Uned 4 Pellach: Deunyddiau Asesu Enghreifftiol

1) a) $\int_0^\infty \frac{dx}{(1+x)^5} = \int_0^\infty (1+x)^{-5}\,dx$

$$= \left[\frac{1}{-4}(1+x)^{-4}\right]_0^\infty$$

$$= \left[-\frac{1}{4(1+x)^4}\right]_0^\infty$$

$$= -\frac{1}{4(1+\infty)^4} - -\frac{1}{4(1+0)^4}$$

$$= -\frac{1}{4(1+\infty)^4} + \frac{1}{4(1)^4}$$

$$= -\frac{1}{4(1+\infty)^4} + \frac{1}{4}$$

Mae $\frac{1}{x} \to 0$ fel mae $x \to \infty$, felly

gwerth yr integryn yw

$$-0 + \frac{1}{4}$$

$$= \frac{1}{4}.$$

b) $\int_2^\infty \frac{dx}{x\ln x}$.

Gadewch i $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

Felly $\int_2^\infty \frac{dx}{x\ln x} = \int_{\ln 2}^{\ln\infty} \frac{\cancel{x}\,du}{\cancel{x}\,u}$

$= \int_{\ln 2}^{\ln\infty} \frac{du}{u}$

$= \Big[\ln u\Big]_{\ln 2}^{\ln\infty}$

$= \ln(\ln(\infty)) - \ln(\ln(2))$

Nid yw hwn yn werth meidraidd gan fod $\ln u \to \infty$ os yw $u \to \infty$

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2)
$$\int_0^1 \frac{dx}{\sqrt{2x^2+4x+6}} = \int_0^1 \frac{1}{\sqrt{2(x^2+2x)+6}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2((x+1)^2-1^2)+6}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2((x+1)^2-1)+6}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2(x+1)^2-2+6}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2(x+1)^2+4}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2[(x+1)^2+2]}}\,dx$$
$$= \int_0^1 \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{(x+1)^2+2}}\,dx$$
$$= \frac{1}{\sqrt{2}} \int_0^1 \frac{1}{\sqrt{(x+1)^2+2}}\,dx$$
$$= \frac{1}{\sqrt{2}} \left[\sinh^{-1}\left(\frac{x+1}{\sqrt{2}}\right) \right]_0^1$$
$$= \frac{1}{\sqrt{2}} \left(\sinh^{-1}\left(\frac{2}{\sqrt{2}}\right) - \sinh^{-1}\left(\frac{1}{\sqrt{2}}\right) \right)$$
$$= 0.3448820598$$
$$= \underline{0.345} \text{ i 3 lle degol}$$

Llyfryn Fformiwlâu: $\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right)$

rydym efo $a = \sqrt{2}$, '$x \to x+1$'

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3) C: $r = 3(2+\cos\theta)$ ar gyfer $0 \leq \theta \leq \pi$.

$$\text{Arwynebedd} = \frac{1}{2}\int_0^{\pi} r^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} [3(2+\cos\theta)]^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 3^2(2+\cos\theta)^2 \, d\theta$$

$$= \frac{9}{2}\int_0^{\pi} (2+\cos\theta)^2 \, d\theta$$

$$= \frac{9}{2}\int_0^{\pi} 4 + 4\cos\theta + \cos^2\theta \, d\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$$

$$= \frac{9}{2}\int_0^{\pi} 4 + 4\cos\theta + \frac{\cos 2\theta + 1}{2} \, d\theta$$

$$= \frac{9}{2}\int_0^{\pi} \frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= \frac{9}{2}\left[\frac{9}{2}\theta + 4\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{9}{2}\left[\left(\frac{9}{2}\pi + 4\sin\pi + \frac{1}{4}\sin(2\pi)\right) - \left(\frac{9}{2}(0) + 4\sin(0) + \frac{1}{4}\sin(2\times 0)\right)\right]$$

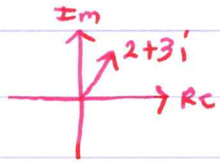
$$= \frac{9}{2}\left[\left(\frac{9}{2}\pi + 4(0) + \frac{1}{4}(0)\right) - \left(0 + 4(0) + \frac{1}{4}(0)\right)\right]$$

$$= \frac{81\pi}{4} \text{ uned sgwâr}$$

Uned 4 Pellach – Deunyddiau Asesu Enghreifftiol

4) $z = \sqrt[3]{2+3i}$

$z^3 = 2+3i$



Ffurf Trigonometreg:

$r = \sqrt{2^2+3^2}$ $\qquad \theta = \tan^{-1}\left(\frac{3}{2}\right)$

$r = \sqrt{4+9}$ $\qquad \theta = 0.9827937232$ rad

$r = \sqrt{13}$

Felly $z^3 = \sqrt{13}\,(\cos\theta + i\sin\theta)$

$z^3 = \sqrt{13}\,(\cos(0.982\cdots) + i\sin(0.982\cdots))$

$z = \left[\sqrt{13}\,(\cos(0.982\cdots) + i\sin(0.982\cdots))\right]^{\frac{1}{3}}$

$z = 13^{\frac{1}{6}}\left[\cos(0.982\cdots) + i\sin(0.982\cdots)\right]^{\frac{1}{3}}$

Nawr

$(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \cos\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right] + i\sin\left[\frac{\theta}{n} + \frac{2(k-1)\pi}{n}\right]$

$(\cos(0.982\cdots) + i\sin(0.982\cdots))^{\frac{1}{3}}$

$= \cos\left[\frac{0.982\cdots}{3} + \frac{2(k-1)\pi}{3}\right]$

$+ i\sin\left[\frac{0.982\cdots}{3} + \frac{2(k-1)\pi}{3}\right]$

$\underline{k=1}$ $\quad z = 13^{\frac{1}{6}}\left(\cos\left(\frac{0.982\cdots}{3}\right) + i\sin\left(\frac{0.982\cdots}{3}\right)\right)$

$z = 1.452 + 0.493i$ i 3 lle degol

$\underline{k=2}$ $\quad z = 13^{\frac{1}{6}}\left(\cos\left(\frac{0.982\cdots}{3} + \frac{2\pi}{3}\right)\right.$

$\left. + \sin\left(\frac{0.982\cdots}{3} + \frac{2\pi}{3}\right)\right)$

$\underline{z = -1.153 + 1.011i}$ i 3 lle degol

$\underline{k=3}$ $z = 13^{\frac{1}{6}}\left(\cos\left(\frac{0.982\cdots}{3} + \frac{4\pi}{3}\right)\right.$

$\left. + i\sin\left(\frac{0.982\cdots}{3} + \frac{4\pi}{3}\right)\right)$

$\underline{z = -0.299 - 1.504i}$ i 3 lle degol

Nodyn: cynllun marcio yn anghywir efo -2.99 yn fan hyn.

Uned 4 Pellach - Deunyddiau Asesu Enghreifftiol

5) $\cos\theta + \cos 3\theta + \cos 5\theta = 0$

Beth yw'r gwreiddiau yn y cyfwng $[0, \pi]$ yn nhermau π?

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos\theta + \cos 5\theta = 2\cos\left(\frac{\theta+5\theta}{2}\right)\cos\left(\frac{\theta-5\theta}{2}\right)$$

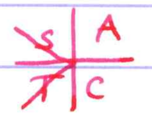
$$= 2\cos 3\theta \cos(-2\theta)$$

$$= 2\cos 3\theta \cos 2\theta$$ trwy gymesuredd graff cos.

Felly $\cos\theta + \cos 3\theta + \cos 5\theta = 0$

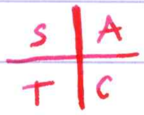
$2\cos 3\theta \cos 2\theta + \cos 3\theta = 0$

$\cos 3\theta(2\cos 2\theta + 1) = 0$



Naill ai $\cos 3\theta = 0$

$3\theta = \cos^{-1}(0)$



$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ ← Mwy na π

$$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$$

neu $2\cos 2\theta + 1 = 0$

$\cos 2\theta = -\frac{1}{2}$

$2\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \ldots$

$\theta = \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{8\pi}{6}, \ldots$ ← Mwy na π

$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}}$$

Atebion

Uned 4 Pellach – Deunyddiau Asesu Enghreifftiol

6) $M = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{pmatrix}$

a) $M^T = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{pmatrix}$

Minorau $= \begin{pmatrix} 11 & -1 & -7 \\ -1 & 1 & 1 \\ -7 & 1 & 5 \end{pmatrix}$

Atgydiol (Cawyddion) $= \begin{pmatrix} 11 & 1 & -7 \\ 1 & 1 & -1 \\ -7 & -1 & 5 \end{pmatrix}$

ii) $\det(M) = 2\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}$

$= 2(11) - 1(-1) + 3(-7)$

$= 22 + 1 - 21$

$= 2$

$M^{-1} = \frac{1}{2}\begin{pmatrix} 11 & 1 & -7 \\ 1 & 1 & -1 \\ -7 & -1 & 5 \end{pmatrix}$

b) Gellir ysgrifennu'r hafaliadau cydamserol fel

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 13 \\ 22 \end{pmatrix}$$

$$M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 13 \\ 22 \end{pmatrix}$$

$$M^{-1}M\begin{pmatrix}x\\y\\z\end{pmatrix} = M^{-1}\begin{pmatrix}13\\13\\22\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}11 & 1 & -7\\1 & 1 & -1\\-7 & -1 & 5\end{pmatrix}\begin{pmatrix}13\\13\\22\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\4\\6\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

Uned 4 Pellach - Deunyddiau Asesu Enghreifftiol

7) $f(x) = \dfrac{8x^2 + x + 5}{(2x+1)(x^2+3)}$

a) $\dfrac{8x^2+x+5}{(2x+1)(x^2+3)} \equiv \dfrac{A}{2x+1} + \dfrac{Bx+C}{x^2+3}$

$\dfrac{8x^2+x+5}{(2x+1)(x^2+3)} \equiv \dfrac{A(x^2+3) + (Bx+C)(2x+1)}{(2x+1)(x^2+3)}$

$8x^2+x+5 \equiv A(x^2+3) + (Bx+C)(2x+1)$

Yn amnewid $x = -\frac{1}{2}$:

$8(-\frac{1}{2})^2 + (-\frac{1}{2}) + 5 \equiv A((-\frac{1}{2})^2+3) + (Bx+C)(0)$

$2 - \frac{1}{2} + 5 \equiv A(\frac{13}{4})$

$\frac{13}{2} \equiv A(\frac{13}{4})$

$\underline{A \equiv 2}$

Yn cymharu cyfernodau x^2:

$8 \equiv A + 2B$

$8 \equiv 2 + 2B$

$6 \equiv 2B$

$\underline{B \equiv 3}$

Yn cymharu cysonion:

$5 \equiv 3A + C$

$5 \equiv 3\times2 + C$

$\underline{C \equiv -1}$

Felly $\dfrac{8x^2+x+5}{(2x+1)(x^2+3)} = \dfrac{2}{2x+1} + \dfrac{3x-1}{x^2+3}$

b) $\int_2^3 f(x)\,dx = \int_2^3 \frac{2}{2x+1} + \frac{3x-1}{x^2+3}\,dx$

$= \int_2^3 \frac{2}{2x+1} + \frac{3x}{x^2+3} - \frac{1}{x^2+(\sqrt{3})^2}\,dx$

$= \left[\ln|2x+1| + \frac{3}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right]_2^3$

$= \left[\left(\ln(7) + \frac{3}{2}\ln(12) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{3}{\sqrt{3}}\right)\right)\right.$

$\left. - \left(\ln(5) + \frac{3}{2}\ln(7) - \frac{1}{\sqrt{3}}\left(\frac{2}{\sqrt{3}}\right)\right)\right]$

$= 5.068670336 - 4.033472416$

$= 1.035197\text{92}$

$= \underline{1.035}$; 3 lle degol

Uned 4 Pellach: Deunyddiau Asesu Enghreifftiol

8) C: $y = 1 + x^3$

a) Cylchdroi o amgyrch yr echelin $\underline{\underline{y}}$ felly

Cynhwysedd = $\pi \int_1^9 x^2 \, dy$

Nawr $y = 1 + x^3 \Rightarrow$

$$y - 1 = x^3$$
$$\sqrt[3]{y-1} = x$$
$$\left(\sqrt[3]{y-1}\right)^2 = x^2$$
$$x^2 = (y-1)^{\frac{2}{3}}$$

Felly'r cynhwysedd $= \pi \int_1^9 (y-1)^{\frac{2}{3}} \, dy$

$$= \pi \left[\frac{3}{5}(y-1)^{\frac{5}{3}} \right]_1^9$$

$$= \pi \left[\left(\frac{3}{5}(9-1)^{\frac{5}{3}} \right) - \left(\frac{3}{5}(1-1)^{\frac{5}{3}} \right) \right]$$

$$= \pi \left[\frac{3}{5}(8)^{\frac{5}{3}} - \frac{3}{5}(0) \right]$$

$$= \frac{3\pi (8)^{\frac{5}{3}}}{5}$$

$$= 19.2\pi$$

$= \underline{60.32}$ uned ciwb i 2 le degol

b) Rydym angen $25 = \pi \int_1^a (y-1)^{\frac{2}{3}} \, dy$

$$25 = \pi \left[\frac{3}{5} (y-1)^{\frac{5}{3}} \right]_1^a$$

$$25 = \pi \left[\left(\frac{3}{5} (a-1)^{\frac{5}{3}} \right) - \left(\frac{3}{5} (1-1)^{\frac{5}{3}} \right) \right]$$

$$25 = \pi \left[\frac{3}{5} (a-1)^{\frac{5}{3}} - 0 \right]$$

$$25 = \frac{3\pi}{5} (a-1)^{\frac{5}{3}}$$

$$\frac{125}{3\pi} = (a-1)^{\frac{5}{3}}$$

$$\left(\frac{125}{3\pi} \right)^{\frac{3}{5}} = a - 1$$

$$a = \left(\frac{125}{3\pi} \right)^{\frac{3}{5}} + 1$$

$$a = 5.716103541$$

$\underline{a = 5.72}$; 2 le degol

Uned 4 Pellach – Deunyddiau Asesu Enghreifftiol

9) Achos $n=1$: Ochr chwith $= (\cos\theta + i\sin\theta)^1$
$= \cos\theta + i\sin\theta$.
Ochr dde $= \cos(1\theta) + i\sin(1\theta)$
$= \cos\theta + i\sin\theta$.

Felly mae'r gosodiad yn wir ar gyfer $n=1$.

Cymerwch bod y gosodiad yn wir ar gyfer $n=k$, fel bod $(\cos\theta + i\sin\theta)^k = \cos(k\theta) + i\sin(k\theta)$.

Edrychwn ar yr achos $n=k+1$.

Ochr chwith $= (\cos\theta + i\sin\theta)^{k+1}$
$= (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)$
$= (\cos(k\theta) + i\sin(k\theta))(\cos\theta + i\sin\theta)$
trwy'r hypothesis anwythol
$= \cos(k\theta)\cos\theta + \cos(k\theta)i\sin\theta$
$+ i\sin(k\theta)\cos\theta + i^2\sin(k\theta)\sin(\theta)$
$= \cos(k\theta)\cos\theta - \sin(k\theta)\sin(\theta)$
$+ i(\cos(k\theta)\sin\theta + \sin(k\theta)\cos\theta)$
$= \cos(k\theta + \theta) + i(\sin(k\theta + \theta))$
$= \cos((k+1)\theta) + i(\sin((k+1)\theta))$
$=$ ochr dde

Rydym wedi dangos bod y gosodiad yn wir ar gyfer $n=k+1$ os yw'r achos $n=k$ yn wir.
Gan fod yr achos $n=1$ yn wir, rydym wedi profi'r gosodiad trwy anwythiad.

b) i) Ystyriwch yr achos $n = 5$.

$$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$$
$$+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$$
$$= \cos 5\theta + i\sin 5\theta$$

$$\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta$$
$$- 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$$
$$= \cos 5\theta + i\sin 5\theta$$

Yn cymharu darnau dychmygol:

$$5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta = \sin 5\theta$$

$$5(1-\sin^2\theta)(1-\sin^2\theta)\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$$
$$= \sin 5\theta$$

$$5(1 - 2\sin^2\theta + \sin^4\theta)\sin\theta + (-10 + 10\sin^2\theta)\sin^3\theta + \sin^5\theta$$
$$= \sin 5\theta$$

$$5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$$
$$= \sin 5\theta$$

$$16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = \sin 5\theta$$

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

Felly $a = 16$, $b = 20$, $c = 5$

ii) $\lim_{\theta \to 0} \frac{\sin 5\theta}{\sin\theta}$

$= \lim_{\theta \to 0} \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin\theta}$

$= \lim_{\theta \to 0} 16\sin^4\theta - 20\sin^2\theta + 5$

Mae $\sin\theta \to 0$ fel mae $\theta \to 0$

felly mae $\lim_{\theta \to 0} \frac{\sin 5\theta}{\sin\theta} = 5$

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10) $\frac{dy}{dx} + 2y \tan x = \sin x, \qquad 0 < x < \frac{\pi}{2}$

a) Ar gyfer $\frac{dy}{dx} + Fy = G$,

ble mae F a G yn ffwythiannau mewn x yn unig,

y ffactor integru yw $I = e^{\int F dx}$

$I = e^{\int 2\tan x\, dx}$

$I = e^{2\ln|\sec x|}$

$I = e^{\ln|\sec^2 x|}$

$I = \sec^2 x$

b) $\frac{dy}{dx} + 2y \tan x = \sin x$.

Lluosi bob ochr yr hafaliad â'r ffactor integru:

$\sec^2 x \frac{dy}{dx} + 2y \tan x \sec^2 x = \sin x \sec^2 x$

$\int \left(\sec^2 x \frac{dy}{dx} + 2y \tan x \sec^2 x \right) dx = \int \sin x \sec^2 x \, dx$

$(\sec^2 x) y = \int \sin x \sec^2 x \, dx$

$(\sec^2 x) y = \int \sin x \frac{1}{\cos^2 x} dx$

$(\sec^2 x) y = \int \tan x \frac{1}{\cos x} dx$

$(\sec^2 x) y = \int \tan x \sec x \, dx$

$(\sec^2 x) y = \sec x + K$

Os yw $x = \frac{\pi}{4}$, mae $y = 0$, felly

$(\sec^2(\frac{\pi}{4}))(0) = \sec(\frac{\pi}{4}) + K$

$0 = \sqrt{2} + K$

$K = -\sqrt{2}$.

Felly $(\sec^2 x)y = \sec x - \sqrt{2}$

$$y = \frac{\sec x - \sqrt{2}}{\sec^2 x}$$

$$y = \frac{1}{\sec x} - \frac{\sqrt{2}}{\sec^2 x}$$

$$y = \cos x - \sqrt{2}\cos^2 x$$

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11) a) $\sinh x = \frac{1}{2}(e^{x} - e^{-x})$
$\cosh x = \frac{1}{2}(e^{x} + e^{-x})$

Gadewch i $y = \tanh^{-1}(x)$

fel bod $\tanh(y) = x$

$$\frac{\sinh(y)}{\cosh(y)} = x$$

$$\frac{\cancel{\frac{1}{2}}(e^{y} - e^{-y})}{\cancel{\frac{1}{2}}(e^{y} + e^{-y})} = x$$

$$e^{y} - e^{-y} = x(e^{y} + e^{-y})$$

$$e^{y} - e^{-y} = xe^{y} + xe^{-y}$$

$$e^{2y} - 1 = xe^{2y} + x$$

$$e^{2y} - xe^{2y} = x + 1$$

$$e^{2y}(1 - x) = x + 1$$

$$e^{2y} = \frac{x+1}{1-x}$$

$$\ln(e^{2y}) = \ln\left(\frac{1+x}{1-x}\right)$$

$$2y = \ln\left(\frac{1+x}{1-x}\right)$$

$$y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad \checkmark$$

(Rhaid bod $-1 < x < 1$ gan mai hwn yw amrediad y ffwythiant $y = \tanh x$.)

b) $a\cosh x + b\sinh x \equiv r\cosh(x+\alpha)$, lle mae $a > b > 0$.
$a\cosh x + b\sinh x \equiv r\cosh x\cosh\alpha + r\sinh x\sinh\alpha$
(Rheol Osborn)

Felly rhaid bod $a = r\cosh\alpha$ $\quad b = r\sinh\alpha$

[r] $\dfrac{a}{\cosh\alpha} = \dfrac{b}{\sinh\alpha}$

$\dfrac{\sinh\alpha}{\cosh\alpha} = \dfrac{b}{a}$

$\tanh\alpha = \dfrac{b}{a}$

$\alpha = \tanh^{-1}\left(\dfrac{b}{a}\right)$

$\alpha = \dfrac{1}{2}\ln\left(\dfrac{1+\frac{b}{a}}{1-\frac{b}{a}}\right)$

$\alpha = \dfrac{1}{2}\ln\left(\dfrac{\frac{a}{a}+\frac{b}{a}}{\frac{a}{a}-\frac{b}{a}}\right)$

$\alpha = \dfrac{1}{2}\ln\left(\dfrac{a+b}{a-b}\right)$ ✓

$[\cosh^2\alpha - \sinh^2\alpha = 1]$

$\left(\dfrac{a}{r}\right)^2 - \left(\dfrac{b}{r}\right)^2 = 1$

$\dfrac{a^2}{r^2} - \dfrac{b^2}{r^2} = 1$

$a^2 - b^2 = r^2$

$r = \sqrt{a^2-b^2}$

c) $5\cosh x + 4\sinh x = 10$

$a = 5,\ b = 4 \Rightarrow \alpha = \dfrac{1}{2}\ln\left(\dfrac{5+4}{5-4}\right)$, $\quad r = \sqrt{5^2-4^2}$

$r = \sqrt{25-16}$

$r = \sqrt{9}$

$r = 3$

$\alpha = \dfrac{1}{2}\ln\left(\dfrac{9}{1}\right)$

$\alpha = \frac{1}{2}\ln(9)$

$\alpha = \ln(\sqrt{9})$

$\alpha = \ln(3)$

Felly $5\cosh x + 4\sinh x = 10$

$$3\cosh(x + \ln(3)) = 10$$

$$\cosh(x + \ln(3)) = \frac{10}{3}$$

$$x + \ln(3) = \pm \cosh^{-1}\left(\frac{10}{3}\right)$$

$$x + \ln(3) = \pm 1.873820243$$

$$x = -\ln(3) \pm 1.873820243$$

Naill ai $x = -\ln(3) + 1.873820243$

$\underline{x = 0.775}$ i 3 ffigur ystyrlon

Neu $x = -\ln(3) - 1.873820243$

$\underline{x = -2.97}$ i 3 ffigur ystyrlon

Uned 4 Pellach: Deunyddiau Asesu Enghreifftiol

12) $f(x) = e^x \cos x$

a) $f'(x) = e^x(-\sin x) + e^x \cos x$
$= -e^x \sin x + e^x \cos x$
$f''(x) = (-e^x \cos x - e^x \sin x) + (-e^x \sin x + e^x \cos x)$
$= -2e^x \sin x.$

b) $f'''(x) = -2e^x \cos x - 2e^x \sin x$
$f^{(4)}(x) = (-2e^x(-\sin x) - 2e^x \cos x) + (-2e^x \cos x - 2e^x \sin x)$
$= -4e^x \cos x$

$f(0) = e^0 \cos(0)$
$= 1$

$f'(0) = -e^0 \sin(0) + e^0 \cos(0)$
$= 0 + 1$
$= 1$

$f''(0) = -2e^0 \sin(0)$
$= 0$

$f'''(0) = -2e^0 \cos(0) - 2e^0 \sin(0)$
$= -2$

$f^{(4)}(0) = -4e^0 \cos(0)$
$= -4$

Cyfres Maclaurin:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(4)}(0) + \cdots$$

$$f(x) = 1 + x(1) + \frac{x^2}{2}(0) + \frac{x^3}{6}(-2) + \frac{x^4}{24}(-4) + \cdots$$

$$f(x) = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \cdots$$

c) $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots$

Differu bob ochr:

$e^x(-\sin x) + e^x \cos x = 1 - \frac{3x^2}{3} - \frac{4x^3}{6} + \dots$

$-e^x \sin x + e^x \cos x = 1 - x^2 - \frac{2}{3}x^3 + \dots$

$-e^x \sin x + \left(1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots\right) = 1 - x^2 - \frac{2}{3}x^3 + \dots$

$\left(\cancel{1} + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots\right) - \left(\cancel{1} - x^2 - \frac{2}{3}x^3 + \dots\right) = e^x \sin x$

$e^x \sin x = x + x^2 - \frac{x^3}{3} + \frac{2}{3}x^3 - \frac{x^4}{6} + \dots$

$e^x \sin x = x + x^2 + \frac{x^3}{3} + \dots$ (hyd at y term mewn x^3)

ch) $10e^x \sin x - 11x = 0$

Amnewid o ran (c):

$10\left(x + x^2 + \frac{x^3}{3} + \dots\right) - 11x = 0$

Chwilio am wreiddyn positif bach felly anwybyddu termau yn y gyfres ar ôl x^3.

$10\left(x + x^2 + \frac{x^3}{3}\right) - 11x = 0$

$10x + 10x^2 + \frac{10}{3}x^3 - 11x = 0$

$-x + 10x^2 + \frac{10}{3}x^3 = 0$

$x\left(-1 + 10x + \frac{10}{3}x^2\right) = 0$

Naill ai $x = 0$ neu $-1 + 10x + \frac{10}{3}x^2 = 0$

$a = \frac{10}{3},\ b = 10,\ c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-10 \pm \sqrt{10^2 - 4\left(\frac{10}{3}\right)(-1)}}{2\left(\frac{10}{3}\right)}$$

$$x = \frac{-10 \pm \sqrt{\frac{340}{3}}}{\frac{20}{3}}$$

Naill ai $x = \dfrac{-10 + \sqrt{\frac{340}{3}}}{\frac{20}{3}}$ neu $x = \dfrac{-10 - \sqrt{\frac{340}{3}}}{\frac{20}{3}}$

$\underline{x = 0.097}$ i 3 lle degol ✓

$x = -3.097$ i 3 lle degol (✗ chwilio am wreiddyn bach)