GCE AS/A Level – LEGACY

0982/01

S18-0982-01

MATHEMATICS – M3
Mechanics

FRIDAY, 22 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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1. A particle of mass 0·3 kg is projected vertically upwards with an initial speed of 40 ms^{-1}. At time t seconds, the resistance due to the atmosphere acting on the particle has magnitude $0{\cdot}12v$ N, where v ms^{-1} is the speed of the particle.

(a) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = -0{\cdot}4(24{\cdot}5 + v)$. [2]

(b) Find an expression for t in terms of v. [5]

(c) Calculate, correct to three significant figures, the time taken by the particle to reach its greatest height. [2]

(d) Write down the differential equation satisfied by v and t on the downward motion of the particle. [1]

2. A particle P is moving in a straight line with Simple Harmonic Motion. It starts from rest from a point A and, 5 seconds later, it reaches its maximum speed of π ms^{-1}.

(a) Show that the amplitude of the motion is 10 m. [4]

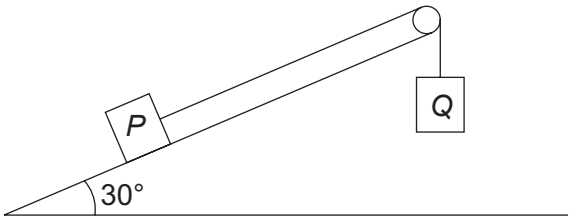
(b) Write down the maximum magnitude of the acceleration of P. [1]

(c) Calculate the speed of the particle when it is 6 m from O, the centre of the motion. [3]

(d) Determine the distance from A of the particle 4 s after the start of motion. [4]

(e) Points X and Y are equidistant from O and are 8 m apart. Calculate the time taken for P to proceed directly from X to Y. [4]

3. The diagram shows a particle P lying on a smooth plane inclined at an angle of 30° to the horizontal. Particle P is connected to another particle Q, by means of a light inextensible string passing over a smooth light pulley fixed at the top of the plane. Particle Q hangs freely. The mass of P is 26 kg and the mass of Q is 23 kg. Initially, the particles are held at rest with the string just taut.



The system is then released. When particle Q has descended a distance of $\frac{49}{16}$ m, it strikes the horizontal floor.

(a) Find the acceleration of the particles immediately after the system is released. [6]

(b) Calculate the speed of P at the instant Q strikes the floor. [3]

(c) Given that P does not reach the pulley in the subsequent motion, determine the speed with which the system starts to move after the string becomes taut again and calculate the impulsive tension in the string. [6]

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4. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} - 15x = 30t - 19 ,$$

where $x = 10$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -31$ when $t = 0$. Hence find the value of x when $t = 1$. [13]

5. A particle of mass 72 kg moves along a straight horizontal line. At time $t = 0$ s, it is at a point O and moving with velocity 1 ms^{-1}. When the particle has a velocity v ms^{-1}, it is subjected to a tractive force of $108v$ N and it experiences a resisting force of $12v^3$ N.

(a) Show that v satisfies the differential equation

$$9v - v^3 = 6v\frac{\mathrm{d}v}{\mathrm{d}x} ,$$

where x m is the distance of the particle from O. [3]

(b) Find an expression for v in terms of x and hence determine the approximate value of v when x is large. [8]

6. A man is climbing a uniform ladder 8 m long. One end A of the ladder rests against a rough vertical wall and the other end B rests on rough horizontal ground. The coefficient of friction between the ladder and the wall is 0·4 and the coefficient of friction between the ladder and the ground is also 0·4. The ladder is inclined at an angle of 60° to the horizontal. The weight of the man is 4 times the weight of the ladder. Determine how far the man can climb before the ladder is on the point of slipping. [10]

END OF PAPER

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