S1: Hapnewidyn Di-dor

Gaeaf 2005

(9) $f(x) = \frac{1}{21}x^2$ ar gyfer $1 \le x \le 4$

$f(x) = 0$ fel arall

(a)
$$E(X) = \int x f(x)\,dx$$
$$= \int_1^4 x\left(\frac{1}{21}x^2\right)dx$$
$$= \int_1^4 \frac{1}{21}x^3\,dx$$
$$= \frac{1}{21}\int_1^4 x^3\,dx$$
$$= \frac{1}{21}\left[\frac{x^4}{4}\right]_1^4$$
$$= \frac{1}{21}\left[\frac{4^4}{4} - \frac{1^4}{4}\right]$$
$$= \frac{1}{21}\left(64 - \frac{1}{4}\right)$$
$$= \frac{85}{28}$$

(b)
$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$
$$= \int_1^x f(t)\,dt$$
$$= \int_1^x \frac{1}{21}t^2\,dt$$
$$= \frac{1}{21}\int_1^x t^2\,dt$$
$$= \frac{1}{21}\left[\frac{t^3}{3}\right]_1^x$$
$$= \frac{1}{21}\left(\frac{x^3}{3} - \frac{1^3}{3}\right)$$
$$= \frac{1}{21}\left(\frac{x^3}{3} - \frac{1}{3}\right)$$
$$= \frac{1}{63}(x^3 - 1)$$

(c)
$$P(2 \le X \le 3)$$
$$= F(3) - F(2)$$
$$= \frac{1}{63}(3^3 - 1) - \frac{1}{63}(2^3 - 1)$$
$$= \frac{26}{63} - \frac{7}{63}$$
$$= \frac{19}{63}$$

(ch) Canolrif X: $F(x) = 0.5$

$$\frac{1}{63}(x^3 - 1) = 0.5$$
$$x^3 - 1 = 0.5 \times 63$$
$$x^3 = 31.5 + 1$$
$$x = \sqrt[3]{32.5}$$
$x = 3.19$ i 2 le degol

Haf 2005

8. $F(x) = 0$ ar gyfer $x < 0$

$F(x) = 4x^3 - 3x^4$ ar gyfer $0 \leq x \leq 1$

$F(x) = 1$ ar gyfer $x > 1$

(a) $P(0.2 \leq X \leq 0.8) = F(0.8) - F(0.2)$
$$= 4(0.8^3) - 3(0.8)^4 - (4(0.2)^3 - 3(0.2)^4)$$
$$= 0.792$$

(b) $F(0.45) = 4(0.45^3) - 3(0.45^4)$
$$= 0.24148125.$$
$F(0.46) = 4(0.46^3) - 3(0.46^4)$
$$= 0.25502032.$$
Mae 0.25 rhwng 0.24148125 a 0.25502032

Felly mae'r chwartel isaf rhwng 0.45 a 0.46.

(c) $f(x) = \frac{d}{dx}(F(x))$
$$= \frac{d}{dx}(4x^3 - 3x^4)$$
$$= 12x^2 - 12x^3$$
(ar gyfer $0 \leq x \leq 1$)

(ch) $E(X) = \int x f(x)\,dx$
$$= \int_0^1 x(12x^2 - 12x^3)\,dx$$
$$= 12\int_0^1 x^3 - x^4\,dx$$
$$= 12\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$
$$= 12\left(\left(\frac{1^4}{4} - \frac{1^5}{5}\right) - \left(\frac{0^4}{4} - \frac{0^5}{5}\right)\right)$$
$$= 12\left(\frac{1}{4} - \frac{1}{5}\right)$$
$$= 0.6$$

Graeaf 2006

9. $f(x) = Kx^2$ ar gyfer $1 \le x \le 4$

$f(x) = 0$ fel arall

(a) (i)

$$\int f(x)\,dx = 1$$
$$\int_1^4 Kx^2\,dx = 1$$
$$K\int_1^4 x^2\,dx = 1$$
$$K\left[\frac{x^3}{3}\right]_1^4 = 1$$
$$K\left(\frac{4^3}{3} - \frac{1^3}{3}\right) = 1$$
$$K\left(\frac{64}{3} - \frac{1}{3}\right) = 1$$
$$\frac{63}{3}K = 1$$
$$K = \frac{3}{63}$$
$$K = \frac{1}{21} \quad ✓$$

(ii)

$$E(X) = \int x\,f(x)\,dx$$
$$= \int_1^4 x(Kx^2)\,dx$$
$$= K\int_1^4 x^3\,dx$$
$$= K\left[\frac{x^4}{4}\right]_1^4$$
$$= K\left(\frac{4^4}{4} - \frac{1^4}{4}\right)$$
$$= K\left(64 - \frac{1}{4}\right)$$
$$= \frac{1}{21}\left(64 - \frac{1}{4}\right)$$
$$= \frac{85}{28}$$

(b) (i)

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$
$$= \int_1^x f(t)\,dt$$
$$= \int_1^x Kt^2\,dt$$
$$= K\int_1^x t^2\,dt$$
$$= K\left[\frac{t^3}{3}\right]_1^x$$
$$= K\left(\frac{x^3}{3} - \frac{1^3}{3}\right)$$
$$= \frac{1}{21}\left(\frac{x^3}{3} - \frac{1}{3}\right)$$
$$= \frac{1}{63}(x^3 - 1)$$

(ii)

$$P(2 \le X \le 3)$$
$$= F(3) - F(2)$$
$$= \frac{1}{63}(3^3 - 1) - \frac{1}{63}(2^3 - 1)$$
$$= \frac{26}{63} - \frac{7}{63}$$
$$= \frac{19}{63}$$

(iii) Canolrif: $F(x) = 0.5$

$$\frac{1}{63}(x^3 - 1) = 0.5$$
$$x^3 - 1 = 0.5 \times 63$$
$$x^3 = 31.5 + 1$$
$$x = \sqrt[3]{32.5}$$
$$x = 3.19 \text{ i 2 le degol}$$

Haf 2006

(8) $F(x) = 0$ ar gyfer $x < 0$

$F(x) = \frac{1}{2}(x^2 + x)$ ar gyfer $0 \le x \le 1$

$F(x) = 1$ ar gyfer $x > 1$

(a) (i) $P(0.25 \le X \le 0.5) = F(0.5) - F(0.25)$

$= \frac{1}{2}(0.5^2 + 0.5) - \frac{1}{2}(0.25^2 + 0.25)$

$= 0.21875$

(ii) canolrif X: $F(x) = 0.5$

$\frac{1}{2}(x^2 + x) = 0.5$

$x^2 + x = 1$

$x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2 \times 1}$$

Unai $x = \frac{-1 + \sqrt{1+4}}{2}$ neu $x = \frac{-1 - \sqrt{1+4}}{2}$

$x = 0.618$ i 3 ll.d. $x = -1.618$ i 3 ll.d.

Gan fod $0 \le x \le 1$ rhaid bod $\underline{x = 0.618}$ i 3 ll.d.

(b) (i) $f(x) = \frac{d}{dx}(F(x))$

$= \frac{d}{dx}(\frac{1}{2}(x^2 + x))$

$= \frac{1}{2}(2x) + \frac{1}{2}$

$= x + \frac{1}{2}$

(ar gyfer $0 \le x \le 1$)

(ii) $E(X) = \int x f(x) dx$

$= \int_0^1 x(x + \frac{1}{2}) dx$

$= \int_0^1 x^2 + \frac{1}{2}x \, dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{4}\right]_0^1$

$= \left[\left(\frac{1^3}{3} + \frac{1^2}{4}\right) - \left(\frac{0^3}{3} + \frac{0^2}{4}\right)\right]$

$= \frac{1}{3} + \frac{1}{4}$

$= \frac{7}{12}$

Gaeaf 2007

7) $f(x) = 20(x^3 - x^4)$ ar gyfer $0 \leq x \leq 1$

$f(x) = 0$ fel arall

(a) $E(X) = \int x f(x) dx$

$= \int_0^1 x(20(x^3 - x^4))dx$

$= \int_0^1 20x^4 - 20x^5 \, dx$

$= \left[\frac{20x^5}{5} - \frac{20x^6}{6} \right]_0^1$

$= \left[\left(\frac{20 \times 1^5}{5} - \frac{20 \times 1^6}{6} \right) - \left(\frac{20 \times 0^5}{5} - \frac{20 \times 0^6}{6} \right) \right]$

$= \frac{20}{5} - \frac{20}{6}$

$= \frac{2}{3}$

(b) (i) $F(x) = \int_{-\infty}^{x} f(t) dt$

$= \int_0^x f(t) dt$

$= \int_0^x 20(t^3 - t^4) dt$

$= \left[\frac{20t^4}{4} - \frac{20t^5}{5} \right]_0^x$

$= [5t^4 - 4t^5]_0^x$

$= [(5x^4 - 4x^5) - (5(0)^4 - 4(0)^5)]$

$= 5x^4 - 4x^5$

(ii) $P(0.4 \leq X \leq 0.6)$

$= F(0.6) - F(0.4)$

$= 5(0.6^4) - 4(0.6^5) - (5(0.4^4) - 4(0.4^5))$

$= \frac{781}{3125}$

$= 0.24992$

(iii) Chwartel uchaf:

$F(x) = 0.75$

Yn defnyddio q,

$F(q) = 0.75$

$5q^4 - 4q^5 = 0.75$

$5q^4 - 4q^5 = \frac{3}{4}$

$20q^4 - 16q^5 = 3$

$0 = 16q^5 - 20q^4 + 3$

$16q^5 - 20q^4 + 3 = 0$ ✓

Haf 2007

7 $f(x) = \frac{6}{5}x(x-1)$ ar gyfer $1 \le x \le 2$

$f(x) = 0$ fel arall

(a) $E\left(\frac{1}{x}\right) = \int \frac{1}{x} f(x)\,dx$

$= \int_1^2 \frac{1}{x}\left(\frac{6}{5}x(x-1)\right)dx$

$= \int_1^2 \frac{6}{5}(x-1)\,dx$

$= \frac{6}{5}\int_1^2 x-1\,dx$

$= \frac{6}{5}\left[\frac{x^2}{2} - x\right]_1^2$

$= \frac{6}{5}\left[\left(\frac{2^2}{2}-2\right)-\left(\frac{1^2}{2}-1\right)\right]$

$= \frac{6}{5}\left[\ 0 - \frac{1}{2} + 1\right]$

$= 0.6$

(b) (i) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$= \int_1^x f(t)\,dt$

$= \int_1^x \frac{6}{5}t(t-1)\,dt$

$= \frac{6}{5}\int_1^x t^2 - t\,dt$

$= \frac{6}{5}\left[\frac{t^3}{3} - \frac{t^2}{2}\right]_1^x$

$= \frac{6}{5}\left[\left(\frac{x^3}{3} - \frac{x^2}{2}\right) - \left(\frac{1^3}{3} - \frac{1^2}{2}\right)\right]$

$= \frac{6}{5}\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{6}\right)$

$= 0.4x^3 - 0.6x^2 + 0.2$

(ii) $P(X \le 1.75) = F(1.75)$

$= 0.4 \times 1.75^3 - 0.6 \times 1.75^2 + 0.2$

$= 0.50625$

(iii) Mae canolrif X yn llai na 1.75 gan fod $F(1.75)$ yn fwy na 0.5.

Graf 2008

(8) $f(x) = 4 - 2x$ ar gyfer $1 \le x \le 2$

$f(x) = 0$ fel arall

(a)
$$E(X) = \int x f(x)\,dx$$
$$= \int_1^2 x(4-2x)\,dx$$
$$= \int_1^2 4x - 2x^2\,dx$$
$$= \left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_1^2$$
$$= \left[\left(2(2^2) - \frac{2}{3}(2^3)\right) - \left(2(1^2) - \frac{2}{3}(1^3)\right)\right]$$
$$= 8 - \frac{16}{3} - 2 + \frac{2}{3}$$
$$= \frac{4}{3}$$

(b)
$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$
$$= \int_1^x f(t)\,dt$$
$$= \int_1^x 4 - 2t\,dt$$
$$= \left[4t - \frac{2t^2}{2}\right]_1^x$$
$$= [(4x - x^2) - (4(1) - 1^2)]$$
$$= 4x - x^2 - 4 + 1$$
$$= 4x - x^2 - 3 \quad \checkmark$$

(c)
$$P(X > 1.2) = 1 - P(X \le 1.2)$$
$$= 1 - F(1.2)$$
$$= 1 - (4(1.2) - (1.2)^2 - 3)$$
$$= 1 - 0.36$$
$$= 0.64$$

(ch) Canolrif X:
$$F(x) = 0.5$$
$$4x - x^2 - 3 = 0.5$$
$$0 = x^2 - 4x + 3 + 0.5$$
$$0 = x^2 - 4x + 3.5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(3.5)}}{2 \times 1}$$
$$x = \frac{4 \pm \sqrt{16 - 14}}{2}$$

unai $x = \frac{4 + \sqrt{2}}{2}$ neu $x = \frac{4 - \sqrt{2}}{2}$

$x = 2.71$ i 2 le degol, $x = 1.29$ i 2 le degol.

Gan fod $1 \le x \le 2$, rhaid bod $\underline{x = 1.29}$ i 2 le degol.

Haf 2008

(8) $F(x) = 0$ ar gyfer $x < 0$

$F(x) = 4x^3 - 3x^4$ ar gyfer $0 \le x \le 1$

$F(x) = 1$ ar gyfer $x > 1$

(a) $P(0.25 \le X \le 0.75) = F(0.75) - F(0.25)$

$= 4(0.75^3) - 3(0.75^4) - (4(0.25^3) - 3(0.25^4))$

$= 0.6875$

(b) $F(0.6) = 4(0.6^3) - 3(0.6^4)$

$= 0.4752.$

Mae canolrif X yn fwy na 0.6 gan fod 0.4752 yn llai na 0.5.

(c) $f(x) = \frac{d}{dx} F(x)$

$= \frac{d}{dx}(4x^3 - 3x^4)$

$= 12x^2 - 12x^3$ (ar gyfer $0 \le x \le 1$)

(ch) $E(X) = \int x.f(x)\,dx$

$= \int_0^1 x(12x^2 - 12x^3)\,dx$

$= \int_0^1 12x^3 - 12x^4\,dx$

$= \left[\frac{12x^4}{4} - \frac{12x^5}{5}\right]_0^1$

$= \left[\left(\frac{12(1^4)}{4} - \frac{12(1^5)}{5}\right) - \left(\frac{12(0^4)}{4} - \frac{12(0^5)}{5}\right)\right]$

$= 3 - \frac{12}{5}$

$= 0.6$

Gaeaf 2009

(9) $F(x) = 0$ ar gyfer $x < 0$

$F(x) = Kx^3$ ar gyfer $0 \le x \le 2$

$F(x) = 1$ ar gyfer $x > 2$

(a) Rhaid bod $F(2) = 1$

$K(2^3) = 1$

$8K = 1$

$K = \frac{1}{8}$ ✓

(b) $P(0.5 \le X \le 1.5) = F(1.5) - F(0.5)$

$= K(1.5^3) - K(0.5^3)$

$= \frac{1}{8} \times 3.375 - \frac{1}{8} \times 0.125$

$= 0.40625$

(c) Canolrif X: $F(x) = 0.5$

$Kx^3 = 0.5$

$\frac{1}{8} x^3 = 0.5$

$x^3 = 4$

$x = \sqrt[3]{4}$

$x = 1.59$ i 2 le degol.

(ch) $f(x) = \frac{d}{dx} F(x)$

$= \frac{d}{dx}(Kx^3)$

$= 3Kx^2$

$= \frac{3}{8}x^2$.

$E(X) = \int x f(x)\,dx$

$= \int_0^2 x\left(\frac{3}{8}x^2\right)dx$

$= \frac{3}{8}\int_0^2 x^3\,dx$

$= \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2$

$= \frac{3}{8}\left[\frac{2^4}{4} - \frac{0^4}{4}\right]$

$= \frac{3}{8} \times \frac{16}{4}$

$= 1.5$

Haf 2009

(8) $f(x) = \frac{1}{2}(1+2x)$ ar gyfer $0 \le x \le 1$

$f(x) = 0$ Fel arall

(a)
$$
\begin{aligned}
E(X) &= \int x f(x)\,dx \\
&= \int_0^1 x\left(\tfrac{1}{2}(1+2x)\right)dx \\
&= \int_0^1 \tfrac{1}{2}(x+2x^2)\,dx \\
&= \int_0^1 \tfrac{1}{2}x + x^2\,dx \\
&= \left[\frac{x^2}{4} + \frac{x^3}{3}\right]_0^1 \\
&= \left[\left(\frac{1^2}{4}+\frac{1^3}{3}\right) - \left(\frac{0^2}{4}+\frac{0^3}{3}\right)\right] \\
&= \frac{1}{4}+\frac{1}{3} \\
&= \frac{7}{12}
\end{aligned}
$$

(b)
$$
\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(t)\,dt \\
&= \int_0^x f(t)\,dt \\
&= \int_0^x \tfrac{1}{2}(1+2t)\,dt \\
&= \int_0^x \tfrac{1}{2} + t\,dt \\
&= \left[\frac{1}{2}t + \frac{t^2}{2}\right]_0^x \\
&= \left[\left(\frac{1}{2}x + \frac{x^2}{2}\right) - \left(\frac{1}{2}(0) + \frac{0^2}{2}\right)\right] \\
&= \tfrac{1}{2}(x+x^2) \\
&= \tfrac{1}{2}x(1+x) \quad (\text{ar gyfer } 0 \le x \le 1).
\end{aligned}
$$

(c) (i) $P(0.4 \le X \le 0.5)$
$$
\begin{aligned}
&= F(0.5) - F(0.4) \\
&= \tfrac{1}{2}(0.5)(1+0.5) - \tfrac{1}{2}(0.4)(1+0.4) \\
&= 0.375 - 0.28 \\
&= 0.095
\end{aligned}
$$

(ii) Canolrif X: $F(x) = 0.5$
$$
\begin{aligned}
\tfrac{1}{2}x(1+x) &= 0.5 \\
x(1+x) &= 1 \\
x + x^2 &= 1 \\
x^2 + x - 1 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2\times 1} \\
x &= \frac{-1 \pm \sqrt{5}}{2}
\end{aligned}
$$

Unai $x = \frac{-1+\sqrt{5}}{2}$ neu $x = \frac{-1-\sqrt{5}}{2}$

$x = 0.62$ i 2 le degol $\qquad x = -1.62$ i 2 le degol

ond mae $0 \le x \le 1$ felly rhaid bod $\underline{x = 0.62}$ i 2 le degol.

Gaeaf 2010

(8) $F(x) = 0$ ar gyfer $x < 1$

$F(x) = \frac{1}{10}(x^2 + x - 2)$ ar gyfer $1 \le x \le 3$

$F(x) = 1$ ar gyfer $x > 3$

(a) (i) $P(2 \le X \le 2.5) = F(2.5) - F(2)$

$= \frac{1}{10}(2.5^2 + 2.5 - 2) - \frac{1}{10}(2^2 + 2 - 2)$

$= 0.675 - 0.4$

$= 0.275$

(ii) canolrif X: $F(x) = 0.5$

$\frac{1}{10}(x^2 + x - 2) = 0.5$

$x^2 + x - 2 = 5$

$x^2 + x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-7)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{29}}{2}$$

Unai $x = \frac{-1 + \sqrt{29}}{2}$ neu $x = \frac{-1 - \sqrt{29}}{2}$

$x = 2.19$ i 2 le degol $x = -3.19$ i 2 le degol.

Ond $1 \le x \le 3$ felly rhaid bod $x = 2.19$ i 2 le degol.

(b) (i) $f(x) = \frac{d}{dx} F(x)$

$= \frac{d}{dx}\left(\frac{1}{10}(x^2 + x - 2)\right)$

$= \frac{1}{10}(2x + 1)$

$= \frac{2}{10}x + \frac{1}{10}$

(ar gyfer $1 \le x \le 3$).

(ii) $f(4) = 0$

gan nad yw 4 yn yr amrediad $1 \le x \le 3$.

(iii) $E(X) = \int x f(x)\,dx$

$= \int_1^3 x\left(\frac{2}{10}x + \frac{1}{10}\right)dx$

$= \int_1^3 \frac{2}{10}x^2 + \frac{1}{10}x\,dx$

$= \left[\frac{2x^3}{30} + \frac{x^2}{20}\right]_1^3$

$= \left[\left(\frac{2 \times 3^3}{30} + \frac{3^2}{20}\right) - \left(\frac{2 \times 1^3}{30} + \frac{1^2}{20}\right)\right]$

$= 2.25 - \frac{7}{60}$

$= \frac{32}{15}$

Haf 2010

⑧ $f(x) = Kx(1-x^2)$ ar gyfer $0 \le x \le 1$

$f(x) = 0$ Fel arall

(a)
$$\int f(x)\,dx = 1$$
$$\int_0^1 Kx(1-x^2)\,dx = 1$$
$$K\int_0^1 x - x^3\,dx = 1$$
$$K\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 1$$
$$K\left[\left(\frac{1^2}{2} - \frac{1^4}{4}\right) - \left(\frac{0^2}{2} - \frac{0^4}{4}\right)\right] = 1$$
$$K\left(\tfrac{1}{2} - \tfrac{1}{4}\right) = 1$$
$$\tfrac{1}{4}K = 1$$
$$K = 4 \quad ✓$$

(b)
$$E(x) = \int x f(x)\,dx$$
$$= \int_0^1 x(Kx(1-x^2))\,dx$$
$$= \int_0^1 4x^2 - 4x^4\,dx$$
$$= \left[\frac{4x^3}{3} - \frac{4x^5}{5}\right]_0^1$$
$$= \left[\left(\frac{4(1^3)}{3} - \frac{4(1^5)}{5}\right) - \left(\frac{4(0)^3}{3} - \frac{4(0)^5}{5}\right)\right]$$
$$= \frac{4}{3} - \frac{4}{5}$$
$$= \frac{8}{15}$$

(c) (i)
$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$
$$= \int_0^x f(t)\,dt$$
$$= \int_0^x Kt(1-t^2)\,dt$$
$$= \int_0^x 4t - 4t^3\,dt$$
$$= \left[\frac{4t^2}{2} - \frac{4t^4}{4}\right]_0^x$$
$$= \left[2t^2 - t^4\right]_0^x$$
$$= \left[(2x^2 - x^4) - (2(0)^2 - (0^4))\right]$$
$$= 2x^2 - x^4$$
(ar gyfer $0 \le x \le 1$).

(ii)
$$P(0.25 \le X \le 0.75)$$
$$= F(0.75) - F(0.25)$$
$$= (2\times 0.75^2 - 0.75^4) - (2\times 0.25^2 - 0.25^4)$$
$$= \frac{207}{256} - \frac{31}{256}$$
$$= 0.6875$$

(iii) Canolrif X: $F(x) = 0.5$
$$2x^2 - x^4 = 0.5$$
$$0 = x^4 - 2x^2 + 0.5$$
Gadewch i $y = x^2$. Yna
$$0 = y^2 - 2y + 0.5$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(0.5)}}{2\times 1}$$

$$y = \frac{2 \pm \sqrt{2}}{2}$$

Unai $y = \frac{2+\sqrt{2}}{2}$ neu $y = \frac{2-\sqrt{2}}{2}$

$y = 1.707106781\ldots$ neu $y = 0.2928932188\ldots$

ond $y = x^2$. Felly $x = \sqrt{y}$

Unai $x = \sqrt{1.707106781\ldots}$ neu $x = \sqrt{0.2928932188\ldots}$

$x = 1.31$ i 2 le degol. $x = 0.54$ i 2 le degol

ond $0 \leq x \leq 1$ felly rhaid bod $\underline{x = 0.54}$ i 2 le degol.

<u>Gaeaf 2011</u>

9 $f(x) = \frac{1}{6}(x+1)$ ar gyfer $1 \leq x \leq 3$.

$f(x) = 0$ fel arall

(a) $E(X) = \int x f(x) dx$

$$= \int_1^3 x\left(\frac{1}{6}(x+1)\right) dx$$

$$= \int_1^3 \frac{1}{6}x^2 + \frac{1}{6}x \, dx$$

$$= \left[\frac{x^3}{18} + \frac{x^2}{12}\right]_1^3$$

$$= \left[\left(\frac{3^3}{18} + \frac{3^2}{12}\right) - \left(\frac{1^3}{18} + \frac{1^2}{12}\right)\right]$$

$$= 2.25 - \frac{5}{36}$$

$$= \frac{19}{9}$$

(b) (i) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$= \int_{1}^{x} f(t)\,dt$

$= \int_{1}^{x} \frac{1}{6}(t+1)\,dt$

$= \frac{1}{6}\int_{1}^{x} t+1\,dt$

$= \frac{1}{6}\left[\frac{t^2}{2}+t\right]_1^x$

$= \frac{1}{6}\left[\left(\frac{x^2}{2}+x\right)-\left(\frac{1^2}{2}+1\right)\right]$

$= \frac{1}{6}\left(\frac{x^2}{2}+x-\frac{3}{2}\right)$

$= \frac{x^2}{12}+\frac{x}{6}-\frac{3}{12}$

$= \frac{x^2}{12}+\frac{x}{6}-\frac{1}{4}$

(ii) $F(4) = 1$ gan fod $4 > 3$.

(iii) $P(1.5 \leq X \leq 2) = F(2) - F(1.5)$

$= \left(\frac{2^2}{12}+\frac{2}{6}-\frac{1}{4}\right)-\left(\frac{1.5^2}{12}+\frac{1.5}{6}-\frac{1}{4}\right)$

$= \frac{5}{12} - \frac{3}{16}$

$= \frac{11}{48}$

(iv) canolrif X: $F(x) = 0.5$

$\frac{x^2}{12}+\frac{x}{6}-\frac{1}{4} = 0.5$

$x^2+2x-3 = 6$ (lluosi efo 12)

$x^2+2x-9=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{2^2-4(1)(-9)}}{2\times 1}$

$x = \frac{-2 \pm \sqrt{40}}{2}$

Unai

$x = \frac{-2+\sqrt{40}}{2}$

$x = 2.16$ i 2 le degol

neu

$x = \frac{-2-\sqrt{40}}{2}$

$x = -4.16$ i 2 le degol

ond $1 \leq x \leq 3$ felly rhaid bod $\underline{x = 2.16}$ i 2 le degol.

Haf 2011

8. $f(x) = 12x^2(1-x)$ ar gyfer $0 \le x \le 1$

$f(x) = 0$ fel arall

(a) (i) $E(X) = \int x f(x)\,dx$

$= \int_0^1 x(12x^2(1-x))\,dx$

$= \int_0^1 12x^3 - 12x^4\,dx$

$= \left[\frac{12x^4}{4} - \frac{12x^5}{5}\right]_0^1$

$= \left[\left(\frac{12(1^4)}{4} - \frac{12(1^5)}{5}\right) - \left(\frac{12(0^4)}{4} - \frac{12(0^5)}{5}\right)\right]$

$= \frac{12}{4} - \frac{12}{5}$

$= 0.6$

(ii) $E\left(\frac{1}{X}\right) = \int \frac{1}{x} f(x)\,dx$

$= \int_0^1 \frac{1}{x}(12x^2(1-x))\,dx$

$= \int_0^1 12x - 12x^2\,dx$

$= \left[\frac{12x^2}{2} - \frac{12x^3}{3}\right]_0^1$

$= \left[6x^2 - 4x^3\right]_0^1$

$= \left[(6(1)^2 - 4(1)^3) - (6(0^2) - 4(0^3))\right]$

$= 6 - 4$

$= 2$

(iii) $P(0.2 \le X \le 0.5) = \int_{0.2}^{0.5} f(x)\,dx$

$= \int_{0.2}^{0.5} 12x^2(1-x)\,dx$

$= \int_{0.2}^{0.5} 12x^2 - 12x^3\,dx$

$= \left[\frac{12x^3}{3} - \frac{12x^4}{4}\right]_{0.2}^{0.5}$

$= \left[4x^3 - 3x^4\right]_{0.2}^{0.5}$

$= \left[(4 \times 0.5^3 - 3 \times 0.5^4) - (4 \times 0.2^3 - 3 \times 0.2^4)\right]$

$= 0.3125 - 0.0272$

$= 0.2853$

(b) $F(y) = ay + by^2$ ar gyfer $1 \leq y \leq 2$.

$F(1) = 0$

$a(1) + b(1^2) = 0$

$a + b = 0$

$a = -b$ — 1

$F(2) = 1$

$a(2) + b(2^2) = 1$

$2a + 4b = 1$ — 2

Yn amnewid am a o 1 i 2:

$2(-b) + 4b = 1$

$-2b + 4b = 1$

$2b = 1$

$\underline{b = \frac{1}{2}}$

Yn amnewid yn ôl i 1:

$\underline{a = -\frac{1}{2}}$

Gaeaf 2012

9

$F(x) = 0$ ar gyfer $x < 1$

$F(x) = K(x^2 - x)$ ar gyfer $1 \leq x \leq 3$

$F(x) = 1$ ar gyfer $x > 3$.

(a) (i) $F(3) = 1$

$K(3^2 - 3) = 1$

$K(9 - 3) = 1$

$6K = 1$

$K = \frac{1}{6}$

(ii) $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - F(2)$

$= 1 - K(2^2 - 2)$

$= 1 - \frac{1}{6}(4 - 2)$

$= 1 - \frac{2}{6}$

$= \frac{2}{3}$

(iii) canolrif X: $F(x) = 0.5$

$$k(x^2 - x) = 0.5$$
$$\frac{1}{6}(x^2 - x) = 0.5$$
$$x^2 - x = 3$$
$$x^2 - x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2 \times 1}$$
$$x = \frac{1 \pm \sqrt{13}}{2}$$

Unai $x = \frac{1+\sqrt{13}}{2}$ neu $x = \frac{1-\sqrt{13}}{2}$

$x = 2.30$ i 2 le degol $\quad x = -1.30$ i 2 le degol

ond $1 \leq x \leq 3$ felly rhaid bod $\underline{x = 2.30}$ i 2 le degol.

(b) (i) $f(x) = \frac{d}{dx} F(x)$

$$= \frac{d}{dx}(k(x^2 - x))$$
$$= \frac{d}{dx}\left(\frac{1}{6}(x^2 - x)\right)$$
$$= \frac{1}{6}(2x - 1)$$
$$= \frac{1}{3}x - \frac{1}{6} \quad \text{(ar gyfer } 1 \leq x \leq 3)$$

(ii) $E(X) = \int_3 x f(x)\, dx$

$$= \int_1 x\left(\frac{1}{3}x - \frac{1}{6}\right) dx$$
$$= \int_1^3 \frac{1}{3}x^2 - \frac{1}{6}x\, dx$$
$$= \left[\frac{x^3}{9} - \frac{x^2}{12}\right]_1^3$$
$$= \left[\left(\frac{3^3}{9} - \frac{3^2}{12}\right) - \left(\frac{1^3}{9} - \frac{1^2}{12}\right)\right]$$
$$= \left(3 - \frac{3}{4}\right) - \left(\frac{1}{9} - \frac{1}{12}\right)$$
$$= \frac{20}{9}$$

Haf 2012

(9) $f(x) = \frac{1}{10}(2x + 3x^2)$ ar gyfer $1 \leq x \leq 2$

$f(x) = 0$ fel arall

(a) (i) $E(X) = \int x f(x) dx$

$$= \int_1^2 x\left(\frac{1}{10}(2x+3x^2)\right) dx$$

$$= \int_1^2 \frac{1}{5}x^2 + \frac{3}{10}x^3 \, dx$$

$$= \left[\frac{x^3}{15} + \frac{3x^4}{40}\right]_1^2$$

$$= \left[\left(\frac{2^3}{15} + \frac{3(2^4)}{40}\right) - \left(\frac{1^3}{15} + \frac{3(1^4)}{40}\right)\right]$$

$$= \frac{8}{15} + \frac{48}{40} - \frac{1}{15} - \frac{3}{40}$$

$$= \frac{191}{120}$$

(ii) $E(X^2) = \int x^2 f(x) dx$

$$= \int_1^2 x^2\left(\frac{1}{10}(2x+3x^2)\right) dx$$

$$= \int_1^2 \frac{1}{5}x^3 + \frac{3}{10}x^4 \, dx$$

$$= \left[\frac{x^4}{20} + \frac{3x^5}{50}\right]_1^2$$

$$= \left[\left(\frac{2^4}{20} + \frac{3(2^5)}{50}\right) - \left(\frac{1^4}{20} + \frac{3(1^5)}{50}\right)\right]$$

$$= \frac{16}{20} + \frac{96}{50} - \frac{1}{20} - \frac{3}{50}$$

$$= 2.61$$

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$$= 2.61 - \left(\frac{191}{120}\right)^2$$

$= 0.08$ i 2 le degol

(b) (i) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$= \int_{1}^{x} f(t)\,dt$

$= \int_{1}^{x} \frac{1}{10}(2t+3t^2)\,dt$

$= \int_{1}^{x} \frac{1}{5}t + \frac{3}{10}t^2\,dt$

$= \left[\frac{t^2}{10} + \frac{3t^3}{30}\right]_1^x$

$= \left[\left(\frac{x^2}{10} + \frac{3x^3}{30}\right) - \left(\frac{1^2}{10} + \frac{3(1^3)}{30}\right)\right]$

$= \frac{x^2}{10} + \frac{x^3}{10} - \frac{1}{10} - \frac{1}{10}$

$= \frac{1}{10}(x^2 + x^3 - 2)$ (ar gyfer $1 \le x \le 2$)

(ii) $P(X \le 1.4) = F(1.4)$

$= \frac{1}{10}(1.4^2 + 1.4^3 - 2)$

$= 0.2704$

(iii) Mae chwartel isaf X yn llai na 1.4 gan fod 0.2704 yn fwy na 0.25

Gaeaf 2013

8 $F(x) = 0$ ar gyfer $x < 0$

$F(x) = 2x^2 - x^4$ ar gyfer $0 \le x \le 1$

$F(x) = 1$ ar gyfer $x > 1$

(a) (i) $P(0.25 \le X \le 0.75) = F(0.75) - F(0.25)$

$= (2(0.75^2) - 0.75^4) - (2(0.25)^2 - 0.25^4)$

$= \frac{207}{256} - \frac{31}{256}$

$= 0.6875$

(ii) Canolrif X: $F(x) = 0.5$

$2x^2 - x^4 = 0.5$

$4x^2 - 2x^4 = 1$

$$0 = 2x^4 - 4x^2 + 1$$
$$2x^4 - 4x^2 + 1 = 0$$

Os yw'r canolrif yn m yna mae'r canolrif yn bodloni

$$2m^4 - 4m^2 + 1 = 0 \quad \checkmark$$

(iii) Gadewch i $y = m^2$. Yna

$$2y^2 - 4y + 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2 \times 2}$$

$$y = \frac{4 \pm \sqrt{8}}{4}$$

Unai $y = \frac{4+\sqrt{8}}{4}$ neu $y = \frac{4-\sqrt{8}}{4}$

$y = 1.70710678\ldots$ $\quad$ $y = 0.2928932188\ldots$

Ond $y = m^2$ felly $m = \sqrt{y}$.

Felly unai $m = \sqrt{1.70710678\ldots}$ neu $m = \sqrt{0.2928932188\ldots}$

$m = 1.31$ i 3 ff. yst. $\quad$ $m = 0.541$ i 3 ff. yst.

Ond $0 \leq m \leq 1$ felly $\underline{m = 0.541}$ i 3 ffigur ystyrlon.

(b) (i) $f(x) = \frac{d}{dx} F(x)$

$$= \frac{d}{dx}(2x^2 - x^4)$$

$$= 4x - 4x^3 \quad \text{(ar gyfer } 0 \leq x \leq 1)$$

(ii) $E(\sqrt{x}) = \int \sqrt{x}\, f(x)\, dx$

$$= \int_0^1 \sqrt{x}(4x - 4x^3)\, dx$$

$$= 4\int_0^1 x^{\frac{1}{2}}(x - x^3)\, dx$$

$$= 4\int x^{\frac{3}{2}} - x^{\frac{7}{2}}\, dx$$

$$= 4\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{9}{2}}}{\frac{9}{2}}\right]_0^1$$

$$= 4\left[\left(\frac{1^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1^{\frac{9}{2}}}{\frac{9}{2}}\right) - \left(\frac{0^{\frac{5}{2}}}{\frac{5}{2}} - \frac{0^{\frac{9}{2}}}{\frac{9}{2}}\right)\right]$$

$$= 4\left(\frac{1}{\frac{5}{2}} - \frac{1}{\frac{9}{2}}\right)$$

$$= 4\left(\frac{2}{5} - \frac{2}{9}\right)$$

$$= \underline{\frac{32}{45}}$$

Haf 2013

(9) $f(x) = k\left(1 - \frac{x^2}{4}\right)$ ar gyfer $0 \leq x \leq 2$

$f(x) = 0$ fel arall

(a) $\int f(x)\,dx = 1$

$$\int_0^2 k\left(1 - \frac{x^2}{4}\right)dx = 1$$

$$k\int_0^2 1 - \frac{x^2}{4}\,dx = 1$$

$$k\left[x - \frac{x^3}{12}\right]_0^2 = 1$$

$$k\left[\left(2 - \frac{2^3}{12}\right) - \left(0 - \frac{0^3}{12}\right)\right] = 1$$

$$k\left(2 - \frac{8}{12}\right) = 1$$

$$k\left(\frac{4}{3}\right) = 1$$

$$k = \frac{3}{4} \quad \checkmark$$

(b) $E(x) = \int x f(x)\,dx$

$$= \int_0^2 x\left(k\left(1 - \frac{x^2}{4}\right)\right)dx$$

$$= \int_0^2 \frac{3}{4}x - \frac{3x^3}{16}\,dx$$

$$= \left[\frac{3x^2}{8} - \frac{3x^4}{64}\right]_0^2$$

$$= \left[\left(\frac{3(2^2)}{8} - \frac{3(2^4)}{64}\right) - \left(\frac{3(0^2)}{8} - \frac{3(0^4)}{64}\right)\right]$$

$$= \frac{12}{8} - \frac{3}{4}$$

$$= \frac{3}{4}$$

(c) (i) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$$= \int_0^x f(t)\,dt$$

$$= \int_0^x k\left(1 - \frac{t^2}{4}\right)dt$$

$$= k\int_0^x 1 - \frac{t^2}{4}\,dt$$

$$= k\left[t - \frac{t^3}{12}\right]_0^x$$

$$= \frac{3}{4}\left[\left(x - \frac{x^3}{12}\right) - \left(0 - \frac{0^3}{12}\right)\right]$$

$$= \frac{3}{4}\left(x - \frac{x^3}{12}\right)$$

(ar gyfer $0 \leq x \leq 2$)

(ii) $P(0.5 \leq X \leq 1.5) = F(1.5) - F(0.5)$

$$= \frac{3}{4}\left(1.5 - \frac{1.5^3}{12}\right) - \frac{3}{4}\left(0.5 - \frac{0.5^3}{12}\right)$$

$$= \frac{117}{128} - \frac{47}{128}$$

$$= 0.546875$$

S1 Gaeaf 2014

(9) $F(x) = 0$ ar gyfer $x < 1$
$F(x) = K(x^3 - x)$ ar gyfer $1 \leq x \leq 2$
$F(x) = 1$ ar gyfer $x > 2$

(a) (i) Rhaid bod $F(2) = 1$

$$K(2^3 - 2) = 1$$
$$K(8-2) = 1$$
$$6K = 1$$
$$K = \frac{1}{6}$$ ✓

(ii) $P(1.25 \leq X \leq 1.75) = F(1.75) - F(1.25)$

$$= \frac{1}{6}(1.75^3 - 1.75) - \frac{1}{6}(1.25^3 - 1.25)$$
$$= \frac{77}{128} - \frac{15}{128}$$
$$= \frac{31}{64}$$

(b) (i) $f(x) = \frac{d}{dx}[F(x)]$

$$= \frac{d}{dx}\left[\frac{1}{6}(x^3 - x)\right]$$
$$= \frac{d}{dx}\left[\frac{1}{6}x^3 - \frac{1}{6}x\right]$$
$$= \left(\frac{1}{6}\right)3x^2 - \frac{1}{6}$$

$f(x) = \frac{1}{2}x^2 - \frac{1}{6}$ ar gyfer $1 \leq x \leq 2$

(ii) $E(X) = \int x f(x)\,dx$

$$= \int_1^2 x\left(\frac{1}{2}x^2 - \frac{1}{6}\right)dx$$
$$= \int_1^2 \frac{1}{2}x^3 - \frac{1}{6}x\,dx$$
$$= \left[\left(\frac{1}{2}\right)\frac{x^4}{4} - \left(\frac{1}{6}\right)\frac{x^2}{2}\right]_1^2$$
$$= \left[\frac{x^4}{8} - \frac{x^2}{12}\right]_1^2$$
$$= \left[\left(\frac{2^4}{8} - \frac{2^2}{12}\right) - \left(\frac{1^4}{8} - \frac{1^2}{12}\right)\right]$$
$$= \left(\frac{16}{8} - \frac{4}{12}\right) - \left(\frac{1}{8} - \frac{1}{12}\right)$$
$$= \frac{5}{3} - \frac{1}{24}$$
$$= \frac{13}{8}$$
$$= 1.625$$

S1 Haf 2014

(9) $F(x) = 0$ ar gyfer $x < 0$
$F(x) = 2x^3 - x^6$ ar gyfer $0 \le x \le 1$
$F(x) = 1$ ar gyfer $x > 1$

(a) (i) $P(0.4 \le X \le 0.6)$
$= F(0.6) - F(0.4)$
$= 2(0.6^3) - 0.6^6 - [2(0.4^3) - 0.4^6]$
$= 0.385344 - 0.123904$
$= 0.26144$

(ii) Canolrif: $F(x) = 0.5$
$2x^3 - x^6 = 0.5$
$2x^3 - x^6 - 0.5 = 0$
$x^6 - 2x^3 + 0.5 = 0$

Gadewch i $y = x^3$. $y^2 = x^6$.
Felly angen datrys $y^2 - 2y + 0.5 = 0$
$2y^2 - 4y + 1 = 0$
~~$(2y \quad)(y \quad) = 0$~~

Fformiwla gwadratig $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \dfrac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2 \times 2}$
Unai $y = \dfrac{4 + \sqrt{16 - 8}}{4}$ neu $y = \dfrac{4 - \sqrt{16 - 8}}{4}$
$y = \dfrac{2 + \sqrt{2}}{2}$ $y = \dfrac{2 - \sqrt{2}}{2}$
Felly $x = \sqrt[3]{\dfrac{2 + \sqrt{2}}{2}}$ $x = \sqrt[3]{\dfrac{2 - \sqrt{2}}{2}}$
$x = 1.195$ i 3 lle degol
Na (Dim rhwng 0 a 1)
$\underline{x = 0.664}$ i 3 lle degol

b) i) $f(x) = \frac{d}{dx}(F(x))$

$= \frac{d}{dx}(2x^3 - x^6)$

$= 6x^2 - 6x^5$ (ar gyfer $0 \leq x \leq 1$)

ii) $E(X^3) = \int x^3 f(x)\,dx$

$= \int_0^1 x^3(6x^2 - 6x^5)\,dx$

$= \int_0^1 6x^5 - 6x^8\,dx$

$= 6\int_0^1 x^5 - x^8\,dx$

$= 6\left[\frac{x^6}{6} - \frac{x^9}{9}\right]_0^1$

$= 6\left[\left(\frac{1^6}{6} - \frac{1^9}{9}\right) - \left(\frac{0^6}{6} - \frac{0^9}{9}\right)\right]$

$= 6\left[\left(\frac{1}{6} - \frac{1}{9}\right) - (0 - 0)\right]$

$= 6 \times \frac{1}{18}$

$E(X^3) = \frac{1}{3}$

S1 Haf 2015

9) $f(x) = \frac{4}{9}(4x - x^3)$ ar gyfer $1 \le x \le 2$

$f(x) = 0$ fel arall

a) $E\left(\frac{1}{X}\right) = \int \frac{1}{x} f(x)\,dx$

$= \int_1^2 \frac{1}{x}\left(\frac{4}{9}(4x - x^3)\right) dx$

$= \frac{4}{9}\int_1^2 4 - x^2\,dx$

$= \frac{4}{9}\left[4x - \frac{x^3}{3}\right]_1^2$

$= \frac{4}{9}\left[\left(4\times 2 - \frac{2^3}{3}\right) - \left(4\times 1 - \frac{1^3}{3}\right)\right]$

$= \frac{4}{9}\left[\left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right)\right]$

$= \frac{4}{9}\left[\frac{16}{3} - \frac{11}{3}\right]$

$= \frac{4}{9}\left[\frac{5}{3}\right]$

$= \underline{\underline{\frac{20}{27}}}$

b) (i) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$= \int_1^x f(t)\,dt$

$= \int_1^x \frac{4}{9}(4t - t^3)\,dt$

$= \frac{4}{9}\int_1^x 4t - t^3\,dt$

$= \frac{4}{9}\left[\frac{4t^2}{2} - \frac{t^4}{4}\right]_1^x$

$= \frac{4}{9}\left[2t^2 - \frac{t^4}{4}\right]_1^x$

$= \frac{4}{9}\left[\left(2x^2 - \frac{x^4}{4}\right) - \left(2\times 1^2 - \frac{1^4}{4}\right)\right]$

$= \frac{4}{9}\left[\left(2x^2 - \frac{x^4}{4}\right) - \left(2 - \frac{1}{4}\right)\right]$

$= \frac{4}{9}\left[2x^2 - \frac{x^4}{4} - \frac{7}{4}\right]$

$$= \frac{8x^2}{9} - \frac{\cancel{4}x^4}{9\times\cancel{4}} - \frac{7\times\cancel{4}}{9\times\cancel{4}}$$

$$= \frac{8x^2}{9} - \frac{x^4}{9} - \frac{7}{9}$$

$$= \frac{1}{9}\left(8x^2 - x^4 - 7\right)$$

(ii) $P(1.25 \le X \le 1.75) = F(1.75) - F(1.25)$

$$= \frac{1}{9}(8\times1.75^2 - 1.75^4 - 7) - \frac{1}{9}(8\times1.25^2 - 1.25^4 - 7)$$

$$= \frac{1}{9}\left(\frac{2079}{256}\right) - \frac{1}{9}\left(\frac{783}{256}\right)$$

$$= \frac{231}{256} - \frac{87}{256}$$

$$= \frac{9}{16}$$

$$= \underline{\underline{0.5625}}$$

(iii) Canolrif X: $F(X) = 0.5$

$$\frac{1}{9}(8x^2 - x^4 - 7) = 0.5$$

$$8x^2 - x^4 - 7 = 0.5\times9$$

$$8x^2 - x^4 - 7 = 4.5$$

$$0 = x^4 - 8x^2 + 7 + 4.5$$

$$0 = x^4 - 8x^2 + 11.5$$

Gadewch i $y = x^2$. Yna

$$0 = y^2 - 8y + 11.5$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{8 \pm \sqrt{(-8)^2 - 4\times1\times11.5}}{2\times1}$$

$$y = \frac{8 \pm \sqrt{18}}{2}$$

Unai $y = \frac{8+\sqrt{18}}{2}$ neu $y = \frac{8-\sqrt{18}}{2}$

ond $y = x^2$. Felly $x = \sqrt{y}$

Unai $x = \sqrt{\frac{8+\sqrt{18}}{2}}$ neu $x = \sqrt{\frac{8-\sqrt{18}}{2}}$

$x = 2.47$ i 2 le degol $x = 1.37$ i 2 le degol.

Ond $1 \leq x \leq 2$ felly rhaid bod $\underline{\underline{x = 1.37}}$ i 2 le degol.
Hwn yw canolrif X.

@mathemateg

S1 Haf 2016

(9) $f(x) = K(2x-1)$ ar gyfer $1 \leq x \leq 2$
$f(x) = 0$ fel arall

a) (i) $F(x) = \int_{-\infty}^{x} f(t)dt$

$= \int_{1}^{x} K(2\ -1)dt$

$= K\int_{1}^{x} 2t-1\,dt$

$= K\left[\frac{2t^2}{2} - t\right]_1^x$

$= K\left[t^2-t\right]_1^x$

$= K\left[(x^2-x)-(1^2-1)\right]$

$= K\left[(x^2-x)-0\right]$

$= K(x^2-x)$

(ii) Yn defnyddio $F(2) = 1$

$K(2^2-2) = 1$

$K(4-2) = 1$

$K = \frac{1}{2}$ ✓

b) (i) $E(X) = \int x f(x)\,dx$

$= \int_{1}^{2} x\left(\frac{1}{2}(2x-1)\right)dx$

$= \int_{1}^{2} x^2 - \frac{1}{2}x\,dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_1^2$

$= \left[\left(\frac{2^3}{3} - \frac{2^2}{4}\right) - \left(\frac{1^3}{3} - \frac{1^2}{4}\right)\right]$

$= \left[\left(\frac{8}{3} - 1\right) - \left(\frac{1}{3} - \frac{1}{4}\right)\right]$

$= \frac{19}{12}$

(ii) Canfrif X: $F(x) = 0.5$

$$\frac{1}{2}(x^2 - x) = 0.5$$

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2 \times 1}$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2}$$

Naill ai $x = \frac{1 + \sqrt{5}}{2}$ neu $x = \frac{1 - \sqrt{5}}{2}$

$\underline{\underline{x = 1.6180}}$ i 4 ll.d. $x = -0.6180$ i 4 ll.d.

(Ddim rhwng 1 a 2)

(iii) $P(X > 1.5) = 1 - P(X \leq 1.5)$

$$= 1 - F(1.5)$$

$$= 1 - \frac{1}{2}(1.5^2 - 1.5)$$

$$= 1 - 0.375$$

$$P(X > 1.5) = 0.625$$

S1 Haf 2017

8) $F(x) = 0$ ar gyfer $x < 1$

$F(x) = K(x^4 - x^2)$ ar gyfer $1 \leq x \leq 2$

$F(x) = 1$ ar gyfer $x > 2$

a) (i) Mae angen $F(2) = 1$

$$K(2^4 - 2^2) = 1$$
$$K(16 - 4) = 1$$
$$12K = 1$$
$$K = \frac{1}{12} \quad \checkmark$$

(ii) 95^{ed} canradd: $F(x) = 0.95$

$$\frac{1}{12}(x^4 - x^2) = 0.95$$
$$x^4 - x^2 = 11.4$$
$$x^4 - x^2 - 11.4 = 0$$

Gadewch i $y = x^2$. Rydym angen datrys

$$y^2 - y - 11.4 = 0$$

Hafaliad cwadratig: $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-11.4)}}{2 \times 1}$$
$$y = \frac{1 \pm \sqrt{46.6}}{2}$$

Naill ai $y = \dfrac{1 + \sqrt{46.6}}{2}$ neu $y = \dfrac{1 - \sqrt{46.6}}{2}$

$y = 3.913209633$ $\qquad$ $y = -2.913209633$

Felly $x = \sqrt{y}$ $\qquad$ $x = \sqrt{y}$

$x = \sqrt{3.913209633}$ $\qquad$ $x = \sqrt{-2.913209633}$

$x = \underline{1.98}$ i 3 ffigur ystyrlon. $\qquad$ Dim datrysiad

(iii) $P(X<1.25 \mid X<1.75)$

$$= \frac{P(X<1.25 \cap X<1.75)}{P(X<1.75)}$$

$$= \frac{P(X<1.25)}{P(X<1.75)}$$

$$= \frac{F(1.25)}{F(1.75)}$$

$$= \frac{\cancel{\frac{1}{12}}(1.25^4-1.25^2)}{\cancel{\frac{1}{12}}(1.75^4-1.75^2)}$$

$$= \frac{1.25^4-1.25^2}{1.75^4-1.75^2}$$

$$= \frac{75}{539}$$

b) (i) $f(x) = \frac{d}{dx}(F(x))$

$$= \frac{d}{dx}\left(\frac{1}{12}(x^4-x^2)\right)$$

$$= \frac{1}{12}(4x^3-2x)$$

$f(x) = \frac{x^3}{3} - \frac{x}{6}$ (dilys ar gyfer $1 \leq x \leq 2$)

(ii) $E(\sqrt{X}) = \int \sqrt{x}\, f(x)\, dx$

$$= \int_1^2 \sqrt{x}\left(\frac{x^3}{3}-\frac{x}{6}\right)dx$$

$$= \int_1^2 x^{\frac{1}{2}}\left(\frac{x^3}{3}-\frac{x}{6}\right)dx$$

$$= \int_1^2 \frac{x^{3\frac{1}{2}}}{3} - \frac{x^{1\frac{1}{2}}}{6}\, dx$$

$$= \left[\frac{x^{4\frac{1}{2}}}{4\frac{1}{2}\times 3} - \frac{x^{2\frac{1}{2}}}{2\frac{1}{2}\times 6}\right]_1^2$$

$$= \left[\frac{2x^{4\frac{1}{2}}}{27} - \frac{x^{2\frac{1}{2}}}{15}\right]_1^2$$

$$= \left(\frac{2(2^{4\frac{1}{2}})}{27} - \frac{2^{2\frac{1}{2}}}{15}\right) - \left(\frac{2(1^{4\frac{1}{2}})}{27} - \frac{1^{2\frac{1}{2}}}{15}\right)$$

$$= 1.298981346 - \frac{1}{135}$$

$$= 1.291573939$$

$$= \underline{\underline{1.2916}} \quad \text{i 4 lle degol}$$

S1 Haf 2018

9) $F(x) = 0$ ar gyfer $x < 1$

$F(x) = \frac{1}{10}(x^2 + x - 2)$ ar gyfer $1 \le x \le 3$

$F(x) = 1$ ar gyfer $x > 3$

a) i) $P(2 < X < 2.5) = F(2.5) - F(2)$

$= \frac{1}{10}(2.5^2 + 2.5 - 2) - \frac{1}{10}(2^2 + 2 - 2)$

$= 0.675 - 0.4$

$= \underline{0.275}$

ii) Chwartel uchaf X: $F(x) = 0.75$

$\frac{1}{10}(x^2 + x - 2) = 0.75$

$x^2 + x - 2 = 7.5$

$x^2 + x - 9.5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-9.5)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{39}}{2}$$

Naill ai $x = \frac{-1 + \sqrt{39}}{2}$ neu $x = \frac{-1 - \sqrt{39}}{2}$

$x = 2.622498999$ $x = -3.622498999$

$\underline{x = 2.6225}$ i 4 lle degol (Ddim yn berthnasol)

b) i) $f(x) = \frac{d}{dx}\left(F(x)\right)$

$$= \frac{d}{dx}\left(\frac{1}{10}(x^2+x-2)\right)$$

$$= \frac{d}{dx}\left(\frac{1}{10}x^2 + \frac{1}{10}x - \frac{2}{10}\right)$$

$f(x) = \frac{2}{10}x + \frac{1}{10}$ (ar gyfer $1 \leq x \leq 3$)

ii) $E(X) = \int x f(x)\,dx$

$$= \int_1^3 x\left(\frac{2}{10}x + \frac{1}{10}\right) dx$$

$$= \int_1^3 \frac{2}{10}x^2 + \frac{1}{10}x\,dx$$

$$= \left[\frac{2}{10}\frac{x^3}{3} + \frac{1}{10}\frac{x^2}{2}\right]_1^3$$

$$= \left[\frac{x^3}{15} + \frac{x^2}{20}\right]_1^3$$

$$= \left(\frac{3^3}{15} + \frac{3^2}{20}\right) - \left(\frac{1^3}{15} + \frac{1^2}{20}\right)$$

$$= \frac{9}{4} - \frac{7}{60}$$

$$= \frac{32}{15}$$