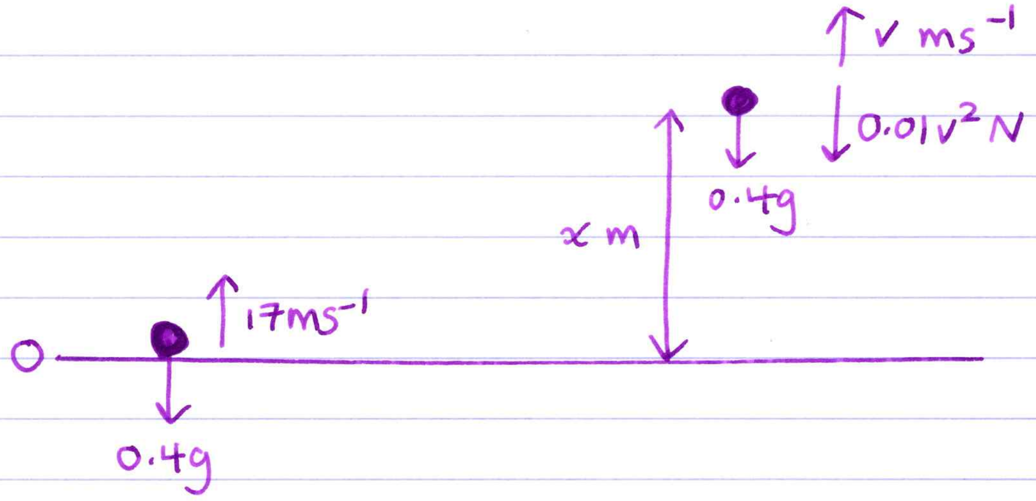
Uned 6 Pellach – Deunyddiau Asesu Enghreifftiol

1)



a) $F=ma$ ar y bêl, yn fertigol, $\uparrow = +$if

$-0.4g - 0.01v^2 = 0.4a$

$-0.4g - 0.01v^2 = 0.4\left(\frac{dv}{dt}\right)$

Nawr $a = \frac{dv}{dt}$

$= \frac{dx}{dt} \times \frac{dv}{dx}$ [rheol y gadwyn]

$= v \times \frac{dv}{dx}$.

Felly

$-0.4g - 0.01v^2 = 0.4v\left(\frac{dv}{dx}\right)$

$-40g - v^2 = 40v\left(\frac{dv}{dx}\right)$

$40v\frac{dv}{dx} = -40 \times 9.8 - v^2$

$40v\frac{dv}{dx} = -(392 + v^2)$ ✓

b) $40v \frac{dv}{dx} = -(392+v^2)$

$40v\,dv = -(392+v^2)dx$

$\frac{40v\,dv}{-(392+v^2)} = dx$

$\int \frac{40v}{-(392+v^2)} dv = \int dx$

$-40 \int \frac{v}{392+v^2} dv = \int dx$

$-40\left(\frac{1}{2}\right) \ln|392+v^2| = x + c$

$-20 \ln|392+v^2| = x + c$

Os yw $t=0s$, $x=0m$ a $v = 17ms^{-1}$.

Felly

$-20 \ln|392+17^2| = 0 + c$

$-20 \ln(681) = c$

Felly $-20 \ln|392+v^2| = x - 20\ln(681)$

$20\ln(681) - 20\ln|392+v^2| = x$

$20 \ln\left|\frac{681}{392+v^2}\right| = x$

$\ln\left|\frac{681}{392+v^2}\right| = \frac{x}{20}$

$\frac{681}{392+v^2} = e^{\frac{x}{20}}$

$681 = (392+v^2)e^{\frac{x}{20}}$

$681e^{-\frac{x}{20}} = 392+v^2$

$v^2 = 681e^{-\frac{x}{20}} - 392$

$v = \sqrt{681e^{-\frac{x}{20}} - 392}$

c) Ar ei uchder mwyaf mae $v = 0 ms^{-1}$

$$0 = \sqrt{681e^{-\frac{x}{20}} - 392}$$

$$0 = 681e^{-\frac{x}{20}} - 392$$

$$\frac{392}{681} = e^{-\frac{x}{20}}$$

$$\ln\left(\frac{392}{681}\right) = -\frac{x}{20}$$

$$-20\ln\left(\frac{392}{681}\right) = x$$

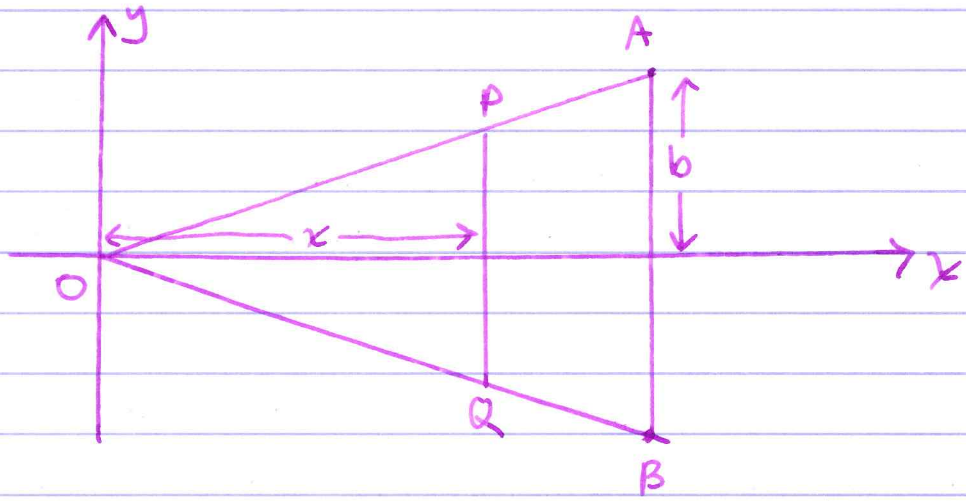
$$x = 11.04600933$$

$$\underline{x = 11.05 m} \text{ i 2 ledegol}$$

ch) Mae buanedd y bêl pan mae'n dychwelyd at O yn llai nag $17ms^{-1}$.
Mae hyn gan fod egni'n cael ei golli wrth oresgyn gwrthiant aer.

Uned 6 Pellach – Deunyddiau Asesu Enghreifftiol

2)



Mae'r côn yn unffurf, fel bod ei ddwysedd ρ yn gyson. (ρ yw'r màs fesul uned cyfaint.)

Trwy gymesuredd, mae'r craidd màs yn gorwedd ar yr echelin-x. Gadewch i ni rannu'r côn yn dafellau yn baralel i'r sylfaen. Ystyriwch dafell PQ, sydd ar bellter x oddi wrth O a thrwch δx.

Radiws y dafell yw $b \times \frac{x}{h}$ (trionglau cyflun)

$$= \frac{bx}{h}.$$

$$\text{Màs y dafell} = \text{cyfaint} \times \text{dwysedd}$$
$$= \pi r^2 \times \text{trwch} \times \text{dwysedd}$$
$$= \pi \left(\frac{bx}{h}\right)^2 \times \delta x \times \rho$$
$$= \frac{\pi b^2 x^2}{h^2}(\delta x)\rho$$

Mae hwn yn gweithredu ar bellter x o O.

Mās y côn = Cyfaint × dwysedd
$= \frac{1}{3}\pi r^2 \times$ uchder × dwysedd
$= \frac{1}{3}\pi b^2 h\rho$

Mae hwn yn gweithredu ar bellter $\bar{x}$ o O.
Yn cymryd momentau o amgylch yr echelin y:

$$\bar{x} = \frac{\Sigma\,(\text{Mās stribed} \times \text{Pellter stribed o O})}{\text{Mās y côn}}$$

$$(\text{Mās y côn})\,\bar{x} = \Sigma\,(\text{Mās stribed} \times \text{Pellter stribed o O})$$

$$\frac{1}{3}\pi b^2 h\rho\,\bar{x} = \Sigma\left(\pi \frac{b^2x^2}{h^2}(\delta x)\rho \times x\right)$$

$$\frac{1}{3}\pi b^2 h\rho\,\bar{x} = \Sigma\,\pi \frac{b^2x^3}{h^2}(\delta x)\rho$$

Fel mae $\delta x \to 0$ mae hwn yn rhoi

$$\frac{1}{3}\pi b^2 h\rho\,\bar{x} = \int_0^h \frac{\pi b^2 x^3}{h^2}\rho\,dx$$

$$\frac{1}{3}\pi b^2 h\rho\,\bar{x} = \frac{\pi b^2\rho}{h^2}\int_0^h x^3\,dx$$

$$\frac{1}{3}h^3\bar{x} = \int_0^h x^3\,dx$$

$$\frac{1}{3}h^3\bar{x} = \left[\frac{x^4}{4}\right]_0^h$$

$$\frac{1}{3}h^3\bar{x} = \frac{h^4}{4} - \frac{0^4}{4}$$

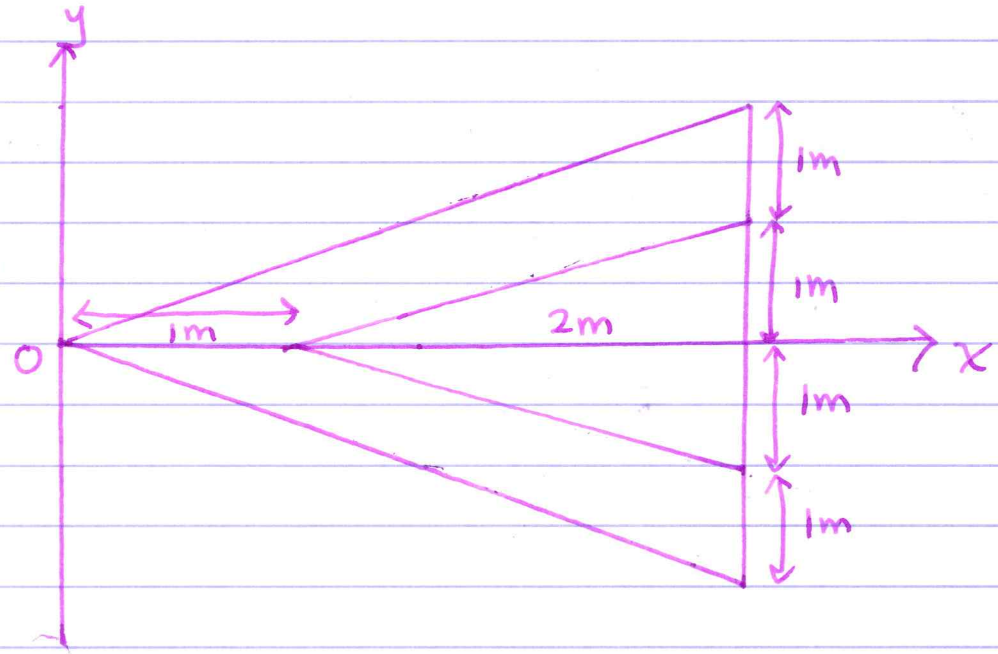
$$\frac{1}{3}h^3\bar{x} = \frac{h^4}{4}$$

$$\bar{x} = \frac{3h^4}{4h^3}$$

$$\bar{x} = \frac{3h}{4}$$

Felly mae'r craidd mās ar uchder o $h - \frac{3h}{4} = \frac{h}{4}$

uwchben y sylfaen. ✓



Gadewch i ρ gynrychioli dwysedd G

Siâp	Màs	Pellter craidd màs o O
C_1	$\frac{1}{3}\pi r^2 h\rho = \frac{1}{3}\pi(2^2)(3)\rho = 4\pi\rho$	$\frac{3}{4}\times 3 = \frac{9}{4}$
C_2	$\frac{1}{3}\pi r^2 h\rho = \frac{1}{3}\pi(1^2)(2)\rho = \frac{2}{3}\pi\rho$	$\frac{3}{4}\times 2 + 1 = \frac{5}{2}$
Gweddill	$4\pi\rho - \frac{2}{3}\pi\rho = \frac{10}{3}\pi\rho$	$\bar{x}$

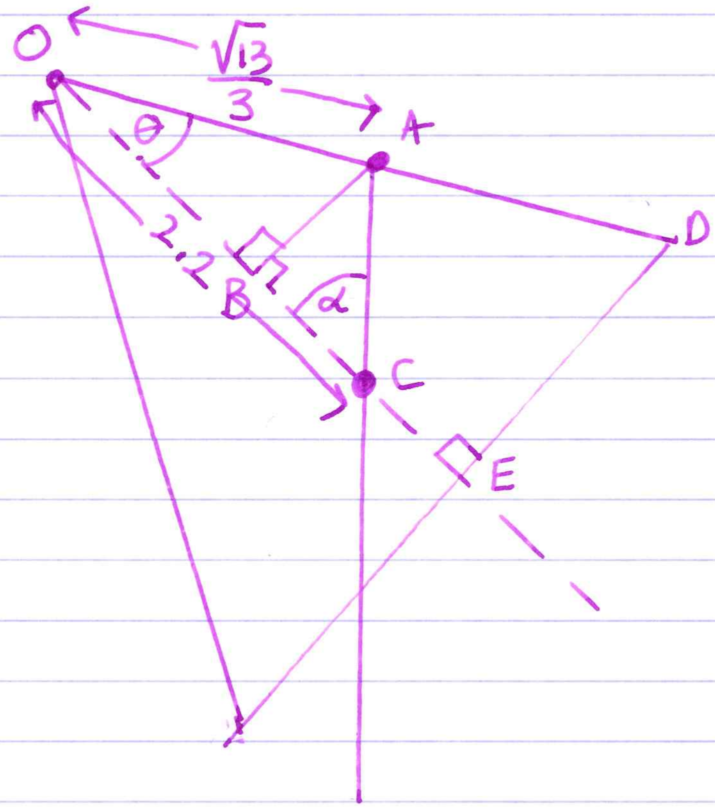
Yn cymryd momentau o amgylch yr echelin-y:

$$\frac{10}{3}\pi\rho(\bar{x}) = 4\pi\rho \times \frac{9}{4} - \frac{2}{3}\pi\rho \times \frac{5}{2}$$

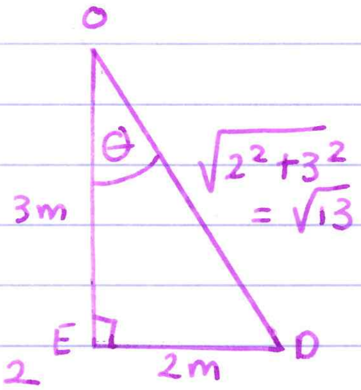
$$\frac{10}{3}\bar{x} = 9 - \frac{10}{6}$$

$$\underline{\bar{x} = 2.2\,\text{m}}$$

c)



Mae'r llinell fertigol yn mynd trwy'r craidd mâs.



Yn defnyddio triongl ODE : $\tan\theta = \frac{2}{3}$

Felly $\sin\theta = \frac{2}{\sqrt{13}}$ a $\cos\theta = \frac{3}{\sqrt{13}}$

Yn defnyddio triongl OAB

$\sin\theta = \frac{AB}{OA}$ $\qquad$ $\cos\theta = \frac{OB}{OA}$

$\frac{2}{\sqrt{13}} = \frac{AB}{\frac{\sqrt{13}}{3}}$ $\qquad$ $\frac{3}{\sqrt{13}} = \frac{OB}{\frac{\sqrt{13}}{3}}$

$\frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{3} = AB$ $\qquad$ $\frac{3}{\sqrt{13}} \times \frac{\sqrt{13}}{3} = OB$

$AB = \frac{2}{3}$ m $\qquad$ $OB = 1$ m

$BC = OC - OB$

$BC = 2.2 - 1$

$BC = 1.2$m

Yn defnyddio triongl ABC

$\tan\alpha = \frac{AB}{BC}$

$\tan\alpha = \frac{\frac{2}{3}}{1.2}$

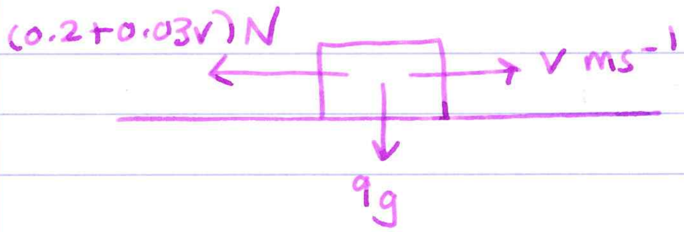
$\tan\alpha = \frac{5}{9}$

Uned 6 Pellach – Deunyddiau Asesu Enghreifftiol

3)

$u = 20\ ms^{-1}$

ar amser $t = 0s$



a) Yn defnyddio $F = ma$ ar y gwrthrych, yn llorweddol, $\rightarrow = +if$

$$-(0.2 + 0.03v) = 9a$$

$$-0.2 - 0.03v = 9\frac{dv}{dt}$$

$$-20 - 3v = 900\frac{dv}{dt}$$

$$900\frac{dv}{dt} = -(20 + 3v)$$

✓

b) $900\frac{dv}{dt} = -(20+3v)$

$$\frac{900}{-(20+3v)}\,dv = dt$$

$$\int \frac{900}{-(20+3v)}\,dv = \int dt$$

$$-900\int \frac{1}{20+3v}\,dv = \int dt$$

$$-900\left(\frac{1}{3}\ln|20+3v|\right) = t + c$$

$$-300\ln|20+3v| = t + c$$

Os yw $t = 0s$, $v = 20ms^{-1}$, felly

$$-300 \ln|20 + 3 \times 20| = 0 + c$$

$$c = -300 \ln(80)$$

Felly $-300 \ln|20 + 3v| = t - 300 \ln(80)$

$$300 \ln(80) - 300 \ln|20 + 3v| = t$$

$$t = 300 \ln\left|\frac{80}{20 + 3v}\right|$$

c) Pan mae'r gwrthrych yn dod i ddisymudedd, mae $v = 0 ms^{-1}$. Felly

$$t = 300 \ln\left|\frac{80}{20 + 3 \times 0}\right|$$

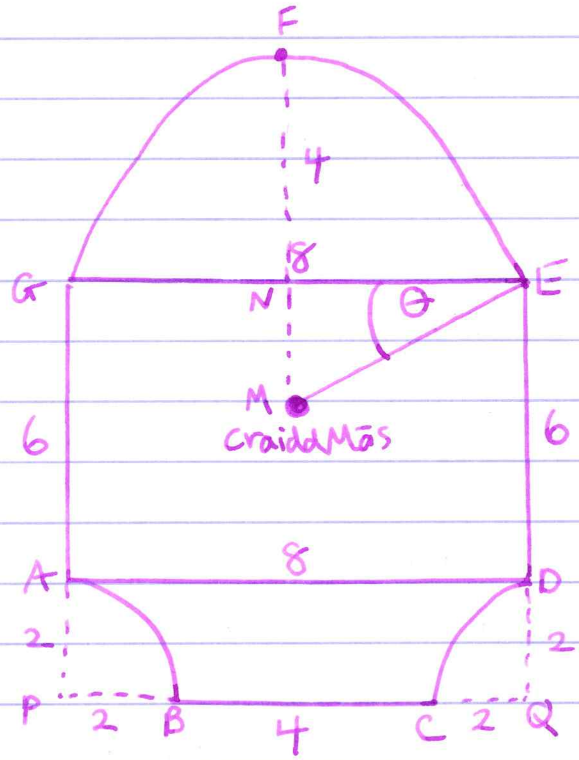
$$t = 300 \ln(4)$$

$$t = 415.8883083$$

$\underline{t = 416s}$ i'r eiliad agosaf

Uned 6 Pellach – Deunyddiau Asesu Enghreifftiol

4)



a) Trwy gymesuredd y siâp, pellter craidd mās y lamina ABCDEFG oddi wrth AG yw 4 cm

b)

Siâp	Mās	Pellter craidd mās o BC
GPQE	$8 \times 8 = 64$	4 cm
EFG	$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 4^2 = 8\pi$	$8 + \frac{4r}{3\pi} = 8 + \frac{16}{3\pi}$ cm
APB	$\frac{1}{4}\pi r^2 = \frac{1}{4} \times \pi \times 2^2 = \pi$	$\frac{4r}{3\pi} = \frac{8}{3\pi}$ cm
CQD	π	$\frac{8}{3\pi}$ cm
Lamina	$64 + 8\pi - \pi - \pi = 64 + 6\pi$	$\bar{y}$

Yn cymryd momentau o amgylch BC:

$$(64+6\pi)\bar{y} = 64\times4 + 8\pi\left(8+\frac{16}{3\pi}\right) - \pi\left(\frac{8}{3\pi}\right) - \pi\left(\frac{8}{3\pi}\right)$$

$$(64+6\pi)\bar{y} = 256 + 64\pi + \frac{128}{3} - \frac{8}{3} - \frac{8}{3}$$

$$(64+6\pi)\bar{y} = 64\pi + \frac{880}{3}$$

$$\bar{y} = \frac{64\pi + \frac{880}{3}}{64+6\pi}$$

$$\bar{y} = 5.967385795$$

$\underline{\bar{y} = 5.967\text{cm}}$ i 3 lle degol

c) Mae'r llinell o E i'r craidd màs yn llinell fertigol.

$$\tan\theta = \frac{MN}{EN}$$

$$\tan\theta = \frac{8-5.967385795}{4}$$

$$\theta = \tan^{-1}\left(\frac{8-5.967385795}{4}\right)$$

$$\theta = 26.937562250^{\circ}$$

$\underline{\theta = 26.938^{\circ}}$ i 3 lle degol

Uned 6 Pellach - Deunyddiau Asesu Enghreifftiol

5) $\underline{v}_A = 2\underline{i} + 7\underline{j}$

$\underline{r}_A = \int 2\underline{i} + 7\underline{j}\ dt$

$\underline{r}_A = (2\underline{i} + 7\underline{j})t + \underline{c}$

Os yw $t = 0s$, mae $\underline{r}_A = 11\underline{i} + 6\underline{j}$, felly

$11\underline{i} + 6\underline{j} = (2\underline{i} + 7\underline{j})(0) + \underline{c}$

$\underline{c} = 11\underline{i} + 6\underline{j}$

Felly $\underline{r}_A = (2\underline{i} + 7\underline{j})t + 11\underline{i} + 6\underline{j}$.

Yn debyg, $\underline{r}_B = (5\underline{i} + 4\underline{j})t + 7\underline{i} + 10\underline{j}$

Os yw'r gronynnau'n gwrthdaro, yna byddent yn yr un safle ar amser penodol.

Cydrannau $\underline{i}$

$$2t + 11 = 5t + 7$$
$$4 = 3t$$
$$t = \frac{4}{3}s$$

Cydrannau $\underline{j}$

$$7t + 6 = 4t + 10$$
$$3t = 4$$
$$t = \frac{4}{3}s$$

Mae'r datrysiadau'n cytuno felly bydd y gronynnau'n gwrthdaro ar amser $t = \frac{4}{3}s$.

Egwyddor Cadwraeth Momentwm:

Momentwm A cyn y gwrthdrawiad

+ Momentwm B cyn y gwrthdrawiad

= Momentwm A a B ar ôl y gwrthdrawiad

$$\cancel{m}(2\underline{i}+7\underline{j})+2\cancel{m}(5\underline{i}+4\underline{j}) = 3\cancel{m}(x\underline{i}+y\underline{j})$$
$$2\underline{i}+10\underline{i}+7\underline{j}+8\underline{j} = 3x\underline{i}+3y\underline{j}$$
$$12\underline{i}+15\underline{j} = 3x\underline{i}+3y\underline{j}$$
$$4\underline{i}+5\underline{j} = x\underline{i}+y\underline{j}$$

Felly cyflymder cyffredin y gronynnau ar ôl y gwrthdrawiad fydd $\underline{4\underline{i}+5\underline{j}}$

b) Ergyd a gwnaeth A ei roi ar B

$= $ Newid momentwm B

$$= 2m(4\underline{i}+5\underline{j}) - 2m(5\underline{i}+4\underline{j})$$
$$= 8m\underline{i}+10m\underline{j}-10m\underline{i}-8m\underline{j}$$
$$= -2m\underline{i}+2m\underline{j}$$
$$= \underline{2m(-\underline{i}+\underline{j})\ \text{Ns}}$$

c) Colled mewn egni cinetig

$= $ EC A gynt $+$ EC B gynt $-$ EC A a B wedyn

$$= \frac{1}{2}(m)\left(\sqrt{2^2+7^2}\right)^2 + \frac{1}{2}(2m)\left(\sqrt{5^2+4^2}\right)^2 - \frac{1}{2}(3m)\left(\sqrt{4^2+5^2}\right)^2$$
$$= \frac{1}{2}m(4+49) + m(25+16) - \frac{3}{2}m(16+25)$$
$$= \frac{53}{2}m + 41m - \frac{123}{2}m$$
$$= \underline{6m\ \text{J}}$$

Uned 6 Pellach - Deunyddiau Asesu Enghreifftiol

6) a) $T = \frac{\lambda x}{l}$

$T = \frac{\lambda(0.05)}{0.75}$

$T = \frac{\lambda}{15}$

Mewn cydbwysedd, mae'r tensiwn yn y sbring yn hafal i bwysau'r sedd.

$12g = T$

$12 \times 9.8 = \frac{\lambda}{15}$

$15 \times 12 \times 9.8 = \lambda$

$\lambda = 1764\,N$

b) Ystyriwch ddadleoliad x o'r safle cydbwysedd.

Yn defnyddio $F=ma$ ar y sedd, yn fertigol, $\downarrow = +if$

$12g - T = 12a$

$12g - \frac{\lambda(0.05+x)}{l} = 12a$

$12 \times 9.8 - \frac{1764(0.05+x)}{0.75} = 12a$

$117.6 - \frac{88.2 + 1764x}{0.75} = 12a$

$117.6 - 117.6 - 2352x = 12a$

$-2352x = 12a$

$-196x = a$

$-(14^2)x = a$

Ond mae $a = \frac{d^2x}{dt^2}$

Felly $-(14^2)x = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -(14^2)x$$

Mae hwn yn hafaliad ar gyfer Mudiant Harmonig Syml.
Wrth gymharu e fo'r hafaliad gyffredinol

$$\frac{d^2x}{dt^2} = -\omega^2 x,$$

gwelwn fod $\omega = 14$.

Osgled y mudiant yw 0.05m.

Y cyfnod yw $\frac{2\pi}{\omega} = \frac{2\pi}{14}$

$$= \frac{\pi}{7} \checkmark$$

c) Mae'r buanedd uchaf ar ganol y mudiant ac mae'n hafal i $a\omega = 0.05 \times 14$

$$= 0.7\ ms^{-1}$$

ch) Yn defnyddio $v^2 = \omega^2(a^2 - x^2)$

gyda $\omega = 14$, $a = 0.05$, $x = 0.03$,

$$v^2 = 14^2(0.05^2 - 0.03^2)$$

$$v^2 = 196(0.0016)$$

$$v^2 = 0.3136$$

$$v = \sqrt{0.3136}$$

$$\underline{v = 0.56\ ms^{-1}}$$

d) Yn defnyddio $x = a\sin(\omega t + \varepsilon)$
Mae'r mudiant yn cychwyn ar y dadleoliad mwyaf.
Felly mae $\varepsilon = \frac{\pi}{2}$

$x = a\sin(\omega t + \frac{\pi}{2})$
$x = a\cos(\omega t)$
$x = 0.05 \times \cos(14 \times 1.6)$
$x = -0.0458\,7890253$

Y pellter o'r safle cydbwysedd yw <u>0.0459 m</u>
i 3 ffigur ystyrlon

dd) Mae'r sedd wedi ei fodelu fel gronyn.
Rydym yn tybio bod y sbring yn ysgafn.