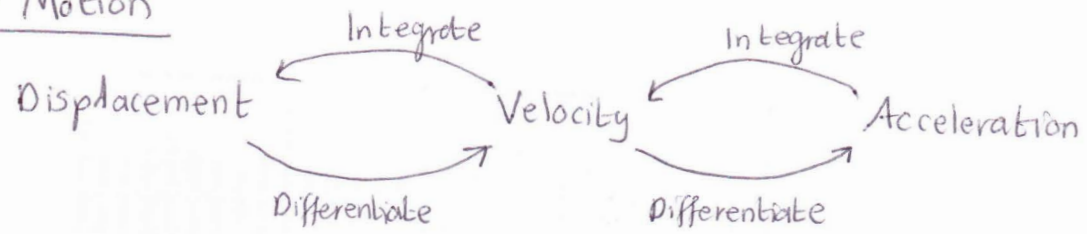
Rectilinear Motion



(with respect to time)

Haf 2006

3) $v = 12t - 3t^2$

(a) $d = \int 12t - 3t^2 \, dt$

$d = 6t^2 - t^3 + K$

When $t = 1$, $d = 0$, so

$0 = 6(1)^2 - (1)^3 + K$

$0 = 6 - 1 + K$

$-5 = K$

So $d = 6t^2 - t^3 - 5$

(b) $a = \frac{d}{dt}(12t - 3t^2)$

$a = 12 - 6t$

(c) When $t = 1.5$,

$v = 12(1.5) - 3(1.5)^2$

$v = 11.25 ms^{-1}$.

Now Power = Force × Velocity

Power = $F \times 11.25$

But $F = ma$

$F = 3(12 - 6(1.5))$

$F = 9 N$

So Power $= 9 \times 11.25$

Power $= 101.25 W$

Haf 2007

1) $v = 3t^2 - 24t + 45$

a) $v = 3(t^2 - 8t + 15)$

$v = 3(t - 3)(t - 5)$

If $v = 0$, then

$0 = 3(t - 3)(t - 5)$

$t = 3$ or $t = 5$

so P first comes to instantaneous rest when $t = 3s$.

b) $a = \frac{d}{dt}(3t^2 - 24t + 45)$

$a = 6t - 24$

c) $d = \int 3t^2 - 24t + 45 \, dt$

$d = t^3 - 12t^2 + 45t + K$

Projected from the origin so $d = 0$ when $t = 0$

$0 = 0^3 - 12(0)^2 + 45(0) + k$

$0 = K$

so $d = t^3 - 12t^2 + 45t$.

d) Displacement after 3s:

$d = 3^3 - 12(3)^2 + 45(3)$

$d = 27 - 108 + 135$

$d = 54m$

e) Displacement after 4s:

$d = 4^3 - 12(4)^2 + 45(4)$

$d = 64 - 12(16) + 180$

$d = 52m$

Distance travelled = $54 + (54 - 52)$

First 3 seconds 3-4

$= 56m$

Haf 2008

3) $F = 15t^2 - 60t$, $t \geq 0$

a) When $t = 2$, $F = 15(4) - 60(2)$

$F = 60 - 120$

$F = -60 N$

Using $F = ma$,

$-60 = 5a$

$a = -12 ms^{-2}$

b) $F = ma$

so $a = \frac{F}{m}$

$a = \frac{15t^2 - 60t}{5}$

$a = 3t^2 - 12t$

so $v = \int 3t^2 - 12t \, dt$

$v = t^3 - 6t^2 + K$

When $t = 0$, $v = 35$, so

$35 = 0^3 - 6(0)^2 + K$

$35 = K$

Therefore $v = t^3 - 6t^2 + 35$

c) Minimum of the velocity function: solve $\frac{dv}{dt} = 0$

or $a = 0$

$3t^2 - 12t = 0$

$3t(t - 4) = 0$

stationary points at $(0, 35)$ and $(4, 3)$

Nature: $6t - 12$ is the '2nd derivative'

MAX at $(0, 35)$; MIN at $(4, 3)$

so the minimum speed is $3 ms^{-1}$

d) $\int_2^8 t^3 - 6t^2 + 35 \, dt$

$= \left[\frac{t^4}{4} - 2t^3 + 35t\right]_2^8$

$= \left[\left(\frac{8^4}{4} - 2(8)^3 + 35(8)\right) - \left(\frac{2^4}{4} - 2(2)^3 + 35(2)\right)\right]$

$= 280 - 58 = 222 m$

M2 Haf 2010

(1) $a = 3 - 4t$ Os yw $t = 0$, mae $v = -1$

a) $v = \int 3 - 4t \, dt$

$v = 3t - 2t^2 + K$

$-1 = 3(0) - 2(0)^2 + K$ (camnewid)

$-1 = K$

Felly $\underline{v = 3t - 2t^2 - 1}$.

b) Mae P yn ddisymud os yw $v = 0$.

$0 = 3t - 2t^2 - 1$

$2t^2 - 3t + 1 = 0$

$(2t - 1)(t - 1) = 0$

Unai $\underline{t = \frac{1}{2}s}$ neu $\underline{t = 1s}$

c) $r = \int 3t - 2t^2 - 1 \, dt$

$r = \frac{3t^2}{2} - \frac{2t^3}{3} - t + K$

Os yw $t = \frac{1}{2}$, mae

$r = \frac{3(\frac{1}{2})^2}{2} - \frac{2(\frac{1}{2})^3}{3} - \frac{1}{2} + K$

$r = \frac{3}{8} - \frac{2}{24} - \frac{1}{2} + K$

$r = -\frac{5}{24} + K$

Os yw $t = 1$, mae

$r = \frac{3(1)^2}{2} - \frac{2(1)^3}{3} - 1 + K$

$r = \frac{3}{2} - \frac{2}{3} - 1 + K$

$r = -\frac{1}{6} + K$

Y pellter rhwng y pwyntiau lle bydd P yn enydaidd ddisymud yw $\left(-\frac{5}{24} + K\right) - \left(-\frac{1}{6} + K\right)$

$= -\frac{1}{24}$

(Y pellter yw $\frac{1}{24}$ m)

M2 Haf 2013

5) $v = 2t(t-6)$ or $v = 2t^2 - 12t$

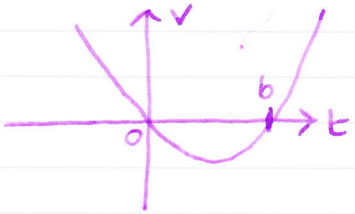
a) We need to solve $v = 0$

$2t(t-6) = 0$

Either $2t = 0$ or $t - 6 = 0$

$t = 0s$ $t = 6s$

So the velocity is negative between $t = 0s$ and $t = 6s$



b) Total distance travelled in the interval $0 \leq t \leq 9$

$= \left| \int_0^6 v \, dt \right| + \int_6^9 v \, dt$

$= \left| \int_0^6 2t^2 - 12t \, dt \right| + \int_6^9 2t^2 - 12t \, dt$

$= \left| \left[\frac{2t^3}{3} - \frac{12t^2}{2} \right]_0^6 \right| + \left[\frac{2t^3}{3} - \frac{12t^2}{2} \right]_6^9$

$= \left| \left[\frac{2t^3}{3} - 6t^2 \right]_0^6 \right| + \left[\frac{2t^3}{3} - 6t^2 \right]_6^9$

$= \left| \left[\left(\frac{2 \times 6^3}{3} - 6 \times 6^2 \right) - \left(\frac{2 \times 0^3}{3} - 6 \times 0^2 \right) \right] \right| + \left[\left(\frac{2 \times 9^3}{3} - 6 \times 9^2 \right) - \left(\frac{2 \times 6^3}{3} - 6 \times 6^2 \right) \right]$

$= \left| \left[(144 - 216) - (0 - 0) \right] \right| + \left[(486 - 486) - (144 - 216) \right]$

$= |-72| + 72$

$= 72 + 72$

$= 144 \text{ m}$

M2Haf 2016

① $v = 12t^2 - 7Kt + 1$

a) $r = \int v\,dt$

$r = \int 12t^2 - 7Kt + 1\,dt$

$r = \frac{12t^3}{3} - \frac{7Kt^2}{2} + t + c$

$r = 4t^3 - \frac{7}{2}Kt^2 + t + c$

When $t = 0$, $r = 3$, so

$3 = 4(0)^3 - \frac{7}{2}K(0)^2 + 0 + c$

$3 = c$

$\therefore\ r = 4t^3 - \frac{7}{2}Kt^2 + t + 3$

When $t = 2$, $r = 16$, so

$16 = 4(2^3) - \frac{7}{2}K(2^2) + 2 + 3$

$16 = 4 \times 8 - \frac{7}{2} \times 4K + 5$

$14K = 32 + 5 - 16$

$K = \frac{21}{14}$

$\underline{K = \frac{3}{2}}$

$\left(\begin{array}{l} \therefore\ r = 4t^3 - \frac{7}{2}\left(\frac{3}{2}\right)t^2 + t + 3 \\ r = 4t^3 - \frac{21}{4}t^2 + t + 3 \end{array} \right)$

b) $a = \frac{d}{dt}(v)$

$a = \frac{d}{dt}(12t^2 - 7Kt + 1)$

$a = 24t - 7K$

$a = 24t - 7\left(\frac{3}{2}\right)$

$a = 24t - \frac{21}{2}$

When $t = 5s$,

$a = 24(5) - \frac{21}{2}$

$a = 109.5\ ms^{-2}$

Using $F = ma$

$F = 4 \times 109.5$

$\underline{F = 438N}$

M1 Haf 2017

2) $v = 4t^3 - 6t + 7$

a) $r = \int v\,dt$

$r = \int 4t^3 - 6t + 7\,dt$

$r = \frac{4t^4}{4} - \frac{6t^2}{2} + 7t + c$

$r = t^4 - 3t^2 + 7t + c$

When $t=0$, $r=5$, so

$5 = 0^4 - 3(0)^2 + 7(0) + c$

$5 = c.$

$\therefore \quad r = t^4 - 3t^2 + 7t + 5$

When $t = 2$, $r = 2^4 - 3(2^2) + 7(2) + 5$

$r = \underline{\underline{23\,m}}$

b) $a = \frac{dv}{dt}$

$a = \frac{d}{dt}(4t^3 - 6t + 7)$

$a = 12t^2 - 6$

When $t = 3$, $a = 12(3)^2 - 6$

$a = 102\,ms^{-2}$

Using $F = ma$ when $t = 3s$

$F = 0.8 \times 102$

$F = \underline{\underline{81.6N}}$

M2 Haf 2018

1) $v = \frac{1}{20}(80 + 16t - t^2)$ $\quad 0 \leq t \leq 20$

a) $a = \frac{d}{dt}(v)$

$a = \frac{d}{dt}\left(\frac{1}{20}(80 + 16t - t^2)\right)$

$a = \frac{d}{dt}\left(4 + \frac{16}{20}t - \frac{1}{20}t^2\right)$

$a = \frac{16}{20} - \frac{2}{20}t$

$a = \frac{4}{5} - \frac{1}{10}t$, $\quad 0 \leq t \leq 20$

b) Cyflymder mwyaf $\Rightarrow \frac{dv}{dt} = 0$

$a = 0$

$\frac{4}{5} - \frac{1}{10}t = 0$

$\frac{4}{5} = \frac{1}{10}t$

$t = \frac{4 \times 10}{5}$

$t = 8s$

Pan fo $t = 8s$, mae $v = \frac{1}{20}(80 + 16 \times 8 - 8^2)$

$v = \frac{1}{20}(80 + 128 - 64)$

$v = \frac{1}{20}(144)$

$v = 7.2\,ms^{-1}$

Ydi hwn yn facsimwm? Mae $\frac{d^2v}{dt^2} = \frac{da}{dt}$

$= -\frac{1}{10}$

yn negatif, felly mae'r cyflymder $7.2ms^{-1}$ yn facsimum.

c) $r = \int v\,dt$

$r = \int \frac{1}{20}(80 + 16t - t^2)\,dt$

$r = \frac{1}{20}\int 80 + 16t - t^2\,dt$

$r = \frac{1}{20}\left(80t + \frac{16t^2}{2} - \frac{t^3}{3}\right) + K$

$r = 4t + \frac{2}{5}t^2 - \frac{t^3}{60} + K$

Os yw $t = 0$, mae'r dadleoliad o A yn sero.

Felly $0 = 4\times 0 + \frac{2}{5}(0)^2 - \frac{(0)^3}{60} + K$

$0 = 0 + 0 - 0 + K$

$K = 0.$

Felly $r = 4t + \frac{2}{5}t^2 - \frac{t^3}{60}$

Os yw $t = 20$, $r = 4\times 20 + \frac{2}{5}(20)^2 - \frac{(20)^3}{60}$

$r = 80 + 160 - \frac{400}{3}$

$r = \frac{320}{3}\ m$

$r = 106\frac{2}{3}\ m$