S3: Egwyddor Sgwariau Lleiaf

Haf 2006

5 (a) STAT MODE:

$\Sigma x = 75 \qquad \Sigma y = 89.2$

$\Sigma xy = 1270.5 \qquad \Sigma x^2 = 1375.$

(b) $y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 1270.5 - \frac{75 \times 89.2}{6}$$

$$= 155.5.$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 1375 - \frac{75^2}{6}$$

$$= 437.5$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{155.5}{437.5}$$

$$= \frac{311}{875}$$

$b = 0.355$ i 3 lle degol.

$$a = \bar{y} - b\bar{x}$$

$$= \frac{89.2}{6} - \frac{311}{875} \times \frac{75}{6}$$

$$= \frac{2189}{210}$$

$a = 10.424$ i 3 lle degol.

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(c) (i) $e_i \sim N(0, 0.4^2)$

$\hat{y}_0 = a + bx_0$
$\hat{y}_0 = 10.424 + 0.355 \times 20$
$= 17.52$ i 2 le degol.

$$SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$
$$= \sqrt{0.4^2\left(\frac{1}{6} + \frac{(20 - \frac{75}{6})^2}{437.5}\right)}$$
$$= 0.4\sqrt{\frac{31}{105}}$$
$$= 0.2173432659\ldots$$

(ii) Cyfwng hyder 95% felly $\frac{5}{2} = 2.5\%$ ym mhob cynffon.
o'r tablau, $P(Z \leq 1.960) = 0.975$.

Cyfwng hyder 95% $= \hat{y}_0 \pm 1.960\,SE$
$= \hat{y}_0 \pm 1.960 \times 0.2173432659$
$= 17.52 \pm 0.4259928012$
$= [17.09, 17.95]$ i 2 le degol.

(ch) $H_0: \beta = 0.4$ yn erbyn
$H_1: \beta \neq 0.4$.

Mae $b = 0.355 < 0.4$ felly'r gwerth-p yw
$2P(X \leq 0.355$ pan fo H_0 yn wir$)$.

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(0.4, \frac{0.4^2}{437.5}\right)$
$= N\left(0.4, \frac{8}{21875}\right)$

felly $SE = \sqrt{\frac{8}{21875}}$.

Y gwerth-p yw $2P(X \leq 0.355$ pan fo H_0 yn wir$)$

$$= 2P\left(Z \leq \frac{0.355 - 0.4}{\sqrt{\frac{8}{21875}}}\right)$$

$$= 2P(Z \leq -2.353106325)$$

$$= 2(1 - P(Z < 2.35))$$

$$= 2(1 - 0.99061)$$

$$= 2 \times 0.00939$$

$$= 0.01878.$$

Gan fod $0.01878 > 0.01$, ar lefel arwyddocâd 1%, nid oes digon o dystiolaeth i gyfiawnhau gwrthod H_0. Felly mae'r canlyniadau yn gyson efo rhagfynegiad Alun.

Haf 2007

7 (a) STAT MODE:

$\Sigma x = 105$ $\Sigma y = 262$

$\Sigma xy = 4760$ $\Sigma x^2 = 2275$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 4760 - \frac{105 \times 262}{7}$$

$$= 830$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 2275 - \frac{105^2}{7}$$

$$= 700$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$= \frac{830}{700}$

$\underline{b = 1.1857}$ i 4 lle degol

$a = \bar{y} - b\bar{x}$

$= \frac{262}{7} - \frac{830}{700} \times \frac{105}{7}$

$= \frac{275}{14}$

$\underline{a = 19.6429}$ i 4 lle degol

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) (i) $H_0 : \beta = 1.2$ yn erbyn
$H_1 : \beta \neq 1.2.$

(ii) Mae $b = 1.1857 < 1.2$ felly'r gwerth-p yw
$2P(X \leq 1.1857$ pan fo H_0 yn wir).

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(1.2, \frac{0.25^2}{700}\right)$
$= N\left(1.2, \frac{1}{11200}\right)$

Felly $SE = \sqrt{\frac{1}{11200}}$.

Y gwerth-p yw $2P(X \leq 1.1857$ pan fo H_0 yn wir)

$= 2P\left(Z \leq \frac{1.1857 - 1.2}{\sqrt{\frac{1}{11200}}}\right)$

$= 2P(Z \leq -1.51336975)$

$= 2(1 - P(Z < 1.51))$

$= 2(1 - 0.93448)$

$= 2 \times 0.06552$

$= 0.13104$

Gan fod $0.13104 > 0.05$ nid oes digon o dystiolaeth i gyfiawnhau gwrthod H_0. Felly rydym yn derbyn mai 1.2 yw gwerth β.

Haf 2008

(7) (a) $\Sigma x = 270$ $\Sigma y = 517$

$\Sigma x^2 = 13900$ $\Sigma xy = 23538$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 23538 - \frac{270 \times 517}{6}$$

$$= 273$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 13900 - \frac{270^2}{6}$$

$$= 1750.$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{273}{1750}$$

$$b = 0.156$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{517}{6} - \left(\frac{273}{1750}\right) \times \frac{270}{6}$$

$$a = 79.14\dot{6}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) Os yw'r tymheredd yn $x_0 = 60°C$ yna

$$\hat{y}_0 = a + bx_0$$
$$= 79.14\dot{6} + 0.156 \times 60$$
$$= 88.50\dot{6} \text{ cm}$$

$$SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$
$$= \sqrt{0.15^2\left(\frac{1}{6} + \frac{\left(60 - \frac{270}{6}\right)^2}{1750}\right)}$$
$$= 0.15\sqrt{\frac{31}{105}}$$
$$= 0.08150372472.$$

Cyfwng hyder 99% felly $\frac{1}{2} = 0.5\%$ ym mhob cynffon.
O'r tablau, $P(Z \leq 2.576) = 0.995$.

$$\text{Cyfwng hyder } 99\% = \hat{y}_0 \pm 2.576 SE$$
$$= 88.50\dot{6} \pm 2.576 \times 0.08150372472$$
$$= 88.50\dot{6} \pm 0.2099535949$$
$$= [88.297, 88.717] \text{ ; 3 lle degol.}$$

Haf 2009

7 (a) $\Sigma x = 105$ $\Sigma y = 262.6$

$\Sigma x^2 = 2275$ $\Sigma xy = 5590.5$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 5590.5 - \frac{105 \times 262.6}{6}$$

$$= 995.$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 2275 - \frac{105^2}{6}$$

$$= 437.5$$

Felly $b = \dfrac{S_{xy}}{S_{xx}}$.

$$= \frac{995}{437.5}$$

$$= \frac{398}{175}$$

$b = 2.2743$ i 4 lle degol.

$$a = \bar{y} - b\bar{x}$$

$$= \frac{262.6}{6} - \frac{398}{175} \times \frac{105}{6}$$

$$= \frac{119}{30}$$

$$= 3.9\dot{6}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) $H_0: \beta = 2.34$ yn erbyn
$H_1: \beta < 2.34$

Y gwerth-p yw $P(X \leq b$ pan fo H_0 yn wir).

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(2.34, \frac{0.5^2}{437.5}\right)$
$= N\left(2.34, \frac{1}{1750}\right)$

Felly $SE = \sqrt{\frac{1}{1750}}$

Y gwerth-p yw $P(X \leq \frac{398}{175}$ pan fo H_0 yn wir)

$$= P\left(Z \leq \frac{\frac{398}{175} - 2.34}{\sqrt{\frac{1}{1750}}}\right)$$

$$= P(Z \leq -2.749025801)$$

$$= P(Z \geq 2.749025801)$$

$$= 1 - P(Z < 2.75)$$

$$= 1 - 0.99702$$

$$= 0.00298$$

Gan fod $0.00298 < 0.01$ mae'r sampl yn darparu tystiolaeth gref iawn ar gyfer gwrthod H_0. Felly rydym yn derbyn bod β yn llai na 2.34.

(c) Mae'n amlwg o'r data bod β (y graddiant) o gwmpas 2 gan fod y yn cynyddu tua 10 bob tro mae x yn cynyddu 5. Felly mae gwerth $\beta = 0.52$ yn amlwg yn rhy isel.

Haf 2010

7 (a) STAT MODE:

$\Sigma x = 210$ $\Sigma y = 14.92$

$\Sigma xy = 554.4$ $\Sigma x^2 = 9100$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$.

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 554.4 - \frac{210 \times 14.92}{6}$$

$$= 32.2$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 9100 - \frac{210^2}{6}$$

$$= 1750$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{32.2}{1750}$$

$$= \frac{23}{1250}$$

$$\underline{b = 0.0184}$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{14.92}{6} - 0.0184 \times \frac{210}{6}$$

$$= \frac{691}{375}$$

$$\underline{a = 1.842\dot{6}}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) Mae a yn arsylw o $N\left(\alpha, \frac{\sigma^2 \Sigma x^2}{n S_{xx}}\right)$

$= N\left(\alpha, \frac{0.02^2 \times 9100}{6 \times 1750}\right)$

$= N\left(\alpha, \frac{13}{37500}\right)$

Felly $SE = \sqrt{\frac{13}{37500}}$

Cyfwng hyder 90% felly $\frac{10}{2} = 5\%$ ym mhob cynffon.
O'r tablau, $P(Z \leq 1.645) = 0.95$

Cyfwng hyder 90% $= a \pm 1.645\, SE$

$= 1.842\dot{6} \pm 1.645 \sqrt{\frac{13}{37500}}$

$= 1.842\dot{6} \pm 0.0306282331\dot{6}$

$= [1.812, 1.873]$ i 3 lle degol.

Haf 2011

(6) (a) $\Sigma x = 90$ $\Sigma y = 169.2$

$\Sigma xy = 2626.2$ $\Sigma x^2 = 1420$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$.

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 2626.2 - \frac{90 \times 169.2}{6}$$

$$= 88.2$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 1420 - \frac{90^2}{6}$$

$$= 70$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{88.2}{70}$$

$$\underline{b = 1.26}$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{169.2}{6} - 1.26 \times \frac{90}{6}$$

$$\underline{a = 9.3}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) Os yw'r tymheredd yn $x_0 = 17°C$ yna

$$\hat{y}_0 = a + bx_0$$
$$= 9.3 + 1.26 \times 17$$
$$= 30.72$$

$$SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$
$$= \sqrt{0.15^2\left(\frac{1}{6} + \frac{(17 - \frac{90}{6})^2}{70}\right)}$$
$$= 0.15\sqrt{\frac{47}{210}}$$
$$= 0.0709627669$$

Cyfwng hyder 99% felly $\frac{1}{2} = 0.5\%$ ym mhob cynffon.
O'r tablau, $P(Z \leq 2.576) = 0.995$.

Cyfwng hyder 99% $= \hat{y}_0 \pm 2.576 SE$
$= 30.72 \pm 2.576 \times 0.0709627669$
$= 30.72 \pm 0.1828000875$
$= [30.537, 30.903]$ i 3 lle degol.

Haf 2012

(5) (a) STAT MODE:

$\sum x = 15$ $\qquad$ $\sum y = 345.5$

$\sum xy = 1131.1$ $\qquad$ $\sum x^2 = 55$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$.

$$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$= 1131.1 - \frac{15 \times 345.5}{6}$$

$$= 267.35$$

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 55 - \frac{15^2}{6}$$

$$= 17.5$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{267.35}{17.5}$$

$$= \frac{5347}{350}$$

$\underline{b = 15.2771}$ i 4 lle degol

$$a = \bar{y} - b\bar{x}$$

$$= \frac{345.5}{6} - \frac{5347}{350} \times \frac{15}{6}$$

$$= \frac{2036}{105}$$

$\underline{a = 19.3905}$ i 4 lle degol.

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) Cyfwng hyder 99% ar gyfer β.

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(\beta, \frac{0.75^2}{17.5}\right)$

$= N\left(\beta, \frac{9}{280}\right)$

Felly $SE = \sqrt{\frac{9}{280}}$.

Cyfwng hyder 99% felly $\frac{1}{2} = 0.5\%$ ym mhob cynffon.
o'r tablau, $P(Z \leq 2.576) = 0.995$.

$$\begin{aligned}
\text{Cyfwng hyder } 99\% &= b \pm 2.576\, SE \\
&= \frac{5347}{350} \pm 2.576 \times \sqrt{\frac{9}{280}} \\
&= \frac{5347}{350} \pm 0.4618363346 \\
&= [14.815,\ 15.739] \text{ i 3 lle degol}
\end{aligned}$$

Haf 2013

6 (a) STAT MODE:

$\Sigma x = 175$ $\Sigma y = 118.1$

$\Sigma xy = 3170$ $\Sigma x^2 = 5075$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$.

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 3170 - \frac{175 \times 118.1}{7}$$

$$= 217.5$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 5075 - \frac{175^2}{7}$$

$$= 700$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{217.5}{700}$$

$$= \frac{87}{280}$$

$\underline{b = 0.3107}$; 4 lle degol

$$a = \bar{y} - b\bar{x}$$

$$= \frac{118.1}{7} - \frac{87}{280} \times \frac{175}{7}$$

$$= \frac{2549}{280}$$

$\underline{a = 9.1036}$; 4 lle degol.

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) Mae a yn arsylw o $N\left(\alpha, \frac{\sigma^2 \Sigma x^2}{n S_{xx}}\right)$

$$= N\left(\alpha, \frac{0.1^2 \times 5075}{7 \times 700}\right)$$

$$= N\left(\alpha, \frac{29}{2800}\right)$$

Felly $SE = \sqrt{\frac{29}{2800}}$.

Cyfwng hyder 95% felly $\frac{5}{2} = 2.5\%$ ym mhob cynffon.
o'r tablau, $P(Z \le 1.960) = 0.975$.

Cyfwng hyder 95% $= a \pm 1.960 SE$

$$= \frac{2549}{280} \pm 1.96 \times \sqrt{\frac{29}{2800}}$$

$$= \frac{2549}{280} \pm 0.1994692959$$

$= [8.904, 9.303]$ i 3 lle degol.

S3 Haf 2014

5 (a) STAT MODE:

$\Sigma x = 42$ $\Sigma y = 340.6$

$\Sigma xy = 2906.4$ $\Sigma x^2 = 364$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 2906.4 - \frac{42 \times 340.6}{6}$$

$$= 522.2$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 364 - \frac{42^2}{6}$$

$$= 70$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$b = \frac{522.2}{70}$$

$$b = 7.46$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{340.6}{6} - 7.46 \times \frac{42}{6}$$

$$= \frac{341}{75}$$

$$a = 4.54\dot{6}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

(b) (i) $e_i \sim N(0, 0.5^2)$

$\hat{y}_0 = a + bx_0$
$\hat{y}_0 = 4.54\dot{6} + 7.46 \times 5$
$\hat{y}_0 = 41.84\dot{6}$

(ii) Cyfwng hyder 95% Felly $\frac{5}{2} = 2.5\%$ ym mhob cynffon.
o'r tablau, $P(Z \leq 1.960) = 0.975$

Cyfwng hyder 95% $= \hat{y}_0 \pm 1.960\,SE$.
Nawr $SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$
$= \sqrt{0.5^2\left(\frac{1}{6} + \frac{\left(5 - \frac{43}{6}\right)^2}{70}\right)}$
$= \sqrt{0.5^2 \times \frac{47}{210}}$
$= 0.2365425563$

Felly cyfwng hyder 95% $= 41.84\dot{6} \pm 1.960 \times 0.2365425563$
$= 41.84\dot{6} \pm 0.4636234104$
$= [41.383, 42.310]$
i 3 lle degol.

(iii) $H_0: \beta = 7.6$ yn erbyn
$H_1: \beta \neq 7.6$

Mae $b = 7.46 < 7.6$ felly'r gwerth-p yw
$2P(X \leq 7.46$ pan fo H_0 yn wir$)$

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(7.6, \frac{0.5^2}{70}\right)$
$= N\left(7.6, \frac{1}{280}\right)$

Felly $SE = \sqrt{\frac{1}{280}}$

Y gwerth-p yw $2P(X \leq 7.46$ pan fo H_0 yn wir)

$$= 2P\left(Z \leq \frac{7.46 - 7.6}{\sqrt{\frac{1}{280}}}\right)$$

$$= 2P(Z \leq -2.342648074)$$

$$= 2(1 - P(Z < 2.34))$$

$$= 2(1 - 0.99036)$$

$$= 2 \times 0.00964$$

$$= 0.01928$$

Gan fod $0.01928 < 0.05$, ar lefel arwyddocâd 5%, mae'r sampl yn darparu tystiolaeth gref ar gyfer gwrthod H_0. Felly nid yw'r gwerthoedd yn y tabl yn gyson efo'r gwerth $\beta = 7.6$.

S3 Haf 2015

5 a) STAT MODE:

$\Sigma x = 100$ $\Sigma y = 1716.6$

$\Sigma xy = 34485$ $\Sigma x^2 = 2250$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 34485 - \frac{100 \times 1716.6}{5}$$

$$= 153$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 2250 - \frac{100^2}{5}$$

$$= 250$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$b = \frac{153}{250}$$

$$b = 0.612$$

$$a = \bar{y} - b\bar{x}$$

$$a = \frac{1716.6}{5} - 0.612 \times \frac{100}{5}$$

$$a = 331.08$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

b) $e_i \sim N(0, 0.25^2)$

Mae a yn arsylw o $N\left(\alpha, \frac{\sigma^2 \Sigma x^2}{n S_{xx}}\right)$

$= N\left(\alpha, \frac{0.25^2 \times 2250}{5 \times 250}\right)$

$= N(\alpha, 0.1125)$

Felly $SE = \sqrt{0.1125}$

Cyfwng hyder 99% felly $\frac{1}{2} = 0.5\%$ ym mhob cynffon.
O'r tablau, $P(Z \leq 2.576) = 0.995$

Cyfwng hyder 99% $= a \pm 2.576 SE$

$= 331.08 \pm 2.576 \times \sqrt{0.1125}$

$= [330.216, 331.944]$ i 3 lle degol.

(ii) $H_0 : \beta = 0.65$ yn erbyn
$H_1 : \beta \neq 0.65$

Mae $b = 0.612 < 0.65$ felly'r gwerth-p yw
$2P(X \leq 0.612$ pan fo H_0 yn wir$)$.

Mae b yn arsylw o $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(0.65, \frac{0.25^2}{250}\right)$

$= N(0.65, 0.00025)$

Felly $SE = \sqrt{0.00025}$

Y gwerth-p yw $2P(X \leq 0.612$ pan fo H_0 yn wir$)$

$= 2P\left(Z \leq \frac{0.612 - 0.65}{\sqrt{0.00025}}\right)$

$= 2P(Z \leq -2.403331022)$

$= 2(1 - P(Z < 2.40))$

$= 2(1 - 0.99180)$

$= 2 \times 0.0082$

$= 0.0164$

Gan fod $0.0164 < 0.05$, ar lefel arwyddocâd 5%, mae'r sampl yn darparu tystiolaeth gref ar gyfer gwrthod H_0. Felly nid yw'r gwerthoedd yn y tabl yn gyson efo'r gwerth $\beta = 0.65$. Maent yn awgrymu $\beta < 0.65$.

S3 Haf 2016

(5) a) STAT MODE:

$\Sigma x = 210$ $\Sigma y = 1286$

$\Sigma xy = 48730$ $\Sigma x^2 = 9100$

$y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 48730 - \frac{210 \times 1286}{6}$$

$$= 3720$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 9100 - \frac{210^2}{6}$$

$$= 1750$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{3720}{1750}$$

$$= \frac{372}{175}$$

$\underline{b = 2.1257}$ i 4 lle degol

$$a = \bar{y} - b\bar{x}$$

$$= \frac{1286}{6} - \frac{372}{175} \times \frac{210}{6}$$

$$= \frac{2099}{15}$$

$\underline{a = 139.9333}$ i 4 lle degol

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

b) Mae b yn arsylwo $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(\beta, \frac{1.5^2}{1750}\right)$
$= N\left(\beta, \frac{9}{7000}\right)$

Felly $SE = \sqrt{\frac{9}{7000}}$

Cyfwng hyder 95% felly 2.5% ym mhob cynffon.
O'r tablau, $P(Z \leq 1.96) = 0.975$

Cyfwng hyder 95% $= b \pm 1.96SE$
$= \frac{372}{175} \pm 1.96 \times \sqrt{\frac{9}{7000}}$
$= \frac{372}{175} \pm 0.07027944223$
$= [2.06, 2.20]$ i 3 ffigur ystyrlon.

ii) $SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$
$\hat{y}_0 = a + bx_0$
Cyfwng hyder 95% $= \hat{y}_0 + 1.96SE$

Mae'r cyfwng hyder ar ei leiaf pan mae SE ar ei leiaf.
Mae SE ar ei leiaf pan mae $x_0 - \bar{x} = 0$
$x_0 = \bar{x}$.

Nawr $\bar{x} = \frac{\Sigma x}{n}$
$\bar{x} = \frac{210}{6}$
$\bar{x} = 35$

Felly mae'r cyfwng hyder ar ei leiaf pan fo $\underline{x_0 = 35}$.

S3 Haf 2017

6) $\Sigma x = 140$ $\quad\quad \Sigma y = 107.3$

$\Sigma x^2 = 3850$ $\quad\quad \Sigma xy = 2744$

a) $y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 2744 - \frac{140 \times 107.3}{6}$$

$$= \frac{721}{3}$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 3850 - \frac{140^2}{6}$$

$$= \frac{1750}{3}$$

Felly $b = \dfrac{S_{xy}}{S_{xx}}$

$$= \frac{\left(\frac{721}{3}\right)}{\left(\frac{1750}{3}\right)}$$

$$= \frac{103}{250}$$

$$\underline{b = 0.412}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

$$a = \bar{y} - b\bar{x}$$

$$= \frac{107.3}{6} - 0.412 \times \frac{140}{6}$$

$$\underline{a = 8.27}$$

Cill dau yn barod yn gywir i dri ffigur ystyrlon)

b) (i) $H_0 : \beta = 0.4$ yn erbyn $H_1 : \beta \neq 0.4$

(ii) Mae $b = 0.412 > 0.4$ felly'r gwerth-p yw
$2P(X \geqslant 0.412$ pan fo H_0 yn wir).

Mae b yn arsylwo $N\left(\beta, \frac{\sigma^2}{S_{xx}}\right) = N\left(0.4, \frac{0.2^2}{\frac{1450}{3}}\right)$
$= N\left(0.4, \frac{3}{43750}\right)$

Felly $SE = \sqrt{\frac{3}{43750}}$

Y gwerth-p yw $2P(X \geqslant 0.412$ pan fo H_0 yn wir)

$$= 2P\left(Z \geqslant \frac{0.412 - 0.4}{\sqrt{\frac{3}{43750}}}\right)$$
$$= 2P(Z \geqslant 1.449137675)$$
$$= 2(1 - P(Z \leqslant 1.45))$$
$$= 2(1 - 0.92647)$$
$$= 2 \times 0.07353$$
$$= 0.14706$$

(iii) Gan fod $0.14706 > 0.05$, ar lefel arwyddocâd 5%, nid oes digon o dystiolaeth i gyfiawnhau gwrthod H_0. Felly mae'r canlyniadau yn cefnogi honiad Emlyn.

S3 Haf 2018

6) $\Sigma x = 150$ $\Sigma y = 164.7$

$\Sigma x^2 = 5500$ $\Sigma xy = 4478$

a) i) $y = \alpha + \beta x$ efo amcangyfrif $y = a + bx$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 4478 - \frac{150 \times 164.7}{6}$$

$$= 360.5$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 5500 - \frac{150^2}{6}$$

$$= 1750$$

Felly $b = \frac{S_{xy}}{S_{xx}}$

$$= \frac{360.5}{1750}$$

$$\underline{b = 0.206}$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{164.7}{6} - 0.206 \times \frac{150}{6}$$

$$\underline{a = 22.3}$$

Gellir gwirio a a b gan ddefnyddio 'Reg' yn STAT MODE.

ii) $\hat{y}_0 = 22.3 + 0.206 x_0$ (amcangyfrif)

Os yw $x_0 = 25$ mae

$\hat{y}_0 = 22.3 + 0.206 \times 25$

$\hat{y}_0 = 27.45$ Kg

iii) $SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$

$SE = \sqrt{0.25^2\left(\frac{1}{6} + \frac{\left(25 - \frac{150}{6}\right)^2}{1750}\right)}$

$= 0.25\sqrt{\frac{1}{6}}$

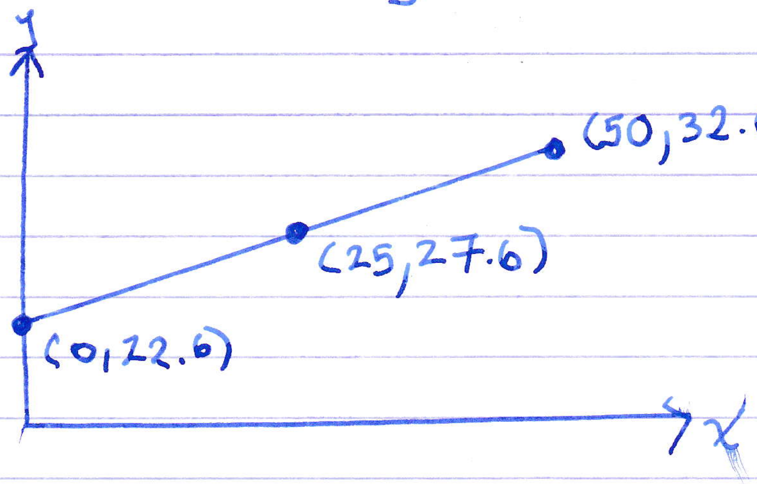
$= \frac{\sqrt{6}}{24}$

$= 0.102$ i 3 lle degol

b) i) Cymedr $= \frac{22.6 + 32.6}{2}$

$= 27.6$ Kg

ii)



$$SE = \sqrt{\sigma^2\left(\frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}\right)}$$

$$= \sqrt{0.25^2\left(\frac{1}{2} + \frac{(25-25)^2}{S_{xx}}\right)}$$

$$= \sqrt{0.25^2\left(\frac{1}{2} + 0\right)}$$

$$= \sqrt{\frac{1}{32}}$$

$$= \frac{\sqrt{2}}{8}$$

$= 0.177$ i 3 lle degol