3 ?

Multiplying two negative numbers gives an answer that is positive. Therefore, $-6 \times -3 = 18$.

Why? Think of the card game from before. Taking away six (-6) red cards (-3) would be the same as adding $6 \times 3 = 18$ to the score.

Example

- | | | |
|-------------------------|-------------------------|-------------------------|
| (a) $8 \times -3 = -24$ | (b) $-4 \times 5 = -20$ | (c) $-6 \times -2 = 12$ |
|-------------------------|-------------------------|-------------------------|

Exercise 10

- | | | | |
|--------------------|---------------------|--------------------|--------------------|
| (a) 7×-3 | (b) -3×7 | (c) -3×-7 | (d) -7×-3 |
| (e) 4×-8 | (f) -2×6 | (g) -4×-6 | (h) 6×-8 |
| (i) -9×3 | (j) -7×-5 | (k) -9×-4 | (l) -5×8 |
| (m) 11×-2 | (n) 4×12 | (o) -9×6 | (p) -8×7 |
| (q) -8×-8 | (r) 12×-6 | (s) -7×9 | (t) -5×-9 |
| (u) 13×-3 | (v) -1×10 | (w) -8×0 | (x) -1×-1 |
| (y) 9×-8 | (z) -14×-3 | (α) -6×5 | (β) 15×-6 |

5

**Exercise 11**

Evaluate the answers to the following. Use your calculator to check your answers.

- | | | | |
|--------------------|---------------|--------------------|--------------------|
| (a) $8 + -5$ | (b) $8 - -5$ | (c) 8×-5 | (d) -8×-5 |
| (e) $-4 + -2$ | (f) $-4 - -2$ | (g) -4×-2 | (h) -4×2 |
| (i) 14×-2 | (j) $14 + -2$ | (k) $-14 + -2$ | (l) $-14 - -2$ |
| (m) $4 - -9$ | (n) $-4 + -9$ | (o) -4×9 | (p) 4×-9 |

**Exercise 12**

Write 10 different sums whose answer is 4.

The rules for dividing directed numbers are the same as for multiplication.

(a) $32 \div 4 = 8$ (b) $32 \div -4 = -8$
(c) $-32 \div 4 = -8$ (d) $-32 \div -4 = 8$

5

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| (a) $16 \div -2$ | (b) $-24 \div 4$ | (c) $-12 \div -3$ | |
| (d) $-20 \div -5$ | (e) $28 \div -4$ | (f) $-30 \div 2$ | |
| (g) $36 \div -9$ | (h) $-40 \div -10$ | (i) $48 \div -6$ | (j) $50 \div -25$ |
| (k) $-39 \div -13$ | (l) $64 \div -8$ | (m) $66 \div -2$ | (n) $-72 \div 8$ |
| (o) $63 \div -7$ | (p) $-80 \div -8$ | (q) $-100 \div 20$ | (r) $-120 \div -40$ |
| (s) $2 \div -4$ | (t) $0 \div -3$ | (u) $-1 \div -10$ | (v) $-7 \div 0$ |

Calculate the following. (Be careful with the order of operations.)

- (a) $8 + 2 \times -3$ (b) $8 - 2 \times -3$ (c) $-8 \times 2 - 3$ (d) $-8 \div -2 - -3$
(e) $3 + 6 \div -3$ (f) $-3 + -6 \div 3$ (g) $3 \times 6 \div -3$ (h) $-3 + -6 - -3$
(i) $(4 + -2) \times -5$ (j) $4 + (-2 \times -5)$ (k) $4 + -2 \times -5$ (l) $(5 \times -6) \div (-20 \div 5)$

(a) Write the following numbers in order, from least to greatest.

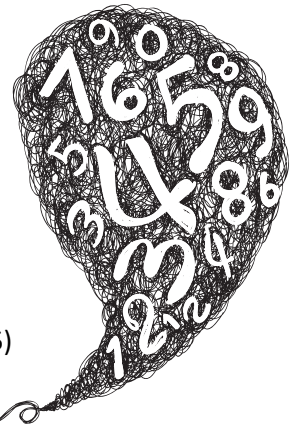
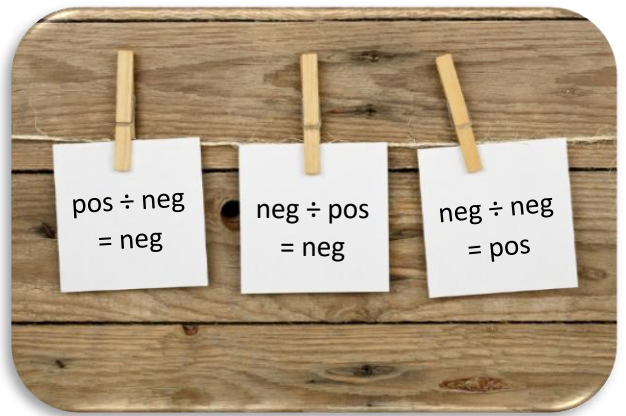
(i) 8, -2, -10, 5, 7 (ii) -3, 6, -5, 8, -7, 0 (iii) 23, -21, 18, 25, -4, -12, 8, -1, 3, -19

- (b) Write the symbol $<$ or $>$ between the following pairs of numbers.

- (i) $-3 \underline{\hspace{1cm}} 2$ (ii) $-8 \underline{\hspace{1cm}} -4$ (iii) $-15 \underline{\hspace{1cm}} 12$
- (c) $8 + -2$ (d) $14 - -3$ (e) 2×-3 (f) $-14 \div -2$
- (g) $-8 + -2$ (h) $-14 - -3$ (i) -2×-3 (j) $14 \div -2$

(k) Copy and complete the following sentences.

- (i) Adding a negative number is the same as _____ (ii) Subtracting a negative number is the same as _____
(iii) Multiplying a negative number by a positive number gives an answer that is _____
(iv) Dividing a negative number by a negative number gives an answer that is _____



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Decimals

Revision

5

Revising Year 7 Work

Exercise 16 (Adding Decimals)

- | | | | |
|---------------------|--------------------|-----------------------|------------------------|
| (a) $37.24 + 42.41$ | (b) $7.17 + 1.45$ | (c) $26.725 + 97.476$ | (d) $13.427 + 42.645$ |
| (e) $8.23 + 24.14$ | (f) $824.4 + 2.45$ | (g) $2834.2 + 25.324$ | (h) $52.498 + 8273.24$ |
| (i) $0.23 + 39.2$ | (j) $34 + 4.28$ | (k) $2989.4 + 399.8$ | (l) $0.23 + 0.062$ |



Exercise 17 (Subtracting Decimals)

- | | | | |
|---------------------|-----------------|---------------------|---------------------|
| (a) $46.87 - 12.31$ | (b) $8.6 - 2.5$ | (c) $274.66 - 52.4$ | (d) $45.2 - 23.8$ |
| (e) $18 - 6.3$ | (f) $39 - 6.23$ | (g) $6.4 - 5.21$ | (h) $700.2 - 263.8$ |
| (i) $200 - 12.625$ | (j) $90 - 27.2$ | (k) $7.2 - 0.06$ | (l) $6.01 - 0.185$ |



Exercise 18 (Multiplying a Decimal by a Whole Number Less Than Ten)

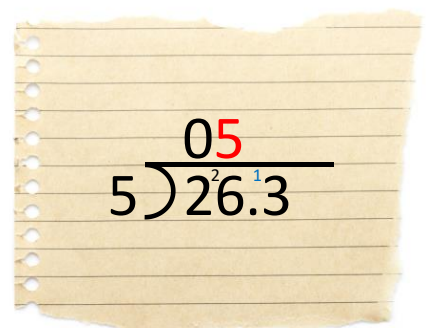
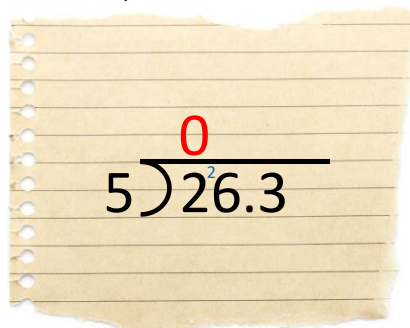
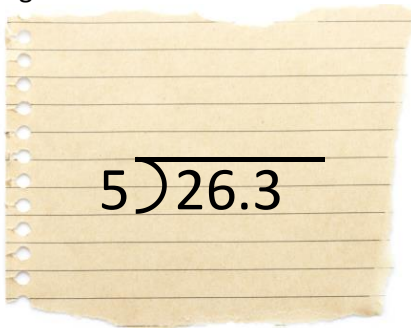
- | | | | |
|-----------------------|-----------------------|-----------------------|------------------------|
| (a) 43.27×5 | (b) 9.3×4 | (c) 2.65×2 | (d) 823.4×6 |
| (e) 723.45×8 | (f) 92.463×9 | (g) 8274.9×7 | (h) 0.264×3 |
| (i) 5.036×6 | (j) 30.46×8 | (k) -4.25×5 | (l) -423.7×-9 |



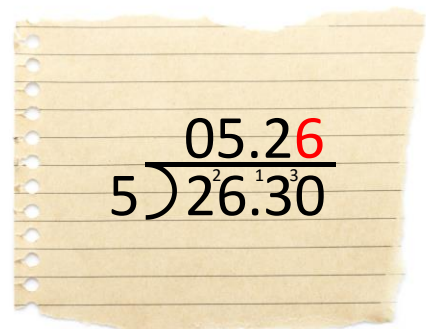
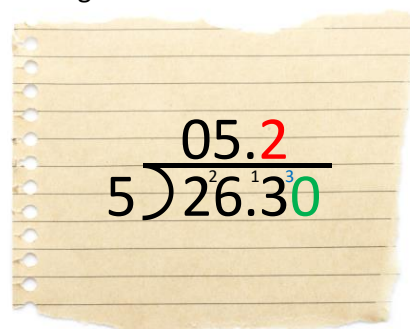
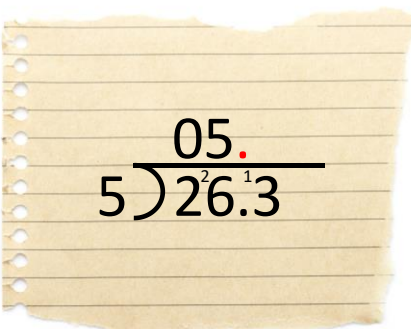
Dividing a Decimal by a Whole Number Less Than Ten

In order to calculate a sum such as $26.3 \div 5$, follow these steps.

1. Set the sum out using a division frame; the 26.3 goes inside and the 5 goes outside.
2. "How many times does 5 fit into 2?" 5 is too big to fit into 2, so it fits in 0 times, with remainder 2.
3. "How many times does 5 fit into 26?" It fits into twenty-six 5 times, with remainder 1.



4. Remember to add the decimal point in the correct position.
5. "How many times does 5 fit into 13?" It fits into thirteen 2 times, with remainder 3. Note that we need to write an extra 0 before writing the remainder.
6. "How many times does 5 fit into 30?" It fits into thirty 6 times, with remainder 0. So, the answer to $26.3 \div 5$ is 5.26.



Exercise 19

- (a) $7.35 \div 3$ (b) $26.88 \div 4$ (c) $842.6 \div 2$ (d) $213.5 \div 5$
 (e) $252.6 \div 6$ (f) $75.6 \div 9$ (g) $341.84 \div 8$ (h) $29.61 \div 7$
 (i) $2531.1 \div 6$ (j) $72.32 \div 8$ (k) $186.06 \div 3$ (l) $371 \div 5$
 (m) $0.85 \div 2$ (n) $297 \div 4$ (o) $37941.75 \div 9$ (p) $305.004 \div 7$

Skill

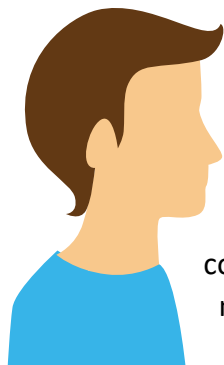
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**Ordering Decimals**

Which number is larger: 4.2 or 4.03?

4.03 is the largest number as 3 is greater than 2.

No, 4.2 is the same as 4.20. Now 20 is greater than 03, so 4.2 is the largest number.



Explain who is correct by using the notepaper on the right.

**Exercise 20**

Write the following numbers in order, from least to greatest.

- (a) 4.2, 4.02, 4.03, 4.3 (b) 8.42, 2.84, 4.82, 8.24, 4.28, 2.48 (c) 7.503, 7.053, 7.53, 7.005, 7.3
 (d) 9.008, 9.8, 9.08, 9 (e) 3.96, 3.09, 3.6, 3.06, 3.096, 3.009 (f) 14.07, 4.71, 7.14, 14.1, 7.04, 4.07
 (g) 0.26, 0.18, 0.3, 0.21, 0.33 (h) 3.8, 3.07, 4.09, 3.54, 4.81, 3.05 (i) 23.7, 20.3, 21.78, 20.09, 22.7
 (j) 0.9, 0.78, 0.98, 0.09, 0.87 (k) 0.821, 0.281, 0.128, 0.218, 0.812 (l) 0.7, 0.68, 0.702, 0.689, 0.73, 0.679
 (m) 5.9, 6.8, 5.09, 5.83, 6.88 (n) 1.8, 1.03, 1.24, 1.043, 1.003, 1.51 (o) 45.08, 5.8, 45.8, 5.004, 45.50, 5.08

Exercise 21

Applying

	Runner 1	Runner 2	Runner 3	Runner 4
Canada	10.31 seconds	10.04 seconds	10.46 seconds	10.09 seconds
Jamaica	9.98 seconds	10.68 seconds	10.10 seconds	10.18 seconds
U.S.A.	10.06 seconds	10.13 seconds	10.24 seconds	10.01 seconds
France	10.53 seconds	10.96 seconds	10.42 seconds	10.18 seconds

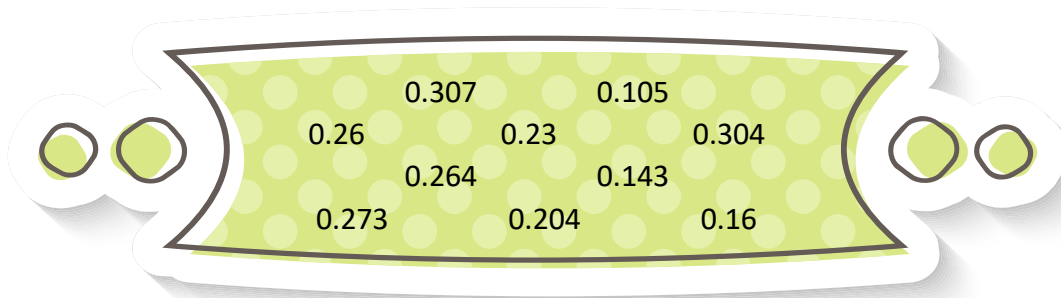
The table above shows the times taken by each runner in a 400 m relay race at an athletics championship.

- (a) How many different teams were competing? (b) How many runners were in each team?
 (c) Which runner, from which team, was fastest? (d) Which runner was slowest?
 (e) Which team won the relay race? (f) Which team came last?



Exercise 22

Add some of the following decimals, aiming to reach a total as close as possible to 1.
You may only use each number once.

**Converting a Percentage to a Decimal**

In order to change a **percentage** to be a **decimal**, divide the percentage by 100.

Example

(a) As a decimal, 62% is $62 \div 100 = 0.62$.

(b) As a decimal, 3% is $3 \div 100 = 0.03$.

Exercise 23

Write the following percentages as decimals.

(a) 27%

(b) 86%

(c) 29%

(d) 35%

(e) 40%

(f) 8%

(g) 70%

(h) 2%

(i) 156%

(j) 250%

(k) 400%

(l) 0%

(m) 25.8%

(n) 4.7%

(o) 0.3%

(p) 0.08%

Skill

6

**Converting a Decimal to a Percentage**

In order to change a **decimal** to be a **percentage**, multiply by 100 and add the % symbol.

Example

(a) As a percentage, 0.45 is $0.45 \times 100 = 45\%$.

(b) As a percentage, 0.06 is $6 \times 100 = 6\%$.

Exercise 24

Write the following decimals as percentages.

(a) 0.31

(b) 0.57

(c) 0.73

(d) 0.66

(e) 0.2

(f) 0.9

(g) 0.04

(h) 0.09

(i) 3.54

(j) 5.5

(k) 6

(l) 12

(m) 0.728

(n) 0.035

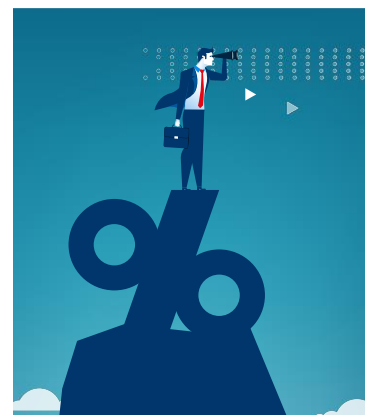
(o) 0.002

(p) 0.0006

Exercise 25

Write the following numbers in order, starting from the smallest.

54% 0.35 7% 0.8 40% 0.09 2.1 92%



Exercise 26

4



Original Number	Round off to the nearest unit	Round off to the nearest 10	Round off to one decimal place	Round off to two decimal places	Round off to three decimal places
432.4385	432	430	432.4	432.44	432.439
273.8924					
348.8753					
219.5541					
199.7502					
537.8025					
1,549.1087					
3,845.6355					
10,293.54					
28,479.5997					

e

- 
- Evaluation**

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Angles 2

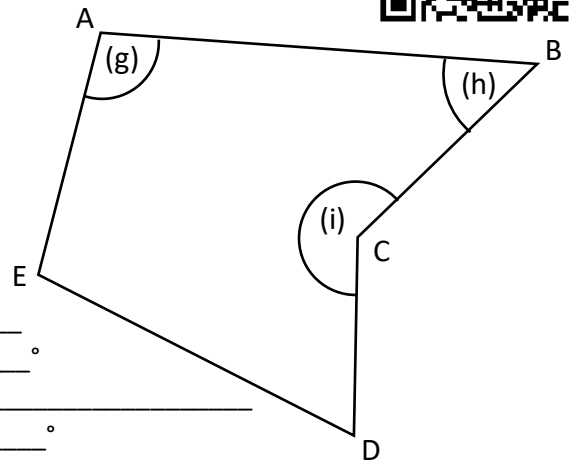


Exercise 28 (Revision of Year 7 work)

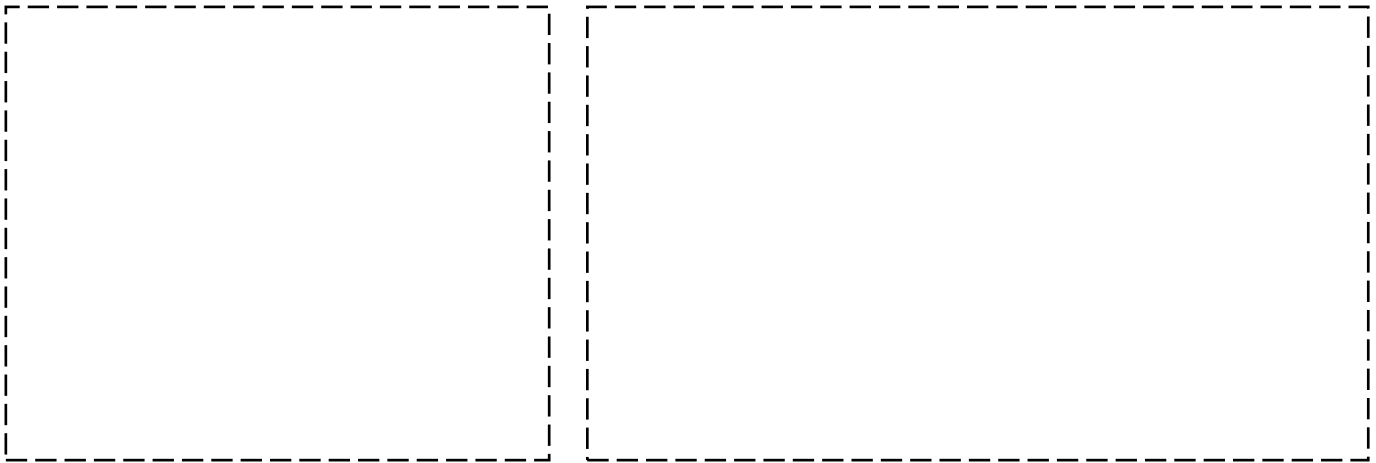
6

- (a) An acute angle is any angle between ____° and ____°.
 (b) A _____ angle is any angle that is exactly 90°.
 (c) An _____ angle is any angle between 90° and 180°.
 (d) A straight line is any angle that is exactly ____°.
 (e) A reflex angle is any angle between ____° and ____°.
 (f) A _____ is any angle that is exactly 360°.

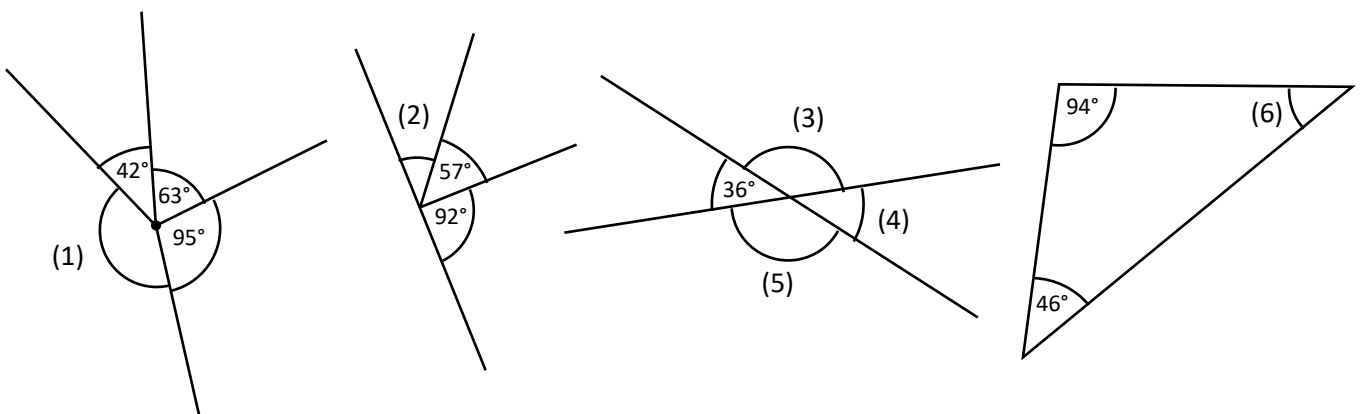
- (g) Name the angle labelled (g) in the diagram to the right. _____
 (h) Measure the angle labelled (h) in the diagram to the right. _____°
 (i) What type of angle is angle (i) in the diagram to the right? _____
 (j) Measure the **reflex** angle $\widehat{CDE}$ in the diagram to the right. _____°



- (k) Draw an angle of 126° in the box below. (l) With a compass and ruler, draw an angle of 30° in the box below.

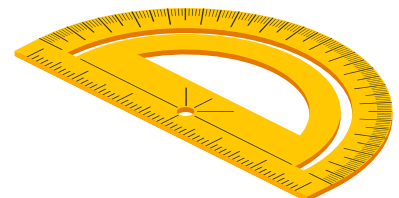


- (m) Calculate the size of the angles in the following diagrams. (The diagrams are not drawn to scale.)



(1) _____° (2) _____° (3) _____° (4) _____° (5) _____° (6) _____°

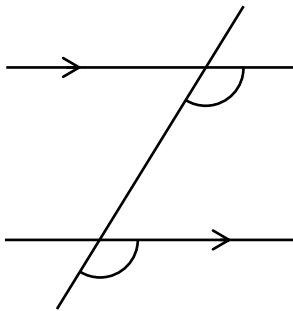
- (n) An isosceles triangle has _____ equal angles.
 (o) The size of each angle in an equilateral triangle is _____°.
 (p) Any triangle with no equal sides is named a _____ triangle.



Angles with Parallel Sides

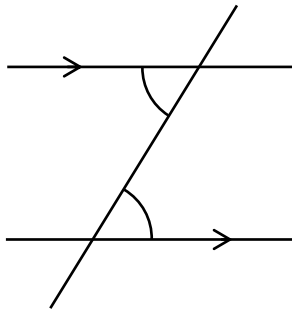
If a line crosses a pair of parallel lines, it is possible to recognise some links between some of the angles.

(1) **Corresponding Angles** 



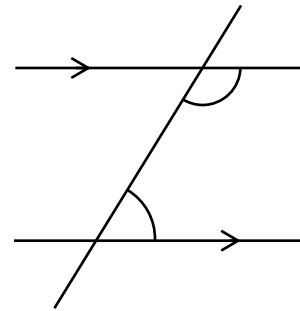
Corresponding angles are **equal**.

(2) **Alternate Angles** 



Alternate angles are **equal**.

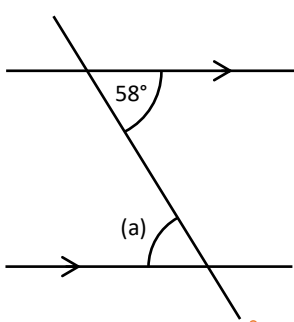
(3) **Internal Angles** 



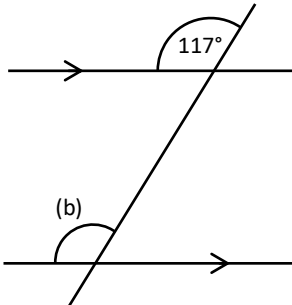
Internal angles **add up to 180°**.

Example

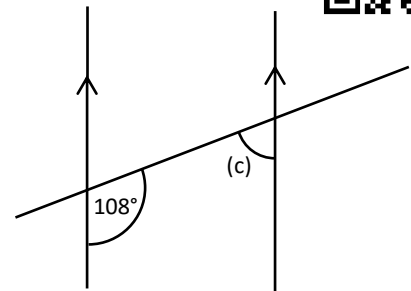
(a) 58° (alternate angles).



(b) 117° (corresponding angles).



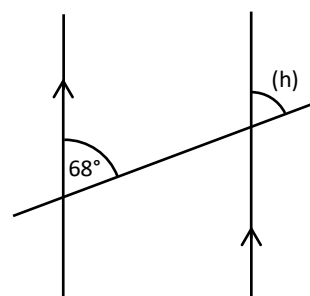
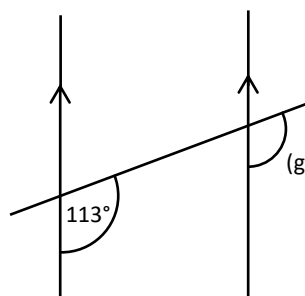
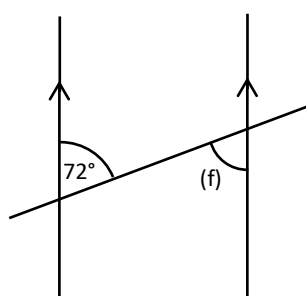
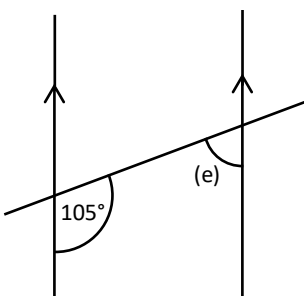
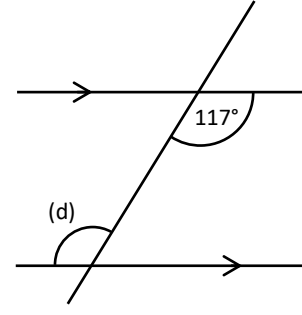
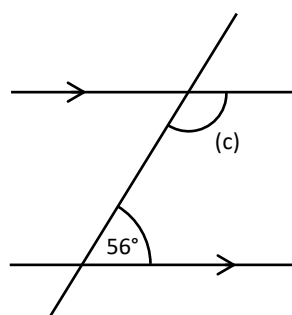
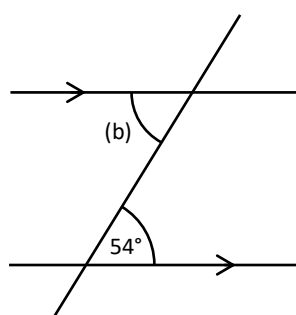
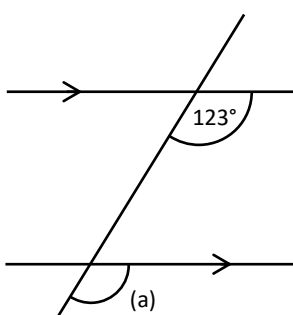
(c) 72° (internal angles).



The **transversal** is the line that intersects the parallel lines.

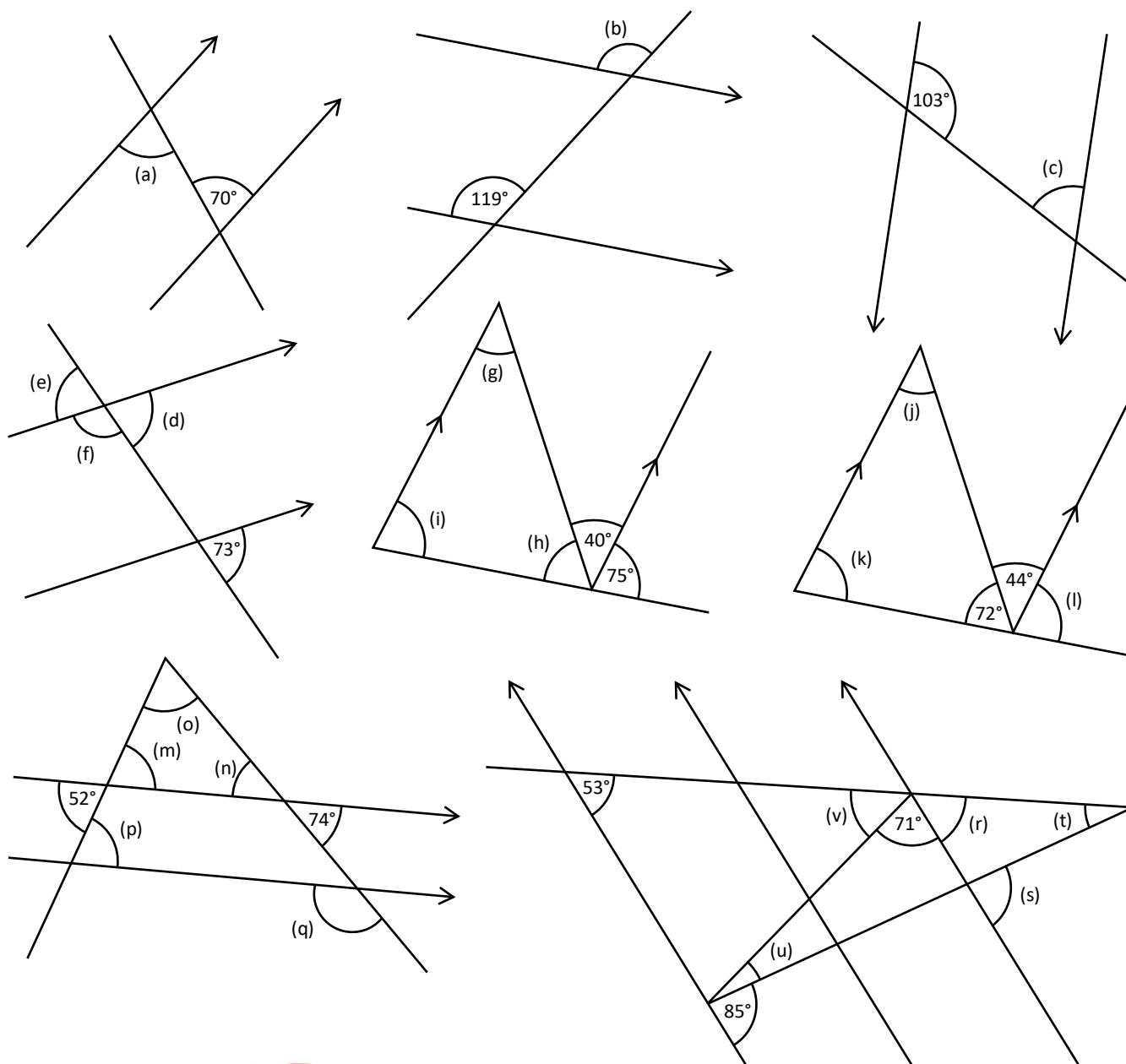
Exercise 29

Calculate the size of the marked angles. (The diagrams are not drawn to scale.)



Exercise 30

Calculate the size of the marked angles. (The diagrams are not drawn to scale.)



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Transformations: Rotation



Rotation is one of the four transformations.

Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	Enlargement

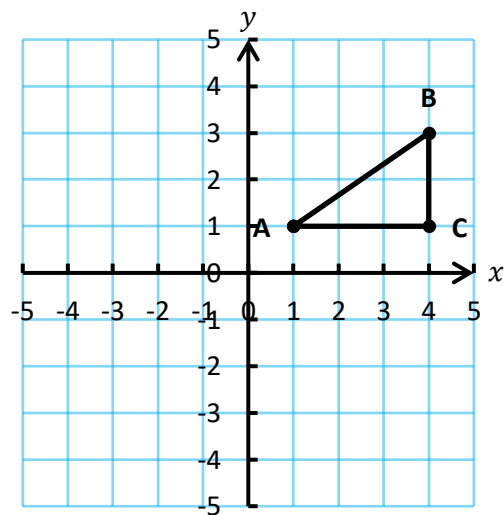
In order to rotate a shape, we need to know

- the **centre of rotation**, e.g. the point (3, 5);
- how** to rotate the shape, e.g. 90° clockwise.

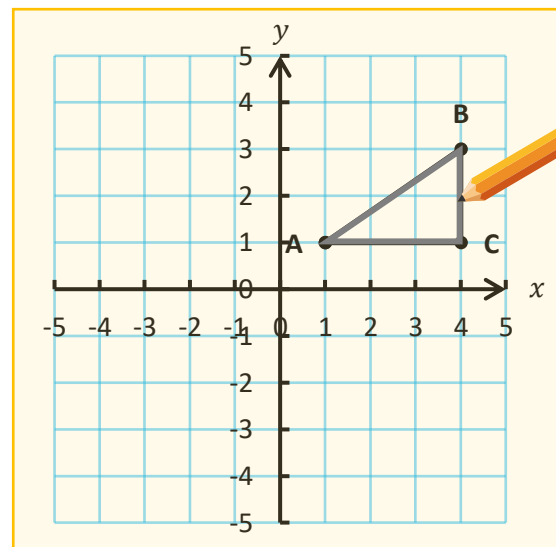
It is good practice to use a piece of tracing paper to complete a rotation question.



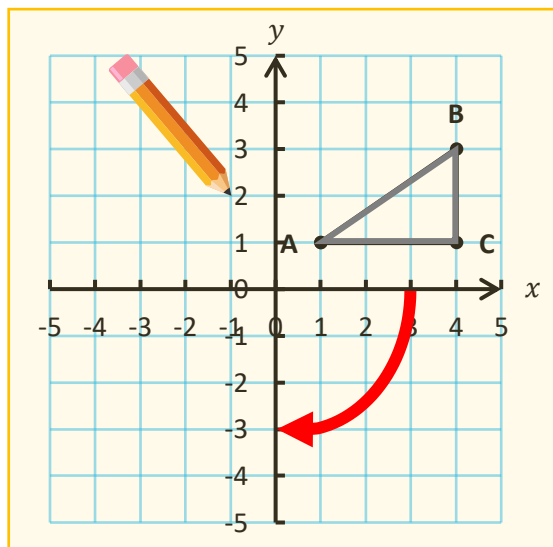
Example: Rotate the triangle **ABC** by 90° clockwise around the point $(-1, 2)$.



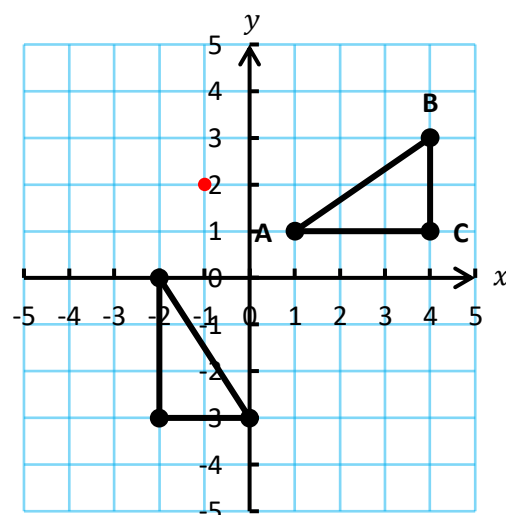
Step 1: Place a piece of tracing paper over the grid and trace the shape **ABC**.



Step 2: Place your pencil on the centre of rotation $(-1, 2)$ and rotate the tracing paper 90° clockwise.



Step 3: Draw the new shape on the squared paper.

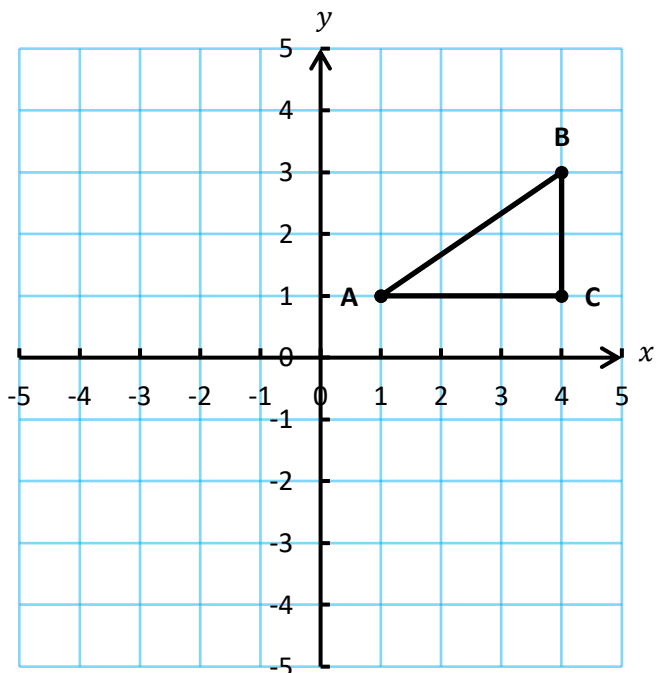


Exercise 31

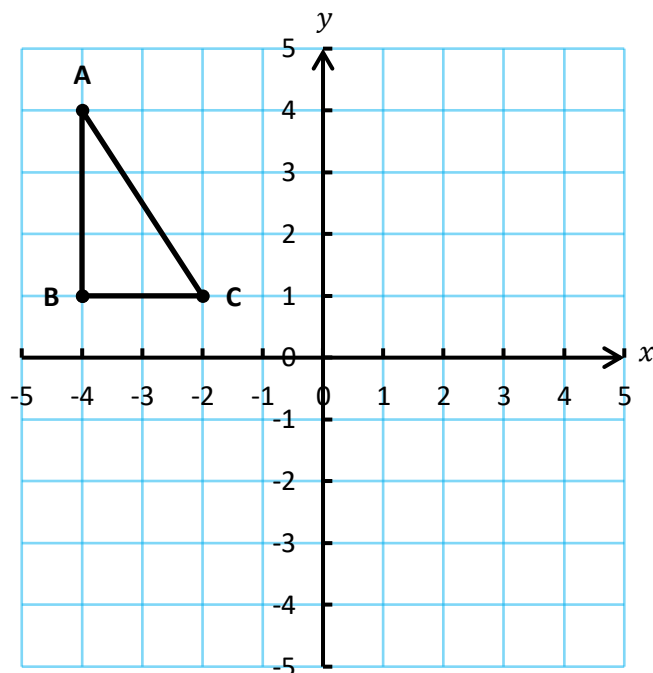
Skill

6

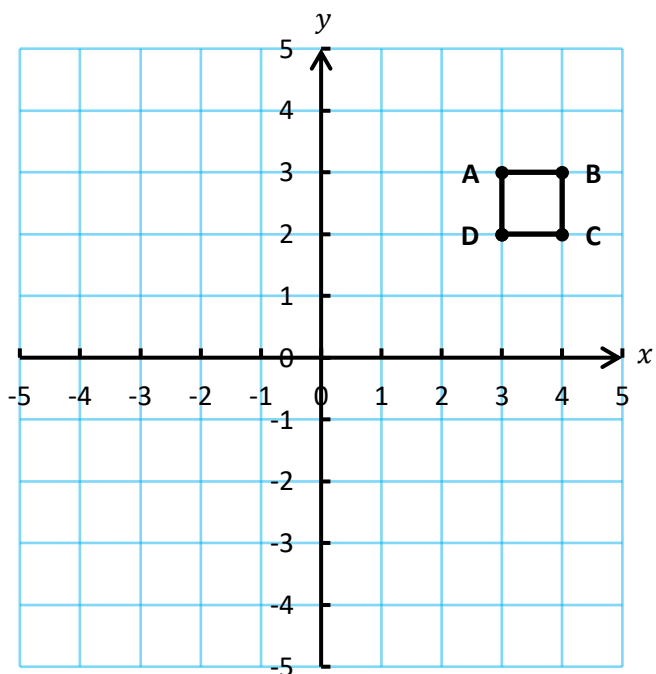
- (a) Rotate the triangle **ABC** by 90° clockwise around the origin.



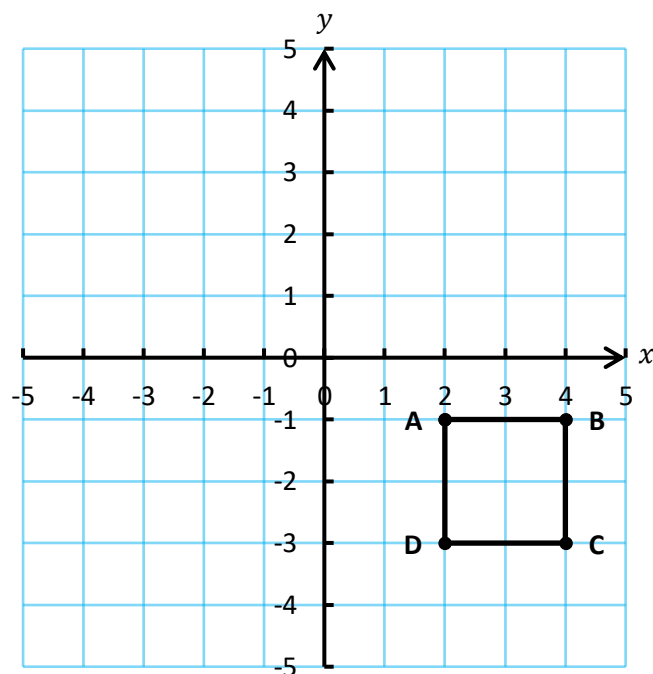
- (b) Rotate the triangle **ABC** by 90° counter-clockwise around the origin.



- (c) Rotate the square **ABCD** by 180° clockwise around the origin.



- (d) Rotate the square **ABCD** by 270° counter-clockwise around the origin.



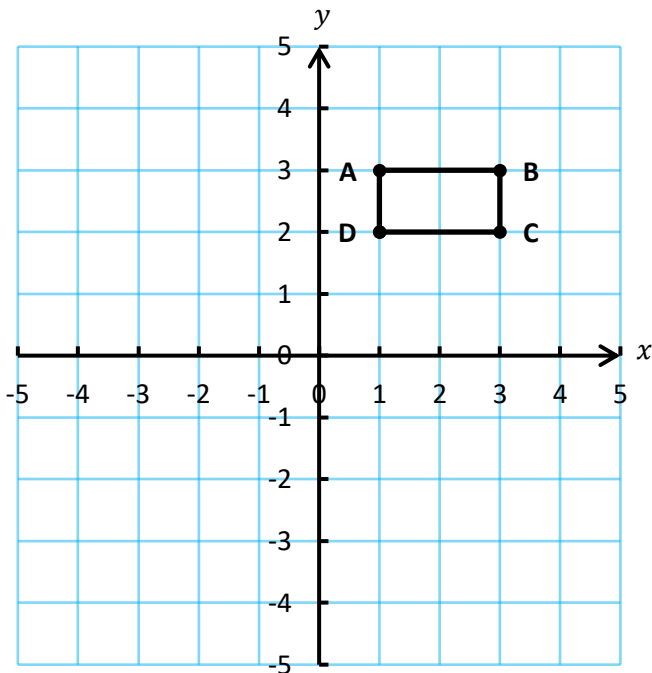
- (e) Would the answer to question (c) change if the word "clockwise" was changed to the word "counter-clockwise"?

- (f) Complete the sentence: a counter-clockwise rotation of 270° is the same as a clockwise rotation of _____.

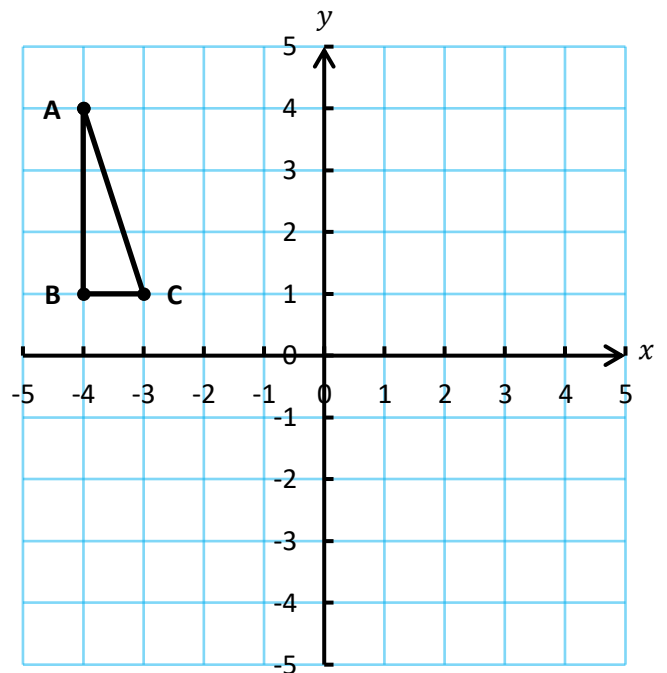
Exercise 32

6

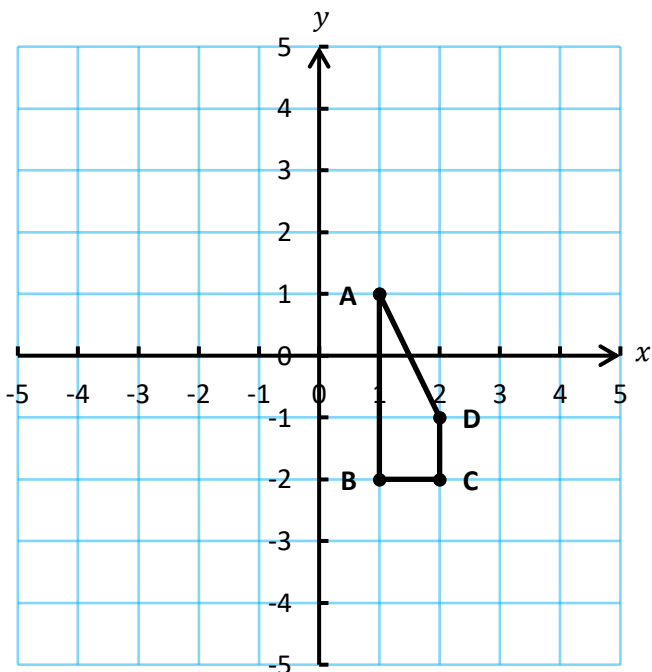
- (a) Rotate the rectangle **ABCD** by 90° clockwise around the point $(-1, 2)$.



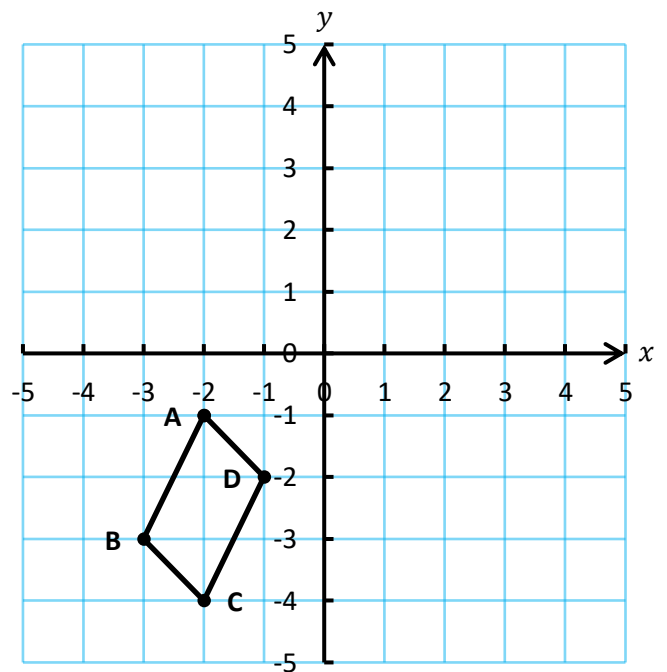
- (b) Rotate the triangle **ABC** by 180° counter-clockwise around the point $(-2, 2)$.



- (c) Rotate the quadrilateral **ABCD** by 90° counter-clockwise around the point $(0, 1)$.



- (d) Rotate the quadrilateral **ABCD** by 270° counter-clockwise around the point $(2, -4)$.

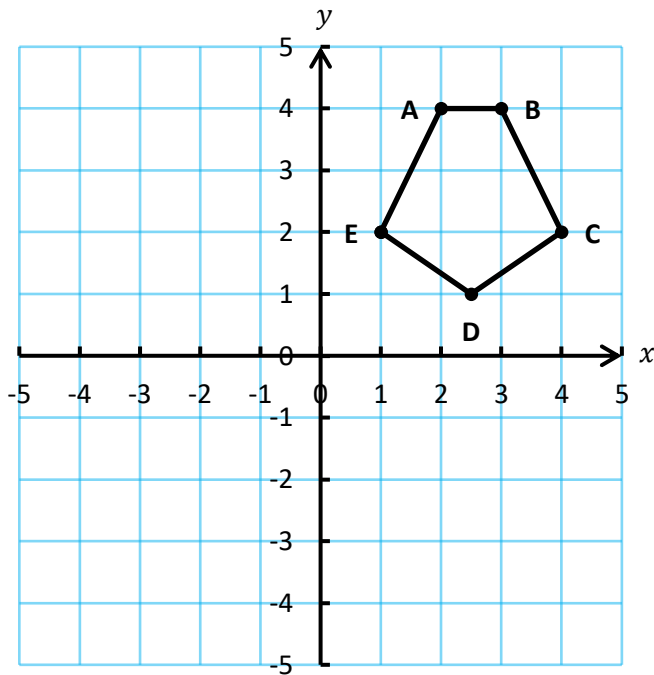


- (e) What is the name of the quadrilateral in question (c)? (f) What is the name of the quadrilateral in question (d)?
 (g) Calculate the area of the quadrilateral in question (c). Give your answer in square units. (h) Calculate the area of the quadrilateral in question (d). Give your answer in square units.

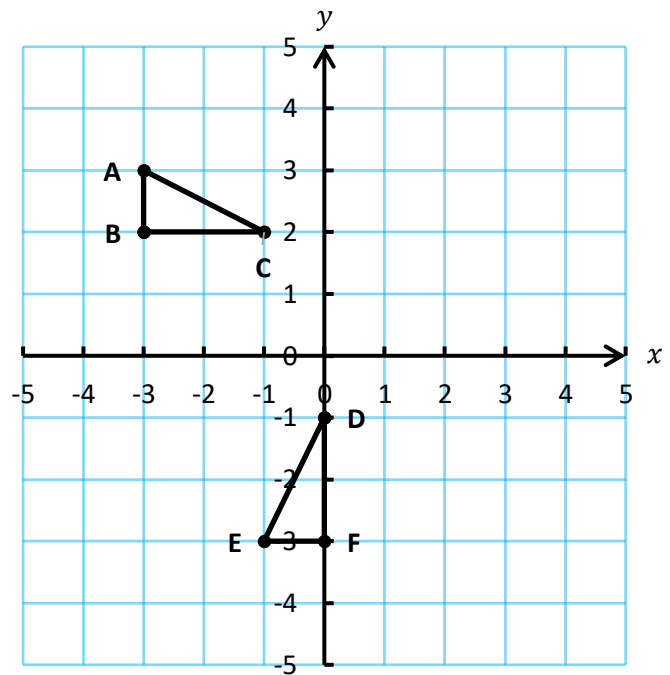
Exercise 33

6

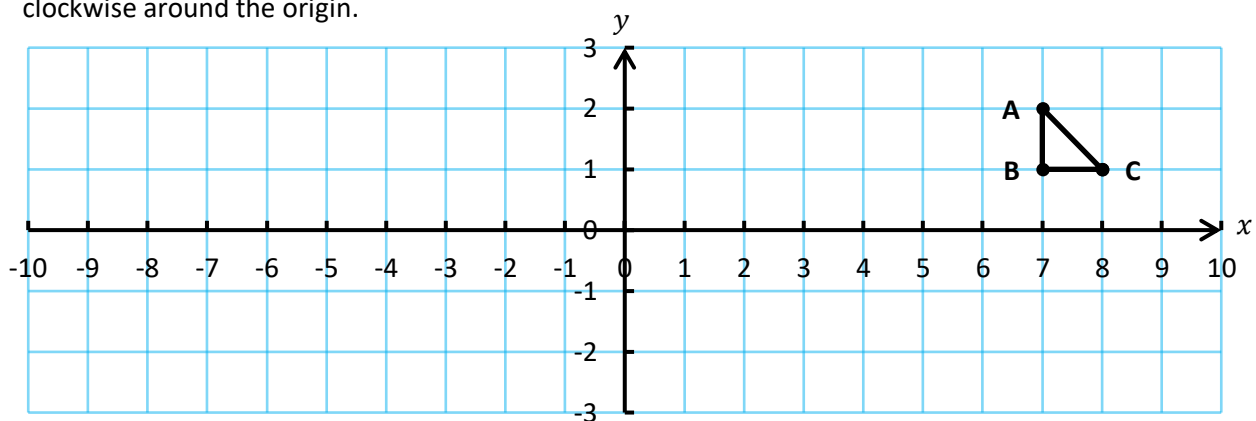
- (a) Rotate the pentagon **ABCDE** by 90° clockwise around the point $(-1, 1)$.



- (b) What **type** of rotation changes the triangle **ABC** to the triangle **DEF**?



- (c) First, translate the triangle **ABC** using the column vector $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$. Then, rotate the translated triangle by 90° clockwise around the origin.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐



Reflection Sheet

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to order positive and negative numbers from least to greatest .			2, 7	
I know when to write the symbols < or > between a pair of numbers, e.g. $5 > -8$.			3	
I can add negative numbers , e.g. $3 + -9$.			4	
I can subtract negative numbers , e.g. $4 - -7$.			4	
I can multiply and divide by directed numbers , e.g. 4×-5 , $-20 \div -10$.			4	
I can add, subtract and multiply using decimals , e.g. $37.24 + 42.41$, $14 - 4.21$, 45.65×6 .			5, 6, 8	
I can divide a decimal by a whole number less than 10 , e.g. $75.6 \div 9$.			6	
I can change a percentage into a decimal .			9	
I can change a decimal into a percentage .			10	
I can round off a number to a specified number of decimal places .			1	
Given parallel lines, I can recognise and use corresponding angles , alternate angles and internal angles .			11	
I can rotate a shape around a specified point .			12, 13	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐

Parts of a Number

★

★

✴

%

Measuring Shapes 2

★

★

✴

%

Level at the start
of the year: _____

End of KS3
target: _____

Tracking Sheet Year 9

Exam:

%

Level after the
examination: _____

Developing Algebra

★

★

✴

%

Data Handling
and Statistics 3

★

★

✴

%



The Mathematics Department

9

Parts of

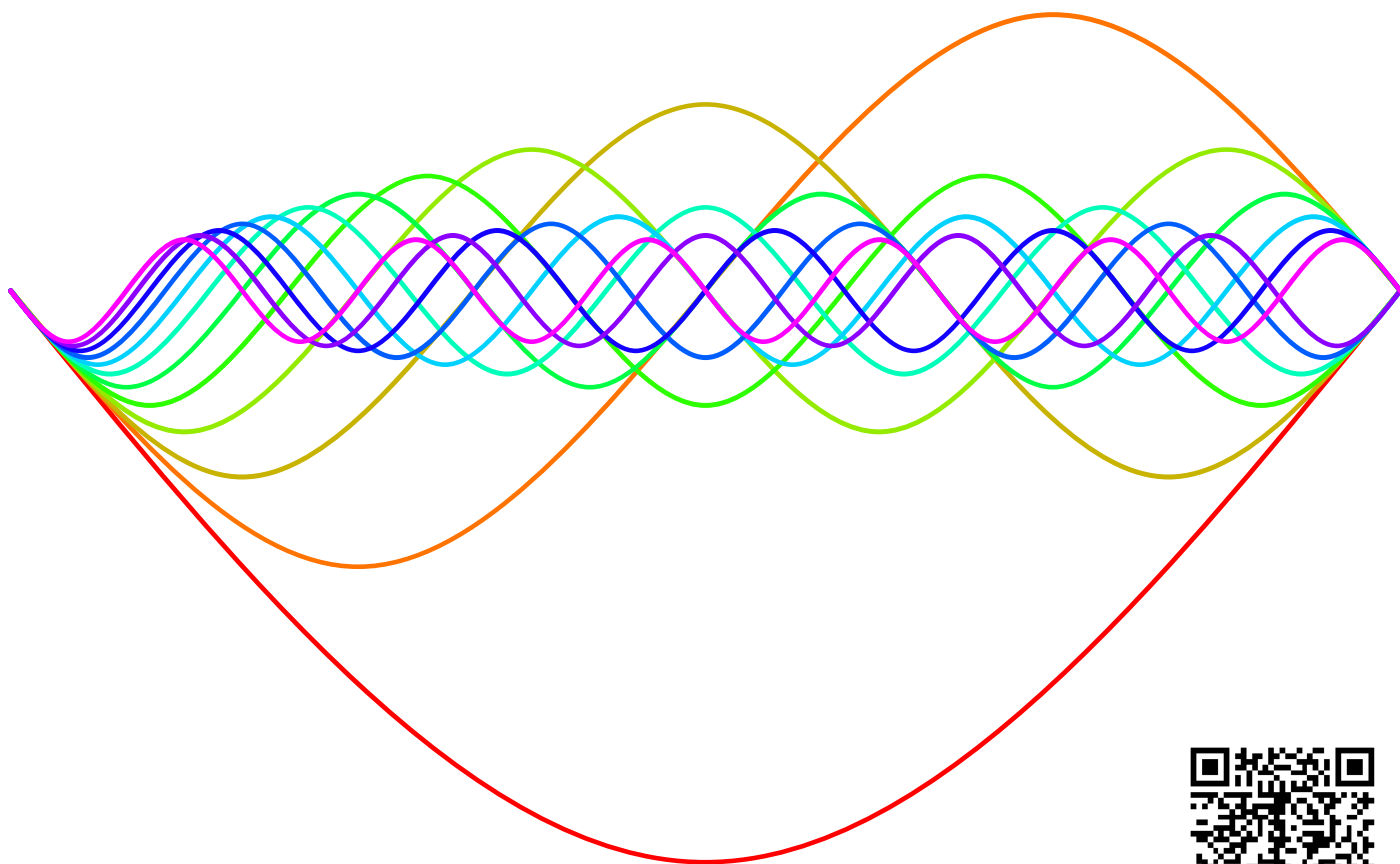
a Number

Name:



Contents

Chapter	Mathematics	Page Number
Introducing Bar Modelling	Addition and subtraction. Problem solving.	3
Ratios	Sharing in a given ratio. Equivalent ratios. Simplifying a ratio. Comparing ratios 1. The unitary method. Comparing ratios 2. Map work.	5
Fractions	Fraction of a shape. Calculating a fraction of a number. Fractions as numbers. Comparing fractions 1. Mixed numbers and improper fractions. Addition and subtraction of fractions with equal denominators. Equivalent fractions. Comparing fractions 2. Simplifying fractions. Addition and subtraction of fractions with different denominators. Multiplying fractions. Dividing fractions. Calculations with mixed numbers. Fractions on a calculator.	14
Reciprocals	Reciprocal of a whole number. Reciprocal of a fraction. Reciprocal of a mixed number.	29



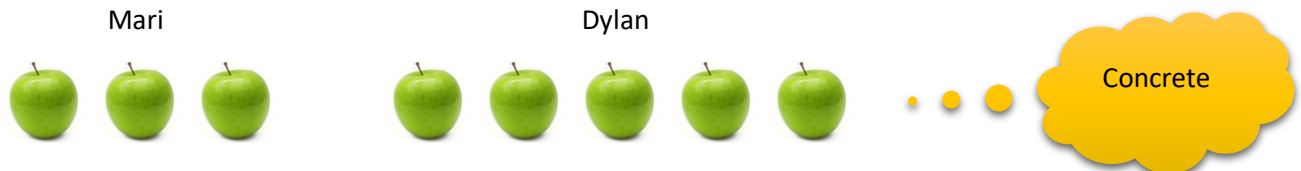
Introducing Bar Modelling



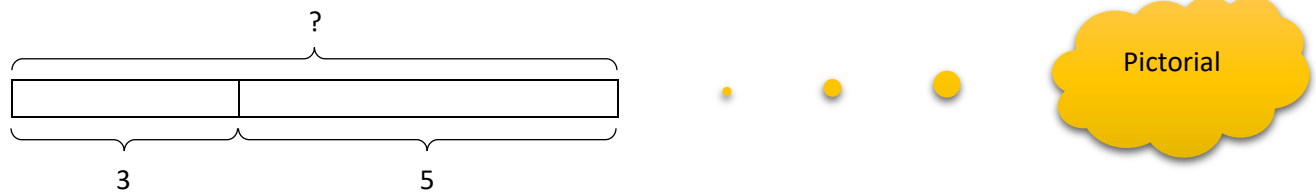
Let us consider the following question.

Mari has 3 apples. Dylan has 5 apples. How many do they have all together?

It would be possible to answer this question by **counting** the apples:



We could use a **bar model** to illustrate the situation:



Both of the above lead us to writing the **sum** for the question:

$$3 + 5 = 8.$$



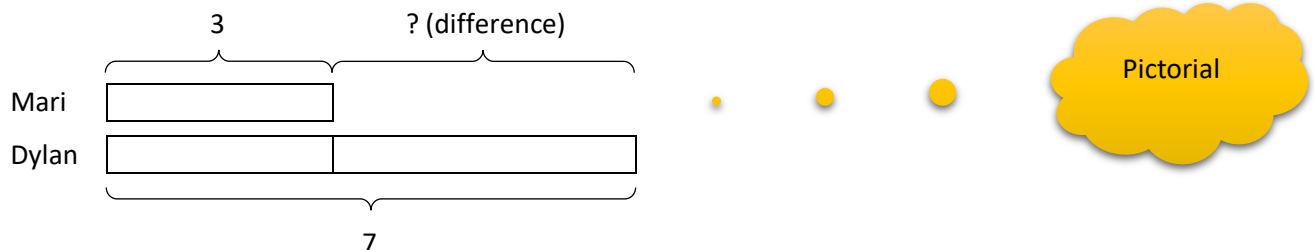
Now consider the following (similar) question.

Mari has 3 apples. Dylan has 7 apples. How many more apples does Dylan have?

It would be possible to answer this question by **pairing** the apples and seeing how many are **left over**.



We could use a **bar model** to illustrate the situation:



Both of the above lead us to writing the **sum** for the question:

$$7 - 3 = 4.$$



In this booklet, bar models will help us to solve complicated problems.

The first bar model (above) is known as a **part-whole** model.

The second bar model is known as a **comparison** model.

Skill

4

(f) A ticket inspector counted 24 boys; 35 girls and 75 adults on a train. How many more adults were on the train compared to boys?



Applying

(c) Ben drives to work from his house. The distance between his house and his work is 32.1 km. After travelling 14.7 km, Ben stops at a petrol station. How much further must Ben travel to reach his work?



(f) 3 shirts and 2 pairs of trousers costs £155. Each pair of trousers costs £10 more than each shirt. What is the cost of each shirt?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Ratios

A **ratio** is used to compare two or more numbers or amounts.

For example, the picture on the right shows the contents of a fruit bowl. We can write the ratio of oranges to apples as $2 : 3$, which is spoken as "two to three".

Exercise 3

The contents of Mr Jones' fruit bowl are shown on the right.

Write the ratio of

- (a) Bananas to oranges.
- (b) Oranges to bananas.
- (c) Bananas to the total number of fruits.
- (d) Oranges to the total number of fruits.

Sharing in a given ratio

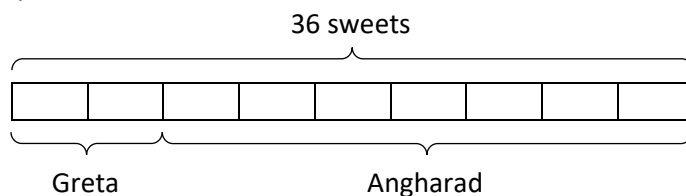
It is possible to divide a number or amount using a given ratio.

Example

A tube of Smarties contains 36 sweets. Share these sweets between Greta and Angharad in the ratio $2 : 7$.

Answer

There are $2 + 7 = 9$ parts in this ratio. We can construct a **part-whole bar model** to represent the situation



Each part of the ratio contains $36 \div 9 = 4$ sweets.

Greta receives $2 \times 4 = 8$ sweets.

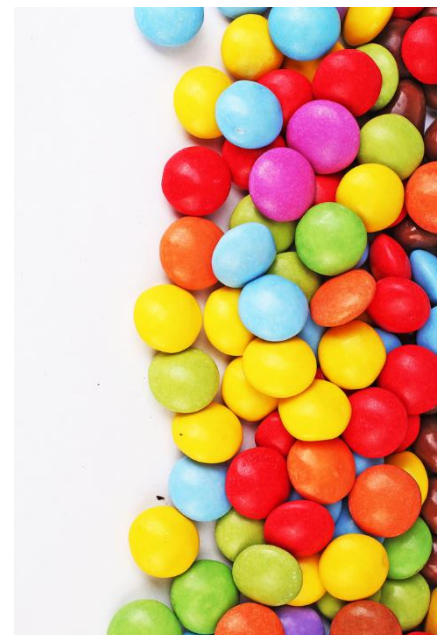
Angharad receives $7 \times 4 = 28$ sweets.

Check the answer: $8 + 28 = 36$ ✓



Exercise 4

- (a) Share £80 between Alun and Bethan in the ratio $3 : 5$.
- (b) Share a 360 g cake between Cai and Delyth in the ratio $1 : 2$.
- (c) Share \$26 between Elsi and Fiona in the ratio $9 : 4$.
- (d) Share a 150 g chocolate bar between Guto and Heledd in the ratio $7 : 8$.
- (e) Iwan wants to make a 500 ml tin of pink paint. How much red and white paint must he buy if the red : white mixing ratio is $1 : 4$?



Exercise 5

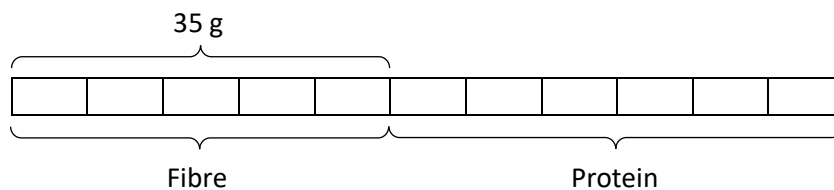
- (a) Share £40 between John, Kenny and Larry in the ratio 2 : 5 : 1.
- (b) Share a 750 ml drink between Menna, Nia and Olwen in the ratio 5 : 3 : 2.
- (c) The length of the edges in a triangle are in the ratio 3 : 4 : 5.
If the perimeter of the triangle is 180 cm, what is the length of the longest side?
- (d) Share €156 between Patrick, Rhydian, Selwyn and Tudur in the ratio 4 : 1 : 2 : 5.

Example

A snack has a fibre to protein ratio of 5 : 6.
If the snack contains 35 g of fibre, how much protein does it contain?

Answer

First, we construct a **part-whole bar model** to represent the situation.



We can see from the bar model that 5 parts of the ratio are worth 35 g.
So, one part of the ratio must be worth $35 \div 5 = 7$ g.
It follows that the snack contains $7 \times 6 = 42$ g of protein.

Exercise 6

- (a) A sum of money was shared between Victoria and William in the ratio 2 : 3.
If Victoria received £16, how much did William receive?
- (b) A set of footballs were shared between Llandudno football club and Bangor City football club in the ratio 5 : 4.
If Bangor received 24 balls, how many balls were shared out in total?
- (c) A long length of cable is cut into the ratio 2 : 3 : 4. The length of the **shortest** piece is 30 cm.
Calculate the length of the two other pieces of cable.
- (d) Jeni, Lora and Lottie buy a cinema season ticket between them. They split the cost using the ratio 1 : 5 : 4. Jeni pays the largest share and Lottie pays the smallest share. Lora pays £30. Calculate the cost of the season ticket.
- (e) A company analyses the number of followers on their social network accounts. They know the ratio of their *Twitter* followers to their *Instagram* followers is 4 : 7. If 360 people follow the company on *Twitter*, how many *Instagram* followers do they have?

Challenge!

The aspect ratio of a regular television is 16 : 9. If the screen on a regular television is 45 cm high, how wide is the screen?



Applying

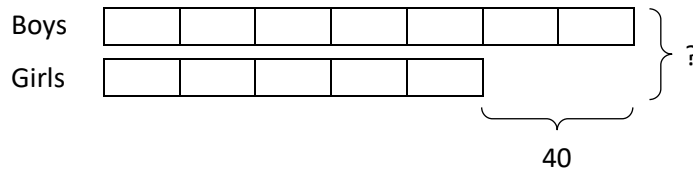


Example

The ratio of boys to girls in a sports club is 7 : 5.
There are 40 more boys than girls in the club.
What is the total number of children in the sports club?

Answer

In this question, we must use a **comparison bar model** to represent the situation.



We can see from the model that two parts of the ratio equals 40 people. So, one part of the ratio is worth $40 \div 2 = 20$ people.
As there are $7 + 5 = 12$ parts in total, there are $20 \times 12 = 240$ children in the sports club.

Exercise 7

6

(a) A farmer mixes fertiliser and water in the ratio 1 : 4. If 15 more litres of water than fertiliser are used in the mixture, how much fertiliser was used?

(b) A manager and deputy manager of a company are paid in the ratio 8 : 5. This year, the manager received £12,000 more than the deputy manager. How much did the manager earn this year?

(c) The ratio of Iestyn's age to Juliet's age is 2 : 3. Juliet is 7 years older than Iestyn. How old is Juliet?

(d) The ratio of adults to children on an aeroplane is 9 : 5. There are 24 more adults than children on the aeroplane. How many people are on the aeroplane in total?

(e) Huw likes to eat sweets. Unfortunately, he has had to have lots of fillings in his teeth because of this! At the moment the ratio of the number of his teeth without fillings to teeth with fillings is 5 : 3. Huw has 8 fewer teeth with fillings than without fillings. How many healthy teeth without fillings does Huw have?

Challenge!

Today, Siôn is 6 years older than Siân. In three years' time, the ratio of Siân's age to Siôn's age will be 2 : 5. How old is Siân today?

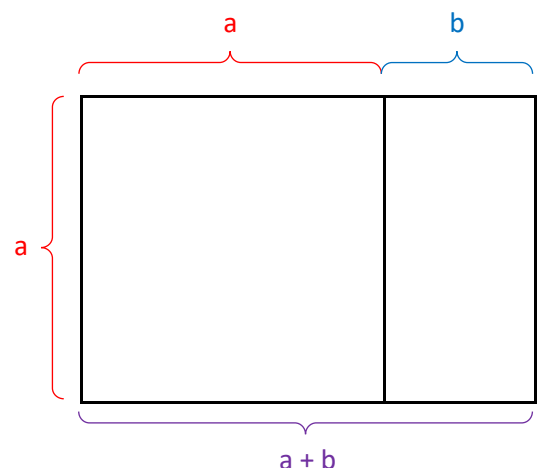
Investigation

Try to draw a rectangle that uses the golden ratio.

Instructions: Choose numbers a and b , and draw a rectangle with a length of $a + b$ cm and a width of a cm.

The sum $(a + b) \div a$ must give the same answer as the sum $a \div b$.
If this is true, then the ratios $a + b : a$ and $a : b$ will be the same as the **golden ratio**.

It is said that a rectangle with the golden ratio is the most beautiful rectangle that can be drawn!



Exercise 8**Skill****6**

Check that you get the same answers to both of the following questions.

- (a) Share £40 between Peredur and Ruth in the ratio 2 : 3.
 (b) Share £40 between Peredur and Ruth in the ratio 4 : 6.

Equivalent Ratios

The ratios 2 : 3 and 4 : 6 above are **equivalent ratios**. This means that they represent the same ratio; sharing any number or amount into either of the two ratios will give the same answer.

We can multiply or divide one of the ratios by 2 to get the other ratio.

$$\begin{array}{c} 2 : 3 \\ \downarrow \times 2 \\ 4 : 6 \end{array}$$

$$\begin{array}{c} 4 : 6 \\ \downarrow \div 2 \\ 2 : 3 \end{array}$$



It is possible to multiply both sides of any ratio with the same whole number to create an **equivalent ratio**. For example, we can multiply both sides of the ratio 5 : 7 by the number 3 to get the equivalent ratio 15 : 21.

Exercise 9

- (a) Write any three ratios that are equivalent to the ratio 1 : 2.
 (b) Write any three ratios that are equivalent to the ratio 3 : 8.
 (c) Write any three ratios that are equivalent to the ratio 10 : 13.

Simplifying a Ratio

We can sometimes **simplify** a ratio by dividing both sides of the ratio by the same number to leave an equivalent ratio that uses smaller whole numbers. For example, we can simplify the ratio 14 : 21, as we can divide both sides of the ratio by the number 7 to give the ratio 2 : 3.

$$\begin{array}{c} 14 : 21 \\ \downarrow \div 7 \\ 2 : 3 \end{array}$$

On the other hand, it is not possible to simplify the ratio 14 : 17, as it is not possible to divide both sides of the ratio by a whole number to give a ratio that contains two smaller whole numbers. When it is not possible to simplify a given ratio, we say that the ratio is in its **simplest form**.

Exercise 10

Simplify the following ratios, giving your answers in their simplest form.

- | | | | |
|----------------|------------------|-------------------------|---------------------|
| (a) 4 : 8 | (b) 6 : 18 | (c) 10 : 25 | (d) 12 : 30 |
| (e) 21 : 28 | (f) 16 : 24 | (g) 40 : 60 | (h) 36 : 60 |
| (i) 28 : 42 | (j) 90 : 120 | (k) 26 : 39 | (l) 45 : 75 |
| (m) 51 : 68 | (n) 24 : 72 | (o) 250 : 300 | (p) 252 : 420 |
| (q) 2 cm : 1 m | (r) £2 : 50p | (s) 0.5 litres : 200 ml | (t) 500 g : 3 kg |
| (u) 6 : 8 : 10 | (v) 36 : 48 : 54 | (w) 65 : 75 : 35 | (x) 22 : 6 : 8 : 18 |

Take care with (q) to (t): Make the units consistent first.

Comparing Ratios 1

Sometimes we can use **equivalent ratios** to compare different ratios.

Example

Delyth goes to the shop. One of the items on her shopping list is semi-skimmed milk. On the shelf at the supermarket, she sees the following three options:



Which option should Delyth choose to get the best value for money?

To answer this question, the options must be compared fairly. One way to do this is to write the options as ratios, and then forming **equivalent ratios** so that each ratio contains 1 pint.

Option 1: 1 pint : 49p

Option 2: 2 pints : 80p

Option 3: 4 pints : 180p

$$\begin{array}{c} \downarrow \div 2 \\ 1 \text{ pint} : 40\text{p} \end{array}$$

$$\begin{array}{c} \downarrow \div 4 \\ 1 \text{ pint} : 45\text{p} \end{array}$$

Why do we need to convert
£1.80 to 180p here?

By writing all of these ratios with 1 pint on the left-hand side, we can see that the option with the best value for money is the 2-pint carton for 80p, as it costs less per pint (40p, compared to 49p and 45p).

Exercise 11

For each of the following, which option gives the best value for money?

Applying

(a) Chocolate



(b) Margarine



The Unitary Method

Sometimes, it is useful to be able to write a ratio in the form $1 : n$ or $n : 1$, where n represents a specific number. In this format, the number 1 appears either on the left-hand side of the ratio, or on the right-hand side of the ratio.

- To write any ratio in the form $1 : n$, divide both sides of the ratio by the number on the **left-hand side** of the ratio.
- To write any ratio in the form $n : 1$, divide both sides of the ratio by the number on the **right-hand side** of the ratio.

Example

To write the ratio $4 : 5$ in the form $1 : n$, divide both sides of the ratio by the number 4.

$$\begin{array}{c} 4 : 5 \\ \downarrow \div 4 \\ 1 : 1.25 \end{array}$$

To write $4 : 5$ in the form $n : 1$, divide both sides of the ratio by the number 5.

$$\begin{array}{c} 4 : 5 \\ \downarrow \div 5 \\ 0.8 : 1 \end{array}$$



Exercise 12

Write the following ratios in the form $1 : n$.

(a) $2 : 5$

(b) $5 : 8$

(c) $4 : 7$

(d) $3 : 15$

(e) $3 : 10$

(f) $6 : 5$

(g) $9 : 11$

(h) $16 : 25$

Exercise 13

Repeat Exercise 12, but this time writing the ratios in the form $n : 1$.

Comparing Ratios 2

Example

Gethin goes to the supermarket. One item on his shopping list is baked beans. On the shelf in the supermarket, he sees two options:



150 g
35p



415 g
95p



Which option should Gethin choose to get the best value for money?

To answer this question, it is not possible to compare equivalent ratios like we did before, as the numbers are not convenient multiples of each other. Gethin must use ratios written using the **unitary method**, making use of one of the following methods.

Method 1: Compare the cost of 1 g of beans in each tin.

Method 2: Compare how much beans you get for every 1p in each tin.

Method 1

$$\begin{array}{r} 150 \text{ g} : 35\text{p} \\ \downarrow \div 150 \\ 1 \text{ g} : 0.233\text{p, to 3 d.p.} \end{array}$$

$$\begin{array}{r} 415 \text{ g} : 95\text{p} \\ \downarrow \div 415 \\ 1 \text{ g} : 0.229\text{p, to 3 d.p.} \end{array}$$

Conclusion

Every 1 g costs **less** in the large tin (0.229p compared to 0.233p), so the large tin (415 g for 95p) gives the best value for money.

Method 2

$$\begin{array}{r} 150 \text{ g} : 35\text{p} \\ \downarrow \div 35 \\ 4.29 \text{ g} : 1\text{p, to 3 d.p.} \end{array}$$

$$\begin{array}{r} 415 \text{ g} : 95\text{p} \\ \downarrow \div 95 \\ 4.37 \text{ g} : 1\text{p, to 3 d.p.} \end{array}$$

Conclusion

For every 1p, the large tin gives us **more** beans (4.37 g compared to 4.29 g), so the large tin (415 g for 95p) gives the best value for money.

Exercise 14

For each of the following, which option gives the best value for money?

Applying

6

(a) Crisps

32 g
45p



70 g
99p

(b) Nutella

750 g
£3.98



400 g
£2.50

(c) Coca-Cola

330 ml
70p



500 ml
£1.05



1.25 litre
£2.75

Exercise 15

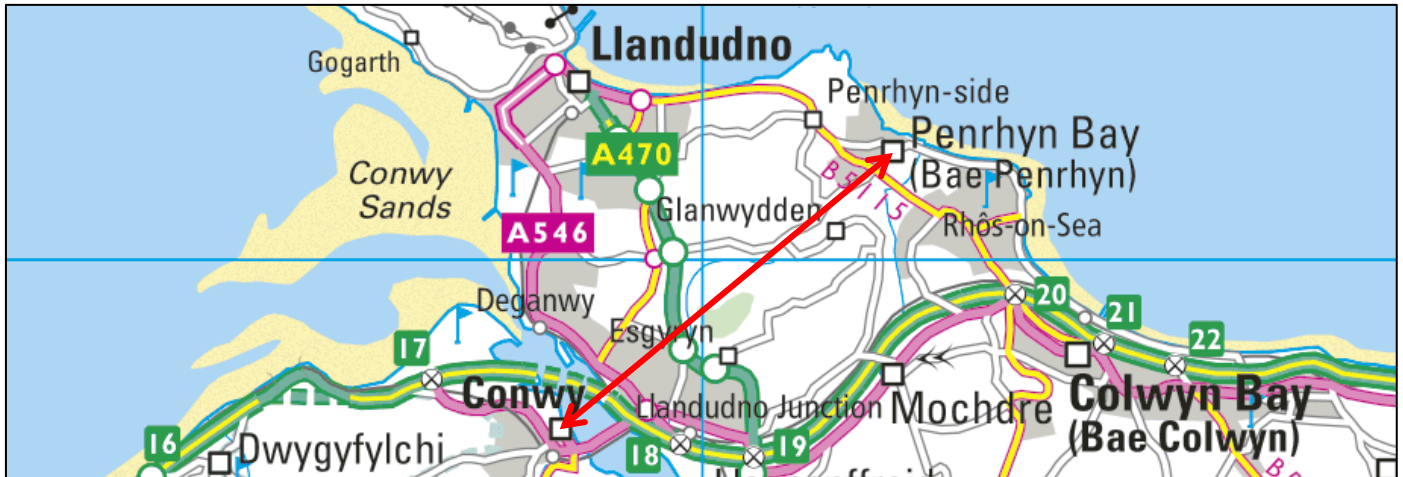
Adolygu

5

- (a) Fill in the blanks: (i) 1 cm = _____ mm (ii) 1 m = _____ cm
 (iii) 1 km = _____ m (iv) 1 mile $\approx$ _____ km
- (b) Convert the measurement 1,400,000 cm into km. (c) Convert the measurement 45 km into cm.

Map Work

Usually, the scale of a map is given as a ratio. For example, the scale of the map below is 1 : 100,000.



On the map, the distance between Penrhyn Bay and the centre of Conwy (as the crow flies) is 5.7 cm (the red arrow).

In reality, this distance would be $5.7 \times 100,000 = 570,000$ cm. This distance does not mean much to us, so it is a good idea to convert this into more useful units.

$$\begin{array}{rcl}
 570,000 \text{ cm} & & \\
 \downarrow \div 100 & & \\
 5,700 \text{ m} & & \\
 \downarrow \div 1,000 & & \\
 5.7 \text{ km} & &
 \end{array}$$

We **multiply** by the map scale to convert the distance on the map to the real-life distance.



In general, for a map with scale 1 : n ,

We **multiply** by n to convert the distance on the map to a real-life distance.
 We **divide** by n to convert a real-life distance to a distance on the map.

Exercise 16

Applying

6

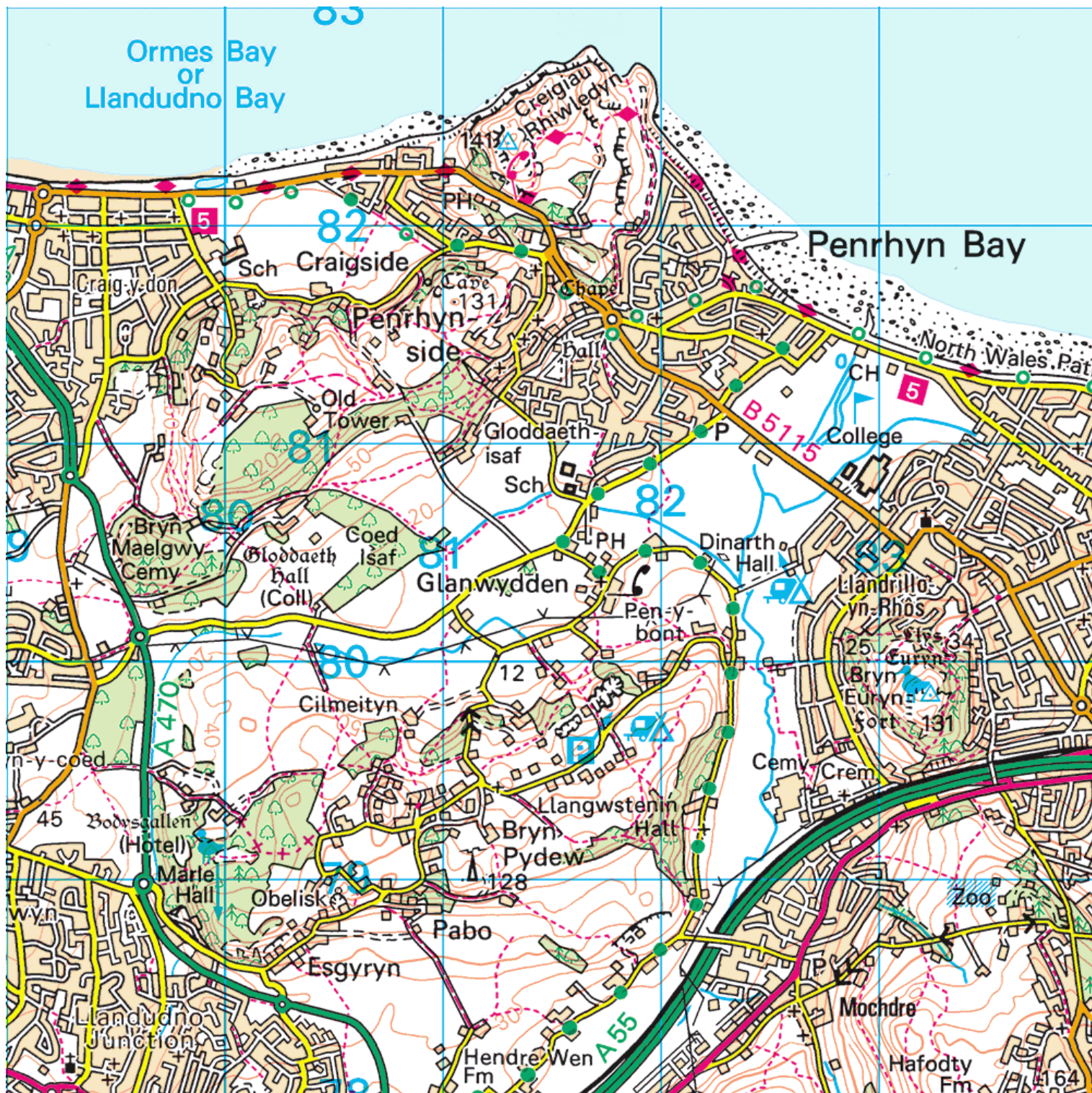
- (a) A map has a scale of 1 : 10,000. If the length of a road is 6 cm on the map, what is the actual length of the road? (Calculate your answer in cm first, then convert to km.)
- (b) A map has a scale of 1 : 10,000. If the actual length of a road is 2 km, what is length of this road on the map? (Convert the actual length of the road into cm first, then use the map scale to calculate the length on the map.)
- (c) A map has a scale of 1 : 5,000.
- (i) If the length of a road is 4 cm on the map, what is the actual length of the road? (Give your answer in meters.)
- (ii) The length of another road is 150 m. Convert this to cm. What is the length of this road on the map?
- (d) A map has a scale of 1 : 2,000.
- (i) If the length of a road is 13.5 cm on the map, what is the actual length of the road? (Give your answer in meters.)
- (ii) The length of another road is 86 m. Convert this to cm. What is the length of this road on the map?

Exercise 17

6

The scale of the map below is 1 : 25,000. Use the map to find the actual distances (in km) between

- | | |
|---|---|
| (a) Ysgol y Creuddyn and Coleg Llandrillo. | (b) Bodysgallen Hotel and Dinarth Hall. |
| (c) The phone box in Glanwydden and the church in Llandudno Junction. | (d) The car park and the zoo. |
| (e) Hendre Wen Farm and the Old Tower. | (f) The cave and the television mast. |
| (g) The crematorium and Ysgol y Gogarth. | (h) The obelisk and the golf clubhouse. |
| (i) The red phone box and the fort. | (j) The two pubs. |



Exercise 18 (Revision)

6

- (a) Megan's ruler is twice as long as Alun's ruler. What is the ratio of the length of Megan's ruler to Alun's ruler?
- (b) Share €600 between Huw, Elin and Fred in the ratio 8 : 1 : 6.
- (c) Two sisters share an inheritance in the ratio 4 : 5. If the eldest sister received £40,000, the larger amount, how much did the youngest sister receive?
- (d) In a fishing competition, the ratio of the number of fish caught by the winner to the number of fish caught by the person who came second was 9 : 5. The winner caught 28 more fish than the person who came second. How many fish did the winner catch?
- (e) Write any two ratios that are equivalent to 4 : 7.
- (f) Simplify the ratio 200 : 240.
- (g) Show which of these is the best value: 5 litres of oil for £18.50 or 2 litres of oil for £7.00.
- (h) Write the ratio 23 : 25 in the form $n : 1$.
- (i) The ratios of the number of teachers to the number of pupils in two schools are 3 : 64 and 4 : 89. By converting these ratios to the form $1 : n$, state which school has the smallest class sizes.
- (j) The ratios of Lycra to other materials in two different fabrics are 2 : 27 and 3 : 39. By converting these ratios to the form $1 : n$, say which fabric contains the most Lycra.
- (k) The scale of a map is 1 : 15,000. If the length of a road is 8 cm on the map, what is the actual length of the road? Give your answer in cm first, then convert to km.
- 



- (I) Kevin has a model of his favourite car, a *Porsche 911*. The scale of the model is 1 : 25. Complete the table.

	<i>Model (cm)</i>	<i>Actual Car (cm)</i>	<i>Actual Car (m)</i>
Height	5 cm		
Length	8.1 cm		
Wheel Diameter			0.4 m
Door Width			0.5 m

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Fractions



A **fraction** is a way of writing a part of a shape, number or amount.

The top of the fraction is called the **numerator**.

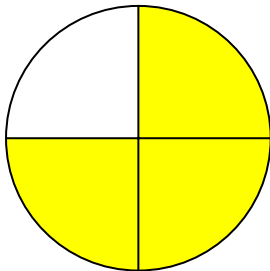
The bottom of the fraction is called the **denominator**.

$$\frac{3}{4}$$

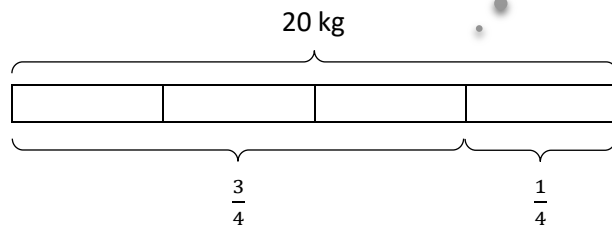
The line of the fraction is called the **vinculum**.

Example

$\frac{3}{4}$ of the following circle is shaded yellow.



$\frac{3}{4}$ of 20 kg is 15 kg.



$$20 \text{ kg} \div 4 = 5 \text{ kg}$$

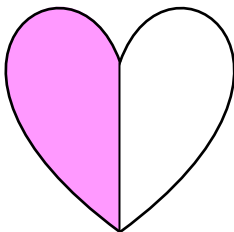
$$5 \text{ kg} \times 3 = 15 \text{ kg}$$

Part-whole bar model

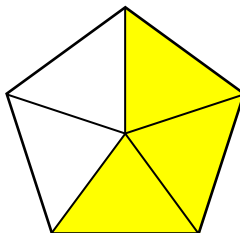
Exercise 19

Which fraction of the following shapes is shaded?

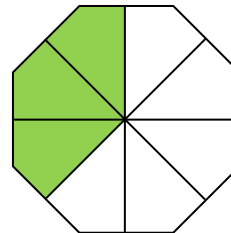
(a)



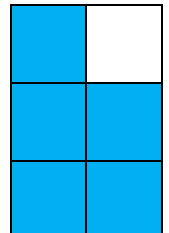
(b)



(c)



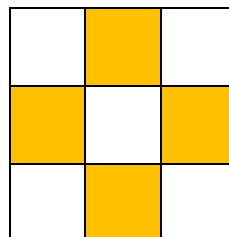
(d)



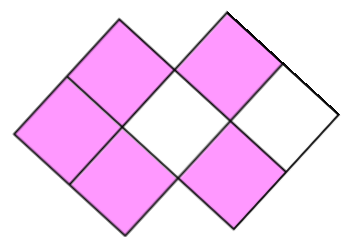
(e)



(f)



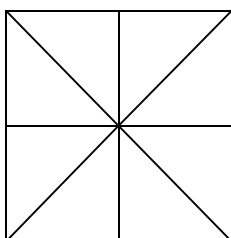
(g)



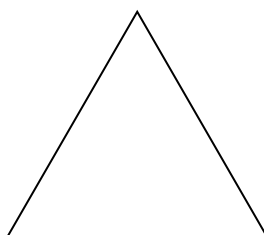
Exercise 20

Shade the following fractions of the shapes.

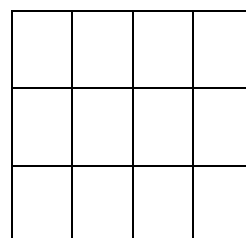
(a) $\frac{5}{8}$



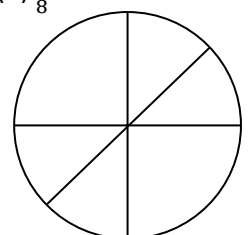
(b) $\frac{1}{3}$



(c) $\frac{1}{4}$



(d) $\frac{3}{8}$



Exercise 21

Use a **part-whole bar model** to calculate the following fractions.

(a) $\frac{1}{5}$ of 40

(b) $\frac{3}{8}$ of £40

(c) $\frac{5}{6}$ of 42 km

(d) $\frac{2}{7}$ of 35 ml



5

Exercise 22

Calculate the following fractions. (Use a part-whole bar model if this helps.)

(a) $\frac{1}{2}$ of 16

(b) $\frac{2}{4}$ of 16

(c) $\frac{1}{8}$ of 16

(d) $\frac{7}{8}$ of 16

(e) $\frac{2}{5}$ of 15

(f) $\frac{2}{5}$ of 10

(g) $\frac{2}{5}$ of 5

(h) $\frac{2}{5}$ of 1

(i) $\frac{3}{7}$ of £14

(j) $\frac{4}{9}$ of £36

(k) $\frac{3}{10}$ of £40

(l) $\frac{7}{8}$ of £24

(m) $\frac{1}{6}$ of 18 mm

(n) $\frac{1}{7}$ of 21 cm

(o) $\frac{1}{8}$ of 72 m

(p) $\frac{1}{9}$ of 18 km

(q) $\frac{1}{2}$ of 7

(r) $\frac{3}{4}$ of 14

(s) $\frac{7}{10}$ of 18

(t) $\frac{3}{8}$ of 2

(u) $\frac{7}{11}$ of £88

(v) $\frac{3}{13}$ of \$39

(w) $\frac{7}{15}$ of €60

(x) $\frac{16}{17}$ of ¥51

(y) $\frac{5}{7}$ of 49

(z) $\frac{5}{9}$ of £72

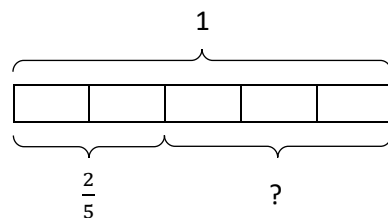
(\alpha) $\frac{10}{11}$ of 132

(\beta) $\frac{1}{2}$ of £2.50

**Example**

What is $1 - \frac{2}{5}$?

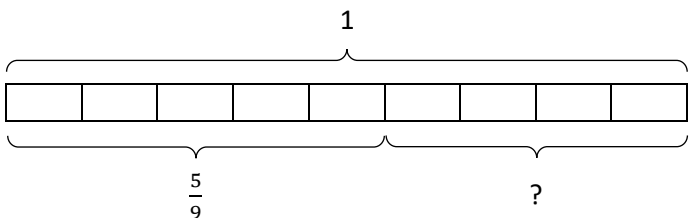
Here is a part-whole bar model for the sum.



We see that $1 - \frac{2}{5} = \frac{3}{5}$.

What is $1 - \frac{5}{9}$?

Here is a part-whole bar model for the sum.



We see that $1 - \frac{5}{9} = \frac{4}{9}$.

Exercise 23

Use a **part-whole bar model** to calculate the following.

(a) $1 - \frac{1}{5}$

(b) $1 - \frac{3}{4}$

(c) $1 - \frac{2}{7}$

(d) $1 - \frac{7}{8}$

**Exercise 24**

Calculate the following. (Use a part-whole bar model if this helps.)

(a) $1 - \frac{4}{5}$

(b) $1 - \frac{1}{4}$

(c) $1 - \frac{5}{7}$

(d) $1 - \frac{1}{8}$

(e) $1 - \frac{5}{6}$

(f) $1 - \frac{2}{9}$

(g) $1 - \frac{1}{3}$

(h) $1 - \frac{3}{10}$

(i) $1 - \frac{2}{9} - \frac{3}{9}$

(j) $1 - \frac{1}{7} - \frac{2}{7}$

(k) $1 - \frac{4}{11} - \frac{5}{11}$

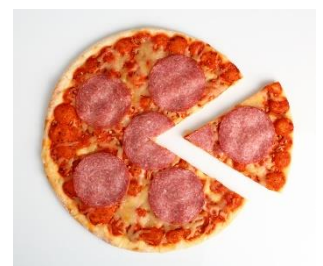
(l) $1 - \frac{7}{20} - \frac{3}{20}$

(m) $1 - \frac{4}{5} + \frac{1}{5}$

(n) $1 - \frac{7}{9} + \frac{2}{9}$

(o) $1 - \frac{7}{10} + \frac{2}{10}$

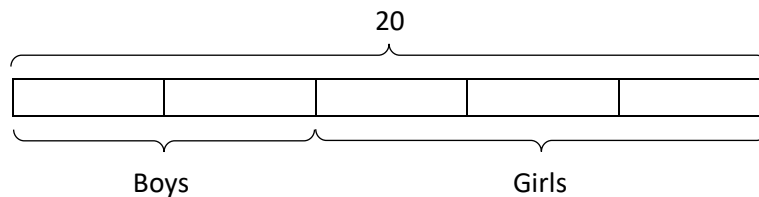
(p) $1 - \frac{8}{8}$



Worted Problems: Example

(a) There are 20 pupils in a class. $\frac{2}{5}$ of the learners are boys and the rest are girls. How many girls are in the class?

Answer: To start, let us construct a **part-whole bar model** to represent the situation.



5 units is 20 pupils.

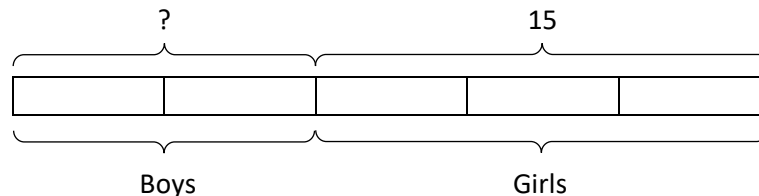
1 unit is $20 \div 5 = 4$ pupils.

3 units is $3 \times 4 = 12$ pupils.

There are 12 girls in the class.

(b) $\frac{2}{5}$ of the learners in the class are boys, and the rest are girls. There are 15 girls in the class. How many boys are in the class?

Answer: To start, let us construct a **part-whole bar model** to represent the situation.



3 units is 15 pupils.

1 unit is $15 \div 3 = 5$ pupils.

2 units is $2 \times 5 = 10$ pupils.

There are 10 boys in the class.

Exercise 25

(a) There are 30 pupils in a class. $\frac{2}{5}$ of the pupils are boys. How many girls are in the class?

(b) $\frac{2}{5}$ of the pupils in a class are boys, and the rest are girls. There are 21 girls in the class.

(i) How many boys are in the class?

(ii) How many pupils are in the class in total?

(c) A group of pupils went on a trip to the local zoo. $\frac{5}{8}$ of the pupils were girls. If 40 girls went on the trip,

(i) How many boys went on the trip?

(ii) How many more girls compared to boys went on the trip?

(d) Emma read $\frac{5}{7}$ of a book on Saturday. She has 36 pages left to read before finishing the book. How many pages did Emma read on Saturday?

(e) A jar was full of 10p and 20p coins. $\frac{3}{10}$ of the coins were 20p coins. The rest were 10p coins. There were 24 more 10p coins than 20p coins.

(i) How many 10p coins were in the jar?

(ii) What was the value of all the coins in the jar?

Applying

5



Fractions as Numbers

In addition to using fractions to calculate part of a shape, number or amount, every fraction is a number in itself. For example, the fraction $\frac{3}{4}$ appears in the following location on the number line.



Comparing the Size of Fractions 1

Which fraction is greatest: $\frac{2}{3}$ or $\frac{4}{5}$?

To answer this question, imagine two number lines placed on top of each other. Let us share the first number line into thirds, and share the second number line into fifths.



Comparing the locations of $\frac{2}{3}$ and $\frac{4}{5}$ above, we can see that the fraction $\frac{4}{5}$ is greater than the fraction $\frac{2}{3}$.



The fraction $\frac{4}{5}$ is greater than the fraction $\frac{2}{3}$, however it is not true that $\frac{4}{5}$ of a number is always greater than $\frac{2}{3}$ of a number. For example, $\frac{4}{5}$ of 10 = 8 is less than $\frac{2}{3}$ of 15 = 10.

Exercise 26

Look at the picture on page 2. Discuss with a partner how the picture can help answer the example above. Discuss how the picture can be used to answer similar questions (like the ones in the next exercise).

Exercise 27

Use the picture on page 2 to decide which fraction is the greatest in each of the following pairs.

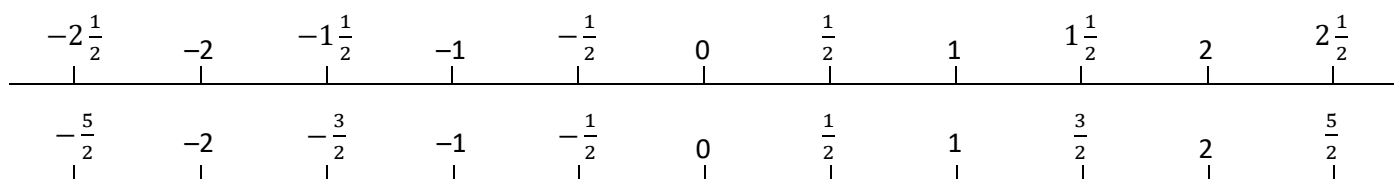
- (a) $\frac{3}{4}$ or $\frac{1}{2}$? (b) $\frac{1}{3}$ or $\frac{1}{4}$? (c) $\frac{3}{4}$ or $\frac{2}{3}$? (d) $\frac{3}{5}$ or $\frac{1}{2}$?
 (e) $\frac{3}{4}$ or $\frac{3}{5}$? (f) $\frac{4}{5}$ or $\frac{5}{6}$? (g) $\frac{3}{10}$ or $\frac{1}{4}$? (h) $\frac{4}{7}$ or $\frac{3}{5}$?

Skill

5

Mixed Numbers and Improper Fractions

Up to now, the fractions we have considered have all appeared on the number line between 0 and 1. This is not true of all fractions. For example, the two number lines below give two different methods of considering the halves between -2.5 and 2.5.



The first number line uses **mixed numbers**, while the second number line uses **improper fractions**.

A **mixed number** is a whole number followed by a fraction.

An **improper fraction** is a fraction whose numerator is **greater** than its denominator.

While we are considering definitions, it is also worth defining the usual type of fraction.

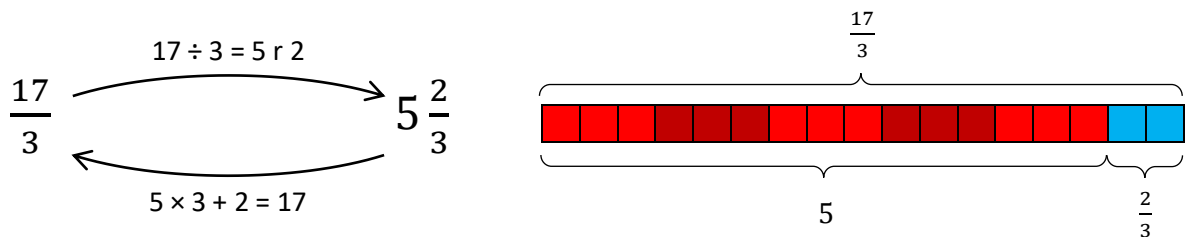
A **proper fraction** is a fraction whose numerator is **less** than its denominator.

It is possible to convert any improper fraction into a mixed number and vice versa.

Example

(a) $\frac{17}{3}$ is an improper fraction. To convert it into a mixed number, think about $\frac{17}{3}$ as the division sum $17 \div 3$, and ask “how many times does 3 fit into 17?”. The answer is **5 times**, **remainder 2**. So the improper fraction $\frac{17}{3}$ is equal to the mixed number $5\frac{2}{3}$.

(b) $5\frac{2}{3}$ is a mixed number. To convert it into an improper fraction, first count how many **thirds** are in **five** wholes: $5 \times 3 = 15$. Then add the other **two** thirds: $15 + 2 = 17$. It follows that the mixed number $5\frac{2}{3}$ is equal to the improper fraction $\frac{17}{3}$.



Exercise 28

Convert the following improper fractions into mixed numbers.

- (a) $\frac{14}{3}$ (b) $\frac{16}{3}$ (c) $\frac{9}{2}$ (d) $\frac{11}{5}$ (e) $\frac{12}{5}$ (f) $\frac{13}{5}$ (g) $\frac{14}{5}$
 (h) $\frac{15}{5}$ (i) $\frac{17}{7}$ (j) $\frac{28}{5}$ (k) $\frac{31}{9}$ (l) $\frac{23}{10}$ (m) $\frac{53}{11}$ (n) $\frac{87}{12}$

Exercise 29

Convert the following mixed numbers into improper fractions.

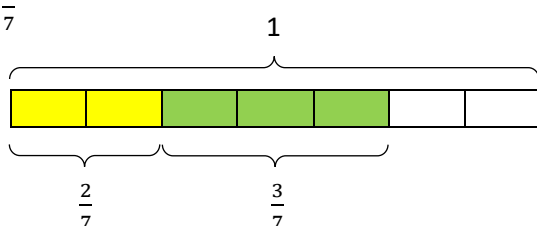
- (a) $4\frac{1}{3}$ (b) $6\frac{2}{3}$ (c) $3\frac{1}{2}$ (d) $2\frac{1}{9}$ (e) $7\frac{2}{5}$ (f) $1\frac{4}{13}$ (g) $8\frac{1}{6}$
 (h) $8\frac{3}{10}$ (i) $13\frac{2}{7}$ (j) $100\frac{6}{13}$ (k) $5\frac{4}{7}$ (l) $5\frac{5}{7}$ (m) $5\frac{6}{7}$ (n) $6\frac{1}{7}$

Addition and Subtraction of Fractions with Equal Denominators

If the denominators of two fractions are equal, then it is simple to add or subtract the two fractions.

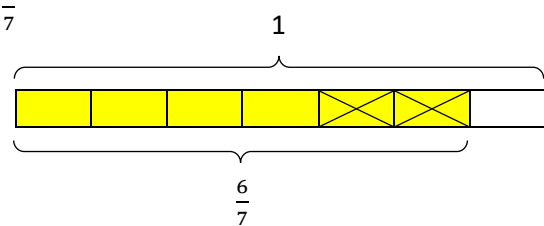
Example

(a) $\frac{2}{7} + \frac{3}{7}$



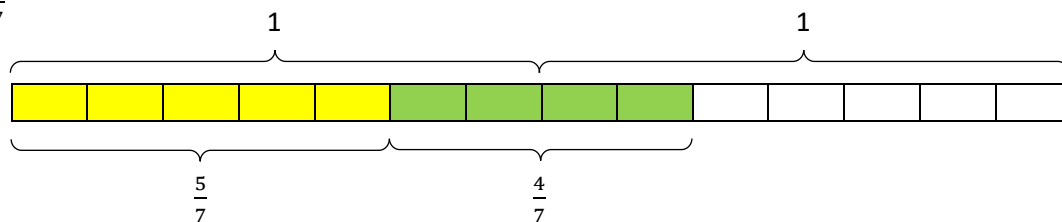
$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

(b) $\frac{6}{7} - \frac{2}{7}$



$$\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$$

(c) $\frac{5}{7} + \frac{4}{7}$



$$\frac{5}{7} + \frac{4}{7} = \frac{9}{7} = 1\frac{2}{7}$$

Exercise 30

8

Use a **part-whole bar model** to calculate the following. Where it is appropriate, give your answer as a mixed number.

(a) $\frac{2}{6} + \frac{3}{6}$

(b) $\frac{7}{8} - \frac{5}{8}$

(c) $\frac{3}{4} + \frac{3}{4}$

Exercise 31

Calculate the following. Where it is appropriate, give your answer as a mixed number.

(a) $\frac{1}{5} + \frac{2}{5}$

(b) $\frac{3}{5} - \frac{1}{5}$

(c) $\frac{3}{5} + \frac{4}{5}$

(d) $\frac{3}{5} + \frac{2}{5}$

(e) $\frac{3}{7} + \frac{2}{7}$

(f) $\frac{2}{9} + \frac{4}{9}$

(g) $\frac{7}{8} - \frac{2}{8}$

(h) $\frac{10}{11} - \frac{3}{11}$

(i) $\frac{2}{13} + \frac{3}{13} + \frac{4}{13}$

(j) $\frac{5}{13} + \frac{6}{13} + \frac{7}{13}$

(k) $\frac{12}{13} - \frac{4}{13} + \frac{6}{13}$

(l) $\frac{11}{13} + \frac{3}{13} - \frac{7}{13}$

(m) $\frac{7}{9} - \frac{3}{9} - \frac{4}{9}$

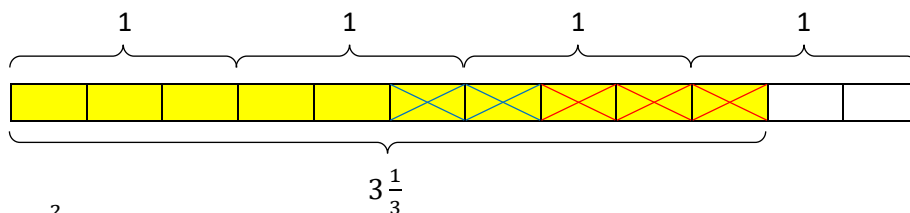
(n) $\frac{10}{7} - \frac{1}{7}$

(o) $\frac{11}{8} + \frac{14}{8}$

(p) $\frac{7}{10} + \frac{15}{10} - \frac{2}{10}$

Example

$3\frac{1}{3} - 1\frac{2}{3}$



$$3\frac{1}{3} - 1\frac{2}{3} = 1\frac{2}{3}$$

Exercise 32

Calculate the following. Where it is appropriate, give your answer as a mixed number.

(a) $3\frac{1}{3} + \frac{1}{3}$

(b) $3\frac{1}{3} - \frac{1}{3}$

(c) $3\frac{1}{3} + 2\frac{1}{3}$

(d) $3\frac{1}{3} - \frac{2}{3}$

(e) $1\frac{2}{5} + \frac{1}{5}$

(f) $1\frac{2}{5} - \frac{3}{5}$

(g) $1\frac{2}{5} + 4\frac{1}{5}$

(h) $1\frac{2}{5} + 2\frac{4}{5}$

(i) $4\frac{3}{7} + 1\frac{2}{7}$

(j) $4\frac{3}{7} - 3$

(k) $4\frac{3}{7} - 1\frac{5}{7}$

(l) $4\frac{3}{7} + 6\frac{5}{7}$

(m) $2\frac{7}{8} - \frac{3}{8}$

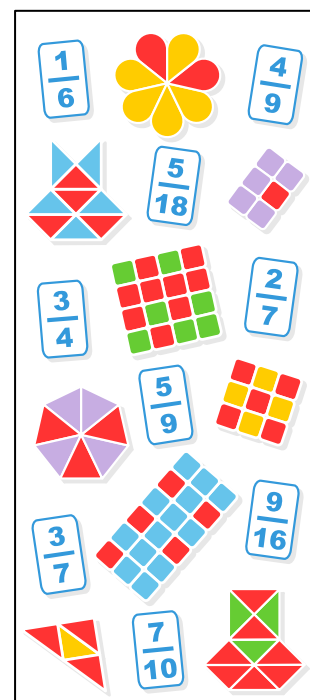
(n) $2\frac{7}{8} + \frac{3}{8}$

(o) $2\frac{7}{8} + 7\frac{5}{8}$

(p) $2\frac{7}{8} - 1\frac{1}{8}$

Challenge!

Using a ruler, pair the fractions to the pictures in the panel on the right.



Exercise 33

5

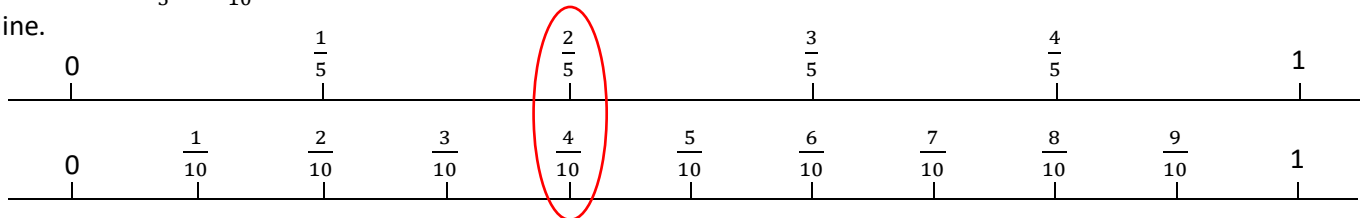
Check that you obtain the same answer to both of the following questions.

(a) Calculate $\frac{2}{5}$ of £40.

(b) Calculate $\frac{4}{10}$ of £40.

**Equivalent Fractions**

The fractions $\frac{2}{5}$ and $\frac{4}{10}$ above are **equivalent fractions**. This means that they appear in the same place on the number line.



We can multiply or divide the numerator and denominator of the fractions by 2 to obtain the other fraction.

$$\frac{2}{5} \xrightarrow{\times 2} \frac{4}{10}$$

$$\frac{4}{10} \xrightarrow{\div 2} \frac{2}{5}$$

It is possible to multiply the numerator and denominator of any fraction by the same whole number to create an **equivalent fraction**. For example, we can multiply the numerator and denominator of $\frac{5}{12}$ by 3 to obtain the equivalent fraction $\frac{15}{36}$.

Exercise 34

- (a) Write any three fractions that are equivalent to the fraction $\frac{1}{3}$.
- (b) Write any three fractions that are equivalent to the fraction $\frac{3}{11}$.
- (c) Write any three fractions that are equivalent to the fraction $\frac{10}{23}$.

**Comparing the Size of Fractions 2**

Which fraction is greatest: $\frac{2}{3}$ or $\frac{4}{5}$?

Earlier, on page 18, we answered this question using **number lines**. Let us now answer the question in a different way, using **equivalent fractions**.

The denominators of the two fractions under consideration are 3 and 5. The **lowest common denominator** of 3 and 5 is 15, since 15 is the lowest common multiple of 3 and 5. By writing equivalent fractions for $\frac{2}{3}$ and $\frac{4}{5}$ so that they have 15 as a new common denominator, we can compare the size of the fractions.

$$\frac{2}{3} \xrightarrow{\times 5} \frac{10}{15}$$

$$\frac{4}{5} \xrightarrow{\times 3} \frac{12}{15}$$

As $\frac{12}{15}$ is greater than $\frac{10}{15}$, we can say that $\frac{4}{5}$ is greater than $\frac{2}{3}$.

The lowest common denominator of two numbers is the lowest common multiple of the two numbers.

Exercise 35

What is the lowest common denominator of the following pairs of numbers?

- (a) 2 and 3 (b) 2 and 4 (c) 2 and 5 (d) 2 and 8
- (e) 4 and 5 (f) 3 and 6 (g) 7 and 8 (h) 8 and 10

This is the lowest common multiple from Year 8 work.

Exercise 36

By writing equivalent fractions with common denominators, decide which fraction is the greatest in each of the following pairs.

(a) $\frac{3}{4}$ or $\frac{1}{2}$?

(b) $\frac{1}{3}$ or $\frac{1}{4}$?

(c) $\frac{3}{4}$ or $\frac{2}{3}$?

(d) $\frac{3}{5}$ or $\frac{1}{2}$?

(e) $\frac{3}{5}$ or $\frac{2}{3}$?

(f) $\frac{4}{5}$ or $\frac{5}{6}$?

(g) $\frac{3}{10}$ or $\frac{1}{4}$?

(h) $\frac{4}{7}$ or $\frac{3}{5}$?

Simplifying Fractions

We can sometimes **simplify** a fraction by dividing the numerator and denominator of the fraction by the same number to give an equivalent fraction that contains smaller whole numbers. For example, we can simplify the fraction $\frac{14}{35}$, as we can divide the numerator and denominator by 7 to give $\frac{2}{5}$.

$$\frac{14}{35} \xrightarrow{\div 7} \frac{2}{5}$$

On the other hand, it is not possible to simplify the fraction $\frac{14}{31}$, as it is not possible to divide the top and bottom of this fraction by a whole number to give a fraction that contains smaller whole numbers. When it is not possible to simplify a given fraction, we say that the fraction is in its **simplest form**.

**Exercise 37**

Simplify the following fractions, giving your answers in their simplest form.

(a) $\frac{4}{8}$

(b) $\frac{6}{18}$

(c) $\frac{10}{25}$

(d) $\frac{12}{30}$

(e) $\frac{21}{28}$

(f) $\frac{16}{24}$

(g) $\frac{40}{60}$

(h) $\frac{36}{60}$

(i) $\frac{28}{42}$

(j) $\frac{90}{120}$

(k) $\frac{26}{39}$

(l) $\frac{45}{75}$

(m) $\frac{51}{68}$

(n) $\frac{24}{72}$

(o) $\frac{250}{300}$

(p) $\frac{252}{420}$

Exercise 38

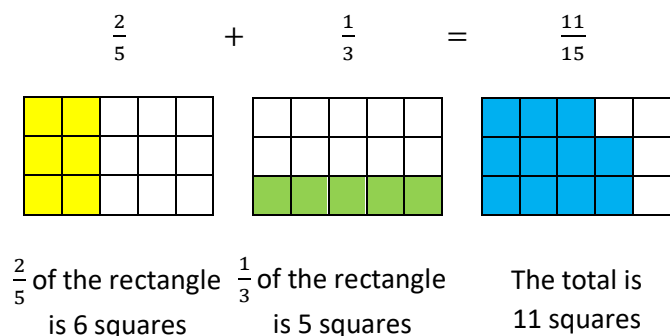
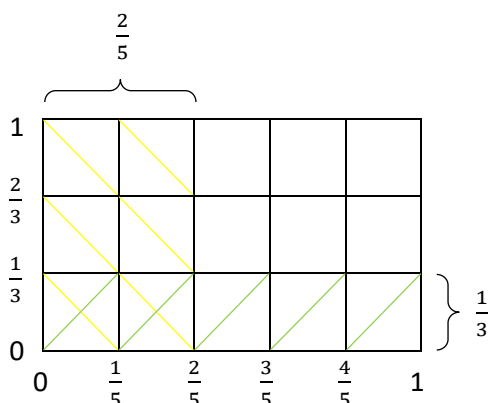
Discuss what is similar and what is different between Exercises 10 and 37.

Extension

Addition and Subtraction of Fractions with Different Denominators

If the denominators of two fractions are different, then adding and subtracting two fractions is more complicated than the technique for equal denominators.

Let us first consider how to calculate the sum $\frac{2}{5} + \frac{1}{3}$. Below, there is a bar model for the sum, where the large rectangle represents 1 unit.



Here are two methods of calculating $\frac{2}{5} + \frac{1}{3}$.

The Traditional Method

Step 1: Find the **lowest common denominator** of the two fractions. Here, the lowest common multiple of 3 and 5 is 15.

Step 2: Write **equivalent fractions** for $\frac{2}{5}$ and $\frac{1}{3}$, using 15 as the common denominator.

$$\frac{2}{5} \xrightarrow{\times 3} \frac{6}{15}$$

$$\frac{1}{3} \xrightarrow{\times 5} \frac{5}{15}$$

Step 3: **Add** the two new fractions.

$$\frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

Step 4: Check to see if it is possible to simplify the answer. (It is not possible in this case.)

Step 1: You can find a common denominator by multiplying the two original denominators, however this might not be the lowest common denominator.

The Peanut Method

Step 1: Draw the following template. (Note that the first fraction goes on the top and the second fraction goes on the left.)

	2	5
1	×	
3		

Step 2: Fill in the gaps in the table by **multiplying** the numbers.

	2	5
1	×	5
3	6	15

Step 3: **Add** the two numbers that form the peanut shape.

	2	5
1	×	5
3	6	15

$$6 + 5 = 11$$

The answer is $\frac{11}{15}$.

Step 4: Check to see if it is possible to simplify the answer. (It is not possible in this case.)

Exercise 39

Calculate the following. Give your answers in their simplest form.

(a) $\frac{3}{5} + \frac{1}{3}$

(b) $\frac{2}{5} + \frac{2}{3}$

(c) $\frac{4}{5} + \frac{1}{3}$

(d) $\frac{6}{5} + \frac{2}{3}$

(e) $\frac{1}{2} + \frac{1}{4}$

(f) $\frac{1}{2} + \frac{3}{8}$

(g) $\frac{3}{4} + \frac{1}{5}$

(h) $\frac{3}{4} + \frac{5}{12}$

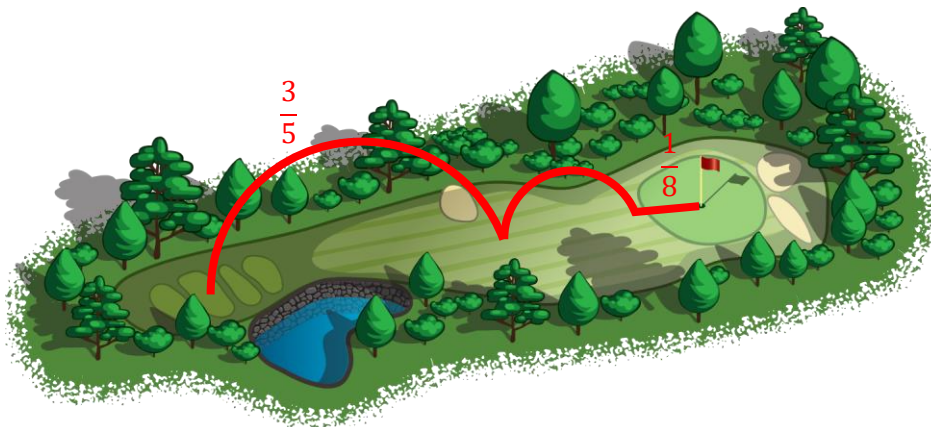
Exercise 40

(a) In a horse race, there was $\frac{1}{3}$ of a horse length between the winning horse and the second placed horse, and $\frac{1}{2}$ of a horse length between the second and third placed horses. What fraction of a horse length was between the winning horse and the third placed horse?

Skill

8

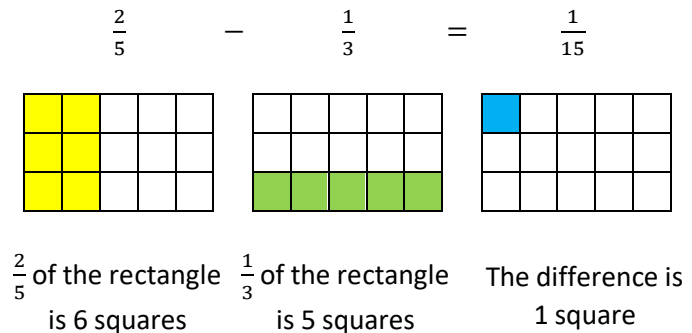
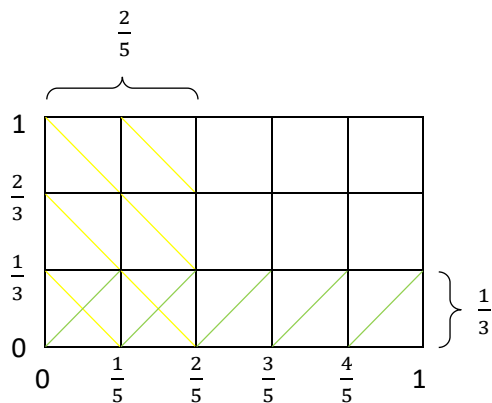
Applying



(b) Whilst playing golf, Sioned estimated that her first shot went $\frac{3}{5}$ of the distance to the hole, while her third (and final) shot was hit $\frac{1}{8}$ of the way. What fraction of the distance to the hole was Sioned's second shot?

Subtracting Fractions

Let us now consider how to calculate the sum $\frac{2}{5} - \frac{1}{3}$. Below, there is a bar model for the sum, where the large rectangle represents 1 unit.



Here are two methods of calculating $\frac{2}{5} - \frac{1}{3}$.

The Traditional Method

$$\frac{2}{5} - \frac{1}{3} = \frac{6}{15} - \frac{5}{15} = \frac{1}{15}$$

Write the fractions over the lowest common denominator

The Peanut Method

	2	5
1	×	5
3	6	15

$$6 - 5 = 1$$

The answer is $\frac{1}{15}$.

Exercise 41

Calculate the following. Give your answers in their simplest form.

(a) $\frac{3}{5} - \frac{1}{3}$

(b) $\frac{4}{5} - \frac{2}{3}$

(c) $\frac{6}{5} - \frac{1}{3}$

(e) $\frac{3}{4} - \frac{1}{2}$

(f) $\frac{1}{2} - \frac{3}{8}$

(g) $\frac{3}{4} - \frac{1}{5}$

(d) $\frac{11}{5} - \frac{2}{3}$

(h) $\frac{3}{4} - \frac{5}{12}$

Exercise 42

(a) In a horse race, there was $\frac{1}{3}$ of a horse length between the winning horse and the second placed horse, and $\frac{1}{2}$ of a horse length between the winning horse and the third placed horse. What fraction of a horse length was between the second and third placed horses?

(b) An airline company undertake short, medium and long distance trips. Last year $\frac{2}{3}$ of the trips were short ones and a $\frac{1}{4}$ were medium trips. $\frac{2}{5}$ of their trips went to Paris, which was considered a short trip.

(i) What fraction of the company's trips did not go to Paris?

(ii) What fraction of the company's trips were **short** trips that did not go to Paris?

(iii) What fraction of the company's trips were long trips?

There were 650 long trips last year.

(iv) Last year, how many trips did the company have in total?

(v) Last year, how many trips did the company take to Paris?

(c) Steven's mum bakes him a birthday cake. After cutting pieces of cake to give to the family, $\frac{3}{4}$ of the cake was left over. Then another $\frac{5}{12}$ of the cake was given to Steven's friends. What fraction of the cake is now left over?



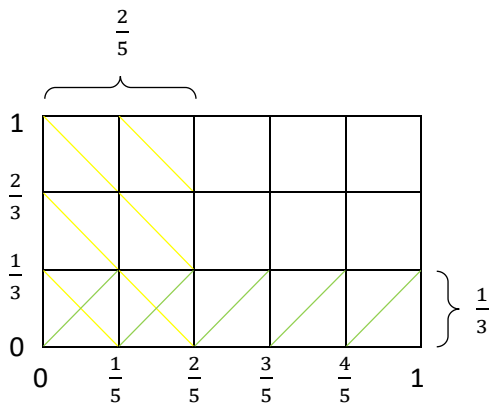
Skill

Applying

Multiplying Fractions

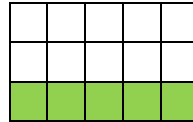
Next, let us consider how to calculate the sum $\frac{2}{5} \times \frac{1}{3}$.

There are **two** ways of thinking about this sum:

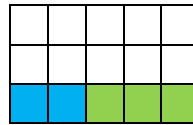


(1) Find $\frac{2}{5}$ of the fraction $\frac{1}{3}$.

Here, we must start with the fraction $\frac{1}{3}$, shown as the 5 green blocks below...



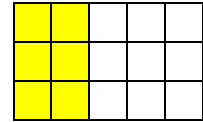
...and colour $\frac{2}{5}$ of the green blocks, shown as the two blue blocks below.



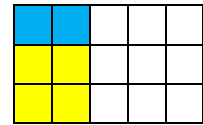
Therefore $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$.

(2) Find $\frac{1}{3}$ of the fraction $\frac{2}{5}$.

Here, we must start with the fraction $\frac{2}{5}$, shown as the 6 yellow blocks below...



...and colour $\frac{1}{3}$ of the yellow blocks, shown as the two blue blocks below.



Therefore $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$.

To calculate $\frac{2}{5} \times \frac{1}{3}$, **multiply the numerators**; **multiply the denominators**; and then **simplify the fraction**, if possible. Therefore $\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \frac{2}{15}$. It is not possible to simplify $\frac{2}{15}$, so this is the final answer.

Exercise 43

Calculate the following. Give your answers in their simplest form.

(a) $\frac{4}{5} \times \frac{1}{3}$

(b) $\frac{3}{5} \times \frac{2}{3}$

(c) $\frac{2}{5} \times \frac{11}{3}$

(e) $\frac{3}{4} \times \frac{1}{2}$

(f) $\frac{3}{4} \times \frac{5}{9}$

(g) $\frac{3}{4} \times \frac{2}{5}$

(d) $\frac{3}{5} \times \frac{11}{3}$

(h) $\frac{3}{4} \times \frac{5}{3}$

Exercise 44

(a) Two fifths of a garden are used to grow vegetables. Potatoes are planted in a quarter of this area. What fraction of the garden is used to grow potatoes?

(b) Lisa claims that three fifths of one quarter is the same amount as three quarters of one fifth. Is Lisa correct?

(c) Deiniol's noticeboard is in the shape of a rectangle, and measures $\frac{9}{10}$ of a metre by $\frac{3}{5}$ of a metre. Calculate the area of Deiniol's noticeboard, giving your answer in m^2 .

(d) Three fifths of a class are boys. Three fifths of the girls in the class play netball. Four girls choose not to play netball. How many boys are in the class?

(e) A snail travels at an average speed of $\frac{3}{4}$ meters per hour for 40 minutes. How far will the snail have travelled during this time?



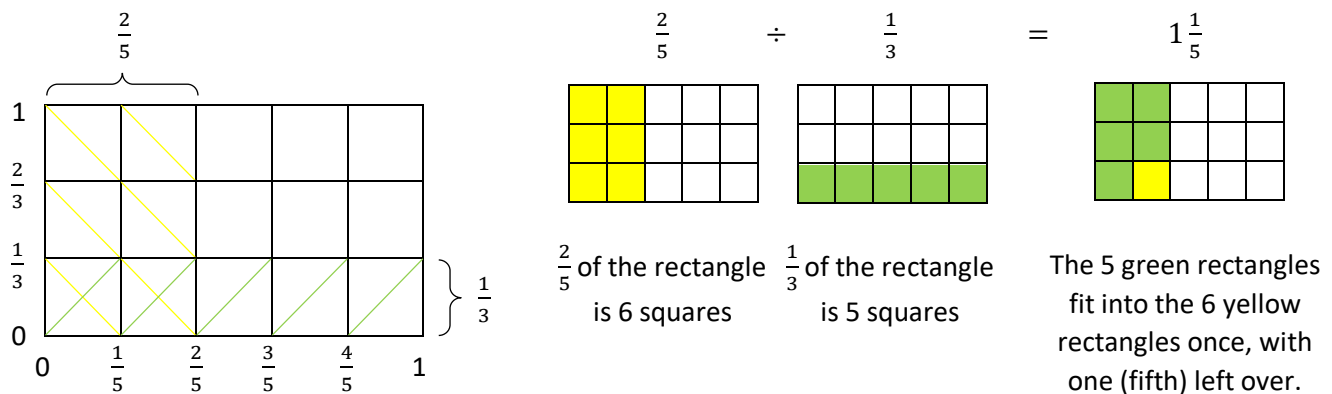
Skill

8

Applying

Dividing Fractions

To finish, let us consider how to calculate the sum $\frac{2}{5} \div \frac{1}{3}$. Here, we must consider how many times the fraction $\frac{1}{3}$ fits into the fraction $\frac{2}{5}$.



Here is the method to calculate $\frac{2}{5} \div \frac{1}{3}$.

Step 1: Turn the second fraction upside down (which is to take the **reciprocal**¹ of the second fraction), and change the sum to be a multiplication sum. $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1}$.

Step 2: Calculate the **multiplication** sum. $\frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$.

Step 3: Check to see if it is possible to **simplify** the answer. Here, we can convert the improper fraction $\frac{6}{5}$ into a mixed number: $\frac{6}{5} = 1 \frac{1}{5}$. This is the final answer.

Exercise 45

Calculate the following. Give your answers in their simplest form.

(a) $\frac{3}{5} \div \frac{2}{3}$

(b) $\frac{2}{5} \div \frac{2}{3}$

(c) $\frac{3}{5} \div \frac{1}{3}$

(d) $\frac{4}{5} \div \frac{1}{3}$

(e) $\frac{3}{4} \div \frac{1}{2}$

(f) $\frac{3}{4} \div \frac{8}{9}$

(g) $\frac{3}{4} \div \frac{2}{5}$

(h) $\frac{3}{4} \div \frac{7}{16}$

Exercise 46

(a) Elsi has $\frac{7}{8}$ pounds of flour left over. A recipe for small cakes asks for $\frac{1}{16}$ pounds of flour for each small cake. How many small cakes can Elsi make using her flour?

(b) There is $\frac{9}{10}$ of a black ink cartridge left in a printer. Every day, Dylan uses $\frac{3}{40}$ of the ink cartridge for work. In how many days will Dylan need to replace the ink cartridge?

(c) A crane can move heavy loads at a speed of $\frac{3}{50}$ meters per second. How much time will the crane take to move a heavy load $\frac{3}{4}$ of a meter?

Challenge!

$\frac{1}{5} = \frac{2697}{12485}$

Each of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 can be used once to form the fraction $\frac{1}{5}$.

Can you form the fractions $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ in the same way?



8

Applying



¹ The meaning of the word *reciprocal* will be explained in the next chapter.

Calculating with Mixed Numbers

Set 1 only

- To add and subtract mixed numbers, work with the whole numbers and the fractions **separately**.
- To multiply and divide mixed numbers, convert the numbers into **improper fractions** before proceeding.

Example

$$\begin{aligned} 4\frac{2}{3} + 1\frac{5}{6} &= 4 + 1 + \frac{2}{3} + \frac{5}{6} \\ &= 5 + \frac{4}{6} + \frac{5}{6} \\ &= 5 + \frac{9}{6} \\ &= 5 + 1\frac{1}{2} \\ &= 6\frac{1}{2} \end{aligned}$$

Split the sum

Add the whole numbers; write the fractions over a common denominator

Add the fractions

Simplify the fraction and write it as a mixed number

Write the final answer as a mixed number

$$\begin{aligned} 4\frac{2}{3} - 1\frac{5}{6} &= 4 - 1 + \frac{2}{3} - \frac{5}{6} \\ &= 3 + \frac{4}{6} - \frac{5}{6} \\ &= 3 - \frac{1}{6} \\ &= 2\frac{5}{6} \end{aligned}$$

Split the sum

Subtract the whole numbers; write the fractions over a common denominator

Subtract the fractions

Write the final answer as a mixed number

$$\begin{aligned} 4\frac{2}{3} \times 1\frac{5}{6} &= \frac{14}{3} \times \frac{11}{6} \\ &= \frac{154}{18} \\ &= 8\frac{5}{9} \end{aligned}$$

Write the mixed numbers as improper fractions

Multiply the improper fractions

Simplify the fraction and write it as a mixed number

$$\begin{aligned} 4\frac{2}{3} \div 1\frac{5}{6} &= \frac{14}{3} \div \frac{11}{6} \\ &= \frac{14}{3} \times \frac{6}{11} \\ &= \frac{84}{33} \\ &= 2\frac{6}{11} \end{aligned}$$

Write the mixed numbers as improper fractions

Turn the second fraction upside down, and change to a multiplication sum

Multiply the improper fractions

Simplify the fraction and write it as a mixed number

Exercise 47

Calculate the following. Give your answers in their simplest form.

(a) $2\frac{2}{3} + 1\frac{1}{4}$

(b) $2\frac{2}{3} - 1\frac{1}{4}$

(c) $2\frac{2}{3} \times 1\frac{1}{4}$

(d) $2\frac{2}{3} \div 1\frac{1}{4}$

(e) $3\frac{1}{2} + 1\frac{5}{8}$

(f) $3\frac{1}{2} - 1\frac{5}{8}$

(g) $3\frac{1}{2} \times 1\frac{5}{8}$

(h) $3\frac{1}{2} \div 1\frac{5}{8}$

(i) $2\frac{1}{4} + \frac{3}{8}$

(j) $2\frac{1}{4} - \frac{3}{8}$

(k) $2\frac{1}{4} \times \frac{3}{8}$

(l) $2\frac{1}{4} \div \frac{3}{8}$

(m) $5\frac{3}{7} + 3\frac{4}{9}$

(n) $5\frac{3}{7} - 3\frac{4}{9}$

(o) $5\frac{3}{7} \times 3\frac{4}{9}$

(p) $5\frac{3}{7} \div 3\frac{4}{9}$

Exercise 48

(a) Bethan works in a café $3\frac{1}{2}$ hours each Saturday and for $2\frac{3}{4}$ hours each Sunday.
How much time does Bethan spend working each weekend?

(b) Complete the magic square on the right.
(Every row, column and diagonal must sum to the same total.)

		$1\frac{2}{5}$
$2\frac{1}{10}$	$2\frac{9}{20}$	$\frac{7}{10}$

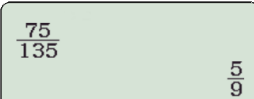
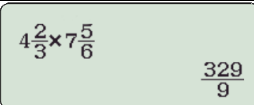
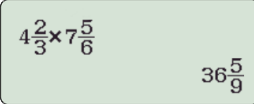
Skill

8

Applying



Example

Sum	Buttons to press	Calculator screen	Answer
Calculate $\frac{5}{8}$ of £120.	        		£75
Simplify the fraction $\frac{75}{135}$.	       		$\frac{5}{9}$
Calculate $\frac{7}{12} + \frac{3}{16}$.	            		$\frac{37}{48}$
Calculate $4\frac{2}{3} \times 7\frac{5}{6}$.	                  	 	$36\frac{5}{9}$

Exercise 49

Use a calculator to calculate the following.

- (a) Calculate $\frac{3}{4}$ of 36.
- (b) Calculate $\frac{7}{9}$ of \$63.
- (c) Simplify the fraction $\frac{60}{84}$.
- (d) Simplify the fraction $\frac{42}{70}$.
- (e) $\frac{4}{5} + \frac{3}{5}$
- (f) $\frac{7}{8} - \frac{5}{6}$
- (g) $\frac{4}{7} \times \frac{3}{4}$
- (h) $\frac{8}{9} \div \frac{2}{5}$
- (i) $3\frac{1}{6} + 1\frac{2}{3}$
- (j) $3\frac{1}{6} - 1\frac{2}{3}$
- (k) $3\frac{1}{6} \times 1\frac{2}{3}$
- (l) $3\frac{1}{6} \div 1\frac{2}{3}$



Exercise 50 (Revision)

Repeat Exercise 49, this time *without* using a calculator. (Questions (i) to (l) are for Set 1 only.)

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Reciprocals

Simply put,

The reciprocal of a number is $\frac{1}{\text{the number}}$.



Example

The reciprocal of 8 is $\frac{1}{8}$, the reciprocal of 10 is $\frac{1}{10}$, and the reciprocal of 42 is $\frac{1}{42}$.

Exercise 51

What is the reciprocal of the following numbers?

- (a) 4 (b) 9 (c) 17 (d) 26 (e) 2947 (f) 1 (g) x

5

The Meaning of a Reciprocal

Multiplying a number by its reciprocal gives an answer of 1.

Number $\times$ Reciprocal of the Number = 1

So, the reciprocal of 8 must be $\frac{1}{8}$, because only $\frac{1}{8}$ can be multiplied by 8 to give 1. ($8 \times \frac{1}{8} = \frac{8}{8} = 1$.)

Reciprocal of a Fraction

What is the reciprocal of the fraction $\frac{2}{3}$?

Here, we must think what can be multiplied by $\frac{2}{3}$ to give an answer of 1.

The answer is $\frac{3}{2}$, as $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$. Generally, the reciprocal of any fraction is the fraction flipped upside down, so the numerator moves to the denominator, and the denominator moves to the numerator.



Exercise 52

What is the reciprocal of each of the following fractions? (If appropriate, give your answer as a mixed number.)

- (a) $\frac{6}{5}$ (b) $\frac{2}{7}$ (c) $\frac{1}{2}$ (d) $\frac{5}{7}$ (e) $\frac{8}{13}$ (f) $\frac{1}{17}$ (g) $\frac{a}{b}$

Reciprocal of a Mixed Number

What is the reciprocal of the mixed number $2\frac{3}{4}$? Here, we must convert the mixed number into an improper fraction, so that we can turn it upside down. So, remembering that $2\frac{3}{4} = \frac{11}{4}$, the reciprocal of $2\frac{3}{4}$ must be $\frac{4}{11}$.

Exercise 53

What is the reciprocal of each of the following mixed numbers?

- (a) $2\frac{1}{4}$ (b) $1\frac{1}{2}$ (c) $3\frac{2}{7}$ (d) $5\frac{7}{8}$ (e) $10\frac{1}{11}$ (f) $4\frac{7}{15}$ (g) $a\frac{b}{c}$

Challenge!

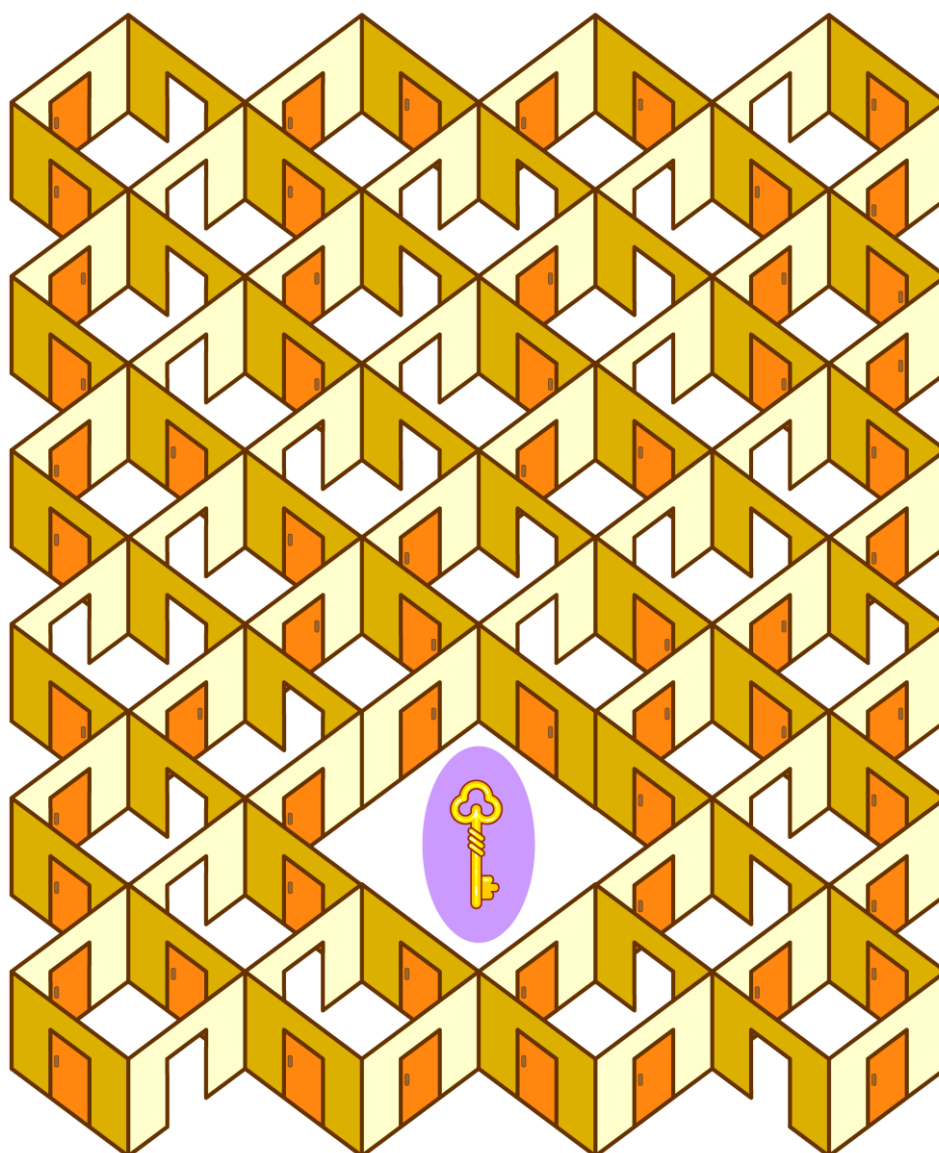
- (a) I add a number to its reciprocal. The answer is $4\frac{1}{4}$. What numbers did I add?
 (b) I add a number to its reciprocal. The answer is $2\frac{9}{10}$. What numbers did I add?
 (c) What is the reciprocal button on a calculator?
 (d) What is the reciprocal of 0? Use the internet to investigate.

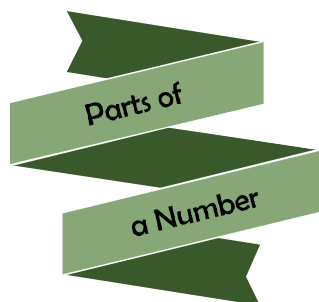
Taking the reciprocal of a reciprocal goes back to the original number.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Puzzle: The key opens *one* door only. Find the path out of the maze.





Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to use a part-whole bar model and a comparison bar model .			1, 16	
I know how to write a ratio .			12	
I know how to share an amount in a given ratio .			7	
I know how to use bar models to solve problems involving ratio .			10	
I know how to write equivalent ratios and how to simplify ratios .			8	
I know how to compare ratios in order to decide which option is best value for money .			11	
I know how to write ratios in the form 1 : n or n : 1 .			9	
I know how to use a map's ratio to calculate distances .			13	
I know how to recognise the fraction of a shape that is shaded .			2	
I know how to calculate the fraction of a number .			4	
I know how to subtract a fraction from a whole .				
I know how to compare the size of fractions using a number line or by using equivalent fractions .			6	
I know how to change between improper fractions and mixed numbers .			14	
I know how to add and subtract fractions that have equal denominators .			15	
I know how to simplify a fraction.			5	
I know how to add and subtract fractions that have different denominators .			15	
I know how to multiply fractions.			15	
I know how to divide fractions.			15	
I know how to work with fractions on a calculator .				
I know how to find the reciprocal of a whole number .			3	
I know how to find the reciprocal of a fraction or a mixed number .			3	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

9

Measuring

Shapes 2

Name:

Contents

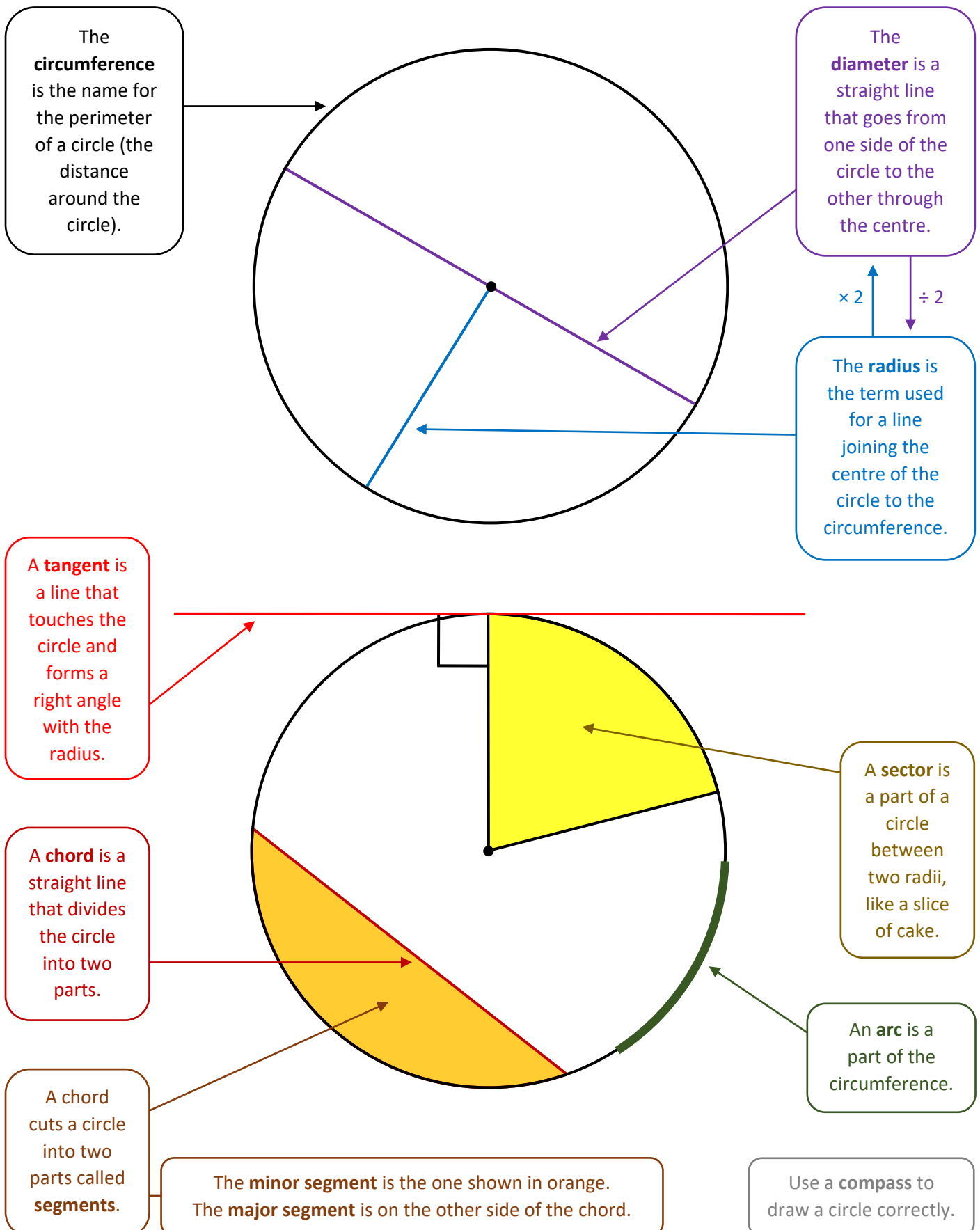
Chapter	Mathematics	Page Number
Circle Terminology	Parts of a circle.	3
Circumference and Area of a Circle	Finding π . Calculating the circumference of a circle. Calculating the area of a circle. Finding the radius from the area.	4
Shapes on Isometric Paper	Drawing shapes on isometric paper.	11
Plans and Elevations	Drawing solids from different plans and elevations.	12
Pythagoras' Theorem (2-D)	The hypotenuse of a right-angled triangle. Pythagoras' Theorem. Pythagorean Triples. Proving whether or not a triangle is right-angled.	16
Scale Drawings	Creating a scale drawing. Interpreting a scale drawing.	21



Circle Terminology



Here is a summary of the terms used to describe parts of a **circle**.



Exercise 1

Skill

4

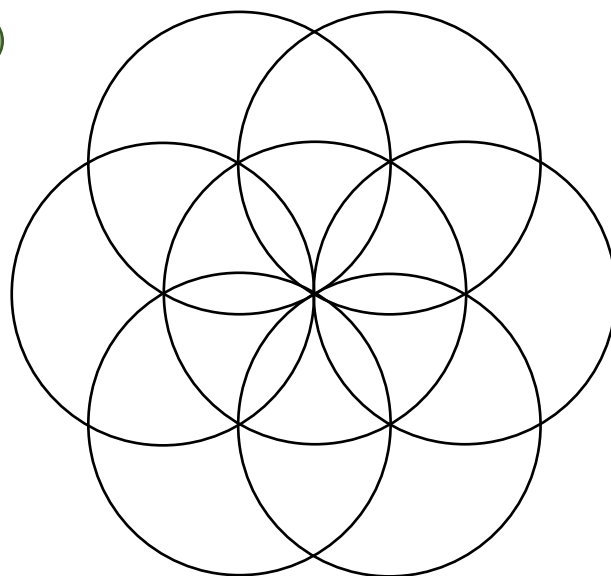
- Draw a circle with a radius of 4 cm.
- Draw a circle with a diameter of 10 cm.
- Draw a semi-circle with a diameter of 9 cm.
- Try to recreate the pattern of circles shown on the right.

Exercise 2

Draw a circle with a radius of 6 cm.

Add the following to the circle.

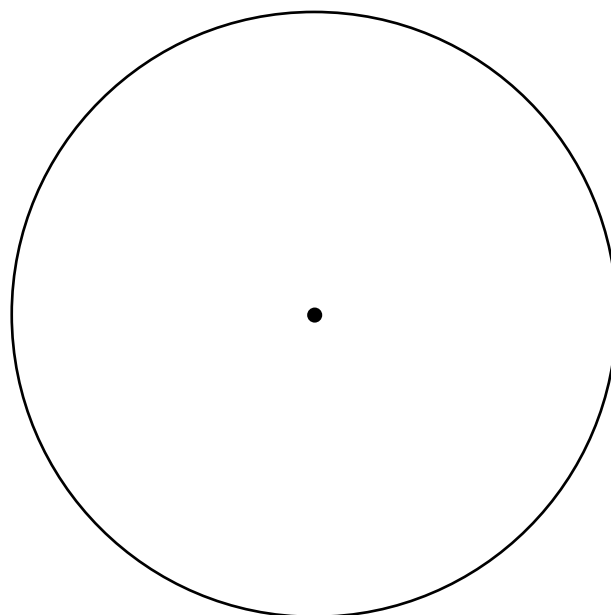
- (a) A chord 7 cm in length.
(b) A sector containing a 70° angle.
(c) An arc that goes a quarter of the way around the circle.
(d) A tangent that touches the circle at the bottom.



Exercise 3

Complete the following table.

Circle Radius	Circle Diameter
12 cm	
	16 cm
2.3 cm	
	6.8 cm
9 m	
	26 mm



Exercise 4 (Revision)

Add the following to the circle on the right:

- (a) Radius
- (b) Segment
- (c) Tangent

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

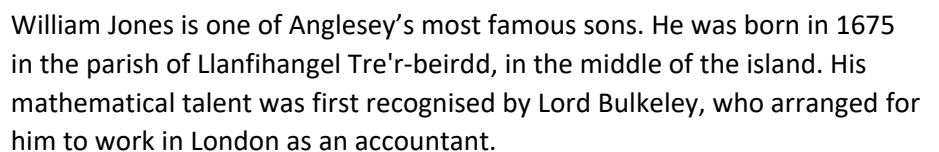
Applying

6

After you have completed the rows, calculate the **mean** of the final column, writing your answer in the yellow box.

Exercise 6

William Jones



Exercise 7

Write a short paragraph summarising your findings.

π

For any circle, of any size, imagine that you could measure the diameter and the circumference perfectly. Then, by writing the ratio Diameter : Circumference in the form $1 : n$, the number n will always be the same.



Diameter : Circumference

$$\begin{array}{c} \downarrow \div \text{Diameter} \\ 1 : \pi \end{array}$$

The special number on the right-hand side of the unitary ratio is the number π which, to 2 decimal places, is equal to 3.14.

Exercise 8

6

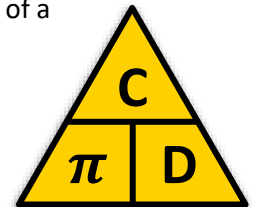
How close to π was your answer to exercise 5 (the yellow box)?

Calculating the Circumference of a Circle

We can use the above ratios to obtain the following formulae for calculating the circumference of a circle, using the diameter; or for calculating the diameter of a circle, using the circumference.

$$\text{Circumference} = \pi \times \text{Diameter}$$

$$\text{Diameter} = \text{Circumference} \div \pi$$

**Exercise 9**

Fill in the spaces in the following table. (Use the π button on your calculator to carry out calculations and, where necessary, round your answers to 2 decimal places.)



Radius	Diameter	Circumference
	6 cm	
5 cm		
		20 cm
	4.8 m	
1.7 mm		
		26 km
	4.3 miles	
9 cm		
		1 cm

 π and the Bible

In the Second Book of Chronicles in the Bible, we get the history of Solomon building the first temple in Jerusalem:

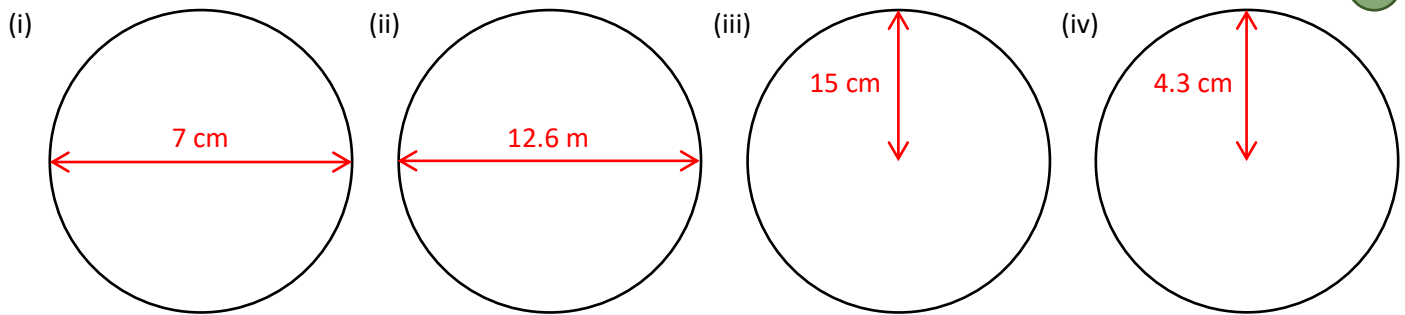
“Also he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about.”

Discuss what the Bible suggests about the value of π .



Exercise 10

(a) Find the circumference of the following circles.

**Applying**

6

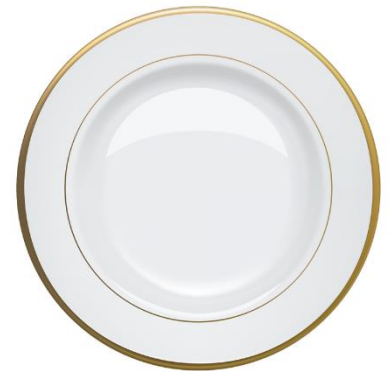
(b) Chris' bike has wheels of diameter 70 cm. What is the circumference of one wheel?

(c) The gold decoration around a plate is shown on the right.
If the circumference of the plate is 62.8 cm, what is the diameter of the plate?

(d) Vauxhall cars have wheels of diameter 48 cm.
Find the circumference of the wheels.

(e) A canvas cover has been made to fit exactly over a circular fish pond.
The pond has a circumference of 8.8 m.

(i) What is the diameter of the cover? (ii) What is the radius of the cover?

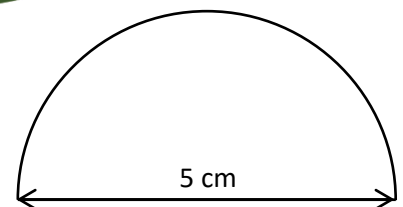
**Exercise 11**

Do not use a calculator for the following exercise. Instead, use 3.14 as the value of π .

- (a) Calculate the circumference of a circle with a diameter of 10 cm.
(b) Calculate the circumference of a circle with a radius of 20 m.
(c) Calculate the circumference of a circle with a radius of 2.5 km.

Skill**Exercise 12**

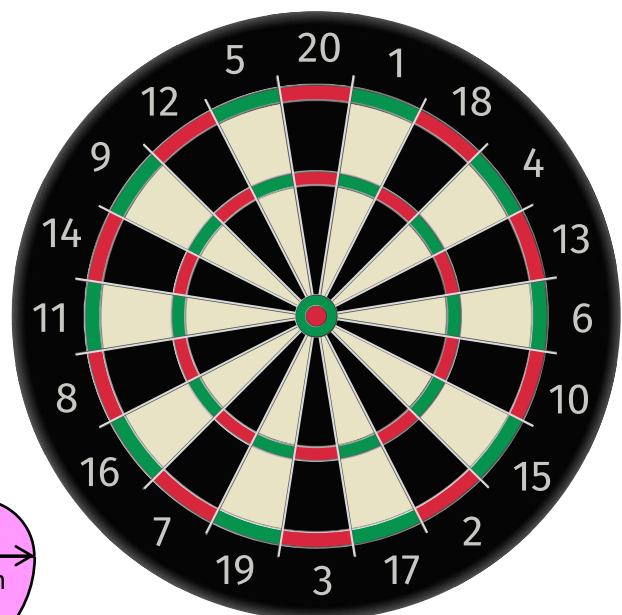
(a) A semicircle is shown on the right.
Calculate the perimeter of the semicircle.

Extension

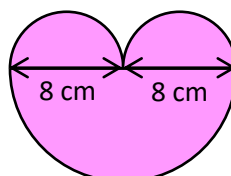
(b) The radius of a bike wheel, including its tyre, is 26 cm. How many whole revolutions will the wheel make whilst travelling 1 km?

(c) All the parts of the shown dartboard are separated by pieces of wire. 6 of the wires are circular. The diameters of these are 33.6 cm; 31.6 cm; 21 cm; 19 cm; 2.4 cm; and 1.3 cm.
The length of each straight piece of wire is 16.2 cm.

- (i) What is the total length of all the straight pieces of wire?
(ii) Find the length of each circular piece of wire.
(iii) What is the total length of all the circular pieces of wire?
(iv) What is the total length of all the pieces of wire on the dart board?



(d) What is the perimeter of the shown heart shape, which is made from three semicircles?



Calculating the Area of a Circle

The formula for calculating the area of a circle is

$$\text{Area of a Circle} = \pi \times \text{Radius}^2$$



Applying

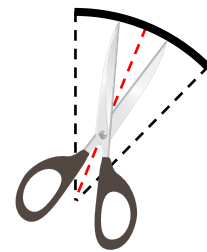
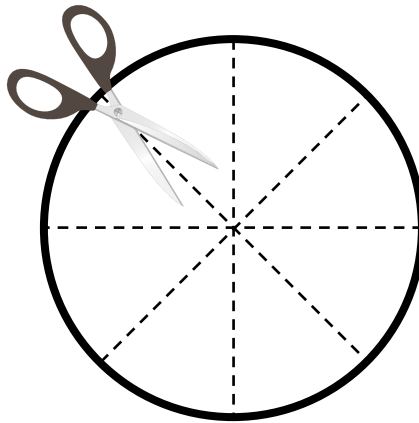
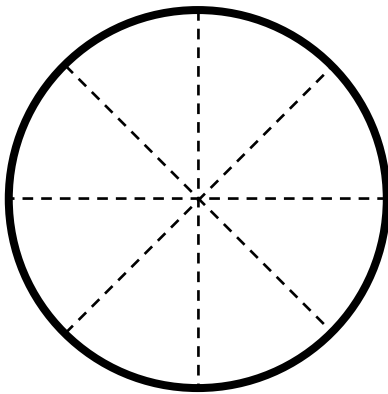
6

The following exercise explains *why* this formula works.

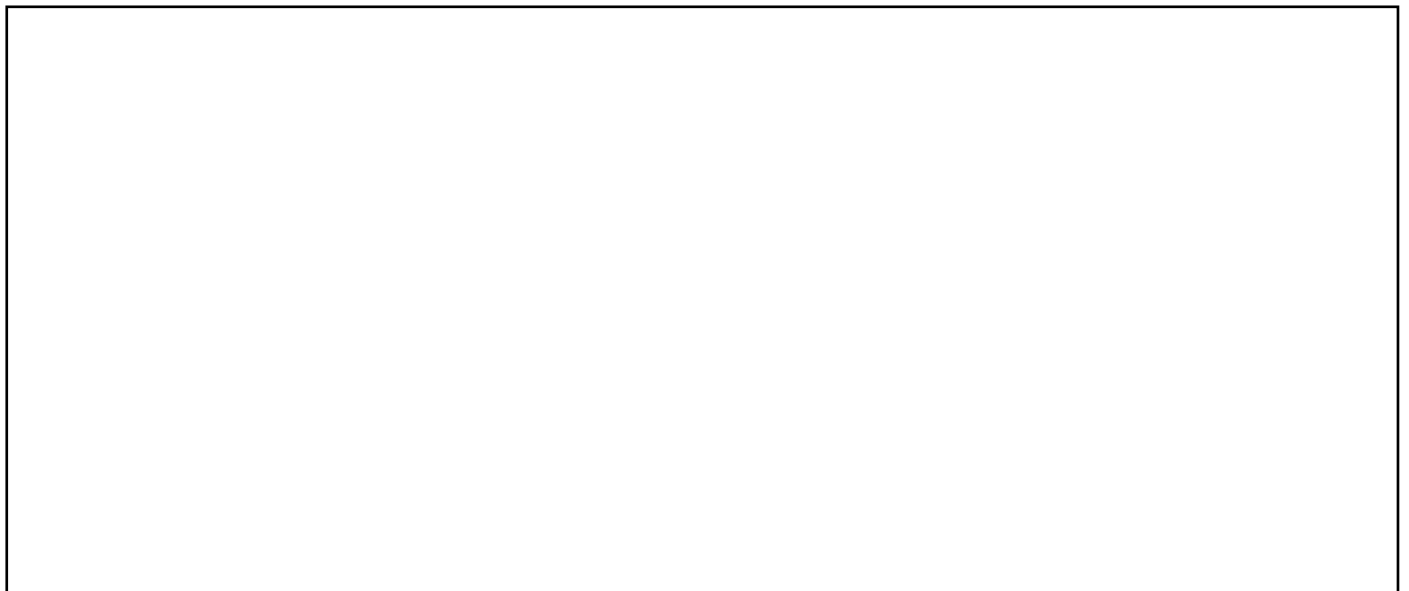
Exercise 13

To do this exercise, you will need a circle of radius 5 cm, and a thick line to emphasize the circumference. Cut out the circle before following the instructions below.

- (1) Fold the circle in half, in half again, and in half again.
- (2) Cut out all of the sectors.
- (3) Cut **one** of the segments in half.



- (4) Glue your pieces in a rectangular shape in the box below. Follow the teacher's instructions to understand where the formula for the area of a circle comes from.



π Day

We celebrate π day on March 14th every year. (Why?)

In the past, the mathematics department has held a cake baking competition, and a competition to memorise π to the greatest number of decimal places.

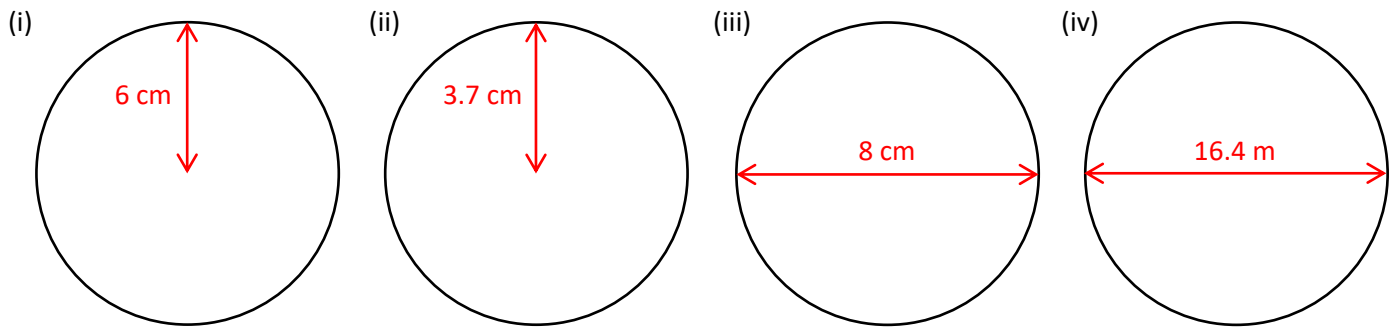
π Estimation Day

This day is celebrated on July 22nd. (Why?)



**Exercise 14**

(a) Find the area of the following circles.



(b) Big Ben's clock face has a radius of 3.5 m. What is the area of Big Ben's clock face?

(c) The diameter of a plate is 10 cm. What is the area of the plate?

(d) The diameter of an Audi car's wheels is 48 cm. Find the area of the wheels.

(e) The radius of a circular badge is 2 cm.

(i) What is the circumference of the badge?

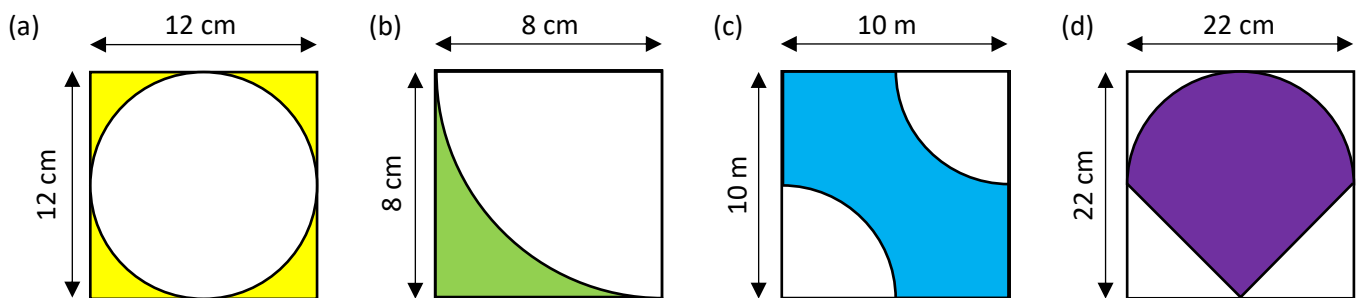
(ii) What is the area of the badge?

(iii) What do you notice about your answers to (i) and (ii)?

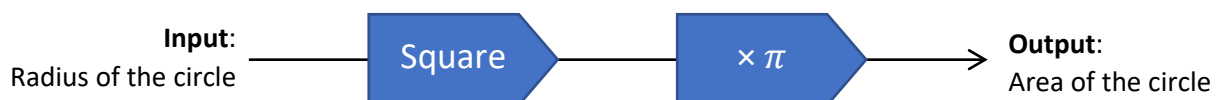
Does this happen again with any other circle?

**Challenge!**

Find the shaded area in each of the following diagrams.

**Finding the radius from the area**

When you know the area of a circle, how do you find its radius? The number machine below may help you answer this question; the number machine explains how to calculate the area of a circle when you know the radius.

**Exercise 15**

If the area of a circle is 50 cm^2 , what is

(a) the radius of the circle;

(b) the diameter of the circle;

(c) the circumference of the circle?

**Skill**

Exercise 16

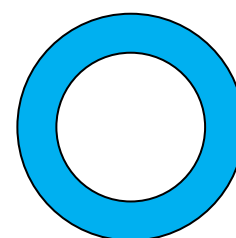
e

Fill in the spaces of the following table. (Use the π button on your calculator to carry out any required calculations, and where necessary round off your answers to 2 decimal places.)

Radius	Diameter	Circumference	Area
26 cm			
	8.8 m		
		200 cm	
			30 cm ²
6.1 m			
	2.7 cm		
		0.5 km	
			54 cm ²

Exercise 17 (Revision)

- (a) Calculate, correct to 3 decimal places, the circumference of a circle with diameter 2.4 cm.
- (b) The circumference of the *London Eye* is 424 m.
What is (i) the diameter of the wheel; (ii) the radius of the wheel?
- (c) Calculate, correct to 2 decimal places, the area of a circle whose radius is 5.3 cm.
- (d) If a circle has an area of 25 m^2 , what is the radius of the circle?
(Give your answer correct to 3 decimal places.)
- (e) The diagram on the right shows the cross-section of a water pipe. The internal diameter is 1.2 cm. The outer diameter is 1.6 cm.
Find the area of the cross-section (the blue region) (i) in cm^2 ; (ii) in mm^2 .



Evaluation

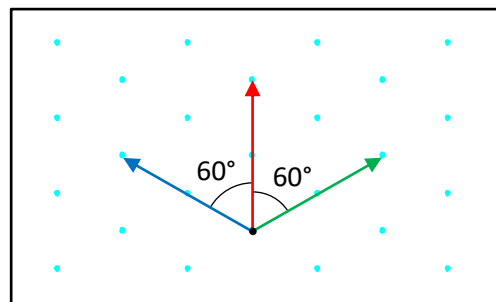
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target



Isometric paper is dotted paper where the dots are equally spaced from each other in three different directions:

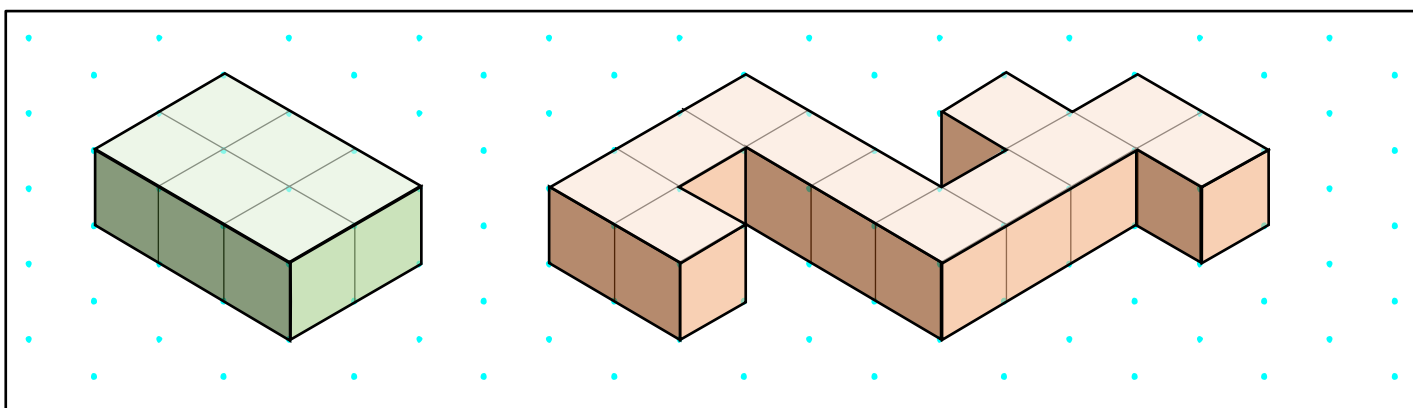
- (1) vertically;
- (2) on a 60° angle, measuring clockwise from the vertical;
- (3) on a 60° angle, measuring anticlockwise from the vertical.

When drawing a shape on isometric paper, only the above three directions may be used.



Example

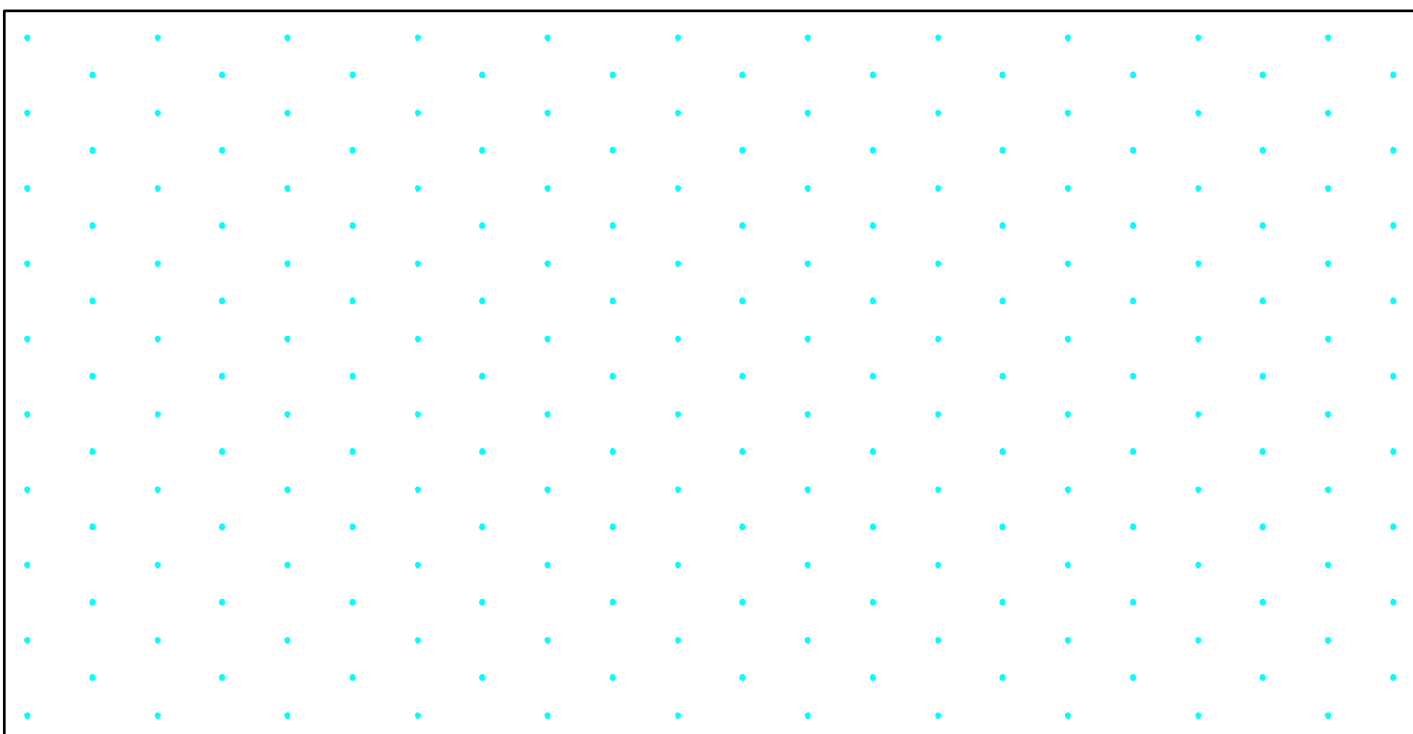
- (a) A cuboid measuring $1\text{ cm} \times 2\text{ cm} \times 3\text{ cm}$. (b) Twelve 1 cm^3 blocks joined together.



Exercise 18

Draw the following shapes on the isometric paper.

- (a) A $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ cube (b) A $2\text{ cm} \times 3\text{ cm} \times 4\text{ cm}$ cuboid (c) Five 1 cm^3 blocks joined together.



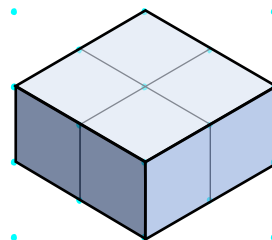
Skill

6

Exercise 19

6

On the right is an example of four 1 cm^3 blocks joined together.
 There are **8** different ways of joining 4 blocks together.
 Can you draw the 7 other ways on the isometric paper below?

**Exercise 20**

Experiment to create different shapes on isometric paper. For example, can you draw your initials on isometric paper?

Challenge!

Use 6 of the solids from Exercise 19, and one other solid, to complete the **SOMA** puzzle. (See the internet for details.)

Evaluation

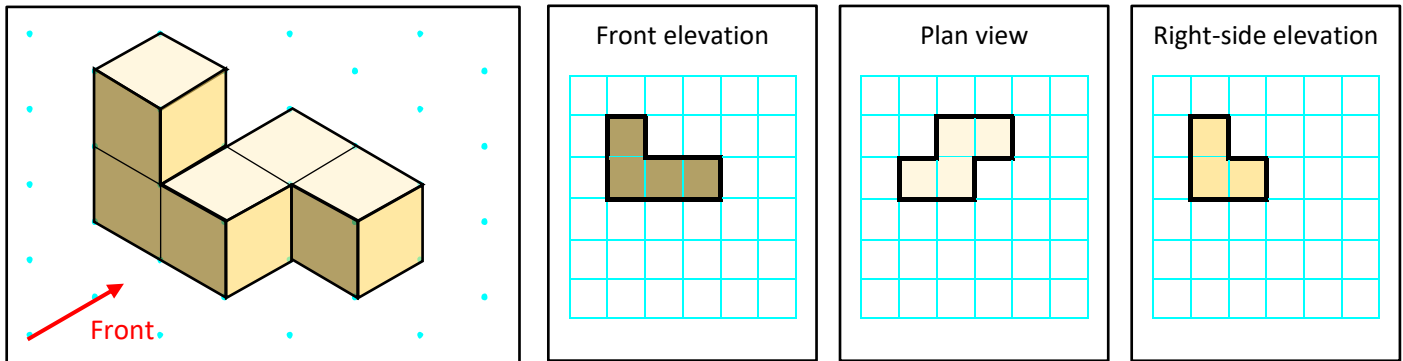
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Plans and Elevations



Given a solid drawn on isometric paper, it is possible to consider what a person would see looking at the solid from a single direction.

Example



Notes

- To be able to draw plans and elevations, we need to know which face is regarded as the **front** face.
- As well as the front elevation; plan view and right-side elevation, it is possible to draw the rear elevation; bottom view and left-side elevation. These are **reflections** of their corresponding plan or elevation.

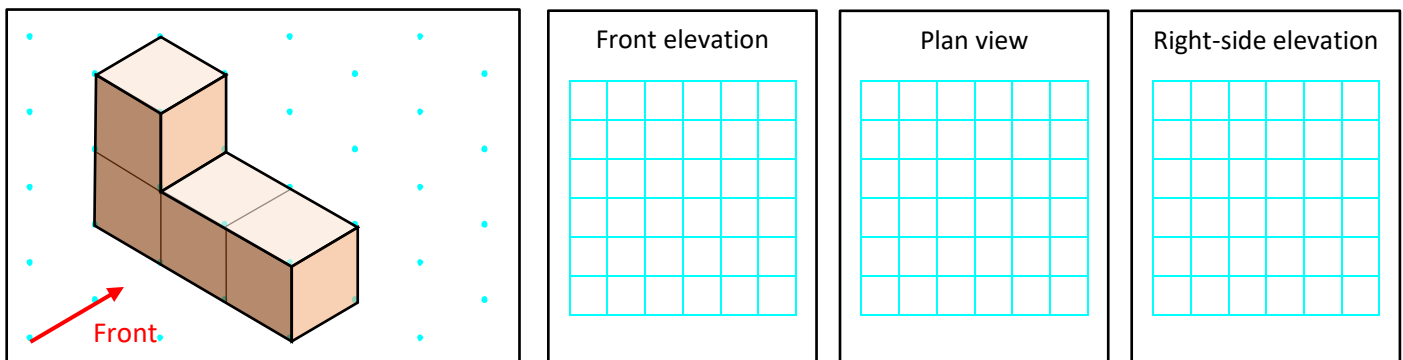
Exercise 21

Draw the front elevation; plan view and right-side elevation for each of the following solids.

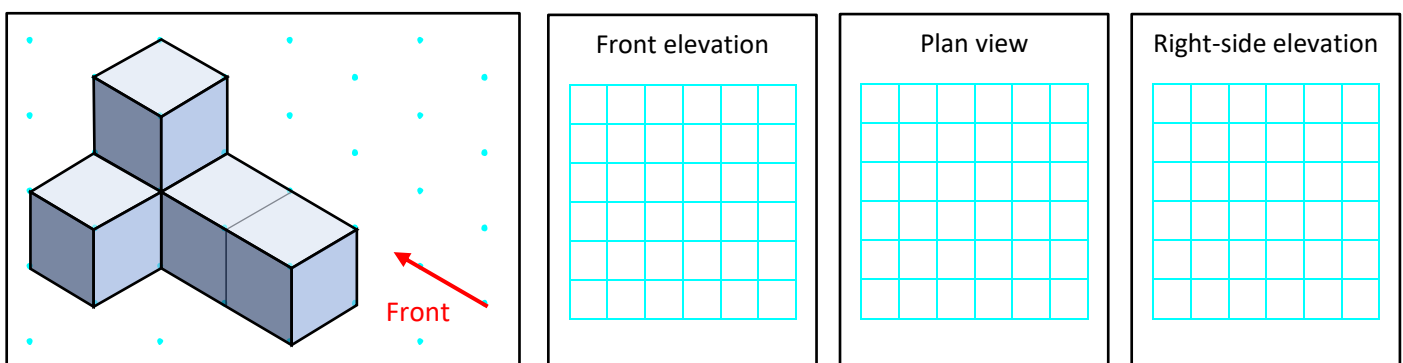
Skill

6

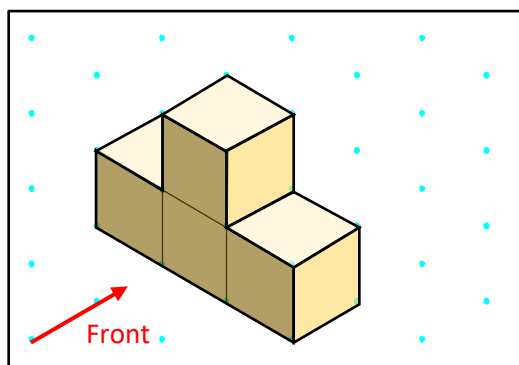
(a)



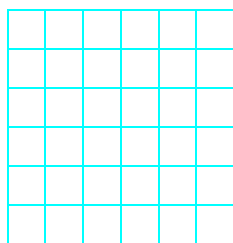
(b)



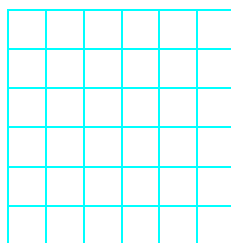
(c)



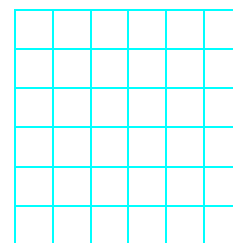
Front elevation



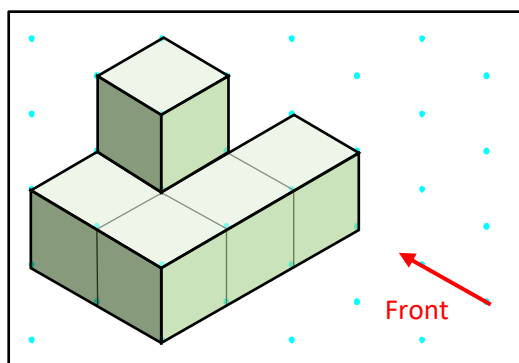
Plan view



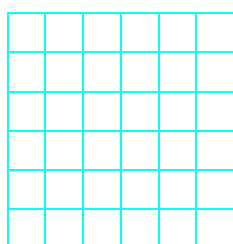
Right-side elevation



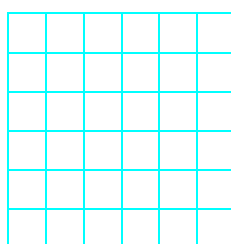
(d)



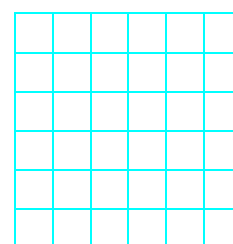
Front elevation



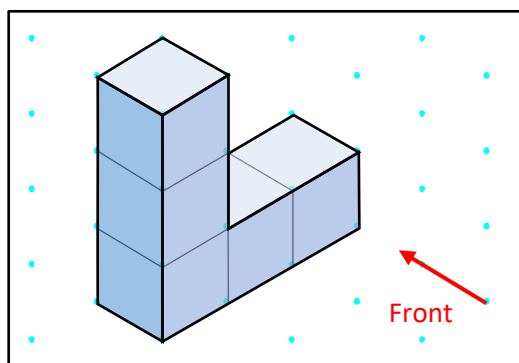
Plan view



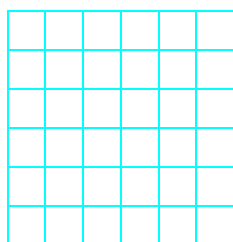
Right-side elevation



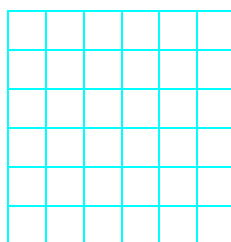
(e)



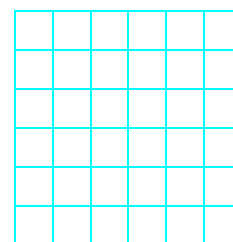
Front elevation



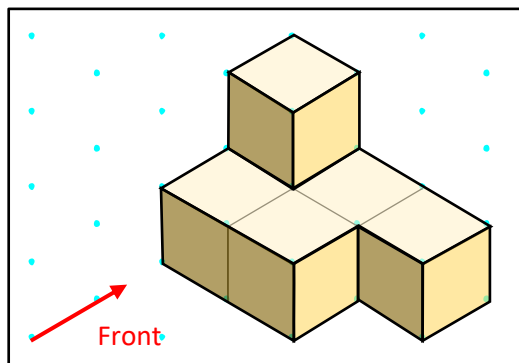
Plan view



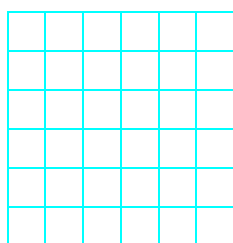
Right-side elevation



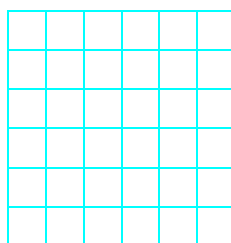
(f)



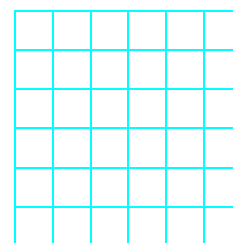
Front elevation



Plan view



Right-side elevation



Challenge!

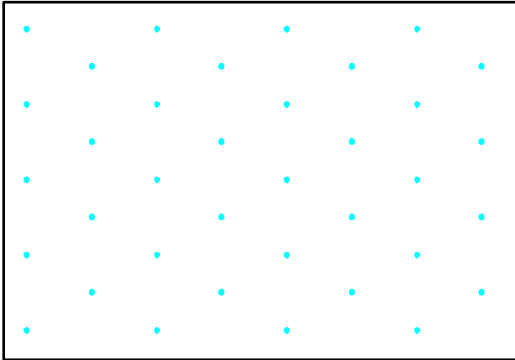
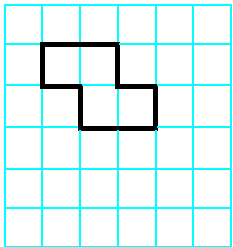
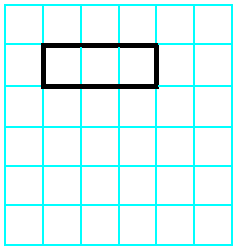
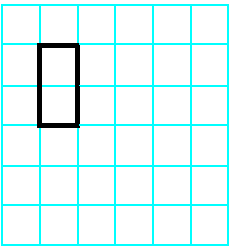
Llion uses more than one 1 cm^3 block to make a solid where the front elevation, the plan view and the right-side elevation are the same. What is the least number of 1 cm^3 blocks that forms Llion's solid?

Exercise 22

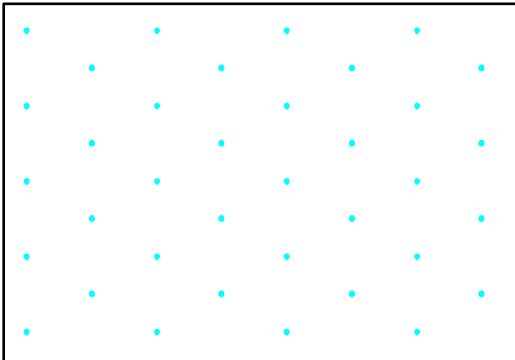
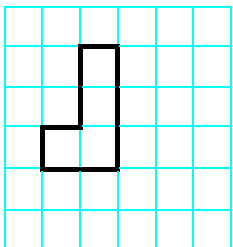
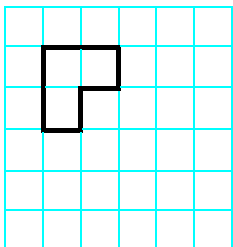
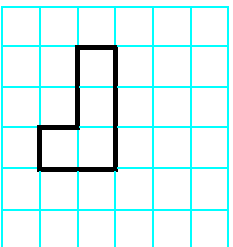
6

What solids come from the plans and elevations shown? Draw each solid on the isometric paper.

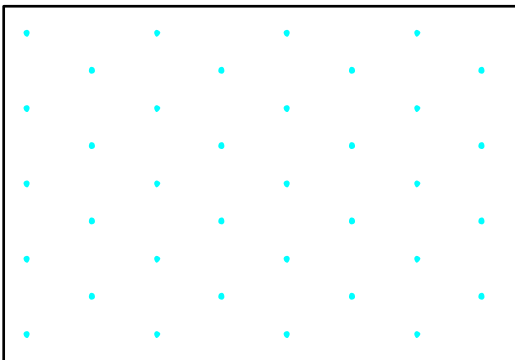
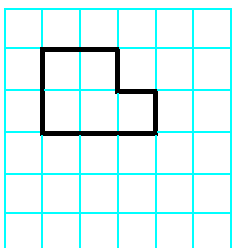
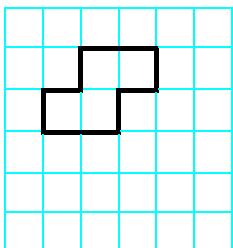
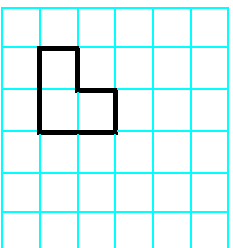
(a)

	Front elevation 	Plan view 	Right-side elevation 
--	--	---	---

(b)

	Front elevation 	Plan view 	Right-side elevation 
---	---	--	--

(c)

	Front elevation 	Plan view 	Right-side elevation 
--	--	---	---



Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

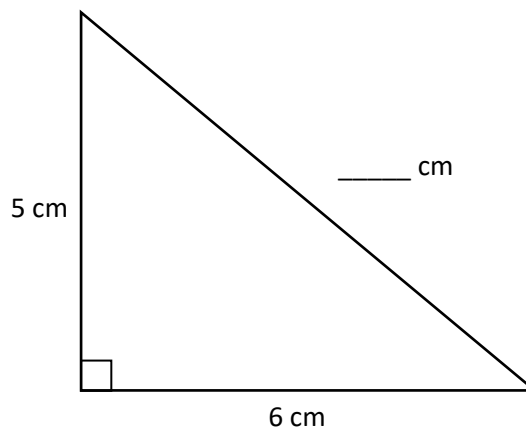
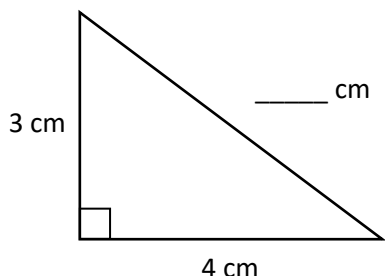
Pythagoras' Theorem (2-D)

Skill

4

Exercise 23

Each of the two triangles below have been drawn to scale. Use a ruler to measure the third side.

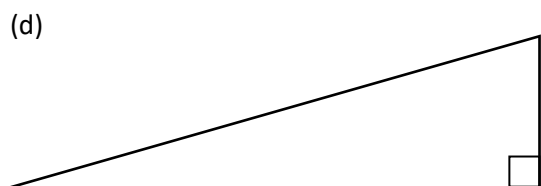
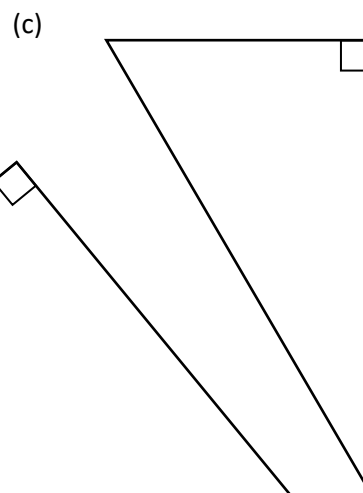
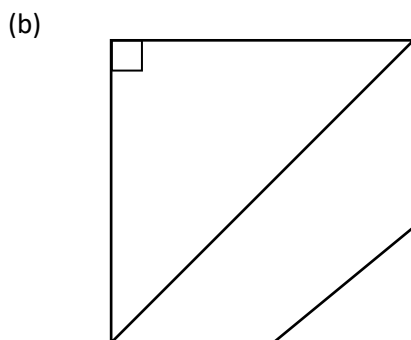
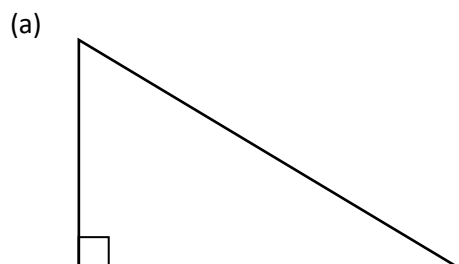


Hypotenuse of a Right-Angled Triangle

The **longest side** of a right-angled triangle is called the **hypotenuse**. The hypotenuse is always opposite the right angle.

Exercise 24

For each of the following right-angled triangles, label the hypotenuse with the letter "H".



Exercise 25

Using a ruler, measure the lengths of the **two shortest sides** of the above triangles. Fill the following table with your measurements (do not fill the last column – you will need this in a future exercise).

Triangle	Side 1 (cm)	Side 2 (cm)	
(a)			
(b)			
(c)			
(d)			
(e)			

Pythagoras' Theorem

Pythagoras' Theorem is a way of calculating the third side of a right-angled triangle, when the lengths of the other two sides are known.

In *symbols*, Pythagoras' Theorem is written like this:

$$c^2 = a^2 + b^2$$

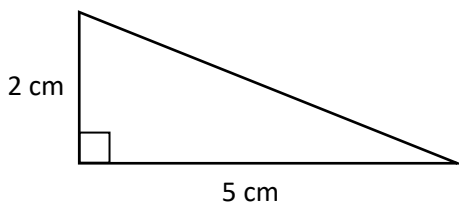
Here c represents the length of the **hypotenuse**, whilst a and b represent the **other lengths**.

In *words*, Pythagoras' Theorem can be written like this:

"The **square of the hypotenuse** is equal to the sum of the squares of the other two sides".

Example

Let us calculate the length of the third side of the following triangle.



By comparing this to the yellow triangle above, we see that $a = 2$ cm, $b = 5$ cm. Following Pythagoras' Theorem, in order to find c^2 we need to find the **sum of the squares of these two sides**.

a^2	$2^2 = 4$
b^2	$5^2 = + 25$
c^2	$\underline{29}$

To find the length of the hypotenuse, c , we need to find the **square root** of 29.

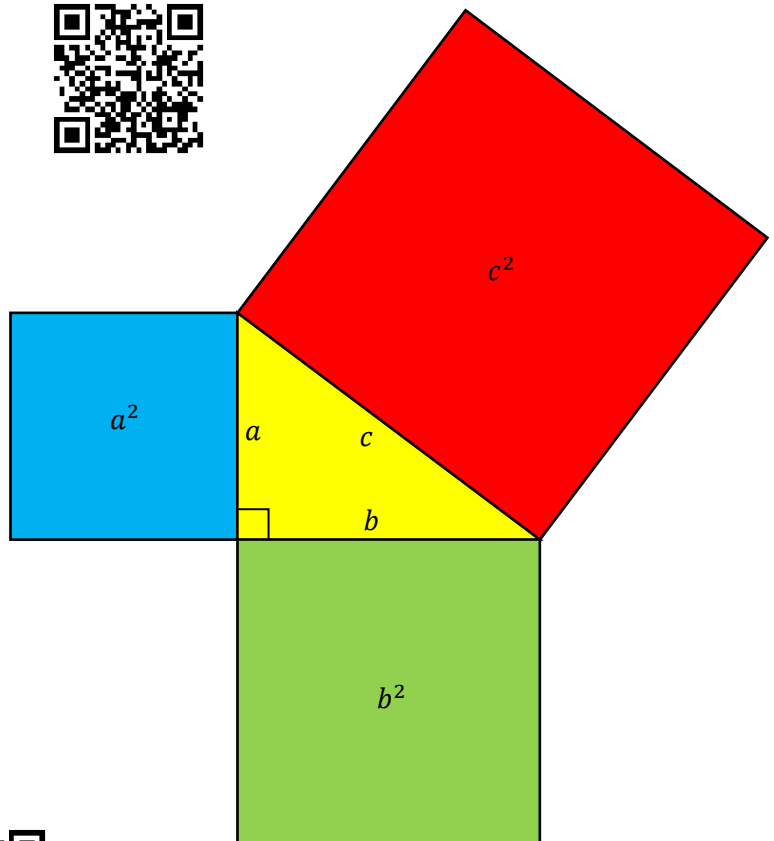
$$\sqrt{29} = 5.39 \text{ cm, to 2 decimal places.}$$

You can check this by measuring the length of the hypotenuse with a ruler.

Exercise 26

Use Pythagoras' Theorem to calculate the length of the hypotenuse for the two triangles in Exercise 23.

When you have calculated your answer, compare the *actual length* with your *measurement* from Exercise 23.



Pythagoras

Pythagoras was born around 570 B.C. on the Island of Samos, in Greece.

He made important developments to mathematics, astronomy and the theory of music.

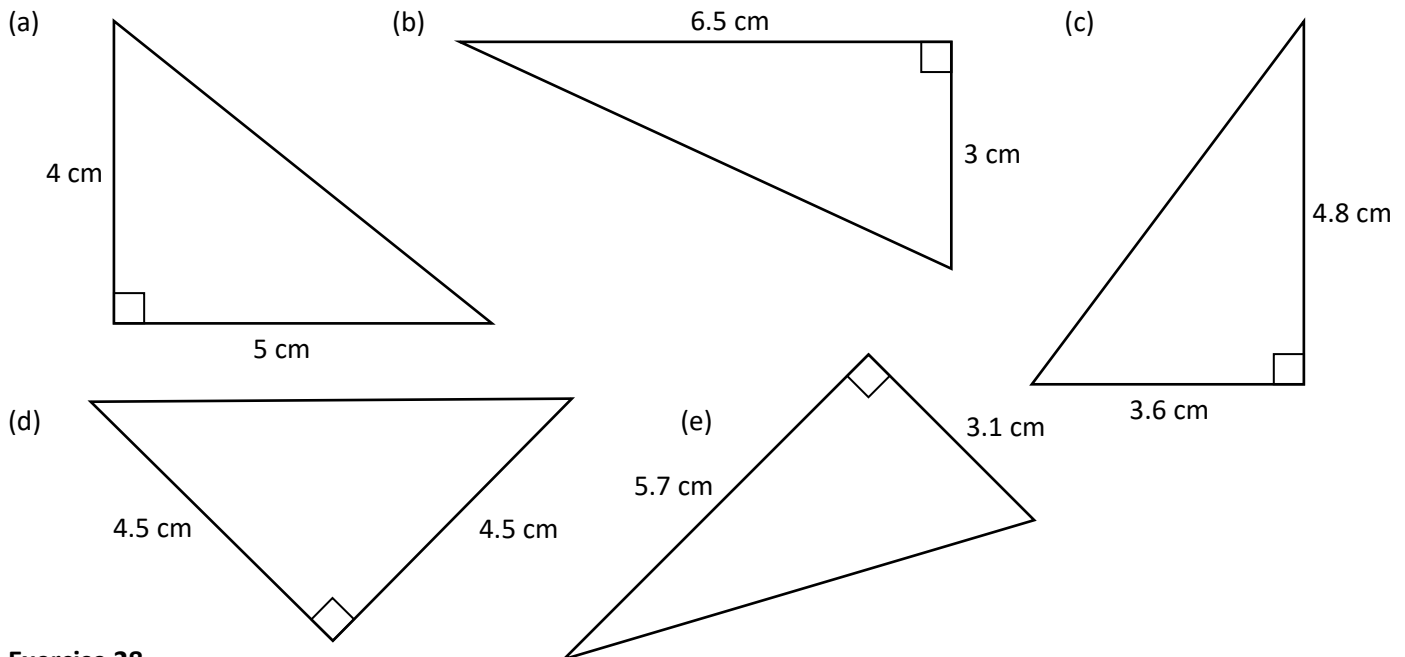
Although the people of Babylon (Iraq today) knew about Pythagoras' Theorem about 1,000 years before Pythagoras' time, Pythagoras was probably the first person to prove the theorem, and hence this is why the theorem is named after him.



Exercise 27

7

Calculate the length of the third side in each of the following triangles. Round your answers to two decimal places.

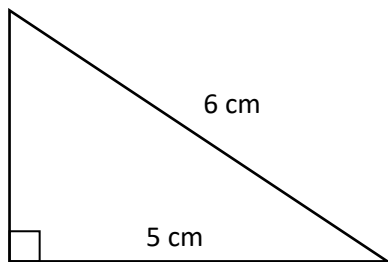


Exercise 28

Go back to Exercise 25 and write “Length of Hypotenuse (cm)” as the title of the last column. Use Pythagoras’ Theorem to complete the last column by calculating the length of the hypotenuse for each triangle, giving your answers correct to one decimal place.

Example

(a) Let us calculate the length of the third side in the following triangle.



By comparing this to the yellow triangle on the previous page, we see that $b = 5$ cm, $c = 6$ cm. We can rearrange Pythagoras’ Theorem as follows.

$$a^2 = c^2 - b^2.$$

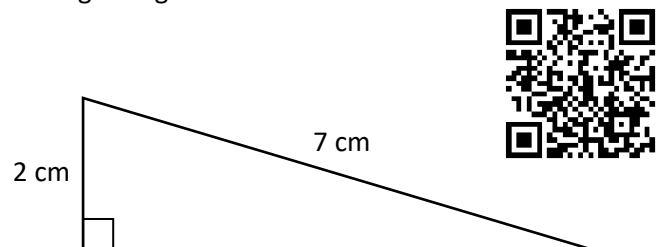
Therefore, to find the value of a^2 , the **difference** between the squares c^2 and b^2 must be calculated.

c^2	$6^2 =$	36
b^2	$5^2 = -$	25
a^2		<u>11</u>

To find the length of the side a , the square root of the difference, 11, must be calculated.

$$\sqrt{11} = 3.32 \text{ cm, to 2 decimal places.}$$

(b) Let us calculate the length of the third side in the following triangle.



By comparing this to the yellow triangle on the previous page, we see that $a = 2$ cm, $c = 7$ cm. We can rearrange Pythagoras’ Theorem as follows.

$$b^2 = c^2 - a^2.$$

Therefore, to find the value of b^2 , the **difference** between the squares c^2 and a^2 must be calculated.

c^2	$7^2 =$	49
a^2	$2^2 = -$	4
b^2		<u>45</u>

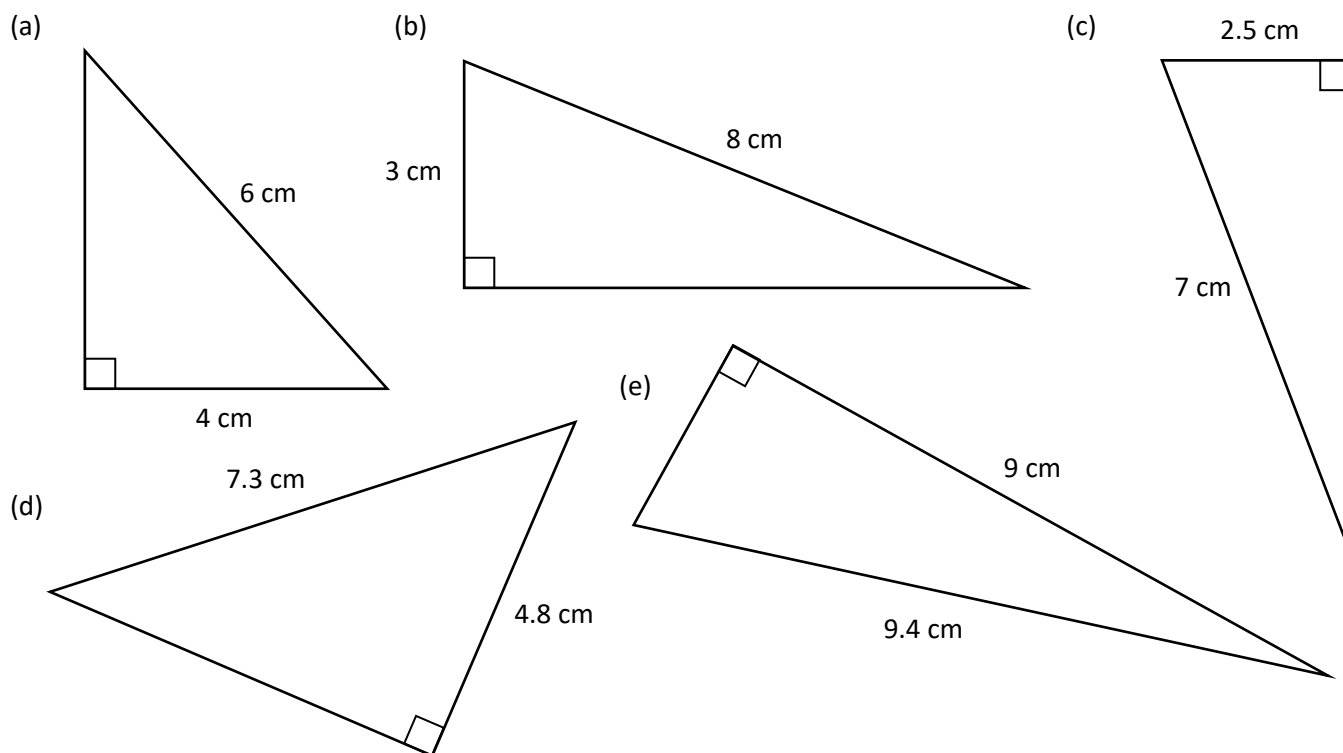
To find the length of the side b , the square root of the difference, 45, must be calculated.

$$\sqrt{45} = 6.71 \text{ cm, to 2 decimal places.}$$

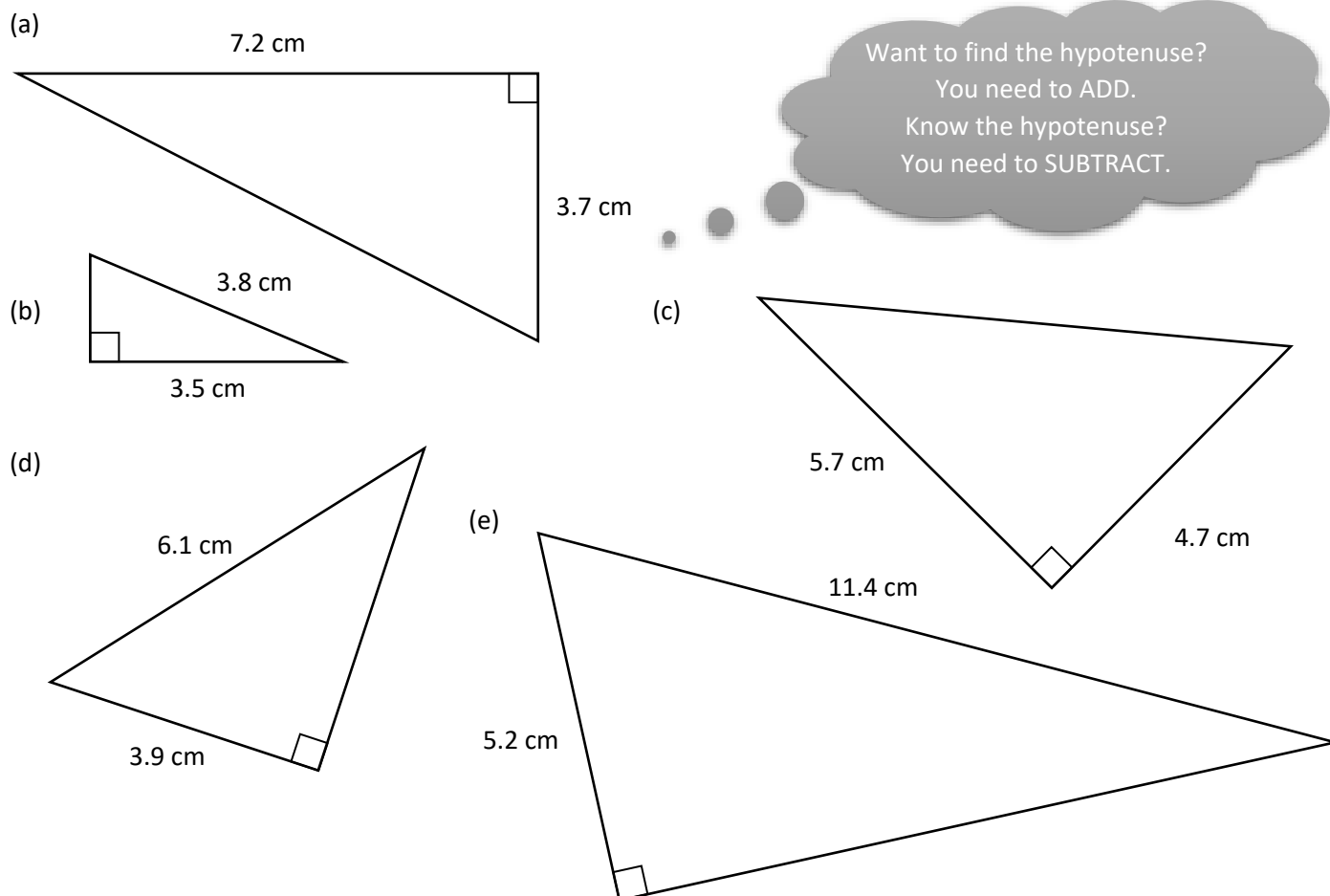
Exercise 29

7

Calculate the length of the third side in each of the following triangles. Round your answers to two decimal places.

**Exercise 30**

Calculate the length of the third side in each of the following triangles. Round your answers to two decimal places.



Applying

7

Exercise 31

(a) The picture on the right shows the measurements of one of the sails of a boat. Fabric has been sewn onto the edge of the sail in order to strengthen it. What is the total length of the fabric?

(b) What is the length of the longest straight line that can be drawn on a piece of A4 paper that measures 21 cm by 29.7 cm?

(c) A helicopter flies 24 km West, then 15 km North. How far is the helicopter from its starting point now?

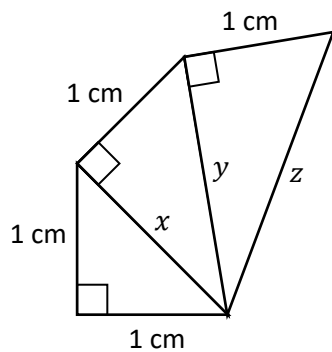
(d) The length of the diagonal bar shown on the gate on the right is 1.2 m. The length of the two horizontal pieces of wood is 0.9 m. What is the length of each of the 8 vertical pieces of wood that form part of the gate?

(e) The size of a television screen is the length of its diagonal. Ceri has a 55-inch television. If the height of the television screen is 27 inches, what is its width?



Challenge 1! ⚠

What are the lengths x , y and z in the diagram below? How would the pattern continue?

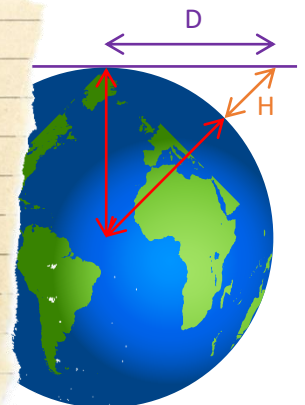


Challenge 2! ⚠

Why does the following method for finding the distance to the horizon work?

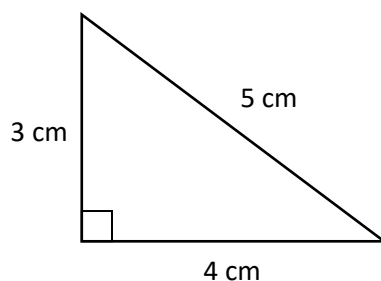
To calculate the distance D (in km) to the horizon,

- Find (or measure) the height H between your eye and sea level, in metres.
- Multiply by 13.
- Calculate the square root.



Pythagorean Triples

A **Pythagorean Triple** is made up of three whole numbers that can be used to draw a right-angled triangle using the numbers as the lengths of each side. For example, the triple (3, 4, 5) is a Pythagorean triple because it is possible to draw a right-angled triangle with the measurements 3cm; 4cm; 5cm.



Exercise 32

Perform an investigation to see what other Pythagorean triples exist.

(Maybe the table on the right will help?)

a	a^2	b	b^2	c	c^2

Proving whether or not a triangle is right-angled

When you know the lengths of all the edges of a specific triangle, Pythagoras' Theorem can be used to see whether or not the triangle is right-angled.



Example

Only one of the triangles on the right is a right-angled triangle. Which one?

For any given right-angled triangle, Pythagoras' Theorem tells us that **the square of the hypotenuse is equal to the sum of the squares of the two other sides.**

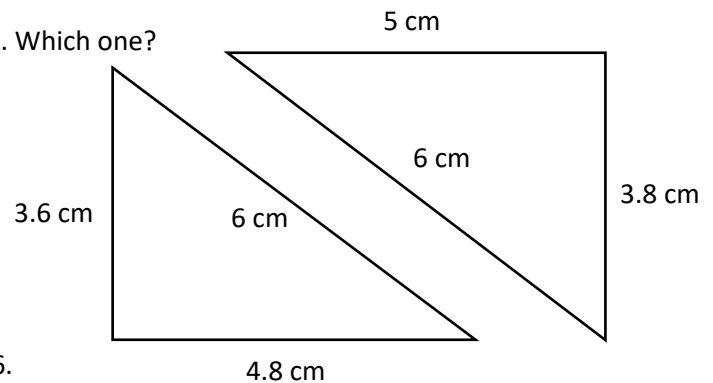
Let us check if this is true for the two triangles on the right.

Triangle 1

The square of the hypotenuse (the longest side): $6^2 = 36$.

The sum of the squares of the other two sides: $3.6^2 + 4.8^2 = 36$.

The square of the hypotenuse **is equal** to the sum of the squares of the other two sides, therefore the first triangle **is** a right-angled triangle.



The diagrams have not been drawn to scale.

Triangle 2

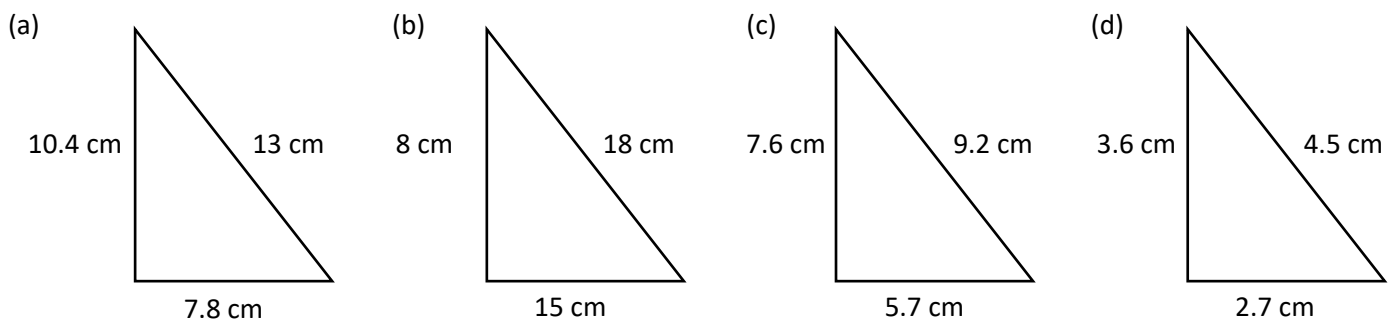
The square of the hypotenuse: $6^2 = 36$.

The sum of the squares of the other two sides: $3.8^2 + 5^2 = 39.44$.

36 is **not equal** to 39.44, therefore the second triangle is **not** a right-angled triangle.

Exercise 33

Which of the triangles below are right-angled? (The diagrams have not been drawn to scale.)



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Scale Drawings



Creating a Scale Drawing

Example

Using the scale 1 cm to represent 5 m, complete a scale drawing of a circle with radius 15 m.

Answer: In this question, 1 cm represents 5 m, so that 3 cm represents 15 m. (*Method:* $15 \div 5 = 3$, or $3 \times 5 = 15$). This means that we need to draw a circle with a radius of 3 cm. This is shown on the right.

Exercise 34

- Using the scale 1 cm to represent 5 m, complete a scale drawing of a circle with radius 20 m.
- Using the scale 1 cm to represent 5 m, complete a scale drawing of a circle with diameter 25 m.
- Using the scale 1 cm to represent 2 m, complete a scale drawing of a rectangle with length 10 m and width 8 m.
- Using the scale 1 cm to represent 10 km, complete a scale drawing of a right-angled triangle with base 70 km and height 40 km.
- Using the scale 1 cm to represent 20 m, complete a scale drawing of a square with edges of length 70 m.
- Using the scale 1 cm to represent 3 m, complete a scale drawing of an equilateral triangle with sides of length 15 m.

Skill

5

Interpreting a Scale Drawing

Example

The diagram on the right is a scale drawing of the cross-section of a house. Use the diagram to estimate, in metres, the width of the whole house, including the garage.

Answer: Using a ruler, we can measure the height of a person in the diagram; this is around 1 cm. In actual fact, the height of a person is around 2 metres, therefore the scale is 1 cm to represent 2 m.

On the diagram, the width of the house (including the garage) is 9 cm. Using the scale 1 cm to represent 2 m, the actual width of the house (including the garage) is $2 \times 9 = 18$ m.

Applying

Exercise 35

Use the diagram to estimate, in **feet**, the width of the whole house, including the garage.



Exercise 36

Write down an estimate for

(a) the actual height of a door in metres;



(b) the actual height of a normal car in feet.



5

Exercise 37

The diagram below shows a scale drawing of a busy street.



Use the diagram to estimate, in metres,

(a) the height of the offices;

(b) the height of the red car;

(c) the height of the clothes shop;

(d) the horizontal distance between the two lamp posts.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Puzzle: Using the stairs and ladders, find a way to go from point 1 to point 2.
Do not jump down or climb up!



Measuring

Shapes 2

Evaluation

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to recognise and draw different parts of a circle.			1	
I know how to calculate a circle's radius given its diameter , or calculate a circle's diameter given its radius .			4	
I know how to calculate a circle's circumference , given its diameter.			4, 8	
I know how to calculate the diameter of a circle, given its circumference.			6	
I know how to calculate a circle's area , given its radius.			4	
I know how to calculate a circle's radius , given its area.			7	
I know how to draw 3-D shapes on isometric paper .			5	
I know how to draw plans and elevations of a solid drawn on isometric paper.			9	
I know how to recognise a solid given different plans and elevations for the solid.				
I know how to recognise the hypotenuse of a right-angled triangle.			10, 11, 12	
I know how to use Pythagoras' Theorem to calculate the hypotenuse of a right-angled triangle, given the length of the other two sides.			10, 11, 12	
I know how to use Pythagoras' Theorem to calculate one of the shorter sides of a right-angled triangle, given the length of the hypotenuse and the other side.			10	
I know that a Pythagorean triple is.				
I know how to prove whether or not a given triangle is right-angled.			13	
I know how to create a scale drawing .			2	
I know how to interpret a scale drawing .			3	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

9

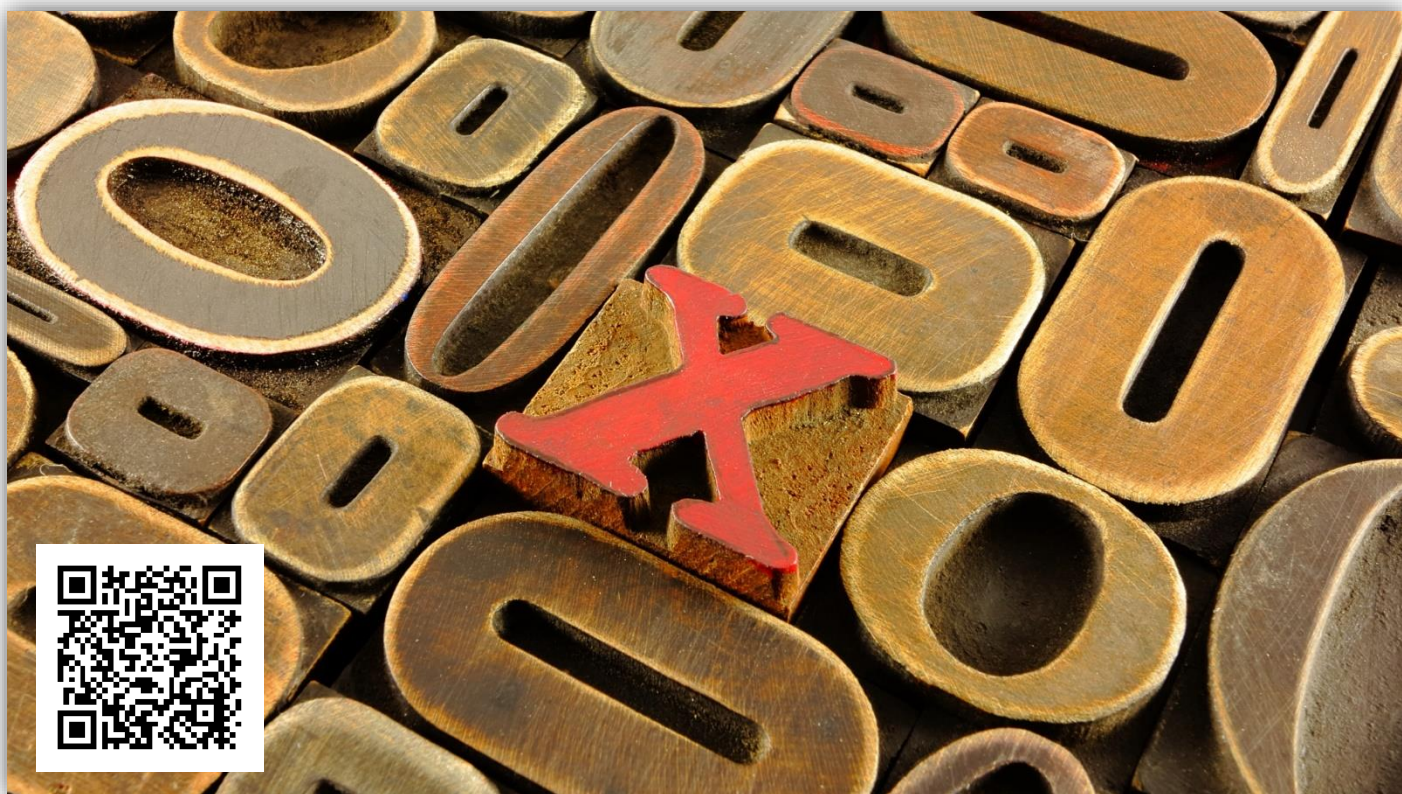
Developing

Algebra

Name:

Contents

Chapter	Mathematics	Page Number
Linear Nth Term	Terms of a sequence. Finding the n th term. Finding the position of a term.	3
Expanding Brackets	Expanding numerical sums. Multiplying algebraic terms. Expanding algebraic expressions. Expanding and simplifying.	7
FOIL	Expanding a double bracket. Expanding and simplifying a double bracket. FOIL with more than one variable. Brackets containing more than two terms.	12
Solving Equations 2	Revision of year 8 work on solving equations. Solving equations containing brackets. Solving equations where a double bracket is equal to zero. Equations containing fractions. Equations containing more than one fraction.	16
Trial and Improvement	Solving equations using trial and improvement (to the nearest unit; to one decimal place; to two decimal places). Using the <i>table mode</i> on a calculator.	22



Linear Nth Term

Revision

4

Sequences

Exercise 1

Find the next two numbers in the following sequences.

- | | | |
|---|--------------------------------------|---|
| (a) 9, 11, 13, 15, 17,, | (b) 85, 80, 75, 70, 65,, | (c) 23, 27, 31, 35, 39,, |
| (d) 48, 57, 66, 75, 84,, | (e) 17, 14, 11, 8, 5,, | (f) -15, -13, -11, -9, -7,, |
| (g) -34, -40, -46, -52, -58,, | (h) 8, 8.5, 9, 9.5, 10,, | (i) 10, 90, 170, 250, 330,, |

Terms of Sequences

Let us consider the sequence 9, 11, 13, 15, 17, ... from Exercise 1.

We say that 9 is the **first term** of the sequence; that 11 is the **second term** of the sequence; that 13 is the **third term**, and so on.

Exercise 2

For each sequence in Exercise 1, find the 10th term of the sequence.

Exercise 3

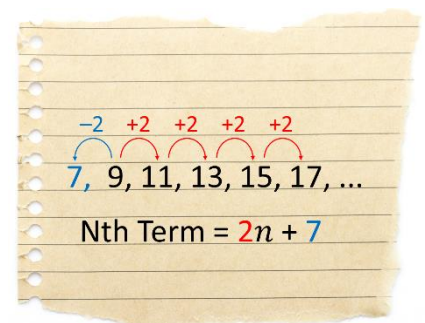
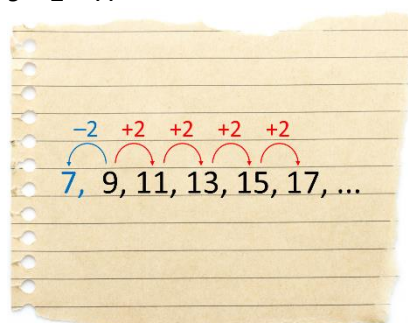
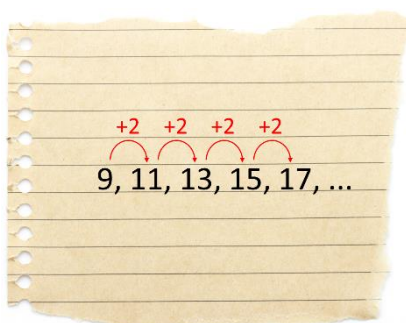
Discuss how to find the 100th term of the sequence 9, 11, 13, 15, 17,



Linear Nth Term

One way of finding the 100th term of the sequence 9, 11, 13, 15, 17, ... would be to find the formula for the **n th term** of the sequence. This is how to find this formula.

1. Consider what the rule is for finding the next number. Here, we must **add two** to find the next number.
2. If another number was added at the start of the sequence, what would this number have to be? Here, the number would have to be $9 - 2 = 7$.
3. The **n th term** for this sequence is $2n + 7$. (The 2 and the 7 come from the previous steps.)



The above method works for sequences where we add or subtract the same number to find the next term in the sequence. The name for this kind of sequence is a **linear** sequence. For example, the sequence 19, 23, 27, 31, 35, ... is a linear sequence (we add four every time), but the sequence 3, 6, 12, 24, 48, ... is not a linear sequence (we double every time).

Exercise 4**Skill****6**Find the n th term for the following linear sequences.

(a) 8, 10, 12, 14, 16,

(b) 5, 7, 9, 11, 13,

(c) 2, 4, 6, 8, 10,

(d) 10, 13, 16, 19, 22,

(e) 5, 8, 11, 14, 17,

(f) 3, 6, 9, 12, 15,

(g) 14, 20, 26, 32, 38,

(h) 3, 9, 15, 21, 27,

(i) 6, 12, 18, 24, 30,

(j) 6, 7, 8, 9, 10,

(k) 31, 32, 33, 34, 35,

(l) 1, 2, 3, 4, 5,

(m) 13, 17, 21, 25, 29,

(n) 100, 104, 108, 112, 116,

(o) 4, 8, 12, 16, 20,

Using the n th term to find the terms of a sequenceIn order to find the **100th term** of the sequence 9, 11, 13, 15, 17, ..., we must **substitute 100** into the formula $2n + 7$ for the n th term of the sequence.

$$n\text{th term} = 2n + 7$$

$$\begin{aligned}
 100\text{th term} &= 2 \times 100 + 7 \\
 &= 200 + 7 \\
 &= 207.
 \end{aligned}$$

We can find any term in a sequence by substituting an appropriate value into the formula for the n th term.

$$\begin{aligned}
 20\text{th term} &= 2 \times 20 + 7 \\
 &= 40 + 7 \\
 &= 47
 \end{aligned}$$

$$\begin{aligned}
 50\text{th term} &= 2 \times 50 + 7 \\
 &= 100 + 7 \\
 &= 107
 \end{aligned}$$

$$\begin{aligned}
 195\text{th term} &= 2 \times 195 + 7 \\
 &= 390 + 7 \\
 &= 397
 \end{aligned}$$

Exercise 5

For each sequence in Exercise 4, find

(i) the 100th term;

(ii) the 50th term;

(iii) the 18th term.

**Example: Introducing Negative Numbers**

Sequence	N th term
45, 48, 51, 54, 57,	$3n + 42$
24, 21, 18, 15, 12,	$-3n + 27$
-24, -21, -18, -15, -12,	$3n - 27$
-11, -14, -17, -20, -23,	$-3n - 8$

Exercise 6Find the n th term of the following linear sequences.

(a) 26, 29, 32, 35, 38,

(b) 14, 11, 8, 5, 2,

(c) -20, -17, -14, -11, -8,

(d) -30, -33, -36, -39, -42,

(e) 45, 43, 41, 39, 37,

(f) -45, -43, -41, -39, -37,

(g) 37, 39, 41, 43, 45,

(h) -37, -39, -41, -43, -45,

(i) 73, 79, 85, 91, 97,

(j) -73, -79, -85, -91, -97,

(k) 73, 67, 61, 55, 49,

(l) -73, -67, -61, -55, -49,

Exercise 7

Find the 100th term of each of the sequences from Exercise 6.

Exercise 8Find the n th term of each of the sequences from Exercise 1.

Finding the position of a term

Let us again consider the sequence 9, 11, 13, 15, 17, ... which has the n th term $2n + 7$. The number 93 forms part of this sequence. Which term of the sequence is 93?

In order to answer this question, we must consider the formula for the n th term as a number machine.



To undo this number machine, we must subtract 7 to begin with, and then divide by 2. Therefore, if 93 has come out of the machine, then by calculating

$$93 - 7 = 86$$

$$86 \div 2 = 43$$

we can say that 93 is the **43rd** term of the sequence.

**Exercise 9**

6

For the following sequences,

(i) find the n th term of the sequence; (ii) calculate which term of the sequence corresponds to the number 86.

(a) 6, 8, 10, 12, 14,

(b) 11, 16, 21, 26, 31,

(c) 14, 22, 30, 38, 46,

Exercise 10

James writes the sequence 15, 21, 27, 33, 39, ...

Which of the following numbers form part of James' sequence? You must give a reason for each answer.

(a) 81

(b) 100

(c) 124

(d) 141

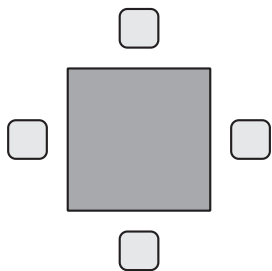
Exercise 11

The n th term of Sarah's sequence is $4n - 3$. The n th term of Tamzin's sequence is $63 - 7n$.

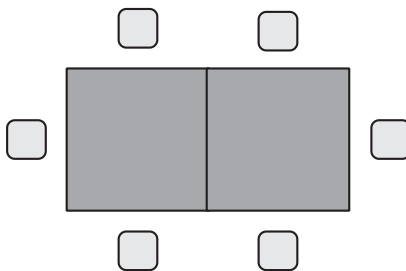
Which numbers appear in both Sarah and Tamzin's sequences?

Exercise 12: Setting tables and chairs in a row

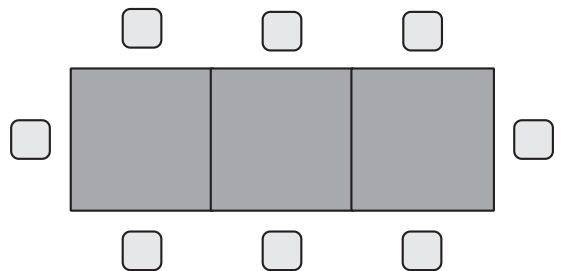
One table



Two tables



Three tables



Applying

(a) How many chairs are required if you have three tables?

(b) How many chairs are required if you have four tables?

(c) Copy and complete the following table.

Number of tables	1	2	3	4	5	6
Number of chairs	4					

(d) How many chairs are required if you have n tables?

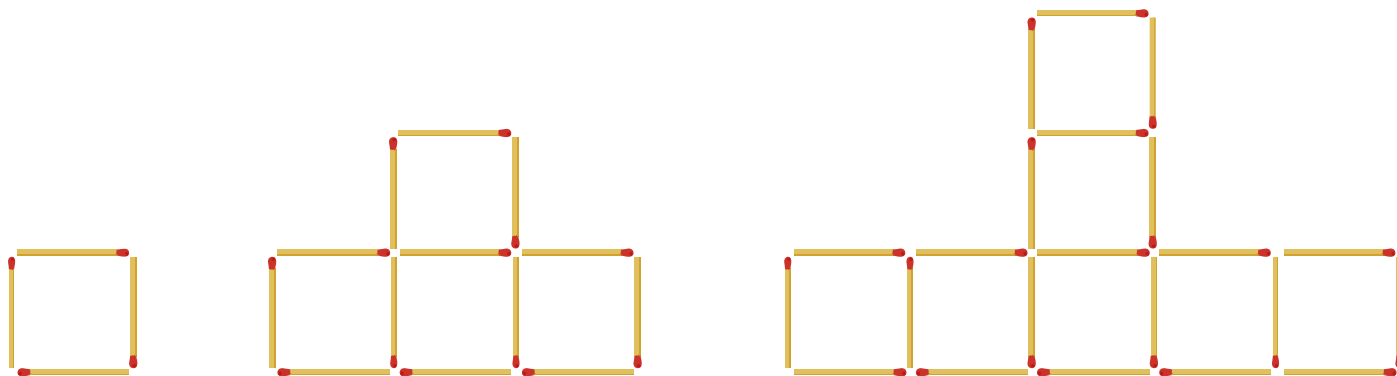
(e) How many chairs are required if you have 100 tables?

(f) How many chairs are required if you have twenty-four tables?

(g) How many tables are required if you have seventy chairs?

Exercise 13

Here is a sequence of matchstick patterns.



Pattern 1

Pattern 2

Pattern 3

- (a) Draw Pattern 4 in your book.
- (b) Copy and complete the following table.

Number of the pattern	1	2	3	4	5	6
Number of matches	4					

- (c) Look at the sequence for the number of matches. What is the n th term of this sequence?
- (d) How many matches would be required in order to make Pattern 25?
- (e) How many **squares** would 76 matches make?

Exercise 14 (Revision)

- (a) Find the n th term of each of the following linear sequences.
- (i) 15, 17, 19, 21, 23, (ii) 48, 45, 42, 39, 36, (iii) $-5, -1, 3, 7, 11, \dots$
- (b) Find the 100th term for each of the sequences from part (a).
- (c) The n th term of a sequence is $3n - 5$. Write down the first five terms of this sequence.
- (d) Which term from part (a)'s sequence corresponds to the number 27?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Expanding Brackets

Exercise 15

4

Discuss what is the same and what is different about the two sums below.

$2(3 + 4)$

$6 + 8$

Expanding Brackets

Above, it is possible to change the first sum to the second sum by performing two multiplication sums. The mathematical name for this is **expanding**.

$2(3 + 4)$
 $2 \times 3 = 6$
 $2 \times 4 = 8$

→

$6 + 8$

Therefore $2(3 + 4) = 6 + 8$.

Exercise 16

Expand the brackets in the following sums. By doing this, find the answer to the sum.

- | | | | |
|----------------|----------------|-----------------|-----------------|
| (a) $2(3 + 5)$ | (b) $2(5 + 4)$ | (c) $5(3 + 4)$ | (d) $4(3 + 2)$ |
| (e) $7(2 + 4)$ | (f) $6(3 + 7)$ | (g) $10(2 + 6)$ | (h) $12(3 + 1)$ |

Exercise 17

Calculate the following.

- | | | | |
|------------------|--------------------|-------------------|--------------------|
| (a) 2×3 | (b) -2×3 | (c) 2×-3 | (d) -2×-3 |
| (e) 7×6 | (f) -7×-6 | (g) 7×-6 | (h) -7×6 |

Exercise 18

Expand the brackets in the following sums. By doing this, find the answer to the sum.

- | | | | |
|-----------------|-----------------|------------------|------------------|
| (a) $3(4 + 5)$ | (b) $3(4 - 5)$ | (c) $3(-4 + 5)$ | (d) $3(-4 - 5)$ |
| (e) $-3(4 + 5)$ | (f) $-3(4 - 5)$ | (g) $-3(-4 + 5)$ | (h) $-3(-4 - 5)$ |

Exercise 19

Pair the expressions in the box.

$2(4 + 6)$

$8 + 24$

$8 + 12$

$-2(4 + 6)$

$4(2 + 6)$

$-8 - 12$

$-4(-2 + 6)$

$8 - 24$

Expanding Algebraic Expressions

When we write a piece of algebra, like $4x + 6$, we say that we write an **algebraic expression**. It is possible to use the idea of expanding brackets with **algebraic expressions**, as well as with numerical sums. Here is the general rule:



$$a(b + c) = ab + ac$$



The
distributive
rule

To expand the bracket, we must **multiply** the term a with **every term** inside the bracket.

Example: Multiplying Algebraic Terms

$$5 \times x = 5x$$

$$y \times 3 = 3y$$

$$x \times x = x^2$$

$$2y \times y = 2y^2$$

$$5x \times 3y = 15xy$$

$$-3y \times 2y = -6y^2$$

Exercise 20

5

Simplify the following multiplication sums.

(a) $4 \times x$

(b) $y \times 7$

(c) $6 \times z$

(d) $a \times 8$

(e) $y \times y$

(f) $t \times t$

(g) $3x \times x$

(h) $y \times 4y$

(i) $12z \times z$

(j) $e \times 14e$

(k) $2x \times 4x$

(l) $7z \times 3z$

(m) $3x \times y$

(n) $6x \times 3y$

(o) $5y \times 2z$

(p) $-3x \times x$

(q) $y \times -5y$

(r) $-z \times 11z$

(s) $-4x \times 3x$

(t) $5m \times -2m$

(u) $-8s \times -3s$

Example: Expanding Algebraic Expressions

$$2(x + 4) = 2x + 8$$

$$2(3x + 4) = 6x + 8$$

$$2(3 + 4x) = 6 + 8x$$

$$2(x - 4) = 2x - 8$$

$$-2(x + 4) = -2x - 8$$

$$-2(x - 4) = -2x + 8$$

$$x(x + 4) = x^2 + 4x$$

$$2x(x + 4) = 2x^2 + 8x$$

$$2x(3 + 4x) = 6x + 8x^2$$

Exercise 21

Expand the following algebraic expressions.

(a) $3(x + 4)$

(b) $5(x + 4)$

(c) $5(x + 3)$

(d) $3(2x + 4)$

(e) $3(4 + 2x)$

(f) $5(2x + 4)$

(g) $3(x - 4)$

(h) $3(4 - x)$

(i) $-3(x + 4)$

(j) $-3(x - 4)$

(k) $-3(4 - x)$

(l) $-5(x - 4)$

(m) $x(x + 2)$

(n) $y(y + 2)$

(o) $f(f + 2)$

(p) $3x(x + 2)$

(q) $3x(2 + x)$

(r) $3x(x - 2)$

(s) $3x(2 - x)$

(t) $-3x(x + 2)$

(u) $-3x(x - 2)$

(v) $-3x(2 - x)$

(w) $3x(2x + 4)$

(x) $3x(2x - 4)$

(y) $3x(4 - 2x)$

(z) $-3x(2x + 4)$

(α) $-3x(4 - 2x)$



Example: Expanding with more than two terms in the bracket

$$2(x + y + 5) = 2x + 2y + 10$$

$$2(3x + 4y + 5) = 6x + 8y + 10$$

$$2x(3x + 4y + 5) = 6x^2 + 8xy + 10x$$

$$-2x(3x - 4y + 5) = -6x^2 + 8xy - 10x$$

Exercise 22

5

Expand the following algebraic expressions.

(a) $3(x + y + 7)$

(b) $4(x + y + 7)$

(c) $4(a + b + 7)$

(d) $3(2x + y + 7)$

(e) $3(2x + 5y + 7)$

(f) $3(2x + 5y - 7)$

(g) $3(x + y + z + 7)$

(h) $3(2x + 5y + 7z)$

(i) $-3(2x + 5y + 7z + 9)$

(j) $4(2x - 5y + z)$

(k) $4(-2x + 5y - z)$

(l) $-4(-2x - 5y + z)$

(m) $3x(x + y + 7)$

(n) $4x(x + y + 7)$

(o) $4y(x + y + 7)$

(p) $3a(a + b + 7)$

(q) $3a(b + 7 + a)$

(r) $3b(b + 7 + a)$

(s) $3x(2x + 5y + 7)$

(t) $3x(2x + 5y - 7z)$

(u) $3x(5y - 2x - 7)$

(v) $-3x(x + y + 7)$

(w) $-3x(2x - 5y - 7)$

(x) $-4x(2x - 5y + 7)$

(y) $-3a(a + b + c)$

(z) $-3a(2a - 5b + 7c)$

(α) $-3a(7c - 5b + 2a - 9d)$

Example: Collecting Like Terms

$$4x + 3x = 7x$$

$$4x + 8y + 3x + 9y = 7x + 17y$$

$$4x^2 + 3x + 8x + 16 = 4x^2 + 11x + 16$$

$$5xy + 10xz - 2xy - xz = 3xy + 9xz$$

Exercise 23

Revision

Simplify the following algebraic expressions.

(a) $5x + 6x$

(b) $9x - 2x$

(c) $3y + 9y$

(d) $23z - 10z$

(e) $7x + x$

(f) $2xy + 4xy$

(g) $3x^2 + 8x^2$

(h) $14y^2 - 6y^2$

(i) $3z^2 - 9z^2$

(j) $6x + 2y + 3x + 12y$

(k) $4a + 6b + 10a + 5b$

(l) $7m + 14n + 9n$

(m) $6x + 2y - 3x + 12y$

(n) $4a - 6b + 10a + 5b$

(o) $7m - 14n - 9n$

(p) $6x - 2y - 3x + 12y$

(q) $-4a - 6b + 10a - 5b$

(r) $-7m + 14n - 9n$

(s) $3x^2 + 2x + 4x^2 + 20x$

(t) $3x^2 + 6x + 9x + 3$

(u) $7a^2 + 14a + 4a + 8$

(v) $3x^2 + 2x + 4x^2 - 20x$

(w) $3x^2 - 6x + 9x + 3$

(x) $-7a^2 - 14a + 4a + 8$

(y) $8xy + 12xz + 3xy + 15xz$

(z) $8xy - 12xz - 3xy + 15xz$

(α) $8xz + 12xy + 3xy - 15xz$

Challenge!

Simplify the following algebraic expressions.

(a) $12x^2 + 16xy + 8x - 5xy + 20x$

(b) $z^2 - z + 15z + 5 - z^2 - 9z$

(c) $5x^3 + 15x + 7x^2 + 21x + 14$

(d) $b^2 + 21ab - 8ab + b^2 + 5$

(e) $13ab^2 + 26b - 6ab^2 - 5a$

(f) $-4x^3 - 9 + 3x^2 + 17$

Example: Expanding and Simplifying

$$2(3x + 4) + 5(6x + 7) = 6x + 8 + 30x + 35$$

$$= 36x + 43$$

$$2(3x + 4) - 5(6x + 7) = 6x + 8 - 30x - 35$$

$$= -24x - 27$$

$$2(3x + 4y) + 5(6x + 7y) = 6x + 8y + 30x + 35y$$

$$= 36x + 43y$$

$$2(3x - 4y) + 5(6x - 7y) = 6x - 8y + 30x - 35y$$

$$= 36x - 43y$$

$$x(2x + 3) + 4x(5x + 6) = 2x^2 + 3x + 20x^2 + 24x$$

$$= 22x^2 + 27x$$

$$x(2x + 3) - 4x(5x - 6) = 2x^2 + 3x - 20x^2 + 24x$$

$$= -18x^2 + 27x$$

$$2x(x + 3) + 4(x + 5) = 2x^2 + 6x + 4x + 20$$

$$= 2x^2 + 10x + 20$$

$$-2x(x + 3) + 4(5 - x) = -2x^2 - 6x + 20 - 4x$$

$$= -2x^2 - 10x + 20$$

**Exercise 24**

Expand and simplify the following expressions.

(a) $3(2x + 5) + 4(3x + 2)$

(b) $6(2x + 5) + 4(3x + 2)$

(c) $6(2x + 5) + 4(5x + 7)$

(d) $3(2x + 5) + 4(3x - 2)$

(e) $6(2x - 5) + 4(3x + 2)$

(f) $6(2x + 5) - 4(5x + 7)$

(g) $3(2x + 5) - 4(3x - 2)$

(h) $6(2x - 5) - 4(3x + 2)$

(i) $-6(2x + 5) - 4(5x - 7)$

(j) $5(3a + 4) + 2(2a + 7)$

(k) $5(3a - 4) + 2(2a - 7)$

(l) $5(-3a + 4) - 2(2a - 7)$

(m) $-3(z + 3) + 4(z + 8)$

(n) $-3(z - 3) + 4(z - 8)$

(o) $3(z - 3) - 4(z - 8)$

Exercise 25

Expand and simplify the following expressions.

(a) $7(3x + 4y) + 3(6x + 4y)$

(b) $2(a + 5b) + 5(3a + b)$

(c) $3(4e + 2f) + 6(e + 5f)$

(d) $7(3x - 4y) + 3(6x + 4y)$

(e) $2(a + 5b) - 5(3a + b)$

(f) $3(4e - 2f) + 6(e - 5f)$

(g) $7(3x - 4y) - 3(6x + 4y)$

(h) $2(a - 5b) - 5(b - 3a)$

(i) $-3(4e - 2f) - 6(e - 5f)$

(j) $9(x + 3z) + 11(2x + 4z)$

(k) $9(x + 3z) + 11(-2x + 4z)$

(l) $-9(x + 3z) + 11(2x + 4z)$

(m) $10(2u + 3v) + 3(5u + 2v)$

(n) $10(-2u + 3v) + 3(5u - 2v)$

(o) $-10(-2u + 3v) - 3(2v - 5u)$

Exercise 26

Expand and simplify the following expressions.

(a) $3x(2x + 3) + 4x(3x + 7)$

(b) $8z(4z + 1) + 3z(9z + 10)$

(c) $2a(3a + 4) + 7a(3a + 5)$

(d) $3x(2x + 3) - 4x(3x - 7)$

(e) $8z(4z - 1) + 3z(9z - 10)$

(f) $2a(3a - 4) + 7a(3a + 5)$

(g) $3x(3 - 2x) - 4x(7 - 3x)$

(h) $8z(1 - 4z) - 3z(10 + 9z)$

(i) $-2a(3a - 4) + 7a(3a + 5)$

Exercise 27

Expand and simplify the following expressions.

(a) $3x(x + 5) + 8(x + 2)$

(b) $4y(2y + 7) + 5(y + 3)$

(c) $6z(3z + 4) + 2(5z + 7)$

(d) $3x(x + 5) - 8(x + 2)$

(e) $-4y(2y + 7) + 5(y + 3)$

(f) $6z(-3z + 4) + 2(5z + 7)$

(g) $3x(x + 5) - 9(2 - x)$

(h) $-4y(2y + 7) + 5(3 - y)$

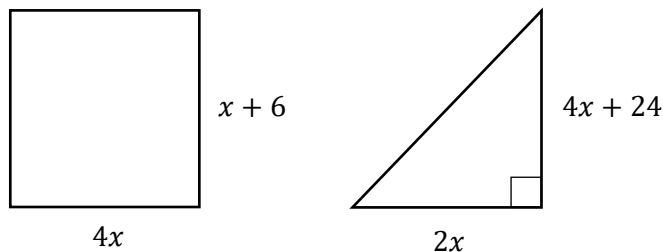
(i) $6z(3z - 4) - 2(5z - 7)$

Exercise 28

Applying

5

Here is a square and a right-angled triangle. (The diagrams are not drawn to scale.)



Show that the square and the triangle have the same area.

Challenge!

Find the areas of the square and triangle in Exercise 28.

Exercise 29 (Revision)

Skill

Expand and simplify the following expressions.

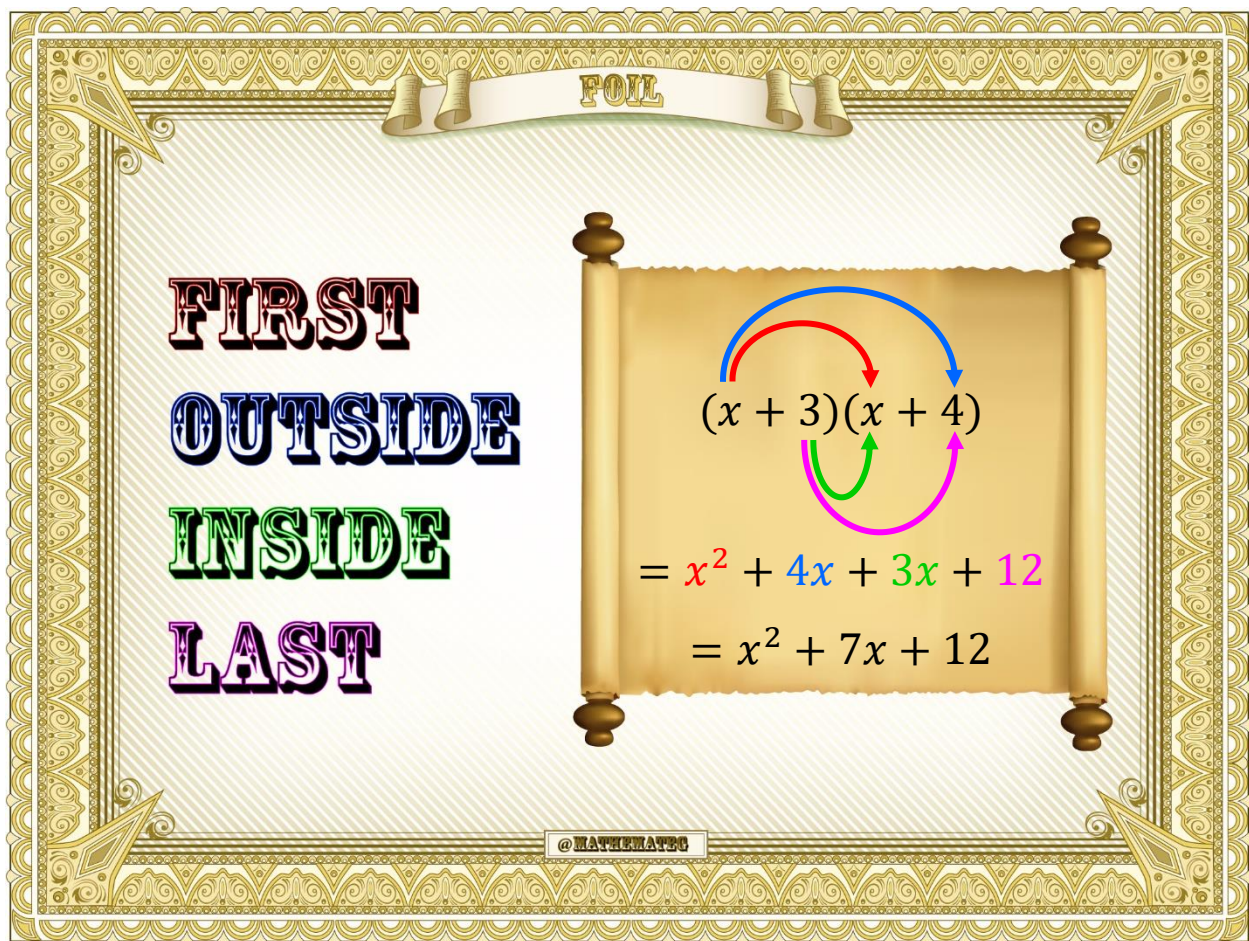
- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| (a) $5(x + 6)$ | (b) $7(y - 2)$ | (c) $14(z + 3)$ |
| (d) $3x(x + 4)$ | (e) $9y(2y - 3)$ | (f) $-5a(3a - 4)$ |
| (g) $2(a + 3b + c)$ | (h) $4x(3x + 2y + 5)$ | (i) $6a(a - 3b + 8c)$ |
| (j) $4(5x + 2) + 3(9x + 6)$ | (k) $4(5x - 2) + 3(9x + 6)$ | (l) $4(5x - 2) + 3(9x - 6)$ |
| (m) $5(2x + 6y) + 3(3x + 5y)$ | (n) $5(2x + 6y) - 3(3x + 5y)$ | (o) $-5(2x + 6y) + 3(3x + 5y)$ |
| (p) $x(3x + 5) + 2x(4x + 7)$ | (q) $3x(x + 5) + 4x(2x - 7)$ | (r) $3x(x - 5) + 4x(7 - 2x)$ |
| (s) $5x(x + 8) + 6(2x + 3)$ | (t) $5x(-x + 8) + 6(2x + 3)$ | (u) $5x(-x + 8) - 6(-2x + 3)$ |
| (v) $4(5k + m) + 3m + 5(k + m)$ | (w) $4(5k - m) + 3m + 5(k - m)$ | (x) $4(5k - m) - 3m - 5(k - m)$ |
| (y) $4x(x + 2) + x(2 + y)$ | (z) $4x(x + 2) - x(2 + y)$ | (α) $4x(x - 2) + x(2 - y)$ |

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐



FOIL is an **acronym** for remembering how to expand a double bracket.



By calculating the four sums suggested by the word FOIL, we multiply every term in the first bracket by every term in the second bracket.

Exercise 30

Expand and simplify the following expressions.

(a) $(x + 2)(x + 3)$

(b) $(x + 2)(x + 4)$

(c) $(x + 2)(x + 5)$

(d) $(x + 3)(x + 3)$

(e) $(x + 3)(x + 5)$

(f) $(x + 3)(x + 10)$

(g) $(x - 2)(x + 3)$

(h) $(x + 2)(x - 3)$

(i) $(x - 2)(x - 3)$

(j) $(x + 7)(x + 8)$

(k) $(x + 7)(x - 8)$

(l) $(7 + x)(x + 8)$

(m) $(x + 1)(x + 9)$

(n) $(1 - x)(x + 9)$

(o) $(1 - x)(9 - x)$

(p) $(y + 2)(y + 8)$

(q) $(y - 2)(y + 8)$

(r) $(y + 2)(y - 8)$

(s) $(z + 12)(z + 3)$

(t) $(z + 12)(z - 3)$

(u) $(z - 12)(3 - z)$

(v) $(w + 6)(w + 8)$

(w) $(w - 6)(w + 8)$

(x) $(w - 6)(w - 8)$

(y) $(f + 9)(f + 8)$

(z) $(9 + f)(8 - f)$

(α) $(f - 9)(8 - f)$

Skill

8

Example

$$(2x + 3)(x + 4) = 2x^2 + 8x + 3x + 12 \\ = 2x^2 + 11x + 12$$

$$(2x + 3)(5x + 4) = 10x^2 + 8x + 15x + 12 \\ = 10x^2 + 23x + 12$$

Exercise 31

8

Expand and simplify the following expressions.

(a) $(2x + 3)(x + 5)$

(b) $(2x + 4)(x + 5)$

(c) $(2x + 5)(x + 4)$

(d) $(3x + 2)(x + 4)$

(e) $(3x + 2)(x + 5)$

(f) $(x + 5)(3x + 2)$

(g) $(4x + 1)(x + 2)$

(h) $(4x + 1)(2 + x)$

(i) $(1 + 4x)(x + 2)$

(j) $(4x + 1)(x - 2)$

(k) $(4x - 1)(x + 2)$

(l) $(4x - 1)(x - 2)$

(m) $(5x + 3)(x + 4)$

(n) $(5x - 3)(x + 4)$

(o) $(3 - 5x)(x + 4)$

(p) $(2y + 4)(y + 7)$

(q) $(2y + 4)(y - 7)$

(r) $(2y - 4)(y - 7)$

(s) $(z + 3)(6z + 3)$

(t) $(6z + 3)(z - 3)$

(u) $(z - 3)(6z - 3)$

(v) $(2x + 4)(3x + 6)$

(w) $(5x + 7)(6x + 1)$

(x) $(8x - 2)(2x + 5)$

(y) $(3y + 5)(4y + 10)$

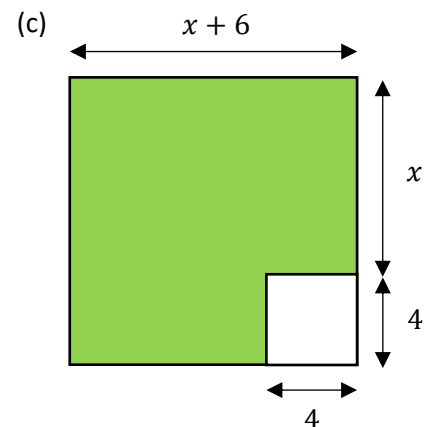
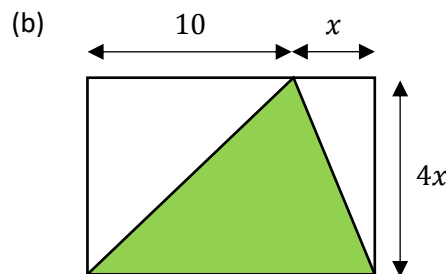
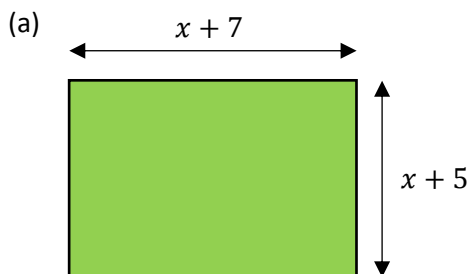
(z) $(7z + 3)(2z - 4)$

(α) $(9w - 6)(4w - 2)$

Exercise 32

In the following shapes, find an expression, expanded and simplified, for the area shaded green. (The diagrams are not drawn to scale.)

Applying

**Example**

$$(x + 5)^2 = (x + 5)(x + 5) \\ = x^2 + 5x + 5x + 25 \\ = x^2 + 10x + 25$$

$$(2x + 3)^2 = (2x + 3)(2x + 3) \\ = 4x^2 + 6x + 6x + 9 \\ = 4x^2 + 12x + 9$$

Exercise 33

Expand and simplify the following expressions.

(a) $(x + 1)^2$

(b) $(x + 2)^2$

(c) $(x + 3)^2$

(d) $(y + 4)^2$

(e) $(x - 1)^2$

(f) $(z - 2)^2$

(g) $(2x + 1)^2$

(h) $(3x + 2)^2$

(i) $(4y + 6)^2$

(j) $(2x - 1)^2$

(k) $(1 - 2x)^2$

(l) $(4z - 3)^2$



Skill

Example: Expanding and Simplifying

$$(x+2)(x+3) + (x+4)(x+5) = x^2 + 3x + 2x + 6 + x^2 + 5x + 4x + 20 \\ = 2x^2 + 14x + 26$$

$$(3x+2)(x+4) - (2x+6)(x+5) = 3x^2 + 12x + 2x + 8 - (2x^2 + 10x + 6x + 30) \\ = 3x^2 + 14x + 8 - (2x^2 + 16x + 30) \\ = 3x^2 + 14x + 8 - 2x^2 - 16x - 30 \\ = x^2 - 2x - 22$$

**Exercise 34**

Expand and simplify the following expressions.

(a) $(x+4)(x+3) + (x+2)(x+1)$

(b) $(x+5)(x+3) + (x+6)(x+2)$

(c) $(x+7)(x+1) + (x+8)(x+4)$

(d) $(x+7)(x-1) + (x+8)(x-4)$

(e) $(2x+3)(x+2) + (x+4)(x+9)$

(f) $(3x+3)(x-2) + (2x+4)(x+9)$

(g) $(x+4)(x+3) - (x+2)(x+1)$

(h) $(x+5)(x+3) - (x+6)(x+2)$

(i) $(2x+3)(x+2) - (x+4)(x+9)$

(j) $(3x+3)(x-2) - (2x+4)(x+9)$

(k) $(4x+1)(x+3) - (2x+3)(x+5)$

(l) $(4x-1)(x+3) - (2x+3)(x-5)$

Example: FOIL with more than one variable

$$(x+2)(x+y) = x^2 + xy + 2x + 2y$$

$$(3x+y)(2x-y) = 6x^2 - 3xy + 2xy - y^2 \\ = 6x^2 - xy - y^2$$

Exercise 35

Expand and simplify the following expressions.

(a) $(x+3)(x+y)$

(b) $(x+y)(x+4)$

(c) $(y+2)(x+y)$

(d) $(x+y)(x+y)$

(e) $(x-y)(x+y)$

(f) $(y-x)(y-x)$

(g) $(2x+4)(x+y)$

(h) $(2x+y)(x+5)$

(i) $(2x+y)(3x+y)$

(j) $(4y+x)(3+2x)$

(k) $(4y+x)(x-2y)$

(l) $(4y+x)(3y-2x)$

Example: Brackets containing more than two terms

$$(x+3)(x+y+5) = x^2 + xy + 5x + 3x + 3y + 15 \\ = x^2 + xy + 8x + 3y + 15$$

Here, FOIL does not work. We must multiply all the terms in the first bracket by all the terms in the second bracket.

Exercise 36

Expand and simplify the following expressions.

(a) $(x+3)(x+y+4)$

(b) $(x+2)(x+y+4)$

(c) $(x+2)(y+4+x)$

(d) $(y+2)(x+y+5)$

(e) $(y+2)(3+y+x)$

(f) $(y+2)(x+7+y)$

(g) $(x+5)(x+y+9)$

(h) $(x+5)(x-y+9)$

(i) $(x-5)(x-y+9)$

(j) $(2x+3)(x+y+4)$

(k) $(2x-3)(x+y+4)$

(l) $(2x-3)(x-y-4)$

(m) $(5y+2)(2x+4y+3)$

(n) $(5y+2)(2x-4y+3)$

(o) $(5y-2)(2x-4y+3)$

(p) $(2x+7y+4)(5x+3y+2)$

(q) $(2x+7y+4)(5x-3y+2)$

(r) $(2x+7y-4)(5x-3y-2)$

Solving Equations 2

Revision

6

To begin, let us revise the equation solving work from year 8.

Exercise 39

(a) $x + 6 = 9$

(b) $2x = 28$

(c) $x - 2 = 9$

(d) $\frac{x}{2} = 10$

(e) $y + 2 = 13$

(f) $3y = 18$

(g) $z - 4 = -1$

(h) $\frac{z}{3} = 6$

(i) $x + 1.5 = 4$

(j) $2x + 1 = 15$

(k) $2x - 1 = 15$

(l) $3x + 4 = 16$

(m) $4y - 5 = 31$

(n) $2z - 3 = 12$

(o) $7x + 10 = 80$

(p) $3x + 1 = 2x + 27$

(q) $3x - 1 = 2x + 27$

(r) $4x + 2 = 2x + 34$

(s) $4x - 6 = 2x + 40$

(t) $5x + 7 = 2x + 37$

(u) $10x - 2 = 5x + 33$

(v) $7x - 3 = 3x - 23$

(w) $10x + 4 = 4x - 38$

(x) $2x + 4 = 5x - 11$

(y) $\frac{x}{2} + 3 = 9$

(z) $\frac{x}{2} - 3 = 1$

(α) $\frac{y}{4} + 7 = 11$

Solving Equations Containing Brackets

There are two methods of solving an equation such as $2(x + 3) = 16$.

Method 1: Start by expanding:

$2(x + 3) = 16$

$2x + 6 = 16$

$2x = 10$

$x = 5$

[Expand the bracket]

[Subtract 6]

[Divide by 2]

Method 2: Start by dividing:

$2(x + 3) = 16$

$x + 3 = 8$

$x = 5$

[Divide by 2]

[Subtract 3]

**Exercise 40**

Solve the following equations.

(a) $2(x + 4) = 16$

(b) $2(x + 4) = 26$

(c) $2(x + 5) = 26$

(d) $3(x + 4) = 18$

(e) $3(x + 4) = 33$

(f) $3(x - 4) = 33$

(g) $4(x + 1) = 24$

(h) $5(x - 2) = 35$

(i) $6(x + 3) = 30$

(j) $7(x - 1) = 49$

(k) $8(x + 5) = 48$

(l) $9(x - 6) = 27$

(m) $10(x + 3) = 80$

(n) $12(x + 1) = 36$

(o) $20(x - 4) = 160$

(p) $-2(x + 3) = 20$

(q) $-2(x - 3) = 20$

(r) $-2(x - 3) = -20$

(s) $-3(x + 4) = 39$

(t) $-3(x - 4) = 39$

(u) $-3(x - 4) = -39$

(v) $4(2x + 3) = 28$

(w) $4(2x - 3) = 28$

(x) $4(2x - 3) = -28$

(y) $3(5x + 2) = 51$

(z) $4(3x - 5) = 40$

(α) $6(4x - 3) = 150$

Skill

With some equations, we have to start by expanding the brackets.

Example

$$3(x + 2) + 2x = 41$$

$$3x + 6 + 2x = 41 \quad [\text{Expand the bracket}]$$

$$5x + 6 = 41 \quad [\text{Collect like terms}]$$

$$5x = 35 \quad [\text{Subtract 6}]$$

$$x = 7 \quad [\text{Divide by 5}]$$

$$4(x + 3) = 2(x - 6)$$

$$4x + 12 = 2x - 12 \quad [\text{Expand the brackets}]$$

$$2x = -24 \quad [\text{Subtract } 2x; \text{ subtract } 12]$$

$$x = -12 \quad [\text{Divide by 2}]$$



Exercise 41

6

Solve the following equations.

(a) $3(x + 2) + 2x = 46$

(b) $3(x + 4) + 2x = 42$

(c) $3(x + 4) + x = 36$

(d) $4(x + 6) + 3x = 31$

(e) $4(x - 6) + 3x = 32$

(f) $4(x - 6) + x = 1$

(g) $2(3x - 5) - 3x + 4 = 9$

(h) $2(3x - 5) + 3x + 4 = 12$

(i) $2(3x - 4) + 4 = 44$

(j) $6x + 9 - 2(x + 4) = 29$

(k) $6x + 9 - 2(x - 4) = 21$

(l) $6x + 9 + 2(x - 4) = 73$

(m) $10(2x - 15) = 5x$

(n) $10(2x + 15) = 5x$

(o) $-10(2x + 5) = 5x$

Exercise 42

Solve the following equations.

(a) $4(x + 2) = 2(x + 10)$

(b) $4(x - 2) = 2(x + 10)$

(c) $4(x - 2) = 2(x - 10)$

(d) $6(2x - 5) = 2(x + 5)$

(e) $6(2x + 5) = 2(x - 5)$

(f) $6(2x + 5) = -2(x - 50)$

(g) $5(1 - 2y) = 4(5 - 3y)$

(h) $7(x + 3) = 7(3 - x)$

(i) $5(x + 3) = 2(3x + 10)$

(j) $3x + 2(x + 1) = 3(x + 6)$

(k) $4 + 2(3 - y) = 6(5 - y)$

(l) $x - 3 - 2(2x - 5) = 2(6 - 2x)$

(m) $5x - 3(2 - x) = 2(3x + 10)$

(n) $3(2z + 1) - 5(3z - 2) = -5$

(o) $2(y + 3) = 5(y - 1) - 7(y - 3)$

Exercise 43

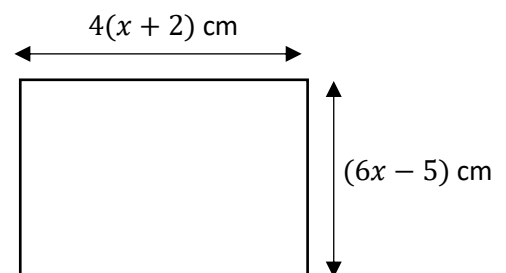
The shown rectangle's perimeter is 46 cm.

(a) Write an expression for the perimeter of the rectangle.

(b) Form and solve an equation containing x .

(c) Find the area of the rectangle.

Applying



Exercise 44

For the right-angled triangle shown on the right, the size of some of the angles and the length of some of the edges are given in terms of x .

(a) Write an expression for the sum of the triangle's angles.

(b) Form and solve an equation containing x .

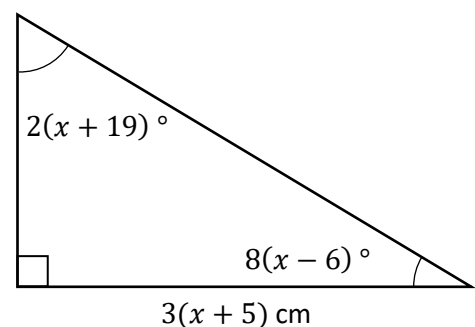
(c) Calculate the length of the triangle's base and height.

(d) Find the area of the triangle.

(e) Calculate the length of the triangle's hypotenuse.

(f) Find the perimeter of the triangle.

$7(x - 6)$ cm



Solving equations where a double bracket is equal to zero

Let us consider how to solve the equation

$$(x + 2)(x - 3) = 0$$

It is tempting to use the FOIL acronym from the previous chapter to expand the double bracket, but this would not help. Instead, we need to consider what we are doing when solving an equation which, in this case, is to try to find a value for x such that the left-hand side of the equation is equal to zero. To do this, we can concentrate on the contents of **one** of the brackets only. If we can find a value for x such that **one** of the brackets is zero, then the **whole left-hand side** of the equation will be zero, as **zero multiplied by anything is zero**. Therefore, to solve this equation, we can **either** find a value for x such that the first bracket is equal to zero, **or** we can find a value for x such that the second bracket is equal to zero. This will give the two solutions for this equation.

$$\begin{array}{l} \text{Either } x + 2 = 0 \text{ or } x - 3 = 0 \\ x = -2 \qquad \qquad x = 3 \end{array}$$

Exercise 45

Solve the following equations.



- | | | |
|----------------------------|-----------------------------|------------------------------|
| (a) $(x + 3)(x - 2) = 0$ | (b) $(x - 3)(x + 2) = 0$ | (c) $(x - 3)(x - 2) = 0$ |
| (d) $(x + 3)(x + 2) = 0$ | (e) $(x - 5)(x + 4) = 0$ | (f) $(x - 6)(x - 7) = 0$ |
| (g) $(x + 8)(x + 10) = 0$ | (h) $(x - 1)(x + 7) = 0$ | (i) $(x + 1.5)(x - 2.5) = 0$ |
| (j) $(2x - 8)(x + 6) = 0$ | (k) $(x - 11)(2x + 6) = 0$ | (l) $(2x - 10)(2x + 12) = 0$ |
| (m) $(2x - 1)(x + 15) = 0$ | (n) $(2x + 1)(x - 15) = 0$ | (o) $(2x - 3)(x - 14) = 0$ |
| (p) $(3x - 6)(4x + 8) = 0$ | (q) $(3x + 6)(4x - 8) = 0$ | (r) $(5x + 20)(4x - 24) = 0$ |
| (s) $(3x - 2)(4x - 1) = 0$ | (t) $(3x + 2)(4x + 1) = 0$ | (u) $(4x - 5)(5x - 4) = 0$ |
| (v) $(3x - 3)(x + 16) = 0$ | (w) $(4x + 3)(7x - 1) = 0$ | (x) $(5x + 2)(3x - 7) = 0$ |
| (y) $(6x + 5)(8x - 2) = 0$ | (z) $(10x - 3)(9x - 3) = 0$ | (α) $(11x + 4)(9x - 6) = 0$ |

Equations containing fractions

Next, let us re-visit how to solve the equation

$$\frac{x}{2} = 4$$



In this equation, we must find a value for x such that this value divided by two gives the answer four. In order to undo the process of dividing by two, we need to multiply by two, so that

$$\begin{array}{l} x = 4 \times 2 \\ x = 8 \end{array}$$

We can use the idea of multiplying to undo division in order to solve a variety of equations containing fractions.

Example

$$\frac{x+2}{4} = 7$$

$$x + 2 = 7 \times 4 \quad \text{[Multiply by 4]}$$

$$x + 2 = 28$$

$$x = 26 \quad \text{[Subtract 2]}$$

$$\frac{8}{x} = 4$$

$$8 = 4 \times x \quad \text{[Multiply by } x \text{]}$$

$$8 = 4x$$

$$4x = 8 \quad \text{[Swap sides]}$$

$$x = 2 \quad \text{[Divide by 4]}$$

Eight divided by which number is equal to four?

Exercise 46

(a) $\frac{x}{2} = 6$

(b) $\frac{x}{2} = 12$

(c) $\frac{x}{2} = 10$

(d) $\frac{x}{3} = 9$

(e) $\frac{x}{4} = 12$

(f) $\frac{x}{5} = 6$

(g) $\frac{x}{2} + 4 = 6$

(h) $\frac{x}{2} - 4 = 6$

(i) $\frac{x}{2} + 3 = 9$

(j) $\frac{x}{3} + 1 = 8$

(k) $\frac{x}{3} - 2 = 8$

(l) $\frac{x}{3} + 2.5 = 7.5$

Exercise 47

(a) $\frac{x+2}{3} = 6$

(b) $\frac{x+2}{2} = 6$

(c) $\frac{x+3}{2} = 6$

(d) $\frac{x-4}{5} = 4$

(e) $\frac{x-4}{3} = 4$

(f) $\frac{x-8}{7} = 4$

(g) $\frac{y+8}{3} = 10$

(h) $\frac{z-5}{4} = 11$

(i) $\frac{d+12}{6} = 5$

(j) $\frac{x+2}{3} + 4 = 6$

(k) $\frac{x+2}{3} - 4 = 6$

(l) $\frac{x+2}{5} + 4 = 6$

(m) $\frac{x-3}{5} + 2 = 7$

(n) $\frac{x+7}{6} - 4 = 5$

(o) $\frac{x-2}{9} + 1.5 = 6.5$

(p) $\frac{2x+6}{4} = 5$

(q) $\frac{2x-6}{4} = 5$

(r) $\frac{2x+6}{10} = 4$

(s) $\frac{4x-1}{3} = 9$

(t) $\frac{5x+3}{6} = 8$

(u) $\frac{6x-3}{9} = 7$

(v) $\frac{2x+4}{3} + 6 = 10$

(w) $\frac{2x+4}{3} - 2 = 8$

(x) $\frac{2x-4}{3} + 6 = 10$

(y) $\frac{3x-1}{2} - 5 = 11$

(z) $\frac{4x+2}{5} + 3 = 13$

(α) $\frac{5x-2}{3} + 13 = 19$

Exercise 48

(a) $\frac{6}{x} = 2$

(b) $\frac{16}{x} = 2$

(c) $\frac{24}{x} = 2$

(d) $\frac{12}{y} = 3$

(e) $\frac{24}{z} = 8$

(f) $\frac{35}{w} = 7$

(g) $\frac{2}{x} = 3$

(h) $\frac{4}{x} = 5$

(i) $\frac{7}{x} = 10$

(j) $\frac{18}{x} + 2 = 8$

(k) $\frac{21}{x} - 1 = 2$

(l) $\frac{24}{x} + 3 = 11$

(m) $\frac{30}{x} - 6 = 9$

(n) $\frac{2}{x} + 6 = 9$

(o) $\frac{3}{z} - 2 = 10$

Exercise 49

(a) $\frac{1}{2x} = 3$

(b) $\frac{1}{3x} = 2$

(c) $\frac{3}{2x} = 1$

(d) $\frac{5}{4x} = 2$

(e) $\frac{5}{4x} = -2$

(f) $\frac{3}{2x} = 5$

(g) $\frac{4}{7y} = 3$

(h) $\frac{2}{11z} = 4$

(i) $\frac{5}{6e} = 1$

(j) $\frac{1}{3x} + 2 = 6$

(k) $\frac{1}{3x} - 2 = 6$

(l) $\frac{1}{3x} - 2 = -6$

(m) $\frac{3}{4x} + 1 = 4$

(n) $\frac{3}{4x} - 1 = 4$

(o) $\frac{4}{3x} - 1 = 4$



Example

$$\frac{x}{2} = 3x - 20$$

$$x = 2(3x - 20) \quad [\text{Multiply by 2}]$$

$$x = 6x - 40 \quad [\text{Expand the bracket}]$$

$$6x - 40 = x \quad [\text{Swap sides}]$$

$$5x = 40 \quad [\text{Subtract } x; \text{ add 40}]$$

$$x = 8 \quad [\text{Divide by 5}]$$

$$\frac{7}{2x+1} = 3$$

$$7 = 3(2x + 1) \quad [\text{Multiply by } 2x + 1]$$

$$7 = 6x + 3 \quad [\text{Expand the bracket}]$$

$$6x + 3 = 7 \quad [\text{Swap sides}]$$

$$6x = 4 \quad [\text{Subtract 3}]$$

$$x = \frac{4}{6} \quad [\text{Divide by 6}]$$

$$x = \frac{2}{3} \quad [\text{Simplify the fraction}]$$

**Exercise 50**

$$(a) \frac{x}{2} = 2x - 12$$

$$(b) \frac{x}{2} = 3x - 15$$

$$(c) \frac{x}{2} = 3x + 5$$

$$(d) \frac{x}{3} = x - 2$$

$$(e) \frac{x}{4} = 5 - x$$

$$(f) \frac{x}{5} = 2x + 18$$

$$(g) \frac{x}{2} + 3 = 2x - 6$$

$$(h) \frac{x}{5} - 2 = 10 - x$$

$$(i) \frac{x}{4} + 1 = 11 - x$$

$$(j) \frac{2x}{3} = 2 - 5x$$

$$(k) \frac{2x}{3} = 5x - 2$$

$$(l) \frac{3x}{2} = 2 + x$$

$$(m) \frac{2x}{5} = 6 - 2x$$

$$(n) \frac{5x}{2} = 4x - 2$$

$$(o) \frac{7x}{2} = 3 + 4x$$

$$(p) \frac{2x}{7} - 1 = 8 - x$$

$$(q) \frac{3x}{4} + 5 = 6x - 2$$

$$(r) \frac{4x}{3} + 2 = 4x + 3$$

Exercise 51

$$(a) \frac{14}{2x+1} = 2$$

$$(b) \frac{14}{2x-1} = 2$$

$$(c) \frac{14}{4x-1} = 2$$

$$(d) \frac{20}{3x+2} = 4$$

$$(e) \frac{18}{4x-6} = 3$$

$$(f) \frac{4}{2x+1} = 2$$

$$(g) \frac{24}{2x-6} + 2 = 8$$

$$(h) \frac{30}{4x+1} - 4 = 11$$

$$(i) \frac{60}{5x+7} + 13 = -7$$

$$(j) \frac{35}{x+4} - 2 = 5$$

$$(k) \frac{11}{2x-1} + 3 = 15$$

$$(l) \frac{4}{2-3x} - 7 = 3$$

Equations containing more than one fraction

The equation

$$\frac{5x-8}{2} = \frac{3x+12}{4}$$

contains two fractions. In order to solve this equation, we can use one of the following two methods.

Method 1: Multiply by each denominator, in turn.

$$\frac{5x-8}{2} = \frac{3x+12}{4}$$

$$5x - 8 = \frac{2(3x+12)}{4} \quad [\text{Multiply by 2}]$$

$$4(5x - 8) = 2(3x + 12) \quad [\text{Multiply by 4}]$$

$$20x - 32 = 6x + 24 \quad [\text{Expand the bracket}]$$

$$14x = 56 \quad [\text{Add 32; subtract 6x}]$$

$$x = 4 \quad [\text{Divide by 14}]$$

Method 2: Multiply by the lowest common denominator.

$$\frac{5x-8}{2} = \frac{3x+12}{4}$$

The lowest common denominator of 2 and 4 is 4.

$$4 \times \left(\frac{5x-8}{2} \right) = 4 \times \left(\frac{3x+12}{4} \right) \quad [\text{Multiply by 4}]$$

$$2(5x - 8) = 3x + 12 \quad [\text{Simplify}]$$

$$10x - 16 = 3x + 12 \quad [\text{Expand the bracket}]$$

$$7x = 28 \quad [\text{Add 16; subtract 3x}]$$

$$x = 4 \quad [\text{Divide by 7}]$$

Remember that the lowest common denominator of two numbers is the smallest number that is a multiple of both numbers.

Trial and Improvement

If we cannot solve an equation by algebraic methods (as in the previous chapter), then we can attempt to solve the equation using **trial and improvement**.

In this method, we start with a sensible guess for the solution (a **trial**), and then **improve** this guess, repeating until we find the solution to a **specified degree of accuracy**.

Example

Let us attempt to solve the equation

$$x^3 + 4x - 29 = 0$$

As we have not seen an equation of this type before (and therefore we do not have an algebraic method of solving the equation), let us use the trial and improvement method to solve the equation. We will find the solution correct to the nearest unit and allow ourselves to use a calculator to perform the calculations.

To begin, let us guess that $x = 5$ is the solution to the equation. To see whether this is correct, we need to substitute $x = 5$ into the left-hand side of the equation, and check to see whether we obtain zero.

Buttons to press	Calculator screen
$5 \ x^3 + 4 \times 5 - 29 =$	$5^3 + 4 \times 5 - 29$ 116

The sum gives the answer 116, which is too **too high** (116 is greater than zero). Therefore, we must trial a number that is less than 5. Let us try $x = 3$.

Buttons to press	Calculator screen
$3 \ x^3 + 4 \times 3 - 29 =$	$3^3 + 4 \times 3 - 29$ 10

The answer is still too high (10 is greater than 0), so we must trial an even smaller number, say $x = 2$.

Buttons to press	Calculator screen
$2 \ x^3 + 4 \times 2 - 29 =$	$2^3 + 4 \times 2 - 29$ -13

This time, we see that the answer is **too low** (-13 is less than 0), so we can say that the solution to the equation is between $x = 2$ and $x = 3$. Because we have decided to solve the equation to the nearest unit, we must now decide whether the true solution is closer to $x = 2$ or to $x = 3$. It *appears* that the solution is closer to $x = 3$ (because 10 is closer to zero than -13), however this is not sufficient evidence to **prove** that the solution (to the nearest unit) is $x = 3$. Instead, we must consider the number that is **half way** between 2 and 3 (namely 2.5) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen
$2.5 \ x^3 + 4 \times 2.5 - 29 =$	$2.5^3 + 4 \times 2.5 - 29$ -3.375

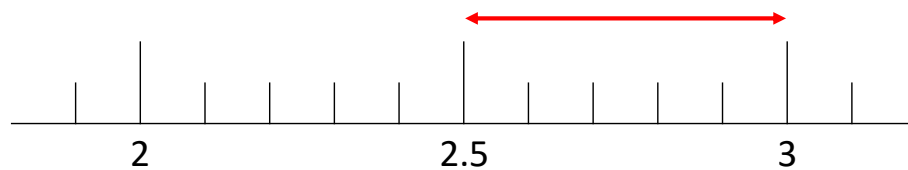
Because -3.375 is less than zero, we can say that the true solution to the equation lies between $x = 2.5$ and $x = 3$. Because every number in this range would round off (to the nearest unit) to be 3, we can now state, without doubt, that the solution to the equation, to the nearest unit, is $x = 3$.

Here's how to set out the solution in the form of a table.

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low

The solution to the equation is between $x = 2.5$ and $x = 3$ so, to the nearest unit, the solution to the equation is $x = 3$.

We can also show the location of the true solution of the equation on a number line.



Exercise 54

The following equations have a solution between $x = 0$ and $x = 10$.

Use the trial and improvement method to find the solution correct to the nearest unit.

- | | | |
|--------------------------|----------------------------|----------------------------|
| (a) $x^3 + 8x - 15 = 0$ | (b) $x^3 - 8x - 15 = 0$ | (c) $x^3 - 2x - 15 = 0$ |
| (d) $x^2 + 7x - 100 = 0$ | (e) $x^2 + 2x - 50 = 0$ | (f) $2x^2 - 11x - 80 = 0$ |
| (g) $2x^3 - 2x - 15 = 0$ | (h) $0.2x^3 - 10x + 4 = 0$ | (i) $x^4 - 60x^2 - 14 = 0$ |

Skill

6

Trial and improvement to one decimal place

Let us return to the previous example of solving the equation $x^3 + 4x - 29 = 0$. We know, from previous work, that the solution to the nearest unit is $x = 3$. Let us now find a more detailed solution, to one decimal place.

From the table on the top of this page, we know that substituting $x = 2.5$ into the equation gives an answer that is too low (-3.375 is less than zero). Let us therefore try to substitute a value that is greater than 2.5, let's say $x = 2.7$.

Buttons to press	Calculator screen
$2 \cdot 7 \times^3 + 4 \times 2 \cdot 7 - 29 =$	$2.7^3 + 4 \times 2.7 - 29$ 1.483

As 1.483 is greater than zero, we must trial a number that is less than 2.7, let's say $x = 2.6$.

Buttons to press	Calculator screen
$2 \cdot 6 \times^3 + 4 \times 2 \cdot 6 - 29 =$	$2.6^3 + 4 \times 2.6 - 29$ -1.024

Now -1.024 is less than zero, so we can say that the solution lies between $x = 2.6$ and $x = 2.7$. As before, we must now decide whether the solution lies closer to $x = 2.6$ or to $x = 2.7$. To do this, we must consider the number that is **half way** between 2.6 and 2.7 (namely 2.65) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen
$2 \cdot 65 \times^3 + 4 \times 2.65 - 29 =$ $\rightarrow$ 0.209625	$2.65^3 + 4 \times 2.65 - 29$ 0.209625

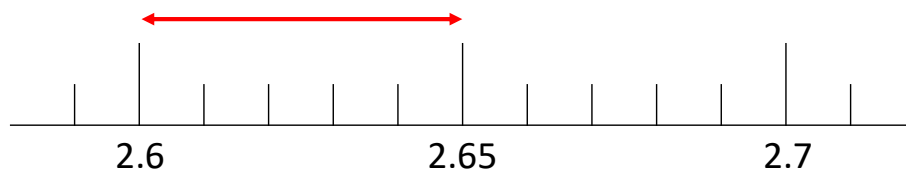
Because 0.209625 is greater than zero, we can say that the true solution to the equation lies between $x = 2.6$ and $x = 2.65$. Because every number in this range would round off (to one decimal place) to be 2.6, we can now state, without doubt, that the solution to the equation, correct to one decimal place, is $x = 2.6$.

Here's how to set out the solution to this question in the form of a table (continuing the previous work).

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low
2.7	1.483	Too high
2.6	-1.024	Too low
2.65	0.209625	Too high

The solution to the equation is between $x = 2.6$ and $x = 2.65$ so, to one decimal place, the solution to the equation is $x = 2.6$.

We can also show the location of the true solution of the equation on a number line.



Exercise 55

The following equations have a solution between $x = 0$ and $x = 10$.

Use the trial and improvement method to find the solution correct to one decimal place.

- | | | |
|--------------------------|--------------------------|----------------------------|
| (a) $x^3 + 8x - 15 = 0$ | (b) $x^3 - 7x - 120 = 0$ | (c) $x^3 - 3x - 93 = 0$ |
| (d) $x^3 + 3x = 15$ | (e) $x^2 + x - 79 = 0$ | (f) $x^2 + 3x = 39$ |
| (g) $2x^3 - 2x - 11 = 0$ | (h) $2x(x + 5) = 110$ | (i) $x^4 - 43x^2 - 25 = 0$ |

Trial and improvement to two decimal places

Let us again return to the example of solving the equation $x^3 + 4x - 29 = 0$. We know, from previous work, that the solution to one decimal place is $x = 2.6$. Let us now find a more detailed solution, to two decimal places.

From the table on the top of this page, we see that substituting $x = 2.65$ into the equation gives an answer that is too high (0.209625 is greater than zero). Let us therefore try to substitute a value that is smaller than 2.65, let's say $x = 2.64$.

Buttons to press	Calculator screen
$2.64^3 + 4 \times 2.64 - 29 =$ $\rightarrow$ -0.040256	$2.64^3 + 4 \times 2.64 - 29$ -0.040256

Now -0.040256 is less than zero, so we can say that the solution lies between $x = 2.64$ and $x = 2.65$. As before, we must now decide whether the solution is closer to $x = 2.64$ or to $x = 2.65$. To do this, we must consider the number that is **half way** between 2.64 and 2.65 (namely 2.645) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen
$2 \cdot 645^3 + 4 \times 2.645 - 29$	$2.645^3 + 4 \times 2.645 - 29$ 0.084486125

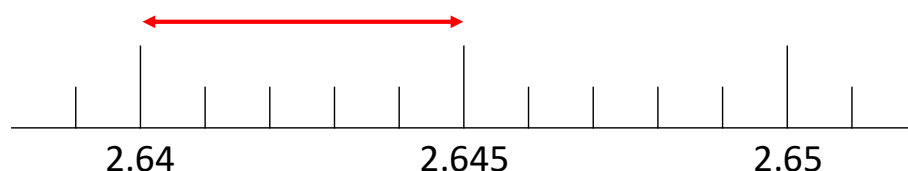
Because 0.084486125 is greater than zero, we can say that the true solution to the equation lies between $x = 2.64$ and $x = 2.645$. Because every number in this range would round off (to two decimal places) to be 2.64, we can now state, without doubt, that the solution to the equation, correct to two decimal places, is $x = 2.64$.

Here's how to set out the solution to this question in the form of a table (continuing the previous work).

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low
2.7	1.483	Too high
2.6	-1.024	Too low
2.65	0.209625	Too high
2.64	-0.040256	Too low
2.645	0.084486125	Too high

The solution to the equation is between $x = 2.64$ and $x = 2.645$ so, to two decimal places, the solution to the equation is $x = 2.64$.

We can also show the location of the true solution of the equation on a number line.



Exercise 56

6

The following equations have a solution between $x = 0$ and $x = 10$.

Use the trial and improvement method to find the solution correct to two decimal places.

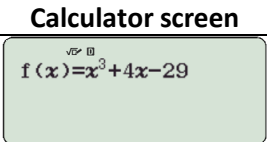
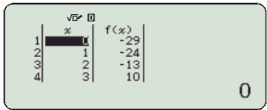
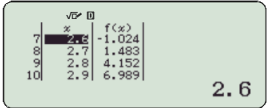
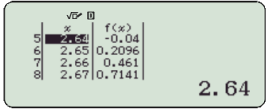
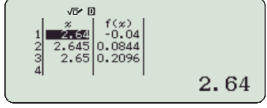
- | | | |
|--------------------------|--------------------------|--------------------------|
| (a) $x^3 + 8x - 15 = 0$ | (b) $x^3 - 5x - 120 = 0$ | (c) $x^3 - 2x - 65 = 0$ |
| (d) $x^3 + 7x = 24$ | (e) $x^2 + x - 43 = 0$ | (f) $x^2 + 6x = 24$ |
| (g) $2x^3 - 3x - 25 = 0$ | (h) $x(24 + x) = 18$ | (i) $x^4 - x^2 - 48 = 0$ |

Using the 'Table Mode'

Some scientific calculators have a 'table mode' that can be used in the solution of trial and improvement questions. The advantage of using the *table mode* is to eliminate the need for any guessing as it can show us the answer to many trials at once.



Let us now use the *table mode* to solve the example from earlier in the chapter, namely to solve the equation $x^3 + 4x - 29 = 0$ correct to two decimal places.

Buttons to press	Calculator screen	Notes
MODE 3 ALPHA) x^3 + 4 ALPHA) - 2 9		Make sure to check that the calculator screen displays the correct equation.
= 0 = 1 0 = =		By starting at 0; ending at 10; and going up in steps of 1, the calculator shows all the trials between $x = 0$ and $x = 10$. We see from the table that trialling $x = 2$ gives an answer that is too low (-13), whilst trialling $x = 3$ gives an answer that is too high (10).
AC = 2 = 3 = 0 . 1 =		Next, we ask the calculator to trial all the numbers between 2 and 3, going up in steps of 0.1. Going down the table, we see that trialling $x = 2.6$ gives an answer that is too low (-1.024), whilst trialling $x = 2.7$ gives an answer that is too high (1.483).
AC = 2 . 6 = 2 . 7 = 0 . 0 1 =		Next, we ask the calculator to trial all the numbers between 2.6 and 2.7, going up in steps of 0.01. Going down the table, we see that trialling $x = 2.64$ gives an answer that is too low (-0.04), whilst trialling $x = 2.65$ gives an answer that is too high (0.2096).
AC = 2 . 6 4 = 2 . 6 5 = 0 . 0 0 5 =		In order to decide whether the true solution is closer to $x = 2.64$ or to $x = 2.65$, we ask the calculator to trial 2.64, 2.645 and 2.65. We see from the screen that trialling $x = 2.645$ gives an answer that is too high (0.0844), so the solution to two decimal places is $x = 2.64$.

Using the calculator's *table mode*, this is what we would have to write as the solution to this question.

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
2	-13	Too low
3	10	Too high
2.6	-1.024	Too low
2.7	1.483	Too high
2.64	-0.040256	Too low
2.65	0.209625	Too high
2.645	0.084486125	Too high

The solution to the equation lies between $x = 2.64$ and $x = 2.645$ so, to two decimal places, the solution to the equation is $x = 2.64$.

Exercise 57

6

The following equations have a solution between $x = 0$ and $x = 10$.

Using the trial and improvement method, and the *table mode* on your calculator, find the solution to the required degree of accuracy.

- (a) $x^2 - 4x - 3 = 0$ to the nearest unit
- (b) $x^3 + 6x - 31 = 0$ to one decimal place
- (c) $x^3 + 8x - 15 = 0$ to two decimal places
- (d) $x^2 - \sqrt{9x} - 1 = 0$ to the nearest whole number
- (e) $x^5 - 63x - 17 = 0$ to one decimal place
- (f) $x^3 - 6x - 19 = 0$ to two decimal places

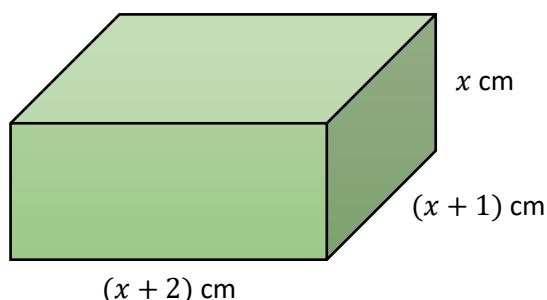


Exercise 58

Applying

6

(a) The length of a cuboid is $(x + 2)$ cm, its width is $(x + 1)$ cm and its height is x cm. The volume of the cuboid is 140 cm^3 .



Calculate the length of the cuboid correct to one decimal place.

(b) Find the whole number that satisfies the equation $x^3 + 4x = 240$.

(c) Huw enters a number into his calculator, before adding the cube of the same number. The calculator screen shows 190.893. Find the number that Huw initially entered into his calculator. Show your method.

(d) Here is a graph of the equation $y = x^3 - 7x^2 + 4x + 3$.

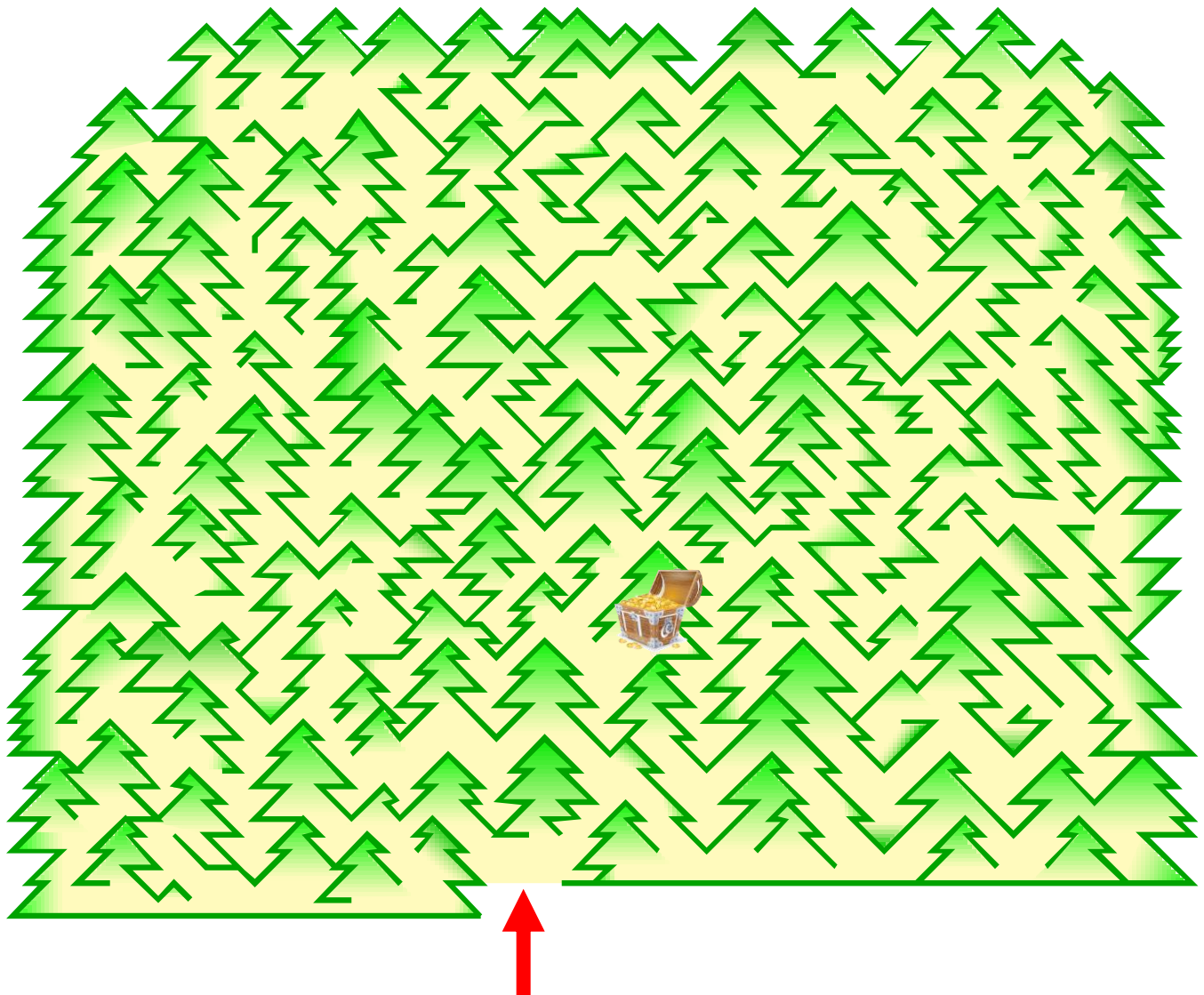


Use the trial and improvement method to find, correct to one decimal place, each of the solutions to the equation $x^3 - 7x^2 + 4x + 3 = 0$.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Puzzle: Plot a path through the forest to find the treasure.



Developing

Algebra

Evaluation

Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the
test:Correct
in the
test?

I know how to find the nth term of a linear sequence , e.g. 9, 13, 17, 21, 25,			2	
I know how to use the formula for the n th term of a sequence to find terms from that sequence.			3	
I know how to find the position of a specific term in a sequence using the formula for the n th term.			4	
I can find the n th term of a sequence of patterns .			5	
I can expand a single bracket , e.g. $6(x - 2)$ or $4y(6y - 8)$.			1	
I can expand an expression where more than two terms appear in the bracket, e.g. $7(3x + y - 4z)$.				
I can expand and simplify an expression that contains two brackets , e.g. $3(x + 2) + 7(x - 3)$.			1	
I can use the acronym FOIL to expand a double bracket , e.g. $(x + 5)(x - 2)$.			10	
I can use the acronym FOIL to expand a single bracket that is squared , e.g. $(2z + 11)^2$.			10	
I can solve equations that contain a single bracket , e.g. $8(x + 4) = 56$.			6	
I can solve equations that contain two brackets , e.g. $8(x + 3) = 6(x - 2)$.			6	
I can solve equations where a double bracket is equal to zero , e.g. $(3x + 6)(x - 5) = 0$.			6	
I can solve equations that contain a single fraction , e.g. $\frac{x+5}{4} = 12$.			6	
I can solve equations containing more than one fraction , e.g. $\frac{4x+1}{3} = \frac{2x-4}{6}$.			7	
I can use the trial and improvement method to solve equations correct to the nearest unit ; to one decimal place ; or to two decimal places .			9	
I can use the trial and improvement method to solve equations , e.g. find the whole number that satisfies the equation $x^3 - 2x = 115$.			8	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

9

Data Handling

and Statistics 3

Name:

Contents

Chapter	Mathematics	Page Number
The Median	Calculating the median. Calculating the median from a table. Comparing two distributions.	3
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The Interquartile Range	Calculating the interquartile range of a set of data.	13
Cumulative Frequency Diagrams	Cumulative frequency tables. Drawing a cumulative frequency diagram. Estimating the median and the interquartile range.	13
Scatter Diagrams	Drawing a scatter diagram. Types of correlation. Line of best fit.	23
Probability as a Fraction	Writing the probability of an event as a fraction. The probability that an event will not happen. Sample space diagrams. Expected frequency.	27



The Median

The **median** is one of the three averages¹ that we study at school.

Year 7	Year 8	Year 9	Year 10
Mean	Mode	Median	Comparing Averages



In order to find the **median** of a set of data:

- (1) Order the data from least to greatest.
- (2) Find the number that is in the middle. (If there are two numbers in the middle, find the **mean** of these two numbers.)

Example

The owner of a café records the number of customers in the café each day for a week.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
28	21	29	16	32	43	24

Use the median to calculate the average number of customers in the café each day.

Answer: To begin, we order the data from least to greatest.

16, 21, 24, **28**, 29, 32, 43

We see that 28 is the number in the middle, so 28 is the median number of customers in the café each day.



Example

Jessica asked her friends how much weekly pocket money they received. Here are her results.

£2, £3, £1, £4, £5, £2, £2, £4.

What is the median amount of weekly pocket money?

Answer: To begin, we order the data from least to greatest.

£1, £2, £2, **£2, £3**, £4, £4, £5.

There are two numbers in the middle, £2 and £3.

The median is the mean of these two numbers.

Add the numbers: $£2 + £3 = £5$.

Divide by two: $£5 \div 2 = £2.50$.

So the median amount of weekly pocket money is £2.50.



Exercise 1

What is the median of the following numbers?

(a) 2, 5, 9, 14, 19

(c) 7, 2, 4, 1, 9

(e) 2, 4, 8, 10

(g) 18, 3, 14, 20, 9, 15

(b) 24, 24, 25, 26, 27, 28, 29

(d) 13, 15, 12, 18, 12, 15, 19

(f) 30, 31, 34, 35, 37, 37, 37, 38, 39, 39

(h) 41, 28, 39, 15, 23, 32, 49, 30

Skill

5

£2.50 is the number half way between £2 and £3.



¹ An average is a number that attempts to represent the middle or typical value of a set of data.

Exercise 2**Applying**

5

Here is some information about five children.

**Dylan**

Height: 135 cm
Weight: 43 kg
Shoe size: 3

Meinir

Height: 136 cm
Weight: 40 kg
Shoe size: 4

Cai

Height: 140 cm
Weight: 41 kg
Shoe size: 5

Gwilym

Height: 147 cm
Weight: 49 kg
Shoe size: 4

Hannah

Height: 149 cm
Weight: 47 kg
Shoe size: 3

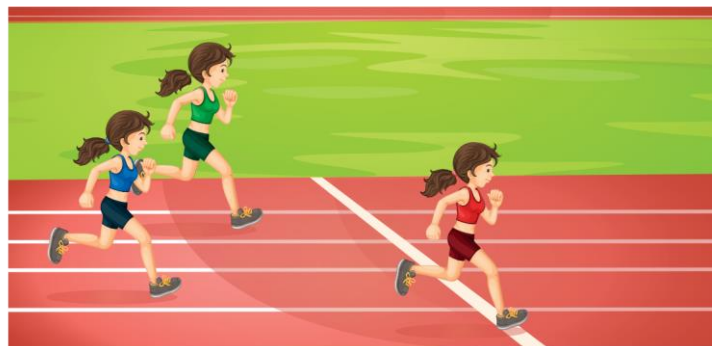
- What is the median height for these 5 children?
- Which children are shorter than the median height?
- What is the median weight for these 5 children?
- Which children are heavier than the median weight?
- What is the median shoe size for these 5 children?
- Which children have a shoe size that is greater than the median value?

**Exercise 3**

Here are the times, in seconds, that the girls in 9R took to run a 100 m race.

14.1, 13.3, 14.5, 16.6, 14.0, 14.7, 13.9,
16.1, 17.8, 14.5, 13.7, 14.6, 18.3, 15.7.

- Find the median time taken to run the race.
- How many girls ran faster than the median time?

**Exercise 4**

During November, Meira recorded the temperature each day at midnight in Abergele. Here are her results.

6°C	5°C	2°C	-1°C	-3°C	2°C	2°C	5°C	7°C	10°C
8°C	1°C	-5°C	-2°C	-2°C	0°C	1°C	4°C	5°C	7°C
4°C	7°C	6°C	3°C	0°C	4°C	2°C	8°C	6°C	5°C

- What was the median temperature at midnight in Abergele during November?
- If the 10°C measurement had been -10°C instead, what would the median temperature have been?
- If the temperature had been 2°C warmer **each day**, what would the median temperature have been?

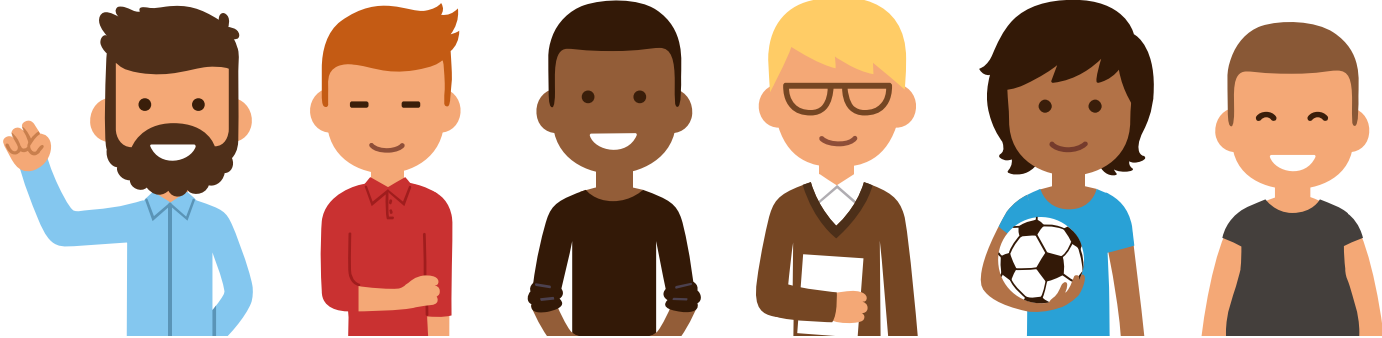


Exercise 5

5

Here is some information about 6 men.

John	Eifion	Steven	Bob	Harry	James
Age: 40	Age: 25	Age: 30	Age: 44	Age: 19	Age: 34
Height: 175 cm	Height: 172 cm	Height: 180 cm	Height: 168 cm	Height: 170 cm	Height: 168 cm
Waist: 34"	Waist: 32"	Waist: 34"	Waist: 36"	Waist: 32"	Waist: 38"



- What is the median age of these six men?
- Who is younger than the median age?
- What is the median height of these six men?
- Who is taller than the median height?
- What is the median waist measurement of these six men?
- Who has a waist measurement equal to the median value?
- A new man named Andrew joins the group above.
- What age must Andrew be in order for the median age of the seven men to be **greater** than the median age of the original six men?
- What height must Andrew be in order for the median height of the seven men to be **less** than the median height of the original six men?
- What waist measurement must Andrew have in order for the median waist measurement of the seven men to be **equal** to the median waist measurement of the original six men?



Calculating the median from a table

Sometimes, when we have a lot of data, it is easier to present the data in the form of a table.

Example

The following table shows the shoe size of the learners of 9E.

Shoe size	Number of learners
5	3
6	4
7	9
8	6
9	4
10	2

One way of calculating the median shoe size for 9E would be to list all the data items using the information given in the table (the three first learners have a shoe size of 5, the next four have a shoe size of 6, and so on).

5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10

From the above list, we see that there are two 7's in the middle of the data, so the median shoe size is 7.

With $3 + 4 + 9 + 6 + 4 + 2 = 28$ data items, it is manageable to list all of the data items in order to find the median, but what would we do with 280 data items? In this case, it would be laborious to list all of the data items, so we need to learn how to find the median straight from the table. The first step is to learn how to quickly find which data items are located in the **middle** of a set of data.

Exercise 6

Complete the following table.

Skill

5

Number of data items	Diagram to represent the data	Which data items are in the middle?
1	•	1
2	• •	1, 2
3	• • •	2
4	• • • •	2, 3
5	• • • • •	3
6	• • • • • •	
7	• • • • • • •	
8	• • • • • • • •	
9	• • • • • • • • •	
10	• • • • • • • • • •	

Exercise 7

Complete the following sentences.

- (a) If you have an odd number of data items, then there is _____ data item in the middle.
 (b) If you have an even number of data items, then there are _____ data items in the middle.



Exercise 8

Complete the following sentences.

- (a) If you have 10 data items, then the data items in the middle are _____.
 (b) If you have 11 data items, then the data items in the middle are _____.
 (c) If you have 12 data items, then the data items in the middle are _____.
 (d) If you have 20 data items, then the data items in the middle are _____.
 (e) If you have 25 data items, then the data items in the middle are _____.

Exercise 9

Complete the paragraphs on the right.

If you have an odd number of data items, then to obtain the data item in the middle

If you have an even number of data items, then to obtain the data items in the middle

Next, we must learn how to locate a specific data item in a set of data.
To do this, let us again consider the data about the shoe size of the learners of 9E.

Shoe size	Number of learners
5	3
6	4
7	9
8	6
9	4
10	2

There will be further information on cumulative frequency on page 13.



Example

What is the 20th data item in the above data?

Answer: Let us add another column to the above table, a **cumulative frequency** column. This column records how many learners we have encountered so far.

Shoe size	Number of learners	Cumulative Frequency
5	3	3
6	4	$3 + 4 = 7$
7	9	$7 + 9 = 16$
8	6	$16 + 6 = 22$
9	4	$22 + 4 = 26$
10	2	$26 + 2 = 28$

3 is less than 20, so the 20th data item is not 5.
7 is less than 20, so the 20th data item is not 6.
16 is less than 20, so the 20th data item is not 7.
22 is greater than 20, so the 20th data item is 8.

Conclusion: The 20th data item is **8**.

Exercise 10

For the above data about 9E's shoe size, find the value of the following data items.

- (a) 2nd (b) 10th (c) 13th (d) 19th (e) 21st (f) 25th (g) 27th

Exercise 11

In a survey, 40 people were asked how many times they had visited a supermarket during the past two weeks.

Number of visits	Frequency
0	3
1	9
2	16
3	7
4	4
5	1



Find the value of the following data items.

- (a) 1st (b) 5th (c) 12th (d) 18th (e) 30th (f) 37th (g) 40th

Example

Let us now return to the problem of finding 9E's median shoe size, using the table at the top of this page. We see, from the table, that there are $3 + 4 + 9 + 6 + 4 + 2 = 28$ learners in 9E. This is an even number, so there are two numbers in the middle of the data, corresponding to data items 14 and 15. ($28 \div 2 = 14$, and $14 + 1 = 15$.) From the table on the middle of this page, we see that 7 is the value of the 14th and 15th data items. **So, 7 is the median.**

Exercise 12

5

Calculate the median of the following sets of data.

(a) 9R's shoe size.

Shoe size	Number of learners
5	2
6	6
7	7
8	8
9	3
10	1

(b) Number of Smarties in 100 tubes.

Number of Smarties in a tube	Number of tubes
35	14
36	24
37	26
38	20
39	16

(c) The number of letters Siwan receives each day for a month.

Number of letters	Frequency
0	15
1	9
2	4
3	2
4	1

(d) How many times 40 people have been to the hairdressers during the past three months.

Number of times	Frequency
0	1
1	19
2	13
3	5
4	2

(e) How many books a group of students have read during the past six months.

Number of books	Frequency
0	22
1	15
2	17
3	13
4	8

(f) Number of M&Ms in a batch of big packets.

Number of M&Ms in a big packet	Number of big packets
96	2
97	18
98	16
99	17
100	6

(g) How many pieces of homework Sam receives, each day, over three weeks.

Number of pieces of homework	Frequency
0	4
1	5
2	3
3	2
4	1

(h) Number of pages in a ream of A4 printing paper.

Number of pages	Number of reams
498	16
499	28
500	31
501	8
502	3

(i) Number of selfies Sophie puts on the internet each day during August.

Number of selfies	Frequency
0	5
1	7
2	10
3	6
4	3

Exercise 13

This exercise refers to the data above from Exercise 12.

(a) How many members of 9R have a shoe size greater than the median value?

(b) What percentage of Smarties tubes contain less than the median amount of Smarties?

(c) How many people went to the hairdressers less than the median amount?

(d) The manufacturer of M&Ms claims that there are 98 sweets in each big packet. Does your median support this claim?

(e) The manufacturer of the A4 printing paper claims that there are 500 sheets in each ream. Does your median support this claim?

(f) Sophie tells her mother that she takes "around 1 selfie per day". Knowing the median of the data, what would you tell Sophie's mother?



5

A cartoon illustration of a blue bowling ball with three finger holes, hitting a set of bowling pins. The pins are white with red stripes. A large, bright yellow and orange explosion with red rays emanates from the point of impact, signifying a successful strike.

Megan	7	9	2	10	3	7	5	2	9	3
Miriam	10	2	6	0	7	9	3	1	7	5

(b) In a survey, a group of boys and girls were asked how many hours they spent on the internet during one week.

Boys	20	17	35	14	18	24	19	38	15
Girls	32	14	8	27	33	17	19	25	32

(c) Here is the number of goals scored by two football teams at the beginning of a season.

Man Utd	2	1	0	3	2	4	1	2
Man City	3	0	2	5	2	3	4	3

Use the median to compare the performance of the two football teams at the beginning of the season.

(d) The heights of 12 boys in a class are measured, in centimetres. Here are the results.

118 135 127 129 131 123 130 128 125 129 131 119

(i) What is the median height of these boys?

The heights of 12 boys from another class are measured, again in centimetres. Here are the results.

137 160 143 151 154 146 148 153 139 147 153 136

(ii) What is the median height for the boys in this new class?

(iii) Use your answers to (i) and (ii) to decide which class is the primary school class, and which class is the secondary school class.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

The Median Class

Given data that is grouped into different classes, is it possible to calculate the **median** of the original data?

Example

The following table shows the number of music albums the learners in 10R have bought during November.

Number of albums	Frequency
0–4	11
5–9	8
10–14	5
15–19	1



What is the **median number** of music albums 10R have bought during November? Without access to the original data (*before* it is grouped), it is **impossible** to state what the **exact** median is, as the frequency table does not give us the required information. For example, the table notes that one person has bought between 15 and 19 music albums, but how many albums is this exactly? It could be 15, but it could also be 16, 17, 18 or 19. Without this information, we cannot calculate the exact median. The best we can do is to find which **class** the median belongs to, following the steps below.

(1) Find the **total number** of data items.

$$11 + 8 + 5 + 1 = 25.$$

How to do this was discussed in a previous chapter.

(2) Find which data item(s) appear(s) in the middle of the data, when the data is written in order. Here, as 25 is an odd number, there is **one** data item in the middle of the data, data item 13. (Method: $25 + 1 = 26$, and $26 \div 2 = 13$.)

(3) Find which class the data item(s) in the middle of the data belong(s) to.

- If there is **one** item in the middle of the data, the **median class** will be the class that this data item belongs to.
- If there are **two** items in the middle of the data, and these two data items appear in the **same class**, then the **median class** will be the class that both of these data items belong to.
- If there are **two** items in the middle of the data, and these two data items appear in **different classes**, then the **estimate of the median** will be the mean of the two limits of the classes. For example, if the two classes are 10–14 and 15–19, then the estimate of the median will be the mean of 14 and 15, namely 14.5.

For the music albums bought by 10R, there is one data item in the middle of the data, so the median class will be the class that the 13th data item belongs to.

Number of albums	Frequency	Cumulative Frequency
0–4	11	11
5–9	8	$11 + 8 = 19$
10–14	5	$19 + 5 = 24$
15–19	1	$24 + 1 = 25$

11 is less than 13, so the 13th data item does not belong to the class 0–4.

19 is greater than 13, so the 13th data item belongs to the class 5–9.

Answer: The median class is the class 5–9.

(We can summarise from this that the actual median is either 5, 6, 7, 8 or 9.)



Exercise 15**Skill**

7

Find the median class (or the estimate of the median) for the following data sets.

(a)

Number of cars sold in a day	Number of days
0–4	18
5–9	11
10–14	6
15–19	4
20–24	1

(b)

Number of flowers per m ²	Number of m ²
0–14	43
15–29	21
30–44	13
45–59	2
60–74	1

(c)

Height of a horse chestnut tree in metres	Frequency
0–3	13
4–7	17
8–11	9
12–15	4
16–19	5
20–23	0
24–27	2

(d)

Number of books sold in a week	Frequency
100–124	1
125–149	9
150–174	8
175–199	5
200–224	2

(e)

Number of customers in a café	Frequency
70–74	3
75–79	21
80–84	30
85–89	28
90–94	14
95–99	4

(f)

Number of sweets in a jar	Frequency
300–324	4
325–349	15
350–374	27
375–399	63
400–424	74
425–449	32
450–474	11
475–499	2

**Exercise 16**

Find the modal class for all the data sets in Exercise 15.

Revision

Skill

7

- (e)
- | Length of a phone call, t , in minutes | Frequency |
|--|-----------|
| $0 \leq t < 5$ | 32 |
| $5 \leq t < 10$ | 42 |
| $10 \leq t < 15$ | 24 |
| $15 \leq t < 20$ | 12 |
| $20 \leq t < 25$ | 7 |
| $25 \leq t < 30$ | 2 |



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

The Interquartile Range

The **interquartile range** is one of the two **measures of spread**² that we study at school.

Range	Interquartile Range
Year 7	Year 9

Exercise 18

Calculate the range of the following data sets.

- (a) 14, 19, 21, 11, 24, 18
- (b) 7, 2, 8, 1, 3, 6, 4, 2, 6, 3
- (c) 28, 29, 43, 25, 21, 48, 34, 32, 27, 32
- (d) -5, 4, -10, 7, 3, 0, 1, -6
- (e) 8.3, 3.5, 7.9, 2.8, 7.6, 4.9, 6.2, 2.9, 8.2

Revision

5



The advantage of the range is that it is easy to calculate. Its disadvantage is that it is affected greatly by **outliers**, which are very high or very low values that do not fit the pattern of the rest of the data set. For example, consider the following data, which shows the number of passengers on a bus service over a period of 10 days.

42, 37, 40, 51, 38, 43, 42, 36, 28, 38

The range of the data is $51 - 28 = 23$, which shows that there is a difference of 23 between the least number of passengers and the greatest number of passengers.

Imagine now that a snowstorm occurred during the following day, so that the bus service was unable to run. The new data set (considering that there were no passengers during the next day) is

42, 37, 40, 51, 38, 43, 42, 36, 28, 38, 0

The range for the 11 days is $51 - 0 = 51$. This range is not a fair reflection of the whole data set, as there was only one day with no passengers. This **outlier** has had a large effect on the range. This will not be true of the interquartile range.

The Interquartile Range

Whilst the range subtracts the least value from the greatest value, the interquartile range subtracts the **lower quartile** from the **upper quartile**.

$$\text{Interquartile Range} = \text{Upper Quartile} - \text{Lower Quartile}$$

What are the quartiles?

The quartiles split a data set into four parts. Here's how to calculate the lower and upper quartiles.

- 1) Order the data from least to greatest.
- 2) If there are an **even number** of data items:
 - The lower quartile is the median of the **first half** of the data.
 - The upper quartile is the median of the **second half** of the data.

- If there are an **odd number** of data items:
- The lower quartile is the median of all the data items **before the one in the middle**.
 - The upper quartile is the median of all the data items **after the one in the middle**.

Note the spelling:
quartile, not quarter.

² A measure of spread attempts to represent how close (or how far) data items are to each other.

Example

(a) Consider again the data about the bus passengers from the previous page. Originally, we had 10 data items:

42, 37, 40, 51, 38, 43, 42, 36, 28, 38

Ordering this data from least to greatest, we obtain the following list.

28, 36, 37, 38, 38, 40, 42, 42, 43, 51

Because 10 is an even number, the lower quartile is the median of the **first 5 data items**, and the upper quartile is the median of the **final 5 data items**. We can illustrate this by colouring the data items:

28, 36, 37, 38, 38, 40, 42, 42, 43, 51

The lower quartile is the median of the **red** data items, namely 37. The upper quartile is the median of the **blue** data items, namely 42. It follows that the interquartile range is $42 - 37 = 5$.

(b) After the day with the snow, we had 11 data items in total:

42, 37, 40, 51, 38, 43, 42, 36, 28, 38, 0

Ordering this data from least to greatest, we obtain the following list.

0, 28, 36, 37, 38, 38, 40, 42, 42, 43, 51

Because 11 is an odd number, the lower quartile will be the median of all the data items **before the one in the middle**, and the upper quartile will be the median of all the data items **after the one in the middle**. We can illustrate this by colouring the data items (the **purple** data item is the one in the middle):

0, 28, 36, 37, 38, 38, 40, 42, 42, 43, 51

The lower quartile is the median of the **red** data items, 36. The upper quartile is the median of the **blue** data items, 42. It follows that the interquartile range is $42 - 36 = 6$. Note that this interquartile range (6) is similar to the interquartile range (5) for the previous 10 days. The outlier (0) has not had such an effect on the interquartile range as it had on the range (the range changed from 23 to 51).

Exercise 19

Calculate the interquartile range of the following data sets.

Skill

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- (a) 2, 4, 6, 9, 10, 13, 14, 18, 18, 19
- (b) 14, 15, 15, 18, 20, 20, 21, 24, 28, 29, 32
- (c) 9, 2, 5, 1, 8, 5, 8, 3
- (d) 43, 48, 41, 45, 49, 44, 45, 48, 40
- (e) 12, 8, 19, 5, 9, 10, 18, 4, 8, 13, 19, 7, 11, 18, 6, 17
- (f) 32, 74, 29, 27, 73, 98, 17, 39, 24, 51, 39, 60, 33
- (g) 3, 2, 4, 3, 1, 3, 2, 4, 2, 1, 3, 2, 4, 3, 1, 2, 3, 2, 3, 2, 4, 3, 2, 3

**Challenge!**

The interquartile range of the following data set is 10. What are the possible values of the missing data item?

23, 17, 32, 19, 24, 28, ?



Note

Mathematicians and software packages vary regarding how they calculate the interquartile range – there isn't an agreement on a standard method! This means that different methods of calculating the interquartile range can be found on the internet, and seen in software packages such as Microsoft Excel.

Challenge!

Investigate the two methods Microsoft Excel uses to calculate the lower and upper quartiles. (You will need to use the formulae `QUARTILE.EXC` and `QUARTILE.INC`.)

Exercise 20

Cerys and Rebecca go together to the gym once a week.

Over the past 8 weeks, here is how far Cerys has run (in miles) on the treadmill.

2.3 2.5 3.3 3.0 3.4 3.6 4.0 4.3

Over the same period, here is how far Rebecca has run (in miles) on a similar treadmill.

2.8 3.1 3.2 2.9 3.1 0.4 2.8 3.2

- Calculate Cerys' range and Rebecca's range.
- Calculate Cerys' interquartile range and Rebecca's interquartile range.
- According to the range, who was most consistent whilst running?
- According to the interquartile range, who was most consistent whilst running?
- Knowing that Rebecca suffered from a heavy cold during one of the sessions at the gym, which measure (the range or the interquartile range) is most suitable for comparing who is most consistent whilst running?

Challenge!

Investigate using the internet what makes a data item an outlier.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Applying**5**

Cumulative Frequency Diagrams

Cumulative Frequency Table

As we have seen already in this workbook, it is sometimes useful to consider the **cumulative frequency** of a set of data, where we record the total frequency as we go through the data set. For example, consider the following data that shows the thickness of books standing on a shelf in a library.



Thickness of book (t mm)	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	14
$20 < t \leq 30$	35
$30 < t \leq 40$	8

We can add a **cumulative frequency** column to the above table, a column that records the total of all the data items encountered so far. (To calculate the next cumulative frequency, we add the current cumulative frequency to the next frequency).

Thickness of book (t mm)	Frequency	Cumulative Frequency
$0 < t \leq 10$	3	3
$10 < t \leq 20$	14	17
$20 < t \leq 30$	35	52
$30 < t \leq 40$	8	60

$$(3 + 14 = 17)$$

$$(17 + 35 = 52)$$

$$(52 + 8 = 60)$$



The cumulative frequency column also appears in a **cumulative frequency table**, where we note the **class interval** that corresponds to each cumulative frequency.

Thickness of book (t mm)	Cumulative Frequency
$t \leq 0$	0
$t \leq 10$	3
$t \leq 20$	17
$t \leq 30$	52
$t \leq 40$	60

The first row is required in order to show where the data starts.

Note the connection between the class intervals of the original table (e.g. $10 < t \leq 20$) and the class intervals of the cumulative frequency table (e.g. $t \leq 20$).

Exercise 21

Draw a cumulative frequency table for each of the following data sets.

(a) The thickness of books in a library. (b) The height of 60 pupils.

(c) The noise of 60 electrical items.

Thickness (t mm)	Frequency
$0 < t \leq 10$	4
$10 < t \leq 20$	14
$20 < t \leq 30$	36
$30 < t \leq 40$	21
$40 < t \leq 50$	5

Height (h cm)	Frequency
$148 < h \leq 152$	2
$152 < h \leq 156$	6
$156 < h \leq 160$	17
$160 < h \leq 164$	22
$164 < h \leq 168$	10
$168 < h \leq 172$	3

Noise (n db)	Frequency
$15 \leq n < 20$	4
$20 \leq n < 25$	12
$25 \leq n < 30$	15
$30 \leq n < 35$	6
$35 \leq n < 40$	8
$40 \leq n < 45$	3
$45 \leq n < 50$	12

Skill

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Example

When forming a cumulative frequency table, we must take care with discrete data that is grouped, or with continuous data that has been rounded off.

(a) The age of members of a netball club.

Age (years)	Frequency		Age (a years)	Cumulative Frequency
11 – 15	12	→	$a < 11$	0
16 – 20	16		$a < 16$	12
21 – 25	11		$a < 21$	28
26 – 30	8		$a < 26$	39
31 – 35	3		$a < 31$	47
			$a < 36$	50



(b) The length of the electrical cord of 100 televisions.

Length, to the nearest cm	Frequency		Length (l cm)	Cumulative Frequency
50 – 69	5	→	$l < 49.5$	0
70 – 89	14		$l < 69.5$	5
90 – 109	28		$l < 89.5$	19
110 – 129	41		$l < 109.5$	47
130 – 149	12		$l < 129.5$	88
			$l < 149.5$	100

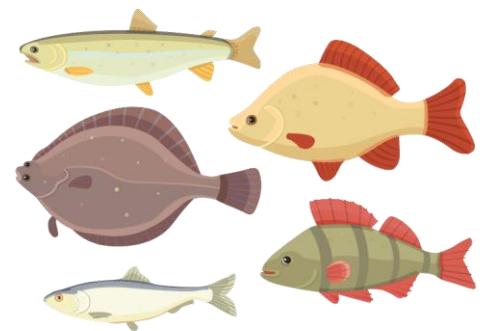
We need to consider which numbers **round off** to give e.g. a length between 70 cm and 89 cm.

Exercise 22

Draw a cumulative frequency table for each of the following data sets.

(a) The age of members of a golf club. (b) The length of 80 fish.

Age (years)	Frequency		Length, to the nearest cm	Frequency
11 – 20	17	→	6 – 10	27
21 – 30	29		11 – 15	21
31 – 40	38		16 – 20	18
41 – 50	25		21 – 25	10
51 – 60	11		26 – 30	4



(c) Cost of a cup of tea in 200 cafes. (d) The height of 60 thirteen-year-old girls.

Cost (£)	Frequency		Height, to the nearest cm	Frequency
£0.40 – £0.79	59	→	141 – 150	12
£0.80 – £1.19	94		151 – 160	29
£1.20 – £1.59	38		161 – 170	17
£1.60 – £1.99	7		171 – 180	2
£2.00 – £2.39	2			



Drawing Cumulative Frequency Diagrams

It is possible to draw a cumulative frequency diagram for the following types of data.

- Discrete quantitative data that has been grouped.
- Continuous quantitative data.



Example

Let us again consider the data about the thickness of books on a shelf in a library, from earlier in this chapter.

Frequency Table

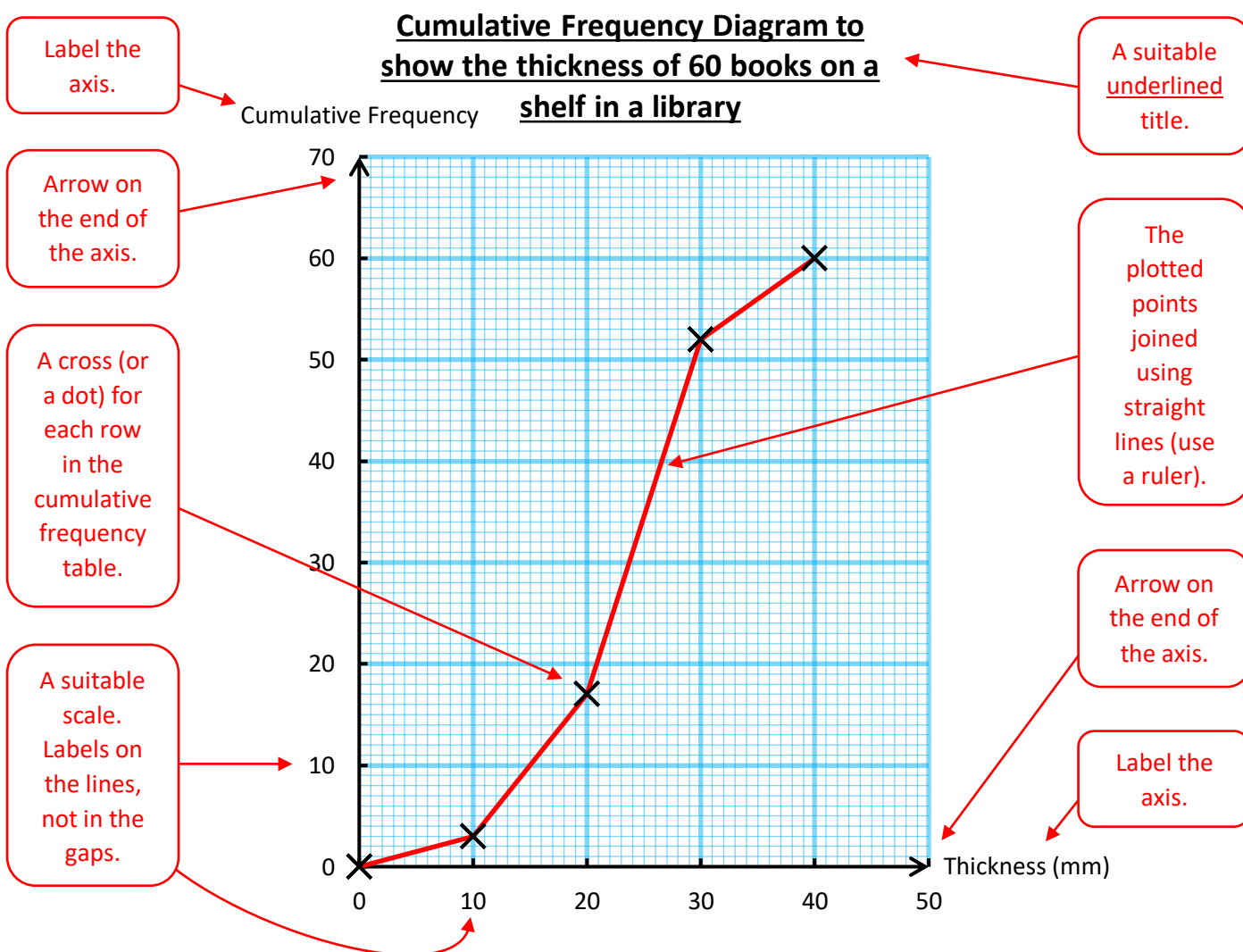
Thickness of book (t mm)	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	14
$20 < t \leq 30$	35
$30 < t \leq 40$	8

Cumulative Frequency Table

Thickness of book (t mm)	Cumulative Frequency
$t \leq 0$	0
$t \leq 10$	3
$t \leq 20$	17
$t \leq 30$	52
$t \leq 40$	60



Here is the cumulative frequency diagram for the above data.



Note: Some textbooks connect the points in a cumulative frequency diagram with a smooth curve, not straight lines.

Exercise 23

8

The following data shows the distance travelled by 200 pupils to school each day. Use the templates to draw a cumulative frequency diagram for the data.

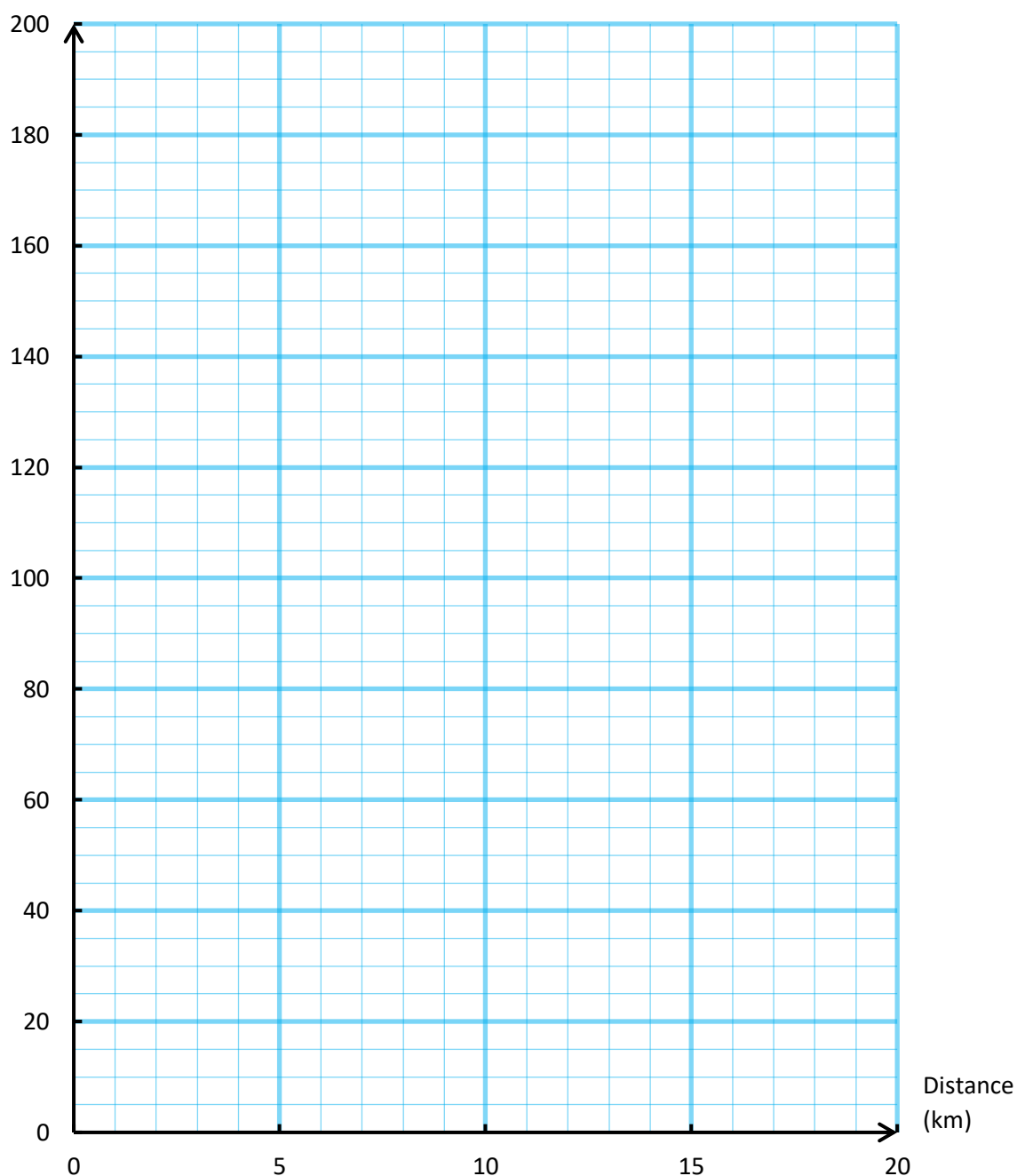
Frequency Table

Distance travelled to school (d km)	Frequency
$0 < d \leq 5$	64
$5 < d \leq 10$	79
$10 < d \leq 15$	45
$15 < d \leq 20$	12

Cumulative Frequency Table

Distance travelled to school (d km)	Cumulative Frequency
$d \leq 0$	
$d \leq 5$	
$d \leq 10$	
$d \leq 15$	
$d \leq 20$	

Cumulative Frequency



Exercise 24

Draw a cumulative frequency diagram for the three data sets in Exercise 21.

Exercise 25

Draw a cumulative frequency diagram for the four data sets in Exercise 22.

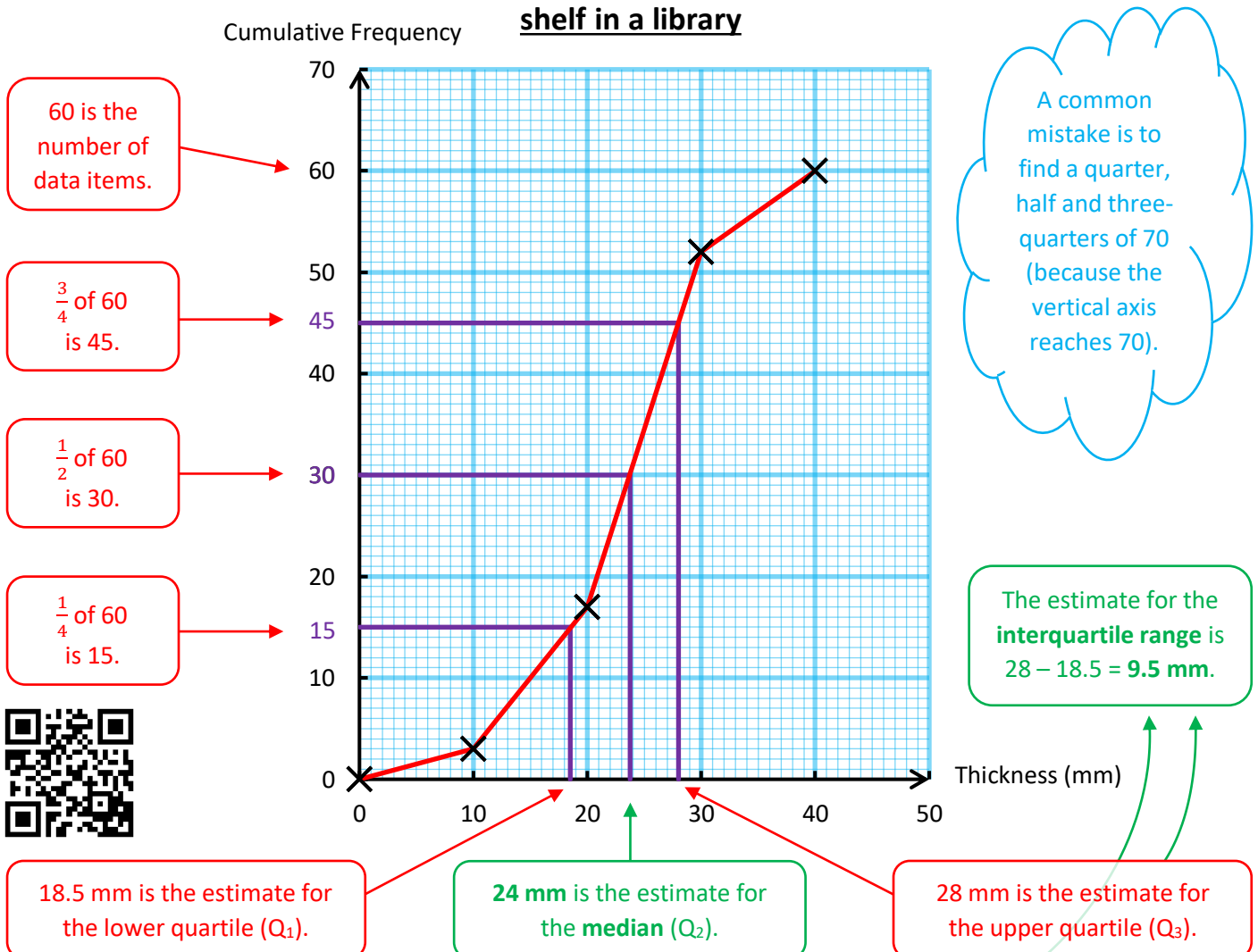
Estimating the median and interquartile range from a cumulative frequency diagram

If we have a large data set (50 items or more), then it is possible to use a cumulative frequency diagram to estimate the median or interquartile range.

- 1) On the vertical axis (cumulative frequency), mark a **quarter**, a **half** and **three-quarters** the number of data items. For example, if there are 500 data items, then mark 125 (a quarter of 500), 250 (a half of 500) and 375 (three-quarters of 500).
- 2) Draw horizontal lines until you reach the line of the cumulative frequency diagram. Then draw vertical lines towards the horizontal axis. The values at the bottom will be the estimates for the **lower quartile** (Q_1), the **median** (Q_2) and the **upper quartile** (Q_3).

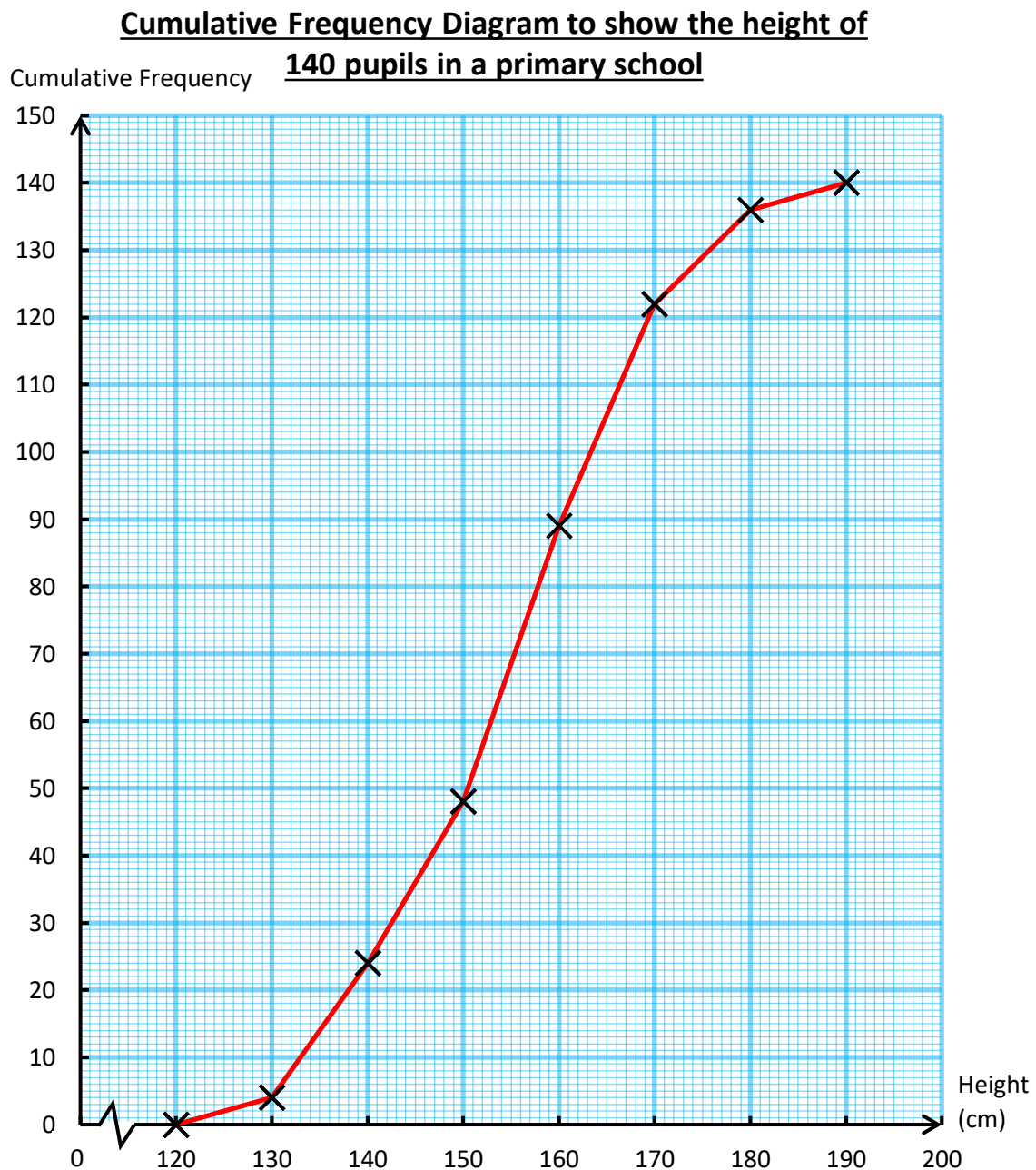
For example, here's how to estimate the median and the interquartile range for the data about books in a library.

**Cumulative Frequency Diagram to
show the thickness of 60 books on a
shelf in a library**



Exercise 26

Here is a cumulative frequency diagram showing the height of 140 pupils in a primary school. Use the diagram to find an estimate for the median and the interquartile range of the height of the pupils.

**Exercise 27**

Use the cumulative frequency diagram from Exercise 23 to estimate the median and the interquartile range of the distance travelled by the 200 pupils to school each day.

Exercise 28

Use the cumulative frequency diagrams from Exercise 24 to estimate the median and the interquartile range of the three sets of data.

Exercise 29

Use the cumulative frequency diagrams from Exercise 25 to estimate the median and the interquartile range of the four sets of data.

Exercise 30 (Revision)

The following data shows the age of people watching a film in a cinema one night.

Applying

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Age (years)	Frequency
15 – 19	14
20 – 24	23
25 – 29	34
30 – 34	21
35 – 39	6
40 – 44	2

- Which age classification does the data suggest the film is?
- Explain why 20 years old is the upper limit of the class 15 – 19.
- Draw a **cumulative frequency table** for the data.
- Draw a **cumulative frequency diagram** for the data.
- Use your cumulative frequency diagram to estimate the **median age** of the people watching the film in the cinema.
- Use your cumulative frequency diagram to estimate the **interquartile range** of the age of the people watching the film in the cinema.
- The manager of the cinema states that the **majority** of the people in the cinema are less than 25 years old. Does the data support this claim?
- Use the cumulative frequency diagram to estimate how many people **older than 32 years old** were watching the film.
- Complete the following sentence: Around _____ people were watching the film.



Evaluation

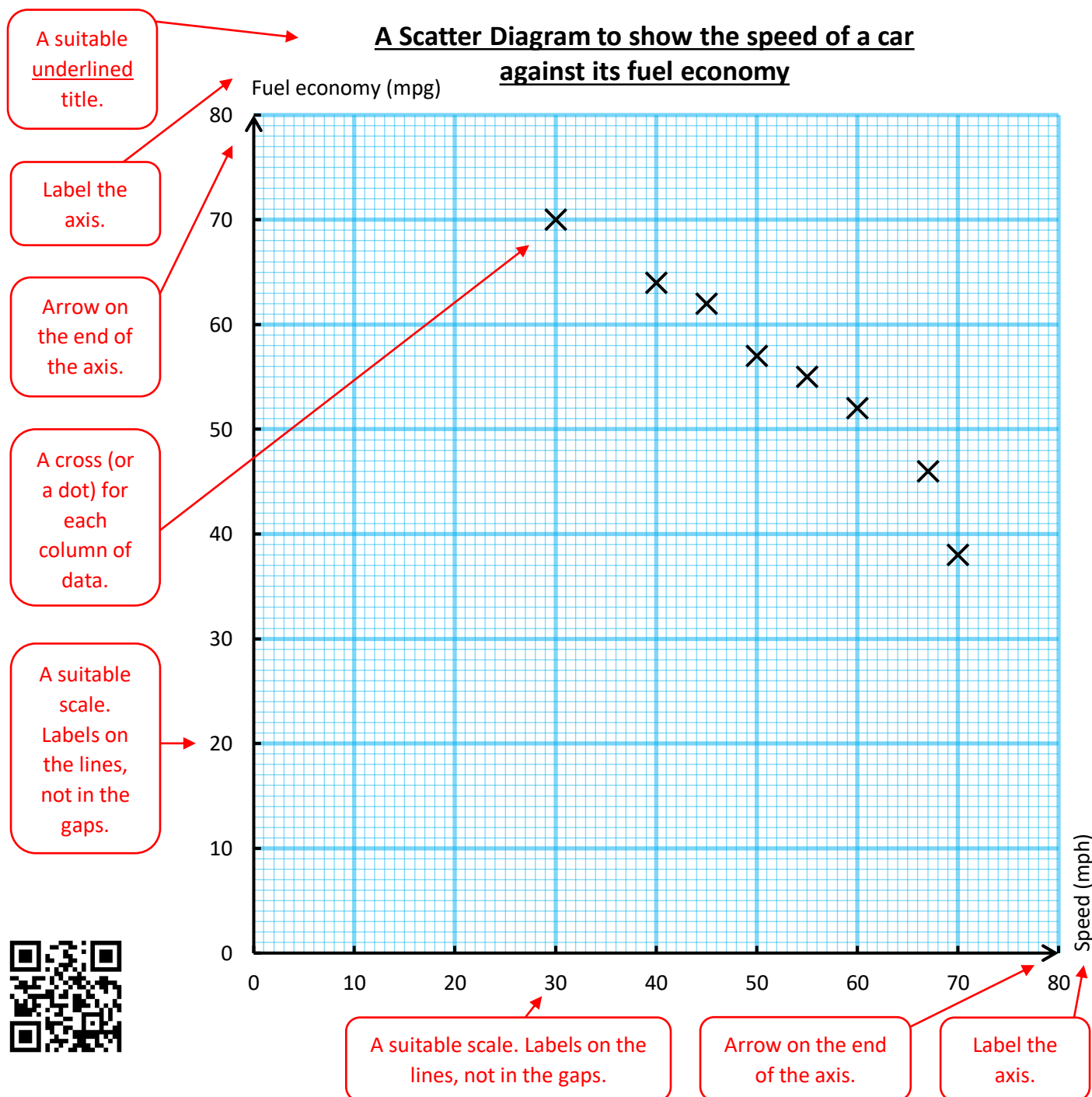
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Scatter Diagrams

A scatter diagram is used to see whether there is a connection between two sets of quantitative data. For example, here is some data about the fuel economy of a specific car at different speeds.

Speed (miles per hour, mph)	30	40	45	50	55	60	67	70
Fuel economy (miles per gallon, mpg)	70	64	62	57	55	52	46	38

We can plot the data on the scatter diagram shown below.

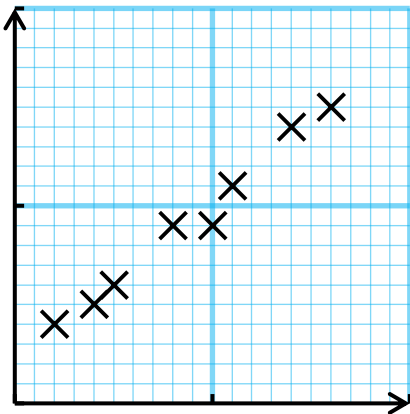


The scatter diagram shows that there is a connection between the two sets of data. As the speed increases, the fuel economy decreases. We say that there is **correlation** between the two sets of data. In this case, **negative correlation** is seen on the scatter diagram.

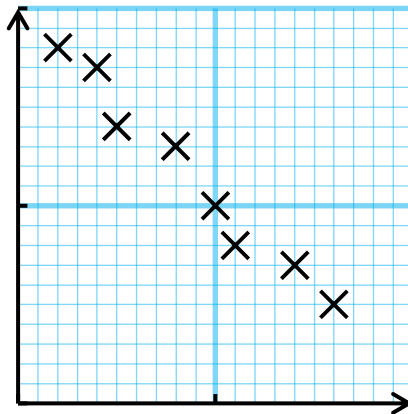


Types of Correlation

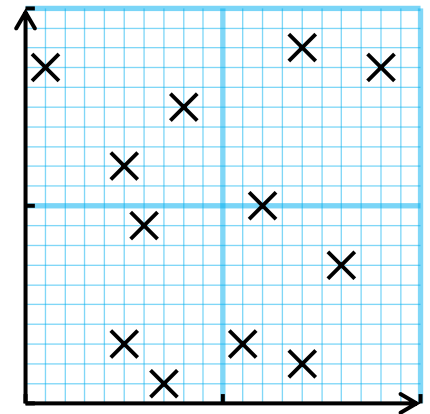
Positive Correlation



Negative Correlation



No Correlation



If a scatter diagram shows correlation, then it is possible to add a **line of best fit** to the diagram. This is a straight line that follows the pattern of the data points. Whilst drawing a line of best fit, try to ensure that there is a balance between the number of data points below and above the line.

There are two ways of drawing a line of best fit:

- 1) By **eye**.
- 2) Using the **mean point**.



If a scatter diagram shows the points closely following a line, then the correlation is said to be **strong**.

In the first method, we plot the line **by eye**, looking at the diagram to **see** where the line of best fit should go.

In the second method, we also look to see where the line of best fit should go, but this time the line of best fit must go through the **mean point**. We find this point by calculating the mean of the two data sets (separately), and then forming a co-ordinate from the two means.

For example, considering the data on the previous page, the mean of the speeds is

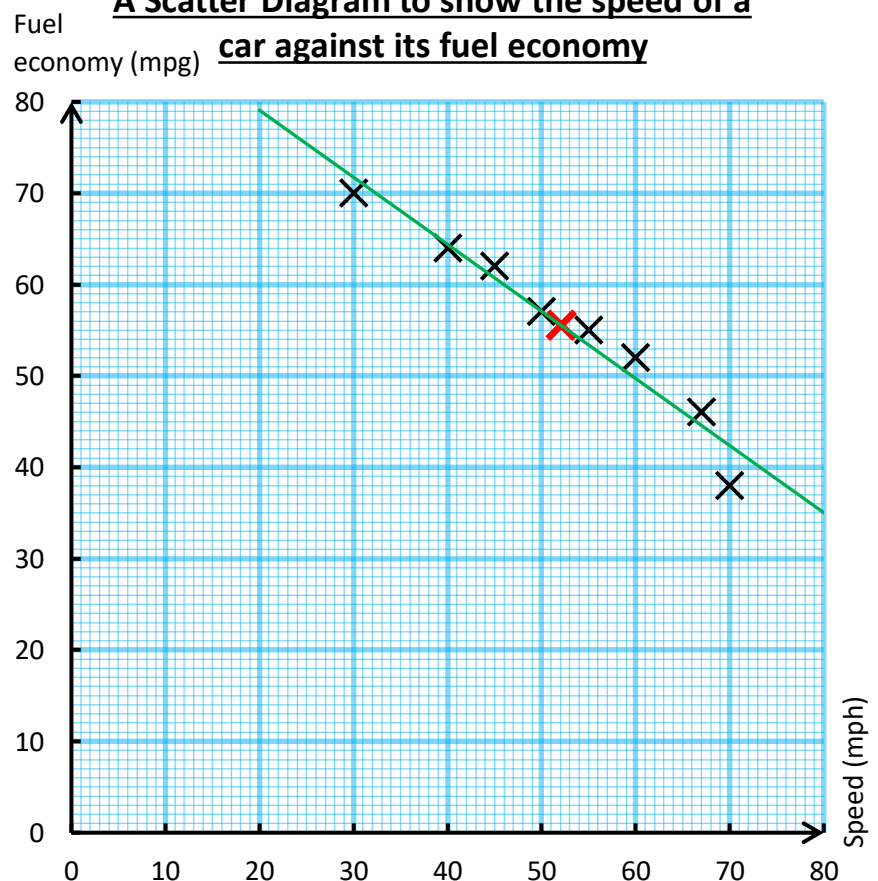
$$\begin{aligned} & (30 + 40 + 45 + 50 + 55 + 60 + 67 + 70) \div 8 \\ &= 417 \div 8 \\ &= 52.125 \end{aligned}$$

whilst the mean of the fuel economy is

$$\begin{aligned} & (70 + 64 + 62 + 57 + 55 + 52 + 46 + 38) \div 8 \\ &= 444 \div 8 \\ &= 55.5. \end{aligned}$$

It follows that the mean point for this data set is (52.125, 55.5). By plotting this point on the scatter diagram (shown in **red** on the right), we can then draw a line of best fit **passing through** this mean point. It would then be possible to use this line (for example) to answer questions about the fuel economy of the car at different speeds, including speeds that do not appear in the original data set.

A Scatter Diagram to show the speed of a car against its fuel economy



Exercise 31

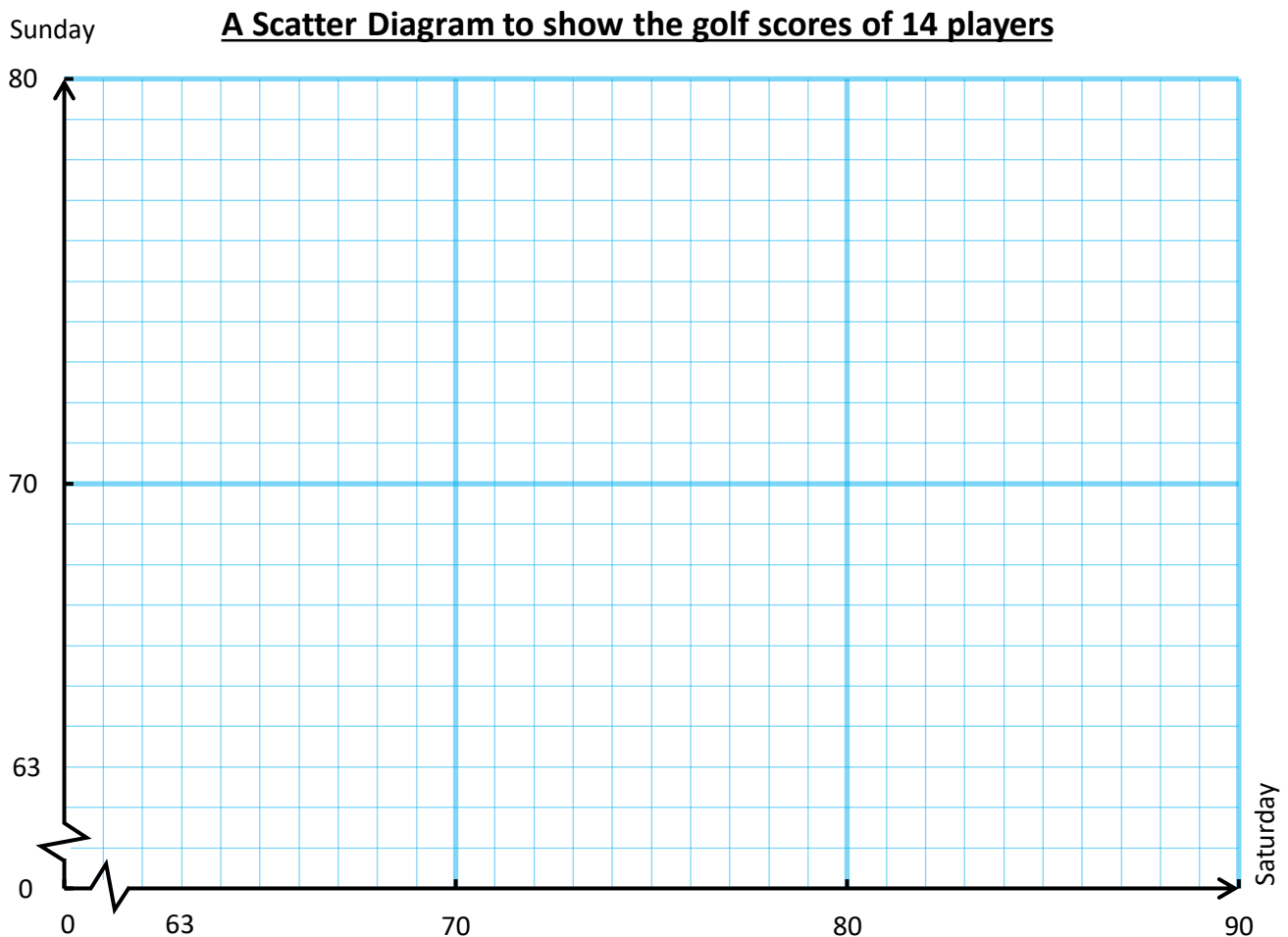
Applying

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The following data records the golf scores of 14 players over two consecutive days.

Saturday	74	79	71	68	81	75	72	69	78	70	81	77	82	75
Sunday	72	76	73	69	77	75	70	71	77	72	79	75	78	74

(a) Plot the data on the following scatter diagram.



- (b) Draw, by eye, a line of best fit on your scatter diagram.
- (c) Estimate the Sunday score of a player scoring 68 on Saturday.
- (d) Estimate the Saturday score of a player scoring 79 on Sunday.

**Exercise 32**

Plot the following data sets on suitable scatter diagrams. Where appropriate, add (by eye) a line of best fit.

(a) The marks of 15 pupils in two mathematics examinations. (The two papers were marked out of 40.)

Paper 1	36	34	23	24	30	40	25	35	20	15	35	34	23	35	27
Paper 2	39	36	27	20	33	35	27	32	28	20	37	35	25	33	30

(b) Head circumference (in cm) and English test result (out of 50).

Head circumference (cm)	600	500	480	570	450	550	600	460	540	430
Mark	43	33	45	31	25	42	23	36	24	39

Exercise 33

7

Plot the following data sets on suitable scatter diagrams. Where appropriate, add (using a mean point) a line of best fit.

(a) The age of Ford Fiesta cars and their second-hand price.

Age (years)	2	4	1	3	5	4.5	3	2	5.5	3.5	5	6
Price (£)	7,000	3,500	8,500	6,000	3,000	4,500	5,250	6,800	2,700	5,600	1,800	3,200

(b) The time taken by people of different ages to complete a crossword in a special competition.

Age (years)	56	60	20	23	43	42	26	24	55	70	30	35	19	54
Time (minutes)	11	8	12	9	16	23	19	14	15	9	21	7	6	19

(c) The percentages obtained by a set of pupils in two Welsh tests.

Test 1 (%)	73	96	58	36	55	83	39	48	86	80	69
Test 2 (%)	50	62	41	23	36	57	28	31	55	54	46

Exercise 34 (Revision)

The number of visitors to a nature park and the number of hours of sunshine was recorded each Saturday for 6 weeks. The following table shows the results.

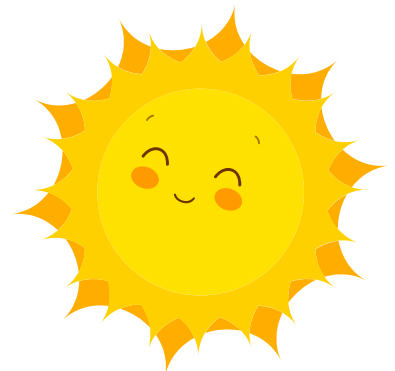
Number of hours of sunshine	3.5	7.5	4.0	4.0	4.5	6.0
Number of visitors	92	170	104	110	115	145

(a) Draw a scatter diagram for these results.

(b) Describe the correlation between the number of hours of sunshine and the number of visitors to the nature park.

(c) Draw, by eye, a line of best fit on your scatter diagram.

(d) Use your line of best fit to find an estimate for the number of visitors to the nature park on a day with 5 hours of sunshine.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Probability as a Fraction

Revision

5

Exercise 35

- (a) Use “no chance”, “low chance”, “even chance”, “good chance” or “certain” to describe the probability that it will snow on at least one day during October this year.
- (b) Draw a probability scale. Mark a point on the scale showing the probability, in your opinion, that someone buys a kettle in a shop in Llandudno tomorrow.
- (c) Answer using a number between 0 and 1: What is the probability that a man will be driving the next car that you will see?



Probability as a Fraction

For outcomes that are **equally likely**, we can write the probability of an event as a **fraction**.

$$\text{Probability of an Event} = \frac{\text{Number of favourable events}}{\text{Number of possible events}}$$

Example

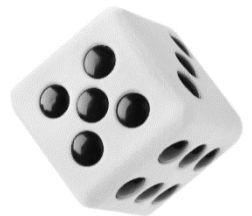
What is the probability of obtaining a factor of 6 when rolling a normal die?

Answer: The **possible events** are 1, 2, 3, 4, 5, 6. Therefore, there are 6 possible events.

The **favourable events** are those numbers that are factors of 6. These are 1, 2, 3 and 6.

Therefore, there are 4 favourable events.

The **probability** is therefore $\frac{4}{6}$.



Exercise 36

What is the probability of obtaining:

- (a) Tails when flipping a coin?
- (b) An odd number when rolling a normal die?
- (c) A number less than 5 when rolling a normal die?

Skill

6

This fraction simplifies to give the fraction $\frac{2}{3}$ but, unless the question asks, we do not have to simplify fractions when answering questions involving probability.

Exercise 37

Morgan has five tins of soup, but the labels have come off. He knows that he has bought three tins of tomato soup and two tins of chicken soup. Morgan opens one of the tins at random. What is the probability that the tin contains:

- (a) tomato soup (b) chicken soup (c) mushroom soup.



Exercise 38

The box on the right contains 8 counters. The counters are either red or blue.

The probability of choosing a red counter without looking is $\frac{3}{8}$.

- (a) How many red counters are in the box?
- (b) How many blue counters are in the box?



Exercise 39

6

One number is chosen at random from the grid shown on the right. What is the probability that the chosen number is:

- (a) Odd? (b) A multiple of 5?
 (c) A multiple of 3? (d) A factor of 10?
 (e) A factor of 16? (f) A one-digit number?
 (g) A prime number? (h) A square number?
 (i) A cube number? (j) An even factor of 60?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Exercise 40

The spinner shown on the right is spun once. What is the probability that the spinner lands on:

- (a) 2? (b) A number less than 5?
 (c) An odd number? (d) A factor of 20?



Applying

Exercise 41

200 raffle tickets are numbered from 1 to 200. All the raffle tickets are sold. One raffle ticket is to be chosen at random to determine who wins the main prize.

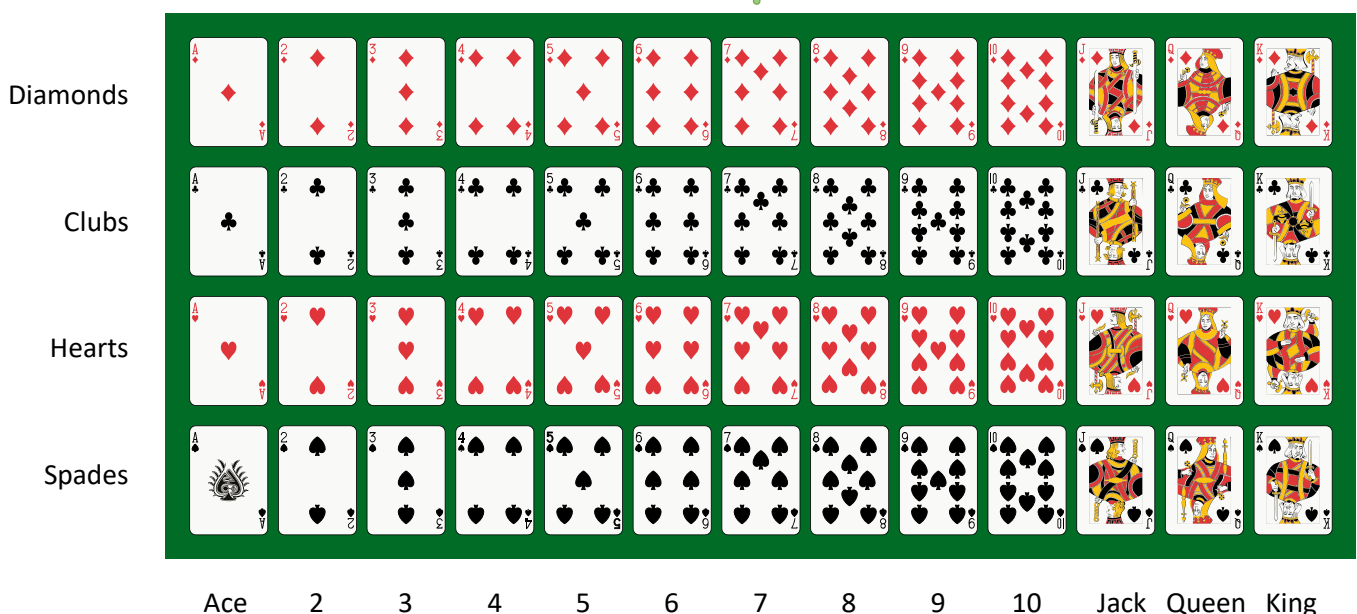
- (a) Karen has ticket number 125. What is Karen's probability of winning the main prize?
 (b) Andrew buys tickets with numbers 81, 82, 83, 84. Siwan buys tickets with numbers 30, 60, 90, 120. Who has the best chance of winning the main prize? Why?
 (c) Rob buys a lot of tickets. Rob's probability of winning the main prize is 5%. How many tickets has Rob bought?



There are no jokers in a standard deck.

Playing Cards

Here are the 52 cards in a standard deck of playing cards.

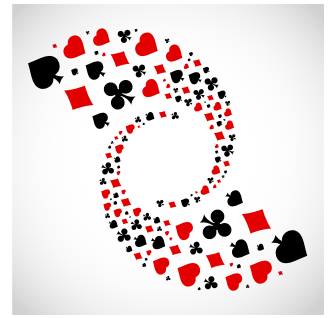


Exercise 42

6

Lisa shuffles the 52 cards in a standard deck of playing cards and chooses one card randomly from the deck. What is the probability that the chosen card is:

- | | |
|-------------------------------|----------------------------------|
| (a) a Jack? | (b) Spades? |
| (c) Red? | (d) 2 or 3? |
| (e) Hearts and an odd number? | (f) A face card? |
| (g) An even number? | (h) Diamonds and a prime number? |

**The probability that an event will not happen**

The probability that an event will not happen = $1 - \text{the probability that the event will happen}$.

Example

The probability of obtaining a 4 when rolling a normal fair die is $\frac{1}{6}$.

Therefore, the probability of **not** obtaining a 4 when rolling a normal fair die is $1 - \frac{1}{6} = \frac{5}{6}$.

**Exercise 43**

What is the probability:

- Of not obtaining a 3 when rolling a normal fair die.
- Of not obtaining a King when choosing 1 card at random from a standard deck of playing cards.
- Of not choosing a blue sock from a drawer containing 6 blue socks and 4 grey socks.
- Of not obtaining a multiple of 5 when rolling a die with 20 faces, numbered from 1 to 20.

**Exercise 44**

(a) The probability that Marc arrives late at school tomorrow is 0.1
What is the probability that Marc is not late tomorrow?

(b) The probability that Anna is successful in her driving test is $\frac{17}{20}$.
What is the probability that Anna is not successful in her driving test?

(c) Black, grey and blue dresses are hung on a rack in a shop.
Ffion chooses a dress at random. The probability of choosing a grey dress is 0.2 and the probability of choosing a black dress is 0.1.
What is the probability of choosing a blue dress?

(d) The probability that the school's rugby team wins their next game is 60%. The probability that they lose is 35%. What is the probability that the game will be drawn?

**Exercise 45**

A bag contains a full set of 22 snooker balls. There are 15 red balls, and one each of the following colours: white, yellow, green, brown, blue, pink and black. One ball is chosen at random from the bag. What is the probability that the chosen ball is:

- | | |
|------------|----------------|
| (a) Black? | (b) Not black? |
| (c) Red? | (d) Not red? |
| (e) Grey? | (f) Not grey? |



Sample Space Diagrams

The **sample space** in a probability question is a list of all the possible outcomes. For example, when flipping a coin, the set of possible outcomes is {head, tail}. We can use a **sample space diagram** to show all the possible outcomes when dealing with **combined events** (where we deal with more than one event at once). For example, consider flipping a coin and rolling a die at the same time. We can use the following sample space diagram to show all of the possible outcomes.

		Number shown on the die					
		1	2	3	4	5	6
Face shown on the coin	Head	Head, 1	Head, 2	Head, 3	Head, 4	Head, 5	Head, 6
	Tail	Tail, 1	Tail, 2	Tail, 3	Tail, 4	Tail, 5	Tail, 6

From the diagram we can see that there are $6 \times 2 = 12$ possible outcomes.

Example

A red die and a blue die are labelled from 1 to 6.

Seren rolls both dice and **multiplies** the numbers shown.

- Use a sample space diagram to list all of the possible outcomes.
- What is the probability that the product of both scores is 4?
- What is the probability that the product of both scores is greater than 24?

Answer: (a) We can draw the following sample space diagram to show all of the possible outcomes. Note that we write the product in the table, and not the result itself³. This will make answering parts (b) and (c) of the question easier.

		Number on the blue die					
		1	2	3	4	5	6
Number on the red die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

(b) There are $6 \times 6 = 36$ possible outcomes. Of these, we see that the number 4 appears **three times** in the table. Therefore, the probability that the product of both scores is 4 is $\frac{3}{36}$.

(c) There are **four numbers** in the table greater than 24, namely 25, 30 (twice) and 36. So the probability that the product of both scores is greater than 24 is $\frac{4}{36}$.

Exercise 46

One red die and one blue die are labelled from 1 to 6. Tegid rolls both dice and **adds** the scores obtained.

- Use a sample space diagram to list all of the possible outcomes.
- What is the probability that the sum of both numbers is 8?
- What sum is most probable? Why?
- What is the probability that the sum of both numbers is a two-digit number?
- What is the probability that the sum of both numbers is less than 4?
- What is the probability that the score on the red die is double the score on the blue die?



³ Technically, this makes the sample space diagram a **two-way table**.

Exercise 47

A glazier randomly places red, green or blue coloured glass in each of two windows.

- (a) Use a sample space diagram to list all of the possible outcomes.
 (b) What is the probability that red glass is placed in any of the two windows?

**Expected Frequency**

We can use probability to predict the **expected frequency** of an outcome, which is how often the outcome occurs.

$$\text{Expected Frequency} = \text{Probability} \times \text{Number of Trials}$$

**Example**

The probability of rolling a 5 on a die is $\frac{1}{6}$. If David rolls the die 120 times, how many 5's is he expected to obtain?

Answer: We need to multiply the probability by the number of times David rolls the die.

$$\begin{aligned} \frac{1}{6} \times 120 &= 120 \div 6 \\ &= 20. \end{aligned}$$

**Exercise 48**

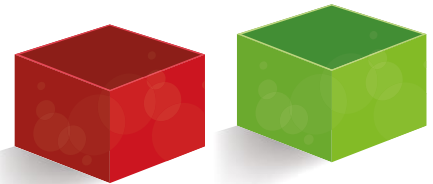
- (a) The probability that it is sunny during any single day in April is $\frac{2}{5}$.
 How many of the 30 days in April would you expect to be sunny?
- (b) The probability that Liverpool football club wins their next game is 0.6.
 How many of their next 20 games would you expect Liverpool to win?
- (c) When Iwan plays darts, the probability that he hits the red bullseye is $\frac{3}{50}$.
 Iwan takes part in a game to raise money for charity and throws 400 darts.
 Each dart that hits the red bullseye earns £5 for the charity.
 How much should we expect Iwan to raise for the charity?
- (d) The probability that Robert goes biking on any day is 0.2. There are 30 days in June. On how many days in June would you expect Robert to go biking?

**Exercise 49**

A red box contains four discs showing the numbers 3, 6, 9 and 12.
 A green box contains four discs showing the numbers 4, 7, 10 and 13.

In a game, a player takes one disc at random from each of the two boxes.
 The score for the game is the **lowest** of the two numbers obtained.

- (a) Use a sample space diagram to show all of the possible outcomes.
- (b) Find the probability that the score is less than 6.
- (c) A player wins if the score is less than 6. If 80 people play the game once each, how many of them would you expect to win?
- (d) It costs 50p to play the game once. The prize for winning is £1. If 80 people play the game once each, how much profit would you expect the game to make?



Exercise 50

6

There are four cards showing the numbers 1, 2, 4 and 5 in bag A.
There are four cards showing the numbers 3, 4, 6 and 7 in bag B.

In a game, one card is randomly chosen from bag A and one from bag B. The score for the game is the **positive difference** between the numbers. For example, if the card from bag A shows 1 and the card from bag B shows 3, then the score is $3 - 1 = 2$.



- (a) Use a sample space diagram to show all of the possible outcomes.
- (b) A player wins a prize by obtaining a score of 2 or less.
- (i) What is the probability that a player wins a prize?
- (ii) 160 people play the game once each.
Approximately how many of them would you expect to win a prize?
- (iii) It costs 90p to play the game once. The prize for obtaining a score of 2 or less is £1.20.
If 160 people play the game once each, how much profit would you expect the game to make?

Exercise 51 (Revision)

- (a) What is the probability of obtaining a number greater than 4 when rolling a normal fair die?
- (b) If 1 card is chosen at random from a normal deck of playing cards, what is the probability of choosing:
- (i) a card showing clubs? (ii) a card that does not show an odd number (3, 5, 7 or 9)?
- (iii) a card that is black and shows a prime number?
- (c) The table shows the probability of obtaining some of the scores when rolling a biased die with six faces.

Score	1	2	3	4	5	6
Probability	0.27	0.16	0.14		0.22	0.1

What is the probability of obtaining 4?

- (d) The probability that Geraint receives a letter on any given day is $\frac{2}{3}$.
During the next 15 days, on how many days would you expect Geraint to receive a letter?



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target



Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the test:Correct
in the
test?

I know how to calculate the median of a data set.			1	
I know how to calculate the median of data given in the form of a table .			2	
I know how to compare two distributions using the median.			1	
I know how to calculate the median class (or the estimate of the median) for grouped data.			9	
I know how to calculate the interquartile range of a data set.			3	
I know how to complete a cumulative frequency table for different types of data .			10	
I know how to draw a cumulative frequency diagram .			10	
I know how to estimate the median or the interquartile range using a cumulative frequency diagram.			10	
I know how to interpret a cumulative frequency diagram.			10	
I know how to draw a scatter diagram .			8	
I know how to recognise the different types of correlation in a scatter diagram.			8	
I know how to draw a line of best fit on a scatter diagram, either by eye or by using the mean point .			8	
I know how to interpret a scatter diagram.			8	
I know how to write the probability of an event as a fraction .			4, 5, 6	
I know how to answer questions involving probability that use a standard deck of playing cards .			6	
I know how to calculate the probability that an event will not happen .			6	
I know how to draw a sample space diagram for combined events .			7	
In a question on probability, I know how to calculate the expected frequency .			7	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐

The Examination

★ ★ ✨

_____ %

Working with Money

★ ★ ✨

_____ %

Level after the examination: _____

End of KS3
target: _____

Tracking Sheet Year 9

Sheet 2

Level at the end
of the year: _____

Straight Line Graphs

★ ★ ✨

_____ %

Movement with Sphero

★ ★ ✨

_____ %



The Mathematics Department

9

Working

with Money

Name:

Contents

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Estimation	Estimating the answer to calculations.	6
Compound Interest	Revising simple interest. The compound interest method of borrowing and investing money.	7
Foreign Money	Exchange rates. Exchanging money.	11
Household Bills	Electricity; gas; water; sewerage and mobile phone bills.	15



Significant Figures

Sarah sees a television for sale on the internet, for £389.

Delyth, who is Sarah's friend, says that the television costs around £400. Greta, another one of Sarah's friends, says that the television costs around £390.

Sarah's two friends have both given an **estimate** for the price of the television. Delyth has given her estimate to **one significant figure**, whilst Greta has given her estimate to **two significant figures**.



£389

When rounding off to **one significant figure**, we need to find an **estimate** that, **at most**, uses **one non-zero digit**.

When rounding off to **two significant figures**, we need to find an **estimate** that, **at most**, uses **two non-zero digits**.

(And so on for three significant figures; four significant figures; ...)



Example

Price	The price rounded off to one significant figure
£389	£400
£1,799	£2,000
£2.35	£2
£0.41	£0.40

Exercise 1

Round off the following prices to one significant figure.

- | | | | |
|------------|------------|-------------|--------------|
| (a) £799 | (b) £899 | (c) £999 | (d) £1,099 |
| (e) £1,846 | (f) £1,500 | (g) £28,381 | (h) £370,000 |
| (i) £4.19 | (j) £7.93 | (k) £16.70 | (l) £42.99 |
| (m) £0.84 | (n) £0.37 | (o) £0.13 | (p) £0.04 |

Exercise 2

Round off the following numbers to one significant figure.

- | | | | |
|------------|-------------|------------|-----------|
| (a) 329 | (b) 698.2 | (c) 450 | (d) 975 |
| (e) 19 | (f) 14 | (g) 7.3 | (h) 2.8 |
| (i) 0.54 | (j) 0.87 | (k) 1.56 | (l) 0.98 |
| (m) 24,528 | (n) 543,590 | (o) 19,829 | (p) 9,502 |

Skill

7



Example

Price	The price rounded off to two significant figures
£389	£390
£1,799	£1,800
£2.35	£2.40
£0.41	£0.41

Exercise 3

7

Round off the following prices to two significant figures.

- (a) £263 (b) £268 (c) £265 (d) £299
- (e) £4,368 (f) £2,809 (g) £54,238 (h) £876,000
- (i) £6.37 (j) £1.62 (k) £14.28 (l) £27.99
- (m) £1.95 (n) £19.99 (o) £0.13 (p) £0.04

**Exercise 4**

Round off the following numbers to two significant figures.

- (a) 836 (b) 278.5 (c) 913 (d) 976
- (e) 76.3 (f) 16.07 (g) 7.92 (h) 5.37
- (i) 0.283 (j) 0.927 (k) 1.8203 (l) 26.07
- (m) 82,573 (n) 903,400 (o) 49,732 (p) 997

Exercise 5

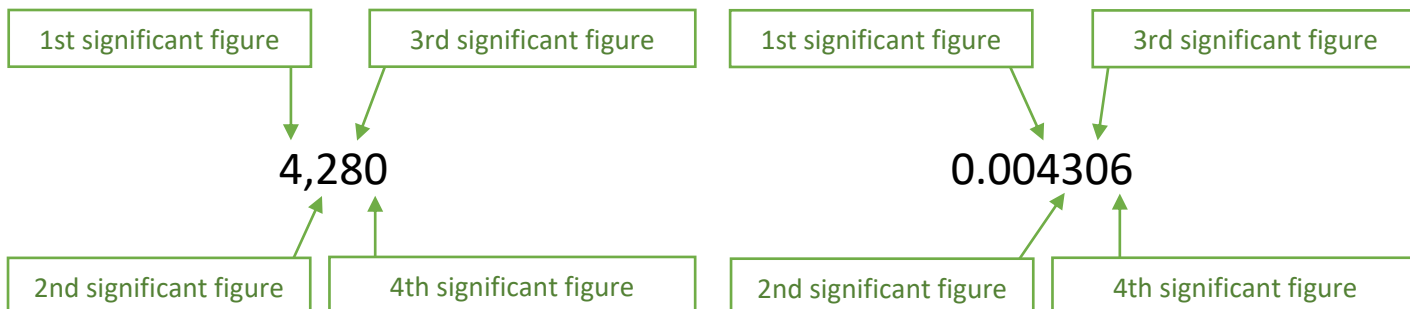
Complete the following table.

	Original number	Round off to one significant figure	Round off to two significant figures	Round off to three significant figures	Round off to four significant figures
(a)	28,539				
(b)	54.591				
(c)	3.0589				
(d)	1,339				
(e)	106,728				
(f)	4,525,925				
(g)	73,524,000				
(h)	183.078				
(i)	8,406.5				
(j)	0.87325				
(k)	0.018393				
(l)	0.0083736				

Labelling the Significant Figures of a Number

When reading a number from left to right, the **first** significant figure is the first digit encountered that is **not zero**. The other significant figures follow on immediately after the first significant figure.

Example



Exercise 6

For the following numbers, circle the **first** significant figure.

- (a) 8,205 (b) 934.2 (c) 387 (d) 46
 (e) 0.0378 (f) 0.982 (g) 0.0400213 (h) 0.000029002

Exercise 7

For the following numbers, circle the **second** significant figure.

- (a) 43,503 (b) 2,043,293 (c) 819 (d) 8.32
 (e) 0.0729 (f) 0.02083 (g) 0.0004893 (h) 0.000000072009



Sometimes, we need to add zeroes to the end of a number to write it correct to a specified number of significant figures. For example, to write 45 correct to three significant figures, we must write 45.0.

Exercise 8

- (a) Write 87 correct to three significant figures. (b) Write 987 correct to four significant figures.
 (c) Write 3,270 correct to six significant figures. (d) Write 7 correct to eight significant figures.



Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Estimation

Dafydd wants to calculate the sum $\frac{482.6 \times 43.5}{2168.12}$, but unfortunately he has lost his calculator. In order to find an approximate value for the correct answer, it would be possible for Dafydd to **estimate** the size of the numbers in the calculation, and perform the calculation with these estimates.

$$\begin{aligned} \frac{482.6 \times 43.5}{2168.12} &\approx \frac{500 \times 40}{2000} \\ &= \frac{20000}{2000} \\ &= 10 \end{aligned}$$



Let us now look at the different parts of the above estimation, explaining each part in turn.

- 1) The first line contains the estimation symbol $\approx$. We say this symbol as 'is around' or 'is approximately', and we need to include the symbol (instead of the equals sign $=$) as the left-hand side is not *equal* to the right-hand side.
- 2) Unless the question tells us otherwise, the convention for estimating numbers is to round them off to **one significant figure**. So, 482.6 rounds off to 500; 43.5 rounds off to 40; and 2168.12 rounds off to 2000.
- 3) Going from the first line to the second line, we perform the calculation $500 \times 40 = 20000$. To perform this sum quickly, note that $5 \times 4 = 20$, and that the **three zeroes** on the left-hand side lead to **three similar zeroes** on the right-hand side.
- 4) Going from the second line to the third line, we perform the calculation $\frac{20000}{2000} = 10$. To perform this sum quickly, note that three of the zeroes **cancel** on the top and bottom: $\frac{20000}{2000} = \frac{20}{2} = 10$. (The process of 'cancelling' zeroes comes from the fact that we can simplify the fraction by dividing the numerator and denominator by 10 each time.)

Exercise 9

Find an estimate for each of the following calculations.

- | | | |
|-----------------------------|------------------------|----------------------------|
| (a) 423×297 | (b) $19 \times 43,926$ | (c) $876 \times 6,875$ |
| (d) $97 \times 1,024$ | (e) 6.4×2.9 | (f) $0.98 \times 60,536.1$ |
| (g) $75,249 \times 342,192$ | (h) $198 \times 5,034$ | (i) $78 \times 452,138$ |

Exercise 10

Find an estimate for each of the following calculations.

- | | | |
|----------------------------|--------------------------------|------------------------------|
| (a) $\frac{7,924}{403}$ | (b) $\frac{324}{18.4}$ | (c) $\frac{879,054}{12,456}$ |
| (d) $\frac{5,783}{1,985}$ | (e) $\frac{98,160}{502}$ | (f) $\frac{87,204}{2,642}$ |
| (g) $\frac{82,457}{23.46}$ | (h) $\frac{7,304,539}{67,045}$ | (i) $\frac{614}{28}$ |

Exercise 11

Find an estimate for each of the following calculations.

- | | | |
|-------------------------------------|-------------------------------------|---|
| (a) $\frac{798 \times 403}{1,980}$ | (b) $\frac{1,975 \times 435}{787}$ | (c) $\frac{61 \times 90,128}{2,975}$ |
| (d) $\frac{3.24 \times 648}{895}$ | (e) $\frac{78,420}{204 \times 4.8}$ | (f) $\frac{4.9 \times 26.4 \times 7.4}{2.4601}$ |
| (g) $\frac{94.2 \times 398}{552.4}$ | (h) $\frac{0.13 \times 783}{19}$ | (i) $\frac{4.15 \times 5.63}{0.53}$ |

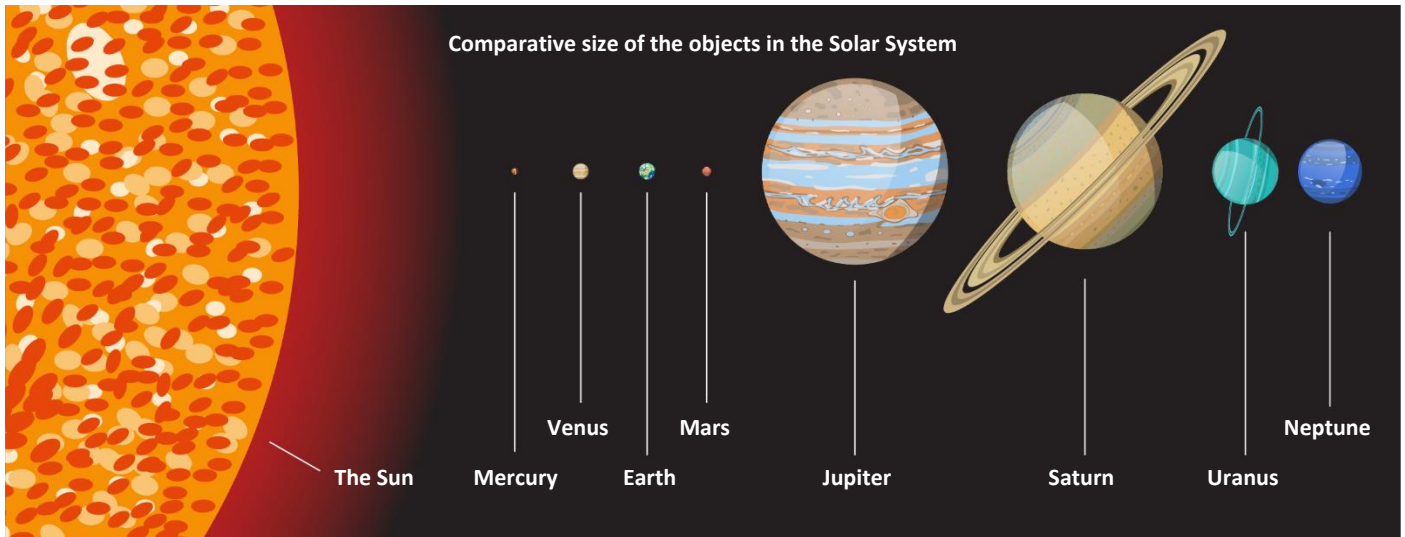


Exercise 12

Applying

7

- (a) A t-shirt costs £5.99 and a skirt costs £12.99. Sophie wants to buy 2 t-shirts and 5 skirts. She has £50. Use an approximation to decide whether Sophie has enough money.
- (b) David saves £3.80 each week. Approximately how much does he save in a year?
- (c) Eleri has a mobile phone contract costing £29.99 a month. Approximately how much does Eleri pay over a year?



- (d) The diameter of the Sun is around 1,391,400 km, and the diameter of Earth is around 12,742 km. Approximately how many times bigger is the diameter of the Sun compared to the diameter of Earth?
- (e) John works in an office and earns £19.75 per hour. He works 39 hours a week. Find an estimate for John's weekly wage.
- (f) Someone has asked the pupils in year 9 to estimate the individual cost of a trip to watch a musical. There are 98 pupils in year 9.
A packed lunch for each pupil costs £3.95.
The total cost of the tickets to the musical is £915.
The transport costs £290.
Estimate the individual cost for each pupil to watch the musical, correct to the nearest pound. Show all of your method.
- 
- A large, illuminated yellow star with a black outline is positioned on the right side of the page. The star is made of many small lights and is set against a background of red curtains. The star is slightly tilted and appears to be part of a stage set.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My target

Compound Interest

When borrowing money from a bank, the bank charges you **interest**. You must pay this interest to the bank as well as paying back all the money borrowed. On the other hand, if you **invest** money into a bank – which is to deposit money into a bank, then the bank pays you interest. The **interest rate** in both cases is usually given as a percentage.

There are several different types of interest. In year 7, we considered one type, namely **simple interest**. In this type of interest, the amount of interest added per year stays the same.



Revision

5

Exercise 13

(a) Arwyn borrows **£15,000** from HSBC bank at a simple interest rate of **4% a year**.

Arwyn wants to pay back all the money after **three years**.

How much money will Arwyn have to pay back after three years?

(b) Ellie wants to invest **£9,000** in Barclays bank at a simple interest rate of **3% a year**.

Ellie wants to withdraw all the money from the bank after **four years**.

How much money will Ellie be able to withdraw after four years?



Compound Interest

Let us now consider a more popular method of interest, namely **compound interest**. In this type of interest, the amount of interest added each year **changes**; it depends on the total amount of money at the start of the year.

Example

Nia borrows **£8,500** from NatWest bank at a compound interest rate of **4% a year**. Nia wants to pay back all the money after **three years**. How much money will Nia have to pay back after three years?

Answer: We must work year-by-year, adding 4% to the sum each year.

Year 1

$$\begin{aligned} &£8,500 \times 4\% \\ &= £340 \\ &£8,500 + £340 \\ &= £8,840 \end{aligned}$$

Year 2

$$\begin{aligned} &£8,840 \times 4\% \\ &= £353.60 \\ &£8,840 + £353.60 \\ &= £9,193.60 \end{aligned}$$

Year 3

$$\begin{aligned} &£9,193.60 \times 4\% \\ &= £367.74, \\ &\text{to the nearest penny.} \\ &£9,193.60 + £367.74 \\ &= £9,561.34 \end{aligned}$$

At the end of the three years, Nia must pay back £9,561.34 to the bank.

Exercise 14

(a) If NatWest has used a **simple interest** rate of 4% a year in the above example, would Nia have had to pay back **more** money or **less** money to the bank?

(b) Calculate **how much less** or **how much more** money Nia pays back in the above example, comparing compound interest at 4% a year to simple interest at 4% a year.

Applying

Exercise 15

5

The following table shows the compound interest rate for borrowing different amounts of money over different periods of time. In each case, calculate how much money needs to be **paid back** to the bank at the end of the period of borrowing.

	Amount to borrow	Compound interest rate per year	Period to borrow the money
(a)	£10,000	4%	3 years
(b)	£18,000	5%	3 years
(c)	£9,500	2%	2 years
(d)	£12,000	3%	4 years
(e)	£2,500	7%	3 years
(f)	£20,000	6%	4 years
(g)	£16,750	3%	2 years
(h)	£2,400	8%	3 years



Example

Owen wants to invest **£4,000** into Lloyds bank at a compound interest rate of **5% a year**. Owen wants to withdraw all the money from the bank after **three years**. How much money will Owen be able to withdraw after three years?

Answer: We need to work year-by-year, adding 5% to the sum each year.

Year 1

$$\begin{aligned} &£4,000 \times 5\% \\ &= £200 \\ &£4,000 + £200 \\ &= £4,200 \end{aligned}$$

Year 2

$$\begin{aligned} &£4,200 \times 5\% \\ &= £210 \\ &£4,200 + £210 \\ &= £4,410 \end{aligned}$$

Year 3

$$\begin{aligned} &£4,410 \times 5\% \\ &= £220.50 \\ &£4,410 + £220.50 \\ &= £4,630.50 \end{aligned}$$

At the end of the three years, Owen can withdraw £4,630.50 from the bank.

Exercise 16

The following table shows the compound interest rate for investing different amounts of money into a bank over different periods of time. In each case, calculate how much money can be **withdrawn** at the end of the period of investment.

	Sum to invest	Compound interest rate per year	Period to invest the money
(a)	£6,000	4%	3 years
(b)	£9,500	6%	2 years
(c)	£18,000	8%	4 years
(d)	£1,200	5%	3 years
(e)	£2,800	2%	2 years
(f)	£100,000	7%	3 years
(g)	£24,600	1%	5 years
(h)	£6,540	3%	3 years



Exercise 17

5

Cadi wants to invest £3,000 into a bank for two years. She sees the following options on the internet. Which bank should Cadi choose?

The image displays four bank offers, each enclosed in a colorful, multi-layered circular frame with a glowing effect. The offers are arranged in a 2x2 grid.

- Vale Bank**
Simple interest
4.1% a year
- City Bank**
Compound interest 2%
every six months
- Bank of Wales**
Compound interest 4%
a year
- People's Bank**
Simple interest
2.06% every
six months

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Foreign Money

Different countries use different currencies. In order to change between one type of currency and another, we need to know and use the **exchange rate**. It is possible to find current exchange rates on websites such as www.xe.com/currencyconverter. For example, the exchange rate for changing pounds into euros, on one day in 2017, is shown below.

XE Currency Converter: GBP to EUR

1 GBP = 1.14085 EUR

British Pound ↔ Euro

1 GBP = 1.14085 EUR 1 EUR = 0.876543 GBP

2017-11-01 15:41 UTC

All figures are based on live mid-market rates. These rates are not available to consumer clients.

[Set up a Rate Alert](#)

GBP - British Pound

↔

EUR - Euro

Example

(a) How many euros would a person get for £120?

Using the above exchange rate (£1 = €1.14085),
£120 would be worth $£120 \times 1.14085 = €136.90$,
correct to the nearest cent.

(b) How many pounds would a person get for €200?

Using the above exchange rate (£1 = €1.14085),
€200 would be worth $€200 \div 1.14085 = £175.31$,
correct to the nearest penny.

Exercise 18

Use the exchange rate £1 = €1.14085 to complete the following table.
(Round off your answers correct to two decimal places.)

Money in £	Money in €
£100	
£24	
£85.40	
£4,050	
£10,824.56	

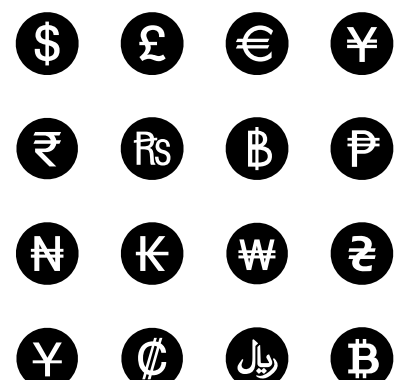
Money in £	Money in €
	€40
	€70.50
	€4,000
	€0.34
	€14,530.38

Exercise 19

Use the internet to investigate the currencies used by different countries.

- Which countries use the same currency?
- In which country can you obtain the greatest numerical value for £1?
- Investigate an unfamiliar currency system.
- Who uses the currencies shown on the right?

Write a paragraph in your book summarising your findings.



Exercise 20

5

(a) Rachel went on holiday to Spain. She changed £300 to euros when the exchange rate was $\text{£}1 = \text{€}1.13$. How many euros did Rachel receive?

(b) Dylan went on holiday to Singapore. He changed £250 to Singapore dollars when the exchange rate was $\text{£}1 = \text{\$}1.78$. How many Singapore dollars did Dylan receive?

(c) Coming back from a holiday in Sweden, Hannah had 945 Swedish krona. She changed this money to pounds when the exchange rate was $\text{£}1 = 11.08$ krona. How many pounds did Hannah receive?

(d) Coming back from a holiday in Australia, Deiniol had 232 Australian dollars. He changed this money back to pounds when the exchange rate was $\text{£}1 = \text{\$}1.72$. How many pounds did Deiniol receive?

(e) Jürgen, who lives in Germany, went on holiday to England. He changed €240 to pounds when the exchange rate was $\text{€}1 = \text{£}1.72$. How many pounds did Jürgen receive?

**Exchanging Money**

Imagine that you are going on holiday to America. As part of the preparations, you go to the Post Office in order to change £175 into dollars. The sign at the Post Office shows the following information:

“We sell dollars at the rate $\text{£}1 = \text{\$}1.35$ ”.



Using this exchange rate, £175 would be worth $\text{£}175 \times 1.35 = \text{\$}236.25$. It is unlikely, however, then the Post Office can give exactly this amount of dollars to you (small denominations of coins are not usually kept in stock). Assuming that a \$5 note is the smallest amount of dollars kept in stock, then the Post Office could give you \$235. This would cost $\text{\$}235 \div 1.35 = \text{£}174.07$, to the nearest penny. Therefore, when handing £175 to the Post Office, you would receive back \$235 in dollars, and £0.93 in change.

Exercise 21

Applying

5

(a) You are going on holiday to France. As part of the preparations, you go to the Post Office in order to change £90 to euros. The sign at the Post Office shows the following information:

“We sell euros at the rate $\text{£}1 = \text{€}1.07$ ”.

The smallest amount of euros available today at the Post Office is a €5 note. What is the maximum number of euros you can get today at the Post Office, and how much change (in pounds) would you receive?

(b) You are going on holiday to Japan. As part of the preparations, you go to the Post Office in order to change £150 into yen. The sign at the Post Office shows the following information:

“We sell yen at the rate $\text{£}1 = \text{¥}143.28$ ”.

The smallest amount of yen available today at the Post Office is a ¥1,000 note. What is the maximum number of yen you can get today at the Post Office, and how much change (in pounds) would you receive?



(c) Carys went on holiday to Brazil. Before going, she went to a holiday money shop to buy Brazilian *real*.

Carys only had £250 to spend on buying *real*. She wanted to buy as much *real* as possible. Unfortunately, only 50 *real* notes were available to buy at the holiday money shop. The exchange rate to buy *real* was £1 = 3.92 *real*.



(i) How many *real* did Carys buy?

(ii) How much did she pay for the *real*?

(iii) During her stay in Brazil, Carys spent 754.25 *real*. After returning from her holiday, she changed the remaining *real* back to pounds. Unfortunately, the holiday money shop was only willing to buy back a whole number of *real*. The exchange rate used by the shop for changing *real* back to pounds was £1 = 3.98 *real*. Calculate how many pounds Carys received back from the holiday money shop. Give your answer correct to the nearest penny.

Exchanging between more than two types of money

Given one exchange rate, e.g. £1 = €1.14, we can divide or multiply to change between the two types of money. Given two exchange rates, we need to consider more carefully how to change between the different currencies.

Example

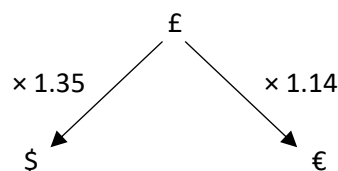
Looking on the internet, Dafydd found the following information about the currencies used in the U.K., America and Europe.

$$£1 = \$1.35 \text{ and } £1 = €1.14.$$

Use the above information to help Dafydd fill in the following blanks.

$$\$1 = €______ \text{ and } €1 = \$______.$$

Answer: To begin, it is useful to draw a diagram representing the situation.



To change \$1 to euros, we need to find a path in the above diagram that goes from dollars to euros. This would be possible by starting at the \$ symbol, going against the arrow on the left to reach the £ symbol, and then following the arrow on the right to reach the € symbol. So, \$1 is worth $1 \div 1.35 \times 1.14 = €0.84$, correct to the nearest cent.

To change €1 into dollars, we need to find a path in the above diagram going from euros to dollars. This would be possible by starting at the € symbol, going against the arrow on the right to reach the £ symbol, and then following the arrow on the left to reach the \$ symbol. So, €1 is worth $1 \div 1.14 \times 1.35 = \1.18 , correct to the nearest cent.

Exercise 22

(a) Looking on the internet, Eryl found the following information about the currencies of the U.K., India and Russia.

$$£1 = 84.7 \text{ Indian rupees and } £1 = 76.5 \text{ Russian roubles.}$$

Use the above information to help Eryl fill in the following blanks.

$$1 \text{ Indian rupee} = ______ \text{ Russian roubles and } 1 \text{ Russian rouble} = ______ \text{ Indian rupees.}$$



(b) Looking on the internet, Geraint found the following information about the currencies used in Europe, Australia and Norway.

€1 = 1.51 Australian dollars and €1 = 9.48 Norwegian krone.

Use the above information to help Geraint fill in the following blanks.

1 Australian dollar = _____ Norwegian krone and
1 Norwegian krone = _____ Australian dollars.

(c) Looking on the internet, Bethan found the following information about the currencies used in Japan, Turkey and Mexico.

¥1 = 0.033 Turkish lira and ¥1 = 0.167 Mexican peso.

Use the above information to help Bethan find how many Mexican peso would correspond to 2,000 Turkish lira. Give your answer correct to the nearest peso.

(d) Kate lives in London. She goes on holiday to Paris with her two friends, Janie who lives in America and Ami who lives in Japan. They meet in London for a few days before flying to Paris together. Janie exchanges 450 American dollars into pounds and Ami exchanges 30,000 Japanese yen into pounds.

In London, Janie and Ami spend £100 each and then exchange the remainder of their money into euros. Kate also exchanges £250 into euros.

£1 = 129.82 Japanese yen
£1 = 1.57 American dollars
£1 = 1.18 euros

Use the above exchange rates to calculate the total number of euros the three women take to Paris. Show all your working, explaining each step as you go.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

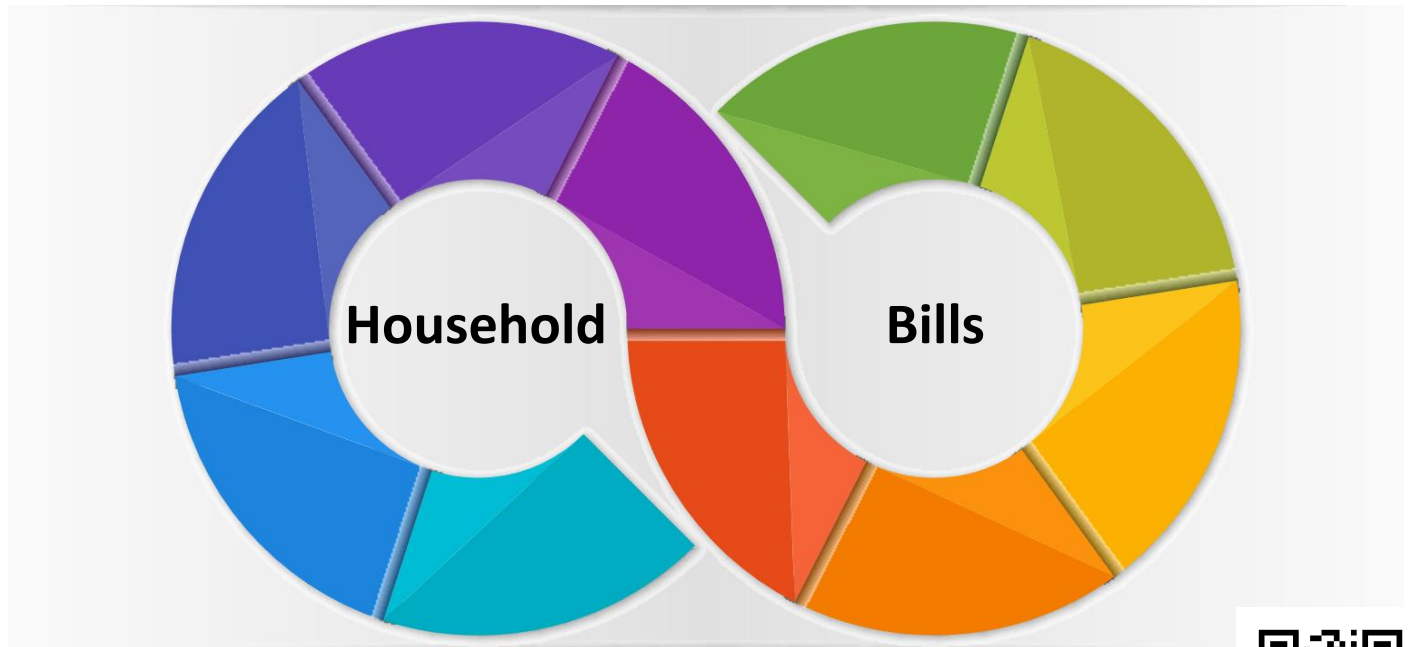
Household Bills

A number of household bills arrive on a weekly; monthly or yearly basis.

Exercise 23

5

Discuss with a partner which bills arrive regularly at a typical home. Fill in the following chart with your ideas.



Terms

Here is an explanation of some of the terms that appear in common household bills.



Term	Explanation
Unit Cost	The cost of one unit of e.g. gas.
Standing Charge	This must be paid no matter how much of the service is used.
Value Added Tax	A tax on goods or services, collected by the government. Usually, it is given as a percentage.
Reading	A number shown on a meter, e.g. electricity meter.
Quarter	Some bills arrive 'quarterly'. This means every quarter of a year, or every three months.
Rateable value	This depends on the type of house you live in. It is used to help determine the value of your bill, e.g. your water bill.

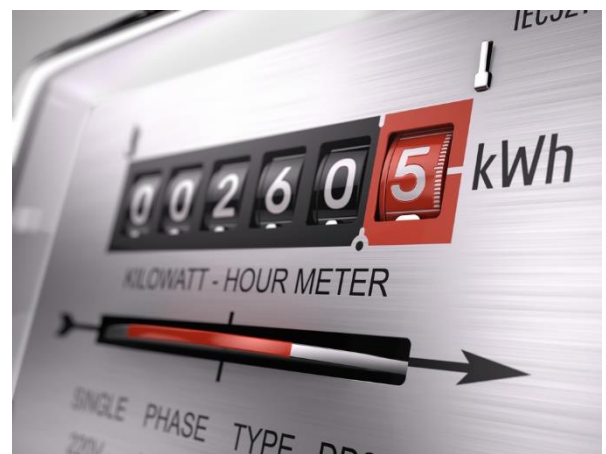
Example

At the end of one quarter, the reading on Mr. Smith's electricity meter was 53,799, whilst the previous reading was 52,471. The electricity company charges 17.4 pence per unit along with a standing charge of £35. On top of this, value added tax is charged at a rate of 5%.

Calculate the total of Mr. Smith's electricity bill.

Answer: To begin, we must calculate how many units of electricity Mr. Smith has used during the quarter (or 3 months).

$$53,799 - 52,471 = 1,328.$$



Each unit of electricity costs 17.4 pence, so the cost of all the electricity is

$$1,328 \times 17.4 = 23,107.2 \text{ pence.}$$

To change the above into pounds, we must divide by 100.

$$23,107.2 \div 100 = \text{£}231.07, \text{ to the nearest penny.}$$

Next, we need to add the standing charge of £35.

$$\text{£}231.07 + \text{£}35 = \text{£}266.07.$$

To finish, we need to add the value added tax, which is 5% of the above total.

$$\text{£}266.07 \times 5\% = \text{£}13.30, \text{ to the nearest penny.}$$

$$\text{£}266.07 + \text{£}13.30 = \text{£}279.37.$$

Therefore, Mr. Smith's bill for the quarter is **£279.37**.

Exercise 24

(a) The electricity meter readings at the start and end of a 3-month period were:

Reading at the start of the period:

6	5	1	9	7
---	---	---	---	---

Reading at the end of the period:

6	4	9	4	7
---	---	---	---	---

The cost of the electricity used was 30p per unit and there was a standing charge of £25.34 for the 3-month period.

Complete the following table to find the total cost.

Reading at the start of the period	
Reading at the end of the period	
Number of units used	
Cost of the units, in £	
Standing charge for the 3-month period	
Total cost	

(b) One quarter, the current reading on Mrs. Roberts' electricity meter was 18,793 and the previous reading was 17,843. The electricity company charges 18.6 pence per unit along with a standing charge of £32.50. On top of this, value added tax is charged at a rate of 5%. Calculate Mrs. Roberts' total electricity bill.

(c) Samantha's electricity bill for a particular quarter is calculated from the following information.

Previous meter reading: 42,764.

Current meter reading: 43,915.

Cost per unit: 18 pence.

Standing charge per quarter: £26

(i) Calculate Samantha's total electricity bill, including value added tax at a rate of 5%.

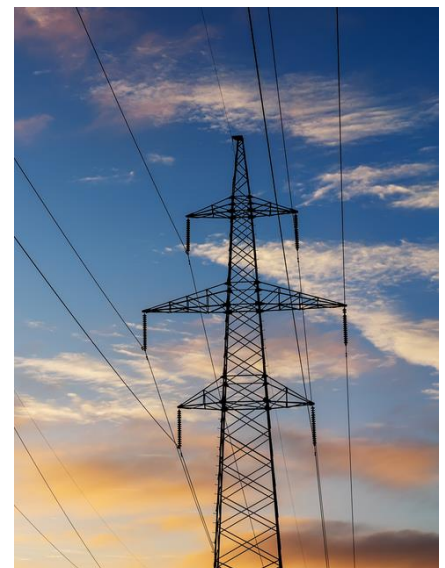
(ii) Samantha saved £17 per week for 12 weeks to help pay off the bill.

How much more does she need to save in order to pay off the electricity bill?



Applying

5



(d) Mr. and Mrs. Hughes receive an electricity bill. The bill is shown below, with some of the entries missing. Use the information given on the bill to complete the missing entries, and hence show the total that Mr. and Mrs. Hughes must pay.

CHARGE ELECTRICITY COMPANY		Bill for the period July 1 st 2020 to September 30 th 2020.		
Mr. and Mrs. Hughes 3 Stryd y Wennol Llandudno LL30 1PX		INVOICE		
Meter reading this time	Meter reading last time	Units that have been used	Price per unit in pence	Sum
4,703	3,425	Units that have been used	18.5p	
		V.A.T. at 5%		
		Amount payable from the previous bill		12.46
		Total to pay		

Example

The following information relates to Emlyn's gas bill for the spring quarter.

Standing charge: 28 pence per day for 91 days.

Gas supplied: 6,458 kilowatt hours.

Cost per kilowatt hour: 3.924 pence.



Calculate the total bill, including value added tax at a rate of 5%. Give your answer correct to the nearest penny.

Answer: The standing charge is 28 pence per day, so for the period of the bill (91 days), the standing charge is

$$91 \times 28 = 2,548 \text{ pence.}$$

To change this into pounds, we need to divide by 100.

$$2,548 \div 100 = \text{£}25.48.$$

Each unit of gas costs 3.924 pence, so the cost of all the gas is

$$6,458 \times 3.924 = 25,341.192 \text{ pence.}$$

To change this into pounds, we need to divide by 100.

$$25,341.192 \div 100 = \text{£}253.41, \text{ to the nearest penny.}$$

The total bill (before tax) is

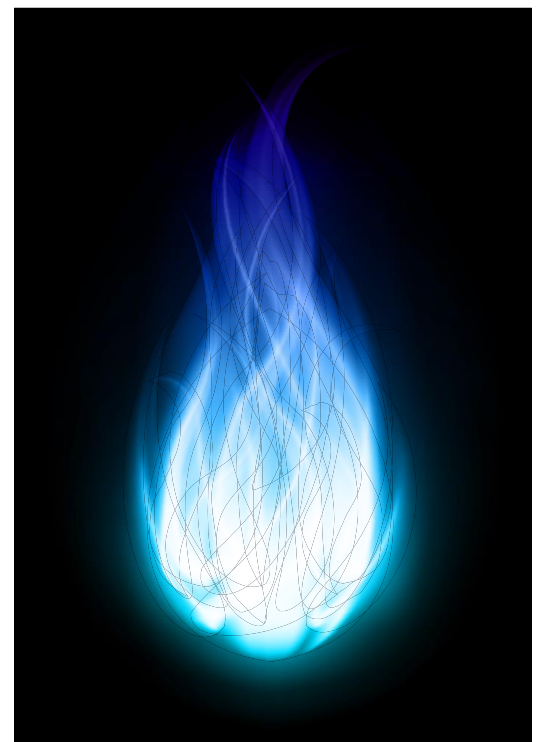
$$\text{£}25.48 + \text{£}253.41 = \text{£}278.89.$$

To finish, we need to add, at a rate of 5%, the value added tax.

$$\text{£}278.89 \times 5\% = \text{£}13.94, \text{ to the nearest penny.}$$

$$\text{£}278.89 + \text{£}13.94 = \text{£}292.83.$$

So, Emlyn's bill for the quarter is **£292.83**.



Exercise 25

5

(a) The following information relates to Mr. Williams' gas bill for the winter quarter.

- Standing charge: 25 pence per day for 91 days.
- Gas supplied: 7,532 kilowatt hours.
- Cost per kilowatt hour: 3.67 pence.

Calculate the total bill, including value added tax at a rate of 5%.
Give your answer correct to the nearest penny.

(b) Gethin's gas bill for a specific quarter is calculated using the following information.

- Number of kW hours used: 9,056.
- Cost per kW hour: 3.52 pence.
- Standing charge: 29 pence per day for 91 days.
- VAT rate: 8%.

Calculate Gethin's gas bill (including the VAT) for the quarter.

(c) Nel has received has gas bill for the period from January to April.
The bill's details are as follows.

- The number of units of gas used is 10,278.
- The cost of one unit of gas is 2.92 pence.
- The number of days for this period is 90.
- The standing charge is 32.4 pence per day.
- VAT at 5% is charged on the total gas bill.

Showing all of your method, calculate Nel's gas bill, including the VAT.

Water and Sewerage

The type of water and sewerage bill a house receives depends upon the presence of a **water meter** in the house. If a water meter is present, then the bill depends upon **how much water** has been used. If no water meter is present, then the bill depends upon the **rateable value** of the house the bill is sent to.

Example

A water bill is calculated by adding the standing charge, the water charge and the sewerage charge. Here are the details for this year:

- Standing charge: 35p per day for the period of the bill.
- Water charge: £1.39 for each m^3 of water that is used.
- Sewerage charge: £1.66 for each m^3 of water that is used.

Calculate the total bill when 75 m^3 of water is used during a period of 180 days.



Answer: The standing charge is 35 pence per day, so over the period of the bill (180 days) the standing charge will be

$$180 \times 35 = 6,300 \text{ pence.}$$

To change the standing charge to be in pounds, we need to divide by 100.

$$6,300 \div 100 = \text{£}63.$$

Each unit of water costs £1.39 per cubic metre, so the cost of all the water is

$$75 \times \text{£}1.39 = \text{£}104.25.$$

The sewerage charge is £1.66 per cubic metre, so the sewerage charge is

$$75 \times \text{£}1.66 = \text{£}124.50.$$

The total bill is

$$\text{£}63 + \text{£}104.25 + \text{£}124.50 = \text{£}291.75.$$

Exercise 26

5

(a) A water bill is calculated by adding the standing charge, the water charge and the sewerage charge. Here are the details for this year:

- A standing charge of 38p per day for the period of the bill.
- A water charge of £1.42 for each m³ of water that is used.
- A sewerage charge of £1.58 for each m³ of water that is used.

Calculate the total bill when 93 m³ of water is used during a period of 180 days.

(b) A water bill is calculated by adding the standing charge, the water charge and the sewerage charge. Here are the details for this year:

- A standing charge of 41p per day for the period of the bill.
- A water charge of £1.20 for each m³ of water that is used.
- A sewerage charge of £1.55 for each m³ of water that is used.

Calculate the total bill when 62 m³ of water is used during a period of 92 days.

(c) The following information relates to Mr. Edwards' bill for water and sewerage. Mr. Edwards does not have a water meter, so the bill uses the rateable value of his house.

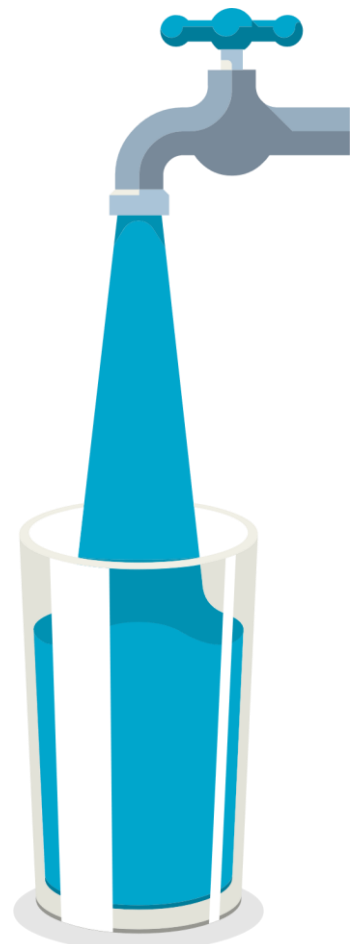
- Rateable Value: £275.
- Water: Standing charge of £126.43 and 80 pence for each £ of rateable value.
- Sewerage: Standing charge of £175.27 and £1.15 for each £ of rateable value.

Calculate the total of Mr. Edwards' water and sewerage bill.

(d) Ben pays his bill from Welsh Water a year in advance using direct debit. The bill is calculated for 2018–19 by using the following information.

- Rateable Value: £345.
- Discount for paying through direct debit: 2%.
- Water: Standing charge of £135 and £0.75 for each £ of rateable value.
- Sewerage: A standing charge of £180 and £1.09 for each £ of rateable value.

Calculate Ben's bill.



Exercise 27

5

Aaron pays £25 a month for his mobile phone contract.

The contract gives a monthly allowance of 120 minutes of talk time plus 400 text messages. The data allowance is unlimited.

If Aaron goes over his allowance, then the following additional charges are incurred, on top of the £25 contract charge.

ADDITIONAL CHARGES

Additional talk time
35 pence per minute

Additional text messages
15 pence per message

Last month, Aaron used 150 minutes of talk time and sent 490 text messages.

A new contract is offered to Aaron. This contains **double** the amount of talk time and **double** the number of text messages, but it costs **£35 per month** (not £25 per month).

Using only the information about last month, decide whether Aaron should accept the new contract.

Give reasons for your answer and show all of your calculations.

Exercise 28

Mrs. Owen requires £800 to buy a new sofa.

She already has £420 in a savings account.

Mrs. Owen earns £2,100 per month.

She invests 8% of her monthly wage into the savings account.

How many months will it take Mrs. Owen to save enough money in order to buy the sofa?

Show all of your calculations.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐



Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the test:Correct
in the
test?

I know how to round off a number to a specified number of significant figures , e.g. to 1 significant figure or to 2 significant figures.			7, 8	
I know how to recognise the significant figures in a given number.			9	
I know how to estimate the answer to a sum, by using rounding off to one significant figure.			10	
I know how to calculate the compound interest on a sum of money over a specific period of time.			1	
I know how to use an exchange rate to change one type of currency to another type of currency.			2	
I know what to do when a restriction is placed on the type of money available at a currency exchange shop.			3	
I know how to change between more than two types of currency.			2	
I know how to calculate an electricity bill .			4	
I know how to calculate a gas bill .			6	
I know how to calculate a water and sewerage bill .			5	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

9

Straight Line

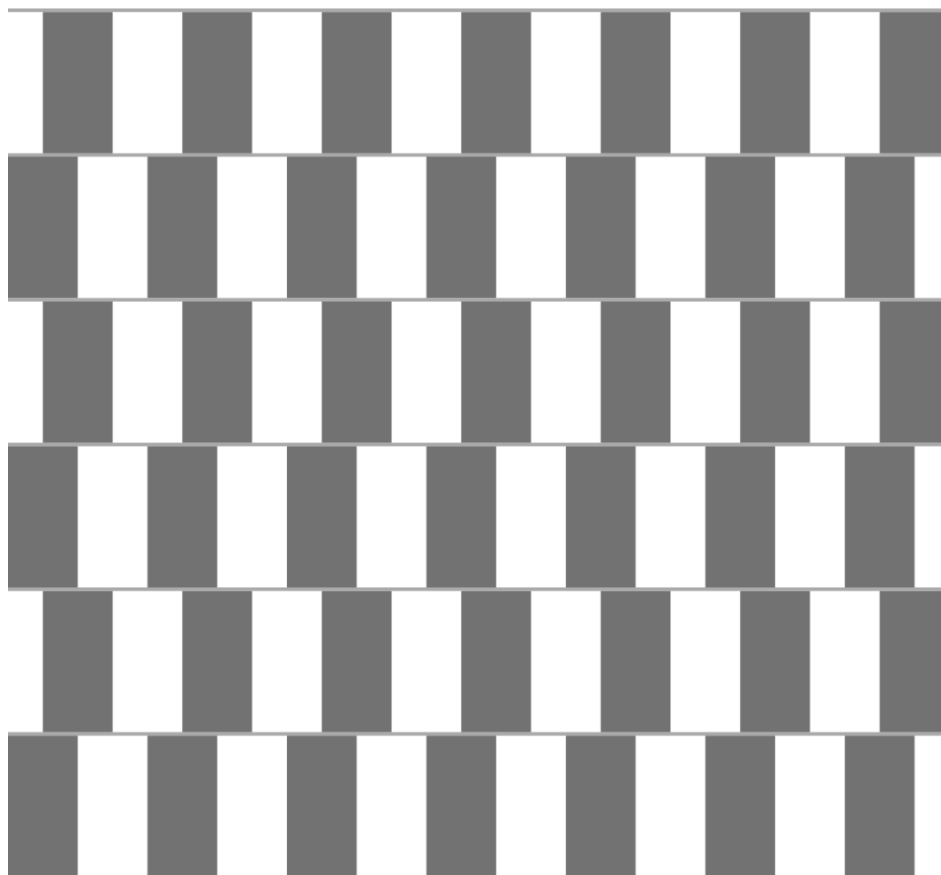
Graphs

Name:



Contents

Chapter	Mathematics	Page Number
Gradient of a Line	Measuring and finding the gradient of straight lines.	3
Straight Line Graphs	Vertical lines of the form $x = a$. Horizontal lines of the form $y = b$. General lines of the form $y = mx + c$. Solving simultaneous equations graphically.	6
The Gradient of Parallel and Perpendicular Lines	The gradient of parallel lines. The gradient of perpendicular lines.	15
Transformations: Reflection	Reflecting shapes in lines of the form $x = a$ and $y = b$. Reflecting shapes in lines of the form $y = mx + c$.	16



The above horizontal lines are parallel lines.



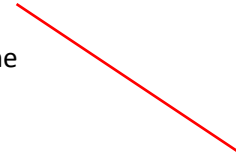
Gradient of a Line

The **gradient** of a line measures how **steep** the line is. Apart from horizontal and vertical lines, every straight line has a gradient that is either **positive** or **negative**.

An example of a straight line with **positive** gradient



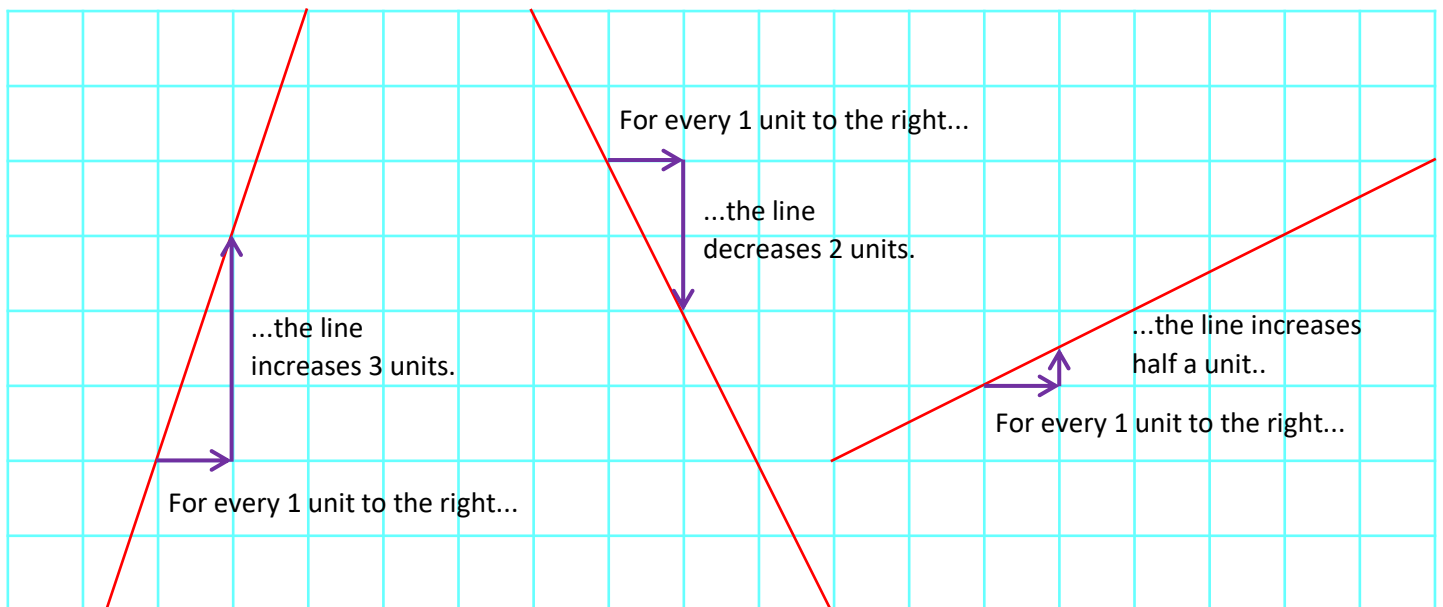
An example of a straight line with **negative** gradient



In order to **measure** the gradient of a line, we need to find how many units the line **increases** or **decreases** when we move **one unit to the right**.

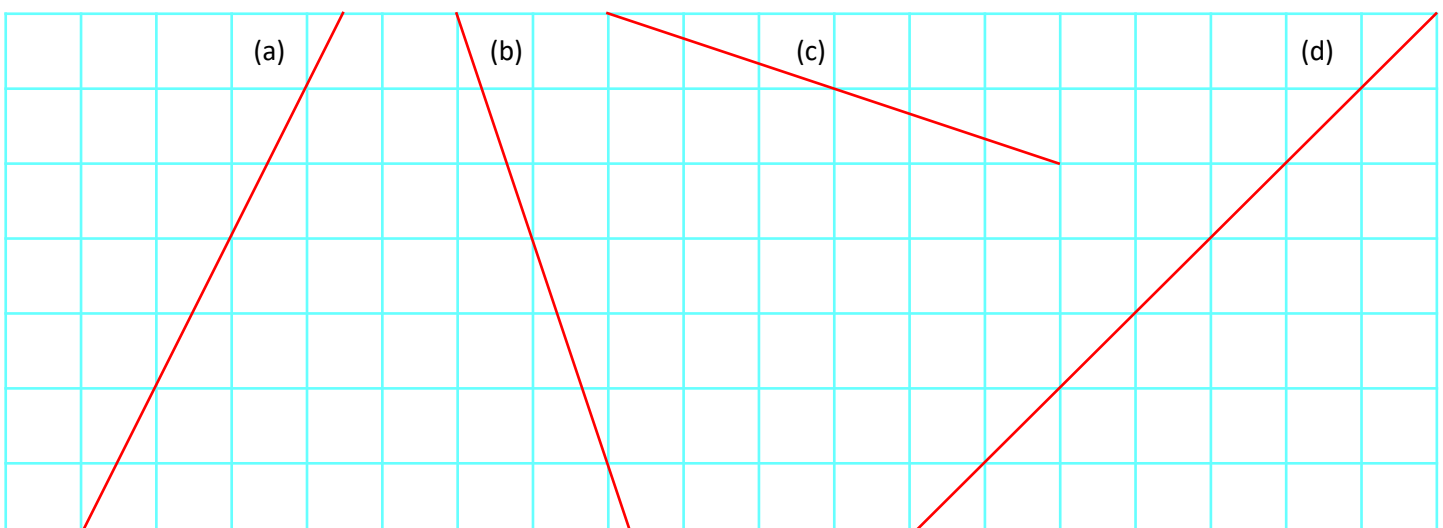
Example

(a) The gradient of the line below is 3. (b) The gradient of the line below is -2 . (c) The gradient of the line below is $\frac{1}{2}$.



Exercise 1

What is the gradient of the following lines?



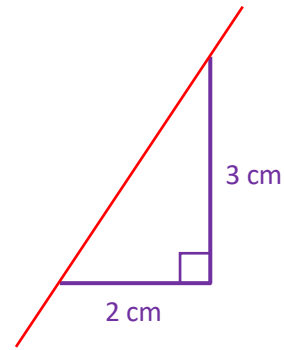
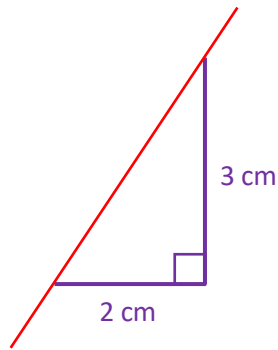
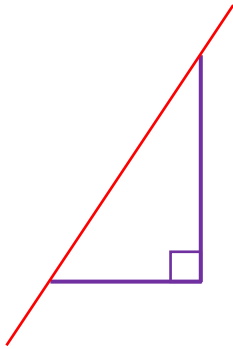
Measuring the Gradient of a Line

If a grid is not available to help, use the following method to measure the gradient of a straight line.

(1) Add a **right-angled triangle** to the **straight line**, ensuring the base of the triangle is horizontal.

(2) Measure the base and height of the triangle using a ruler.

(3) Divide the vertical distance by the horizontal distance.



$$3 \div 2 = 1.5$$

The gradient of the line is 1.5.

Checking

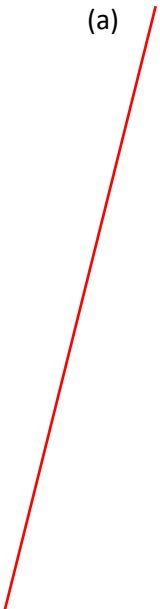
- Is the gradient of your line positive or negative? Check that you have used the correct sign in your answer.
- Check that you have divided the numbers in the correct order (vertical $\div$ horizontal, *not* horizontal $\div$ vertical).

Exercise 2

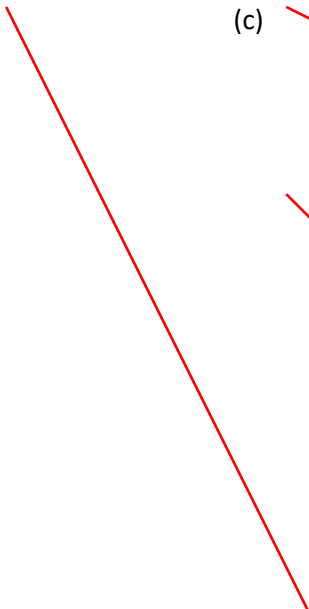
4

Find the gradient of the following lines.

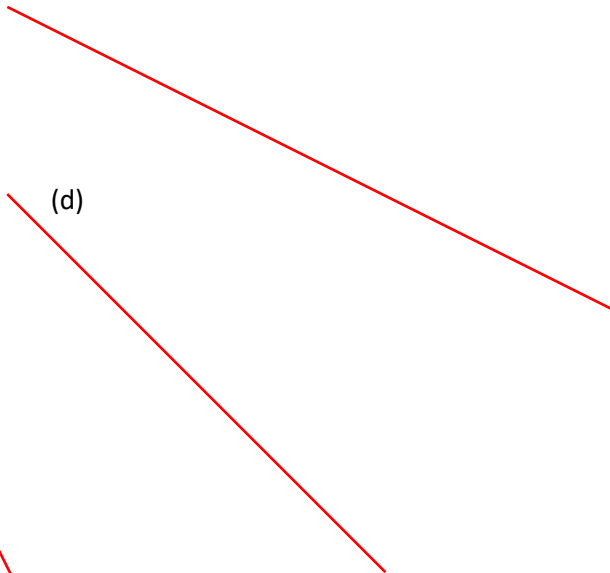
(a)



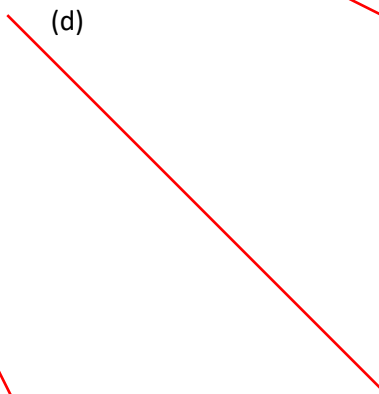
(b)



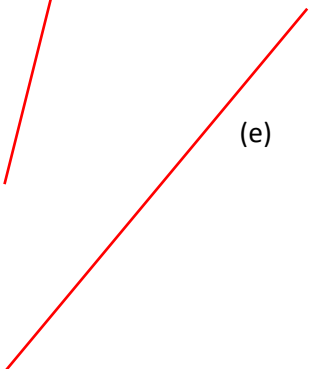
(c)



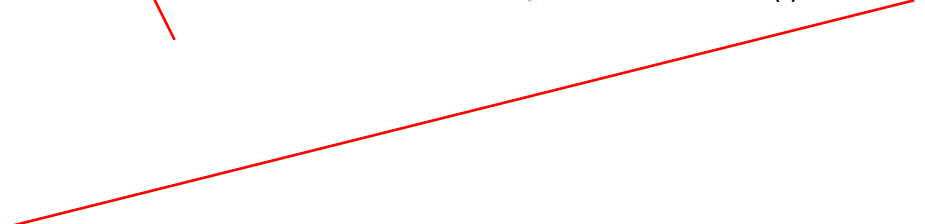
(d)



(e)



(f)

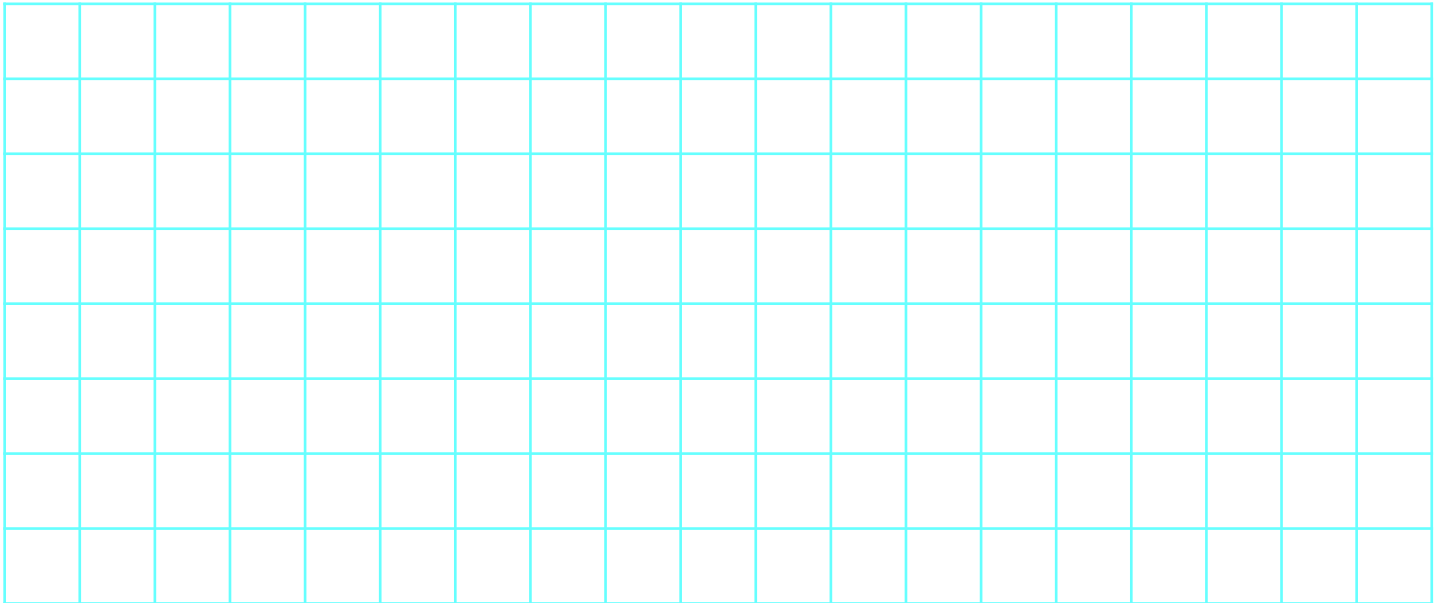


Exercise 3

4



On the squared paper below, draw lines with gradient (a) 5; (b) -1 ; (c) -4 ; (d) $\frac{2}{3}$.



Exercise 4

(a) What is the gradient of any horizontal line?

(b) What is the gradient of any vertical line?

(c) Two road signs are shown on the right; the gradient of the roads are shown as percentages. Write the gradient of the roads as normal numbers.



Challenge!

How could you measure the gradient of any point on a curve?



Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Straight Line Graphs

Vertical lines of the form $x = a$

Let us consider plotting co-ordinates on a set of axes.

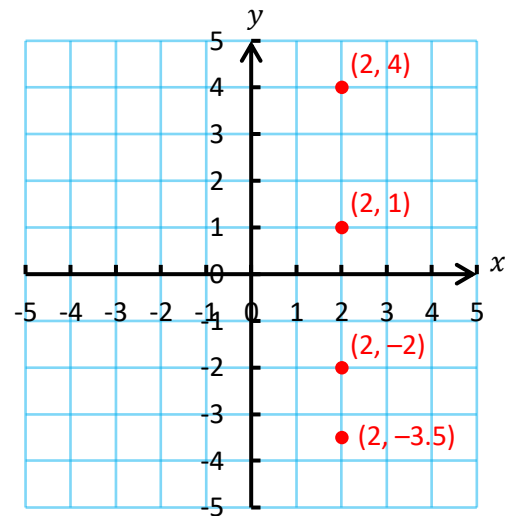
On the right, some co-ordinates are shown where the x -coordinate is always two.

Imagine what would happen if **all** the possible co-ordinates whose x -coordinate is two were plotted on the axes shown on the right.

The co-ordinates would come together to form a single vertical line, a line that goes through the point $(2, 0)$.

The equation of this line would be $x = 2$.

In general, the equation $x = a$ represents a **vertical** line that passes through the point $(a, 0)$.



Exercise 5

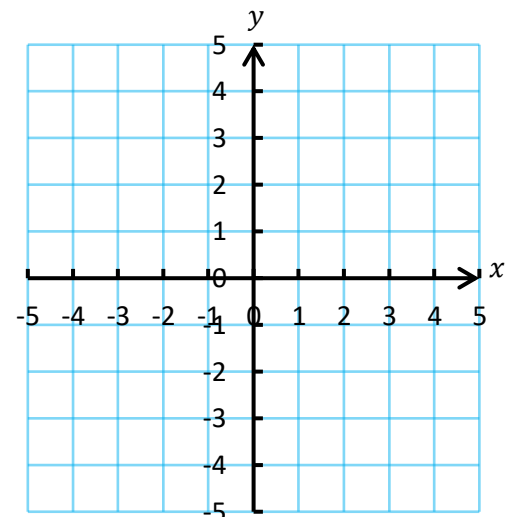
On the axes shown on the right, plot the following lines.

- (a) $x = 3$
- (b) $x = -2$
- (c) $x = 4.5$
- (d) $x = -1.5$
- (e) $x = -4$



Skill

6



Horizontal lines of the form $y = b$

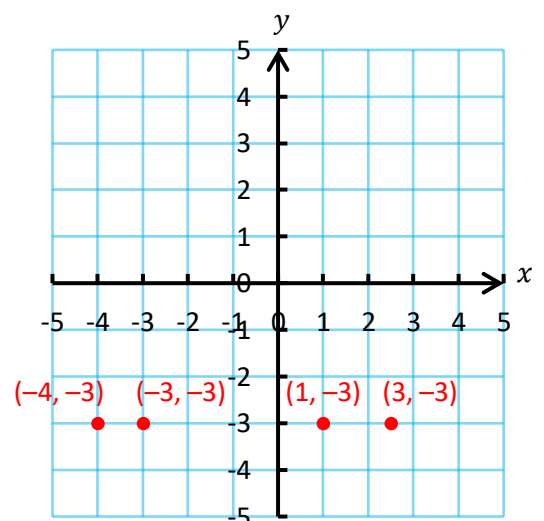
On the right, some co-ordinates are shown where the y -coordinate is always -3 .

Imagine what would happen if **all** the possible co-ordinates whose y -coordinate is -3 were plotted on the axes shown on the right.

The co-ordinates would come together to form a single horizontal line, a line that goes through the point $(0, -3)$.

The equation of this line would be $y = -3$.

In general, the equation $y = b$ represents a **horizontal** line that passes through the point $(0, b)$.



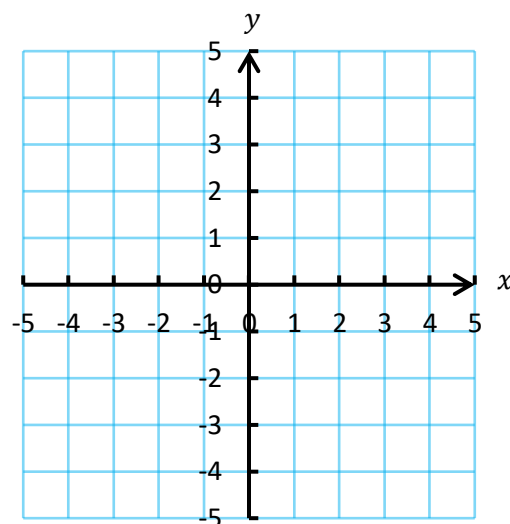
Exercise 6

6

On the axes shown on the right, plot the following lines.

- (a) $y = 2$
- (b) $y = -4$
- (c) $y = 2.5$
- (d) $y = -3.5$
- (e) $y = 4$

Remember to
use a ruler!

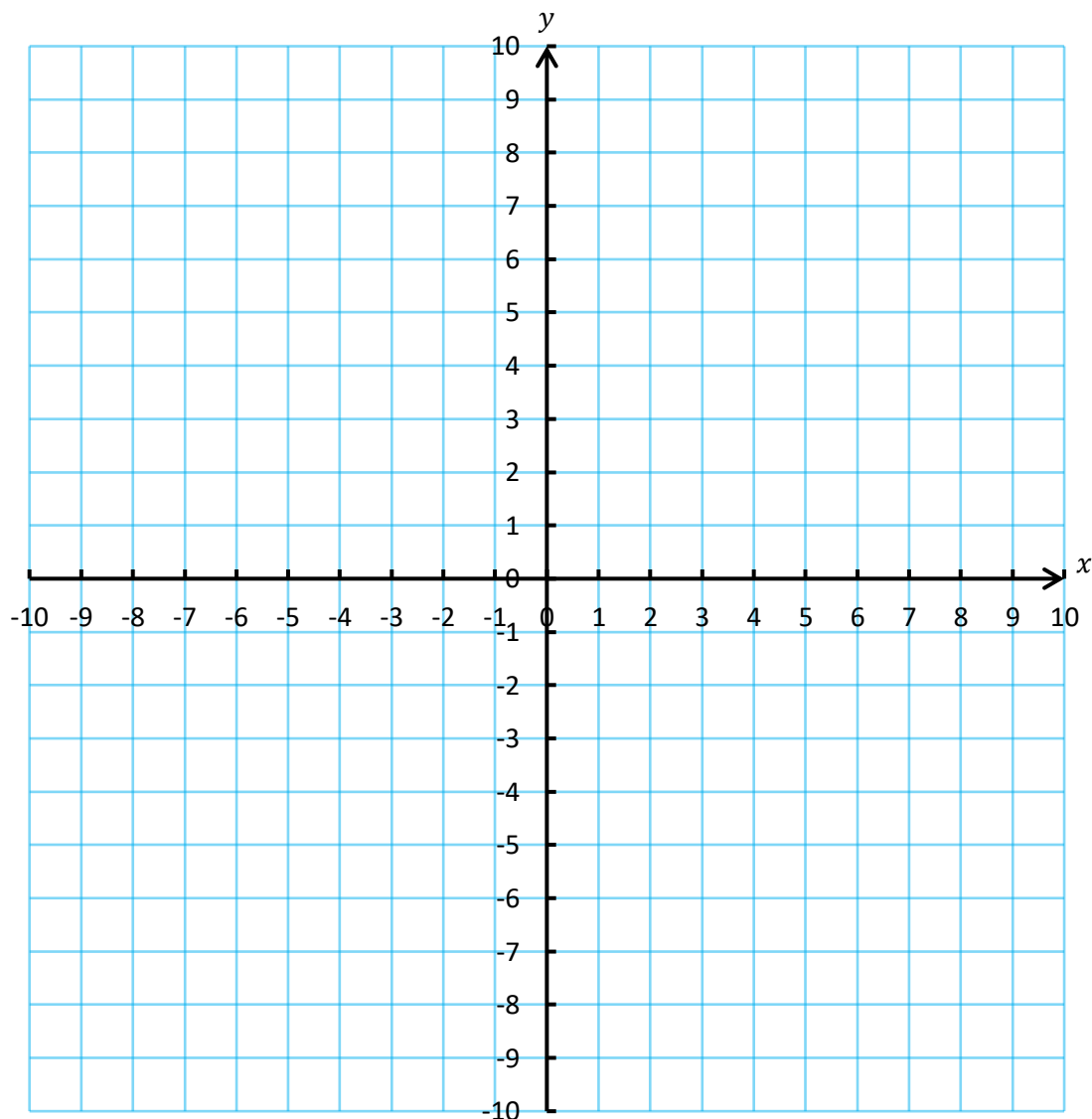
**Exercise 7**

- (a) What is the equation of the x -axis?
- (b) What is the equation of the y -axis?

Exercise 8

On the axes shown below, plot the following lines.

- | | | | |
|----------------|---------------|----------------|-----------------|
| (a) $x = 7$ | (b) $y = 6$ | (c) $x = -5$ | (d) $y = -9$ |
| (e) $x = 3.25$ | (f) $y = 8.5$ | (g) $x = -7.5$ | (h) $y = -2.75$ |



General lines of the form $y = mx + c$

If we have a straight line that is not vertical or horizontal, then it is possible to write an equation for the line in the form $y = mx + c$.



TO LEARN:

m is the gradient of the line. For each 1 unit we go to the right, we need to go m units up/down.

$y = mx + c$

**c is the y -intercept of the line.
The line passes through the point $(0, c)$.**

@MATHMATE8

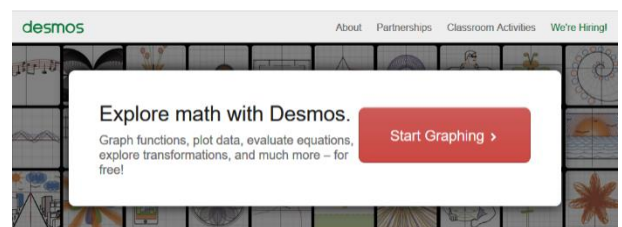
Exercise 9

6

You will need to use the website www.desmos.com/calculator to complete this exercise.

Type $y = mx + c$ into the box. When the “add slider” option appears click “all”.

- Which values for m and c does the computer set?
- What happens as you change the value of m ?
- What stays the same as you change the value of m ?
- What happens as you change the value of c ?
- What stays the same as you change the value of c ?
- In box 4 type $(0, 3)$ and in box 5 type $(2, 9)$. Change the values of m and c until the straight line passes through the two points. What is the equation of this line?
- Change your points to $(0, -3)$ and $(-2, 7)$. What is the equation of the straight line that passes through these points?
- What is the equation of the straight line that passes through the points $(0, -5)$ and $(10, 0)$?

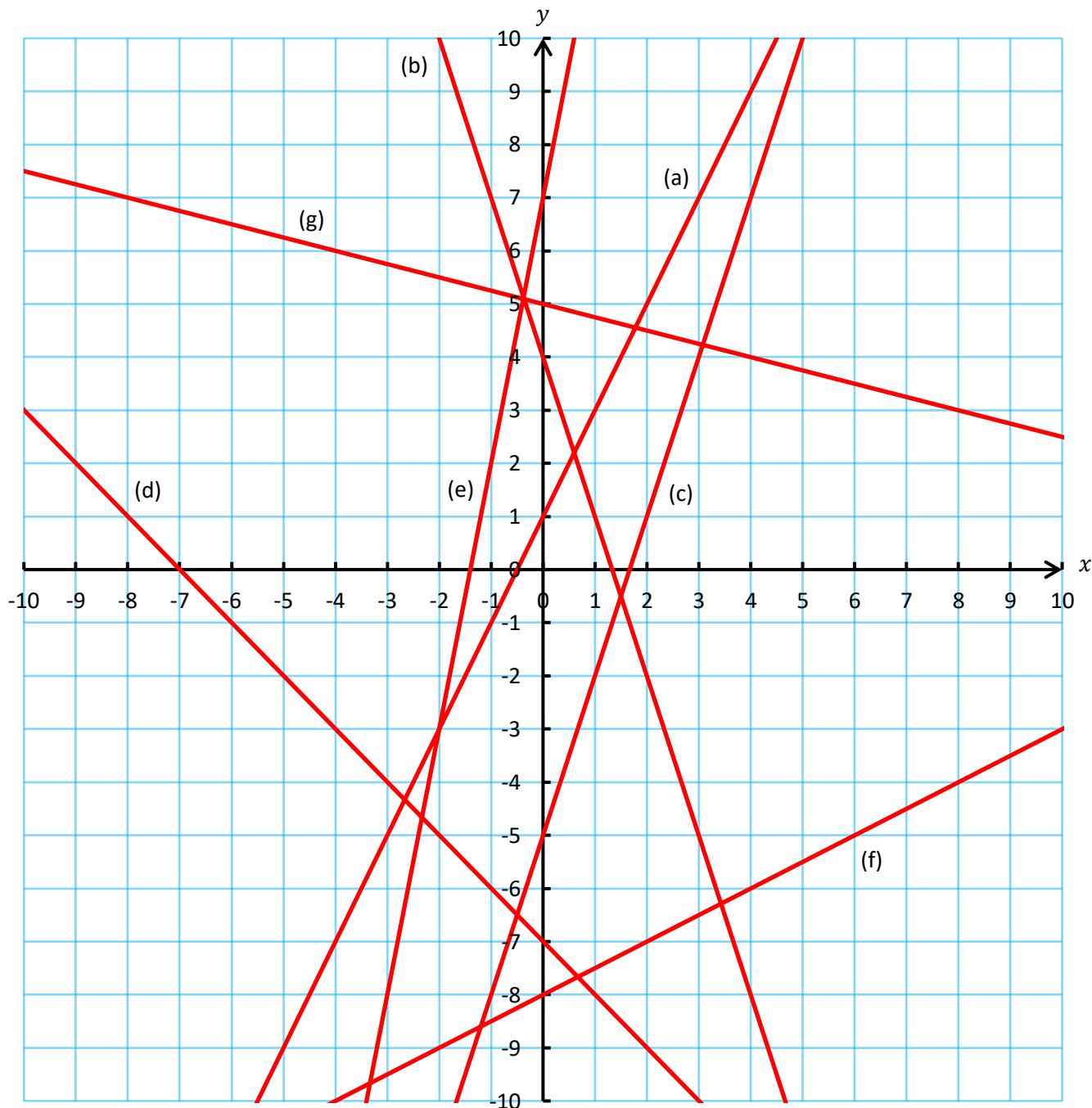


Exercise 10

6

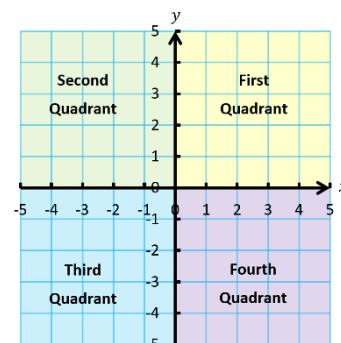
For each of the following lines,

- Write down the gradient of the line;
- Write down the y -intercept of the line;
- Write down the equation of the line in the form $y = mx + c$.

**Exercise 11**

Consider again the lines (a) to (g) from Exercise 10.

- Which line is the steepest?
- Which line is the least steep?
- In which quadrant do the lines (a) and (e) intersect?
- In which quadrant will the lines (b) and (d) intersect?



Plotting a straight line of the form $y = mx + c$

Given a straight line of the form $y = mx + c$, for example $y = 2x - 1$, here are two ways of plotting the line.

Method 1: Using a table

(a) Substitute different values of x into the equation in order to create a table of values.

x	0	1	2	3
y	-1	1	3	5
	↑	↑	↑	↑
	$2 \times 0 - 1$	$2 \times 1 - 1$	$2 \times 2 - 1$	$2 \times 3 - 1$
	$= 0 - 1$	$= 2 - 1$	$= 4 - 1$	$= 6 - 1$
	$= -1$	$= 1$	$= 3$	$= 5$

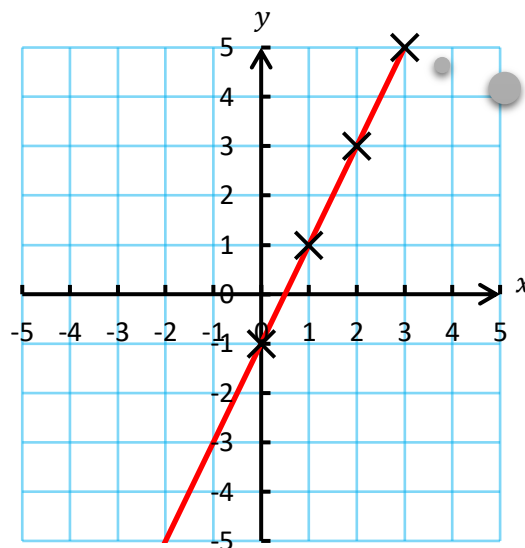
(b) Plot the values from the table on graph paper before connecting the points with a straight line.

Method 2: Use the gradient and y -intercept

(a) For the line $y = 2x - 1$, the y -intercept is -1 , so the line passes through the point $(0, -1)$.

Plot this point on graph paper.

(b) The gradient is 2, so for each **one** unit we go to the right (starting from the point $(0, -1)$), we must move **two** units up. Plot some of these points before connecting the points with a straight line.



It is possible to plot using a cross (as shown here) or using a dot.

Exercise 12

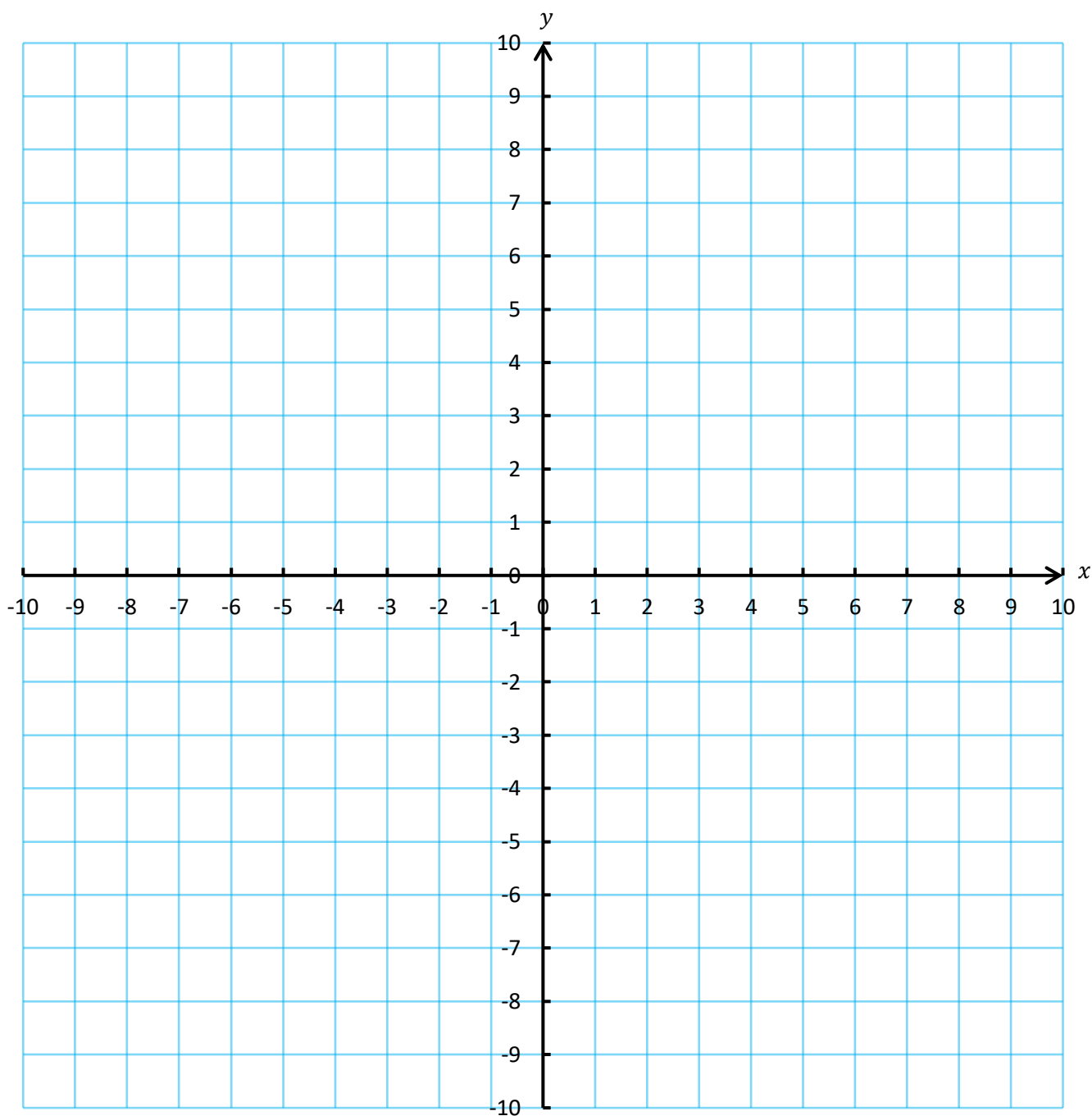
Complete the following table.

Question	Equation	Table of values	y -intercept	Gradient										
(a)	$y = 2x + 3$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(b)	$y = 3x + 1$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(c)	$y = 3x - 1$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(d)	$y = -2x + 1$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(e)	$y = -2x - 4$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(f)	$y = 5x - 6$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(g)	$y = -4x + 10$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														
(h)	$y = \frac{1}{2}x + 4$	<table> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr><td>y</td><td></td><td></td><td></td><td></td></tr> </table>	x	0	1	2	3	y						
x	0	1	2	3										
y														

Exercise 13

6

On the following graph paper, plot the lines from Exercise 12.

**Exercise 14**

In your book, draw x and y axes going from -10 to 10 . Plot the following lines on the axes.

(a) $y = 2x + 4$

(b) $y = 2x - 4$

(c) $y = -2x + 4$

(d) $y = -2x - 4$

(e) $y = 3x - 1$

(f) $y = -4x + 5$

(g) $y = 5x - 8$

(h) $y = -3x + 9$

(i) $y = \frac{1}{2}x - 8$

(j) $y = -\frac{1}{2}x - 4$

(k) $y = x$

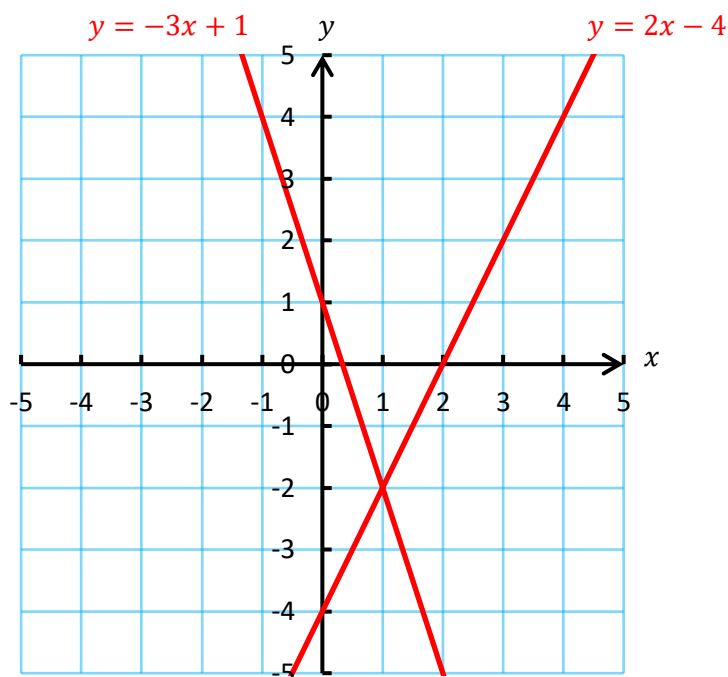
(l) $y = -x$

Challenge! 

Plot the lines (a) $y = \frac{2}{3}x + \frac{1}{2}$; (b) $y = -\frac{3}{4}x - \frac{3}{2}$ on suitable graph paper.

Solving simultaneous equations graphically

Let us consider the equations $y = 2x - 4$ and $y = -3x + 1$. It is possible to plot these equations on graph paper, as shown below.



We note that the lines intersect at the point $(1, -2)$. This means that the values $x = 1$, $y = -2$ satisfy both equations, at the same time. We say that $x = 1$, $y = -2$ is the **solution to the simultaneous equations** $y = 2x - 4$, $y = -3x + 1$.

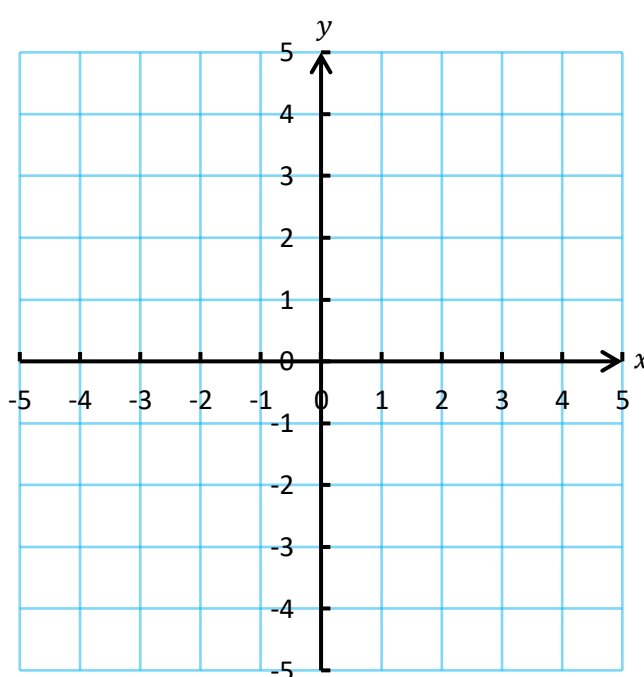
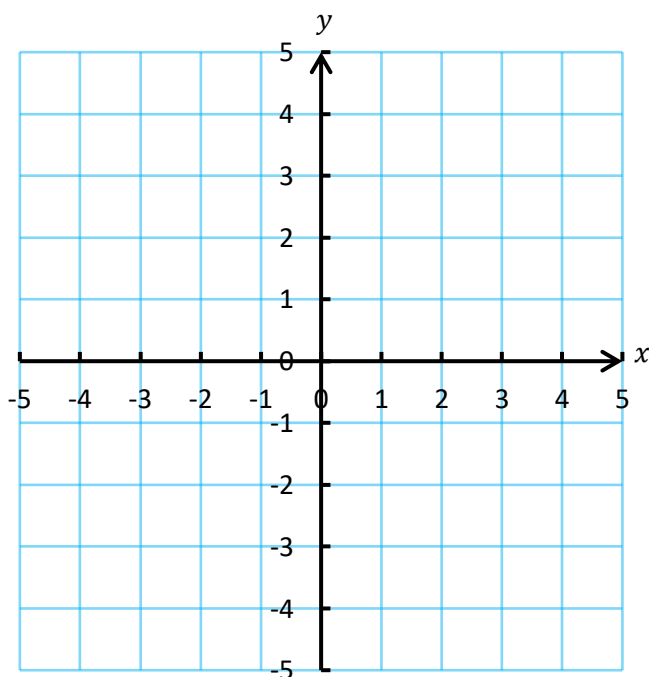
Exercise 15

7

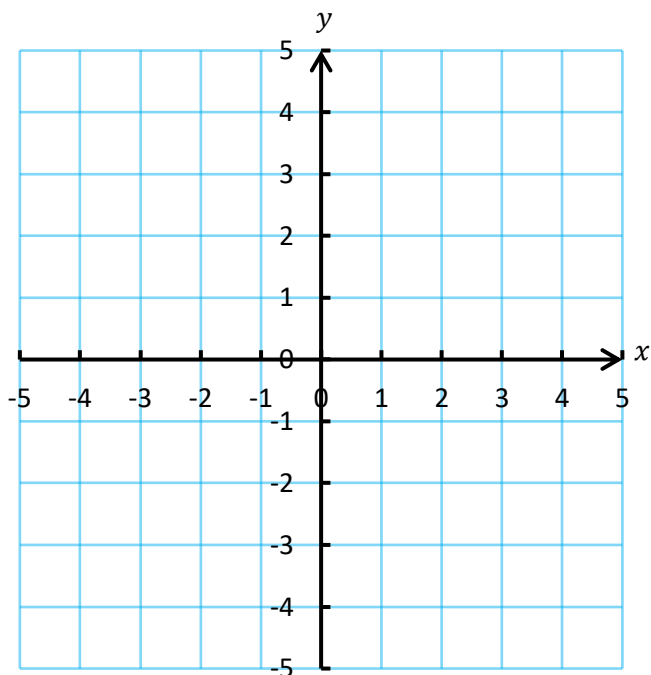
Use the graph paper below to solve the following simultaneous equations.

(a) $y = 2x - 1$, $y = 3x - 2$

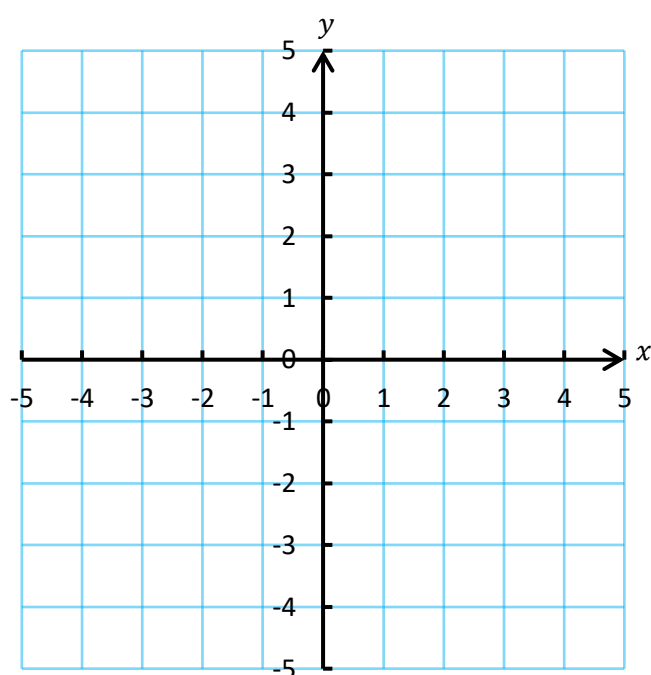
(b) $y = -2x + 3$, $y = x - 3$



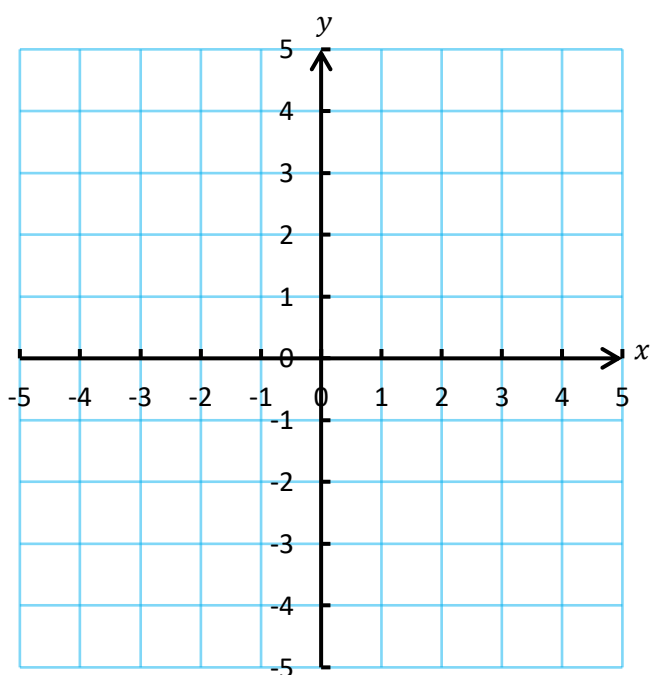
(c) $y = x + 2, y = -x - 4$



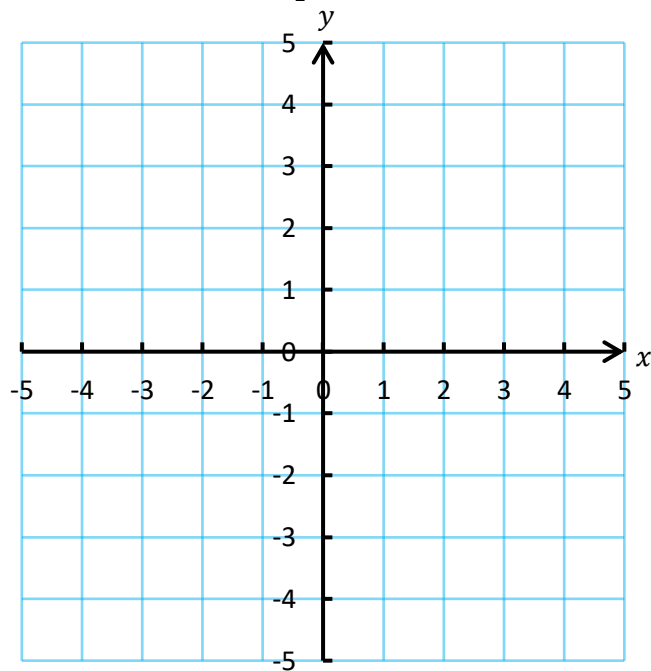
(d) $y = 4x - 2, y = -2x + 4$



(e) $y = \frac{1}{2}x + 3, y = 4x - 4$



(f) $y = -x - 3, y = -\frac{1}{2}x - 2$

**Exercise 16**

Draw suitable axes in order to solve the following simultaneous equations graphically.

(a) $y = x - 1, y = 2x + 1$

(b) $y = -2x + 4, y = -x + 1$

(c) $y = 3x + 4, y = -3x + 4$

(d) $y = 4, y = 6x - 2$

(e) $y = -x + 5, x = 3$

(f) $y = \frac{1}{2}x + 2, y = -\frac{1}{4}x - 1$

Exercise 17

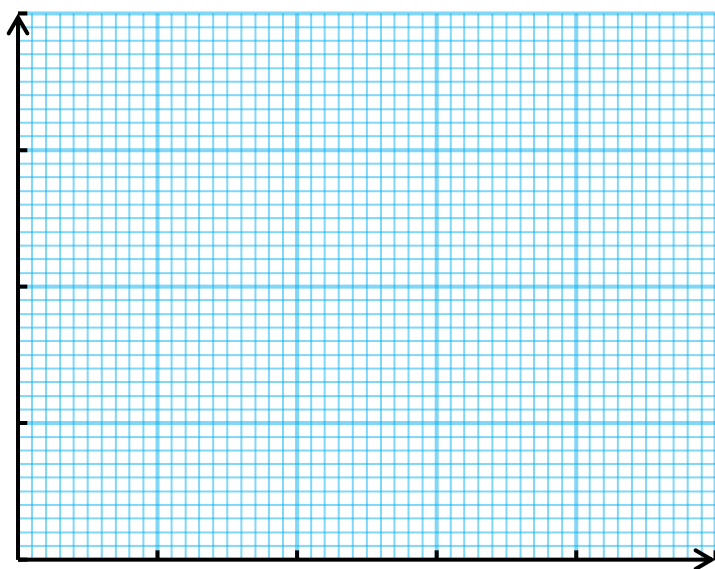
Applying

7

(a) Charlotte is arranging a prom for her year group. The number of people attending the prom is likely to be between 20 and 80. Here is the cost of holding the prom in Afonwen hotel.

- Room hire: £100
- Food: £15 per person
- A drink on arrival: £3 per person
- Decorations: £2 per person

Draw a graph to show the cost of holding the prom for between 20 and 80 people. Use the graph paper below.



(b) In reality, the costs for arranging the prom were £1,500.
Use the graph from part (a) to find how many people attended the prom.



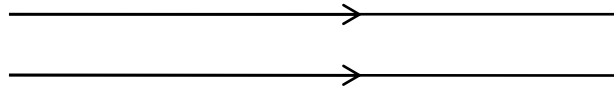
Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

Parallel, Perpendicular

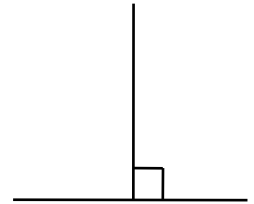


Parallel lines do not meet (or intersect).



We use *arrows* to show when two lines are parallel.

Two lines are **perpendicular** if they meet at a right angle.



Exercise 18

Revision

3

The picture on the right shows Conwy suspension bridge. Discuss where it is possible to see parallel and perpendicular lines in the picture.



The gradient of parallel and perpendicular lines

Parallel lines have the **same** gradient.

The gradients of **perpendicular** lines multiply to give **-1**. Or, one gradient is the **negative reciprocal** of the other gradient.

Example

The lines L_1 , L_2 , L_3 and L_4 have respective gradients 2 , $-\frac{1}{2}$, $\frac{1}{2}$ and 2 .

The lines L_1 and L_4 are parallel as they have the same gradient (2).

The lines L_1 and L_2 are perpendicular as their gradients multiply to give -1 (we have $2 \times -\frac{1}{2} = -1$).

Exercise 19

The lines L_1 , L_2 , L_3 , L_4 and L_5 have respective gradients $\frac{1}{3}$, 3 , $-\frac{1}{3}$, $\frac{1}{3}$ and -3 .

- Which pairs of lines are parallel?
- Which pairs of lines are perpendicular?

Skill

5

Exercise 20

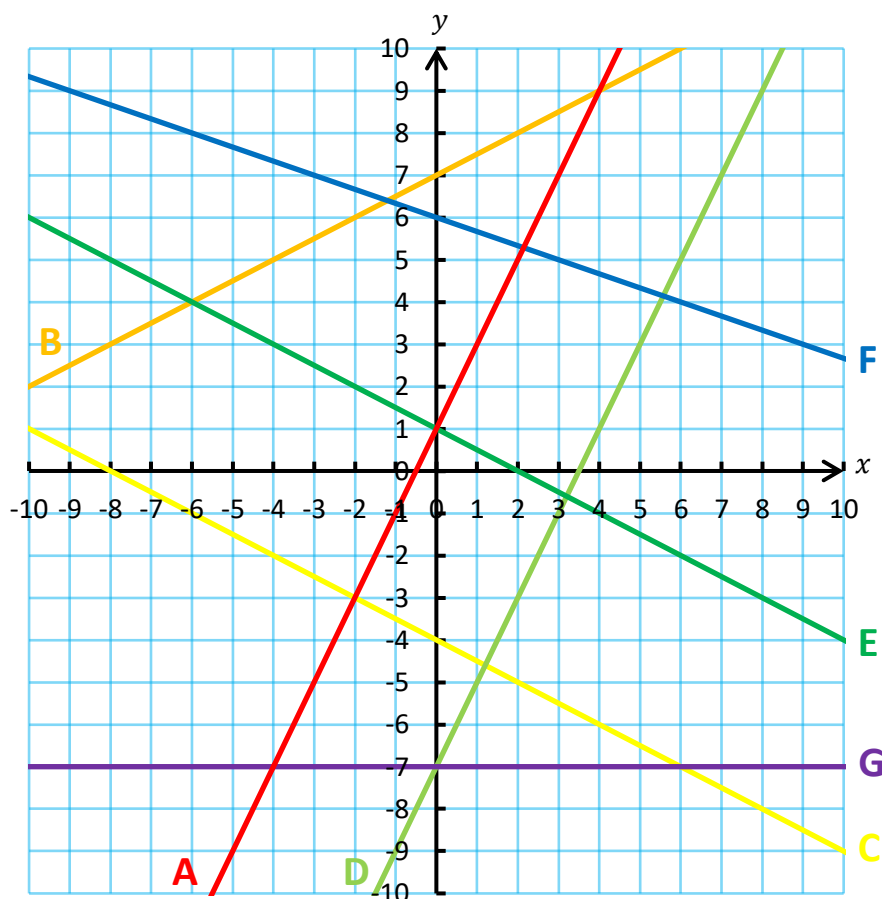
Complete the following table.

Gradient of a line	Gradient of a perpendicular line
4	
6	
$\frac{1}{2}$	
-3	
$-\frac{1}{5}$	

Gradient of a line	Gradient of a perpendicular line
$\frac{2}{3}$	
$-\frac{3}{4}$	
$2\frac{1}{2}$	
$-3\frac{1}{4}$	
$6\frac{4}{5}$	

Exercise 21

6



- What is the gradient of A?
- What is the gradient of C?
- Which lines are perpendicular to C?
- What is the equation of D?
- What is the equation of B?
- Write down an equation of a line that would be parallel to F.
- Which line is parallel to A?
- Which line is parallel to C?
- Which line is parallel to the x -axis?
- Prove that D and F are not perpendicular.
- Write down an equation of a line that would be perpendicular to B.
- What is the area of the triangle formed by the lines A, C and G?

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Transformations: Reflection

Reflection is one of the four transformations.

Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	Enlargement

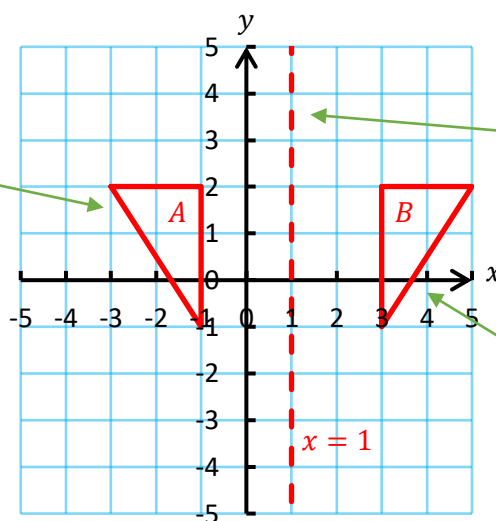
When a shape is reflected, a **mirror line** or **reflection line** is used. Often, this line is given in the form of an equation.

Example

In the following diagram, the triangle *A* has been reflected in the line $x = 1$ to give the triangle *B*. We say that the triangle *B* is the **image** of the reflection.



Triangle *A* is the **original shape**.



The line $x = 1$ is the **mirror line** or **reflection line**. The convention is to draw this line using a dotted line.

The triangle *B* is the **image** of the reflection.



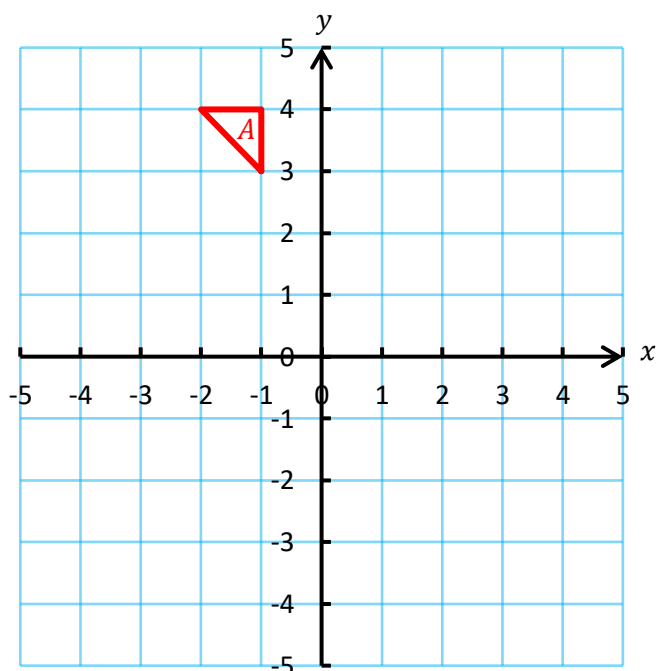
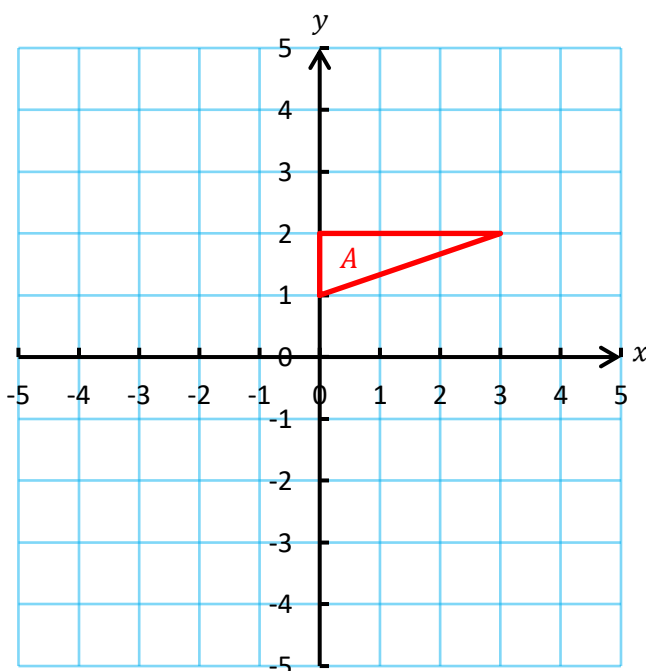
Exercise 22

- (a) Reflect the triangle *A* in the line $x = -1$.
Label the image with the letter *B*.

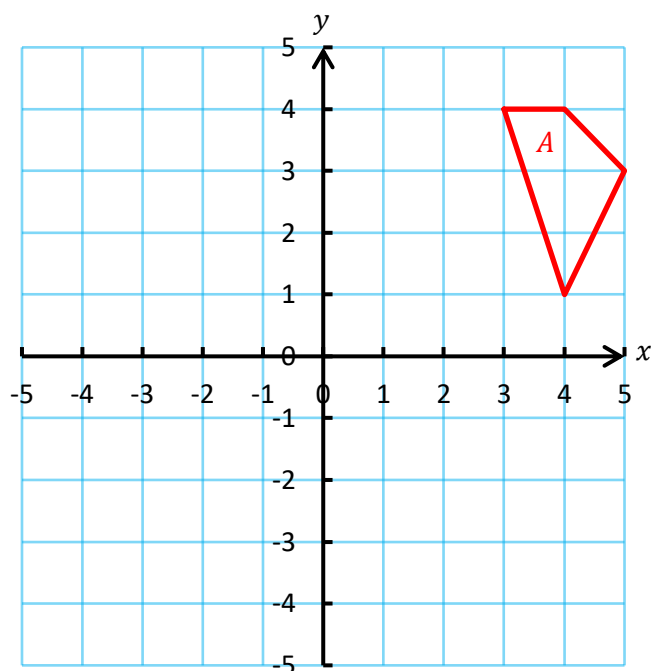
- (b) Reflect the triangle *A* in the line $y = 1$.
Label the image with the letter *B*.

Skill

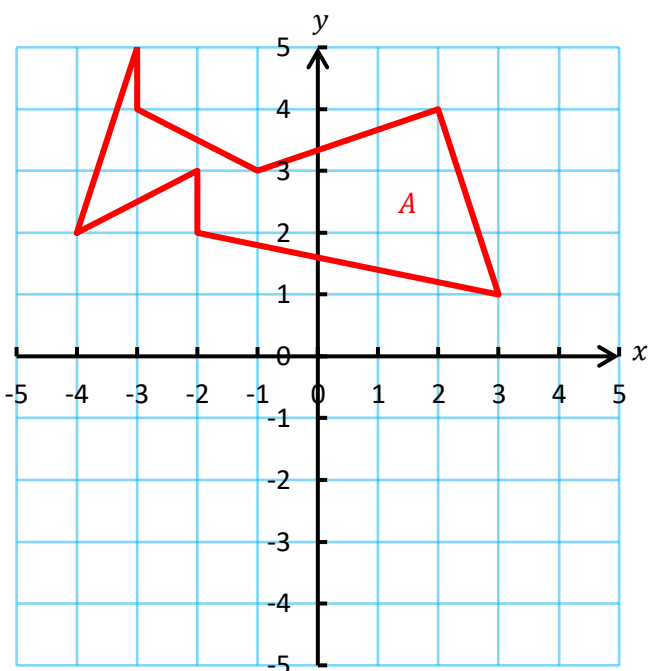
6



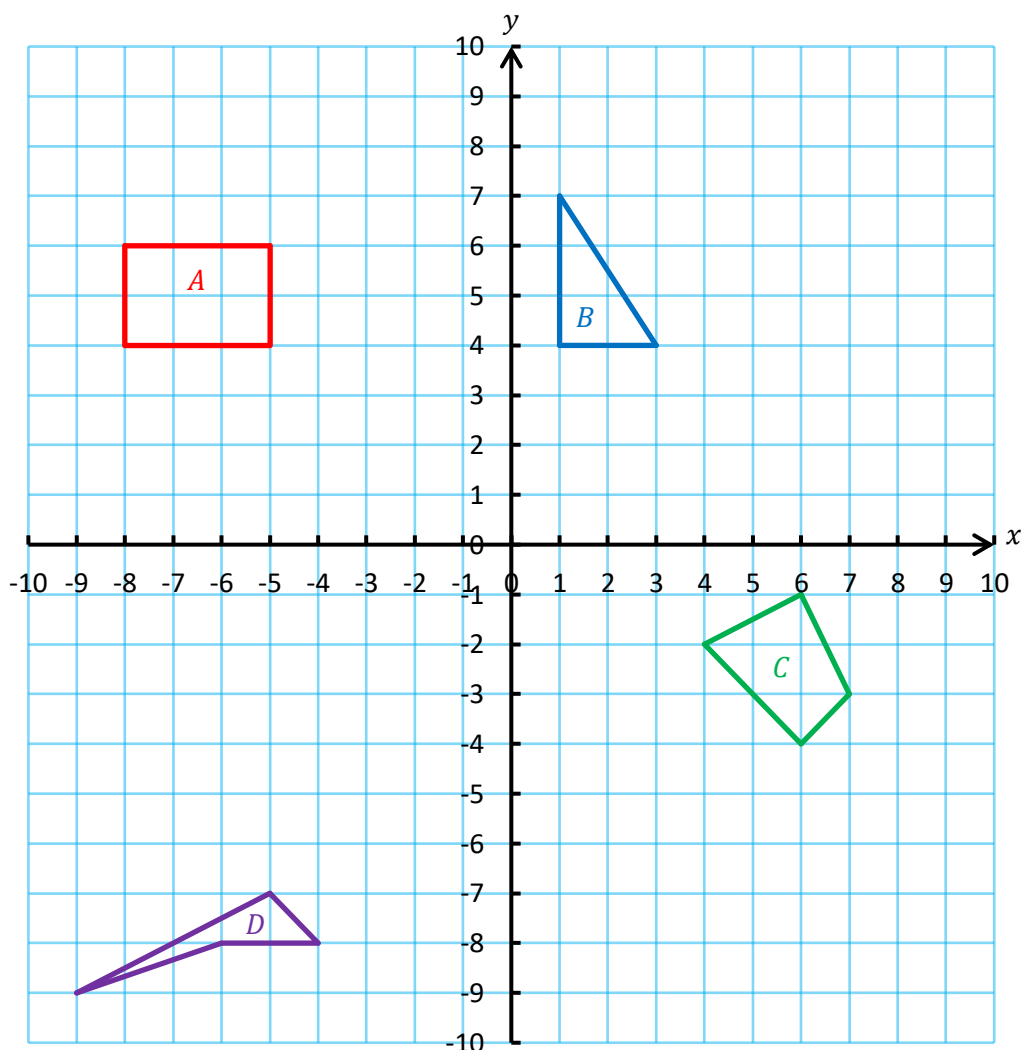
- (c) Reflect the quadrilateral A in the line $x = 2$.
Label the image with the letter B .



- (d) Reflect the octagon A in the x -axis.
Label the image with the letter B .

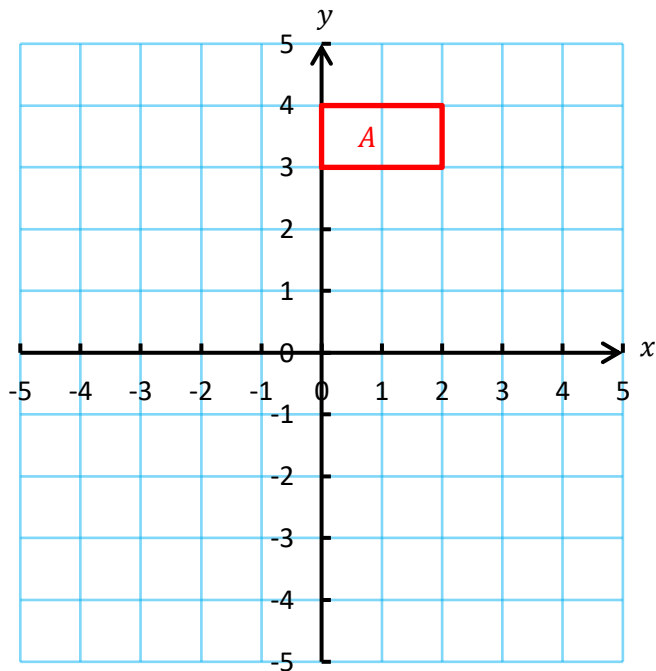


- (e) (i) Reflect the rectangle A in the line $y = 1$. Label the image with the letter A' .
(ii) Reflect the triangle B in the line $x = 5$. Label the image with the letter B' .
(iii) Reflect the quadrilateral C in the line $y = -5$. Label the image with the letter C' .
(iv) Reflect the quadrilateral D in the line $x = -3.5$. Label the image with the letter D' .

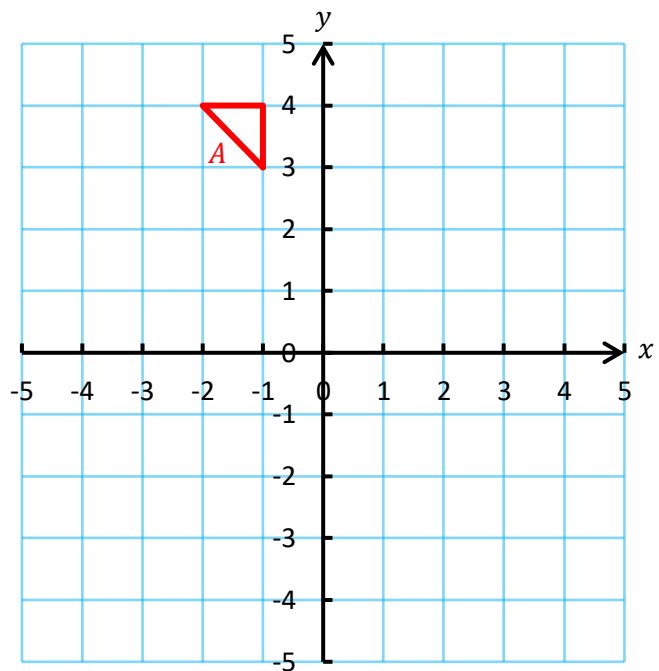


Exercise 23

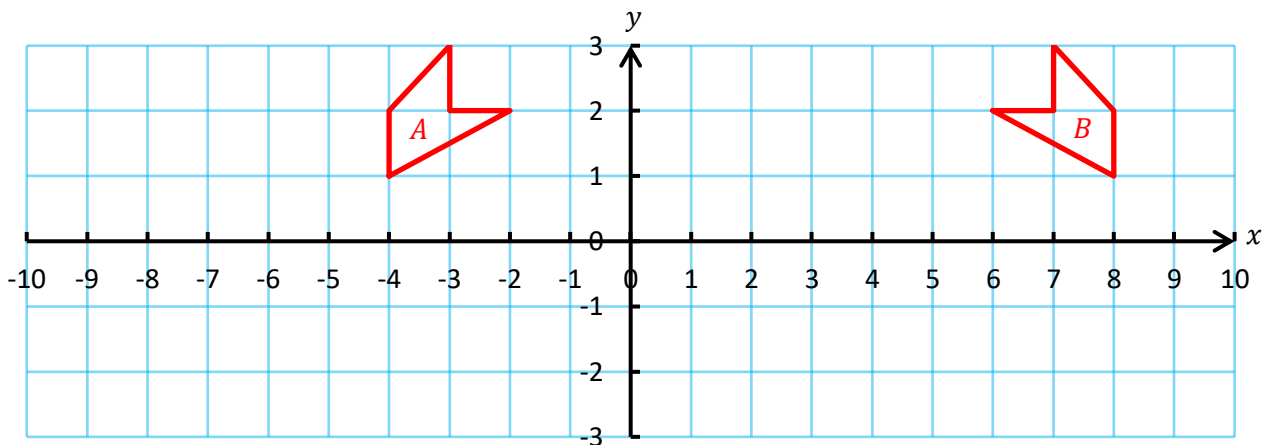
- (a) Reflect the rectangle A in the line $y = x$.
Label the image with the letter B .



- (b) Reflect the triangle A in the line $y = -x$.
Label the image with the letter B .



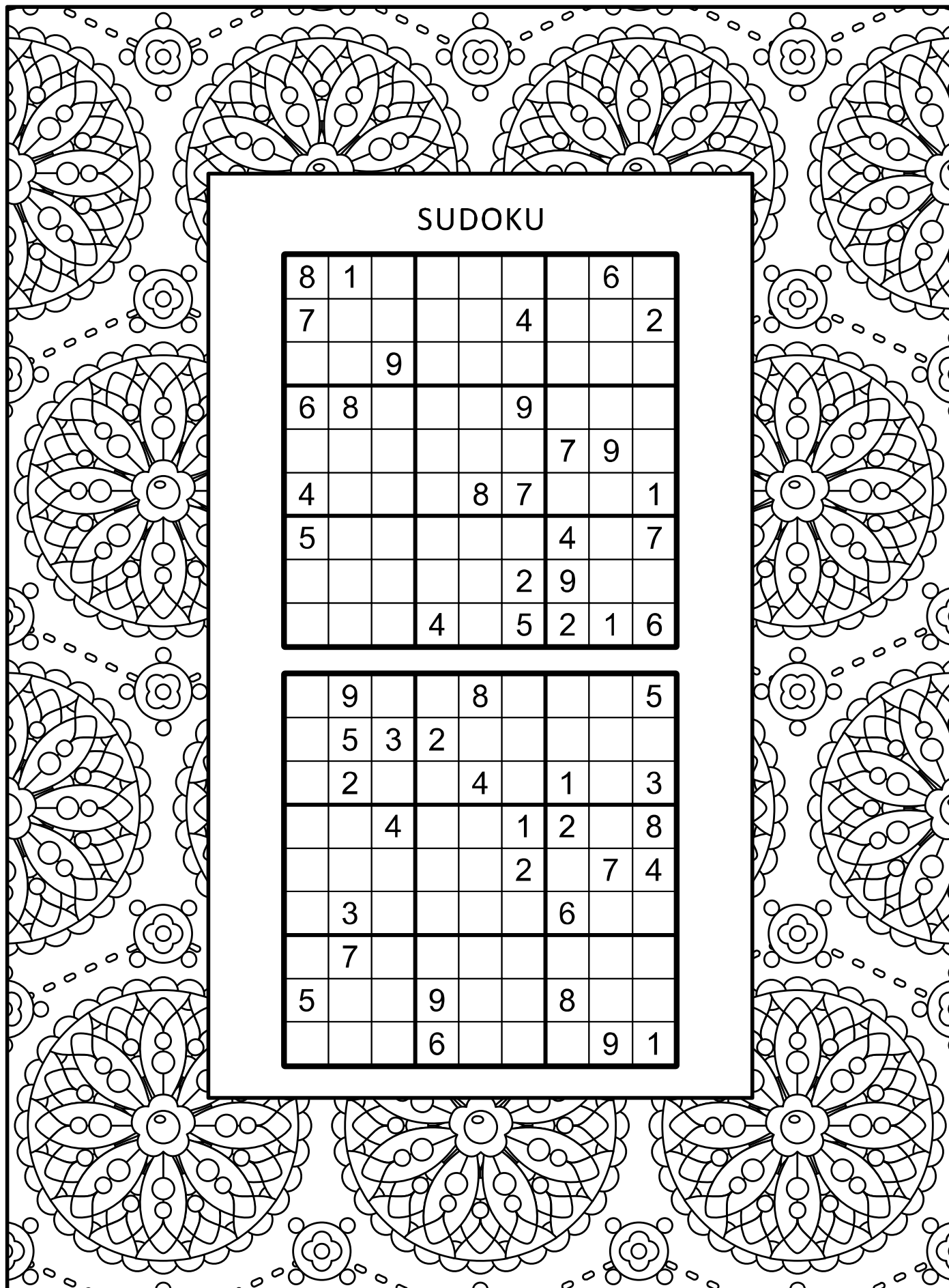
- (c) Describe the reflection that transforms the pentagon A to the pentagon B in the diagram below.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level ☐ My Target ☐

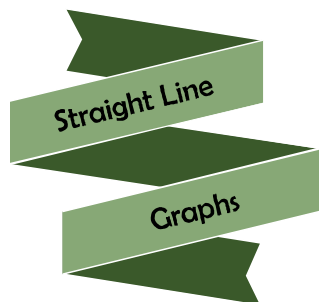
Puzzle: Write the numbers between 1 and 9 in each row, column and 3×3 square.



SUDOKU

8	1					6	
7				4			2
		9					
6	8				9		
						7	9
4				8	7		1
5						4	7
				2	9		
			4	5	2	1	6

	9			8			5
	5	3	2				
	2			4		1	3
		4			1	2	8
					2		7
	3					6	
	7						
5			9			8	
			6				9



Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the test:Correct
in the
test?

I know how to measure the gradient of a line.			1, 5	
I know how to draw a line with a specific gradient .			1	
I know how to draw a straight line of the form $x = a$.			3	
Given a vertical line on graph paper, I can write an equation for the line in the form $x = a$.			5	
I know how to draw a straight line of the form $y = b$.			3	
Given a horizontal line on graph paper, I can write an equation for the line in the form $y = b$.			5	
I know the meaning of the symbols m and c in the equation $y = mx + c$.			5	
I know how to draw a straight line of the form $y = mx + c$.			3, 4	
Given a straight line on graph paper, I know how to write an equation for the line in the form $y = mx + c$.			5	
I know how to solve simultaneous equations graphically .			8	
I can solve problems involving plotting and interpreting straight lines.			7	
I know the condition for two lines to be parallel .			5	
I know the condition for two lines to be perpendicular .			2, 5	
I can reflect shapes in the lines $x = a$ or $y = b$.			6	
I can reflect shapes in the lines $y = x$ or $y = -x$.			6	
I can describe the reflection that transforms one shape to another shape.			6	



Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



The Mathematics Department

9

Movement

with Sphero

Name:

Contents

Chapter	Mathematics	Page Number
Programming Sphero	Working Systematically. Thinking Logically.	3
Travel Graphs	Distance, Time and Speed. Distance-Time Graphs. Speed-Time Graphs.	5
Bearings	Directions of the Compass. Moving on a Plane. Finding Bearings. Locating a Point.	13
Loci	Distance from a Point. Distance from a Line. Perpendicular Bisector. Angle Bisector. Drawing the Perpendicular from a Point to a Line.	20



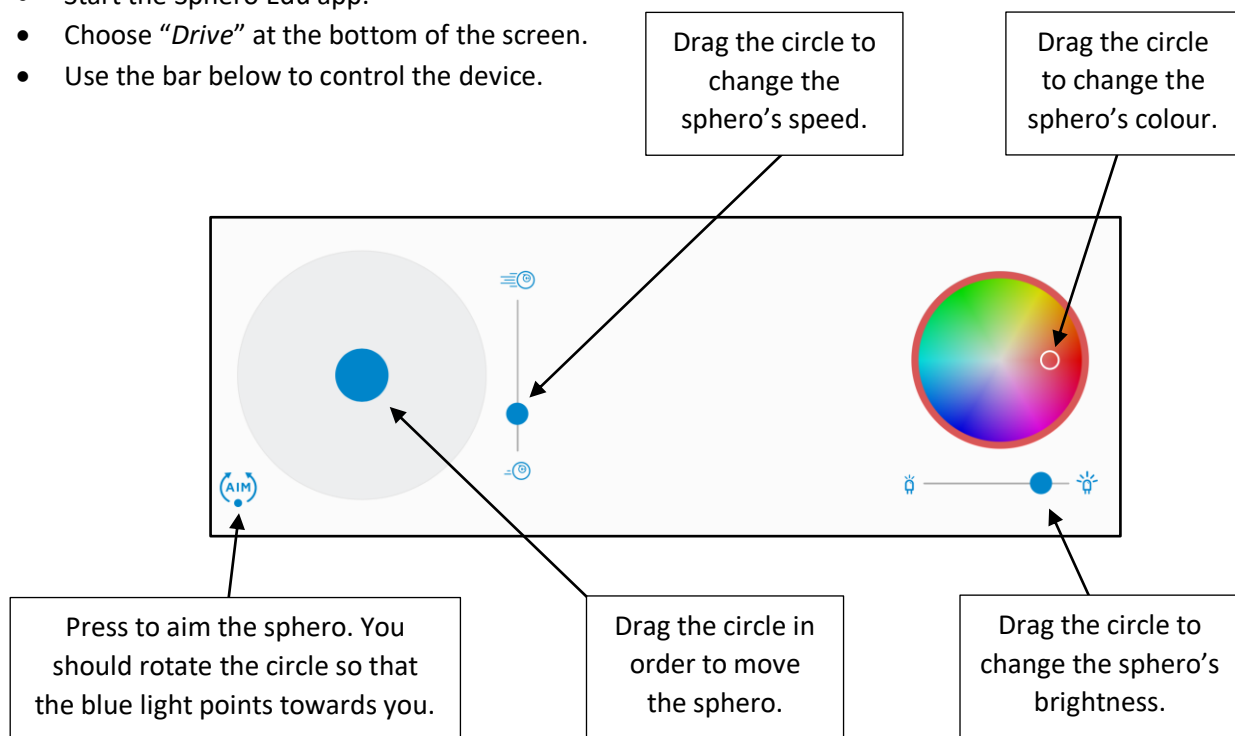
Programming Sphero

In this workbook, you will be working with **Sphero**, a ball that can move under the instructions of the **Sphero Edu** app.

The sphero balls connect to the school iPad using Bluetooth. It is possible to connect a Sphero Bolt or a Sphero SPRK directly to the app, but a Sphero 2.0 must be connected to the iPad using the Bluetooth settings of the iPad.

In order to control sphero:

- Start the Sphero Edu app.
- Choose “Drive” at the bottom of the screen.
- Use the bar below to control the device.



Take a few minutes to become used to controlling the sphero successfully.

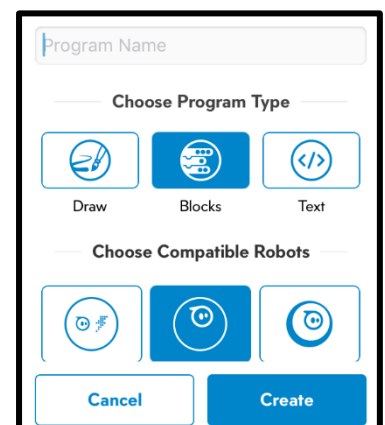
Rules when using the sphero:

- Do not throw the sphero.
- Try not to drop the sphero – it will not break if it falls from a table, but try to catch it first!
- Follow the instructions of the teacher at the end of the lesson to keep the iPad and sphero tidily.



To create a program:

- Click “Programs” at the bottom of the screen.
- Press the button to create a new program.
- It is possible to create three types of program: “Draw”, “Blocks” or “Text”. Most of the time, we will be using the “Blocks” type.
- Under “Choose Compatible Robots”, make sure that you choose “Sphero BOLT” and “Sphero”.



Applying

4

Exercise 1

Create a new program based on blocks.

Copy the program that is shown on the right into your program. Run the program (make sure to aim the sphero first). How does the sphero move under the effect of the program?

Exercise 2

Modify the program from Exercise 1 to move around your table, around 10 cm from the edge.

After finishing, write down the program's instructions in your book.

Exercise 3

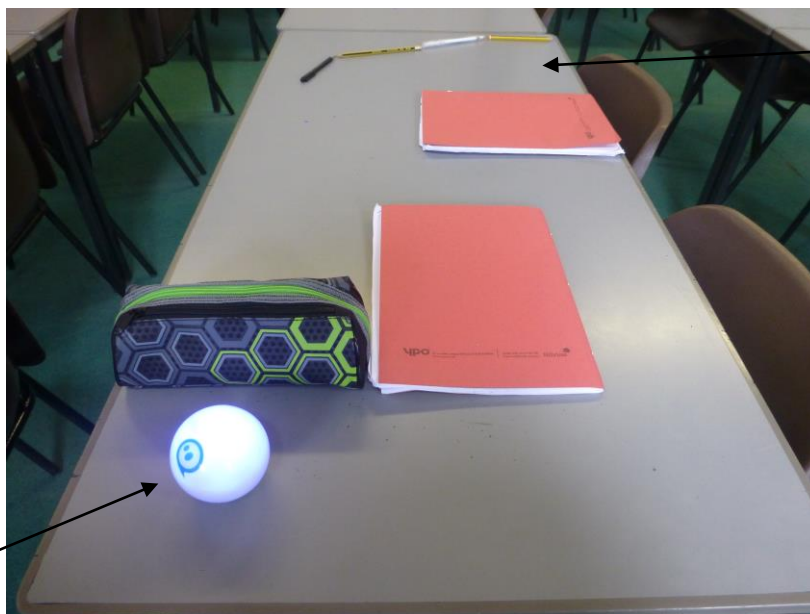
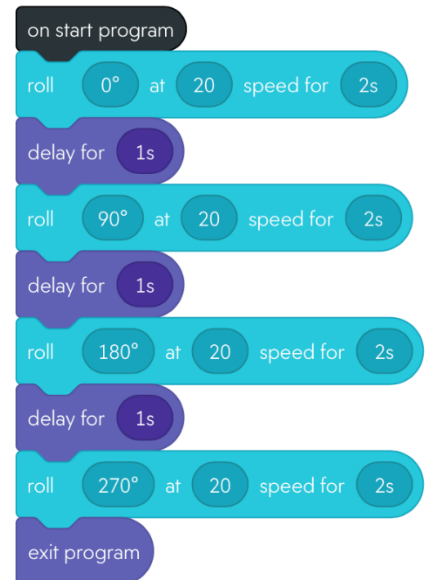
Program the sphero to move around your table three times in succession, using the "Loop" command.

Exercise 4

Using objects such as your pencil case, create an obstacle course on your table. Program the sphero to move from one corner of the table to the opposite corner,

- using a "Draw" program;
- using a "Blocks" program.

After finishing, sketch the obstacle course in your book, and write down the program's instructions.



Sphero to finish here.

Sphero to start here.

Exercise 5

Experiment with the instructions available in the programming language. Can you:

- Change the sphero's colour?
- Change the sphero's direction after it hits something?
- Make sphero move randomly?

Challenge!

Can you program the sphero to move in a circle?
Is there more than one way of doing this?

[YouTube/adolygumathemateg](https://www.youtube.com/channel/UCadolygumathemateg)



Games

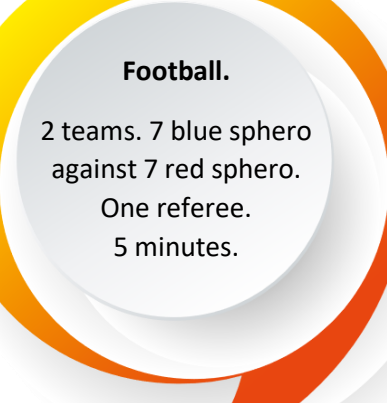


Football.

2 teams. 7 blue sphero
against 7 red sphero.

One referee.

5 minutes.



Football.

2 teams. 7 blue sphero
against 7 red sphero.

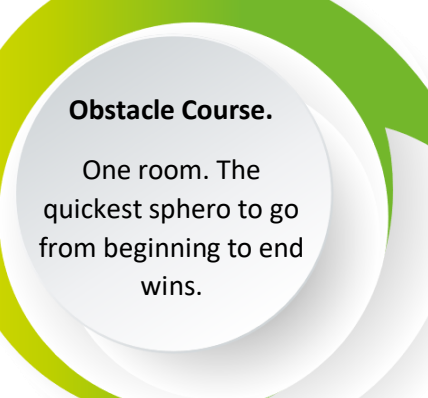
One referee.

5 minutes.

A graphic featuring a large, thick, lime-green circular arrow that curves around a central grey circle. The grey circle contains the text 'Obstacle Course.' and a paragraph. The background is white with a blue curved shape at the bottom right.

Obstacle Course.

One room. The quickest sphero to go from beginning to end wins.

A graphic featuring a large, thick, lime-green circular arrow that curves around a central grey circle. The grey circle contains the text 'Obstacle Course.' and a paragraph. The background is white with a blue curved shape at the bottom right.

Obstacle Course.

One room. The quickest sphero to go from beginning to end wins.



Darts.

Program the sphero to land on the dart board. The first to 301 wins.

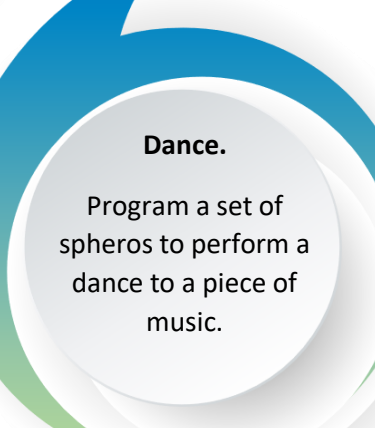


Darts.

Program the sphero to land on the dart board. The first to 301 wins.

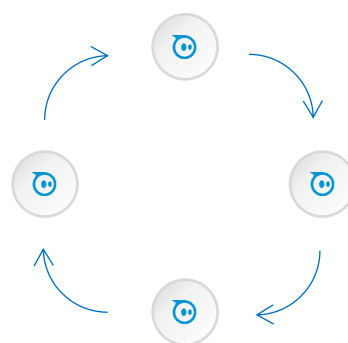
Dance.

Program a set of spheros to perform a dance to a piece of music.



Dance.

Program a set of spheros to perform a dance to a piece of music.



Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Travel Graphs

Applying

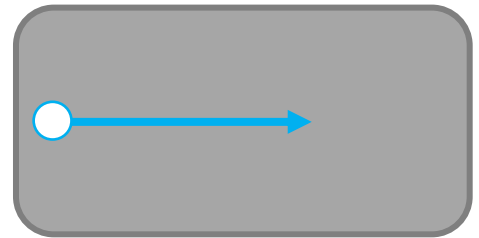
4

Exercise 6

Aim your sphero to travel along the longest side of your table, as shown in the diagram on the right.

Use your ruler to measure how **far** the sphero travels under the effect of the following program.

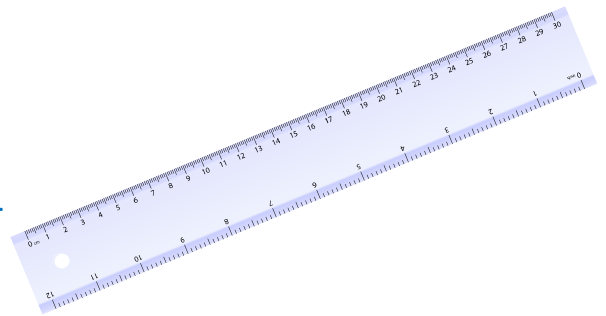
Roll the sphero at an angle 0° ; at a speed of 100%; for 1 second.



Exercise 7

Repeat Exercise 6, but using the following programs.

- (a) Roll the sphero at an angle 0° ; at a speed of 50%; for 1 second.
- (b) Roll the sphero at an angle 0° ; at a speed of 25%; for 1 second.
- (c) Roll the sphero at an angle 0° ; at a speed of 25%; for 2 seconds.
- (d) Roll the sphero at an angle 0° ; at a speed of 50%; for 2 seconds.



Exercise 8

Complete the following sentences.

- (a) Rolling the sphero for the same time but at half the previous speed _____ the distance travelled.
- (b) Rolling the sphero at the same time but at double the previous speed _____ the distance travelled.
- (c) Rolling the sphero for half the previous time but at the same speed _____ the distance travelled.
- (d) Rolling the sphero for double the previous time but at the same speed _____ the distance travelled.

Exercise 9

Write, in your book, two new programs for rolling the sphero the same distance that it travelled during Exercise 6. Check that your programs work.

Exercise 10

Consider the following program.

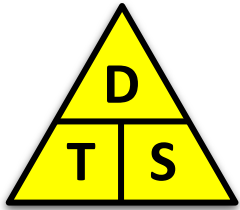
- Roll the sphero at an angle 0° ; at a speed of 100%; for 1 second.
- Delay for 1 second.
- Roll the sphero at an angle 90° ; at a speed of 25%; for 1 second.
- Delay for 1 second.
- Roll the sphero at an angle 180° ; at a speed of 50%; for 2 seconds.
- Delay for 1 second.
- Roll the sphero at an angle 270° ; at a speed of 50%; for 0.5 seconds.

- (a) **WITHOUT** running the program, estimate the distance that the sphero will travel under the effect of the program. Show your method.
- (b) Will the sphero return to its original starting position under the effect of the program?
- (c) Run the program to check your answers to (a) and (b). If your answers were different, try to explain why.



Distance, Time and Speed

When completing Exercises 6–10, you should have noticed a connection between the three measures **distance**; **time** and **speed**. The formula triangle shown below displays the connection between these three measures.



Distance = Time × Speed
Time = Distance ÷ Speed
Speed = Distance ÷ Time

To use the triangle,
hide the letter that you
 want to use.

Skill

7

Exercise 11

- A sphero travels 6 metres in 3 seconds. Calculate the speed of the sphero, in metres per second.
- A sphero travels 9 metres at a speed of 1.5 metres per second. For how long was the sphero travelling?
- A sphero travels at a speed of 0.5 metres per second for 9 seconds. How far has the sphero travelled?

Exercise 12

- A train travels 120 miles in 2 hours. Calculate the average speed of the train, in miles per hour.
- A car travels for an hour and a half at an average speed of 40 miles per hour. How far did the car travel?
- Iestyn walks for 2 km at an average speed of 4 km/hour. For how long was Iestyn walking?
- A boat sails 21 km in 3 hours. Calculate its average speed.
- A car travels 192 miles on a motorway in 3 hours. Calculate its average speed.
- Katie walks at an average speed of 3.5 km/hour for an hour and a half. How far did Katie walk?
- How long does a boat travelling at 12 km/hour take to travel 60 km?

**Exercise 13**

- Sioned walks 2 miles at an average speed of 3 miles per hour. For how long was Sioned walking? Give your answer in minutes.
- Usain Bolt runs a 100 m race in 9.58 seconds. What was his average speed, in metres per second? Give your answer correct to 2 decimal places.
- The manufacturer states that the maximum speed of a sphero is seven feet per second.
 - Change this speed to be in metres per second.
 - How far would the sphero travel in 3 seconds at its maximum speed? Give your answer in cm.
- A football player sprints for 4 seconds, at an average speed of 7.5 metres per second. How far did the player sprint?
- A motorbike travels for 1 hour 20 minutes at an average speed of 30 km/hour. How far did the motorbike travel?

Challenge!

When driving a vehicle on the road, the maximum possible speed is usually 60 miles per hour. What is this speed in metres per second?



Exercise 14

Two spheros are placed 1 metre away from each other.

Applying

7



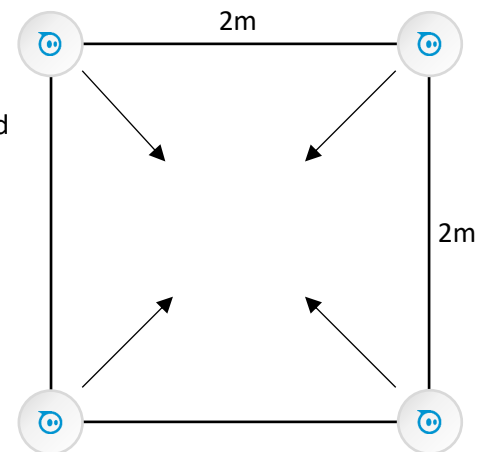
1 m



The spheros travel towards each other at different speeds and meet after 4 seconds. The first sphero travels at a constant speed of **0.15 metres per second**. At which speed is the second sphero travelling?

Challenge!

Four spheros stand at the corners of a square of length 2 metres. At the same time, the spheros start to travel towards the centre of the square. At which speed should the spheros travel in order to reach the centre after 1 second? (Give your answer in metres per second, correct to one decimal place.)

**Travel Graphs for Rectilinear Motion**

Consider the following program.

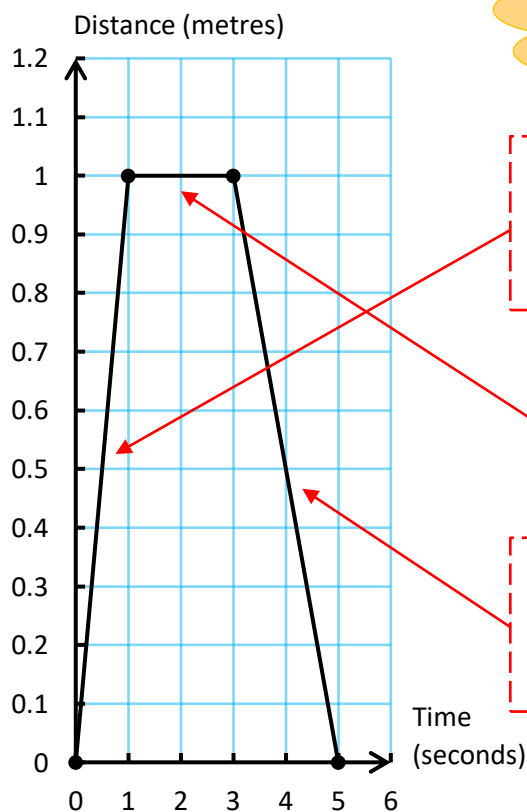
Roll the sphero at an angle 0° ; at a speed of 100%; for 1 second.

Delay for 2 seconds.

Roll the sphero at an angle 180° ; at a speed of 50%; for 2 seconds.

It is possible to draw the sphero's journey in the form of a **distance-time** graph. Here is the graph for this program.

Distance-time graph for the motion of the sphero



Rectilinear motion is motion that moves in one direction only (forwards and backwards).

Roll the sphero at an angle 0° ; at a speed of 100%; for 1 second.

The sphero rolls forwards 1 metre in 1 second.
The gradient of the graph is **positive**.

Delay for 2 seconds.

The sphero is stationary for 2 seconds, at a distance of 1 m away.
The graph is horizontal (with gradient **zero**).

Roll the sphero at an angle 180° ; at a speed of 50%; for 2 seconds.

The sphero rolls backwards 1 metre in 2 seconds.
The gradient of the graph is **negative**.



Exercise 15

7

Here are some different programs for moving the sphero. Draw the distance-time graph for each program.

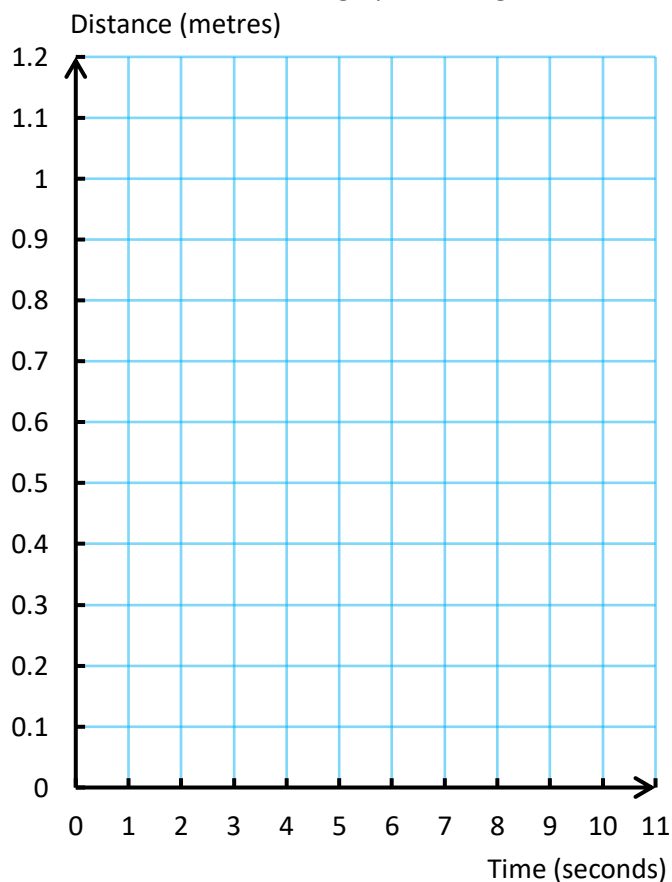
PROGRAM 1

Roll the sphero at an angle 0° ; at a speed of 100%; for 1 second.

Delay for 3 seconds.

Roll the sphero at an angle 180° ; at a speed of 25%; for 4 seconds.

Distance-time graph for Program 1

**PROGRAM 2**

Roll the sphero at an angle 0° ; at a speed of 20%; for 2 seconds.

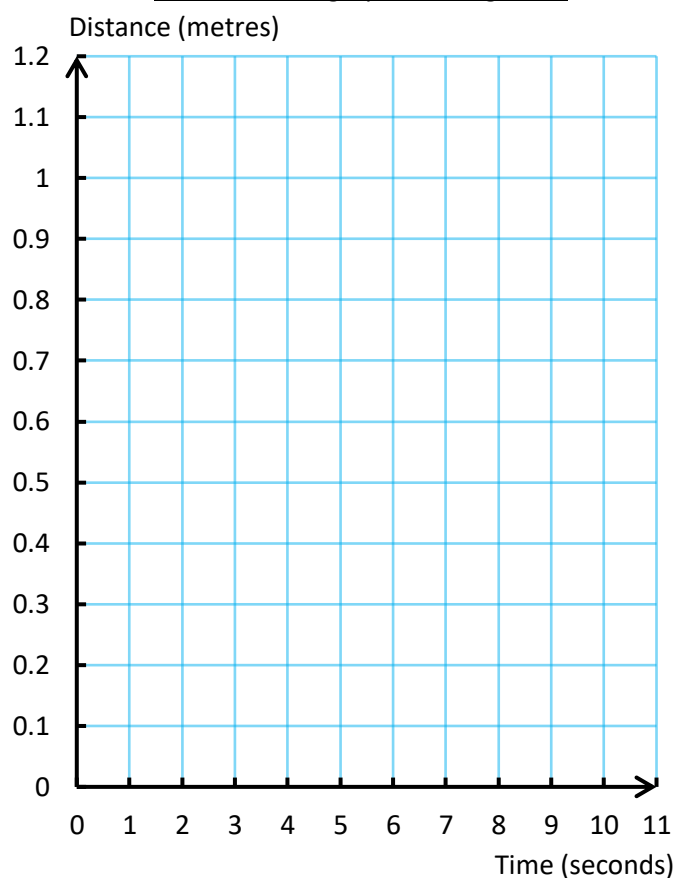
Delay for 1 second.

Roll the sphero at an angle 0° ; at a speed of 30%; for 2 seconds.

Delay for 3 seconds.

Roll the sphero at an angle 180° ; at a speed of 50%; for 2 seconds.

Distance-time graph for Program 2



(Draw the graphs for programs 3 and 4 on a piece of squared paper).

PROGRAM 3

Roll the sphero at an angle 0° ; at a speed of 50%; for 1 second.

Delay for 2 seconds.

Roll the sphero at an angle 180° ; at a speed of 25%; for 2 seconds.

Delay for 1 second.

Roll the sphero at an angle 0° ; at a speed of 60%; for 1 second.

Delay for 1 second.

Roll the sphero at an angle 180° ; at a speed of 30%; for 2 seconds.

PROGRAM 4

Roll the sphero at an angle 0° ; at a speed of 80%; for 1 second.

Delay for 1.5 seconds.

Roll the sphero at an angle 180° ; at a speed of 80%; for 0.5 seconds.

Delay for 2 seconds.

Roll the sphero at an angle 0° ; at a speed of 50%; for 1 second.

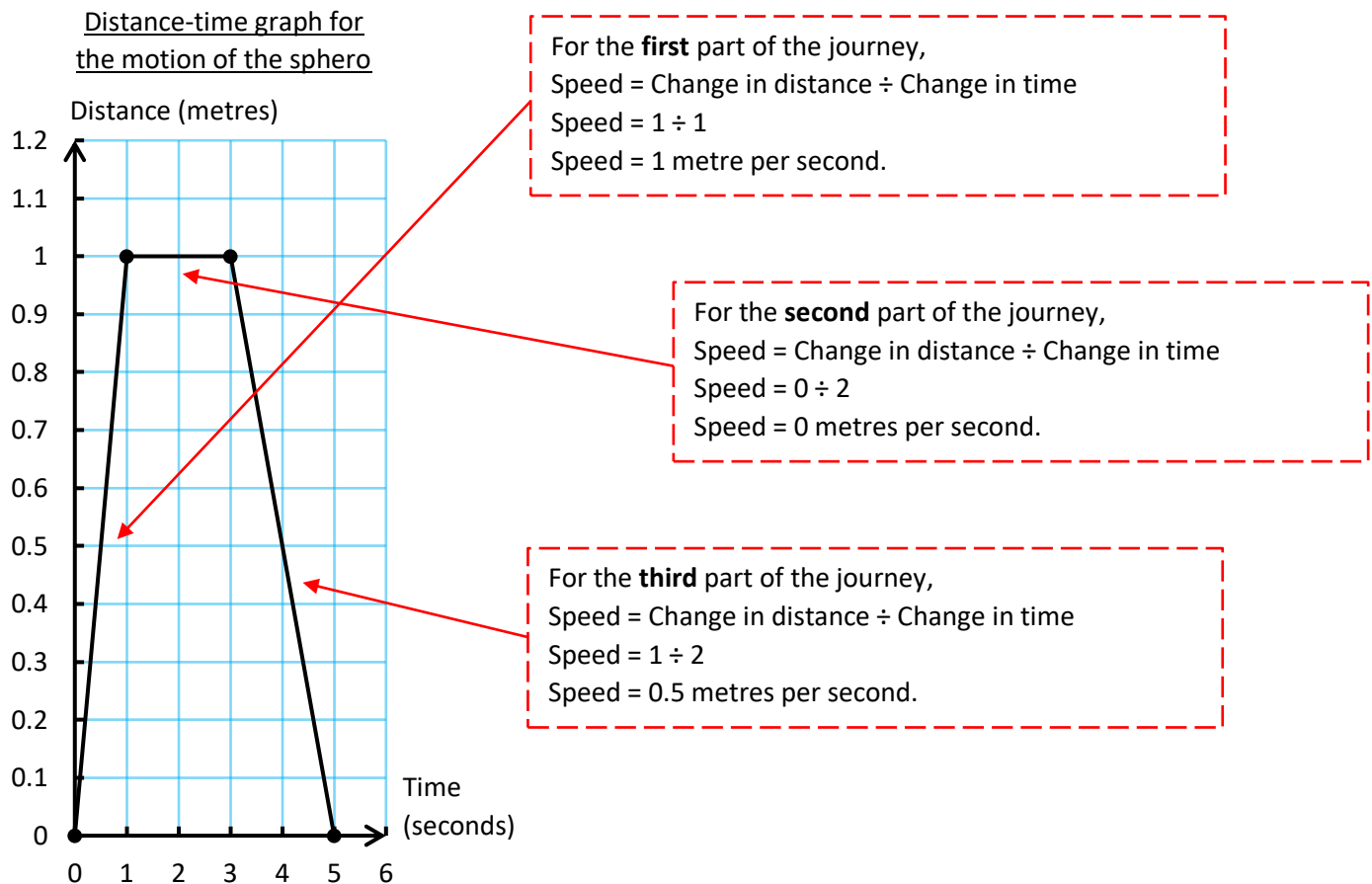
Delay for 1 second.

Roll the sphero at an angle 180° ; at a speed of 10%; for 2 seconds.

Calculating the Speed from a Distance-Time graph

It is possible to use a distance-time graph to calculate the speed for any part of the journey. Use the formula **Speed = Change in distance ÷ Change in time** to do this.

Let us consider again the example from page 8.



Exercise 16

7

For each graph from Exercise 15, calculate the speed for each part of the journey. (Perhaps you would like to label the parts of each journey with the letters A, B, C, ...)

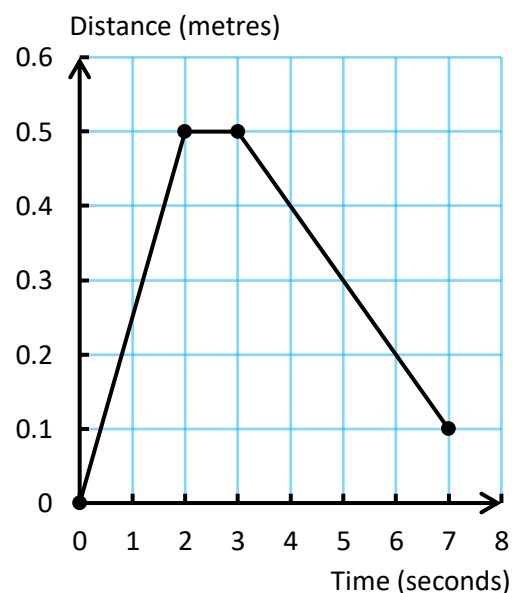
Exercise 17

- For the distance-time graph on the right, calculate the speed for each part of the journey.
- Write the sphero program that gives the distance-time graph on the right.

Exercise 18

- Write your own program to control the sphero. (Use angles 0° and 180° only; exactly 10 seconds of time; and at least 4 commands.)
- Draw the distance-time graph for your program.
- Calculate the speed for each part of your program.

Distance-time graph for Exercise 17

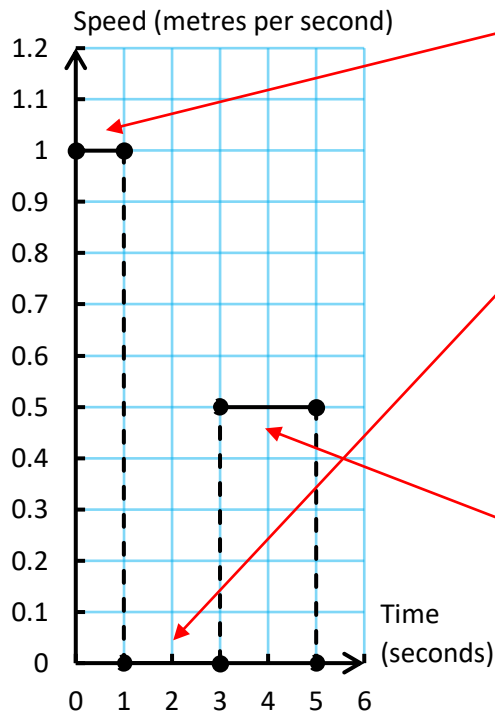


Speed-Time Graphs

During the top half of the previous page, we calculated the speed of a sphero during the three parts of its journey. It is possible to show this information in the form of a **speed-time graph**.



Speed-time graph for the motion of the sphero



For the **first** part of the journey, the sphero travels at 1 metre per second for 1 second.

For the **second** part of the journey, the sphero travels at 0 metres per second for 2 seconds. (It is stationary.)

For the **third** part of the journey, the sphero travels at 0.5 metres per second for 2 seconds.

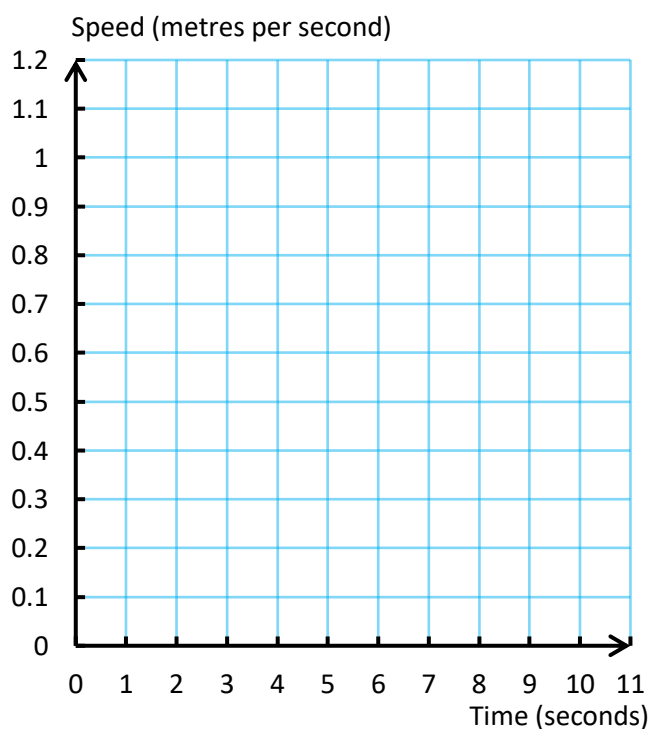
Why are the **vertical** lines in the speed-time graph **dotted lines**, and not solid lines?

Exercise 19

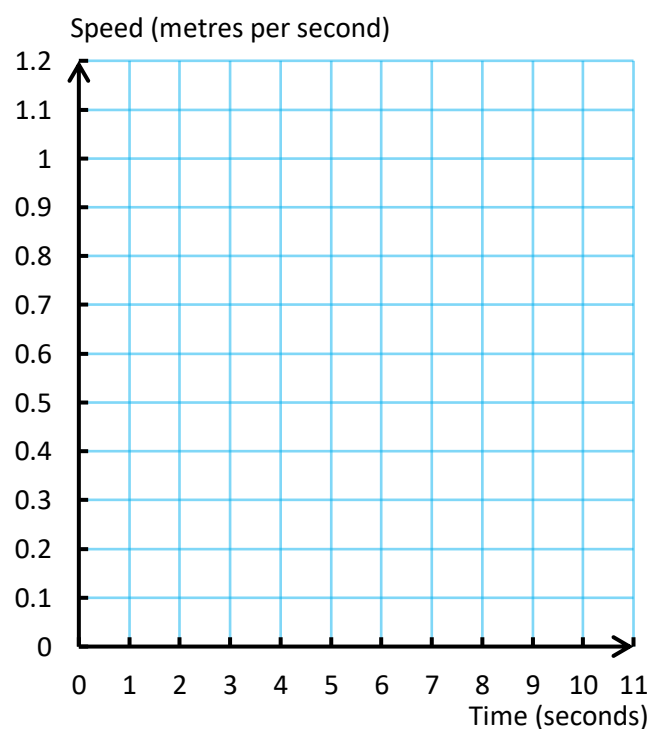
7

Draw a speed-time graph for each program in Exercise 15. (Use the squared paper below for programs 1 and 2.)

Speed-time graph for Program 1



Speed-time graph for Program 2

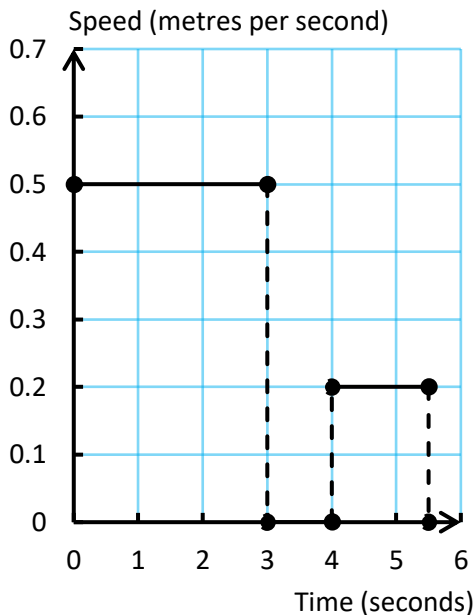


Exercise 20

7

Here is a speed-time graph for a sphero journey that lasts 5.5 seconds.

Speed-time graph for Exercise 20



- Write a program to control the sphero that would produce this graph. (Use 3 exactly commands, and rectilinear motion.)
- Write a **different** program to control the sphero that would produce the same graph. (Again, use exactly 3 commands, and rectilinear motion.)
- How many different (3 command) programs is it possible to write in order to produce this graph? Why is it possible to write more than one?
- Draw a distance-time graph for your program from part (a).
- Draw a distance-time graph for your program from part (b).
- CHALLENGE!** ⚠ How many different distance-time graphs would give the speed-time graph on the left?



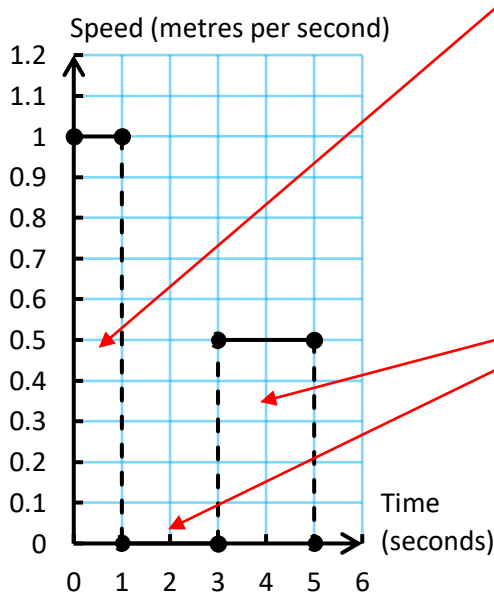
Distance travelled from a Speed-time graph

It is possible to use a speed-time graph to calculate the distance an object has travelled during its journey. To do this, we must find the total area underneath the graph.

Example

Let us consider, once again, the speed-time graph from the previous page.

Speed-time graph for the motion of the sphero



The area of this rectangle is $1 \times 1 = 1$.
So, during the first second of the journey, the sphero has travelled **1 metre**.

Between one and three seconds, the speed of the sphero is 0 metres per second, so the sphero **does not move**.

The area of this rectangle is $2 \times 0.5 = 1$.
So, between three and five seconds, the sphero has travelled **1 metre**.

The total distance travelled during the whole journey is $1 + 1 = 2$ metres. This is the total area underneath the graph.

Exercise 21

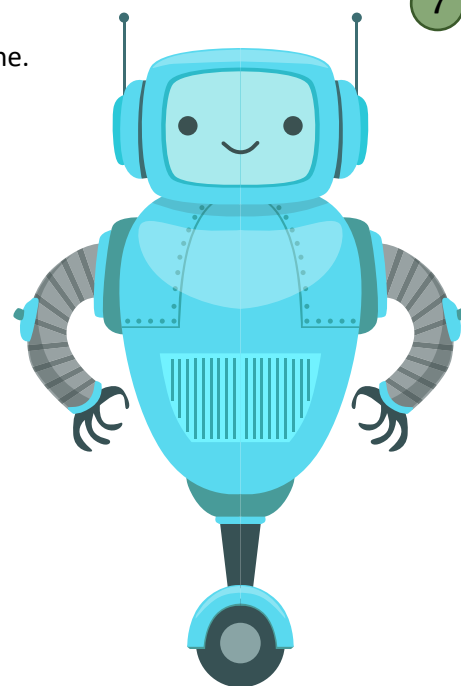
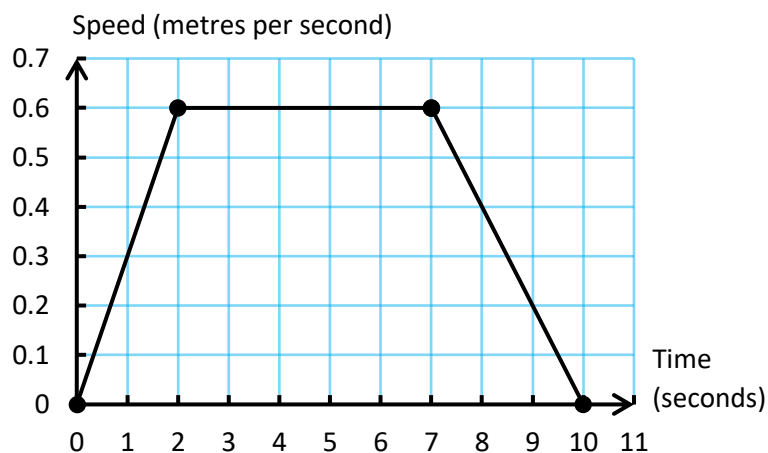
7

Using your speed-time graphs from Exercise 19, calculate the total distance travelled for each of the 4 programs.

Exercise 22

The following graph shows the speed of a robot as it travels along a straight line.

Speed-time graph for the journey of a robot



- (a) How fast is the robot moving at time 3 seconds?
- (b) How fast is the robot moving at time 1 second?
- (c) For how many seconds is the robot travelling at constant speed during the middle part of its journey?
- (d) How far did the robot travel during its journey?
- (e) **CHALLENGE!**  Draw a possible distance-time graph for the journey of the robot.



Summary

The **gradient** of a distance-time graph gives the **speed**.

The **area** underneath a speed-time graph gives the **distance travelled**.

Evaluation

Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

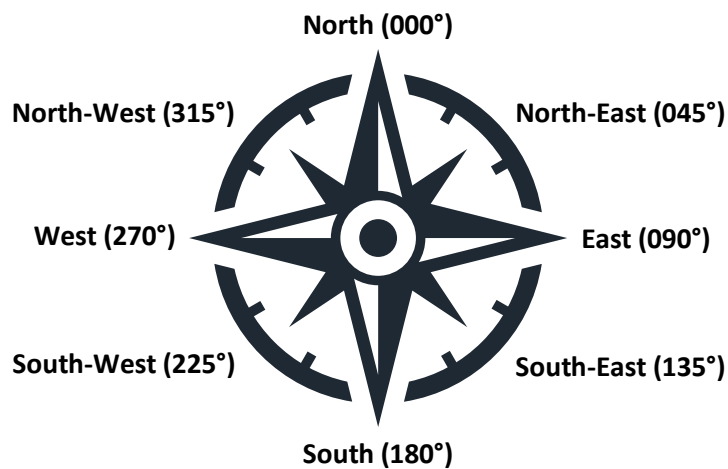
Bearings

When programming a sphero, we must note the angle that we would like to use. After aiming the sphero correctly, the 0° (for example) means that the sphero moves away from us, and 90° means that the sphero starts to travel to the right.

Bearings use a similar system, but the angle is measured from the direction of the **North** each time.

Directions of the compass

The needle on a compass always points towards the magnetic north pole. The following diagram shows some of the directions of the compass; along with their **bearing**, given as an angle. Note that each bearing uses 3 digits, and that they increase in a clockwise direction.



A sentence to remember

Bearings are measured **clockwise** from the **North** in **degrees**.

Exercise 23

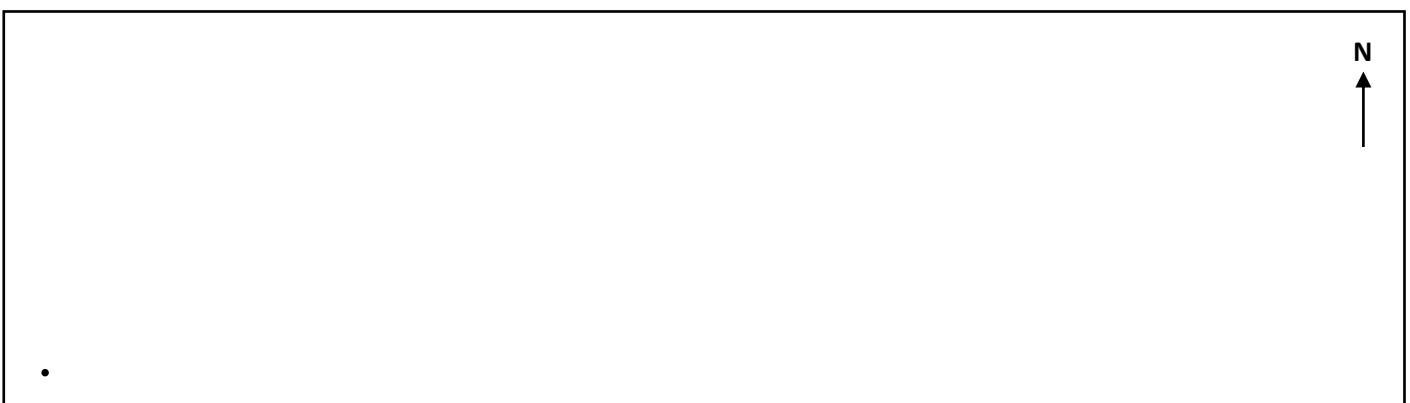
Starting from the dot in the box below, draw, to scale, the following journey.

- Travel 4 cm at a bearing of 000° .
- Travel 10 cm at a bearing of 090° .
- Travel 3 cm at a bearing of 135° .
- Travel 5 cm at a bearing of 270° .



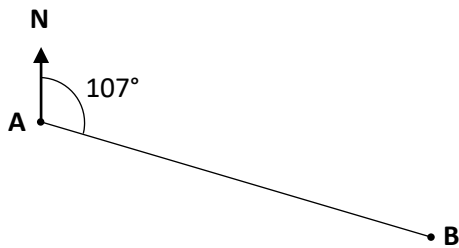
Applying

7



Example

The bearing of B from A below is 107° .

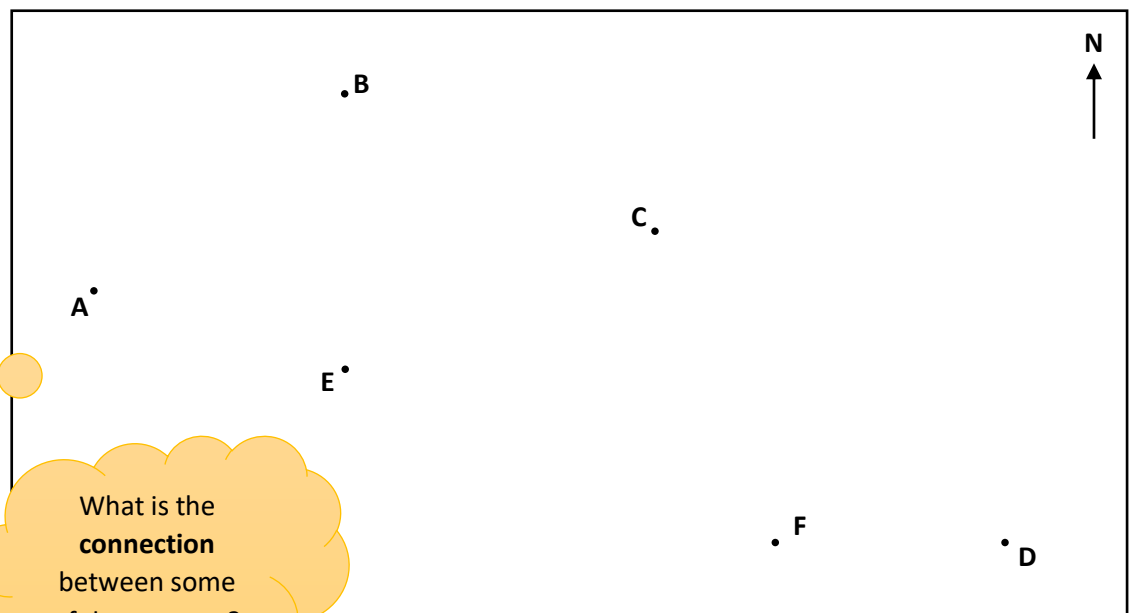


The bearing of A from B below is 266° .

**Exercise 24**

Use a protractor and a ruler to measure the following bearings.

- (a) B from A
- (b) A from B
- (c) D from C
- (d) C from D
- (e) F from E
- (f) E from F
- (g) E from B
- (h) B from E
- (i) D from F
- (j) F from D
- (k) C from E
- (l) E from C
- (m) F from C
- (n) C from F
- (o) E from A
- (p) A from E

**Skill**

7

Exercise 25

Using the scale 1 cm = 1 m, draw the journey of a sphero that follows the program below.

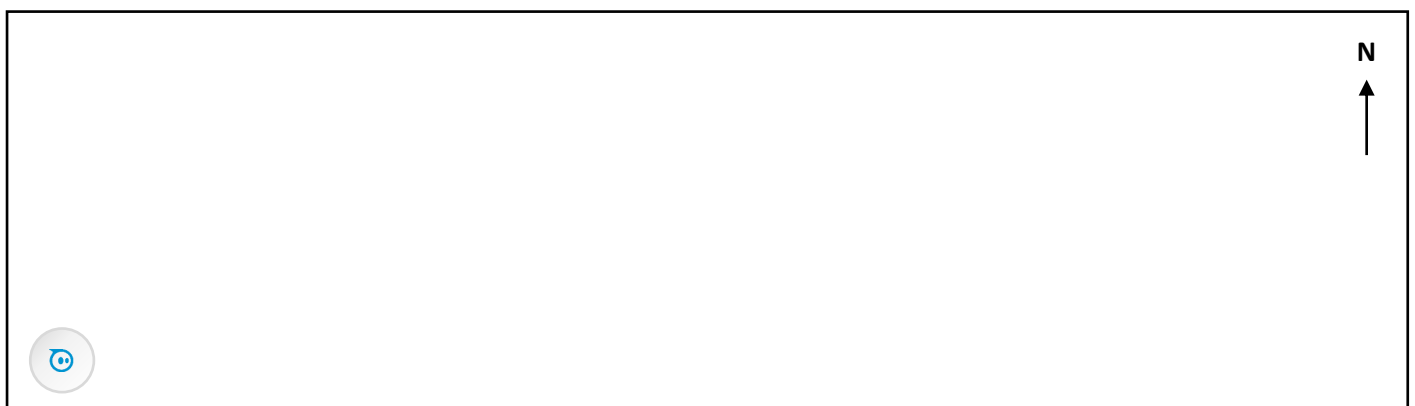
Roll the sphero at an angle 20° ; at a speed of 100%; for 3 seconds.

Delay for 1 second.

Roll the sphero at an angle 125° ; at a speed of 50%; for 10 seconds.

Delay for 1 second.

Roll the sphero at an angle 70° ; at a speed of 25%; for 48 seconds.

Applying

Exercise 26

7

A ship sails at a bearing of 065° for 6 miles, then at a bearing of 200° for 3 miles.

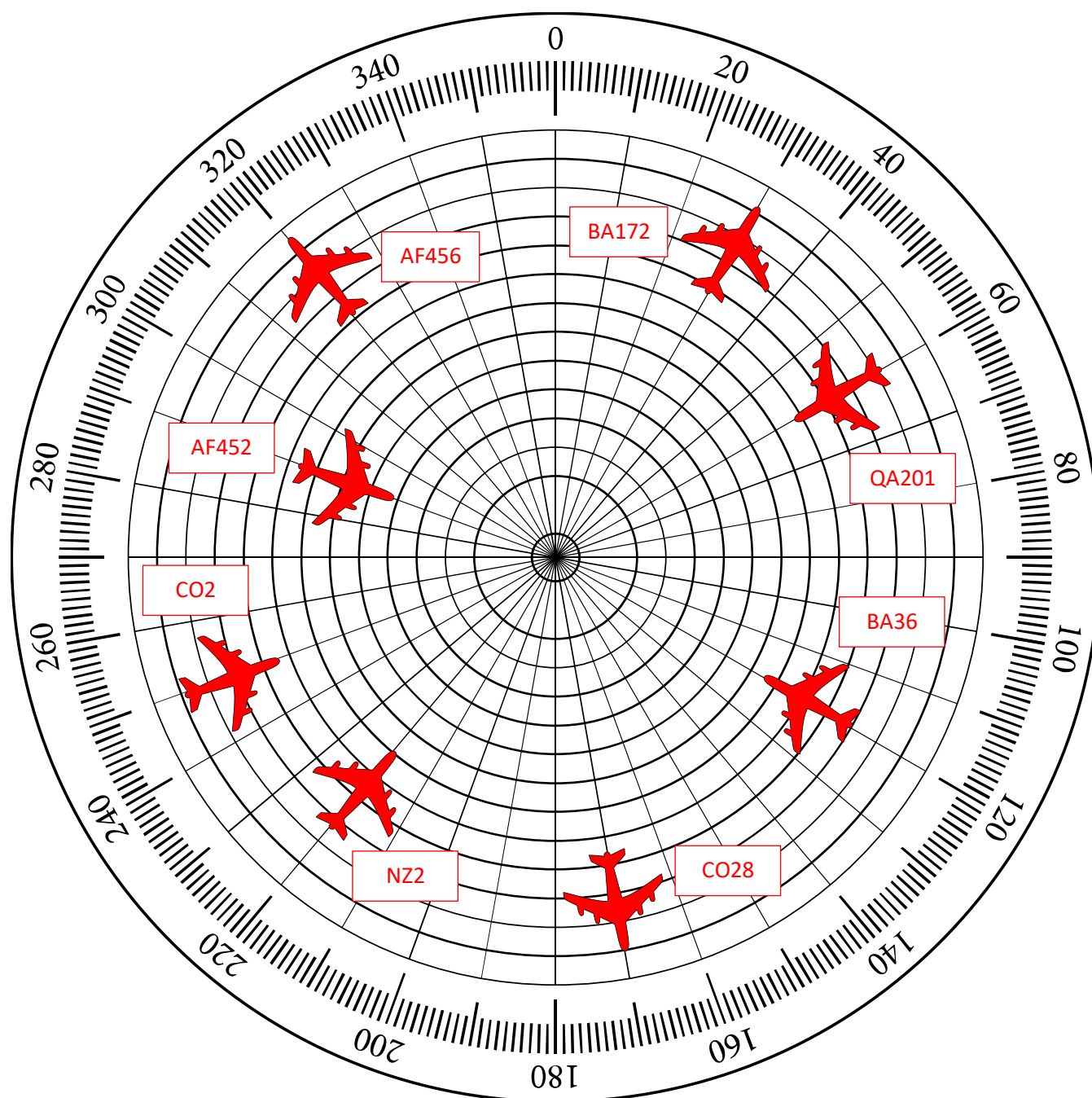
(a) Using the scale $1 \text{ cm} = 1 \text{ mile}$, draw a scale drawing of the ship's journey.

(b) How far is the ship from its starting point at the end of its journey?

(c) Which bearing would take the ship back to its starting point?

**Exercise 27**

Complete the following table to show the bearings of the aeroplanes.



Aeroplane	BA172	BA36		CO28	NZ2		AF452	
Bearing			320°			070°		240°

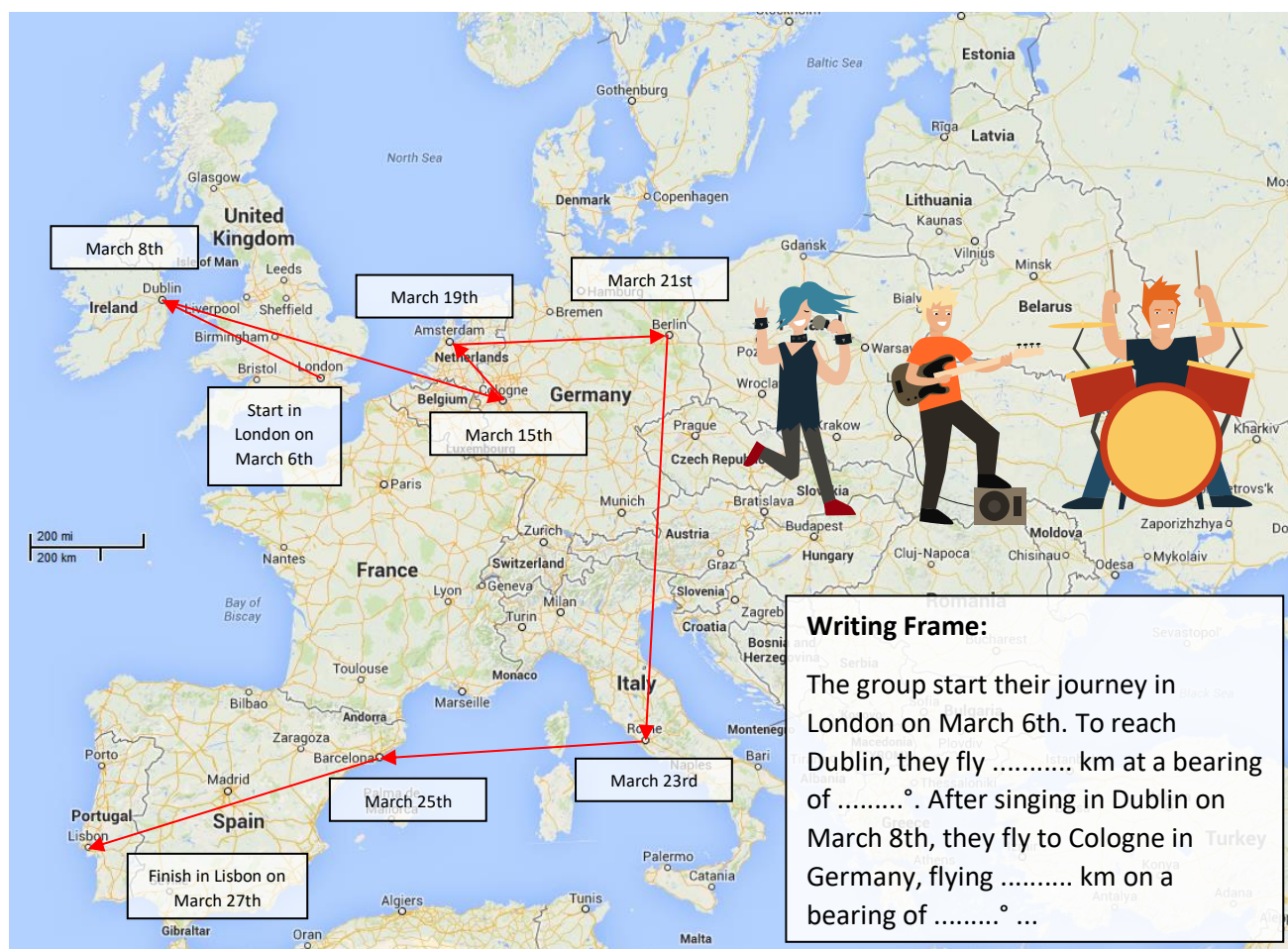
Exercise 28

7

A famous pop group embark on a European tour during the month of March.

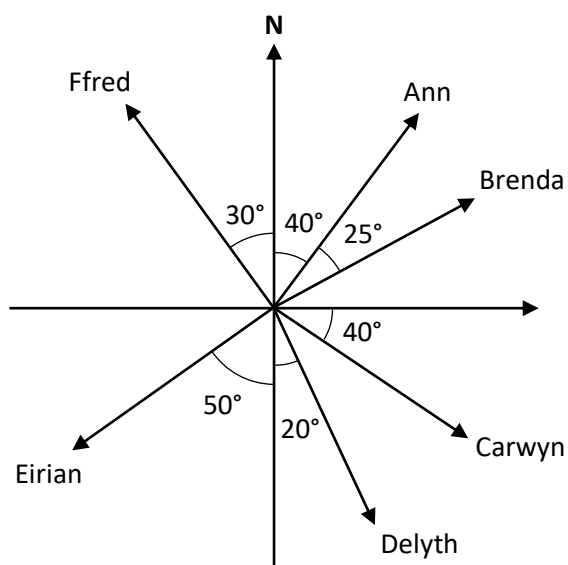
Copy and complete the following writing frame to describe their journey around Europe.

Give the bearings correct to the nearest 5° and the distances correct to the nearest 20 km.



Exercise 29

Complete the following table to show who travels at which bearing. (The diagram is not drawn to scale.)



Name	Bearing
Ann	
Brenda	
Carwyn	
Delyth	
Eirian	
Ffred	

Exercise 30

7

The following diagram shows the plan view of a house.



A sphero is located where the red arrow points to on the diagram.

Plot the journey of the sphero, using the following instructions.

- Move 2 m at a bearing of 140° .
- Move 4 m at a bearing of 270° .
- Move 1.2 m at a bearing of 200° .
- Move 3 m at a bearing of 265° .
- Move 1 m at a bearing of 320° .
- Move 2 m at a bearing of 280° .
- Move 1.5 m at a bearing of 040° .

In which room does the sphero finish its journey?

**Exercise 31**

The bearing of a ship in the Irish Sea is measured from two coastal locations.

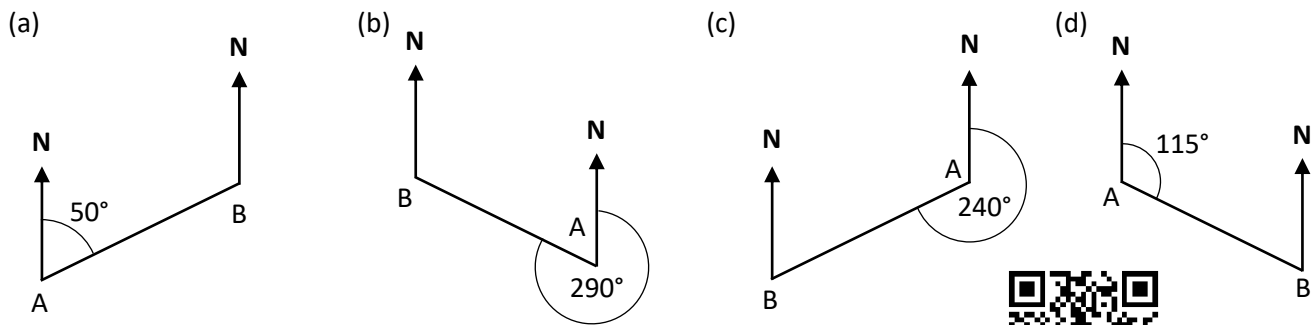
The ship is at a bearing of 040° from Moelfre and at a bearing of 335° from Hoylake.

- By drawing appropriate lines on the diagram, mark the location of the ship.
- Write the bearing of the ship from Douglas.



Exercise 32

Find the bearing of A from B in each case. (The diagrams are not drawn to scale.)



CHALLENGE 1

Re-arrange the tables in your mathematics classroom to create an obstacle course for a sphero.

Program your sphero to go around the obstacle course in the quickest time, being careful not to touch any objects or fall on the floor.



CHALLENGE 2

Look at the map of the grounds of Ysgol y Creuddyn. Design a treasure hunt around the school, in order to form a word or short sentence. For example, here are the instructions for the Welsh word “ONGL”:

- Start at the letter O.
- Walk 90 m at a bearing of 180° . The letter is _____.
- Walk 80 m at a bearing of 290° . The letter is _____.
- Walk 60 m at a bearing of 160° .
- Walk 45 m at a bearing of 130° . The letter is _____.

Verify your instructions outside using a compass and measuring wheel.



Evaluation

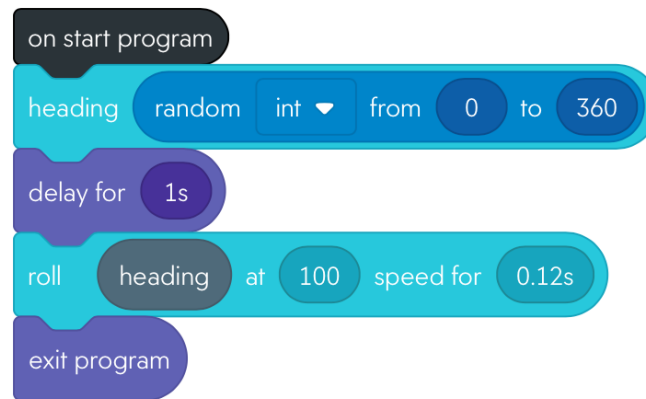
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target



A **locus** is a set of points satisfying some property. A locus can be a point; a line; a curve; an area or a space.

Example

Draw a dot in the middle of a piece of A3 paper. Input the following program into the [Sphero Edu](#) app.



Run the program 20 times, starting the sphero from the dot each time, and recording (with a cross) where the sphero ends.

Exercise 33

On running the program an infinite number of times, what would be the pattern of crosses at the end?

Applying

7

Distance from a point

The above program describes one type of locus, namely the locus of all points that are at a constant distance from a fixed point. This type of locus has a shape in the form of a **circle**.



Example

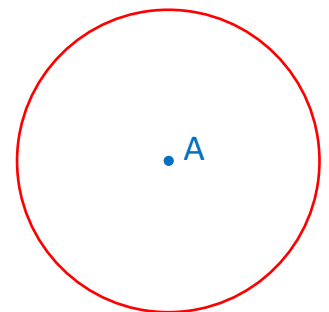
On the right, the locus of a point **P** is shown, where **P** moves so that it is always at a distance of 2 cm from the fixed point **A**.

Exercise 34

Draw the following loci in your book.

- The locus of the point **P** that moves so that it is always 6 cm from a fixed point **A**.
- The locus of the point **P** that moves so that it is always less than 6 cm from a fixed point **A**.
- The locus of the point **P** that moves so that it is always greater than 6 cm from a fixed point **A**.

Skill



Exercise 35

The length of the minute hand on a clock is 8 cm. The length of the hour hand is 3 cm less. Draw the locus of the end of these hands as the time changes from 12:00 pm to 12:20 pm.

Applying

Challenge!

For an hour, a helicopter travelled at a constant speed of 100 mph. During this time, the distance of the helicopter from Liverpool Airport was exactly 50 miles. Draw, to scale, the locus of the helicopter's journey.

Distance from a line

The diagram on the right shows the locus of the point **P** that moves so that it is always 2 cm from the line **AB**, which is 3 cm in length.

Exercise 36

7

Discuss with a partner:

- (a) How would you draw the above locus accurately?
 (b) Which sports use this type of locus?

Exercise 37

Draw the following loci in your book.

- (a) The locus of the point **P** that moves so that it is always 4 cm from a line **AB**, which is 5 cm in length.
 (b) The locus of the point **P** that moves so that it is always 3.5 cm from a line **AB**, which is 8 cm in length.
 (c) The locus of the point **P** that moves so that it is always 5 cm from a line **AB**, which is 2 cm in length.

Exercise 38

Applying

On the right, a plan for a garden is shown, with a fence encircling the garden.

The scale of the drawing is 1 cm = 1 m.

The owner wants to sow grass seeds in the garden.

The grass must be at least 2 m from the tree and at least 0.5 m from the fence.

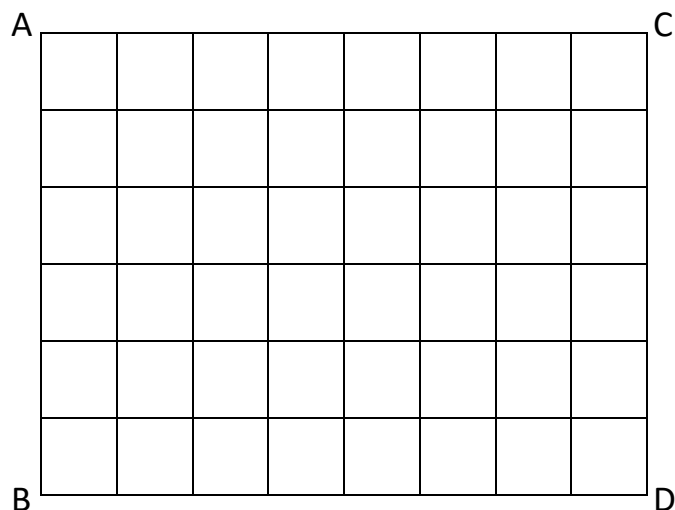
Colour, in green on the plan, where the owner can sow grass seeds.

• Tree

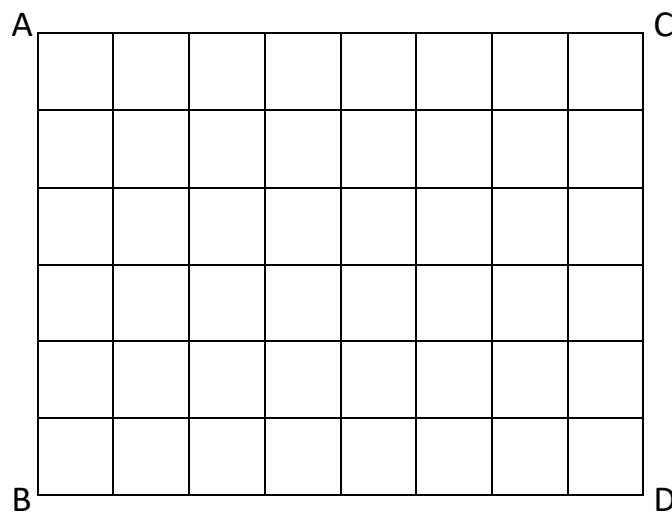
Exercise 39

The scale for this exercise is 1 cm = 1 m.

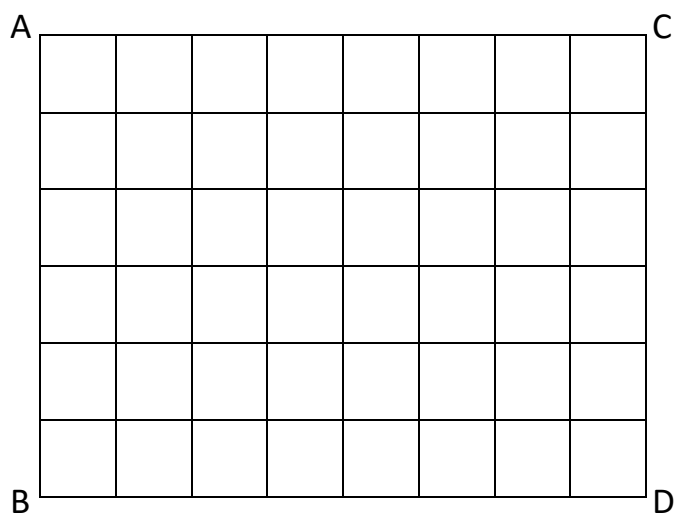
- (a) Draw the locus of all the points that are less than 3 m from the point **A**.



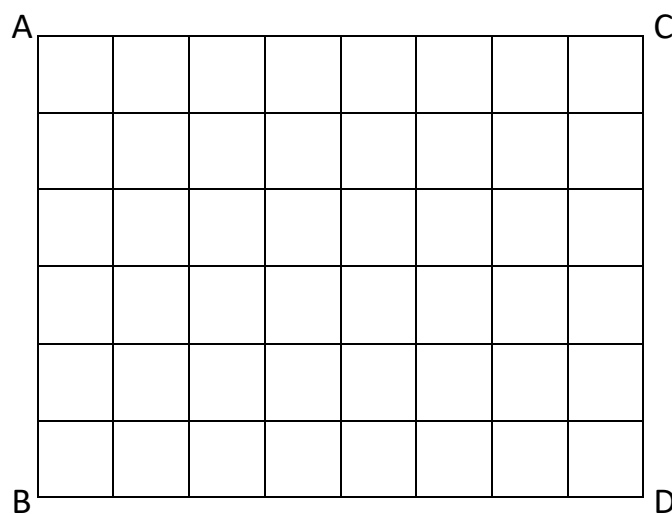
- (b) Draw the locus of all the points that are less than 2 m from the line **AB**.



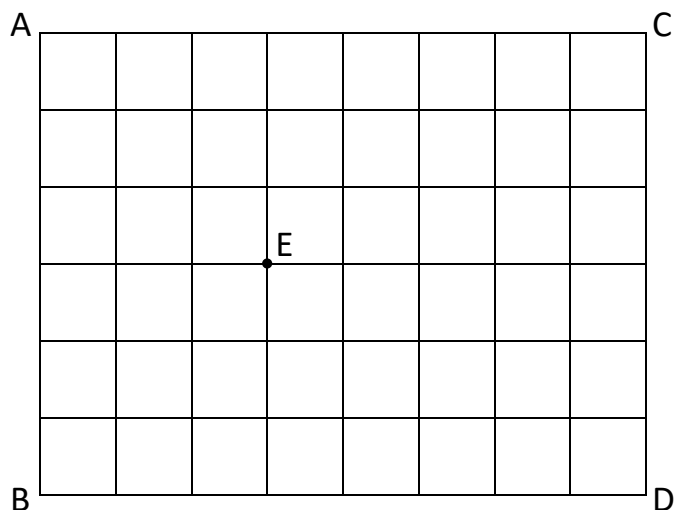
(c) Draw the locus of all the points that are less than 4 m from the point A and more than 4 m from C.



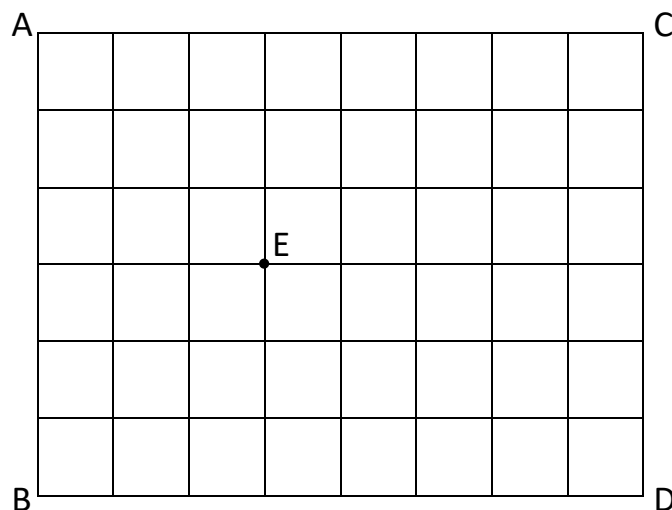
(d) Draw the locus of all the points that are less than 2 m from the line AD.



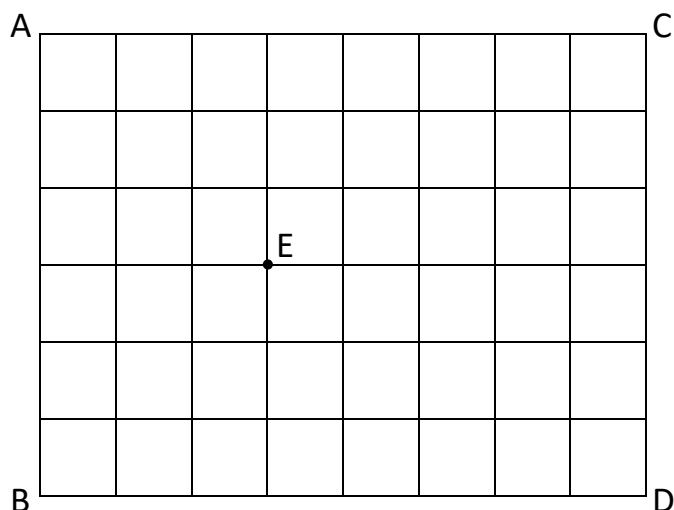
(e) Draw the locus of all the points that are less than 3 m from the point E.



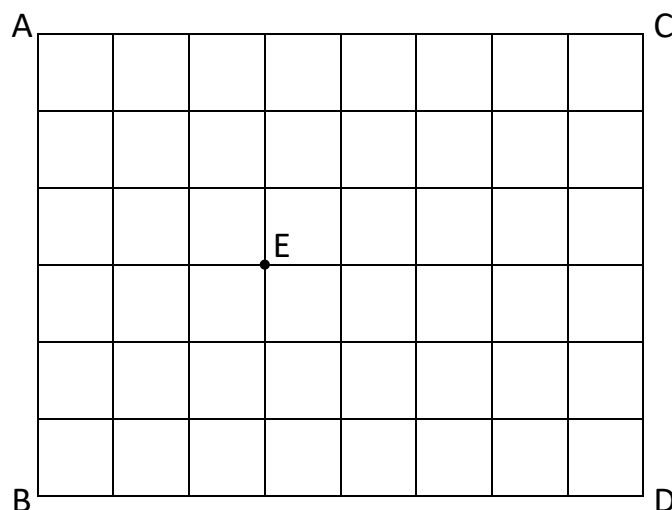
(f) Draw the locus of all the points that are more than 2 m from the point E and more than 5 m from C.



(g) Draw the locus of all the points that are less than 3 m from the point E and more than 3 m from the line AB.



(h) Draw the locus of all the points that are more than 1 m from the triangle AEB.



Perpendicular Bisector

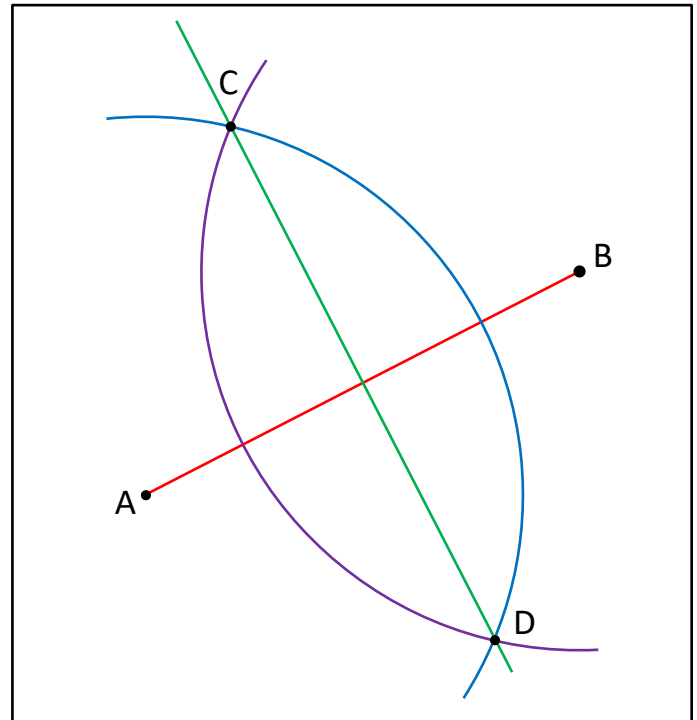


In order to draw the locus of all the points that are **equidistant** from two different points (the same distance from the points), we need to use a compass and a ruler to draw the **perpendicular bisector**.

Example

Let us draw a **perpendicular bisector** for the points A and B shown in the diagram on the right.

- With a ruler, connect A to B with a **straight line**.
- Open your compass to be **more than half** the length of the **line AB**. Put the point of your compass on A and draw a **half circle** as shown.
- Without changing** the compass, put the point on B and draw a **half circle** as shown.
- The two half circles will **intersect** at two new points C and D. Use a ruler to **connect these two points**. This line is the **perpendicular bisector** of the points A and B (any point on the line is equidistant from A and B). Note that the lines AB and CD intersect at a right angle.



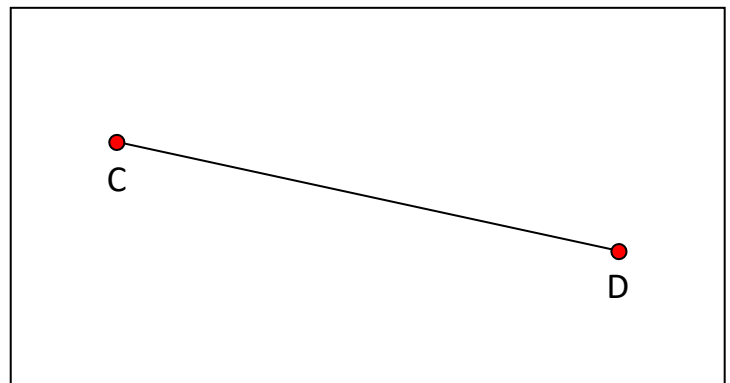
7

Exercise 40

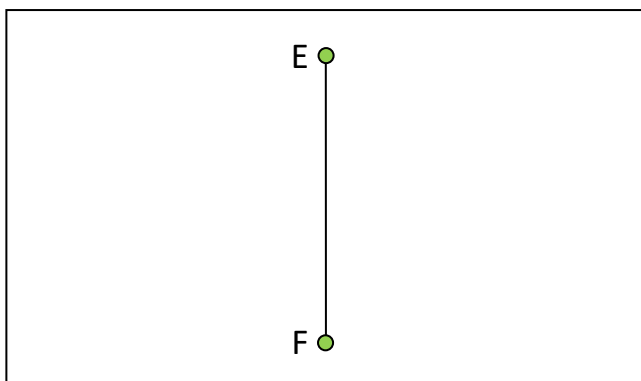
- (a) Draw the locus of all the points that are half way between A and B.



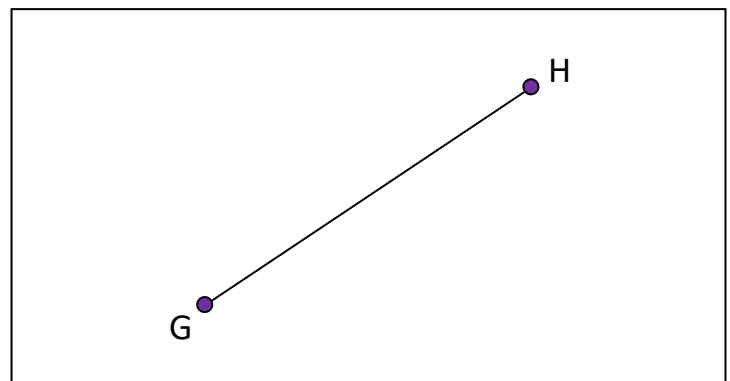
- (b) Draw the locus of all the points that are half way between C and D.



- (c) Draw the locus of all the points that are half way between E and F.



- (d) Draw the locus of all the points that are half way between G and H.



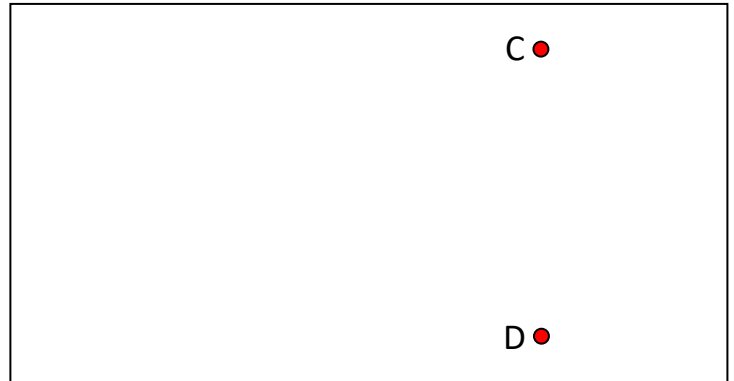
Exercise 41

7

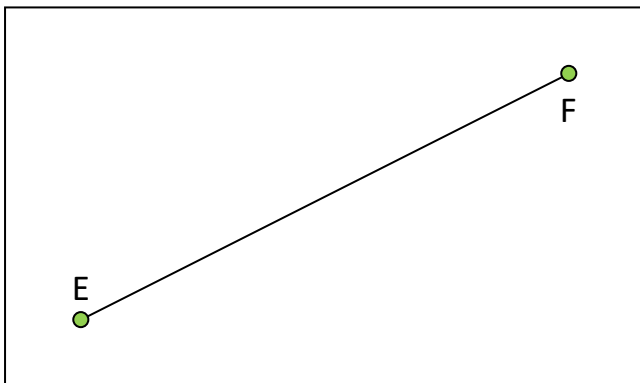
(a) Draw the locus of all the points that are closer to A than to B.



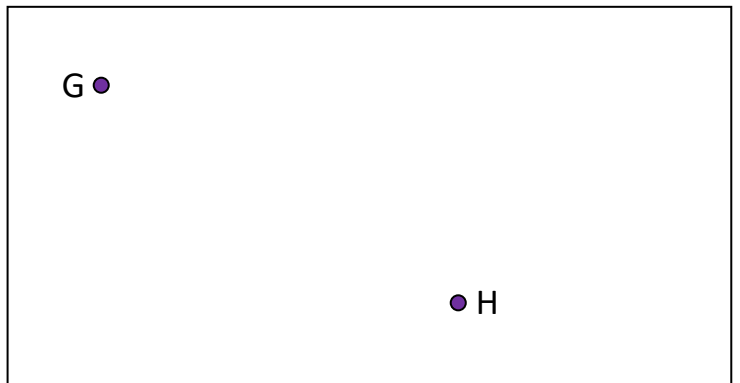
(b) Draw the locus of all the points that are closer to C than to D.



(c) Draw the locus of all the points that are closer to F than to E.



(d) Draw the locus of all the points that are closer to G than to H.



Angle Bisector

Given an angle $\hat{A}BC$, it is possible to use a compass and a ruler to draw the line that **halves** the angle. Like this, we draw the line that is **equidistant** from the lines AB and BC that form the angle.

Example

Let us halve the angle $\hat{A}BC$ shown on the right.

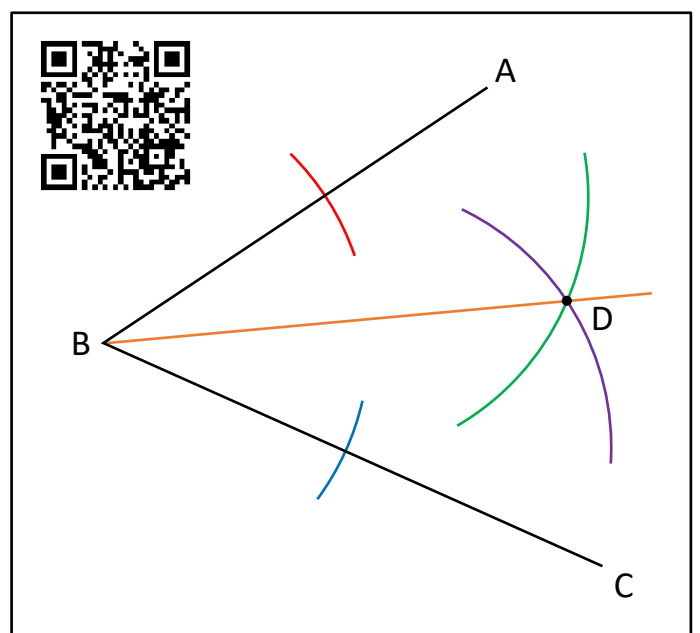
(a) Open your compass to be around half the length of the shortest line, here AB.

(b) Put the point of your compass on B, and draw arcs that cross the lines AB and BC.

(c) **Without changing** the compass, move its point to the location where the **first arc** intersects the line AB. Draw a **new arc** as shown.

(d) **Without changing** the compass, move its point to the location where the **second arc** intersects the line BC. Draw a **new arc** as shown.

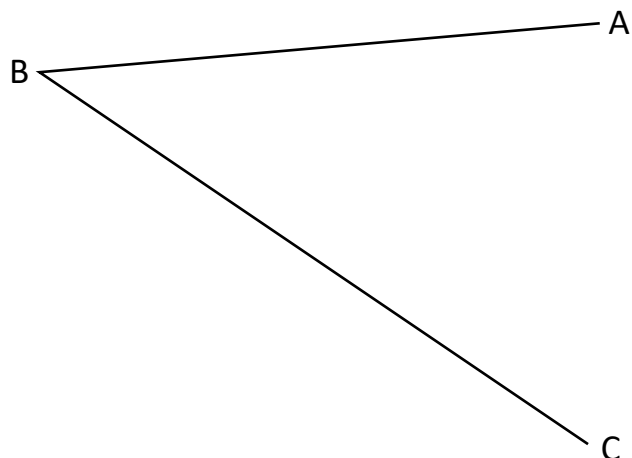
(e) The **two new arcs** intersect at the point D. With a ruler, draw the **line BD** in order to halve the angle $\hat{A}BC$.



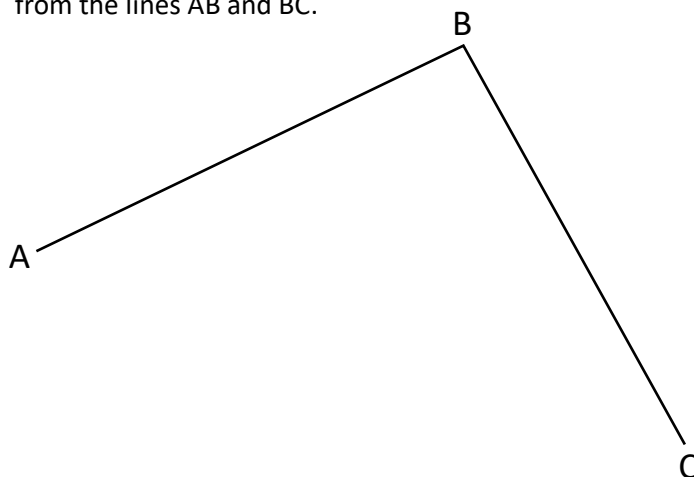
Exercise 42

7

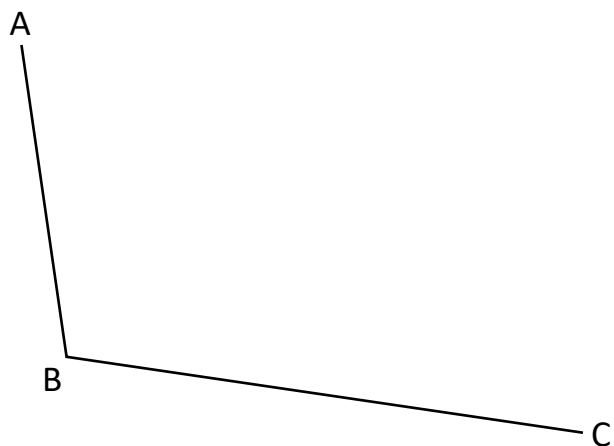
(a) Draw the locus of all the points that are equidistant from the lines AB and AC.



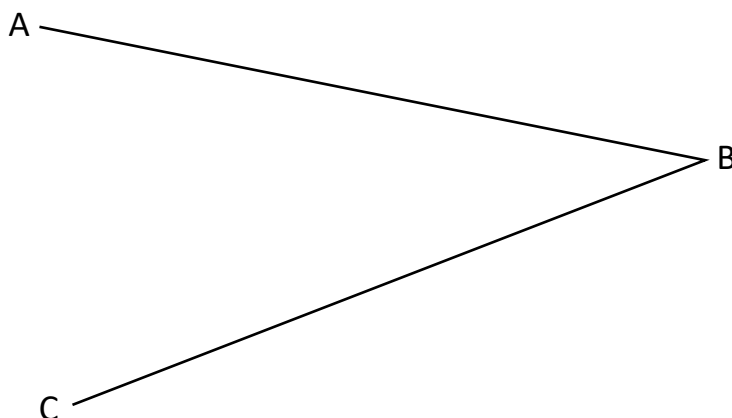
(b) Draw the locus of all the points that are equidistant from the lines AB and BC.



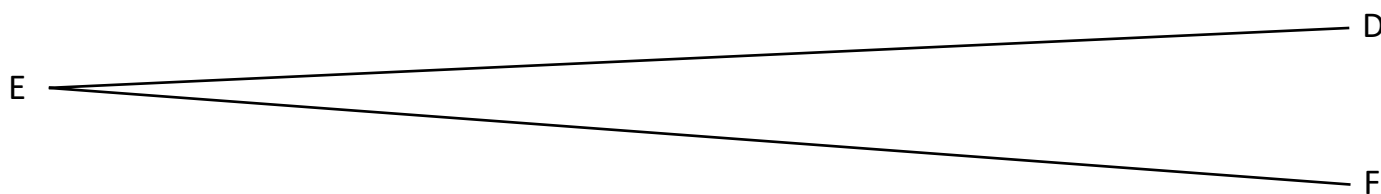
(c) Draw the locus of all the points that are closer to the line AB than to the line BC.



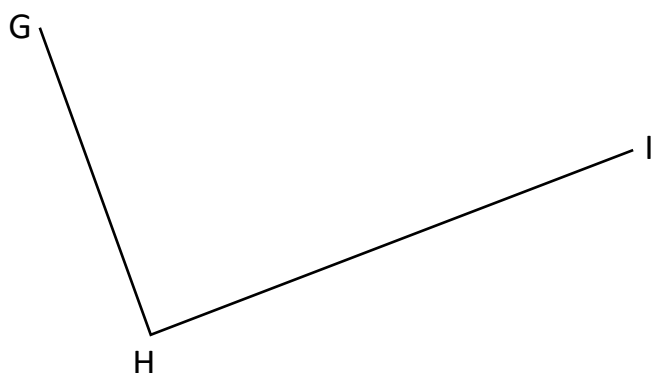
(d) Draw the locus of all the points that are closer to the line BC than to the line AB.



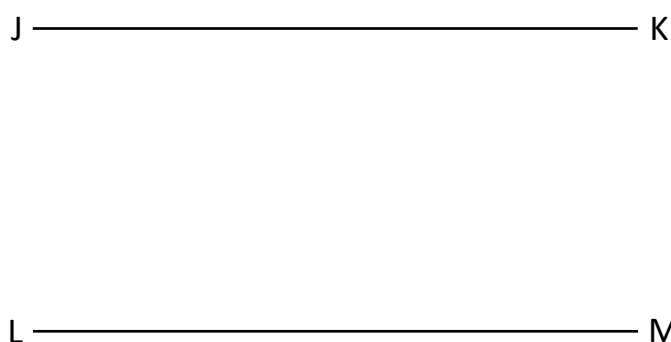
(e) Draw the locus of all the points that are closer to the line DE than to the line EF.



(f) Draw the locus of all the points that are closer to the line GH than to the line HI.



(g) Draw the locus of all the points that are equidistant from the lines JK and LM.



Exercise 43

Applying

7

A large company wishes to base their headquarters in Wales.

The manager tells Ffion to find a location for the headquarters.

Here are the instructions Ffion has received from her manager.

“Find the point that is

- an equal distance between Wrexham and Aberaeron, and*
- an equal distance between Caernarfon and Swansea.*

The new headquarters needs to be located within 20 miles of this point.”

On the map shown on the right, shade the region, **in Wales**, that Ffion should present to her manager.



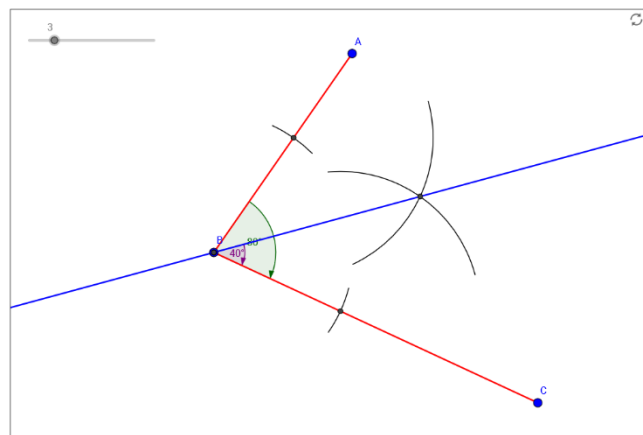
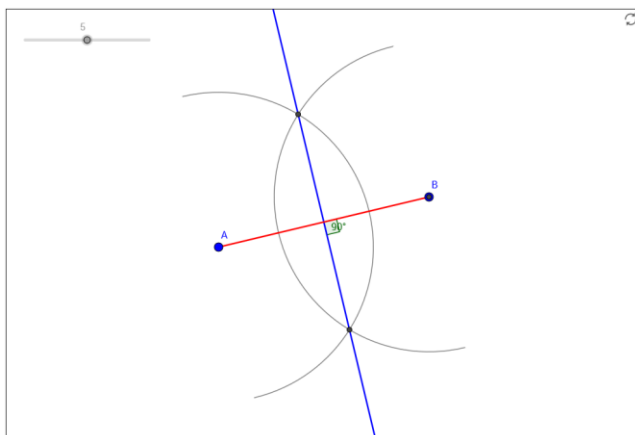
Exercise 44

It is possible to experiment with *GeoGebra* to create different loci.

Try to use the software to create the following types of loci.

- (a) [Distance from a point](#)
 (c) [Perpendicular bisector](#)

- (b) [Distance from a line](#)
 (d) [Angle bisector](#)





Drawing a Perpendicular from a Point to a Line

It is possible to adapt the technique for drawing a perpendicular bisector in order to draw a perpendicular from any point to a line.

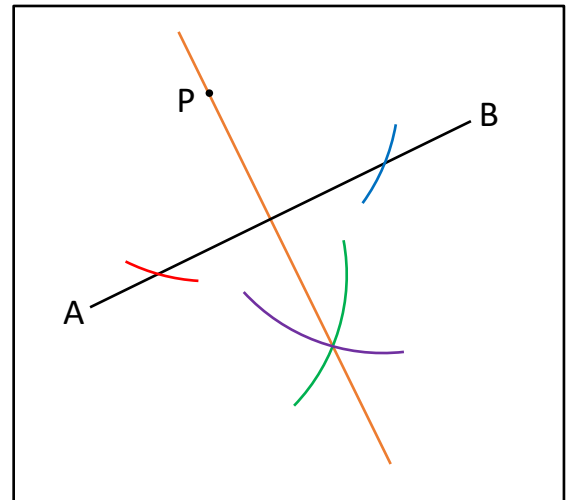
Example

Let us draw a perpendicular line from the point P to the line AB in the diagram shown on the right.

(a) Put the point of your compass on the point P, and extend the length of the compass past the line AB. Draw **two arcs** that cross the line AB.

(b) **Without changing** your compass, move its point to the location where the **first arc** intersects the line AB. Draw a **new arc** as shown.

(c) **Without changing** your compass, move its point to the location where the **second arc** intersects the line AB. Draw a **new arc** as shown.



(d) With a ruler, draw a **straight line** from the point P to where the final **two arcs** drawn intersect. This line is the perpendicular from the point P to the line AB.

Exercise 45

Draw the perpendicular from the point P to the line AB in the following boxes.



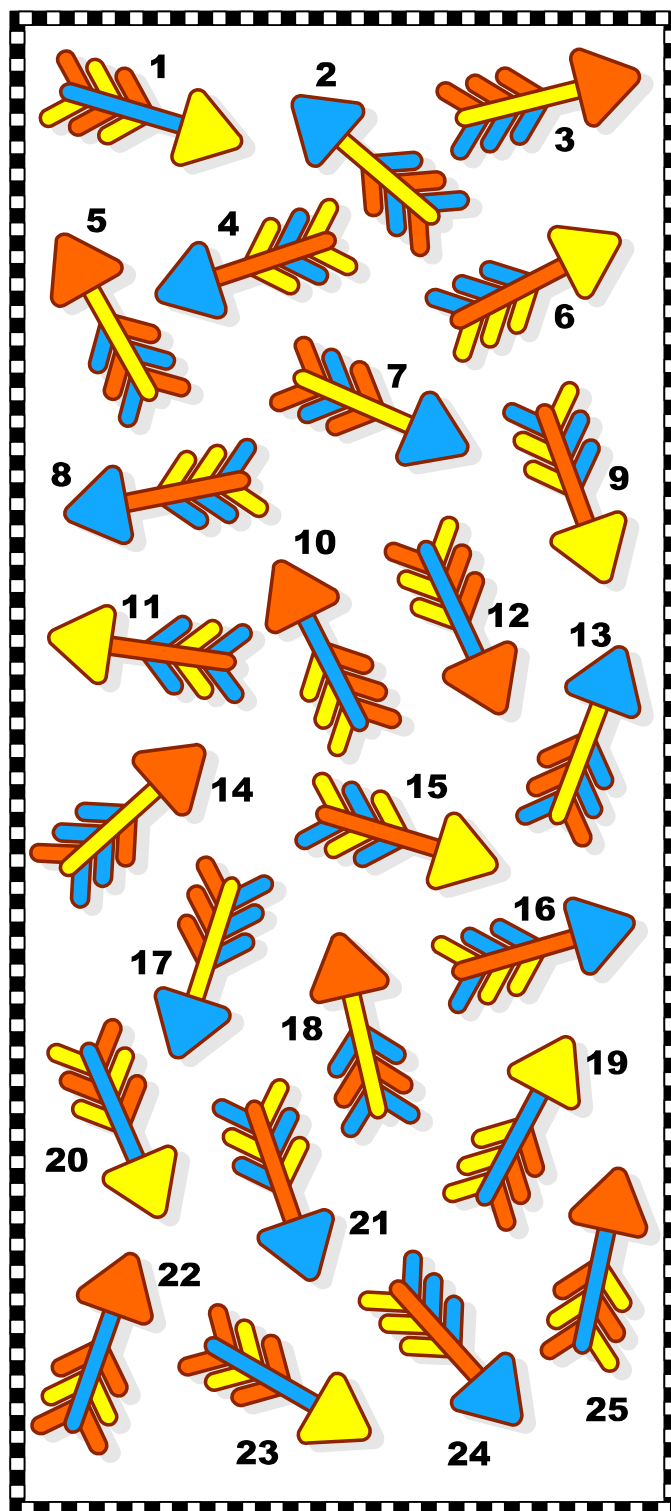
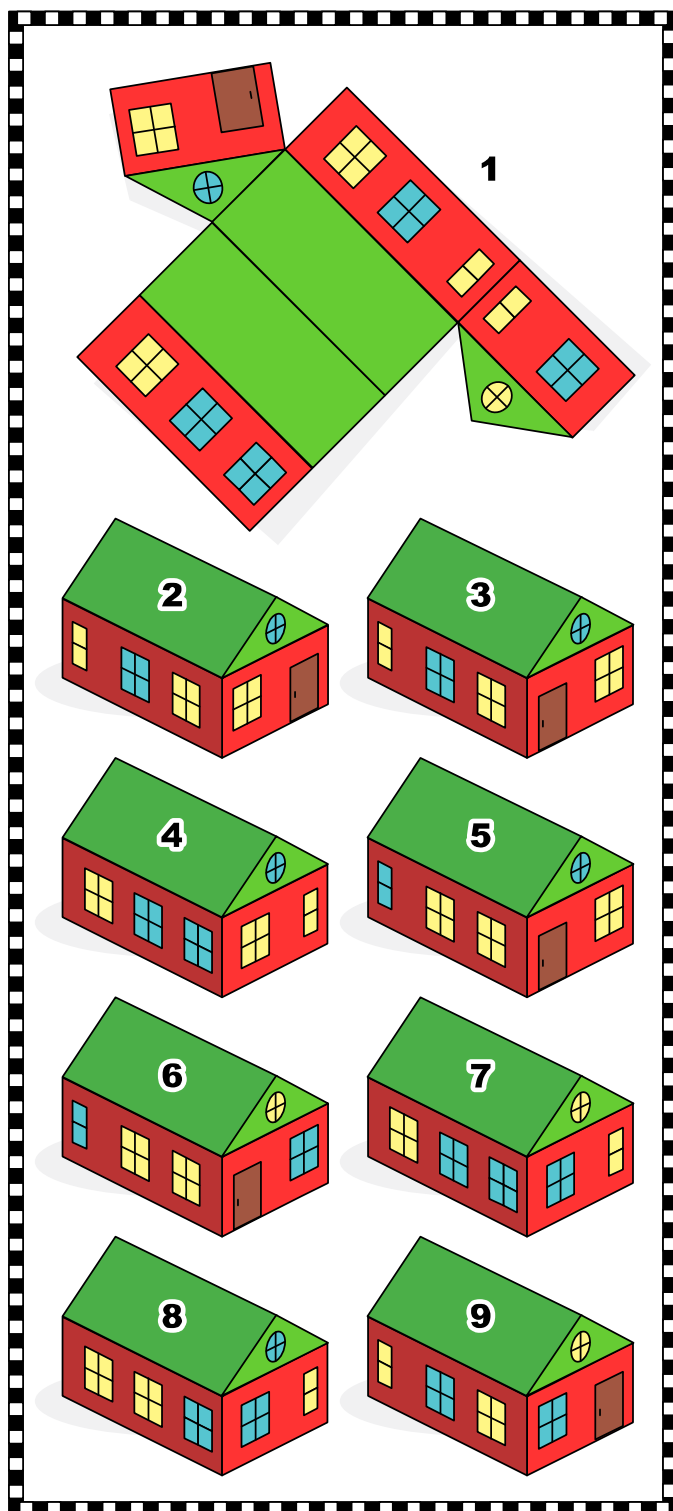
Evaluation

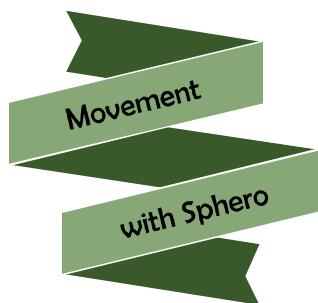
Key Words	Further Questions	What went well?	To reach my target level I will...
			My Level My Target

Puzzles

If 1 is folded to make a house,
which of 2–9 are formed?

Which two arrows are the same?





Name:

Percentage in the test:

I know
this.I need to
revise this.Question
in the test:Correct
in the
test?

I know how to program a sphero to move on a table.			1	
I know how to use the formulae for distance, time and speed .			2, 3	
I know how to draw a distance-time graph.			5	
I know how to interpret a distance-time graph.			5	
I know how to calculate the speed from a distance-time graph.			5	
I know how to draw a speed-time graph.				
I know how to interpret a speed-time graph.			9	
I know how to calculate the distance travelled from a speed-time graph.			9	
I know what the directions of a compass are.			11	
I know how to measure the bearing of one point from another point .			7	
I know how to draw a journey using bearings.			4	
I know how to determine the location of a point using bearings.				
I know how to calculate a bearing from given information.				
I know how to draw a locus that is a specific distance from a fixed point .			6, 11	
I know how to draw a locus that is a specific distance from a fixed line .			6	
I know how to draw the perpendicular bisector of any two points.			11	
I know how to draw the angle bisector of any two lines meeting at a point.			8	
I know how to draw the perpendicular from a fixed point to any line .			10	

Am I ready for the test?
Tick the boxes below...

I have revised the work in my mathematics book.

☐

I have revised the workbook.

☐

I have watched the relevant videos on YouTube.

☐

I have completed the Diagnostic Questions quiz.

☐

I have completed at least 4 pages in my revision book.

☐



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