

Old Exam Questions – Old Course
Continuous Random Variables

(S1 Winter 2005)

9. The continuous random variable X has probability density function f where

$$\begin{aligned} f(x) &= \frac{1}{21} x^2, & \text{for } 1 \leq x \leq 4, \\ f(x) &= 0, & \text{otherwise.} \end{aligned}$$

- (a) Evaluate $E(X)$. [4]
 (b) Find an expression for $F(x)$, valid for $1 \leq x \leq 4$, where F denotes the cumulative distribution function of X . [3]
 (c) Calculate $P(2 \leq X \leq 3)$. [3]
 (d) Find the median of X correct to two decimal places. [3]

(S1 Summer 2005)

8. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 & \text{for } x < 0, \\ F(x) &= 4x^3 - 3x^4 & \text{for } 0 \leq x \leq 1, \\ F(x) &= 1 & \text{for } x > 1. \end{aligned}$$

- (a) Evaluate $P(0.2 \leq X \leq 0.8)$. [3]
 (b) Show that the lower quartile of X lies between 0.45 and 0.46. [3]
 (c) Find an expression for $f(x)$, valid for $0 \leq x \leq 1$, where f denotes the probability density function of X . [2]
 (d) Evaluate $E(X)$. [4]

(S1 Winter 2006)

9. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= kx^2 & \text{for } 1 \leq x \leq 4, \\ f(x) &= 0 & \text{otherwise,} \end{aligned}$$

where k is a constant.

- (a) (i) Show that $k = \frac{1}{21}$.
 (ii) Evaluate $E(X)$. [6]
 (b) (i) Obtain an expression for $F(x)$, valid for $1 \leq x \leq 4$, where F denotes the cumulative distribution function of X .
 (ii) Evaluate $P(2 \leq X \leq 3)$.
 (iii) Find the median of X . [8]

(S1 Summer 2006)

8. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \quad \text{for } x < 0,$$

$$F(x) = \frac{1}{2}(x^2 + x) \quad \text{for } 0 \leq x \leq 1,$$

$$F(x) = 1 \quad \text{for } x > 1.$$

(a) Find

(i) $P(0.25 \leq X \leq 0.5),$

(ii) the median of X . [7]

(b) (i) Obtain an expression for $f(x)$ that is valid for $0 \leq x \leq 1$, where f denotes the probability density function of X .

(ii) Evaluate $E(X)$. [6]

(S1 Winter 2007)

7. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= 20(x^3 - x^4), & \text{for } 0 \leq x \leq 1, \\ f(x) &= 0, & \text{otherwise,} \end{aligned}$$

(a) Find $E(X)$. [4]

(b) (i) Obtain an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X .

(ii) Evaluate $P(0.4 \leq X \leq 0.6)$.

(iii) The upper quartile of X is denoted by q . Show that

$$16q^5 - 20q^4 + 3 = 0. \quad [8]$$

(S1 Summer 2007)

7. The continuous random variable X has probability density function f given by

$$f(x) = \frac{6}{5}x(x-1), \quad \text{for } 1 \leq x \leq 2,$$

$$f(x) = 0, \quad \text{otherwise.}$$

(a) Evaluate

$$E\left(\frac{1}{X}\right). \quad [4]$$

(b) (i) Find an expression for $F(x)$, for $1 \leq x \leq 2$, where F denotes the cumulative distribution function of X .

(ii) Evaluate $P(X \leq 1.75)$.

(iii) Hence state, with a reason, whether the median of X is greater than or less than 1.75.

[8]

(S1 Winter 2008)

8. The continuous random variable X has probability density function f given by

$$f(x) = 4 - 2x, \quad \text{for } 1 \leq x \leq 2,$$

$$f(x) = 0, \quad \text{otherwise.}$$

- (a) Evaluate $E(X)$. [4]

- (b) Show that, for $1 \leq x \leq 2$,

$$F(x) = 4x - x^2 - 3$$

where F denotes the cumulative distribution function of X . [3]

- (c) Evaluate $P(X > 1.2)$. [3]

- (d) Calculate the median of X , giving your answer correct to two decimal places. [3]

(S1 Summer 2008)

8. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \quad \text{for } x < 0,$$

$$F(x) = 4x^3 - 3x^4 \quad \text{for } 0 \leq x \leq 1,$$

$$F(x) = 1 \quad \text{for } x > 1.$$

- (a) Evaluate $P(0.25 \leq X \leq 0.75)$. [3]

- (b) By evaluating $F(0.6)$, determine whether the median of X is greater or less than 0.6. [3]

- (c) Obtain an expression for $f(x)$, valid for $0 \leq x \leq 1$, where f denotes the probability density function of X . [2]

- (d) Evaluate $E(X)$. [4]

(S1 Winter 2009)

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0, & \text{for } x < 0, \\ F(x) &= kx^3, & \text{for } 0 \leq x \leq 2, \\ F(x) &= 1, & \text{for } x > 2, \end{aligned}$$

where k is a constant.

- (a) Show that $k = \frac{1}{8}$. [2]

- (b) Find the probability that the value of X lies between 0.5 and 1.5. [2]

- (c) Find the median of X . [2]

- (d) Evaluate $E(X)$. [6]

(S1 Summer 2009)

8. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= \frac{1}{2} (1 + 2x) && \text{for } 0 \leq x \leq 1, \\ f(x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Calculate $E(X)$. [4]
- (b) Obtain an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X . [3]
- (c) Calculate
- (i) $P(0.4 \leq X \leq 0.5)$,
- (ii) the median of X . [5]

(S1 Winter 2010)

8. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 1, \\ F(x) &= \frac{1}{10} (x^2 + x - 2) && \text{for } 1 \leq x \leq 3, \\ F(x) &= 1 && \text{for } x > 3. \end{aligned}$$

- (a) (i) Evaluate $P(2 \leq X \leq 2.5)$.
- (ii) Find the median of X . [7]
- (b) (i) Obtain an expression for $f(x)$, valid for $1 \leq x \leq 3$, where f denotes the probability density function of X .
- (ii) Write down the value of $f(4)$.
- (iii) Calculate $E(X)$. [7]

(S1 Summer 2010)

8. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= kx(1 - x^2) && \text{for } 0 \leq x \leq 1, \\ f(x) &= 0 && \text{otherwise,} \end{aligned}$$

where k is a constant.

- (a) Show that $k = 4$. [3]
- (b) Calculate $E(X)$. [4]
- (c) (i) Find an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X .
- (ii) Evaluate $P(0.25 \leq X \leq 0.75)$.
- (iii) Find the median of X . [9]

(S1 Winter 2011)

9. The continuous random variable X has probability density function f given by

$$f(x) = \frac{1}{6}(x+1) \quad \text{for } 1 \leq x \leq 3,$$

$$f(x) = 0 \quad \text{otherwise.}$$

- (a) Calculate $E(X)$. [4]
- (b) (i) Find an expression for $F(x)$, valid for $1 \leq x \leq 3$, where F denotes the cumulative distribution function of X .
- (ii) State the value of $F(4)$.
- (iii) Evaluate $P(1.5 \leq X \leq 2)$.
- (iv) Find the median of X . [10]

(S1 Summer 2011)

8. (a) The continuous random variable X has probability density function f given by

$$f(x) = 12x^2(1-x) \quad \text{for } 0 \leq x \leq 1,$$

$$f(x) = 0 \quad \text{otherwise.}$$

Calculate

- (i) $E(X)$,
- (ii) $E\left(\frac{1}{X}\right)$,
- (iii) $P(0.2 \leq X \leq 0.5)$. [9]
- (b) The continuous random variable Y takes values between 1 and 2 and its cumulative distribution function F is given, for $1 \leq y \leq 2$, by

$$F(y) = ay + by^2.$$

Find the values of the constants a and b . [4]

(S1 Winter 2012)

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 & \text{for } x < 1, \\ F(x) &= k(x^2 - x) & \text{for } 1 \leq x \leq 3, \\ F(x) &= 1 & \text{for } x > 3, \end{aligned}$$

where k is a constant.

- (a) (i) Show that $k = \frac{1}{6}$.
- (ii) Find the probability that the value of X is greater than 2.
- (iii) Find the median of X . [8]
- (b) (i) Find an expression for $f(x)$, valid for $1 \leq x \leq 3$, where f denotes the probability density function of X .
- (ii) Determine $E(X)$. [6]

(S1 Summer 2012)

9. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= \frac{1}{10} (2x + 3x^2) && \text{for } 1 \leq x \leq 2, \\ f(x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) (i) Determine $E(X)$.
(ii) Show that

$$E(X^2) = 2.61$$

and hence calculate the variance of X correct to 2 decimal places. [8]

- (b) (i) Find an expression for $F(x)$, valid for $1 \leq x \leq 2$, where F denotes the cumulative distribution function of X .
(ii) Determine $P(X \leq 1.4)$.
(iii) Hence, giving a reason, state whether the lower quartile of X is less than or greater than 1.4. [7]

(S1 Winter 2013)

8. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 0, \\ F(x) &= 2x^2 - x^4 && \text{for } 0 \leq x \leq 1, \\ F(x) &= 1 && \text{for } x > 1. \end{aligned}$$

- (a) (i) Evaluate $P(0.25 \leq X \leq 0.75)$.
(ii) Show that the median m satisfies the equation $2m^4 - 4m^2 + 1 = 0$.
(iii) Hence find the value of m , giving your answer correct to 3 significant figures. [7]
- (b) (i) Find an expression for $f(x)$, valid for $0 \leq x \leq 1$, where f denotes the probability density function of X .
(ii) Evaluate $E(\sqrt{X})$. [6]

(S1 Summer 2013)

9. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= k \left(1 - \frac{x^2}{4} \right) && \text{for } 0 \leq x \leq 2, \\ f(x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Show that $k = \frac{3}{4}$. [3]
(b) Calculate $E(X)$. [4]
(c) (i) Find an expression for $F(x)$, valid for $0 \leq x \leq 2$, where F denotes the cumulative distribution function of X .
(ii) Hence evaluate $P(0.5 \leq X \leq 1.5)$. [5]

(S1 Winter 2014)

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 1, \\ F(x) &= k(x^3 - x) && \text{for } 1 \leq x \leq 2, \\ F(x) &= 1 && \text{for } x > 2, \end{aligned}$$

where k is a constant.

- (a) (i) Show that $k = \frac{1}{6}$.
 (ii) Evaluate $P(1.25 \leq X \leq 1.75)$. [5]
- (b) (i) Find an expression for $f(x)$, valid for $1 \leq x \leq 2$, where f denotes the probability density function of X .
 (ii) Hence determine $E(X)$. [6]

(S1 Summer 2014)

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 0, \\ F(x) &= 2x^3 - x^6 && \text{for } 0 \leq x \leq 1, \\ F(x) &= 1 && \text{for } x > 1. \end{aligned}$$

- (a) (i) Determine $P(0.4 \leq X \leq 0.6)$.
 (ii) Find the median of X . [6]
- (b) (i) Find an expression for $f(x)$, valid for $0 \leq x \leq 1$, where f denotes the probability density function of X .
 (ii) Calculate $E(X^3)$. [6]

(S1 Summer 2015)

9. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= \frac{4}{9} (4x - x^3) && \text{for } 1 \leq x \leq 2, \\ f(x) &= 0 && \text{otherwise.} \end{aligned}$$

- (a) Determine $E\left(\frac{1}{X}\right)$. [4]
- (b) (i) Find an expression for $F(x)$, valid for $1 \leq x \leq 2$, where F denotes the cumulative distribution function of X .
 (ii) Hence calculate $P(1.25 \leq X \leq 1.75)$.
 (iii) Calculate the median of X . [9]

(S1 Summer 2016)

9. The time, X hours, in the evening that Bill spends on his homework has probability density function f given by

$$\begin{aligned} f(x) &= k(2x - 1) && \text{for } 1 \leq x \leq 2, \\ f(x) &= 0 && \text{otherwise,} \end{aligned}$$

where k is a constant.

- (a) (i) Find an expression in terms of k and x for $F(x)$, valid for $1 \leq x \leq 2$, where F denotes the cumulative distribution function of X .
- (ii) Hence show that $k = \frac{1}{2}$. [5]
- (b) Determine
- (i) $E(X)$,
- (ii) the median of X ,
- (iii) the probability that, on a randomly chosen evening, Bill spends longer than 1.5 hours on his homework. [9]

(S1 Summer 2017)

8. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 1, \\ F(x) &= k(x^4 - x^2) && \text{for } 1 \leq x \leq 2, \\ F(x) &= 1 && \text{for } x > 2, \end{aligned}$$

where k is a constant.

- (a) (i) Show that $k = \frac{1}{12}$.
- (ii) Find the 95th percentile of X , giving your answer correct to three significant figures.
- (iii) Evaluate $P(X < 1.25 | X < 1.75)$. [9]
- (b) (i) Find an expression for $f(x)$, valid for $1 \leq x \leq 2$, where f denotes the probability density function of X .
- (ii) Calculate $E(\sqrt{X})$. [6]

(S1 Summer 2018)

9. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 && \text{for } x < 1, \\ F(x) &= \frac{1}{10}(x^2 + x - 2) && \text{for } 1 \leq x \leq 3, \\ F(x) &= 1 && \text{for } x > 3. \end{aligned}$$

- (a) (i) Evaluate $P(2 < X < 2.5)$.
(ii) Find the upper quartile of X . [6]
- (b) (i) Find an expression for $f(x)$, valid for $1 \leq x \leq 3$, where f denotes the probability density function of X .
(ii) Calculate $E(X)$. [6]