Old Exam Questions – Old Course

Least Squares Regression Lines

(S3 Summer 2006)

5. Alan is investigating the relationship between the resistance, y ohms, of a new type of electrical component and the temperature, x°C. He obtains the following results.

x	0	5	10	15	20	25
y	10·3	11·8	14·2	16·6	17·4	18·9

(a) Evaluate $\Sigma x, \Sigma y, \Sigma xy$ and Σx^2. [2]

(b) Assuming a linear relationship $y = \alpha + \beta x$, calculate a and b, the least squares estimates of α and β. [6]

The values of x are exact whereas the values of y are subject to independent normally distributed errors with zero mean and standard deviation 0·4.

(c) (i) Use your values of a and b to estimate the true value of the resistance at 20°C. Determine the standard error of your estimate.

(ii) Hence find a 95% confidence interval for the true value of the resistance at 20°C.

(d) Alan predicted beforehand that the value of β would be 0·4. Determine, at the 1% significance level, whether or not his results are consistent with this prediction. [12]

(S3 Summer 2007)

7. The following table shows the temperature, y°C, of the water in a boiler at various times, x minutes, after switching on.

x	0	5	10	15	20	25	30
y	20	25	31	38	44	49	55

[You may assume that $\Sigma xy = 4760$, $\Sigma x^2 = 2275$]

(a) Assuming a linear relationship $y = \alpha + \beta x$, calculate least squares estimates a, b for α, β. Give your answers correct to four decimal places. [7]

(b) The boiler is being tested to determine whether or not the value of β is 1·2. Assuming that the values of x are exact whereas the values of y are subject to independent normally distributed errors with zero mean and standard deviation 0·25,

(i) state suitable hypotheses,

(ii) calculate the p-value of your value of b and state your conclusion. [9]

(S3 Summer 2008)

7. The length, y cm, of a wire is related to its temperature, x°C, by the equation $y = \alpha + \beta x$. The values of x can be controlled exactly whereas the measured values of y are subject to independent normally distributed errors with mean zero and standard deviation 0·15 cm. The following results were obtained for a particular wire.

Temperature (x°C)	20	30	40	50	60	70
Measured length (y cm)	82·3	83·9	85·3	86·8	88·6	90·1

[You are given that $\Sigma x = 270$, $\Sigma y = 517$, $\Sigma x^2 = 13\,900$, $\Sigma xy = 23\,538$]

(a) Calculate least squares estimates for α and β. [6]

(b) Calculate a 99% confidence inverval for the actual length of the wire when its temperature is 60°C. [6]

(S3 Summer 2009)

7. The variables x and y are known to be related by an equation of the form $y = \alpha + \beta x$. In order to estimate the values of α and β, the values of y were measured for six different values of x. The following results were obtained.

x	5	10	15	20	25	30
y	15·5	27·2	37·4	49·1	60·8	72·6

[You are given that $\Sigma x = 105$, $\Sigma y = 262{\cdot}6$, $\Sigma x^2 = 2275$, $\Sigma xy = 5590{\cdot}5$]

The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·5.

(a) Calculate least squares estimates for α and β. [6]

(b) The value of β is thought to be 2·34. The following hypotheses are therefore defined:

$$H_0: \beta = 2{\cdot}34 \text{ versus } H_1: \beta < 2{\cdot}34$$

Calculate the p-value of your result and interpret it. [6]

(c) Alun is given the same data and he evaluates the least squares estimate of β as 0·52. Explain briefly why this answer is obviously incorrect. [1]

(S3 Summer 2010)

7. The length, y metres, of an elastic string and its tension, x Newtons, are related by an equation of the form $y = \alpha + \beta x$. In order to estimate the values of α and β, the values of y were measured for six different values of x. The following results were obtained.

x	10	20	30	40	50	60
y	2·02	2·23	2·39	2·56	2·77	2·95

The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·02 metres.

(a) Calculate least squares estimates for α and β. [8]

(b) Determine a 90% confidence interval for α. [5]

(S3 Summer 2011)

6. The solubility y, in appropriate units, of a certain chemical in water is related to the temperature, x °C, by an equation of the form $y = \alpha + \beta x$. In order to estimate α and β, the following measurements were made.

x	10	12	14	16	18	20
y	21·7	24·4	27·3	29·6	31·7	34·5

[You are given that $\sum x = 90$, $\sum x^2 = 1420$, $\sum y = 169{\cdot}2$, $\sum xy = 2626{\cdot}2$]

(a) Calculate least squares estimates for α and β. [6]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·15. Determine a 99% confidence interval for the solubility of the chemical in water at 17 °C. [7]

(S3 Summer 2012)

5. The temperature y°C in an oven x minutes after switching on the oven can be assumed to satisfy the equation $y = \alpha + \beta x$. In order to estimate α and β, the following measurements were made.

x	0	1	2	3	4	5
y	20·0	34·4	49·3	65·6	79·7	96·5

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·75. Determine a 99% confidence interval for β. [5]

(S3 Summer 2013)

6. The resistance, y ohms, of an electrical component is related to the temperature, x°C, by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β , the following measurements were made.

x	10	15	20	25	30	35	40
y	12·3	13·9	15·1	16·6	18·6	20·1	21·5

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·1. Determine a 95% confidence interval for α. [5]

(S3 Summer 2014)

5. The variables x and y are related by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	2	4	6	8	10	12
y	19·8	33·9	49·9	64·1	77·9	95·0

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·5.

(i) Calculate an unbiased estimate of the value of y when $x = 5$.

(ii) Determine a 95% confidence interval for the value of y when $x = 5$.

(iii) It was thought beforehand that the value of β was 7·6. Determine whether or not, at the 5% significance level, the values in the table above are consistent with this value of β. [10]

(S3 Summer 2015)

5. The speed of sound in air, y ms^{-1}, and the air temperature, x °C, may be assumed to be related by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	10	15	20	25	30
y	337·1	340·7	343·0	346·1	349·7

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·25.

(i) Determine a 99% confidence interval for α, giving your answer correct to one decimal place. [5]

(ii) Test, at the 5% significance level, the null hypothesis $H_0 : \beta = 0{\cdot}65$ against a two-sided alternative. [7]

(S3 Summer 2016)

5. The amount, y grams, of chemical that dissolves in 1 litre of water at a temperature of x °C satisfies the relationship $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	10	20	30	40	50	60
y	162	183	201	225	248	267

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 1·5.

(i) Determine a 95% confidence interval for β, giving your limits correct to three significant figures.

(ii) A 95% confidence interval is to be determined for the value of y when $x = x_0$. Giving a reason, state the value of x_0 for which the confidence interval has minimum width. [7]

(S3 Summer 2017)

6. The length, y cm, of a spring subjected to a tension of x Newtons satisfies the relationship $y = \alpha + \beta x$, where α and β are unknown constants. In order to estimate α and β, the following measurements were made.

x	10	15	20	25	30	40
y	12·4	14·3	16·4	18·9	20·7	24·6

You are given that $\sum x = 140,\ \sum y = 107{\cdot}3,\ \sum x^2 = 3850,\ \sum xy = 2744.$

(a) Calculate least squares estimates for α and β, giving your answers correct to three significant figures. [6]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·2. Before the measurements were made, Emlyn believed that the value of β was 0·4.

(i) State suitable hypotheses to carry out a two-sided test of Emlyn's belief.

(ii) Calculate the *p*-value of the above results.

(iii) State whether or not the data support Emlyn's belief. [9]

(S3 Summer 2018)

6. A market gardener wishes to investigate the effect of a new fertiliser on his tomato yield. He therefore plants the same number of tomato plants in six different greenhouses and he applies different amounts of fertiliser in each greenhouse during the growing season. He records the total yield obtained from each greenhouse. He assumes that the total yield, y kg, and the amount of fertiliser applied, x units, satisfy a relationship of the form $y = \alpha + \beta x$, where α and β are unknown constants. The following table gives the results of his investigation.

x	0	10	20	30	40	50
y	22·6	24·1	26·2	28·5	30·7	32·6

You are given that $\Sigma x = 150, \quad \Sigma y = 164{\cdot}7, \quad \Sigma x^2 = 5500, \quad \Sigma xy = 4478.$

You may assume that the values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·25.

(a) (i) Calculate least squares estimates for α and β.

(ii) Hence estimate the total yield that would have been obtained with an application of 25 units of fertiliser.

(iii) Determine the standard error of this estimate. [10]

(b) An alternative estimate of the total yield defined in *(a)*(ii) can be found by calculating the arithmetic mean of the recorded values of y corresponding to $x = 0$ and $x = 50$.

(i) Calculate this estimate.

(ii) Determine its standard error. [3]