Uned 3 – Deunyddiau Asesu Enghreifftiol

1) Ar gyfer gwerthoedd bach o θ, $\sin\theta \approx \theta$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

Felly, ar gyfer $\cos x - 4\sin x = x^2$,
gellid defnyddio $1 - \frac{x^2}{2} - 4x = x^2$

os yw x yn fach. Datrys:

$$2 - x^2 - 8x = 2x^2$$
$$0 = 3x^2 + 8x - 2$$

Fformiwla Gwadratig: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-8 \pm \sqrt{8^2-4(3)(-2)}}{2\times 3}$$

$$x = \frac{-8 \pm \sqrt{88}}{6}$$

Naill ai $x = \frac{-8+\sqrt{88}}{6}$ neu $x = \frac{-8-\sqrt{88}}{6}$

$x = 0.230138\ldots$ $\quad$ $x = -2.8968\ldots$

Rydym yn chwilio am werth bach ar gyfer x, felly dewis $\underline{x = 0.23}$, yn gywir i 2 le degol.

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2) Cyfaint sffêr = $\frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi r^3$

Mae'r cyfaint yn newid 250cm^3 yr eiliad, felly rydym angen darganfod $\frac{dV}{dt}$.

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \qquad \text{(rheol y gadwyn)}$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \times \frac{dr}{dt}$$

$$250 = 3\left(\frac{4}{3}\pi r^2\right) \times \frac{dr}{dt}$$

$$250 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{250}{4\pi r^2} = \frac{dr}{dt}$$

Os yw'r radiws yn 15cm, cyfradd cynnydd y radiws yw

$$\frac{dr}{dt} = \frac{250}{4 \times \pi \times 15^2}$$

$$\frac{dr}{dt} = \frac{5}{18\pi}$$

$$\frac{dr}{dt} \approx \underline{0.088} \text{ cm/eiliad i 3 lle degol}$$

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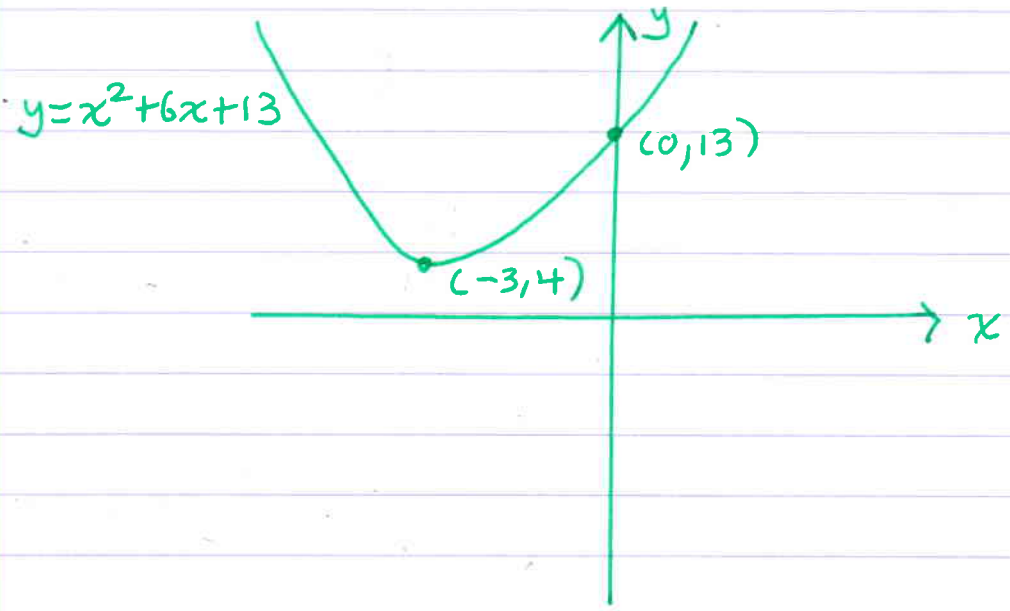
3) a) $y = x^2 + 6x + 13$ Siâp $\cup$ oherwydd $1x^2$

$y = (x+3)^2 - 3^2 + 13$

$y = (x+3)^2 + 4$

Y pwynt minimwm yw $(-3, 4)$



b) $f(x) = x^2 + 6x + 13$ parth (a, b)

i) Parth $(-10, 10)$

Nid yw $f^{-1}(x)$ yn bodoli gan nad yw $f(x)$ yn ffwythiant un-i-un. Er enghraifft, mae $f(-6) = 13$ ac mae $f(0) = 13$.

ii) Dewis $a = -3$, $b = \infty$

Fel bod y parth yn $(-3, \infty)$.

(Fel yma, mae'r ffwythiant $f(x)$ yn ffwythiant un-i-un, fel y gellid gweld o'r graff.)

Beth yw $f^{-1}(x)$?

$f(x) = x^2 + 6x + 13$

Gadewch i $y = x^2 + 6x + 13$

$y = (x+3)^2 - 3^2 + 13$

$y = (x+3)^2 + 4$

$y - 4 = (x+3)^2$

$(x+3)^2 = y - 4$

$x + 3 = \pm\sqrt{y-4}$

$x = -3 \pm \sqrt{y-4}$

Felly $f^{-1}(x) = -3 \pm \sqrt{x-4}$

Gan fod $x > -3$ yn ein parth, maer arwydd positif yn briodol.

Felly $f^{-1}(x) = -3 + \sqrt{x-4}$

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4) a) $(1-x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2\times 1}(-x)^2 + \cdots$

$= 1 + \frac{x}{2} + \frac{3}{8}x^2 + \cdots$

Mae'r ehangiad yn ddilys ar gyfer $|-x| < 1$

$|-1| \times |x| < 1$

$|x| < 1$

Felly $-1 < x < 1$

b) $\left(1-x\right)^{-\frac{1}{2}} \approx 1 + \frac{x}{2} + \frac{3}{8}x^2$

Gyda $x = \frac{1}{10}$,

$\left(1-\frac{1}{10}\right)^{-\frac{1}{2}} \approx 1 + \frac{\frac{1}{10}}{2} + \frac{3}{8}(\frac{1}{10})^2$

$\left(\frac{9}{10}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{20} + \frac{3}{800}$

$\left(\frac{10}{9}\right)^{\frac{1}{2}} \approx 1 + \frac{1}{20} + \frac{3}{800}$

$\sqrt{\frac{10}{9}} \approx \frac{843}{800}$

$\frac{\sqrt{10}}{3} \approx \frac{843}{800}$

$\sqrt{10} \approx \frac{2529}{800}$

(Felly $a = 2,529$, $b = 800$.)

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5) Mae cynydda swm 8% yn golygu lluosi efo 108%, neu 1.08.

Dros 3 blynedd, mae £1000 o arian Aled yn y cynllun cynilo am 3 blynedd (lluosi efo 1.08^3), mae £1000 o'i arian yn y cynllun cynilo am 2 flynedd (lluosi efo 1.08^2), ac mae £1000 o'i arian yn y cynllun am flwyddyn (lluosi efo 1.08).

Felly mae gan Aled
$1000 \times 1.08 + 1000 \times 1.08^2 + 1000 \times 1.08^3$
o arian ar ddiwedd tair blynedd. ✓

Yn ymestyn dros 30 mlynedd, bydd gan arian
$1000 \times 1.08 + 1000 \times 1.08^2 + 1000 \times 1.08^3$
$+ 1000 \times 1.08^4 + \cdots + 1000 \times 1.08^{20}$
o arian ar ddiwedd 30 mlynedd

Mae'n bosib ysgrifennu hyn fel
$1000(1.08 + 1.08^2 + 1.08^3 + \cdots + 1.08^{20})$

Mae'r swm yn y cromfachau yn swm dilyniant/cyfres geometrig efo $a = 1.08$, $r = 1.08$
Yn defnyddio $S_n = \frac{a(1-r^n)}{1-r}$

$$S_{30} = \frac{1.08(1-1.08^{30})}{1-1.08}$$

$$S_{30} = 122.345868$$

Felly, ar ddiwedd 30 o flynyddoedd, mae gan Aled
$1000 \times 122.345868 = £122,345.87$
$= £122,346$ i'r bunt agosaf.

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6) Cyfres Rifyddol

Term 1af = a

Nfed term = $a + (n-1)d$

Gwahaniaeth cyffredin = d

$S_n = \frac{n}{2}[2a + (n-1)d]$

Gwybodaeth:

$S_{15} = 270$

$\frac{15}{2}[2a + 14d] = 270$

$15(2a + 14d) = 540$

$2a + 14d = 36$

$a + 7d = 18$ — (2)

$t_{15} = 8t_1$

$a + 14d = 8a$

$14d = 7a$

$2d = a$ — (1)

Yn amnewid am a o (1) i (2):

$2d + 7d = 18$

$9d = 18$

$d = 2$

Felly $a = 2 \times 2$

$a = 4$

Hyd yr ochr leiaf yw 4 cm

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7) $y = ax^4 + bx^3 + 18x^2$ pwynt ffurfdro $(1, 11)$.

a) Pwynt ffurfdro $\Rightarrow \frac{d^2y}{dx^2} = 0$.

Nawr $\frac{dy}{dx} = 4ax^3 + 3bx^2 + 36x$

$\frac{d^2y}{dx^2} = 12ax^2 + 6bx + 36$

Amnewid $\frac{d^2y}{dx^2} = 0$, $x = 1$:

$0 = 12a(1)^2 + 6b(1) + 36$

$0 = 12a + 6b + 36$

$0 = 2a + b + 6$

$2a + b + 6 = 0$ ✓

b) Yn amnewid $x = 1$, $y = 11$ i mewn i

$y = ax^4 + bx^3 + 18x^2$

$11 = a(1)^4 + b(1)^3 + 18(1)^2$

$11 = a + b + 18$

$0 = a + b + 7$

$-b - 7 = a$

Yn amnewid am a i mewn i $2a + b + 6 = 0$

$2(-b - 7) + b + 6 = 0$

$-2b - 14 + b + 6 = 0$

$-b = 8$

$b = -8$

Felly $a = -b - 7$

$a = 8 - 7$

$a = 1$

Felly $y = x^4 - 8x^3 + 18x^2$.

$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x$

$\frac{d^2y}{dx^2} = 12x^2 - 48x + 36$

$= 12(x^2 - 4x + 3)$

$= 12(x-1)(x-3)$

Felly mae $\frac{d^2y}{dx^2} = 0$ os yw $12(x-1)(x-3) = 0$

$(x-1)(x-3) = 0$

Naill ai $x-1=0$ neu $x-3=0$

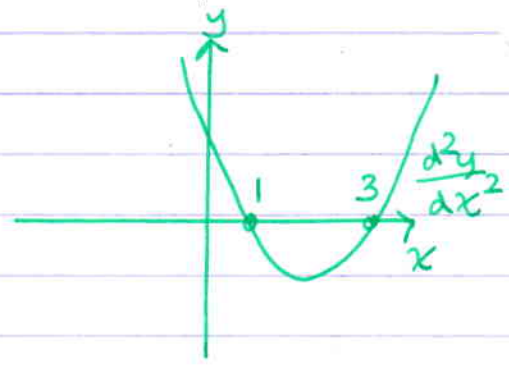
$x = 1$ $\quad$ $x = 3$

Os yw $x=3$, mae $y = 3^4 - 8(3)^3 + 18(3)^2$

$y = 27$.

Ydi (3, 27) yn bwynt ffurfdro?

Ydi, mae $\frac{d^2y}{dx^2}$ yn negatif cyn 3 ac yn bositif ar ôl 3. Maer newid arwydd yn profi bod (3, 27) yn bwynt ffurfdro.



c) $y = x^4 - 8x^3 + 18x^2$.

Pwyntiau ffurfdro yn (1, 11) a (3, 27).

$$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x$$
$$= 4x(x^2 - 6x + 9)$$
$$= 4x(x-3)(x-3)$$

Pwyntiau arhosol os yw $\frac{dy}{dx} = 0$

$$4x(x-3)^2 = 0$$

Naill ai $4x = 0$ neu $x - 3 = 0$

$x = 0$ $\quad$ $x = 3$ (Pwynt ffurfdro)

Os yw $x = 0$ mae $\frac{d^2y}{dx^2} = 12(0)^2 - 48(0) + 36$

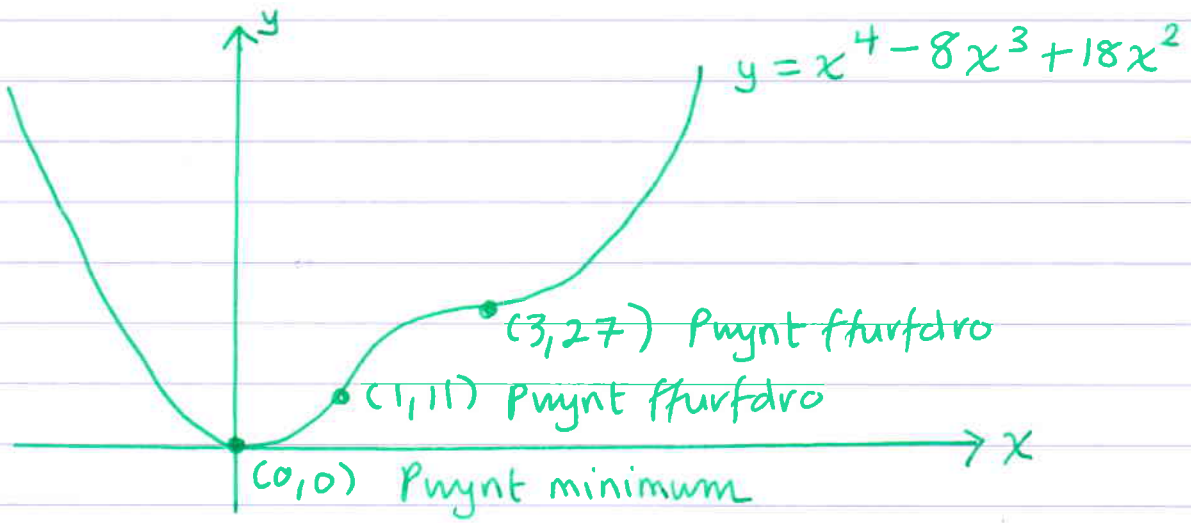
$$= 36$$

Felly mae gennym bwynt minimwm.

Os yw $x = 0$ mae $y = 0^4 - 8(0)^3 + 18(0)^2$

$$y = 0$$

Felly pwynt minimum yn (0, 0).



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8) a) i) $\int e^{-3x+5}\,dx$

$= -\frac{1}{3}e^{-3x+5} + K$

ii) $\int x^2 \ln x\,dx$

Gadewch i $u = \ln x$ $\quad \frac{dv}{dx} = x^2$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = \frac{x^3}{3}$

$$\int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx} v\,dx$$

$$\int x^2 \ln x\,dx = (\ln x)\left(\frac{x^3}{3}\right) - \int \frac{1}{x}\left(\frac{x^3}{3}\right)dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\,dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + K$$

b) $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$. Gadewch i $x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\,d\theta$

Terfannau: $x = 0 \Rightarrow 0 = \sin\theta$

$\theta = \sin^{-1}(0)$

$\theta = 0$

$$x = \tfrac{1}{2} \Rightarrow \tfrac{1}{2} = \sin\theta$$
$$\theta = \sin^{-1}\left(\tfrac{1}{2}\right)$$
$$\theta = \frac{\pi}{6}$$

Felly $\displaystyle\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} \cos\theta\,d\theta$

Nawr $\sin^2\theta + \cos^2\theta = 1$
$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$
$$\cos\theta = \sqrt{1-\sin^2\theta}$$

Felly'r integryn yw $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cancel{\cos\theta}} \cancel{\cos\theta}\,d\theta$
$$= \int_0^{\frac{\pi}{6}} \sin^2\theta\,d\theta$$

Nawr $\cos 2\theta = 1 - 2\sin^2\theta$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Felly'r integryn yw $\displaystyle\int_0^{\frac{\pi}{6}} \frac{1-\cos 2\theta}{2}\,d\theta$
$$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}}$$
$$= \frac{1}{2}\left[\left(\frac{\pi}{6} - \frac{1}{2}\sin\left(\frac{2\pi}{6}\right)\right) - \left(0 - \frac{1}{2}\sin(2\times 0)\right)\right]$$
$$= \frac{1}{2}\left[\left(\frac{\pi}{6} - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right) - (0-0)\right]$$
$$= \frac{\pi}{12} - \frac{\sqrt{3}}{8} \quad \checkmark$$

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9) Ble maer cromlinau croesi?

Rydym angen $x^2+4=12-x^2$

$2x^2=8$

$x^2=4$

$x=\pm\sqrt{4}$

$x=\pm 2$

Yr arwynebedd wedi'i ffinio gan y ddwy gromlin yw

$$\left(\int_{-2}^{2} 12-x^2\,dx\right)-\left(\int_{-2}^{2} x^2+4\,dx\right)$$

$$=\int_{-2}^{2} 8-2x^2\,dx$$

$$=\left[8x-\frac{2x^3}{3}\right]_{-2}^{2}$$

$$=\left(8\times 2-\frac{2\times 2^3}{3}\right)-\left(8\times -2-\frac{2(-2)^3}{3}\right)$$

$$=\left(16-\frac{16}{3}\right)-\left(-16+\frac{16}{3}\right)$$

$$=32-\frac{32}{3}$$

$$=\frac{64}{3}\text{ uned sgwâr}$$

Uned 3 – Deunyddiau Asesu Enghreifftiol

10) $1 + 5x - x^4 = 0$ Gwreiddyn positif α

a) Gadewch i $f(x) = 1 + 5x - x^4$.

Mae $f(1) = 1 + 5(1) - 1^4$
$= 5$

Mae $f(2) = 1 + 5(2) - 2^4$
$= -5$

Mae'r newid ar wyddyn dangos bod gwreiddyn rhwng $x = 1$ ac $x = 2$.

b) $x = \sqrt[4]{1+5x}$

NODYN: Cwestiwn a chynllun marcio'n anghywir: $\sqrt[4]{1+5x}$, nid $\frac{1}{4}\sqrt{1+5x}$

Gadewch i $x_{n+1} = \sqrt[4]{1+5x_n}$

$x_0 = 1.5$
$x_1 = 1.707476485$
$x_2 = 1.757346089$
$x_3 = 1.768721303$
$x_4 = 1.771285475$
$x_5 = 1.771861948$
$x_6 = 1.771991473$
⋮

I ddau le degol, y gwreiddyn yw $\alpha = 1.77$

c) Dull Newton–Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{1 + 5x_n - x_n^4}{5 - 4x_n^3}$$

$x_0 = 1.5$
$x_1 = 1.904411765$
$x_2 = 1.788115338$
$x_3 = 1.772305155$
$x_4 = 1.772029085$
$x_5 = 1.772029002$
$x_6 = 1.772029002$
$\vdots$

İ 6 'lı değol, $\alpha = 1.772029$

Uned 3 – Deunyddiau Asesu Enghreifftiol

11) $C: x^4 + x^2y + y^2 = 13$

a) Differu:

$$4x^3 + x^2(1)\frac{dy}{dx} + 2xy + 2y\left(\frac{dy}{dx}\right) = 0$$

$$4x^3 + 2xy + \left(\frac{dy}{dx}\right)(x^2 + 2y) = 0$$

$$\frac{dy}{dx}(x^2 + 2y) = -4x^3 - 2xy$$

$$\frac{dy}{dx} = \frac{-4x^3 - 2xy}{x^2 + 2y}$$

Os yw $x = -1$, $y = 3$ mae

$$\frac{dy}{dx} = \frac{-4(-1)^3 - 2(-1)(3)}{(-1)^2 + 2(3)}$$

$$\frac{dy}{dx} = \frac{4 + 6}{1 + 6}$$

$$\frac{dy}{dx} = \frac{10}{7}$$

b) $y^2 = 4x$

Differu: $2y\left(\frac{dy}{dx}\right) = 4$

$$\frac{dy}{dx} = \frac{4}{2y}$$

$$\frac{dy}{dx} = \frac{2}{y}$$

Felly graddiant y normal yw $-\frac{y}{2}$ (negatif y cilydd)

Hafaliad y normal yn y pwynt $P(p^2, 2p)$:

$$y - y_1 = m(x - x_1)$$
$$y - 2p = \left(-\frac{2p}{2}\right)(x - p^2)$$
$$y - 2p = -p(x - p^2)$$
$$y - 2p = -px + p^3$$
$$y + px = 2p + p^3 \quad \checkmark$$

Os yw $y = 0$ (torrir echelin-x), mae $x = b$

Felly $0 + pb = 2p + p^3$

$$b = 2 + p^2$$

Beth bynnag yw gwerth p, mae $p^2 > 0$ (gan fod $p \neq 0$), felly mae $b > 2$.

Uned 3 – Deunyddiau Asesu Enghreifftiol

12) a) Gadewch i $y = \cos x$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{\cos(x+\delta x) - \cos(x)}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{\cos(x)\cos(\delta x) - \sin(x)\sin(\delta x) - \cos(x)}{\delta x}$$

Wrth i $\delta x \to 0$, mae $\cos(\delta x) \to 1 - \frac{(\delta x)^2}{2}$

ac mae $\sin(\delta x) \to \delta x$.

Felly mae

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\cos(x)\left(1 - \frac{(\delta x)^2}{2}\right) - \sin(x)(\delta x) - \cos(x)}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{\cos(x) - \cos(x)\left(\frac{(\delta x)^2}{2}\right) - \delta x \sin(x) - \cos(x)}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{-\cos(x)\left(\frac{(\delta x)^2}{2}\right) - \delta x \sin(x)}{\delta x}$$

$$= \lim_{\delta x \to 0} -\cos(x)\left(\frac{\delta x}{2}\right) - \sin(x)$$

$$= -\cos(x)\left(\frac{0}{2}\right) - \sin(x)$$

$$\frac{dy}{dx} = -\sin(x)$$

b) i) $y = \frac{3x^2}{x^3+1}$ $\quad u = 3x^2$, $v = x^3+1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(6x) - 3x^2(3x^2)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{6x^4+6x-9x^4}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{6x-3x^4}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{3x(2-x^3)}{(x^3+1)^2}$$

ii) $y = x^3\tan(3x)$ $\quad u = x^3$, $v = \tan(3x)$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = x^3(3\sec^2(3x)) + \tan(3x)(3x^2)$$

$$\frac{dy}{dx} = 3x^4\sec^2(3x) + 3x^2\tan(3x)$$

$$\frac{dy}{dx} = 3x^2(x^2\sec^2(3x) + \tan(3x))$$

Uned 3 – Deunyddiau Asesu Enghreifftiol

13) a) $\operatorname{cosec}^2 x + \cot^2 x = 3$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$1 + \cot^2 + \cot^2 x = 3$

$2\cot^2 x = 2$

$\cot^2 x = 1$

$\left(\frac{1}{\tan x}\right)^2 = 1$

$\frac{1}{\tan^2 x} = 1$

$\tan^2 x = 1$

$\tan x = \pm 1$

Nodyn: Maer cynllun marcio'n dabrys $\operatorname{cosec}^2 x + \cot^2 x = 5$, nid $\operatorname{cosec}^2 x + \cot^2 x = 3$

$\tan x = 1$ (S | A / T | C)

$x = \tan^{-1}(1)$

$x = 45^\circ, 225^\circ$

$\tan x = -1$

$x = \tan^{-1}(-1)$

$x = -45^\circ, 135^\circ, 315^\circ$

Ateb: $x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$.

b) (i) $4\sin\theta + 3\cos\theta$

$R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

Felly $R\cos\alpha = 4$, $R\sin\alpha = 3$

[R] $\frac{4}{\cos\alpha} = \frac{3}{\sin\alpha}$

$\tan\alpha = \frac{3}{4}$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$\alpha = 36.87^\circ$ i 2le degol

[$\cos^2\alpha + \sin^2\alpha = 1$]

$\left(\frac{4}{R}\right)^2 + \left(\frac{3}{R}\right)^2 = 1$

$\frac{16}{R^2} + \frac{9}{R^2} = 1$

$16 + 9 = R^2$

$25 = R^2$

$R = 5$

Felly $4\sin\theta + 3\cos\theta = 5\sin(\theta + 36.87^\circ)$ i 2le degol

ii) $4\sin\theta + 3\cos\theta = 2$

$5\sin(\theta + 36.87^\circ) = 2$

$\sin(\theta + 36.87^\circ) = \frac{2}{5}$

$\theta + 36.87^\circ = \sin^{-1}\left(\frac{2}{5}\right)$

S | A

T | C

$\theta + 36.87^\circ = 23.58^\circ, 156.42^\circ, 383.58^\circ, \ldots$

$\theta = \ldots, -13.29^\circ, 119.55^\circ, 346.71^\circ, \ldots$

I'r radd agosaf, ac yn yr amrediad $0^\circ \leq \theta \leq 360^\circ$,

$\theta = 120^\circ, 347^\circ$

Uned 3 – Deunyddiau Asesu Enghreifftiol

14) Cyfaint y tanc silindrog: $V = 4h$

a) Bob eiliad, mae cyfaint y dŵr yn cynyddu $(0.004 - 0.0008h)\ m^3$

Felly $\frac{dV}{dt} = 0.004 - 0.0008h$

Ond mae $V = 4h$ felly $\frac{dV}{dt} = 4\frac{dh}{dt}$

Felly $4\frac{dh}{dt} = 0.004 - 0.0008h$

$\frac{dh}{dt} = 0.001 - 0.0002h$

$5000\frac{dh}{dt} = 5 - h$ ✓

b) Os yw $t = 0$, mae $V = 0$, ac felly $h = 0$.

$\int 5000\frac{dh}{dt} = \int 5 - h$

$\int \frac{5000}{5-h}\,dh = \int dt$

$5000\int \frac{1}{5-h}\,dh = \int dt$

$5000\left(-\ln|5-h|\right) = t + K$

Amnewid $t = 0$, $h = 0$:

$5000(-\ln|5-0|) = 0 + K$

$K = -5000\ln(5)$

Felly $-5000 \ln|5-h| = t - 5000 \ln(5)$

$5000 \ln(5) - 5000 \ln|5-h| = t$

$5000\left(\ln(5) - \ln|5-h|\right) = t$

$5000 \ln\left|\frac{5}{5-h}\right| = t$

$\ln\left|\frac{5}{5-h}\right| = \frac{t}{5000}$

$\frac{5}{5-h} = e^{\frac{t}{5000}}$

$5 = (5-h)e^{\frac{t}{5000}}$

$5e^{-\frac{t}{5000}} = 5-h$

$h = 5 - 5e^{-\frac{t}{5000}}$

$h = 5(1-e^{-\frac{t}{5000}})$.

c) Os yw $t = 3600s$, mae

$h = 5(1-e^{-\frac{3600}{5000}})$

$h = 2.56623872$

$h = 2.57$ m i 2 le dego|

Uned 3 – Deunyddiau Asesu Enghreifftiol

15) Pan fydd x yn real a phositif, mae $4x + \frac{9}{x} \geq 12$.

Prawf

Cymerwch yn ganiatáol bod gan x werth positif a gwerth real sy'n bodloni $4x + \frac{9}{x} < 12$.

Yna
$$4x^2 + 9 < 12x$$
$$4x^2 - 12x + 9 < 0$$
$$4 \times 9 = 36$$
$$4x^2 - 6x - 6x + 9 < 0$$
$$2x(2x-3) - 3(2x-3)$$
$$(2x-3)(2x-3) < 0$$
$$(2x-3)^2 < 0$$

Os yw x yn real, yna mae'n rhaid bod $(2x-3)^2 \geq 0$. Felly mae'n amhosib bod $(2x-3)^2 < 0$.

Mae hwn yn wrthddywediad i frawddeg gyntaf y prawf, ac felly mae'n rhaid bod $4x + \frac{9}{x} \geq 12$ pan fydd x yn real a phositif.