2.1 AS UNIT 1

Unit 1: Pure Mathematics A
Written examination: 2 hours 30 minutes
25% of A level qualification (62.5% of AS qualification)
120 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
2.1.1 Proof	
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including (a) proof by deduction, (b) proof by exhaustion, (c) disproof by counter example.	Proof by deduction to include the proofs of the laws of logarithms.
2.1.2 Algebra and Functions	
Understand and use the laws of indices for all rational exponents. Use and manipulate surds, including rationalising the denominator.	To include rationalising fractions such as $\frac{2+3\sqrt{5}}{3-2\sqrt{5}}$ and $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$.
Work with quadratic functions and their graphs. The discriminant of a quadratic function, including the conditions for real roots and repeated roots. Completing the square. Solution of quadratic equations in a function of the unknown.	The nature of the roots of a quadratic equation. To include finding the maximum or minimum value of a quadratic function. To include by factorisation, use of the formula and completing the square.

Topics	Guidance
Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.	To include finding the points of intersection or the point of contact of a line and a curve.
Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. Express solutions through the correct use of 'and' and 'or', or through set notation. Represent linear and quadratic inequalities graphically.	To include the solution of inequalities such as $1 - 2x < 4x + 7$, $\frac{x}{2} \geq 2(1-3x)$ and $x^2 - 6x + 8 \geq 0$. To include, for example, $y > x + 1$ (a strict inequality) and $y \geq ax^2 + bx + c$ (a non-strict inequality).
Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem.	The use of the Factor Theorem will be restricted to cubic polynomials and the solution of cubic equations.
Understand and use graphs of functions; sketch curves defined by simple equations, including polynomials. $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, including their vertical and horizontal asymptotes. Interpret algebraic solutions of equations graphically. Use intersection points of graphs of curves to solve equations. Understand and use proportional relationships and their graphs.	The equations will be restricted to the form $y = f(x)$.
Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	

Topics	Guidance
2.1.3 Coordinate geometry in the (x, y) plane	
Understand and use the equation of a straight line, including the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts.	To include • finding the gradient, equation, length and midpoint of a line joining two given points; • the equations of lines which are parallel or perpendicular to a given line.
Understand and use the coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle. Use of the following circle properties: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.	To also be familiar with the equation of a circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$. To include: • finding the equations of tangents, • the condition for two circles to touch internally or externally, • finding the points of intersection or the point of contact of a line and a circle,
2.1.4 Sequences and Series - The Binomial Theorem	
Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n . The notations $n!$, $\binom{n}{r}$ and $n\mathrm{C}r$. Link to binomial probabilities.	To include use of Pascal's triangle.

Topics	Guidance
2.1.5 Trigonometry	
Understand and use the definitions of sine, cosine and tangent for all arguments.	Use of the exact values of the sine, cosine and tangent of 30°, 45° and 60°.
Understand and use the sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.	To include the use of the sine rule in the ambiguous case.
Understand and use the sine, cosine and tangent functions. Understand and use their graphs, symmetries and periodicity.	
Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Understand and use $\cos^2\theta + \sin^2\theta = 1$.	These identities may be used to solve trigonometric equations or prove trigonometric identities.
Solve simple trigonometric equations in a given interval, including quadratic equations in $\sin$, $\cos$ and $\tan$, and equations involving multiples of the unknown angle.	To include the solution of equations such as $3\sin\theta = 1$, $\tan\theta = \frac{\sqrt{3}}{2}$, $3\cos 2\theta = -1$ and $2\cos^2\theta + \sin\theta - 1 = 0$.
2.1.6 Exponentials and logarithms	
Know and use the function a^x and its graph, where a is positive. Know and use the function e^x and its graph.	
Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.	Realise that when the rate of change is proportional to the y value, an exponential model should be used.

Topics	Guidance
Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$. Know and use the function $\ln x$ and its graph. Know and use $\ln x$ as the inverse function of e^x.	
Understand and use the laws of logarithms. $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $k \log_a x = \log_a (x^k)$ (including, for example $k = -1$, $k =$ -½)	To include the proof of the laws of logarithms. Use of the laws of logarithms. e.g. Simplify $\log_2 36 - 2\log_2 15 + \log_2 100 + 1$. Change of base will not be required.
Solve equations in the form $a^x = b$.	The use of a calculator to solve equations such as (i) $3^x = 2$, (ii) $25^x - 4 \times 5^x + 3 = 0$. (iii) $4^{2x+1} = 5^x$
Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.	Link to laws of logarithms. Understand that on a graph of $\log y$ against $\log x$, the gradient is n and the intercept is $\log a$, and that on a graph of $\log y$ against x, the gradient is $\log b$ and the intercept is $\log k$.
Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as model for population growth.) Consideration of limitations and refinements of exponential models.	The formal differentiation and integration of formulae involving e^x and/or a^x will not be required.

Topics	Guidance
2.1.7 Differentiation	
Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second order derivatives. Differentiation from first principles for small positive integer powers of x. Understand and use the second derivative as the rate of change of gradient.	The notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $f'(x)$ may be used. Up to and including power of 3. To include polynomials up to and including a maximum degree of 3.
Differentiate x^n for rational n, and related constant multiples, sums and differences.	To include polynomials.
Apply differentiation to find gradients, tangents and normals, maxima and minima, and stationary points. Identify where functions are increasing or decreasing.	To include finding the equations of tangents and normals. The use of maxima and minima in simple optimisation problems. To include simple curve sketching.
2.1.8 Integration	
Know and use the Fundamental Theorem of Calculus.	Integration as the reverse of differentiation.
Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.	To include polynomials.
Evaluate definite integrals. Use a definite integral to find the area under a curve.	To include finding the area of a region between a straight line and a curve.

Topics	Guidance
2.1.9 Vectors	
Use vectors in two dimensions.	To include the use of the unit vectors, $\mathbf{i}$ and $\mathbf{j}$.
Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.	Condition for two vectors to be parallel.
Understand and use position vectors; calculate the distance between points represented by position vectors. Use vectors to solve problems in pure mathematics.	Use of $\mathbf{AB} = \mathbf{b} - \mathbf{a}$. To include the use of position vectors given in terms of unit vectors. To include the use and derivation of the position vector of a point dividing a line in a given ratio.