Old Exam Questions – Old Course

The Binomial Distribution

(S1 Winter 2005)

2. The discrete random variable X has the binomial distribution B (48, 0·25). The random variable Y is defined by

$$Y = 2X - 1.$$

Find the mean and the standard deviation of Y. [7]

5. It is estimated that 0·7% of the population of university students have a rare blood disease. In a faculty of 550 such students, use a Poisson approximation to find the probability that the number of students with this disease is

(a) exactly 4, [4]

(b) more than 2. [4]

7. A scientist researching a new breed of chicken knows that the probability of a newly born chick of the breed being female is 0·6. Let X denote the number of female chicks in a batch of 20 randomly chosen newly born chicks. Find

(a) $P(X = 12)$, [3]

(b) $P(9 \leqslant X \leqslant 15)$. [4]

(S1 Summer 2005)

6. *(a)* A fair die is tossed 10 times and X denotes the number of times a '6' is obtained.

(i) State the distribution of X.

(ii) Find the mean and variance of X.

(iii) Calculate $P(X \leqslant 2)$. [6]

(b) Two fair dice are tossed 81 times and Y denotes the number of times a total of 12 is obtained. Use a Poisson approximation to evaluate $P(Y = 4)$. [4]

(S1 Winter 2006)

5. The random variable X has the binomial distribution $B(n,p)$. The mean and standard deviation of X are 20 and 4 respectively. Find the values of n and p. [6]

7. Wine glasses are mass produced. There is a probability of 0·05 that a randomly selected glass is defective, independently of all other glasses.

(a) **Without using tables,** find the probability that a set of 24 glasses contains exactly 2 defective glasses. [3]

(b) **Using tables,** find the probability that a set of 50 glasses contains between 3 and 5 (both inclusive) defective glasses. [3]

(c) Use a Poisson approximation to find the probability that a set of 120 glasses contains fewer than 8 defective glasses. [3]

(S1 Summer 2006)

6. *(a)* When a drawing pin is thrown onto a table, the probability that it will fall 'point upwards' is 0·2. All 50 drawing pins in a packet are thrown onto a table. Given that X denotes the number falling 'point upwards',

(i) identify the distribution of X,

(ii) find the mean and standard deviation of X,

(iii) find $P(8 \leqslant X \leqslant 12)$. [8]

(b) It is known that 1% of drawing pins are defective. A shopkeeper buys 1000 drawing pins. Use an appropriate distributional approximation to find the probability that less than 10 of them are defective. [4]

(S1 Winter 2007)

3. The random variable X has the distribution $B(n, 0{\cdot}1)$. Given that the mean and standard deviation of X are equal, find the value of n. [5]

(S1 Summer 2007)

5. Alan and Brenda play Scrabble against each other regularly.

(a) The probability that Alan wins a game is 0·6 and the probability that Brenda wins a game is 0·4, independently of all other games. During a weekend, they play 5 games. Let X denote the number of games won by Brenda.

(i) State the distribution of X.

(ii) Determine the mean and standard deviation of X.

(iii) Find the probability that Brenda wins at least 3 of the games. [6]

(b) The probability that one of their games takes more than 2 hours to complete is 0·05. During a school holiday, they play 24 games. Use a Poisson approximation to find the probability that less than 3 of these games take more than 2 hours to complete. [4]

(S1 Winter 2008)

4. The random variable X has the binomial distribution $B(10, 0{\cdot}3)$. Given that $Y = 3X + 4$, evaluate

(a) $E(Y)$, [4]

(b) $\text{Var}(Y)$, [2]

(c) $P(Y = 16)$. [3]

7. On a farm, chickens are bred from eggs under strictly controlled conditions.

(a) The probability that an egg will produce a female chick is 0·3. When 20 eggs are kept under the controlled conditions, find the probability that the number of female chicks produced will be

(i) exactly 8,

(ii) more than 5. [5]

(b) The probability that an egg will fail to hatch is 0·01. When 1000 eggs are kept under the controlled conditions, use a Poisson approximation to find the probability that the number of eggs failing to hatch will be less than 9. [3]

(S1 Summer 2008)

7. A salesman makes 50 house calls during a particular week. You may assume that, independently for each house visited, the probability of a sale is 0·2.

(a) Find the probability that, during this week, he makes

(i) exactly 12 sales,

(ii) between 10 and 14 (both inclusive) sales.

(iii) his first sale on the third house visited. [9]

(b) At the end of the week, he is paid £100 plus a commission of £50 for every sale. Find the mean and standard deviation of his total pay for this week. [5]

(S1 Winter 2009)

6. For a certain type of tulip bulb, the probability of producing a red flower is 0·6. A gardener plants 20 of these bulbs.

(a) Find the probability that the number of red flowers produced is

(i) exactly 10,

(ii) at least 12. [6]

(b) The probability that this type of tulip bulb fails to produce a flower of any colour is 0·04. A park-keeper plants 80 of these bulbs. Use a Poisson approximation to find the probability that fewer than 5 bulbs fail to produce a flower of any colour. [3]

(S1 Summer 2009)

3. The random variable X has the binomial distribution $B(25, 0{\cdot}8)$.

(a) State the mean and variance of X. [2]

(b) The random variable Y is defined by

$$Y = aX - b$$

where a, b are positive constants.

(i) Given that $a = 2$, $b = 3$, find the mean and variance of Y.

(ii) Given that $E(Y) = 0$ and $\mathrm{Var}(Y) = 1$, find the values of a and b. [8]

(S1 Winter 2010)

3. *(a)* The random variable X has the binomial distribution $B(n, p)$.
Given that the mean and standard deviation of X are 10 and 3 respectively, find the values of n and p. [5]

(b) The random variable Y has the binomial distribution $B(380, 0{\cdot}016)$.
Use a Poisson approximation to find an approximate value for the probability that Y is less than 3. [4]

5. When seeds of a certain variety of flower are planted, the probability of each seed germinating is $0{\cdot}8$, independently of all other seeds.

(a) David plants 20 of these seeds. Find the probability that

(i) exactly 15 seeds germinate,

(ii) at least 15 seeds germinate. [6]

(b) Beti plants n of these seeds and she correctly calculates that the probability that they all germinate is $0{\cdot}10737$, correct to five decimal places. Find the value of n. [3]

(S1 Summer 2010)

7. Sheila buys two biased dice in a shop. Each time either of the dice is thrown, the probability of obtaining a six is $0{\cdot}2$.

(a) She throws one of the dice 50 times. Determine the probability that she obtains

(i) exactly 12 sixes,

(ii) at least 10 sixes. [5]

(b) She now throws the two dice simultaneously 200 times. Use a Poisson approximation to find the probability that between 5 and 10 (both inclusive) double sixes are obtained. [5]

(S1 Winter 2011)

4. The random variable X has the binomial distribution B(n, 0·2). Given that the mean of X is twice its standard deviation, find the value of n. [5]

8. Wine glasses are packed in boxes, each containing 20 glasses. Each glass has a probability of 0·05 of being broken in transit, independently of all other glasses.

(a) Let X denote the number of glasses in a box broken in transit.

(i) State the distribution of X.

(ii) **Without** the use of tables, calculate $P(X = 1)$.

(iii) **Using tables**, determine the value of $P(X \geqslant 3)$. [5]

(b) A retailer buys 10 of these boxes. Use a Poisson approximation to find the probability that less than 5 of the 200 glasses are broken in transit. [3]

(S1 Summer 2011)

7. *(a)* A series of trials is carried out, each resulting in either success or failure. State **two** conditions that have to be satisfied in order for the total number of successes to be modelled by the binomial distribution. [2]

(b) Each time Ann shoots an arrow at a target, she hits it with probability 0·4. She shoots 20 arrows at the target. Determine the probability that she hits it

(i) exactly 8 times,

(ii) between 6 and 10 times (both inclusive). [5]

(c) Each time she shoots an arrow, she hits the centre of the target with probability 0·04. She shoots 100 arrows at the target. Use a Poisson approximation to find the probability that she hits the centre of the target less than 5 times. [3]

(S1 Winter 2012)

3. Alun and Ben are snooker players. When they play a game against each other, Alun wins with probability 0·6 and successive games are independent.

(a) One evening they play 10 games against each other. Determine the probability that Alun wins

(i) exactly 7 games,

(ii) at least 6 games. [5]

(b) On another evening, find the probability that Alun wins for the first time on the fourth game. [3]

8. The random variable X has the binomial distribution B(16, p), where $p < 0{\cdot}5$.
Given that the variance of X is 2·56,

(a) calculate the value of p, [4]

(b) for this value of p, calculate $E(X^2)$. [3]

(S1 Summer 2012)

4. Charlie and Dave regularly play chess against each other. When they play each other, Charlie wins with probability 0·75 and successive games are independent.

(a) One weekend they play 10 games against each other. Determine the probability that Charlie wins

(i) exactly 4 games,

(ii) more than 5 games. [5]

(b) The probability that a game lasts for less than one hour is 0·08.
They play 45 games against each other over a holiday period. Use a Poisson approximation to determine the probability that more than 6 of these games last for less than one hour. [3]

(S1 Winter 2013)

2. The random variable X has the binomial distribution B(16, 0·2). The random variable Y is defined by

$$Y = 2X + 5.$$

(a) Find the mean and variance of Y. [6]

(b) Evaluate $P(Y = 11)$. [3]

5. *(a)* When a certain type of seed is planted, there is a probability of 0·7 that it produces red flowers. A gardener plants 20 of these seeds. Calculate the probability that

(i) exactly 15 seeds produce red flowers,

(ii) more than 12 seeds produce red flowers. [6]

(b) When a different type of seed is planted, there is a probability of 0·09 that it produces white flowers. The gardener plants 150 of these seeds. Use an appropriate Poisson distribution to determine, approximately, the probability that exactly 10 seeds produce white flowers. [3]

(S1 Summer 2013)

3. The random variable X has a binomial distribution with parameters $n = 25$, $p = 0{\cdot}8$. The random variable Y is defined by $Y = aX + b$, where $a, b > 0$.
Given that the mean and standard deviation of Y are 65 and 6 respectively, find the values of a and b. [6]

4. Bethan has two fair dice, each in the shape of a regular tetrahedron. The four faces of each dice are numbered 1, 2, 3, 4 respectively.

(a) She throws one of the dice 20 times and her score on each throw is defined as the number appearing on the face in contact with the table. Let X denote the number of throws resulting in a score of 4.

(i) Write down the distribution of X.

(ii) Determine $P(3 \leqslant X \leqslant 9)$.

(iii) **Without the use of tables**, calculate $P(X = 6)$. [6]

(b) She now throws the two dice simultaneously 160 times and her score on each throw is defined as the sum of the numbers on the two faces in contact with the table. Use a Poisson approximation to determine the probability that the number of throws resulting in a score of 8 is

(i) equal to 12,

(ii) between 6 and 14 (both inclusive). [6]

(S1 Winter 2014)

4. *(a)* The random variable X has the binomial distribution B(20, 0·2).

(i) Without the use of tables, calculate $P(X = 6)$,

(ii) Determine $P(2 \leqslant X \leqslant 8)$. [5]

(b) The random variable Y has the binomial distribution B(200, 0·0123).
Use the Poisson distribution to determine the approximate value of $P(Y = 3)$. [3]

6. Jim takes part in a quiz in which he has to answer 10 questions on his chosen topic. You may assume that he answers each question correctly with probability 0·75 and that answers to successive questions are independent.
Let X denote the number of questions that he answers correctly.

(a) (i) Find the mean and the variance of X.

(ii) Find the most likely value of X. [7]

(b) Jim wins £10 for each question answered correctly but loses £2 for each question not answered correctly. Let £W denote the total amount that Jim wins.

(i) Show that $W = aX - b$, where a, b are positive integers whose values are to be found.

(ii) Find the mean and the variance of W. [4]

(S1 Summer 2014)

2. The random variable X has the binomial distribution $B(n, p)$. Given that the mean and the standard deviation of X are both equal to 0·9, find the value of n and the value of p. [5]

5. A zoologist is studying a certain breed of dog.

(a) He knows from past experience that the probability of a newly born puppy being female is 0·55. He selects a random sample of 20 newly born puppies. Calculate the probability that the number of females in the sample is

(i) exactly 12,

(ii) between 8 and 16 (both inclusive). [8]

(b) The probability of a newly born puppy being yellow is 0·05. Use an approximating distribution to find the probability that less than 5 out of a random sample of 60 newly born puppies are yellow. [3]

(S1 Summer 2015)

1. The random variable X has the binomial distribution B(10, 0·3) and $Y = 2X + 1$. Calculate

(a) the mean and the variance of Y, [5]

(b) $P(Y = 7)$. [3]

6. *(a)* A factory manufactures cups. The manager knows from past experience that 5% of the cups produced are defective. Given a random sample of 50 of these cups, determine the probability that the number of defective cups in this sample is

(i) exactly 2,

(ii) between 3 and 8 (both inclusive). [6]

(b) The factory also manufactures plates. The manager knows that 1·5% of the plates produced are defective. Use an appropriate Poisson distribution to find, approximately, the probability that a random sample of 250 of these plates contains exactly 4 defective plates. [3]

(S1 Summer 2016)

6. In a shooting range at a country fair, customers pay £5 to fire 8 shots at a target. Let X denote the number of shots which hit the target. Prizes are awarded according to the following rules.

If $X < 2$, no prize is awarded.
If $X = 2$, a prize of £10 is awarded.
If $X > 2$, a prize of £25 is awarded.

Jim decides to spend £5 to fire 8 shots. You may assume that the probability of one of his shots hitting the target is 0·12 and that successive shots are independent.

(a) Calculate the probability that he wins

(i) no prize,

(ii) a £10 prize,

(iii) a £25 prize. [5]

(b) Calculate his expected profit, giving your answer correct to two decimal places. [2]

8. Jane is solving a problem in which she has to calculate $P(X = 2)$ where X has a Poisson distribution with mean 3. Unfortunately, she has no statistical tables with her and her simple calculator has no e^x button and it can only carry out arithmetic operations. She decides to use an appropriate binomial distribution to give an approximate value for $P(X = 2)$. She takes $n = 50$.

(a) What value of p should she take? [2]

(b) Write down and evaluate an arithmetic expression giving her approximate value correct to four decimal places. [2]

(c) Show that the approximation is within 1% of the value obtained from the appropriate Poisson table. [3]

(S1 Summer 2017)

5. Anne and Brian play a board game against each other regularly.

(a) The probability that Anne wins a game is 0·7 and the probability that Brian wins a game is 0·3, independently of all other games. One day, they play 10 games. Let X denote the number of games won by Anne on that day.

(i) State the distribution of X, including any parameters.

(ii) Determine the mean and the standard deviation of X.

(iii) Find the probability that Anne wins more games than Brian. [7]

(b) The probability that one of their games takes more than 1 hour to complete is 0·06. During a school holiday, they play 44 games. Use a Poisson approximation to find the probability that more than 2 of these games take more than 1 hour to complete. [3]

(S1 Summer 2018)

4. The random variable X has the binomial distribution B(10, p). Find the set of values of p for which the standard deviation of X is greater than the mean of X. [5]

8. *(a)* Seeds of a certain variety germinate independently with probability 0·6. A batch of 20 of these seeds is planted and X denotes the number which germinate.

(i) State the distribution of X, including any parameters.

(ii) Without the use of tables, calculate the probability that exactly 15 of these seeds germinate.

(iii) Determine the probability that at least 15 of these seeds germinate. [8]

(b) Seeds of a different variety germinate independently with probability 0·05. A batch of 200 of these seeds is planted and Y denotes the number which germinate. Use an appropriate Poisson approximation to determine, approximately, the probability that

(i) exactly 8 of these seeds germinate,

(ii) more than 12 of these seeds germinate. [5]